

Lecture XII - Passenger Flow Control in Urban Rail

Applied Optimization with Julia

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Introduction

FIFA World Cup 2022 in Qatar



Public transport

- FIFA Worldcup 2022 took place in a **small region**
- The capacity of the metro system was **finite**
- More than **1 million tourists** where expected
- Metro usage was **free for all ticket holders**
- Transport methods were expected to be **overloaded**

Question: What could become an issue?

Crowd Disasters

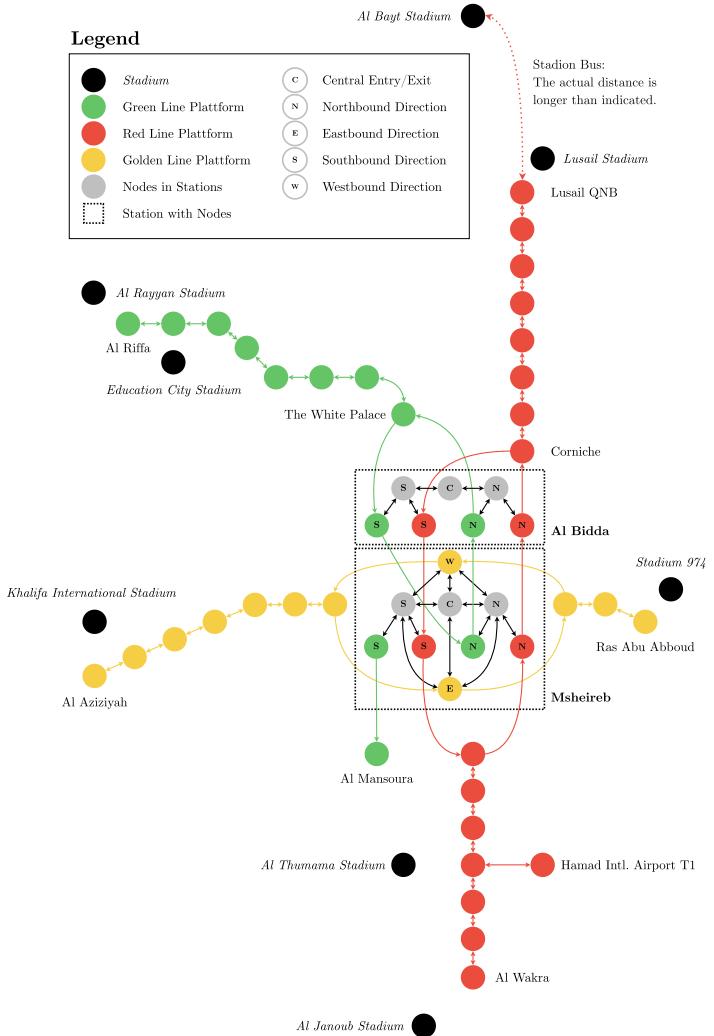


Potential locations

Question: Where could crowd disasters happen?

- At event venues and in waiting areas
- At intersections of multiple pedestrian flows
- In narrow passages and entrances
- On crowded metro platforms and transfer stations
- At ticket/turnstile bottlenecks and emergency exits

Metrosystem of Doha



Main Bottleneck

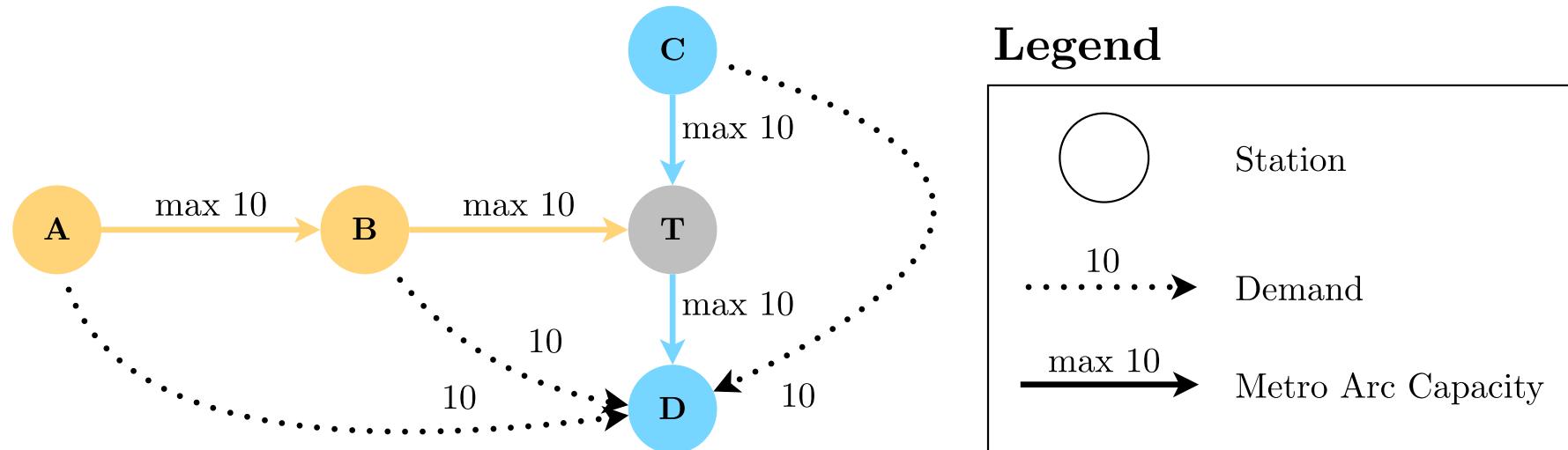
Question: What could be a potential bottleneck?

