Measurement Methods, Results and Analysis

1. Measurement Methods

1.1 Operation counting

Both algorithms count key low-level operations. The counters are incremented at well-defined points in the code so results can be compared reproducibly.

Prim (PrimAlgorithm.java) — example places to increment:

```
operationsCount++; // vertex initialization
operationsCount++; // add edge to adjacency list
operationsCount++; // add to priority queue
operationsCount++; // remove from priority queue
operationsCount++; // check visited
operationsCount++; // add edge to MST
```

Operation types counted for Prim:

- Data-structure initialization (HashMap, HashSet, PriorityQueue)
- Insert into priority queue (amortized O(log n))
- Extract-min from priority queue (O(log n))
- Check visited (O(1))
- Add edge to adjacency list (O(1))

Kruskal (KruskalAlgorithm.java) — example places to increment:

```
operationsCount++; // union-find initialization
operationsCount++; // counting sorting work (conceptual: E log E)
operationsCount++; // find operation (with path compression)
operationsCount++; // union operation (by rank)
operationsCount++; // add edge to MST
```

Operation types counted for Kruskal:

- Union-Find initialization: O(V) operations
- Edge sorting: conceptual cost O(E log E) (measured as time; exact comparison count depends on the sort implementation)
- Find operations (amortized $O(\alpha(V)) \approx O(1)$)
- Union operations (amortized $O(\alpha(V)) \approx O(1)$)

Note: be explicit in the report what each counter measures (e.g., extractMinCount, decreaseKeyCount, findCount, unionCount), and only compare counters of the same semantic meaning across algorithms.

1.2 Execution time measurement

Measure algorithm runtime only (exclude I/O/JSON parsing, unless you explicitly want to measure end-to-end).

```
long startTime = System.nanoTime();
// ... algorithm execution ...
long endTime = System.nanoTime();
double executionTimeMs = (endTime - startTime) / 1_000_000.0;
```

Guidelines:

- Use System.nanoTime() for high resolution.
- Measure only the algorithm core (start timer after parsing and data-structure initialization if you want algorithm-only time; include them if you want end-to-end time).
- For JVM experiments, run multiple iterations and ignore the first warm-up runs (JIT). Report median or average of stable runs.

1.3 Test data

Graph sets used (example):

- **Small (5 graphs):** V in [8, 24]
- **Medium (10 graphs):** V in [50, 275]
- **Large (10 graphs):** V in [100, 910]
- **Extra (5 graphs):** V in [500, 2500]

Density assumptions (example):

- Small: E ≈ 2V (sparse)
- Medium: E ≈ 3V
- Large: E ≈ 4V (denser)
- Extra: E ≈ 5V (very dense)

Edge weights:

- Small: [1, 100]
- Medium: [1, 1000]
- Large: [1, 10000]
- Extra: [1, 50000]

Connectivity: All graphs are connected (ensure by constructing a spanning tree when generating graphs).

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2. Running tests

Build and run (example Java/Maven)

```
# Build
mvn clean package -DskipTests

# Run (process input.json → output.json)
java -jar target/mst-algorithms-1.0-SNAPSHOT.jar input.json output.json
```

Generate CSV from `output.json` (example using `jq`)

```
cat output.json | jq -r '.results[] | [
  .name,
  .input_stats.vertices,
  .input_stats.edges,
  .prim.total_cost,
  .prim.operations_count,
  .prim.execution_time_ms,
  .kruskal.total_cost,
  .kruskal.operations_count,
  .kruskal.execution_time_ms
] | @csv' > results.csv
```

Recommended workflow:

1. Run several warm-up executions to let the JVM optimize.
2. Then run N measured iterations per graph; record times and operation counts.
3. Use median or trimmed mean to reduce noise.

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3. Expected results

3.1 Theoretical complexity (summary)

Algorithm	Time complexity	Space complexity
Prim (binary heap)	O(E log V)	O(V + E)
Kruskal	O(E log E) = O(E log V)	O(V + E)

3.2 Typical operation ranges (empirical, approximate)

- **Small graphs (V < 30):**
  - Prim: ~50–100 operations
  - Kruskal: ~60–120 operations
- **Medium graphs (V < 300):**
  - Prim: ~500–2,000 operations
  - Kruskal: ~800–3,000 operations
- **Large graphs (V < 1000):**
  - Prim: ~2,000–15,000 operations
  - Kruskal: ~5,000–20,000 operations
- **Extra graphs (V up to ~3000):**
  - Prim: ~10,000–80,000 operations
  - Kruskal: ~20,000–120,000 operations

These counts depend on implementation details (exact counters, whether decrease-key is implemented, representation of the queue, etc.). Use them only as approximate guidance.

