Adaptive Quantization for Deep Neural Network

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Abstract

In recent years Deep Neural Networks (DNNs) have been rapidly developed in various applications, together with increasingly complex architectures. The performance gain of these DNNs generally comes with high computational costs and large memory consumption, which may not be affordable for mobile platforms. Deep model quantization can be used for reducing the computation and memory costs of DNNs, and deploying complex DNNs on mobile equipment. In this work, we propose an optimization framework for deep model quantization. First, we propose a measurement to estimate the effect of parameter quantization errors in individual layers on the overall model prediction accuracy. Then, we propose an optimization process based on this measurement for finding optimal quantization bit-width for each layer. This is the first work that theoretically analyse the relationship between parameter quantization errors of individual layers and model accuracy. Our new quantization algorithm outperforms previous quantization optimization methods, and achieves 20-40% higher compression rate compared to equal bit-width quantization at the same model prediction accuracy.

Introduction

Deep neural networks (DNNs) have achieved significant success in various machine learning applications, including image classification (Krizhevsky, Sutskever, and Hinton 2012; Simonyan and Zisserman 2014; Szegedy et al. 2015), image retrieval (Hoang et al. 2017; Do, Doan, and Cheung 2016), and natural language processing (Deng, Hinton, and Kingsbury 2013). These achievements come with increasing computational and memory cost, as the neural networks are becoming deeper (He et al. 2016), and contain more filters per single layer (Zeiler and Fergus 2014).

While the DNNs are powerful for various tasks, the increasing computational and memory costs make it difficult to apply on mobile platforms, considering the limited storage space, computation power, energy supply of mobile devices (Han, Mao, and Dally 2015), and the real-time processing requirements of mobile applications. There is clearly a need to reduce the computational resource requirements of DNN models so that they can be deployed on mobile devices (Zhou et al. 2016).

In order to reduce the resource requirement of DNN models, one approach relies on model pruning. By pruning some parameters in the model (Han, Mao, and Dally 2015), or skipping some operations during the evaluation (Figurnov et al. 2016), the storage space and/or the computational cost of DNN models can be reduced. Another approach consists in parameter quantization (Han, Mao, and Dally 2015). By applying quantization on model parameters, these parameters can be stored and computed under lower bit-width. The model size can be reduced, and the computation becomes more efficient under hardware support (Han et al. 2016). It is worth noting that model pruning and parameter quantization can be applied at the same time, without interfering with each other (Han, Mao, and Dally 2015); we can apply both approaches to achieve higher compression rates.

Many deep model compression works have also considered using parameter quantization (Gupta et al. 2015; Han, Mao, and Dally 2015; Wu et al. 2016) together with other compression techniques, and achieve good results. However, these works usually assign the same bit-width for quantization in the different layers of the deep network. In DNN models, the layers have different structures, which lead to the different properties related to quantization. By applying the same quantization bit-width for all layers, the results could be sub-optimal. It is however possible to assign different bit-width for different layers to achieve optimal quantization result (Hwang and Sung 2014).

In this work, we propose an accurate and efficient method to find the optimal bit-width for coefficient quantization on each DNN layer. Inspired by the analysis in (Fawzi, Moosavi-Dezfooli, and Frossard 2016), we propose a method to measure the effect of parameter quantization errors in individual layers on the overall model prediction accuracy. Then, by combining the effect caused by all layers, the optimal bit-width is decided for each layer. By this method we avoid the exhaustive search for optimal bit-width on each layer, and make the quantization process more efficient. We apply this method to quantize different models that have been pre-trained on ImageNet dataset and achieve good quantization results on all models. Our method constantly outperforms recent state-of-the-art, i.e., the SQNR-based method (Lin, Talathi, and Annapureddy 2016) on different models, and achieves 20-40% higher compression rate compared to equal bit-width quantization. Furthermore, we give a theoretical analysis on how the quantization on layers affects DNN accuracy. To the best of our knowledge, this is the first work that theoretically analyses the relationship between coefficient quantization effect of individual layers and DNN accuracy.

Related works

Parameter quantization has been widely used for DNN model compression (Gupta et al. 2015; Han, Mao, and Dally 2015; Wu et al. 2016). The work in (Gupta et al. 2015) limits the bit-width of DNN models for both training and testing, and stochastic rounding scheme is proposed for quantization to improve the model training performance under low bit-width. The authors in (Han, Mao, and Dally 2015) use k-means to train the quantization centroids, and use these centroids to quantize the parameters. The authors in (Wu et al. 2016) separate the parameter vectors into sub-vectors, and find sub-codebook of each sub-vectors for quantization. In these works, all (or a majority of) layers are quantized with the same bit-width. However, as the layers in DNN have various structures, these layers may have different properties with respect to quantization. It is possible to achieve better compression result by optimizing quantization bit-width for each layer.

Previous works have been done for optimizing quantization bit-width for DNN models (Hwang and Sung 2014; Anwar, Hwang, and Sung 2015; Lin, Talathi, and Annapureddy 2016; Sun, Lin, and Wang 2016). The authors in (Hwang and Sung 2014) propose an exhaustive search approach to find optimal bit-width for a fully-connected network. In (Sun, Lin, and Wang 2016), the authors first use exhaustive search to find optimal bit-width for uniform or non-uniform quantization; then two schemes are proposed to reduce the memory consumption during model testing. The exhaustive search approach only works for a relatively small network with few layers, while it is not practical for deep networks. As the number of layers increases, the complexity of exhaustive search increases exponentially. The authors in (Anwar, Hwang, and Sung 2015) use mean square quantization error (MSQE) (L_2 error) on layer weights to measure the sensitivity of DNN layers to quantization, and manually set the quantization bit-width for each layer. The work in (Lin, Talathi, and Annapureddy 2016) use the signal-to-quantization-noise ratio (SQNR) on layer weights to measure the effect of quantization error in each layer. These MSQE and SQNR are good metrics for measuring the quantization loss on model weights. However, there is no theoretical analysis to show how these measurements relate to the accuracy of the DNN model, but only empirical results are shown. The MSQE-based approach in (Anwar, Hwang, and Sung 2015) minimizes the L_2 error on quantized weight, indicating that the L_2 error in different layer has the equal effect on the model accuracy. Similarly, in (Lin, Talathi, and Annapureddy 2016), the authors maximize the overall SQNR, and suggest that quantization on different layers has equal contribution to the overall SQNR, thus has equal effect on model accuracy. Both works ignore that the various structure and position of different layers may lead to different robustness on quantization, and thus render the two approaches suboptimal.

In this work, we follow the analysis in (Fawzi, Moosavi-Dezfooli, and Frossard 2016), and propose a method to measure the effect of quantization error in each DNN layers. Different from (Anwar, Hwang, and Sung 2015; Lin, Talathi, and Annapureddy 2016), which use empirical results to show the relationship between the measurement and DNN accuracy, we conduct a theoretical analysis to show how our proposed method relates to the model accuracy. Furthermore, we show that our bit-width optimization method is more general than the method in (Lin, Talathi, and Annapureddy 2016), which makes our optimization more accurate.

There are also works (Hinton, Vinyals, and Dean 2015; Romero et al. 2014) that use knowledge distillation to train a smaller network using original complex models. It is also possible to combine our quantization framework with knowledge distillation to achieve yet better compression results.

Measuring the effect of quantization noise

In this section, we analyse the effect of quantization on the accuracy of a DNN model. Parameter quantization can result in quantization noise that would affect the performance of the model. Previous works have been done for analyzing the effect of input noise on the DNN model (Fawzi, Moosavi-Dezfooli, and Frossard 2016); here we use this idea to analyse the effect of noise in intermediate feature maps in the DNN model.

Quantization optimization

The goal of our paper is to find a way to achieve optimal quantization result to compress a DNN model. After the quantization, under controlled accuracy penalty, we would like the model size to be as small as possible. Suppose that we have a DNN $\mathcal{F}$ with N layers. Each layer i has s_i parameters, and we apply b_i bit-width quantization in the parameters of layer i to obtain a quantized model $\mathcal{F}'$. Our optimization objective is:

$$\begin{aligned} min \sum_{i=1}^{N} s_i \cdot b_i \\ s.t.\ acc_{\mathcal{F}} - acc_{\mathcal{F}'} \leq \Delta_{acc}, \end{aligned} \tag{1}$$

where $acc_{\mathcal{F}}$ is the accuracy of the model $\mathcal{F}$, and Δ_{acc} is the maximum accuracy degradation. Note that it requires enormous computation to calculate the accuracy of the model for all quantization cases. To solve the problem more efficiently, we propose a method to estimate the value of the performance penalty given by $acc_{\mathcal{F}} - acc_{\mathcal{F}'}$.

Quantization noise

Value quantization is a simple yet effective way to compress a model (Han, Mao, and Dally 2015). Here we evaluate the effect of using value quantization on model parameters.

Assume that conducting quantization on a value is equivalent to adding noise to the value:

$$w_q = w + r_w \tag{2}$$

Here w is the original value, $w \in W$, with W the set of weights in a layer. Then, w_q is the quantized value, and r_w is the quantization noise. Assume that we use a uniform quantizer, and that the stepsize of the quantized interval is fixed.