- Red Line has twice the capacity of the Green and Gold Line
- People use the metro to get to the event venues

Question: What could be the issue?

- Imagine a match at a stadium along the Gold line
- People from Red and Green line will try to get there
- This will cause a **massive crowd at transfer stations**

Visualization

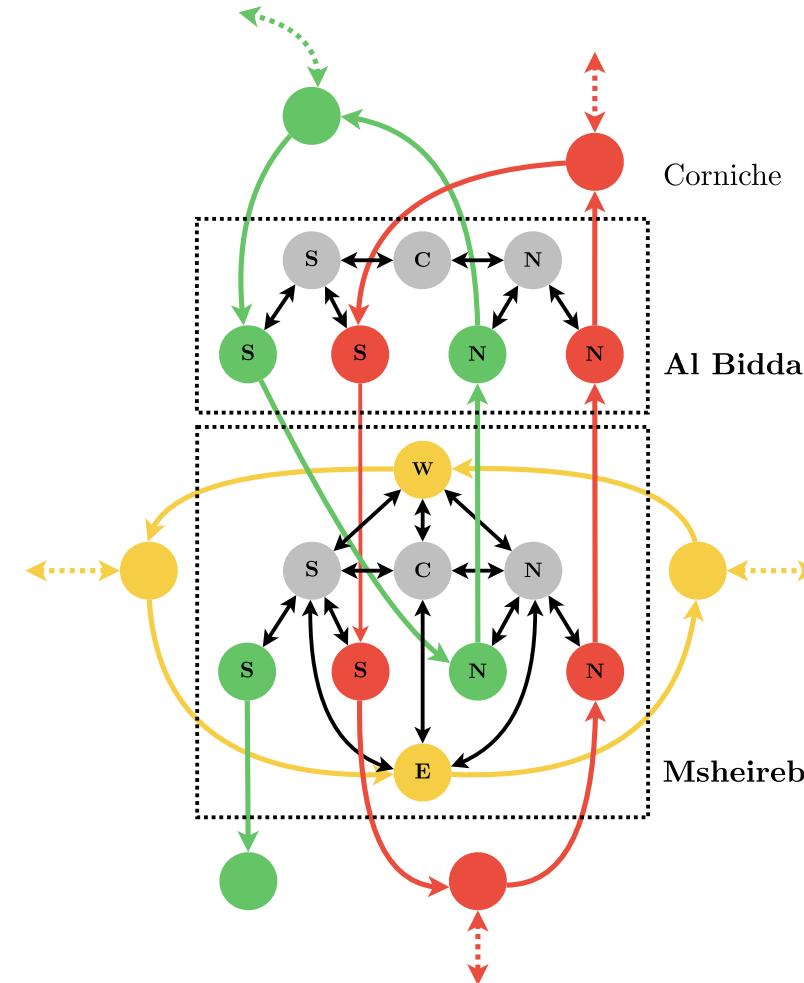


- 10 people want to start per minute at **each station**
- Everybody wants to get to **station D**

Closer look at the Metro System

Legend

●	Green Line Plattform
●	Red Line Plattform
●	Golden Line Plattform
	Nodes in Stations
	Expanded Station
	Central Entry/Exit
	Northbound Direction
	Eastbound Direction
	Southbound Direction
	Westbound Direction



Capacities inside the stations

- Capacities **inside the stations** are limited as well
- These include the escalators, stairs, and elevators
- But also the platforms and the ticket gates



Warning

This can lead to **overcrowding and potential crowd disasters!**

General Issues

Question: What could also be an issue?

- Often unknown **how many people** exactly will participate
- Gathering data is **extremely importand and difficult**
- Crowd behavior can be **unpredictable and dynamic**
- Weather conditions may affect **transportation preferences**
- Cultural factors influence **crowd movement patterns**
- Emergency situations require **flexible contingency plans**

Let's take a
closer look at the
problem structure!

Problem Structure

Objective

Question: What could be the objectives of the authorities?

- A **safe and successful event** as host country
- Good **publicity and a positive recognition** worldwide
- **Satisfied visitors** that enjoyed their time



Note: We can model none of the above directly!

But we can assist in the estimation of transport demands and the design of operational plans to ensure public **safety** and a **smooth transportation** through the city.

Underlying Problem

Question: Can we just start modeling?

- First, we need to understand the **movement patterns**
- Event is **unprecedented**, movement patterns are unknown
- Multiple **concurrent events** affect flow patterns



Tip: We need to estimate the data first!

