# CENG 798 QUANTUM COMPUTING ASSIGNMENT

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## Answer 1

### a.

Below, $|\psi\rangle$ is given in the canonical basis, i.e., $\{|0\rangle, |1\rangle\}$.

$$\begin{aligned} |\psi\rangle &= \cos\left(\frac{\pi}{3}\right)|0\rangle + \sin\left(\frac{\pi}{3}\right)|1\rangle \\ &= \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle \end{aligned} \tag{1}$$

### b.

$|\phi\rangle$ is found as follows.

$$\begin{aligned} |\phi\rangle &= NOT\ |\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} \\ &= \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle \end{aligned} \tag{2}$$

Then, we find $|\psi\rangle \otimes |\phi\rangle$ below.

$$\begin{aligned} |\psi\rangle \otimes |\phi\rangle &= \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix} \otimes \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/4 \\ 1/4 \\ 3/4 \\ \sqrt{3}/4 \end{bmatrix} \\ &= \frac{\sqrt{3}}{4}|00\rangle + \frac{1}{4}|01\rangle + \frac{3}{4}|10\rangle + \frac{\sqrt{3}}{4}|11\rangle \end{aligned} \tag{3}$$

# Answer 2

## a.

First, we write $|0\rangle$ and $|1\rangle$ in the $\{|u\rangle, |w\rangle\}$ basis.

$$|0\rangle = \frac{1}{\sqrt{2}}|u\rangle + \frac{1}{\sqrt{2}}|w\rangle \tag{4}$$

$$|1\rangle = \frac{1}{\sqrt{2}}|u\rangle - \frac{1}{\sqrt{2}}|w\rangle \tag{5}$$

Using Eqn. 4 and Eqn. 5, we write $|\psi\rangle$ in the $\{|u\rangle, |w\rangle\}$ basis.

$$\begin{aligned}
|\psi\rangle &= \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle \\
&= \frac{1}{2}\left(\frac{1}{\sqrt{2}}|u\rangle + \frac{1}{\sqrt{2}}|w\rangle\right) + \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}|u\rangle - \frac{1}{\sqrt{2}}|w\rangle\right) \\
&= \frac{(1+\sqrt{3})}{2\sqrt{2}}|u\rangle + \frac{(1-\sqrt{3})}{2\sqrt{2}}|w\rangle
\end{aligned} \tag{6}$$

## b.

The probability of observing $|u\rangle$ as the new state after a measurement is given below.

$$\left(\frac{(1+\sqrt{3})}{2\sqrt{2}}\right)^2 = \frac{(2+\sqrt{3})}{4} \tag{7}$$