Following the uniform quantization analysis in (You 2010), if we consider $\mathbf{r}_w = (r_{w,1}, \cdots, r_{w,N_W})$ as the quantization noise on all weights in W, we have the expectation of $||\mathbf{r}_w||_2^2$ as

$$E(||\mathbf{r}_w||_2^2) = p'_w \cdot e^{-\alpha \cdot b}, \tag{3}$$

where $p'_w = N_W \frac{(w_{min} - w_{max})^2}{12}$, N_W is the number of weights in W, and $\alpha = ln(4)$ (You 2010). Detailed analysis can be found in Supplementary Material. Eq. (3) indicates that every time we reduce the bit-width by 1 bit, $E(||\mathbf{r}_w||_2^2)$ will increase by 4 times. This is equivalent to the quantization efficiency of 6dB/bit in (Gray and Neuhoff 2006).

Measurement for quantization noise

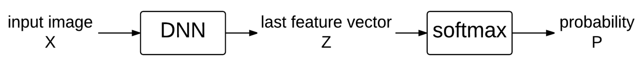


Figure 1: Simple DNN model architecture.

From weight domain to feature domain Eq. (3) shows the quantization noise in the weight domain; here we show how the noise on weight domain can link to the noise in the feature domain.

A simplified DNN classifier architecture is shown in Fig. 1.

Here we define W_i as the weights of layer i in the DNN model $\mathcal{F}$. And Z is the last feature map (vector) of the DNN model $\mathcal{F}$. As we quantize W_i, the quantization noise is $\mathbf{r}_{W_i}$, and there would be a resulting noise $\mathbf{r}_{Z_i}$ on the last feature map Z. Here we define $\mathbf{r}_{Z_i}$ as the noise on last feature map Z that is caused by the quantization only on a single layer i.

As the value of $||\mathbf{r}_{Z_i}||_2^2$ is proportional to the value of $||\mathbf{r}_{W_i}||_2^2$, similar to Eq. (3), the expectation of resulting noise on $\mathbf{r}_{Z_i}$ is:

$$E(||\mathbf{r}_{Z_i}||_2^2) = p_i \cdot e^{-\alpha \cdot b_i} \tag{4}$$

This is proved in later sections, empirical results are shown in Fig. 4.

The effect of quantization noise Similarly to the analysis in (Pang, Du, and Zhu 2017), we can see that the softmax classifier has a linear decision boundary in the last feature vectors Z (Pang, Du, and Zhu 2017) in Fig. 1. The analysis can be found in the Supplementary Material. Then we apply the result of (Fawzi, Moosavi-Dezfooli, and Frossard 2016) to bound the robustness of the classifier with respect to manipulation of weights in different layers.

We define $\mathbf{r}^*$ to be the adversarial noise, which represents the minimum noise to cause misclassification. For a certain input vector $\mathbf{z} = (z_1, \cdots, z_L)$, where L is the number of element in $\mathbf{z}$, the $||\mathbf{r}^*||_2$ is the distance from the datapoint to the decision boundary, which is a fixed value. We define a sorted vector of $\mathbf{z}$ as $\mathbf{z}_{sorted} = (z_{(1)}, \cdots, z_{(L)})$, where the max value is $z_{(1)}$, and second max value is $z_{(2)}$. The result for softmax classifier (or max classifier) can be expressed as: $max(\mathbf{z}) = z_{(1)}$, which is picking up the maximum value in the vector$\mathbf{z}$.

As adversarial noise is the minimum noise that can change the result of a classifier, we can get the adversarial noise for softmax classifier $max(\mathbf{z})$ as $\mathbf{r}^*_{\mathbf{z}_{sorted}} = (\frac{z_{(2)} - z_{(1)}}{2}, \frac{z_{(1)} - z_{(2)}}{2}, 0, \cdots, 0)$, then the norm square of adversarial noise $||\mathbf{r}^*||_2^2 = (z_{(1)} - z_{(2)})^2/2$.

Here we define $\mathbf{r}_Z$ as the noise that we directly add on last feature map Z. We can consider $\mathbf{r}_Z$ as the collective effect of all $\mathbf{r}_{Z_i}$ that caused by the quantization on all layers $i \in \{1, \cdots, N\}$, where N is the number of layers.

As mentioned in (Fawzi, Moosavi-Dezfooli, and Frossard 2016), if we apply random noise $\|\mathbf{r}_Z\|_2^2$ rather than adversarial noise $\|\mathbf{r}^*\|_2$ on the input vector $\mathbf{z}$ for a softmax classifier $max(\mathbf{z})$, it requires higher norm for random noise $\|\mathbf{r}_Z\|_2^2$ to causes prediction error with same probability, compared to adversarial noise $\|\mathbf{r}^*\|_2$.

The following result shows the relationship between the random noise $\|\mathbf{r}_Z\|_2^2$ and adversarial noise $\|\mathbf{r}^*\|_2$, under softmax classifier with a number of classes equal to d:

Lemma 1. *Let $\gamma(\delta) = 5 + 4\ln(1/\delta)$. The following inequalities hold between the norm square of random noise $\|\mathbf{r}_Z\|_2^2$ and adversarial noise $\frac{(z_{(1)} - z_{(2)})^2}{2}$.*

$$\frac{\ln d}{d}\gamma(\delta)\|\mathbf{r}_Z\|_2^2 \geq \frac{(z_{(1)} - z_{(2)})^2}{2} \tag{5}$$

with probability exceeding $1 - 2\delta$.

The proof of Lemma 1 can be found in the Supplementary Material. The lemma states that if the norm of random noise is $o\left((z_{(1)} - z_{(2)})\sqrt{d/\ln d}\right)$, it does not change the classifier decision with high probability.

Based on Lemma 1, we can rewrite our optimization problem. Assume that we have a model $\mathcal{F}$ with accuracy $acc_{\mathcal{F}}$. After adding random noise $\mathbf{r}_Z$ on the last feature map Z, the model accuracy drops by Δ_{acc}. If we have

$$\theta(\Delta_{acc}) = \frac{d}{\gamma(\frac{\Delta_{acc}}{2acc_{\mathcal{F}}})\ln d}, \tag{6}$$

we have the relation between accuracy degradation and noise $\mathbf{r}_Z$ as:

$$\begin{aligned} acc_{\mathcal{F}} - acc_{\mathcal{F}'} \leq \Delta_{acc} \Rightarrow \\ \|\mathbf{r}_Z\|_2^2 < \theta(\Delta_{acc})\frac{(z_{(1)} - z_{(2)})^2}{2} \end{aligned} \tag{7}$$

The detailed analysis can be found in the Supplementary Material. Eq. (7) shows the bound of noise on last feature map Z. However, adding quantization noise to different layers may have different effect on model accuracy. Suppose we have model $\mathcal{F}$ for quantization. By adding noise $\mathbf{r}_{W_i}$ on weights of layer i, we induce the noise $\mathbf{r}_{Z_i}$ on last feature map Z. By quantizing earlier layers, the noise needs to pass through more layers to get $\mathbf{r}_{Z_i}$, which results in a low rank noise $\mathbf{r}_{Z_i}$. For example, when quantizing the first layer, is results in $\mathbf{r}_{Z_1} = (e_1, \cdots, e_{d'}, 0, \cdots, 0)$, and

$rank(\mathbf{r}_{Z_1}) = d' < d$. When quantizing the last layer, it results in $\mathbf{r}_{Z_N} = (e_1, \cdots, e_d)$, and $rank(\mathbf{r}_{Z_N}) = d$. In order to let $\mathbf{r}_{Z_N}$ have equivalent effect on model accuracy as $\mathbf{r}_{Z_1}$, $\|\mathbf{r}_{Z_N}\|_2$ should be larger than $\|\mathbf{r}_{Z_1}\|_2$.

By considering the different effects of $\mathbf{r}_{Z_i}$ caused by quantization in different layers, we rewrite Eq. (7) in a more precise form:

$$\begin{aligned} &acc_{\mathcal{F}} - acc_{\mathcal{F}'} = \Delta_{acc} \Rightarrow \\ &\|\mathbf{r}_{Z_i}\|_2^2 = t_i(\Delta_{acc})\frac{(z_{(1)} - z_{(2)})^2}{2}. \end{aligned} \tag{8}$$

Here $t_i(\Delta_{acc})$ is the robustness parameter of layer i under accuracy degradation Δ_{acc}.

Eq. (8) shows a precise relationship between $\mathbf{r}_{Z_i}$ and Δ_{acc}. If we add quantization noise to layer i of model $\mathcal{F}$, and get noise $\mathbf{r}_{Z_i}$ on last feature map Z, then the model accuracy decreases by Δ_{acc}.

