Mathematical Economics: Game Theory

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Contents

1 Tues January 13

Tests on Feb 10, March 26, and April 23. No final.

1.1 Individual Decision Making

> **Definition 1.1**
>
> Given a set X, a binary relation on X is a subset $\succsim$ of X^2. We write $x \succsim y$ to denote $(x, y) \in \succsim$.

We say a relation on X is transitive if $x \succsim y$ and $y \succsim z \implies x \succsim z$. We say a relation is **complete** if $\forall x, y$ we have $x \succsim y$ or $y \succsim x$ or both. Then a **rational preference relation** is just a complete, transitive relation.

2 Thu Jan 15

Given a relation $\succsim$, we can define other relations that are useful with the obvious definitions: $\not\succsim, \succ, \precsim, \prec, \sim$.

Given a set X, a **utility** is any function $u : X \to \mathbb{R}$. Any utility generates a binary relation, with $x \succsim y \iff u(x) \geq u(y)$.

> **Proposition 2.1**
>
> If u represents $\succsim$ and $f : u(X) \to \mathbb{R}$ is strictly increasing, then $f \circ u$ also represents $\succsim$.

Proof. $x \succsim y \iff u(x) \geq u(y) \iff f(u(x)) \geq f(u(y))$.

So the actual values of the utility don't matter that much! But also:

> **Proposition 2.2**
>
> If X is finite, then $\succsim$ has a utility representation.

Proof. Just have $u(x)$ be the number of elements in X such that $x \succsim y$.

> **Proposition 2.3**
>
> If X is countable, then $\succsim$ has a utility representation.

Proof. Same as above, but sum 2^{-n} for each element less than.