This is a **huge challenge**, but we can use **simulation** to estimate the data.

Simulation

- Based on **publicly available data** from the area
- Simulates **all individuals** participating at the event
- Includes **all transportation infrastructure**
- **Individual mode choice** based on a choice model



Tip: Julia is a great tool for this!

Simulations was build in Julia. With 1,000,000 individuals walking, using cars, busses, and the metro, a day took **less than 5 minutes**.

Results of the Simulation



- Detailed movement patterns throughout the region
- Potential sections at risk in the transport infrastructure
- Potential capacity overloads at event locations

But it is still
based on a lot
of assumptions!

Main risk: Metro



Idea

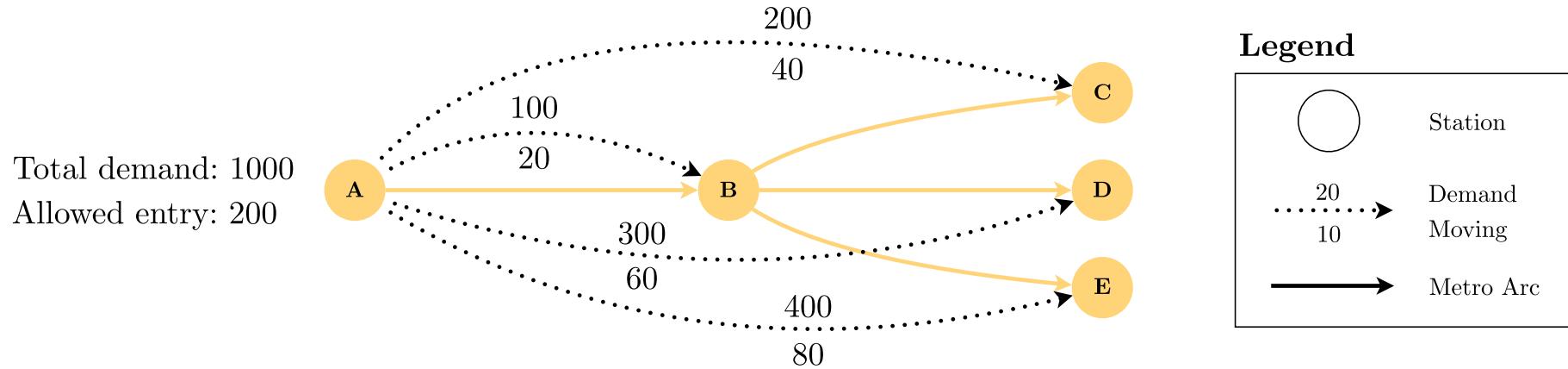
Question: What can we do to prevent crowd disasters?

- Regulate the **inflow** at each individual station
- Ensure utilized capacity is **always within bounds**

Question: What could we try to model?

- **Minimize the queues** outside of the metro stations
- Based on the **allowed inflow** and origin-destination data
- Adhering to the **capacity constraints**

Difficulty

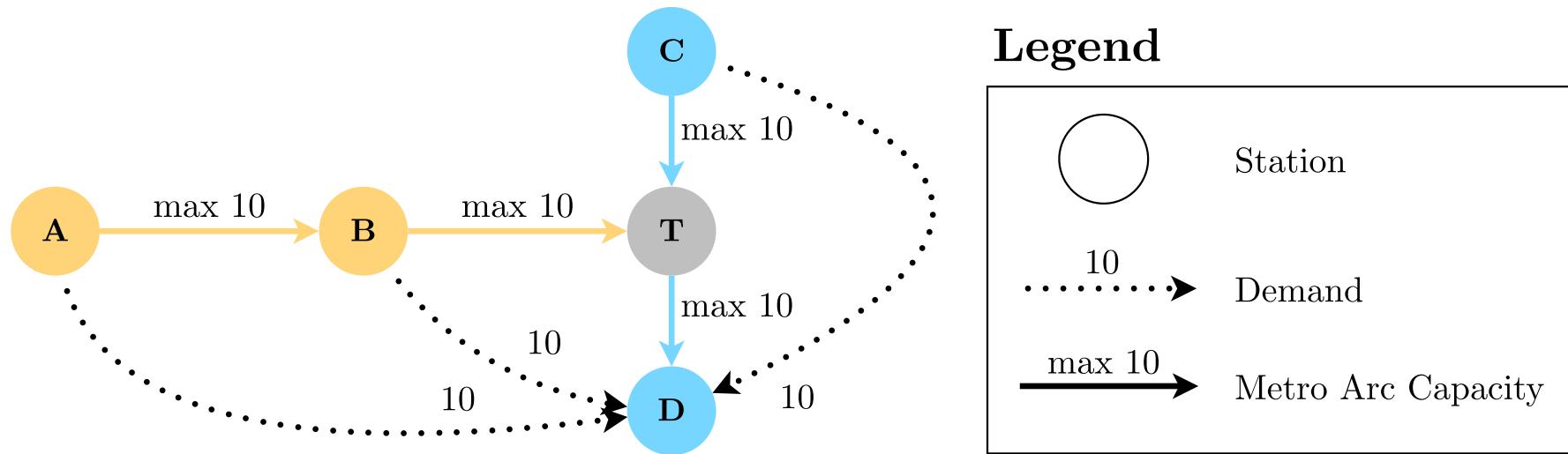


- Movement patterns have different origin-destination pairs
- Regulating the inflow does not affect the destination

Model Formulation

Graph Sets?