We consider the layer i in model $\mathcal{F}$ as $\mathbf{y}_i = \mathcal{F}_i(\mathbf{y}_{i-1})$, where $\mathbf{y}_i$ is the feature map after layer i. Here we consider that the noise $\mathbf{r}_{y_i}$ would transfer through layers under almost linear transformation (to be discussed in later sections). If we add random noise in the weights of layer i, we have the rank of the resulting noise $\mathbf{r}_{Z_i}$ on last feature map Z given as:

$$rank(\mathbf{r}_{Z_i}) = rank(\prod_i^N \mathcal{F}_i) \leq min\{rank(\mathcal{F}_i)\} \tag{9}$$

Based on Eq. (9), we have:

$$rank(\mathbf{r}_{Z_1}) \leq \cdots \leq rank(\mathbf{r}_{Z_i}) \leq \cdots \leq rank(\mathbf{r}_{Z_N}) \tag{10}$$

Eq. (10) suggests that the noise on earlier layers of DNN needs to pass through more layers to affect the last feature map Z, the noise $\mathbf{r}_{Z_i}$ on Z would have lower rank, resulting in a lower value of $t_i(\Delta_{acc})$.

From Eq. (8), we can see in particular that when

$$\frac{\|\mathbf{r}_{Z_i}\|_2^2}{t_i(\Delta_{acc})} = \frac{\|\mathbf{r}_{Z_j}\|_2^2}{t_j(\Delta_{acc})}, \tag{11}$$

the quantization on layer i and j have same effect on model accuracy. Based on Eq. (11), $\frac{\|\mathbf{r}_{Z_i}\|_2^2}{t_i(\Delta_{acc})}$ can be a good measurement for estimating the accuracy degradation caused by quantization noise, regardless of which layer to quantize. Consider $\mathbf{x} \in \mathcal{D}$ as the input in dataset $\mathcal{D}$, we have the corresponding feature vector $\mathbf{z} = \mathcal{G}(\mathbf{x}, W)$ in the last feature map Z. By quantizing layer i in model $\mathcal{F}$, we get noise $\mathbf{r}_{\mathbf{z}_i}$ on $\mathbf{z}$. We define the accuracy measurement on layer i as:

$$m_i = \frac{\frac{1}{|\mathcal{D}|}\sum_{\mathbf{x}\in\mathcal{D}}(||\mathbf{r}_{\mathbf{z}_i}||_2^2)}{t_i(\Delta_{acc})} = \frac{||\mathbf{r}_{Z_i}||_2^2}{t_i(\Delta_{acc})} \tag{12}$$

The way to calculate $t_i(\Delta_{acc})$ is given by:

$$\begin{aligned} t_i(\Delta_{acc}) = \frac{mean_{\mathbf{r}_{\mathbf{z}_i}}}{mean_{\mathbf{r}^*}} &= \frac{\frac{1}{|\mathcal{D}|}\sum_{\mathbf{x}\in\mathcal{D}} ||\mathbf{r}_{\mathbf{z}_i}||_2^2}{\frac{1}{|\mathcal{D}|}\sum_{\mathbf{x}\in\mathcal{D}} \frac{(z_{(1)}-z_{(2)})^2}{2}} \\ s.t. \quad & acc_{\mathcal{F}} - acc_{\mathcal{F}'} = \Delta_{acc} \end{aligned} \tag{13}$$

The detailed method to calculate $t_i(\Delta_{acc})$ will be discussed in the experiment section. Note that, based on the optimization result in Eq. (22), the selected value of Δ_{acc} does not matter for the optimization result, as long as the the value of $\frac{t_i(\Delta_{acc})}{t_j(\Delta_{acc})}$ is almost independent w.r.t. Δ_{acc}, which is true according to Fig. 3. So choosing different value of Δ_{acc} does not change the optimization result. In later sections, we use t_i instead of $t_i(\Delta_{acc})$ for simplicity.

From Eq. (12), based on the linearity and additivity of the proposed estimation method (shown in later sections), the measurement of the effect of quantization error in all the layers of the DNN model is shown in Eq. (20).

After we define the accuracy measurement for each layer of model, based on Eq. (8), we can then rewrite the optimization in Eq. (1) as

$$\begin{aligned} & min \sum_{i=1}^N s_i \cdot b_i \\ & s.t. \ m_{all} = \sum_{i=1}^N m_i \leq C, \end{aligned} \tag{14}$$

where m_{all} is the accuracy measurement for all layers, and C is a constant related to model accuracy degradation Δ_{acc}, with higher C indicating higher Δ_{acc}.

Linearity of the measurements

In this section we will show that the DNN model are locally linear to the quantization noise measurement $||\mathbf{r}_W||_2$, under the assumption that the quantization noise is much smaller than the original value: $||r_w||_2 \ll ||w||_2$. That is, if a quantization noise $||\mathbf{r}_{W_i}||_2$ on layer i leads to $||\mathbf{r}_{Z_i}||_2$ on last feature vector Z, then we have a quantization noise $\alpha \cdot ||\mathbf{r}_{W_i}||_2$ on layer i leads to $\alpha \cdot ||\mathbf{r}_{Z_i}||_2$ on last feature vector Z.

For linear layers like convolutional layers and fully connected layers in the DNN model, the linearity for noise is obvious. Here we mainly focus on the non-linear layers in the DNN model, such as ReLU and Max-pooling layers.

ReLU layers The ReLU layers is widely used to provide nonlinear activation for DNN. Given the input $a \in A$ to a ReLU layer, the output value $z \in Z$ is calculated as:

$$z = ReLU(a) = \begin{cases} a, & if\ a > 0 \\ 0, & if\ a <= 0 \end{cases} \tag{15}$$

From Eq. (15) we can see that the ReLU layer is linear to noise in most cases. The non-linearity happens only when the noise r_w crossing the zero point, which has small probability when the noise is sufficiently small.

Max-pooling layers Max-pooling is a nonlinear downsampling layer that reduces the input dimension and controls overfitting. We can consider that max-pooling acts as a max filter to the feature maps.

Similarly to the ReLU layer, which can be described as $z = ReLU(a) = max(0, a)$, the max-pooling layer can be describes as $z = maxpool(\{a_i\}) = max(\{a_i\})$, where $i = 1, \cdots, P$, with P the kernel size for max-pooling. The linearity for noise holds when the noises are sufficiently small and do not alter the order for $\{a_i\}$.

Other layers For other non-linear layers like Sigmoid and PReLU, the linearity for small noises still holds under the assumptions that the function is smooth along most input ranges, and the noise has very low probability to cross the non-linear region.

Based on the linearity assumption, as we model the quantization noise on weight as Eq. (3), the resulting noise on last feature vector Z can be modeled as:

$$||\mathbf{r}_{Z_i}||_2^2 = p_i \cdot e^{-\alpha \cdot b_i} \tag{16}$$

Additivity of the measurements

Noise on single multiplication Pairwise multiplication is a basic operation in the convolutional layers and fully connected layers of DNN. Given one value in the input $a \in A$, one value in the weight matrix $w \in W$, we have the pairwise multiplication as $a \cdot w$. If we consider noise in both input $a \in A$ and $w \in W$, we have noised value $a_q \in A_q$ and $w_q \in W_q$, and finally $a_q \cdot w_q = (a + r_a) \cdot (w + r_w)$.

Noise on one layer Given a convolutional layer input A with size $M \times N \times C$, conv kernel K with size $M_k \times N_k \times C \times D$, and stride size s, we have the output feature map Z with size $(M/s) \times (N/s) \times D$. Here M and N are the height and width of input, C is the number of channels of input. M_k and N_k are the height and width of the conv kernel, D is the depth of output feature map.

The analysis on fully connected layers will be similar to the analysis on convolutional layers. It can be considered as a special case of convolutional layers when M, N, M_k, and N_k are equal to 1. For a single value $z_{p,q,d} \in Z$, the noise term of $z_{p,q,d}$ can be expressed as:

$$\begin{aligned} r_{z_{p,q,d}} &= \sum_{i=1}^{M_k \cdot N_k \cdot C} (a_i \cdot r_{w_{i,d}} + r_{a_i} \cdot w_{i,d} + r_{a_i} \cdot r_{w_{i,d}}) + r_{b_d} \\ &\approx \sum_{i=1}^{M_k \cdot N_k \cdot C} (a_i \cdot r_{w_{i,d}} + r_{a_i} \cdot w_{i,d}) \end{aligned} \tag{17}$$

The calculation details can be found in Supplementary Material. Note that the term r_{b_d} can be ignored under the assumption that w and b have same bit-width quantization. The term $r_{a_i} \cdot r_{w_{i,d}}$ can be ignored under the assumption that $||r_a||_2 \ll ||a||_2$ and $||r_w||_2 \ll ||w||_2$.

From Eq. (17) we can see that: 1) adding noise to input feature maps and weights separately and independently, is equivalent to adding noise to both input feature maps and weights; 2) regarding the output feature map $\mathbf{z} \in Z$, adding noise to the input feature maps and weights and doing layer operation (pairwise product), is equivalent to adding the noise directly to the output feature map. We will use these two properties in later sections.

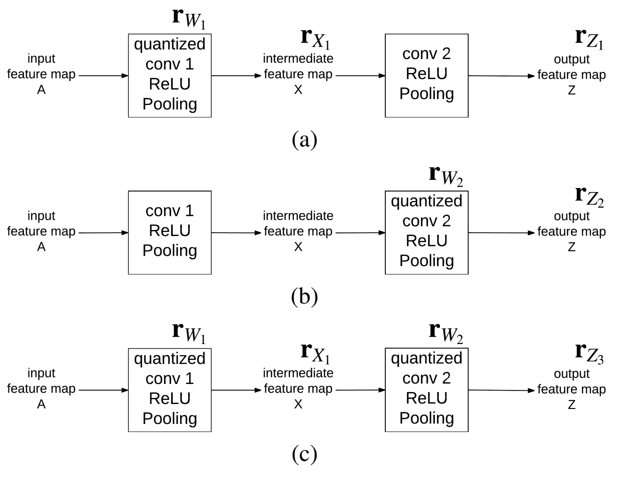


Figure 2: Effect of adding noise to multiple layers.