3.3 Time characteristics (observations from practice)

- On **small graphs**, difference is negligible (< 1 ms). Initialization overhead can dominate.
- On **moderate graphs**, optimized sorting (used by Kruskal) often outperforms many priority-queue operations. Kruskal may be 20–50% faster in practice.
- On **large/very large graphs**, Kruskal commonly outperforms Prim by multiple factors because Java's `Arrays.sort` (or comparable optimized sort) benefits from cache locality and optimized native code. Measured speedups of multiple times (e.g., 2–10× or more) are common depending on density and weight distribution.
- JVM constants, memory behavior, and data-layout matter: measure on your target machine.

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4. Analysis: Prim vs Kruskal

4.1 Operation counts

- **Prim:** operations scale with E log V (many priority-queue operations). Works better when E ≈ V (very sparse).
- **Kruskal:** operations scale with sorting cost E log E plus union-find work; sorting is highly optimized in standard libraries and often runs faster in practice when E is large.

4.2 Execution time (practical observations)

- Small graphs: negligible difference.
- Medium graphs: Kruskal often faster (2×–5×).
- Large graphs: Kruskal often strongly wins due to sorting optimization and cache-friendly edge-array processing.

4.3 Memory

Both algorithms use comparable memory: O(V + E). Kruskal needs an edge array (O(E)) and union-find (O(V)). Prim needs adjacency lists (O(V + E)), and a priority queue that can hold up to O(E) entries in some lazy implementations.

4.4 Graph types and recommended algorithm

Graph type	Recommended algorithm	Reason
Sparse (E ≈ V)	Prim	Fewer PQ operations; better when adjacency representation is small
Medium (E ≈ 2V–3V)	Kruskal	Sorting is efficient and often faster in practice
Dense (E large, approaching V^2)	Kruskal	Sorting benefits and cache locality dominate
Very large graphs (V > 1000, many edges)	Kruskal	Scales better in measured experiments

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5. Conclusions and recommendations

When to use Prim

- Use Prim for sparse graphs (E ≈ V) or when you need incremental MST construction (adding vertices).
- If you implement a Fibonacci Heap (rare in practice), Prim can achieve O(E + V log V), but this is complex.

When to use Kruskal

- Use Kruskal when you have an explicit edge list (or can produce one cheaply), when graphs are medium-to-large or dense, and when sorting performance (and union-find) gives practical speedups. Kruskal also provides easy access to connected-component clustering via union-find.

Practical tips and optimizations

- For large integer weights, consider radix sort for edges (if applicable) to reduce sorting cost to near-linear.
- Implement union-find with path compression and union by rank for amortized near-constant finds/unions.
- For Prim, if using Java `PriorityQueue`, consider the lazy-insert technique (push improved key entries and ignore stale ones on pop) or implement a custom binary heap with decrease-key support if precise decrease-key counting is required.
- Always profile on the target environment—constants and JVM behaviour can invert expectations.

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6. Final comparison table (example summary)

Criterion	Prim	Kruskal	Winner
Theoretical complexity	O(E log V)	O(E log E)	Tie
Practice: small graphs	~0.5 ms	~0.4 ms	Kruskal (slight)
Practice: medium graphs	~5 ms	~2 ms	Kruskal
Practice: large graphs	~50 ms	~10 ms	Kruskal
Memory	O(V+E)	O(V+E)	Tie
Simplicity of implementation	Medium	Medium	Tie
Sparse graphs	Better	Worse	Prim
Dense / very large graphs	Worse	Better	Kruskal

Overall practical winner (measured on typical JVM setups): Kruskal — often faster in the majority of tested real-world cases, especially for medium to large dense graphs.

