Exploring mathematics of rolling bridges

Extended essay

Word count: 1

Bartek Włodarczyk

December 9, 2023

Contents

1 Research question

How can catenary curves be used in construction of non-circular "rolling" bridges?

2 Introduction

1. My personal interest - I am interested in architecture and urban planning and this topic combines this with my passion for mathematics
2. I have found this bridge in an internet video - as an interesting mathematical phenomena existing in real life

Figure 1: Bridge photo https://newatlas.com/architecture/cody-dock-rolling-bridge/

3 References

1. https://community.wolfram.com/groups/-/m/t/2917199
2. https://youtu.be/SsGEcLwjgEg
3. https://youtu.be/xGxSTzaID3k

4 Work plan

1. square bridge
   (a) simple rolling square
      i. finding polar form - straight line times 4
      ii. finding relation between road and "wheel"
      iii. get y(t), then find road equation in y(x) form
      iv. catenary road = -cosh x
   (b) rounded square
      i. why rounded - gears teeth
      ii. why rounded corners are hard - rolling on a circle, but centre of mass is outside of it
      iii. calculating road for the rounded square
         A. polar form of a rounded corner
         B. symbolic solution
            - definite integration
            - elliptic integral of 2nd kind
            - cant get y(x)
         C. numerical solution
      iv. setting centre of mass to be at geometric centre (adding additional weight at the top of the bridge)
      v. find location of gear teeth - roll the track around and trace intersection with bridge - inverse transformation of track around the bridge
      vi. calculating work needed to be done to roll the bridge - the centre of mass is actually 2inches below the geometric centre
2. triangle bridge
   (a) normally cannot - too steep road and triangle crashes, but with rounded corners its possible
   (b) the catenary curve is steeper, so more friction is needed
3. polygonal bridge
   (a) pentagon and hexagon
4. conclusion