Adding noise to multiple layers Fig. 2 shows a 2-layer module inside a DNN model. Given input feature map A, after the first conv layer, an intermediate feature map X is generated, then after the second conv layer, output feature map Z is generated. Fig. 2(a) and 2(b) show the effect of noise on layer 1 and 2, respectively. And Fig. 2(c) shows the effect of noise on both layer 1 and 2. By analysing the additivity of $\|\mathbf{r}_Z\|_2^2$, we have:

$$||\mathbf{r}_{\mathbf{z}_3}||_2^2 \doteq ||\mathbf{r}_{\mathbf{z}_1}||_2^2 + ||\mathbf{r}_{\mathbf{z}_2}||_2^2 \tag{18}$$

Detailed analysis can be found in Supplementary Material. Eq. (18) holds under the assumption that $\mathbf{r}_{\mathbf{z}_1}$ and $\mathbf{r}_{\mathbf{z}_2}$ are independent. This is reasonable in our case, as $\mathbf{r}_{\mathbf{z}_1}$ and $\mathbf{r}_{\mathbf{z}_2}$ are caused by $\mathbf{r}_{W_1}$ and $\mathbf{r}_{W_2}$ which are two independent quantization noises. This independence between $\mathbf{r}_{\mathbf{z}_1}$ and $\mathbf{r}_{\mathbf{z}_2}$ is also important for our proposed estimation method.

We can extend Eq. (18) to the situation of N layers:

$$||\mathbf{r}_{\mathbf{z}}||_2^2 = \sum_{i=1}^{N} ||\mathbf{r}_{\mathbf{z}_i}||_2^2 \tag{19}$$

If we consider the linearity and additivity of the proposed measurement, from Eq. (12) and Eq. (19), as well as the independence of the measurement among different layers, we have the measurement of the effect of quantization errors in all layers in DNN model:

$$m_{all} = \sum_{i=1}^{N} m_i = \sum_{i=1}^{N} \frac{||\mathbf{r}_{Z_i}||_2^2}{t_i} \tag{20}$$

Eq. (20) suggests that the noise effect of adding noise to each layer separately and independently, is equivalent to the effect of adding noise to all layers simultaneously. We use Eq. (12) as the measurement for noise effect on layer i, and the effect of adding noise to all layers can be predicted using Eq. (20).

Layer-wise bit-width optimization

In this section we show the approach for optimizing the layer-wise bit-width quantization to achieve an optimal compression ratio under certain accuracy loss.

Following the discussion from the optimization problem in Eq. (14), our goal is to constraint Eq. (20) to be a small value while minimizing the model size.

Adaptive quantization on multiple layers

Based on Eq. (16) and (20), the optimization Eq. (14) can be expressed as:

$$\begin{aligned} &min \sum_{i=1}^{N} s_i \cdot b_i \\ &s.t. \sum_{i=1}^{N} \frac{p_i}{t_i} \cdot e^{-\alpha \cdot b_i} \leq C \end{aligned} \tag{21}$$

The optimal value of Eq. (21) can be reached when:

$$\frac{p_1 \cdot e^{-\alpha \cdot b_1}}{t_1 \cdot s_1} = \frac{p_2 \cdot e^{-\alpha \cdot b_2}}{t_2 \cdot s_2} = \cdots = \frac{p_N \cdot e^{-\alpha \cdot b_N}}{t_N \cdot s_N} \tag{22}$$

The detailed analysis can be found in Supplementary Material.

Optimal bit-width for each layer

From Eq. (22) we can directly find the optimal bit-width for each layer using the following procedure:

- Calculate t_i (Eq. (13)):
  - First, calculate the mean value of adversarial noise for the dataset: $mean_{\mathbf{r}^*} = \frac{1}{|\mathcal{D}|} \sum_{\mathbf{x} \in \mathcal{D}} \frac{(z_{(1)} - z_{(2)})^2}{2}$.
  - Then, fix Δ_{acc} value. For example, $\Delta_{acc} = 10\%$. Note that the selection of Δ_{acc} value does not affect the optimization result.
  - For each layer i, change the amount of noise $\mathbf{r}_{W_i}$ added in weight W_i, until the accuracy degradation equals to Δ_{acc}. Then, record the mean value of noise $\mathbf{r}_{\mathbf{z}_i}$ on the last feature map Z: $mean_{\mathbf{r}_{\mathbf{z}_i}} = \frac{1}{|\mathcal{D}|} \sum_{\mathbf{x} \in \mathcal{D}} ||\mathbf{r}_{\mathbf{z}_i}||_2^2$.
  - The value t_i can be calculated as: $t_i(\Delta_{acc}) = \frac{mean_{\mathbf{r}_{\mathbf{z}_i}}}{mean_{\mathbf{r}^*}}$.
  - The details for the calculation of t_i can be found in Fig. 3.
- Calculate p_i:
  - First, for each layer i, fix b_i value. For example, use $b_i = 10$.
  - Then, record the mean value of noise $\mathbf{r}_{\mathbf{z}_i}$ on the last feature map Z: $mean_{\mathbf{r}_{\mathbf{z}_i}} = \frac{1}{|\mathcal{D}|} \sum_{\mathbf{x} \in \mathcal{D}} ||\mathbf{r}_{\mathbf{z}_i}||_2^2$.
  - The value p_i can be calculated using Eq. (16): $mean_{\mathbf{r}_{\mathbf{z}_i}} = ||\mathbf{r}_{Z_i}||_2^2 = p_i \cdot e^{-\alpha \cdot b_i}$.
- Calculate b_i:
  - Fix the bitwidth for first layer b_1, for example, $b_1 = 10$. Then bitwidth for layer i can be calculated using the Eq. (22): $\frac{p_1 \cdot e^{-\alpha \cdot b_1}}{t_1 \cdot s_1} = \frac{p_i \cdot e^{-\alpha \cdot b_i}}{t_i \cdot s_i}$

The detailed algorithm about the above procedure can be found in Supplementary Material. Note that, by selecting different b_1, we achieve different quantization result. A lower value of b_1 results in higher compression rate, as well as higher accuracy degradation.

Comparison with SQNR-based approach

Based on the SQNR-based approach (Lin, Talathi, and Annapureddy 2016), the optimal bit-width is reached when:

$$\frac{e^{-\alpha \cdot b_1}}{s_1} = \frac{e^{-\alpha \cdot b_2}}{s_2} = \cdots = \frac{e^{-\alpha \cdot b_N}}{s_N} \tag{23}$$

The proof can be found in Supplementary Material. Note that compared with our result in Eq. (22), the parameters p_i and t_i are missing. This is consistent with the assumption of the SQNR-based approach, where two layers having the same bit-width for quantization would have the same SQNR value; hence the effects on accuracy are equal. This makes the SQNR-based approach a special case of our approach, when all layers in the DNN model have the equal effect on model accuracy under the same bit-width.

Experimental results

In this section we show empirical results that validate our assumptions in previous sections, and evaluate the proposed bit-width optimization approach.

All codes are implemented using MatConvNet (Vedaldi and Lenc 2015). All experiments are conducted using a Dell workstation with E5-2630 CPU and Titan X Pascal GPU.

Empirical results about measurements

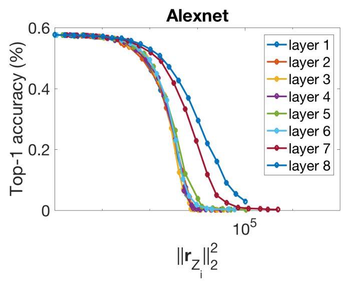
To validate the effectiveness of the proposed accuracy estimation method, we conduct several experiments. These experiments validate the relationship between the estimated accuracy, the linearity of the measurement, and the additivity of the measurement.

Here we use Alexnet (Krizhevsky, Sutskever, and Hinton 2012), VGG-16 (Simonyan and Zisserman 2014), GoogleNet (Szegedy et al. 2015), and Resnet (He et al. 2016) as the model for quantization. Each layer of the model is quantized separately using uniform quantization, but possibly with different bit-width. The quantized model is then tested on the validation set of Imagenet (Krizhevsky, Sutskever, and Hinton 2012), which contains 50000 images in 1000 classes.

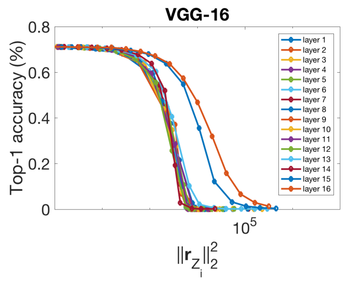
Calculate t_i As Eq. (12) is proposed to measure the robustness of each layer, we conduct an experiment to find t_i value. We use Alexnet as an example.