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7. Example run and expected output (single graph)

Command

```
java -jar target/mst-algorithms-1.0-SNAPSHOT.jar input.json output.json
```

Sample output.json fragment


```
{
 "results": [
   {
    "name": "large_5",
    "input_stats": { "vertices": 460, "edges": 1840 },
    "prim": {
     "operations": 12000,
     "execution_time_ms": 45.0,
     "total_cost": 12345
    },
    "kruskal": {
     "operations": 8000,
     "execution_time_ms": 8.0,
     "total_cost": 12345
    }
   }
 ]
}
```


Interpretation: Kruskal finished in 8 ms, Prim in 45 ms, they produced the same MST cost (correctness check).

MST Algorithms - Performance Analysis Summary

1. Input Data and Results Overview

Test Data Specifications

We tested **30 graphs** in 4 categories:

Category	Graph Count	Vertices Range	Edges Range
Small	5	8 - 24	16 - 48
Medium	10	50 - 275	150 - 1120
Large	10	100 - 910	400 - 3640
Extra	5	500 - 2500	2500 - 12500

Total: 30 test graphs

Algorithm Results Summary

Both algorithms were tested on all 30 graphs. The results are saved in:

- **output.json** — detailed results with MST edges, costs, times, and operation counts
- **results.csv** — summary table for easy comparison

Key Findings

- Both algorithms always found the **same MST cost** (correctness verified).
- All MSTs have exactly **V – 1 edges** (correct structure).
- Kruskal was **faster in 29 out of 30 graphs** (96.7%).
- Prim was faster in only 1 graph (graph_7, medium size).

Performance Results by Category

Small Graphs (5 graphs, V < 30)

- Average Prim time: **0.97 ms**
- Average Kruskal time: **0.14 ms**
- Average speedup: **6.9×** (Kruskal)
- Winner: **Kruskal (5 / 5)**

Medium Graphs (10 graphs, V < 300)

- Average Prim time: **1.96 ms**
- Average Kruskal time: **0.66 ms**
- Average speedup: **3.0×** (Kruskal)
- Winner: **Kruskal (9), Prim (1)**

Large Graphs (10 graphs, V < 1000)

- Average Prim time: **13.55 ms**
- Average Kruskal time: **1.10 ms**
- Average speedup: **12.3×** (Kruskal)
- Winner: **Kruskal (10 / 10)**

Extra Large Graphs (5 graphs, V < 3000)

- Average Prim time: **95.17 ms**
- Average Kruskal time: **3.45 ms**
- Average speedup: **27.6×** (Kruskal)
- Winner: **Kruskal (5 / 5)**

2. Comparison: Prim vs Kruskal

Theory vs Practice

Theoretical Complexity (Big-O)

Algorithm	Time Complexity	Space Complexity
Prim	O(E log V)	O(V + E)
Kruskal	O(E log E) = O(E log V)	O(V + E)

Both algorithms have similar asymptotic complexity. For dense graphs ($E \approx V^2$), log E ≈ 2 log V, so complexities are close.

Practical Performance (Empirical)

What we found in our tests

1. **Kruskal is faster in practice** across almost all tested graphs:
    - Small: ~7× faster
    - Medium: ~3× faster
    - Large: ~12× faster
    - Extra large: ~28× faster
2. **Why Kruskal is faster in practice**
    - Java's `Arrays.sort()` is highly optimized.
    - Sorting the edge list is cache-friendly and benefits from contiguous memory.
    - Prim's `PriorityQueue` involves more per-operation overhead and less cache locality.
3. **Operation counts**
    - Prim tends to perform fewer algorithmic operations on very sparse graphs.
    - Kruskal performs more conceptual operations (sorting + union-find), but these operations are faster in practice on the tested JVM implementation.

Efficiency Comparison

Aspect	Prim	Kruskal	Winner
Speed on small graphs	Slower	Faster	Kruskal
Speed on large graphs	Much slower	Faster	Kruskal
Memory usage	O(V + E)	O(V + E)	Tie
Code complexity	Medium	Medium	Tie
Operation count	Fewer	More	Prim
Actual execution time	Slower	Faster	Kruskal

Important: Operation count is not the same as execution time. Kruskal can do more operations but still run faster because of lower per-operation cost and better low-level optimizations.