Question: What could be the sets for the graph?



Graph Sets?

- \mathcal{G} - Connected digraph of the metro network $\mathcal{G} = (\mathcal{O}, \mathcal{E})$
- \mathcal{O} - Set of metro stations, indexed by o
- \mathcal{E} - Set of directed arcs between connected stations



Note: These are the easier sets.

These simply help us to represent the entire metro system as a graph with different nodes \mathcal{O} and arcs \mathcal{E} between connected stations.

Time Sets?

Question: What could be the time sets?

- \mathcal{T} - Set of minutes in the time horizon, indexed by t
- \mathcal{P} - Set of periods in the observed time horizon, where
 $p \in \{p_1, p_2, \dots, p_n\}$



Note: Number of periods

We further define n as the number of periods in observed time horizon.

Periods

Question: Why do we add periods here?

- Staff needs **clear**, consistent instructions
- Frequent changes in flow **increase risk of errors**
- **Easier** to manage and communicate for planners



Note: Period Length

We define the period length as the length of the period in minutes measured as m minutes, which is the **same for all periods**.

Mapping Minutes to Periods

- We can define an **additional set** with their relation
- It specifies the relation of **periods p to minutes t .**

$$I_p = \{t \in \mathcal{T} | (p - 1) \times m + 1 \leq t \leq p \times m\} \quad \forall p \in \mathcal{P}$$

Question: Can anybody explain it?

- I_p is the set of minutes t that belong to period p
- Minutes are **not overlapping**, but they are **continuous**

Parameters?

Question: What could be possible parameters?

- $q_{o,d,p}$ - Demand from station o to d with $o, d \in \mathcal{O}$ in p
- d_e - Travel time (min) of the arcs $e \in \mathcal{E}$
- c_e - Max. allowed arc entry rate e per minute with $e \in \mathcal{E}$
- c_o^{\min} - Min. station entry rate o per minute with $o \in \mathcal{O}$
- c_o^{\max} - Max. station entry rate o per minute with $o \in \mathcal{O}$
- α - Maximal allowed arc utilization ($0 < \alpha < 1$)

Metro Movement

Question: How do people move inside the metro?

- Simple network: assume people use the **shortest path**
- Use **Djikstra's algorithm** to compute it
- From each station to all other stations



Tip: We can save the results in a new set $\mathcal{C}_{o,d}$

This will help us to model the movement inside the metro.

Shortest Paths (SP)

- $\mathcal{C}_{o,d}$ - Set of arcs $e \in \mathcal{E}$ on the SP from $o, d \in \mathcal{O}$

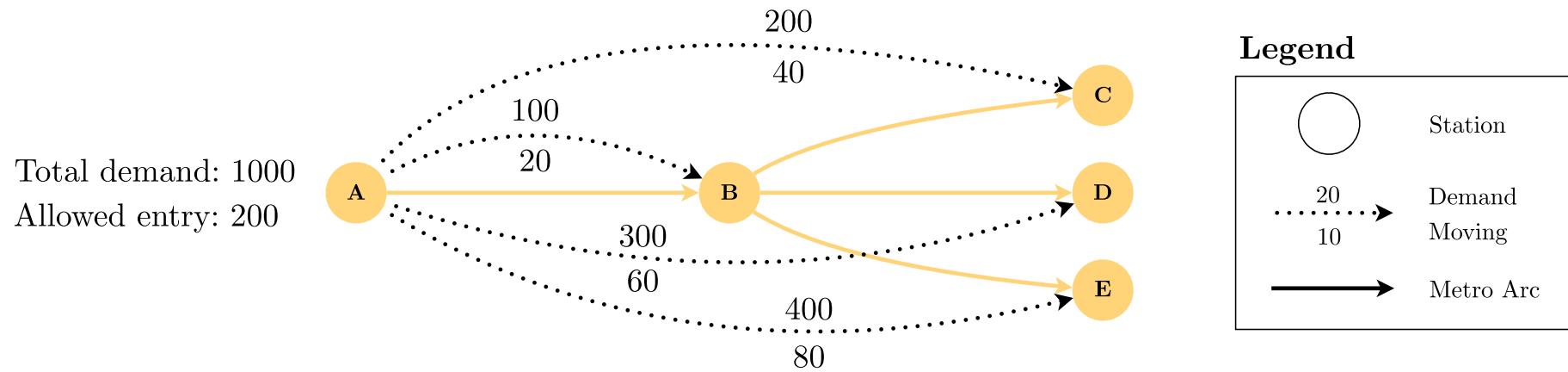
 Note

Now we compute the **travel time on the shortest paths**. One parameter for the SP from **station to station** and one for the SP from **station to arc**.