First, we calculate the adversarial noise for Alexnet on the last feature vector Z. The calculation is based on Eq. (13). The mean value of $||\mathbf{r}^*||_2^2$ for Alexnet is $mean_{\mathbf{r}^*} = 5.33$. The distribution of $||\mathbf{r}^*||_2^2$ for Alexnet on Imagenet validation set can be found in Supplementary Material.

After finding $||\mathbf{r}^*||_2^2$ value, the value of t_i is calculated based on Fig. 3(a) and Eq. (13). We set the accuracy degradation to be roughly half of original accuracy (57%), which is 28%. Based on the values in Fig. 3(a), t_1 to t_6 are equal to 5.2×10^2, $t_7 = 1.3 \times 10^3$, and $t_8 = 2.0 \times 10^3$.

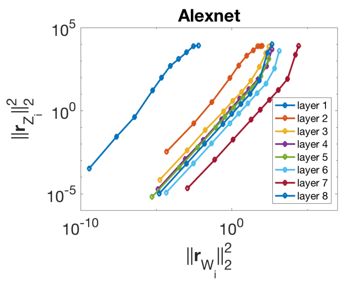


(a)

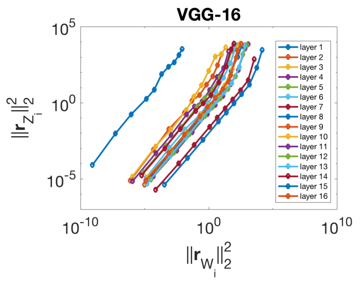


(b)

Figure 3: The relationship between different $||\mathbf{r}_{Z_i}||_2^2$ and model accuracy.



(a)



(b)

Figure 4: The relationship between $||\mathbf{r}_{W_i}||_2^2$ and $||\mathbf{r}_{Z_i}||_2^2$.

Here we show the example for Alexnet of how to calculate the t_i value. Note that for other networks like VGG-16, GoogleNet, and Resnet, we also observe that only the t_i value for the last 1 or 2 layers are obviously different than the other t_i values. During our calculation, we can thus focus on the t_i values for the last several layers. Furthermore, in Fig. 3(a), we find the $||\mathbf{r}_Z||_2^2 - accuracy$ relationship for different amounts of noise, which requires a lot of calculations. In real cases, we use binary search to find appropriate points under the same accuracy degradation. This makes the process to calculate t_i fast and efficient. Typically, for a deep model with N layers, and dataset with $|\mathcal{D}|$ size, we require $O(\tau N|\mathcal{D}|)$ forward passes to calculate accuracy. Here τ is the trial times over one layer. We can reduce it to $O(\tau N'|\mathcal{D}|)$(with $N' << N$) by only calculating t_i values for the last N' layers.

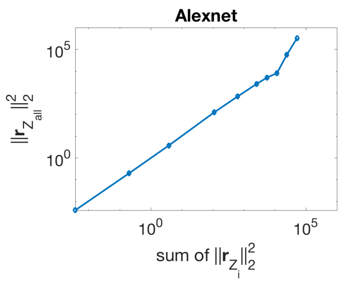
In our experiments, the calculation of t_i is the most time-consuming part of our algorithm. We use around 15 mins to calculate the t_i value for Alexnet (30 sec for forward pass on the whole dataset), and around 6 hours to calculate the t_i value for Resnet-50 (2 min for forward pass on the whole dataset). This time can be reduced if we only calculate t_i values for the last few layers.

Linearity of measurements Fig. 4 shows the relationship between the norm square of noise on quantized weight $||\mathbf{r}_{W_i}||_2^2$ and $||\mathbf{r}_{Z_i}||_2^2$ on different layers. When the quantization noise on weight is small, we can observe linear relationships. While it is interesting to see that, when the quantization noise is large, the curve does not follow exact linearity, and curves for earlier layers are not as linear as later layers. One possible explanation is that earlier layers in a DNN model are affected by more non-linear layers, such as ReLU and Max-pooling layers. When the noise is large enough to reach the non-linear part of the layer functions (i.e. the zero point of the ReLU function), the curves become non-linear. It is worth noting that, when the non-linearity in most layers happens, the accuracy of the model is already heavily affected (become near zero). So this non-linearity would not affect our quantization optimization process.

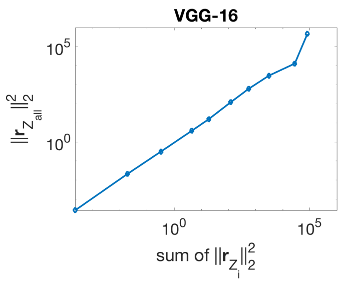
Additivity of measurements Fig. 5 shows the relationship between $\sum_i^N ||\mathbf{r}_{Z_i|}||_2^2$ when we quantize each layer separately, and the value $||\mathbf{r}_Z||_2^2$ when we quantize all layers together. We can see that when the quantization noise is small, the result closely follows our analysis that $||\mathbf{r}_Z||_2^2 = ||\mathbf{r}_{Z_i}||_2^2$; it validates the additivity of $||\mathbf{r}_Z||_2^2$. When the quantization noise is large, the additivity of $||\mathbf{r}_Z||_2^2$ is not accurate. This result fits our assumption in Eq. (17), where the additivity holds under the condition $||\mathbf{r}_{W_i}||_2 \ll ||W_i||_2$ for all layer i in the DNN model. When the noise is too high and we observe the inaccuracy of additivity, the model accuracy is already heavily degraded (near zero). Hence it does not affect the quantization optimization process which rather works in low noise regime.

Optimal bit-width for models

After the validation of the proposed measurement, we conduct experiments to show the results on adaptive quantization. Here we use Alexnet (Krizhevsky, Sutskever, and Hinton 2012), VGG-16 (Simonyan and Zisserman 2014), GoogleNet (Szegedy et al. 2015), and Resnet-50 (He et al. 2016) to test our bit-width optimization approach. Similarly to the last experiments, the validation set of Imagenet is used. As the SQNR-based method (Lin, Talathi, and Anna-



(a)



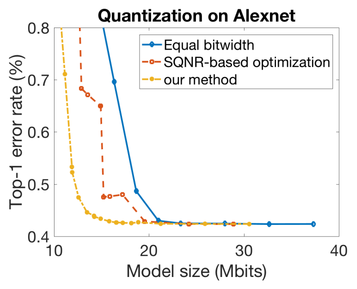
(b)

Figure 5: The value of $\sum_i^N ||\mathbf{r}_{Z_i}||_2^2$ when quantize each layer separately, compare to $||\mathbf{r}_Z||_2^2$ when quantize all layers simultaneously.

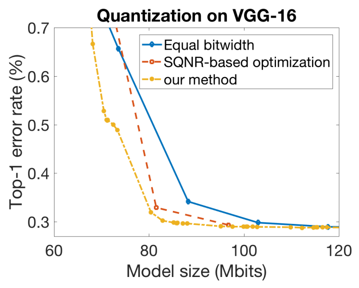
pureddy 2016) only works for convolutional layers, here we keep the fully connected layers with 16 bits .

Fig. 6 shows the quantization results using our method, SQNR-based method (Lin, Talathi, and Annapureddy 2016), and equal bit-width quantization. The equal bit-width quantization means that the number of quantization intervals in all layers are the same. For all three methods, we use uniform quantization for each layer. We can see that for all networks, our proposed method outperforms SQNR-based method, and achieves smaller model size for the same accuracy degradation. It is interesting to see that the SQNR-based method does not obviously outperform equal quantization on the Resnet-50 model. One possible reason is that Resnet-50 contains 1×1 convolutional layers in its "bottleneck" structure, which is similar to fully connected layers. As the authors claim in (Lin, Talathi, and Annapureddy 2016), the SQNR-based method does not work for fully connected layers. Note that our method generates more datapoints on the figure, because the optimal bit-width for different layers may contain different decimals. And by rounding the optimal bit-width in different ways, we can generate more bit-width combinations than the SQNR-based methods.

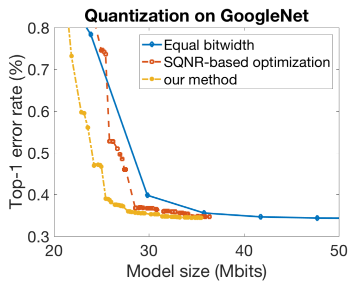
The results for quantization on all layers are shown in Supplementary Material. For Alexnet and VGG-16 model, our method achieves $30-40\%$ smaller model size with the same accuracy degradation, while for GoogleNet and Resnet-50, our method achieves $15-20\%$ smaller model size with the same accuracy degradation. These results indicate that our proposed quantization method works better for models with more diverse layer size and structures, like Alexnet and VGG.



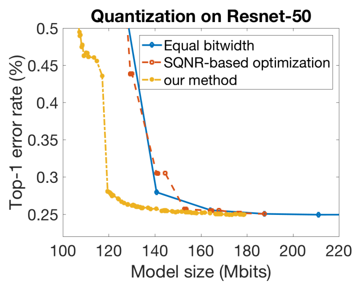
(a)



(b)



(c)



(d)

Figure 6: Model size after quantization, v.s. accuracy. To compare with SQNR-based method (Lin, Talathi, and Annapureddy 2016), only convolutional layers are quantized.

