

# Exploring mathematics of rolling bridges

Extended essay

Word count: 1

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# 1 Research question

How can catenary curves be used in construction of non-circular "rolling" bridges?

## 2 Introduction

1. My personal interest - I am interested in architecture and urban planning and this topic combines this with my passion for mathematics
2. I have found this bridge in an internet video - as an interesting mathematical phenomena existing in real life



Figure 1: Bridge photo

<https://newatlas.com/architecture/cody-dock-rolling-bridge/>

## 3 References

1. <https://community.wolfram.com/groups/-/m/t/2917199>
2. <https://youtu.be/SsGEcLwjgEg>
3. <https://youtu.be/xGxSTzaID3k>

## 4 Work plan

### 1. square bridge

#### (a) simple rolling square

- i. finding polar form - straight line times 4
- ii. finding relation between road and "wheel"
- iii. get  $y(t)$ , then find road equation in  $y(x)$  form
- iv. catenary road =  $-\cosh x$

#### (b) rounded square

- i. why rounded - gears teeth
- ii. why rounded corners are hard - rolling on a circle, but centre of mass is outside of it
- iii. calculating road for the rounded square
  - A. polar form of a rounded corner
  - B. symbolic solution
    - definite integration
    - elliptic integral of 2nd kind
    - cant get  $y(x)$
  - C. numerical solution
- iv. setting centre of mass to be at geometric centre (adding additional weight at the top of the bridge)
- v. find location of gear teeth - roll the track around and trace intersection with bridge - inverse transformation of track around the bridge
- vi. calculating work needed to be done to roll the bridge - the centre of mass is actually 2inches below the geometric centre

### 2. triangle bridge

- (a) normally cannot - too steep road and triangle crashes, but with rounded corners its possible
- (b) the catenary curve is steeper, so more friction is needed

### 3. polygonal bridge

(a) pentagon and hexagon

4. conclusion

## 5 “Smoothly” rolling square