- $d_{o,d}$ - travel time (min) on SP from $o \in \mathcal{O}$ to $d \in \mathcal{O}$
- $d_{o,e}$ - travel time (min) on SP from $o \in \mathcal{O}$ to $e \in \mathcal{E}$

People Spreading

Question: How do people spread?



Ratio of Origin-Destination

- We **cannot control** the destination of the passengers
- Thus we assume that people spread **based on destination**

$$\frac{q_{o,d,p}}{\sum_{d \in \mathcal{O}} q_{o,d,p}} \quad \forall o, d \in \mathcal{O}, p \in \mathcal{P}$$



Note

Based on the ratio of the different destinations d to the total queue for each station $o \in \mathcal{O}$ in each period $p \in \mathcal{P}$.

Variables and Objective

Decision Variable?

! Our goal is to:

Regulate the **inflow** at **each individual station** to **minimize the queues** outside the metro stations based on the allowed inflow while **adhering to the capacity constraints**.



Tip

There is **one queue for all passenger flow directions**, as managing multiple queues would be too complex for the planners!

Decision Variable

 We need the following sets:

- All the metro stations, $o \in \mathcal{O}$
- All periods under observation, $p \in \mathcal{P}$

Question: What could be our decision variable?

- $X_{o,p}$ - Allowed inflow (per minute) at station o in period p

Objective Function?

! Our main objective is to:

Regulate the **inflow** at each individual station to minimize the queues outside the metro stations based on the allowed inflow while **adhering to the capacity constraints**.

Question: How again are queues minimized?

- By the allowed inflow $X_{o,p}$ subtracted from the queue

Objective Function

 We need the following parameters and variables:

- $q_{o,d,p}$ - People queued to travel from station o to d with $o, d \in \mathcal{O}$ in period p
- $X_{o,p}$ - Allowed inflow (per minute) at station o in period p

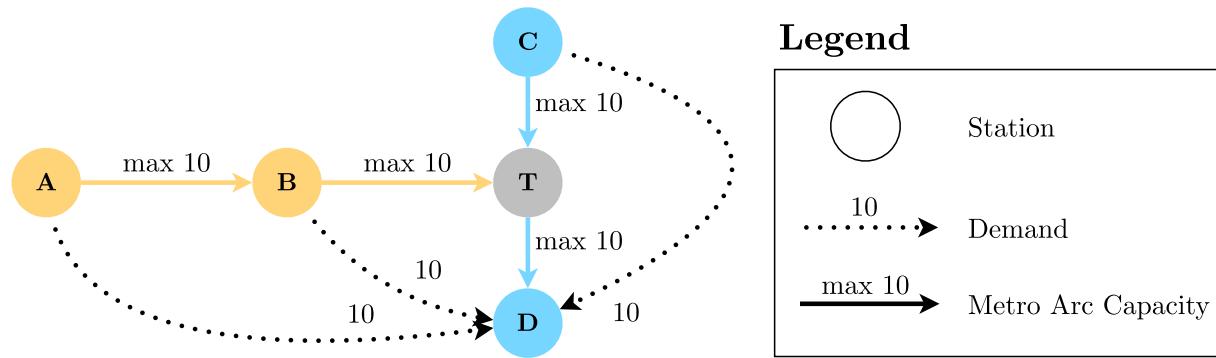
Question: What could be our objective function?

$$\text{minimize} \quad \sum_{o \in \mathcal{O}} \sum_{p \in \mathcal{P}} \left(\sum_{d \in \mathcal{O}} q_{o,d,p} - m \times X_{o,p} \right)$$

Constraints

Necessary Constraints

Question: What constraints do we need?



- The capacity of each arc is **not exceeded**
- Do not dispatch more people than are queued
- Do not dispatch less than the minimum allowed inflow

Things are going to
get a little
complicated now!

Central Question

Question: When do people flowing into the metro system change the arcs?

- People enter metro station at o with destination d
- They will lead to a usage of an arc e on their SP
- This usage depends on their path and the travel times



Tip: You'll likely know what we need now!

We can add a new set $\mathcal{R}_{e,t}$ to help us with this.