Conclusions

Parameter quantization is an important process to reduce the computation and memory costs of DNNs, and to deploy complex DNNs on mobile equipments. In this work, we propose an efficient approach to optimize layer-wise bit-width for parameter quantization. We propose a method that relates quantization to model accuracy, and theoretically analyses this method. We show that the proposed approach is more general and accurate than previous quantization optimization approaches. Experimental results show that our method outperforms previous works, and achieves $20 - 40\%$ higher compression rate than SQNR-based methods and equal bit-width quantization. For future works, we will consider combining our method with fine-tuning and other model compression methods to achieve better model compression results.

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Supplementary Material: Adaptive Quantization for Deep Neural Network

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Measuring the effect of quantization noise

Quantization noise

Assume that conducting quantization on a value is equivalent to adding noise to the value:

$$w_q = w + r_w \qquad \text{(2 revisited)}$$

Here w is the original value, $w \in W$, W is all weights in a layer. w_q is the quantized value, and r_w is the quantization noise. Assume we use a uniform quantizer, that the stepsize of the quantized interval is fixed. Then the quantization noise r_w follows a uniform distribution in range $(-\frac{B}{2}, \frac{B}{2})$, where B is the quantized interval. Based on this, r_w has zero mean, and the variance of the noise $var(r_w) = \frac{B^2}{12}$. Then we have $E(r_w^2) = var(r_w) + E(r_w)^2 = \frac{B^2}{12}$.

Follow the uniform quantization analysis in (You 2010), given weights $w \in W$ in a layer, $w \in (w_{min}, w_{max})$. If we quantize the weights by b bits, the total number of interval would be $M = 2^b$, and quantization interval would be $\frac{w_{min} - w_{max}}{2^b}$. If we consider $\mathbf{r}_w = (r_{w,1}, \cdots, r_{w,N_W})$ as the quantization noise on all weights in W, The expectation of noise square:

$$E(||\mathbf{r}_w||_2^2) = \sum_i^{N_W} E(r_w^2) = N_W \frac{(w_{min} - w_{max})^2}{12} \cdot 4^{-b}$$
$$= p'_w \cdot e^{-\alpha \cdot b} \qquad \text{(3 revisited)}$$

Where $p'_w = N_W \frac{(w_{min} - w_{max})^2}{12}$, N_W is the number of weights in W, and $\alpha = ln(4)$. Eq. (3) indicates that every time we reduce the bit-width by 1 bit, $E(\mathbf{r}_w^2)$ will increase by 4 times. This is equivalent to the quantization efficiency of 6dB/bit mentioned in (Lin, Talathi, and Annapureddy 2016).

The property of softmax classifier

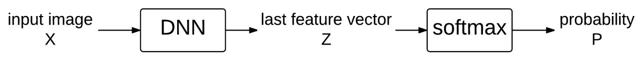


Figure 1: Simple DNN model architecture.

Similar to the analysis in (Pang, Du, and Zhu 2017), we analyse the property of softmax classifier.

A DNN classifier can be expressed as a mapping function $\mathcal{F}(X, W) : \mathscr{R}^d \to \mathscr{R}^L$, where $X \in \mathscr{R}^d$ is the input variable, W is the parameters, and L denotes the number of classes.

From Fig. 1, here we divide the DNN into two parts. In the first part, we have a mapping function $\mathcal{G}(X, W) : \mathscr{R}^d \to \mathscr{R}^L$, which maps input variables $X \in \mathscr{R}^d$ into the feature vectors $Z \in \mathscr{R}^L$ for the last layer of DNN. In the second part, we have the softmax function $softmax(\mathbf{z}) : \mathscr{R}^L \to \mathscr{R}^L$ as $softmax(z_i) = exp(z_i)/\sum_{i=1}^L exp(z_i)$, $i \in [L]$, where $[L] := 1, ..., L$.

The final classification result can be calculated by picking the maximum value of the softmax value: $\text{argmax}_i\, softmax(z_i)$, $i \in [L]$. Note that this is equivalent to picking the maximum value for feature vector $\mathbf{z}$: $\text{argmax}_i\, z_i$, $i \in [L]$. So we can see that the softmax classifier has a linear decision boundary in the feature vectors Z (Pang, Du, and Zhu 2017).

Proof of Lemma 1

Lemma 1. *Let $\gamma(\delta) = 5 + 4\ln(1/\delta)$. The following inequalities hold between the norm square of random noise $\|\boldsymbol{r}_Z\|_2^2$ and adversarial noise $\frac{(z_{(1)} - z_{(2)})^2}{2}$.*

$$\frac{\ln d}{d}\gamma(\delta)\|\boldsymbol{r}_Z\|_2^2 \geq \frac{(z_{(1)} - z_{(2)})^2}{2} \qquad (5)$$

with probability exceeding $1 - 2\delta$.

Proof. Based on Theorem 1 in (Fawzi, Moosavi-Dezfooli, and Frossard 2016), for an L-class classifier, the norm of a random noise to fool the classifier can be bounded from below by

$$d\|\mathbf{r}^*\|_2^2 \leq \beta(\delta')\|\mathbf{r}_Z\|_2^2, \qquad (24)$$

with a probability exceeding $1 - 2(L+1)\delta'$, where $\beta(\delta') = 1 + 2\sqrt{\ln(1/\delta')} + 2\ln(1/\delta')$. For the softmax layer, $L = d - 1$, therefore one can write

$$\mathbb{P}\left(d\|\mathbf{r}^*\|_2^2 \leq \beta(\delta')\|\mathbf{r}_Z\|_2^2\right) \geq 1 - 2d\delta'. \qquad (25)$$

Furthermore, $\|\mathbf{r}^*\|_2^2 = (z_{(1)} - z_{(2)})^2/2$, where $z_{(i)}$ is i^{th} largest element of z. Put $\delta' = \delta/d$, hence

$$\mathbb{P}\left(d\|\mathbf{r}^*\|_2^2 \leq \beta(\delta/d)\|\mathbf{r}_Z\|_2^2\right) \geq 1 - 2\delta. \qquad (26)$$

From the other hand,

$$\begin{aligned}\beta(\delta/d) &= 1 + 2\sqrt{\ln(d/\delta)} + 2\ln(d/\delta) \\ &\geq 1 + 4\ln(d/\delta) \\ &\geq \ln d\,(5 + 4\ln(1/\delta)) = \gamma(\delta)\ln d.\end{aligned} \tag{27}$$

Therefore,

$$\begin{aligned}\mathbb{P}\left(\frac{d}{\ln d}\|\mathbf{r}^*\|_2^2 \leq \gamma(\delta)\|\mathbf{r}_Z\|_2^2\right) &\geq \mathbb{P}\left(d\|\mathbf{r}^*\|_2^2 \leq \beta(\delta/d)\|\mathbf{r}_Z\|_2^2\right) \\ &\geq 1 - 2\delta,\end{aligned} \tag{28}$$

which concludes the proof. □

Relationship between accuracy and noise

The original quantization optimization problem:

$$\begin{aligned}&min\sum_{i=1}^{N} s_i \cdot b_i \\ &s.t.\ \ acc_{\mathcal{F}} - acc_{\mathcal{F}'} \leq \Delta_{acc}\end{aligned} \tag{1 revisited}$$

Lemma 1 states that if the norm of random noise is $o\left((z_{(1)} - z_{(2)})\sqrt{d/\ln d}\right)$, it does not change the classifier decision with high probability. In particular, from Lemma 1 (Eq. (28) in specific), the probability of misclassification can be expressed as:

$$\mathbb{P}(\|\mathbf{r}_Z\|_2^2 \leq \frac{d}{\gamma(\delta)\ln d}\frac{(z_{(1)} - z_{(2)})^2)}{2}) \leq 2\delta \tag{29}$$

Eq. (29) suggest that as we limit the noise to be less than $\frac{d}{\gamma(\delta)\ln d}\frac{(z_{(1)}-z_{(2)})^2}{2}$, the probability of misclassification should be less than 2δ.

Based on Lemma 1 and Eq. (29), we formulate the relationship between the noise and model accuracy. Assume that we have a model $\mathcal{F}$ with accuracy $acc_{\mathcal{F}}$. After adding random noise $\mathbf{r}_Z$ on the last feature map Z, the model accuracy drops Δ_{acc}. If we assume that the accuracy degradation is caused by the noise $\mathbf{r}_Z$, we can see that the δ value in Eq. (29) is closely related to Δ_{acc}:

$$\begin{aligned}&2\delta \sim \frac{\Delta_{acc}}{acc_{\mathcal{F}}} \\ \Rightarrow&\gamma(\delta) \sim \gamma(\frac{\Delta_{acc}}{2acc_{\mathcal{F}}})\end{aligned} \tag{30}$$

If we have:

$$\theta(\Delta_{acc}) = \frac{d}{\gamma(\frac{\Delta_{acc}}{2acc_{\mathcal{F}}})\ln d} \tag{6 revisited}$$

From Eq. (29), we have:

$$\mathbb{P}(\|\mathbf{r}_Z\|_2^2 < \theta(\Delta_{acc})\frac{(z_{(1)} - z_{(2)})^2}{2}) \leq 2\delta \sim \frac{\Delta_{acc}}{acc_{\mathcal{F}}} \tag{31}$$

Eq. (31) indicates that by limiting noise r_z to be less than $\theta(\Delta_{acc})\frac{(z_{(1)}-z_{(2)})^2}{2}$, we can approximately assume that model accuracy drops less than Δ_{acc}. As $\gamma(\delta)$ is strictly decreasing, we can see that $\theta(\Delta_{acc})$ is strictly increasing w.r.t. Δ_{acc}. So as the model has higher accuracy degradation Δ_{acc}, the noise limitation also increase.

