Lecture V - Production Planning in Breweries

Applied Optimization with Julia

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Introduction

Case Study

- Large brewery
- Brews and sells beverages
- Production planning by hand
- Planner has a lot of experience
- But will retire soon

Challenges

- Strong competition
- Customer demand is changing
- Craft beer gains popularity
- Variety of drinks is increasing
- Batch sizes are getting smaller

Different costs

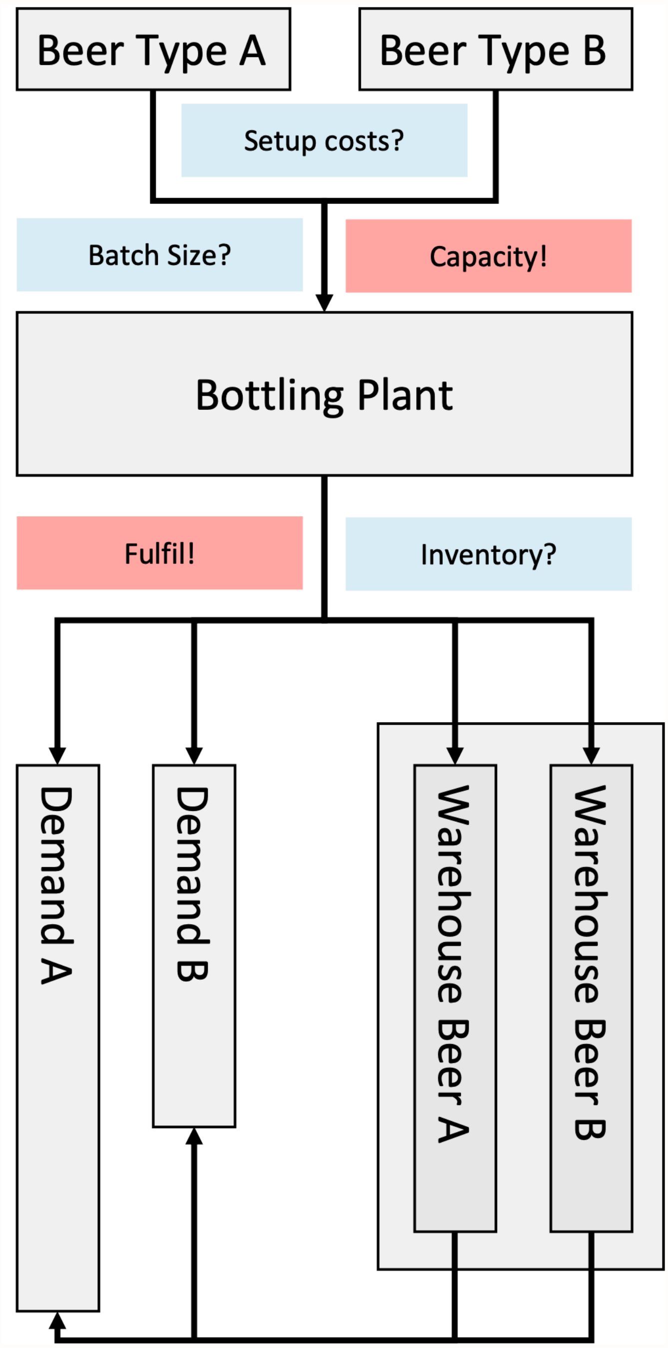
- Plant can fill multiple types
- Time depends on type and batch
- Changing type leads to set-up costs for preparation and cleaning
- Unsold beer bottles can be stored in a warehouse
- This leads to inventory costs

Where is the

challenge?

Problem Structure

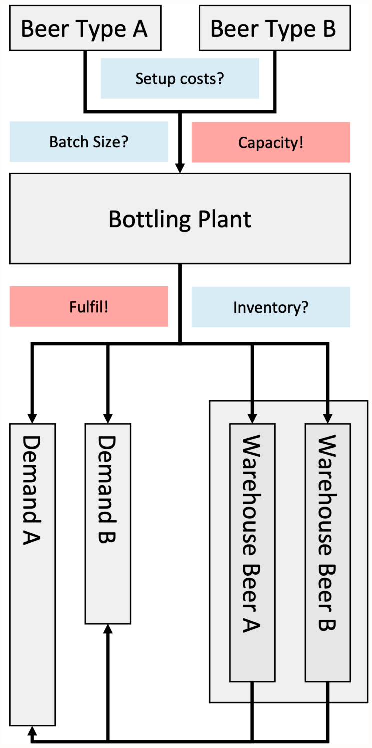
Objective



Question: What could be the objective?

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

Trade-Off



Question: What is the trade-off?

Larger batches require less setup cost per bottle, but increase the storage cost.

Available Sets

Question: What are sets again?

. . .