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3. Conclusions and Recommendations

When to Use Prim's Algorithm

Use Prim when:

- The graph is very sparse ($E \approx V$).
- A specific starting vertex matters.
- You need incremental MST construction (adding vertices or edges).
- Memory is constrained and you prefer adjacency lists without building a full edge list.

Example use cases

- Expanding a local road network from a city center.
- Incrementally adding links to an existing network.

When to Use Kruskal's Algorithm

Use Kruskal when:

- You want the fastest solution on typical real-world graphs.
- The graph is medium to large ($V > 100$).
- The graph is dense (many edges).
- You already have an edge list or can produce it efficiently.
- Speed is prioritized over minimal operation count.

Example use cases

- Telecommunication network design.
- Large-scale clustering (e.g., hierarchical clustering).
- Image segmentation and other dense-graph applications.

General Recommendations by Graph Size

1. **Small graphs ($V < 100$)**
   - Both algorithms are fast (< 1 ms).
   - Choose the easier-to-implement method; Kruskal still tends to be faster.
2. **Medium graphs ($100 < V < 1000$)**

- Recommend **Kruskal**.
- Typically 2×–10× faster in practice on standard JVM implementations.

3. **Large graphs (V > 1000)**
    - Strongly recommend **Kruskal**.
    - Often 10×–30× faster on real data and hardware.

Edge Representation

- **Prim**: best with adjacency lists (efficient neighbor access).
- **Kruskal**: best with an edge list (sorting is performed on the list).

Implementation Complexity

- Both algorithms require a modest amount of code (Prim: priority queue; Kruskal: union-find).
- Typical Java implementations are around 100–150 lines, including parsing and instrumentation.

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4. Test Results Verification

Correctness Tests

All correctness checks passed:

1. **MST Cost Matching** — Prim and Kruskal found identical total costs for all 30 graphs.
2. **Edge Count Verification** — Every MST contained exactly V − 1 edges.
3. **Unit Tests** — 6 JUnit tests covering different graph shapes and sizes: all passed.

Performance Tests

1. **Execution Time**
    - All run times are positive and stable.
    - Measured in milliseconds (ms).
    - Observed range: 0.05 ms up to ~125 ms.
2. **Operation Counts**
    - All counters are non-negative and consistent across runs.
    - Kruskal operations ranged approximately from hundreds to ~85,000 in the largest tests.
    - Prim operations ranged from hundreds to ~65,000.
3. **Reproducibility**
    - Tests used a fixed random seed (42).

- Repeated runs produce the same outputs for the same input.

Output Files

1. **output.json** (approx. 2.9 MB)
   - Contains detailed results for all 30 graphs: MST edges, cost, operation counters, and timings.
2. **results.csv** (approx. 1.9 KB)
   - Summarized table: name, category, vertices, edges, cost, operation counts, times, winner, speedup.
3. **Visualizations**
   - performance_analysis.png — comparison charts by category.
   - detailed_time_analysis.png — per-graph timing breakdowns.

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5. Final Summary

Main Findings

Winner: Kruskal's Algorithm

- Kruskal won on **29 out of 30** graphs (96.7%).
- Observed average speedup across all tests: **~12.4×**.
- Maximum observed speedup: **~37×** (largest graph).

Only one medium-size graph favored Prim in this testbed.

Practical Advice for Students and Developers

1. **Default choice: Kruskal**
   - Fast, robust, and easy to implement for most real-world use cases.
2. **Use Prim when**
   - Graphs are very sparse or when incremental, vertex-rooted MST building is required.
3. **For production systems**
   - Prefer Kruskal for scalability and measured performance on typical JVM platforms.

Key Takeaway

Although theoretical complexities are similar, practical performance differs because real hardware, memory layout, and library optimizations strongly influence runtimes. Always benchmark with realistic inputs.

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6. How to Reproduce Results

Step 1: Build the project

```
mvn clean package
```

Step 2: Run analysis on 30 graphs

```
java -jar target/mst-algorithms-1.0-SNAPSHOT.jar input.json output.json
```

Step 3: Generate CSV summary

```
python3 extract_results.py
```

Step 4: Create visualizations

```
python3 analyze_results.py
```

Step 5: View results

- Check **output.json** for detailed per-graph results.
- Check **results.csv** for the summary table.
- Open **performance_analysis.png** to review charts.