Based on Eq. (31), we have the relation between accuracy degradation and noise $\mathbf{r}_Z$ as:

$$\begin{aligned}&acc_{\mathcal{F}} - acc_{\mathcal{F}'} \leq \Delta_{acc} \Rightarrow \\ &\|\mathbf{r}_Z\|_2^2 < \theta(\Delta_{acc})\frac{(z_{(1)} - z_{(2)})^2}{2}\end{aligned} \tag{7 revisited}$$

Calculation of noise on convolutional layer

Given a convolutional layer input A with size $M \times N \times C$, conv kernel K with size $M_k \times N_k \times C \times D$, and stride size s, we have the output feature map Z with size $(M/s) \times (N/s) \times D$. Here M and N are the height and width of input, C is the number of channel of input. M_k and N_k are the height and width of conv kernel, D is the depth of output feature map.

The analysis on fully connected layers will be similar to the analysis on convolutional layers. It can be considered as a special case of convolutional layers when M, N, M_k, and N_k are equal to 1.

Based on the definition of convolutional operation, for a single value $z \in Z$, the value is calculated as:

$$\begin{aligned}z_{p,q,d} &= \sum_{m_k=1}^{M_k}\sum_{n_k=1}^{N_k}\sum_{c=1}^{C} a_{m_k+p\cdot s,n_k+q\cdot s,c} \cdot w_{m_k,n_k,c,d} + b_d \\ &= \sum_{i=1}^{M_k\cdot N_k\cdot C} a_i \cdot w_{i,d} + b_d\end{aligned} \tag{32}$$

where w is the weight, and b is the bias. $p \in \{1, ..., M/s\}$ and $q \in \{1, ..., N/s\}$.

As we consider noise on both input feature maps and weights, the Eq. (32) will become:

$$\begin{aligned}z_{p,q,d} = &(\sum_{i=1}^{M_k\cdot N_k\cdot C} a_i \cdot w_{i,d} + b_d) \\ &+ (\sum_{i=1}^{M_k\cdot N_k\cdot C} (a_i \cdot r_{w_{i,d}} + r_{a_i} \cdot w_{i,d} + r_{a_i} \cdot r_{w_{i,d}}) + r_{b_d})\end{aligned} \tag{33}$$

Then the noise term of $z_{p,q,d}$ can be expressed as:

$$\begin{aligned}r_{z_{p,q,d}} &= \sum_{i=1}^{M_k\cdot N_k\cdot C} (a_i \cdot r_{w_{i,d}} + r_{a_i} \cdot w_{i,d} + r_{a_i} \cdot r_{w_{i,d}}) + r_{b_d} \\ &\approx \sum_{i=1}^{M_k\cdot N_k\cdot C} (a_i \cdot r_{w_{i,d}} + r_{a_i} \cdot w_{i,d})\end{aligned} \tag{17 revisited}$$

Note that the term r_{b_d} can be ignored under the assumption that w and b have same bit-width quantization. The term $r_{a_i} \cdot r_{w_{i,d}}$ can be ignored under the assumption that $||r_a||_2 \ll ||a||_2$ and $||r_w||_2 \ll ||w||_2$.

Additivity of $||\mathbf{r}_Z||_2^2$

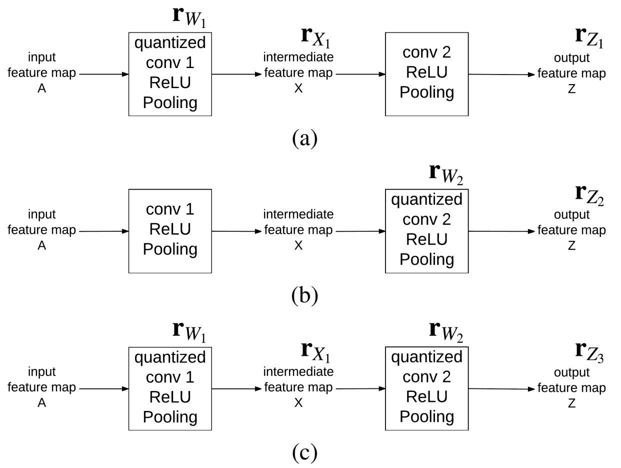


Figure 2: Effect of adding noise to multiple layers.

Fig. 2 shows a 2-layer module inside a DNN model. Given input feature map A, after the first conv layer, an intermediate feature map X is generated, then after the second conv layer, output feature map Z is generated.

In Fig. 2(a), the weight of the first conv layer W_1 is quantized, result in the quantization noise $\mathbf{r}_{W_1}$ on weight W_1, then the noise passes to feature map X with noise $\mathbf{r}_{X_1}$, and feature map Z with $\mathbf{r}_{Z_1}$.

In Fig. 2(b), the weight of the second conv layer W_2 is quantized, result in the quantization noise $\mathbf{r}_{W_2}$ on weight W_2, then the noise passes to feature map Z with $\mathbf{r}_{Z_2}$.

In Fig. 2(c), we quantize the weight of both layer W_1 and W_2, with quantization noise $\mathbf{r}_{W_1}$ and $\mathbf{r}_{W_2}$, the noise passes to Z with noise $\mathbf{r}_{Z_3}$. Based on the discussion in previous section, $\mathbf{r}_{Z_3} \approx \mathbf{r}_{Z_1} + \mathbf{r}_{Z_2}$. Given a particular $\mathbf{z} \in Z$, we have $\mathbf{z} = (z_1, \cdots, z_L)$, where $\mathbf{z} \in \mathbb{R}^L$. Then $\mathbf{r}_{\mathbf{z}_1} = (r_{z_1,1}, \cdots, r_{z_1,L})$, $\mathbf{r}_{\mathbf{z}_2} = (r_{z_2,1}, \cdots, r_{z_2,L})$, and $\mathbf{r}_{\mathbf{z}_3} \approx (r_{z_1,1} + r_{z_2,1}, \cdots, r_{z_1,L} + r_{z_2,L})$. $||\mathbf{r}_{\mathbf{z}_3}||_2^2$ can be calculated as:

$$\begin{aligned} ||\mathbf{r}_{\mathbf{z}_3}||_2^2 &= \sum_{i=1}^{L} r_{z_3,i}^2 \\ &\approx \sum_{i=1}^{L} (r_{z_1,i} + r_{z_2,i})^2 \\ &= \sum_{i=1}^{L} r_{z_1,i}^2 + \sum_{i=1}^{L} r_{z_2,i}^2 + 2 \cdot \sum_{i=1}^{L} r_{z_1,i} \cdot r_{z_2,i} \\ &= ||\mathbf{r}_{\mathbf{z}_1}||_2^2 + ||\mathbf{r}_{\mathbf{z}_2}||_2^2 + 2 \cdot \sum_{i=1}^{L} r_{z_1,i} \cdot r_{z_2,i} \\ &\doteq ||\mathbf{r}_{\mathbf{z}_1}||_2^2 + ||\mathbf{r}_{\mathbf{z}_2}||_2^2 \end{aligned} \tag{18 revisited}$$

The last equality holds under the assumption that $\mathbf{r}_{\mathbf{z}_1}$ and $\mathbf{r}_{\mathbf{z}_2}$ are independent. Which is reasonable in our case, as $\mathbf{r}_{\mathbf{z}_1}$ and $\mathbf{r}_{\mathbf{z}_2}$ are caused by $\mathbf{r}_{W_1}$ and $\mathbf{r}_{W_2}$ which are two independent quantization noise.

Layer-wise bit-width optimization

Solving the optimization problem

Consider the optimization problem:

$$\begin{aligned} &min \sum_{i=1}^{N} s_i \cdot b_i \\ &s.t. \sum_{i=1}^{N} \frac{p_i}{t_i} \cdot e^{-\alpha \cdot b_i} \leq C \end{aligned} \tag{21 revisited}$$

The form can be written as a dual problem:

$$\begin{aligned} &min \sum_{i=1}^{N} \frac{p_i}{t_i} \cdot e^{-\alpha \cdot b_i} \\ &s.t. \sum_{i=1}^{N} s_i \cdot b_i = C' \end{aligned} \tag{34}$$

We convert this to Lagrange function:

$$min \sum_{i=1}^{N} \frac{p_i}{t_i} \cdot e^{-\alpha \cdot b_i} - \lambda \cdot \sum_{i=1}^{N} s_i \cdot b_i \tag{35}$$

Using KKT conditions, we have that the optimal value can be reached when:

$$\frac{p_1 \cdot e^{-\alpha \cdot b_1}}{t_1 \cdot s_1} = \frac{p_2 \cdot e^{-\alpha \cdot b_2}}{t_2 \cdot s_2} = \cdots = \frac{p_N \cdot e^{-\alpha \cdot b_N}}{t_N \cdot s_N} \tag{22 revisited}$$

Algorithm to calculate optimal bit-width

The algorithms to calculate t_i, p_i, and b_i is shown in Alg. 1, 2, and 3, respectively.