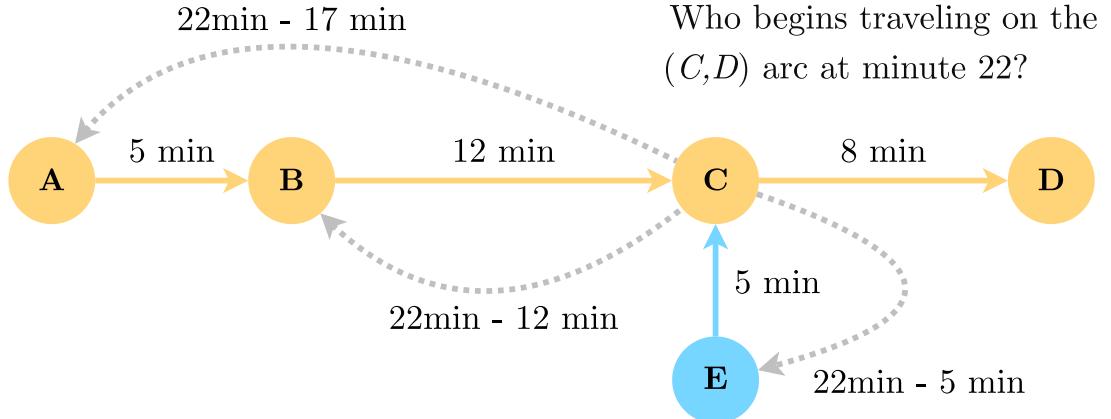
Set of Time-Delays

$$\begin{aligned} & (o, d) | o, d \in \mathcal{O}, \\ & q_{o,d,p} > 0, \\ \mathcal{R}_{e,t} = & \{(o, d, p) \mid e \in \mathcal{C}_{o,d}, \quad \forall e \in \mathcal{E}, t \in \mathcal{T} \\ & t - d_{o,e} \in I_p, \\ & p \in \mathcal{P}\} \end{aligned}$$

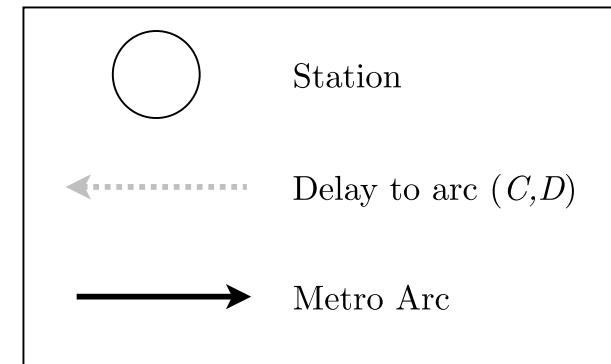
Question: Who can explain this set?

The set $\mathcal{R}_{e,t}$ contains all combinations (o, d, p) which trigger a capacity utilization of arc e in period t .

Small Example



Legend



The set contains all possible o-d pairs and periods, that result in passengers starting at arc (C, D) at minute 22. For $m=2$, it would be:

$$\{((A,D),3), ((B,D),5), ((C,D),11), ((E,D),9)\}.$$

Essentially, we can use this set to compute the utilization of an arc at each minute based on the inflow level at the different stations and periods.

Ensure Capacity Utilization?

! The goal of this constraint is to:

Ensure that the capacity of each arc is not exceeded at any minute.

i We need the following variable, parameter and set:

- $q_{o,d,p}$ - people waiting to travel from station $o \in \mathcal{O}$ to station $d \in \mathcal{O}$ in period p
- c_e - people max. allowed to enter arc e per minute with $e \in \mathcal{E}$
- $\mathcal{R}_{e,t}$ - mapping of station entries to arc e in time t with $e \in \mathcal{E}$ and $t \in \mathcal{T}$
- α - maximal allowed arc utilization ($0 < \alpha < 1$)
- $X_{o,p}$ - the allowed inflow per minute at metro station o in the period p

Ensure Capacity Utilization

Question: What could be the constraint?

$$\sum_{(o,d,p) \in \mathcal{R}_{e,t}} X_{o,p} \times \frac{q_{o,d,p}}{\sum_{f \in \mathcal{O}} q_{o,f,p}} \leq \alpha \times c_e \quad \forall e \in \mathcal{E}, t \in \mathcal{T}$$



Note

Here, we combine the inflow at each station based on the o-d proportion and let the people spread through the network, checking that no arc is over-utilized at any minute.

Dispatch Only Available People?

! The goal of this constraint is to:

Ensure that we do not dispatch more people than are queued and less than the minimum allowed inflow, preventing over- and under-dispatching.

i We need the following variables and parameters:

- $X_{o,p}$ - Allowed inflow (per minute) at station o in period p
- $q_{o,d,p}$ - People queued to travel from station o to d with $o, d \in \mathcal{O}$ in p
- m - period length in minutes

Dispatch Only Available People

Question: What could be the constraint?

$$\min\left\{\frac{\sum_{d \in \mathcal{O}} q_{o,d,p}}{m}, c_o^{\min}\right\} \leq X_{o,p} \leq \max\left\{\frac{\sum_{d \in \mathcal{O}} q_{o,d,p}}{m}, c_o^{\max}\right\} \quad \forall o \in \mathcal{O}, p \in \mathcal{P}$$

 Note

We need this constraint for **two reasons**:

1. We could also dispatch more people than there are as this would **minimize the objective value**.
2. We also need to ensure that we can dispatch at **least the minimum allowed inflow** or less if the queue is smaller than the minimum allowed inflow.

