Computational Fluid Dynamics EML4722 **Name:** *Benjamin Jones*
Homework 1, Numerical
September 12, 2023

1. Below is the plot for C_L vs. α for the NACA0012 airfoil at $M = 0.70$.

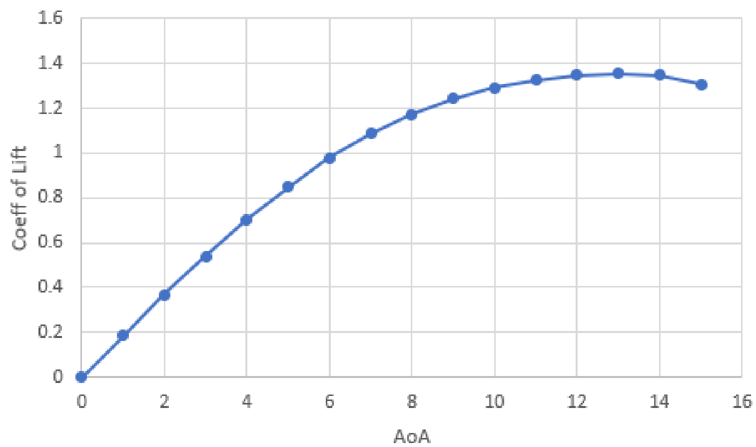

Figure 1: Coeff. of Lift vs. Angle of Attack for NACA 0012 airfoil at Mach 0.70

The simulation does not converge at an angle of attack of 15 degrees. This is likely because the airfoil is stalling at that point. As the airplane stalls, turbulent eddies are created in the wing's wake, which are unstable and unable to be explicitly resolved by the solver.

The maximum Coefficient of Lift achieved is at an angle of attack of 13 degrees, achieving a C_L of 1.3535.

