CENG 222

Statistical Methods for Computer Engineering

Spring '2017-2018

Take Home Exam 1

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Answer 3.8

When the user tries to find her password among the 4 possible alternatives, it is equally likely that she finds the right one at the first, second, third or fourth trial. Thus, the pmf of $X = 0$, $X = 1$, $X = 2$, and $X = 3$ are all equal as follows,

$$P(0) = P(1) = P(2) = P(3) = 0.25 \tag{1}$$

$$E(X) = \sum_{x=0}^{3} xP(x) = \big(0 + 1 + 2 + 3\big)(0.25) = 1.5 \tag{2}$$

$$Var(X) = \sum_{x=0}^{3} (x - 1.5)^2 P(x) = \big((-1.5)^2 + (-0.5)^2 + (0.5)^2 + (1.5)^2\big)(0.25) = 1.25 \tag{3}$$

Answer 3.15

(a)

Probability of at least one hardware failure can be computed as follows,

$$1 - P_{(X,Y)}(0, 0) = 1 - 0.52 = 0.48 \tag{4}$$

(b)

It is known that X and Y are independent if $P_{(X,Y)}(x,y) = P_X(x)P_Y(y)$ for all values of x and y. By the Addition Rule we can find $P_X(0)$ and $P_Y(0)$ as follows,

$$\begin{aligned} P_X(0) = P\{X=0\} &= \sum_{y=0}^{2} P_{(X,Y)}(0,y) = 0.52 + 0.14 + 0.06 = 0.72 \\ P_Y(0) = P\{Y=0\} &= \sum_{x=0}^{2} P_{(X,Y)}(x,0) = 0.52 + 0.20 + 0.04 = 0.76 \end{aligned} \tag{5}$$

Now, let's test the statement $P_{(X,Y)}(x,y) = P_X(x)P_Y(y)$ for $x = 0$ and $y = 0$,

$$\begin{gathered} P_{(X,Y)}(x,y) = P_X(x)P_Y(y) \\ P_{(X,Y)}(0,0) = P_X(0)P_Y(0) \\ 0.52 \neq 0.5472 \end{gathered} \tag{6}$$

Hence, X and Y are not independent, it follows that X and Y are dependent.

Answer 3.19

Before starting further analysis on the given cases, we need to calculate $E(X)$, $Var(X)$, $E(Y)$, and $Var(Y)$ as follows,

$$\begin{aligned} E(X) &= (2)(0.5) + (-2)(0.5) = 0 \\ Var(X) &= (2)^2(0.5) + (-2)^2(0.5) = 4 \\ E(Y) &= (4)(0.2) + (-1)(0.8) = 0 \\ Var(Y) &= (4)^2(0.2) + (-1)^2(0.8) = 4 \end{aligned} \tag{7}$$

Expected value and variance of the total profit for strategies are computed below.
(a) Buying 100 share of A means collecting a profit of $A = 100X$. $E(A)$ and $Var(A)$ can be computed as follows,

$$\begin{aligned} E(A) &= (100)E(X) = (100)(0) = 0 \\ Var(A) &= (100)^2 Var(X) = (100)^2(4) = 40000 \end{aligned} \tag{8}$$

(b) Buying 100 share of B means collecting a profit of $B = 100Y$. $E(B)$ and $Var(B)$ can be computed as follows,

$$\begin{aligned} E(B) &= (100)E(Y) = (100)(0) = 0 \\ Var(B) &= (100)^2 Var(Y) = (100)^2(4) = 40000 \end{aligned} \tag{9}$$

(c) Buying 50 share of A and 50 share of B means collecting a profit of $C = 50X + 50Y$. $E(C)$ and $Var(C)$ can be computed as follows,

$$\begin{aligned} E(C) &= (50)E(X) + (50)E(Y) = (50)(0) + (50)(0) = 0 \\ Var(C) &= (50)^2 Var(X) + (50)^2 Var(Y) = (50)^2(4) + (50)^2(4) = 20000 \end{aligned} \tag{10}$$

Answer 3.29

In this problem, we will use Poisson distribution and Bayes Rule for two events. $P\{H\}$ denotes probability of high risk driver and $P\{L\}$ denotes probability of low risk driver. In the problem it is given that $P\{H\} = 0.2$, $P\{L\} = 0.8$. Using Poisson distribution, we can find $P\{0|H\}$ and $P\{0|L\}$. Note that, for $P\{0|H\}$, $\lambda_H = 1$ and $x_H = 0$ and for $P\{0|L\}$, $\lambda_L = 0.1$ and $x_L = 0$.

$$
\begin{aligned}
P\{0|H\} &= e^{-\lambda_H}\frac{\lambda_H^{x_H}}{x_H!} = e^{-1}\frac{1^0}{0!} = e^{-1} \\
P\{0|L\} &= e^{-\lambda_L}\frac{\lambda_L^{x_L}}{x_L!} = e^{-0.1}\frac{0.1^0}{0!} = e^{-0.1}
\end{aligned} \tag{11}
$$

After finding all necessary values, using Bayes Rule for two events, we can find $P\{H|0\}$ as follows,

$$
\begin{aligned}
P\{H|0\} &= \frac{P\{0|H\}P\{H\}}{P\{0|H\}P\{H\} + P\{0|L\}P\{L\}} \\
P\{H|0\} &= \frac{(e^{-1})(0.2)}{(e^{-1})(0.2) + (e^{-0.1})(0.8)} \\
P\{H|0\} &= 0.0923
\end{aligned} \tag{12}
$$