Metro Inflow Model

$$\text{minimize} \quad \sum_{o \in \mathcal{O}} \sum_{p \in \mathcal{P}} \left(\sum_{d \in \mathcal{O}} q_{o,d,p} - m \times X_{o,p} \right)$$

subject to:

$$\sum_{(o,d,p) \in \mathcal{R}_{e,t}} X_{o,p} \times \frac{q_{o,d,p}}{\sum_{f \in \mathcal{O}} q_{o,f,p}} \leq \alpha \times c_e \quad \forall e \in \mathcal{E}, t \in \mathcal{T}$$

$$\min\left\{\frac{\sum_{d \in \mathcal{O}} q_{o,d,p}}{m}, c_o^{\min}\right\} \leq X_{o,p} \leq \max\left\{\frac{\sum_{d \in \mathcal{O}} q_{o,d,p}}{m}, c_o^{\max}\right\} \quad \forall o \in \mathcal{O}, p \in \mathcal{P}$$

Model Characteristics

Characteristics

Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?

Model Assumptions

Questions: On model assumptions

- What assumptions have we made?
- What are likely issues that can arise if applied?
- Have we thought in detail about queues?
- Are shortest paths a feasible assumption?

Implementation and Impact

Metro Inflow Optimization

Question: Can this be applied?



Metro Inflow Problem

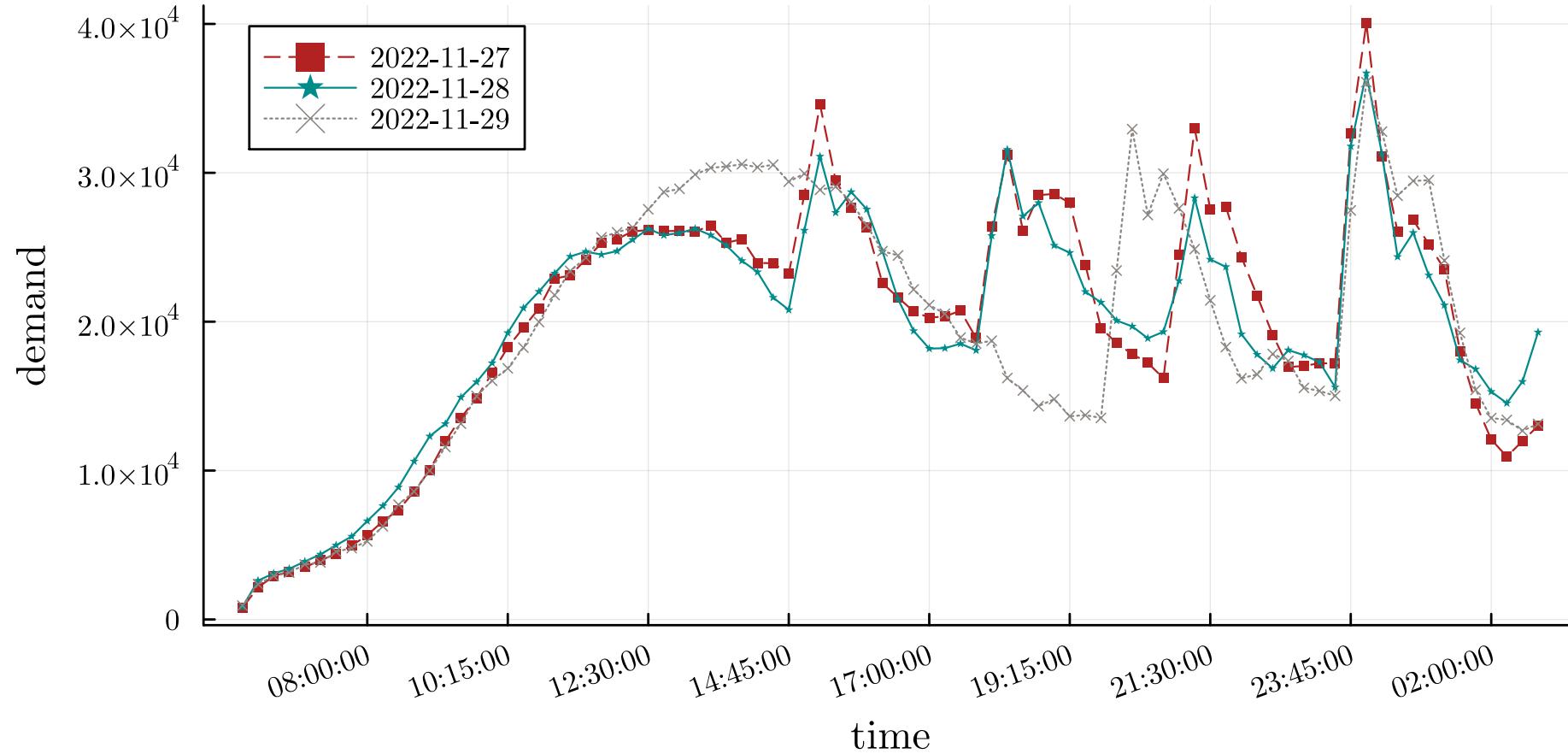
- Solved very fast **within seconds** for realistic problem sizes
- **But** we cannot plan or control the metro inflow
- Queues are **too simplified** with passengers disappearing

Question: Any ideas, how the current model could be improved or how it could be embedded into a heuristic?

