Student Information

Full Name : Beyazıt Yalçınkaya
Id Number : 2172138

Answer 1

In the question the following properties are given for a graph $G = (V, E)$:

- $|E| = 23$
- $deg(v) \geq 4,\ for\ all\ v \in V$

Since $deg(v) \geq 4,\ for\ all\ v \in V$ is given, the following can be concluded:

$$\sum_{v \in V} deg(v) \geq 4 \times |V|$$

By The Handshaking Theorem the following equation can be derived:

$$\begin{aligned}
2 \times |E| &= \sum_{v \in V} deg(v) \quad (The\ Handshaking\ Theorem) \\
2 \times |E| &= \sum_{v \in V} deg(v) \geq 4|V| \\
2 \times |E| &\geq 4 \times |V| \\
2 \times 23 &\geq 4 \times |V| \\
46 &\geq 4 \times |V| \\
11.5 &\geq |V| \\
Largest\ value\ of\ |V|\ is\ |V| &= 11.
\end{aligned}$$

By the equation above $|V|$ must be less than or equal to 11.5, then the largest value of $|V|$ is $|V| = 11$. Thus, the largest possible number of vertices is 11.

Answer 2

Say $G = (V, E)$ is a simple graph with n vertices with $n \geq 2$ such that the degree of every vertex in G is at least $\frac{n-1}{2}$. Let's add a vertex v' to the graph G and connect it to all other vertices in the graph and name the new graph as G'. Since we connected v' with all other vertices, the degree of all other vertices has been incremented by one and note that the degree of v' is n. Then the degree of every vertex in G' is at least $\frac{n-1}{2} + 1$ which is equal to $\frac{n+1}{2}$. Note that $\frac{n+1}{2} > \frac{n}{2}$ and $n \geq 3$, then by the Dirac's Theorem, G' has a Hamiltonian circuit. Let's denote this Hamiltonian circuit as a sequence of vertices as follows:

$$v_1 v_2 v_3 v_4 .... v_{n-1} v_n v' v_1$$

Now, let's remove the vertex v' and all edges incident to it from G'. Note that after this removal the initial graph G has been obtained. The Hamiltonian circuit denoted as a sequence of vertices is no longer valid since the vertex v' is not in G. However, all other vertices and the edges connecting them are in the graph since the removing operation of v' and edges incident to it did not affect other vertices and edges. Then the sequence of vertices written above changed as follows:

$$v_1 v_2 v_3 v_4 .... v_{n-1} v_n$$

It can be observed that the sequence of vertices above is a Hamiltonian path since all vertices of G is visited exactly once by the path. Note that, for $n = 1$ case, since G contains only 1 vertex, trivially it has a Hamiltonian path. Thus, a simple graph $G = (V, E)$ with n vertices such that the degree of every vertex in G is at least $\frac{n-1}{2}$ contains a Hamiltonian path.

Answer 3

Say $G = (V, E)$ is a bipartite graph with adjacency matrix A^{37}, then $|V| = 37$. Since G is a bipartite graph, its vertices can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2. Assume that there exists a vertex $v_k \in V = \{v_1, v_2, ..., v_{36}, v_{37}\}$ and an edge $e_k \in E$ such that e_k connects v_k with itself. This implies in the adjacency matrix A^{37}, the entry A^{37}_{kk} is 1. Since an edge in a bipartite graph must connect a vertex in V_1 and a vertex in V_2, v_k must be in both of the partition sets V_1 and V_2. However, partition sets V_1 and V_2 are disjoint sets, so there cannot be a vertex which is in both sets. Thus, this is a contradiction, assumption is discharged, there does not exists a vertex $v_k \in V$ and an edge $e_k \in E$ such that e_k connects v_k with itself, so the entry A^{37}_{kk} is not 1. Note that A^{37}_{kk} is on the diagonal of A^{37} for all values of k, this implies in the adjacency matrix 1 cannot appear on the diagonal of the matrix. Hence, the diagonal entries of A^{37} are equal to 0.

Answer 4

a.

Choice	Edge	Weight
1	{e, f}	1
2	{a, d}	2
3	{e, h}	2
4	{g, h}	2
5	{g, d}	3
6	{c, f}	3
7	{d, b}	3
8	{h, i}	4

b.

Choice	Edge	Weight
1	{e, f}	1
2	{e, h}	2
3	{g, h}	2
4	{g, d}	3
5	{a, d}	2
6	{c, f}	3
7	{d, b}	3
8	{h, i}	4

Figure 1: Obtained MST in both part **a.** and part **b.**