Symbolic System Dynamics Deriving Algorithm

bexultan.rakhim@nu.edu.kz

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1 Introduction

this algorithm calculates system dynamics equation with help of MATLAB. It uses concept of DH table introduced in the book "Introduction to Robotics" by Jhon J. Craig. At this moment algorithm is able to find Dynamic Model of any chained Open Loop robotic manipulator with Rotational Joints. However, with further improvements it is possible to achieve model of any manipulator dynamics. This algorithm is done in order to reduce time to calculate dynamic model of manipulators and be able to obtain this models automatically.

2 Basics of Manipulator Dynamics

Any Manipulator Dynamics can be represented as:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = f \tag{1}$$

If we can obtain this matrices M,C,G then we can be sure that we are able to calculate dynamic model of manipulator. Here f is inputs to the system and θ is joint angles. In order to achieve this equation you should solve Euler-Lagrange equation, which states that if L = T-V , T -kinetic energy V - potential energy:

$$\frac{d}{dt}(\nabla_{\dot{\theta}} L)^T - (\nabla_{\theta} L)^T = f \tag{2}$$

In majority of situations it is true that V is only function of θ and T is function of both θ and $\dot{\theta}$. So if it is a case then from (1) and (2) it is true that:

$$M(\theta) = \frac{\partial}{\partial\dot{\theta}}\Big(\frac{\partial T}{\partial\dot{\theta}}\Big)^T \tag{3}$$

$$C(\theta, \dot{\theta}) = \frac{\partial}{\partial\theta}\Big(\frac{\partial T}{\partial\dot{\theta}}\Big)^T \tag{4}$$

$$G(\theta) = \frac{\partial V}{\partial\theta} - \frac{\partial T}{\partial\theta} \tag{5}$$

This only possible if system is chosen to be time invariant

However, the kinetic and potential energies of system are not easy to compute. They Highly depend on the way you choose the reference frames, and coordinate system. If we assume that we know the velocities of centre of mass with respect to reference frame and angular velocities with respect to its own frame then it is true that:

$$T = \frac{1}{2} \sum_{i=1}^{N} m_i \vartheta_{C_i}^T \vartheta_{C_i} + {}^i\omega_i^{T\,C_i} I_i {}^i\omega_i^T \tag{6}$$

m - mass of the Link I - moment of inertia of the link represented in frame of centre of mass This matrix is diagonal matrix if it's primal axis orientation and orientation of frame of the link is the same.

Potential energy is:

$$V = \sum_{i=1}^{N} m_i g^0 z_i = \sum_{i=1}^{N} m_i g [0, 0, 1] * {}^0P_{C_i} \tag{7}$$

Here g - gravitational force, P_{C_i} vector of centres of mass represented in the reference frame 0.

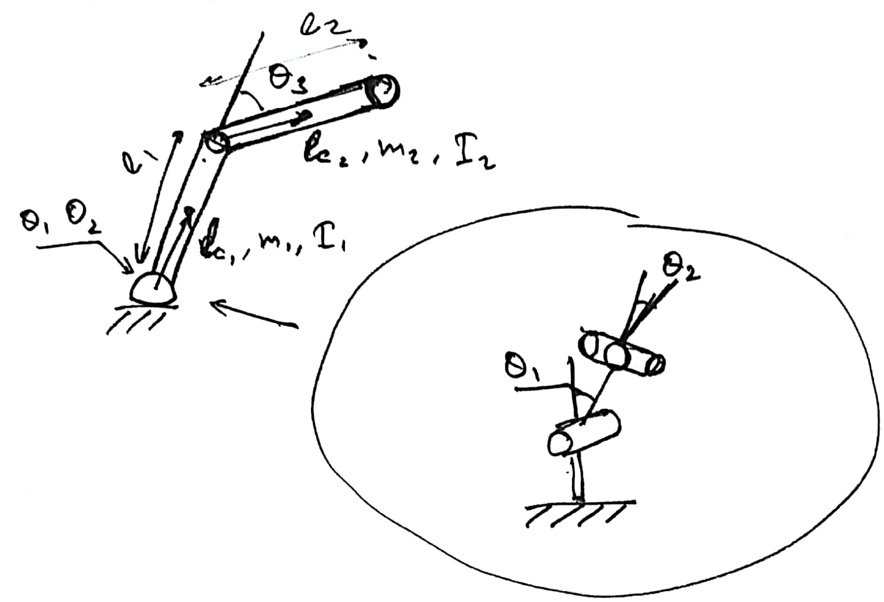
In order to compute velocities and positions in appropriate frame, the frames are chosen as it was introduced in "Introduction to Robotics" book. This is simplifies process of derivation of velocities and makes it much easier to derive them symbolically.

3 Algorithm description

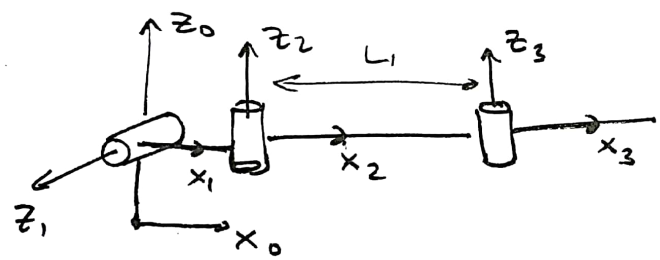
Algorithm relies on MATLAB's symbolic parameters and operation with help of symbolic toolbox. For the moment of time it was not tested for high dimensional calculations, but for manipulator with less than 5 links it works fine.

As you probably noticed in equations (3) by (5) none of this equations require derivative of function by time, and all of them are partial derivatives of join angle and angle velocities.

You can find in folder the matlab script named DHtabledtest.m of following example shown in figure(1). The other example is simple Cart-Pole system.



DH table:



	α_{i-1}	a_{i-1}	d_i	θ_i
1	90°	0	0	θ_1+90°
2	-90°	0	0	θ_2
3	0	L_1	0	θ_3

Figure 1: Manipulator with following DH table