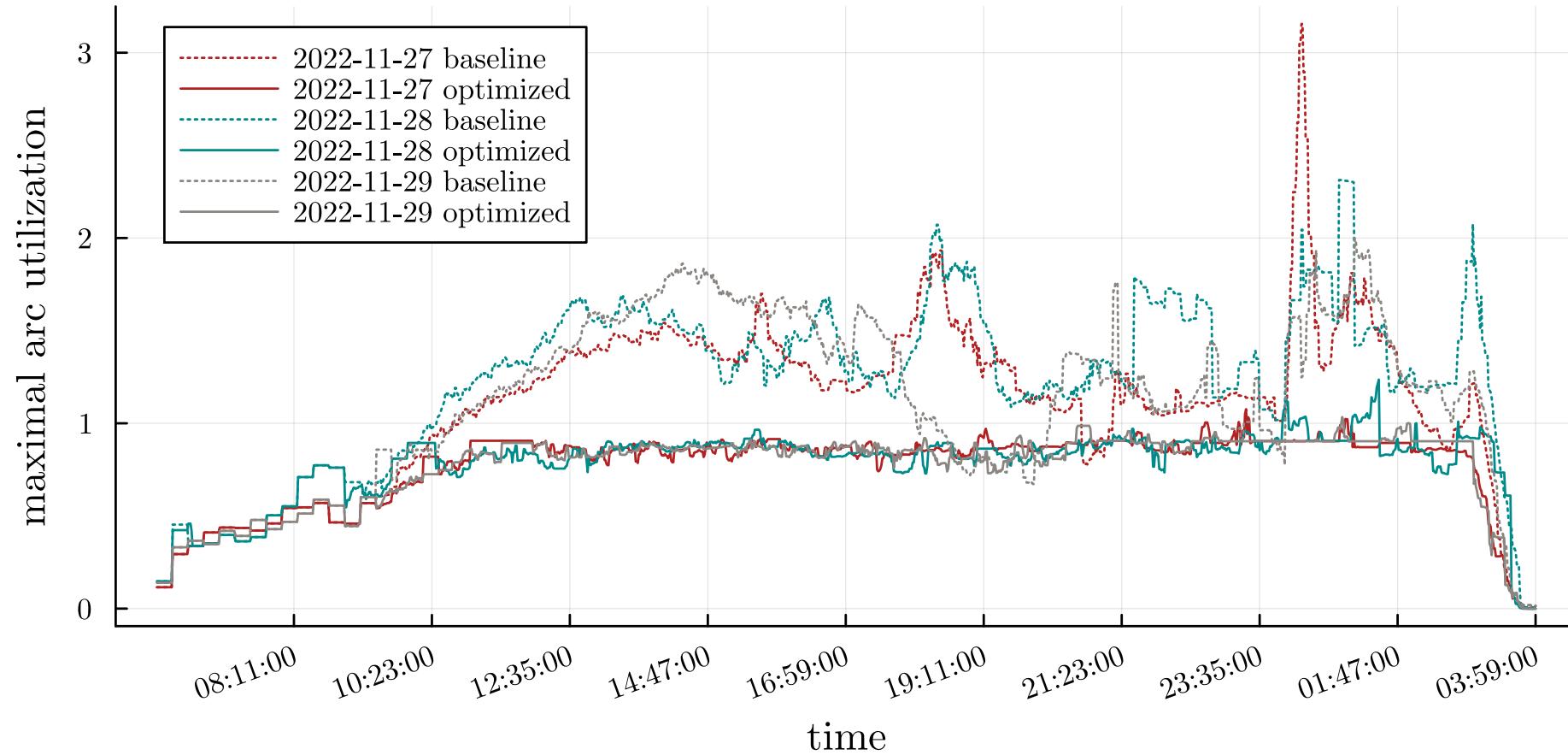
Heuristic: Step-Wise Optimization

- Solve the model for the time-horizon of a **few periods**
- Fix the inflow in the **current first period**
- Decrease capacity in the network **based on the inflow**
- Transfer **remaining queues** into the subsequent period
- **Solve the model again**
- **Repeat**, until the inflow is computed for all periods

Transport Demand



Utilization Analysis



Implementation

- Assumption of known destinations based is strong
- Movements seemed to follow our forecasts
- We did achieve our goal of **metro inflow control**
- Simulation was used to **estimate the inflows**

 Note

Few dangerous situations, especially at the FIFA Fan Fest, were **handled well** by the authorities.

Wrap Up

- Model can help to achieve a **good balance**
- Can be adapted easily to **any metro system worldwide**
- Especially interesting for **larger Asian cities**



And that's it for todays lecture!

We now have covered a metro inflow control problem based on a real-world application and are ready to start solving some new tasks in the upcoming tutorial.

Questions?

Literature

Literature I

For more interesting literature to learn more about Julia, take a look at the [literature list](#) of this course.