2. Below are contour maps of a NACA 0012 airfoil at varying Mach numbers at 0 angle of attack.

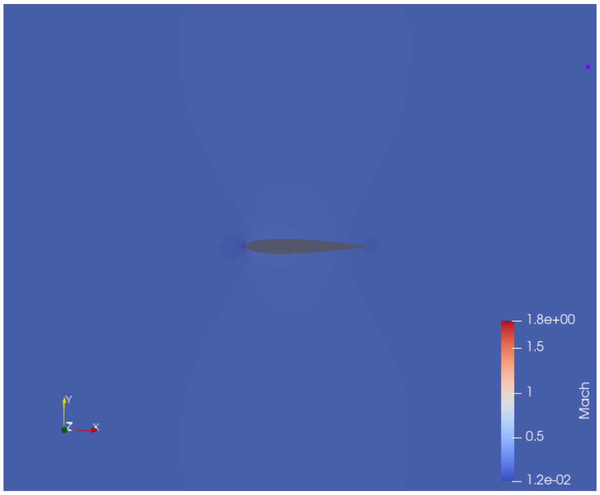

Figure 2: M=0.2

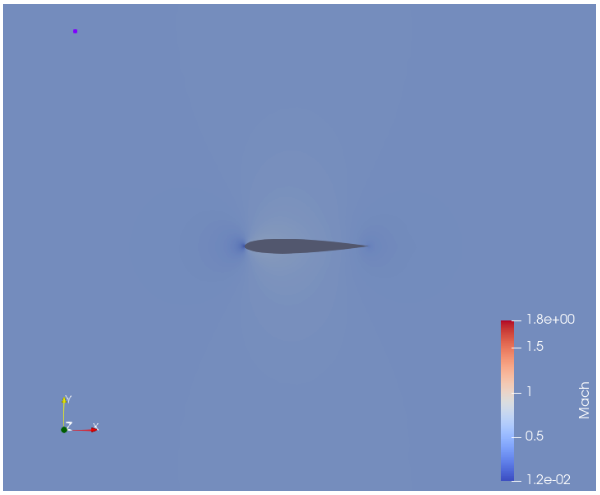

Figure 3: M=0.5

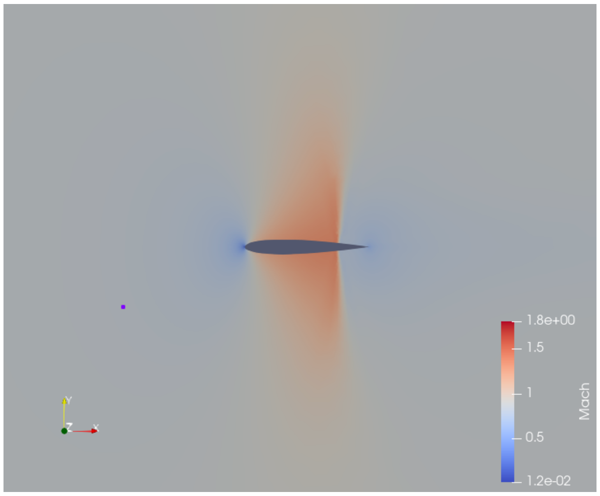

Figure 4: M=0.85

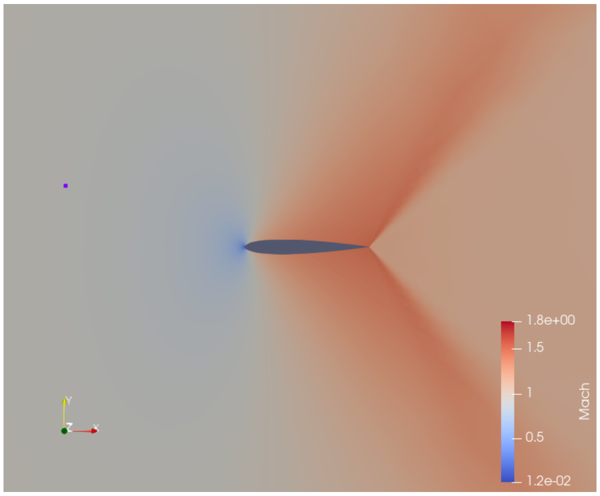

Figure 5: M=1

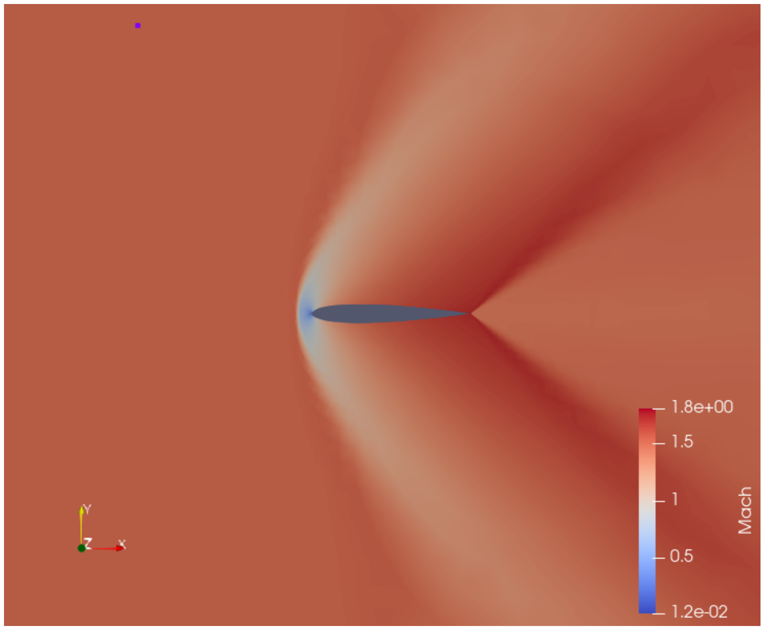

Figure 6: M=1.5

As Mach number increases, the first shockwave to form is a normal shockwave, first appearing in the $M = 0.85$ example. This shock is located part-way down the airfoil. At $M = 1$ two shocks appear: One at the leading edge and one at the trailing edge. At Mach 1.5 it is the same pattern, but the shockwaves are now at a more oblique angle relative to the flow.

At $M = 0.85$, the shock appears at approximately a location of $x/c = 0.75$. At $M = 1$ The shocks occur at the leading and trailing edge.