Algorithm 1 Calculate t_i

```
1: procedure CAL_T
       Input:
       Dataset $\mathcal{D}$; label $\mathcal{L}$; accuracy degradation $\Delta_{acc}$;
       model $\mathcal{F}$; model before softmax: $\mathcal{G}$;
       weights $W_i \in W$ in layer $i$;
       $mean_{\mathbf{r}^*} \leftarrow \frac{1}{|\mathcal{D}|}\sum_{\mathbf{x}\in\mathcal{D}} \frac{(z_{(1)}-z_{(2)})^2}{2}$;
       $acc_{\mathcal{F}} \leftarrow \frac{|\mathcal{F}(\mathcal{D},W)=\mathcal{L}|}{|\mathcal{D}|}$.
       Output:
       $t_i, i = \{1, \cdots, N\}$
2:     for layer $i = 1 \to N$ do
3:         $\mathbf{r}'_{W_i} = \mathcal{U}(-0.5, 0.5)$
4:         $k = k_{min} = 10^{-5}, k_{max} = 10^3$
5:         while $acc_{\mathcal{F}} - acc_{\mathcal{F}'} \neq \Delta_{acc}$ do
6:             if $acc_{\mathcal{F}} - acc_{\mathcal{F}'} < \Delta_{acc}$ then $k_{min} \leftarrow k$
7:             else $k_{max} \leftarrow k$
8:             $k = \sqrt{k_{min} \cdot k_{max}}$
9:             $\mathbf{r}_{W_i} \leftarrow k \cdot \mathbf{r}'_{W_i}$
10:            $acc_{\mathcal{F}'} \leftarrow \frac{|\mathcal{F}(\mathcal{D},W_i+\mathbf{r}_{W_i})=\mathcal{L}|}{|\mathcal{D}|}$
11:        $\mathbf{r}_{\mathbf{z}_i} = \mathcal{G}(\mathbf{x}, W_i) - \mathcal{G}(\mathbf{x}, W_i + \mathbf{r}_{W_i})$
12:        $mean_{\mathbf{r}_{\mathbf{z}_i}} = \frac{1}{|\mathcal{D}|}\sum_{\mathbf{x}\in\mathcal{D}} ||\mathbf{r}_{\mathbf{z}_i}||_2^2$
13:        $t_i = \frac{mean_{\mathbf{r}_{\mathbf{z}_i}}}{mean_{\mathbf{r}^*}}$
       return $t_i, i = \{1, \cdots, N\}$
```

Algorithm 2 Calculate p_i

```
1: procedure CAL_P
       Input:
       Dataset $\mathcal{D}$; $\alpha = ln(4)$;
       model before softmax: $\mathcal{G}$;
       weights $W_i \in W$ in layer $i$;
       quantization bit-width $b_i$ for layer $i$.
       Output:
       $p_i, i = \{1, \cdots, N\}$
2:     for layer $i = 1 \to N$ do
3:         $W_{i,q} = quantize(W_i, b_i)$
4:         $\mathbf{r}_{\mathbf{z}_i} = \mathcal{G}(\mathbf{x}, W_i) - \mathcal{G}(\mathbf{x}, W_{i,q})$
5:         $mean_{\mathbf{r}_{\mathbf{z}_i}} = \frac{1}{|\mathcal{D}|}\sum_{\mathbf{x}\in\mathcal{D}} ||\mathbf{r}_{\mathbf{z}_i}||_2^2$
6:         $p_i = \frac{mean_{\mathbf{r}_{\mathbf{z}_i}}}{e^{-\alpha \cdot b_i}}$
       return $p_i, i = \{1, \cdots, N\}$
```

Algorithm 3 Calculate b_i

```
1: procedure CAL_B
       Input:
       Parameter $t_i, p_i, i = \{1, \cdots, N\}$; $\alpha = ln(4)$;
       size $s_i$ of layer $i$;
       quantization bit-width $b_1$ for layer 1.
       Output:
       $b_i, i = \{2, \cdots, N\}$
2:     for layer $i = 2 \to N$ do
3:         $b_i = b_1 + \frac{1}{\alpha} \cdot ln(\frac{p_i \cdot t_1 \cdot s_1}{p_1 \cdot t_i \cdot s_i})$
       return $b_i, i = \{2, \cdots, N\}$
```

Comparison with SQNR-based approach

From our work, the optimal quantization bit-width for each layer is reached when Eq. (22) is fulfilled.

From the paper we have that $||r_{Z_i}||_2^2 = p_i \cdot e^{-\alpha \cdot b_i}$ is the noise norm on last feature map Z when we apply b_i-bit quantization on layer i, s_i is the size of layer i, and t_i is the robustness parameter for layer i.

Now we focus on the optimization result for SQNR-based method (Lin, Talathi, and Annapureddy 2016). For the SQNR-based method, the optimal quantization bit-width is achieved when

$$\beta_i - \beta_j = \frac{10log(\rho_j/\rho_i)}{\kappa}, \tag{36}$$

where β_i is the bit-width for layer i, ρ_i is the number of parameter in layer i. κ is the quantization efficiency, which corresponds to the α parameter in our work.

If we rewrite Eq. (36) using our notation, the optimization result for SQNR-based method becomes

$$\begin{aligned} b_i - b_j &= \frac{10log(s_j/s_i)}{\alpha'} \\ \Rightarrow \frac{\alpha'}{10}b_i - \frac{\alpha'}{10}b_j &= log(\frac{s_j}{s_i}) \\ \Rightarrow \frac{e^{\frac{\alpha'}{10}b_i}}{e^{\frac{\alpha'}{10}b_j}} &= \frac{s_j}{s_i} \\ \Rightarrow \frac{e^{-\frac{\alpha'}{10}b_i}}{s_i} &= \frac{e^{-\frac{\alpha'}{10}b_j}}{s_j} \end{aligned} \tag{37}$$

Let $\alpha = \alpha'/10$, we have the optimization result for SQNR-based method as

$$\frac{e^{-\alpha \cdot b_1}}{s_1} = \frac{e^{-\alpha \cdot b_2}}{s_2} = \cdots = \frac{e^{-\alpha \cdot b_N}}{s_N} \tag{23 revisited}$$

Note that compared with our result in Eq. (22), the parameters p_i and t_i are missing. This is consistent with the assumption of the SQNR-based approach, that if two layers have the same bit-width for quantization, they would have the same SQNR value, thus the affection on accuracy are equal. This makes the SQNR-based approach a special case of our approach, when all layers in the DNN model have the equal affection on model accuracy under the same bit-width.

Experimental results

Fig. 8 shows the quantization results using our method, compared to equal quantization. For Alexnet and VGG-16 model, our method achieves 40% less model size with same accuracy degradation, while for GoogleNet and Resnet-50, our method achieves 15 – 20% less model size with same accuracy degradation. These results indicates that our proposed quantization method works better for models with more diverse layer-wise size and structures, like Alexnet and VGG.

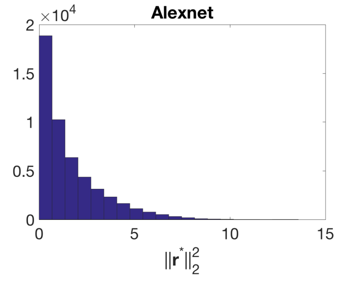
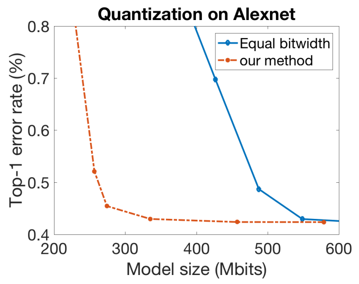
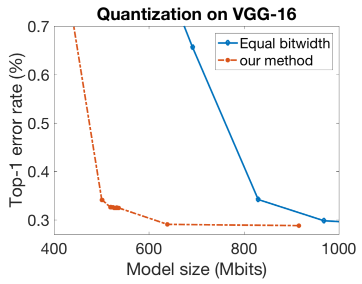


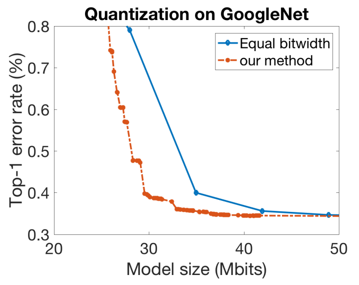
Figure 7: Histogram of $||\mathbf{r}^*||_2^2$ value for Alexnet on Imagenet validation set.



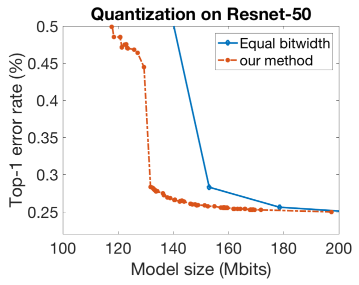
(a)



(b)



(c)



(d)

Figure 8: Model size after quantization, v.s. model accuracy. All layers are quantized.