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Answer 1

Table 1: Answer 1.1

p	q	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$
T	T	T
T	F	T
F	T	T
F	F	T

Table 2: Answer 1.2

p	q	r	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	T	T
T	T	F	T
T	F	F	T
F	T	F	T
F	F	F	T

Answer 2

$$\begin{aligned}(p \rightarrow q) \vee (p \rightarrow r) &\equiv p \rightarrow (q \vee r) \qquad \textit{Using Table 7} \\ &\equiv \neg(q \vee r) \rightarrow \neg p \qquad \textit{Using Table 7} \\ &\equiv (\neg q \wedge \neg r) \rightarrow \neg p \qquad \textit{De Morgan's Laws}\end{aligned} \tag{1}$$

We started from the first equation and we managed to get the second equation by using some logical equivalences from given tables. Thus, it can be concluded that $(p \rightarrow q) \vee (p \rightarrow r)$ and $(\neg q \wedge \neg r) \rightarrow \neg p$ are logically equivalent.

Answer 3

1. (a) *Every cat has at least one dog friend.*
   (b) *Some cats are friends with all dogs.*

2. (a) $\neg(\exists x \exists y(\neg Customer(x) \land Meal(y) \land Eats(x,\ y)))$
   (b) $\exists x \exists y(Chef(x) \land Meal(y) \land \neg Cooks(x,\ y))$
   (c) $\exists x(Customer(x) \land \exists y(Chef(y) \land \neg(\exists z(Meal(z) \land Cooks(y,\ z) \land \neg Eats(x,\ z)))))$
   (d) $\forall x(Chef(x) \rightarrow \exists y(Chef(y) \land Knows(x,\ y) \land \neg(\exists z(Meal(z) \land \neg Cooks(x,\ z) \land \neg Cooks(y,\ z)))))$

Answer 4

The truth value of $\neg p$ is given as T, so it can be concluded that the truth value of p is F, since $\neg p \equiv \neg F$ and $\neg F \equiv T$. Thus, the following truth table can be constructed.

Table 3: Answer 4

p	q	$p \rightarrow q$
F	T	T
F	F	T

From the table it can be seen that, the truth value of $\neg q$ cannot be determined as T. Since the compound proposition is a tautology, the truth value of the compound proposition $p \rightarrow q$ does not depends on the truth value of q. The propositions $p \rightarrow q$ and $\neg p$ being T does not concludes that $\neg q$ is also T. Hence, it cannot be a deduction rule.

Answer 5

Table 4: Answer 5

1.	$p \to q$	*premise*
2.	$q \to r$	*premise*
3.	$r \to p$	*premise*
4.	q	*assumption*
5.	r	$\to$*e, 2, 4*
6.	p	$\to$*e, 3, 5*
7.	$q \to p$	$\to$*i, 4-6*
8.	p	*assumption*
9.	q	$\to$*e, 1, 8*
10.	r	$\to$*e, 2, 9*
11.	$p \to r$	$\to$*i, 8-10*
12.	$p \leftrightarrow q$	$\leftrightarrow$*i, 1, 7*
13.	$p \leftrightarrow r$	$\leftrightarrow$*i, 3, 11*
14.	$(p \leftrightarrow q) \wedge (p \leftrightarrow r)$	$\wedge$*i, 12, 13*

Answer 6

Table 5: Answer 6

1.		$\forall x(Q(x) \to R(x))$	*premise*
2.		$\exists x(P(x) \to Q(x))$	*premise*
3.		$\forall x P(x)$	*premise*
4.	c	$P(c) \to Q(c)$	*assumption*
5.		$P(c)$	$\forall$*e, 3*
6.		$Q(c) \to R(c)$	$\forall$*e, 1*
7.		$Q(c)$	$\to$*e, 4, 5*
8.		$R(c)$	$\to$*e, 6, 7*
9.		$P(c) \wedge R(c)$	$\wedge$*i, 5, 8*
10.		$\exists x(P(x) \wedge R(x)$	$\exists$*i, 9*
11.		$\exists x(P(x) \wedge R(x))$	$\exists$*e, 2, 4-10*