It is possible to have a standing normal shock at $M = 0.85$. As the flow passes over the wing it accelerates, leading to locally supersonic flow over the wing. This represents transonic flow.

The distance to the bow shock is approximately $x/c = -0.08$.

3. Below is a countour plot of temperature for a NACA 0012 airfoil at $M = 10$.

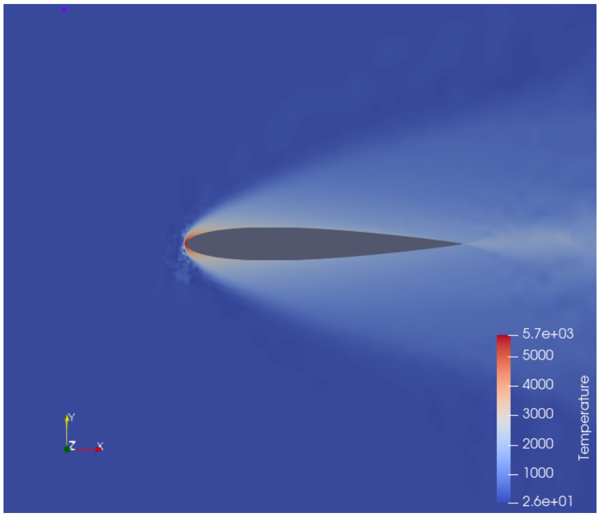

Figure 7: Temperature contour at $M = 10$

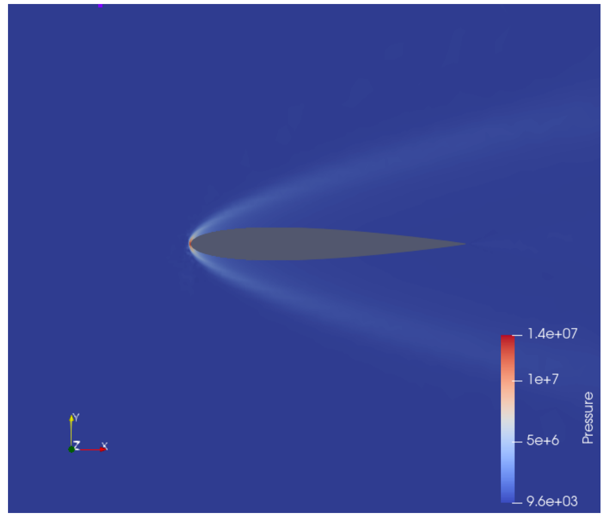

Figure 8: Pressure contour at $M = 10$

The maximum temperature is 5707 Kelvin. It is located at the leading edge of the airfoil. The maximum pressure reached is 1.367e7 Pascal.

Using compressible flow theory, one may assume stagnation conditions at the leading edge of the airfoil, following a normal shock. Assuming air to be a calorically perfect gas, the following equations may be used to calculate stagnation conditions for temperature and pressure respectively.

$$\frac{p_0}{p} = (1 + \frac{\gamma - 1}{2} M^2)^{\frac{\gamma}{\gamma - 1}} \tag{1}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{2}$$

The equations to calculate conditions after a normal shock are below:

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma + 1}(M_1^2 - 1) + 1 \tag{3}$$

$$\frac{T_2}{T_1} = [1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)][\frac{2 + (\gamma - 1)M_1^2}{(\gamma + 1)M_1^2}] \tag{4}$$

$$M_2^2 = \frac{1 + 1/2(\gamma - 1)M_1^2}{\gamma M_1^2 - 1/2(\gamma - 1)} \tag{5}$$

Using these equations with the starting conditions of 1.01e6 Pascal and 273.15K, we can arrive at stagnation conditions of 1.25e7 Pascal and 4643 Kelvin. These results are both off from expected. The temperatures vary by bout 1000 K, and the pressure by one order of magnitude. This deviation may be due to lack of grid refinement around the shock, although I do not know much about how accurate CFD simulation is around shockwaves when the shocks are not well resolved.