Sets are collections of objects.

. . .

Question: What could be the sets here?

. . .

- $\mathcal{I}$ - Set of beer types indexed by $i \in \{1, 2, ..., |\mathcal{I}|\}$
- $\mathcal{T}$ - Set of time periods indexed by $t \in \{1, 2, ..., |\mathcal{T}|\}$

Available Parameters

Question: What are possible parameters?

. . .

- a_t - Available time on the bottling plant in period $t \in \mathcal{T}$
- b_i - Time used for bottling one unit of beer type $i \in \mathcal{I}$
- g_i - Setup time for beer type $i \in \mathcal{I}$
- f_i - Setup cost of beer type $i \in \mathcal{I}$
- c_i - Inventory holding cost for one unit of beer type $i \in \mathcal{I}$

- $d_{i,t}$ - Demand of beer type $i \in \mathcal{I}$ in period $t \in \mathcal{T}$

Decision Variables?

i We have the following sets:

- Beer types indexed by $i \in \{1, 2, ..., |\mathcal{I}|\}$
- Time periods of the planning horizon indexed by $t \in \{1, 2, ..., |\mathcal{T}|\}$

...

! Our objective is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

...

Question: What could be our decision variable/s?

Decision Variables

- $W_{i,t}$ - Inventory of type $i \in \mathcal{I}$ at the end of $t \in \mathcal{T}$
- $Y_{i,t}$ - 1, if type $i \in \mathcal{I}$ is bottled in $t \in \mathcal{T}$, 0 otherwise
- $X_{i,t}$ - Batch size of type $i \in \mathcal{I}$ in $t \in \mathcal{T}$

Model Formulation

Objective Function?

! Our objective is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

...

Question: What could be our objective function?

...

i We need the following variables:

- $W_{i,t}$ - Inventory of type $i \in \mathcal{I}$ at the end of $t \in \mathcal{T}$
- $Y_{i,t}$ - 1, if type $i \in \mathcal{I}$ is bottled in $t \in \mathcal{T}$, 0 otherwise

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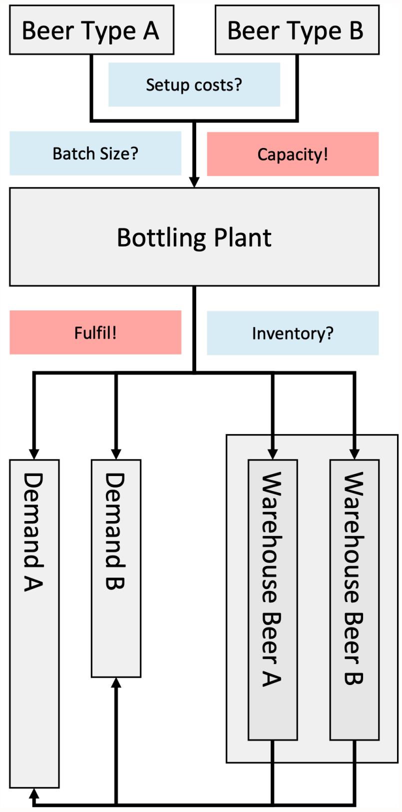
Objective Function

> ℹ We need the following parameters:
>
> - f_i - Setup cost of beer type $i \in \mathcal{I}$
> - c_i - Inventory holding cost for one unit of beer type $i \in \mathcal{I}$

...

$$\text{Minimize} \quad \sum_{i=1}^{\mathcal{I}} \sum_{t=1}^{\mathcal{T}} (c_i \times W_{i,t} + f_i \times Y_{i,t})$$

Constraints



Question: What constraints?

- Transfer unused inventory
- Fulfill the customer demand
- Set up beer types
- Calculate the batch size per set-up
- Compute remaining inventory
- Limit the bottling plant

Demand/Inventory Constraints?

! The goal of these constraints is to:

Consider the current inventory and batch sizes and compute the remaining inventory.

...

i We need the following variables and parameters:

- $W_{i,t}$ - Inventory of beer type $i \in \mathcal{I}$ at the end of period $t \in \mathcal{T}$
- $X_{i,t}$ - Batch size of beer type $i \in \mathcal{I}$ in $t \in \mathcal{T}$
- $d_{i,t}$ - Demand of beer type $i \in \mathcal{I}$ in period $t \in \mathcal{T}$

...

Question: What could the constraint look like?

Demand/Inventory Constraints

$$W_{i,t-1} + X_{i,t} - W_{i,t} = d_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \mid t > 1$$

...

i Remember, these are the variables and parameters:

- $W_{i,t}$ - Inventory of beer type $i \in \mathcal{I}$ at the end of period $t \in \mathcal{T}$
- $X_{i,t}$ - Batch size of beer type $i \in \mathcal{I}$ in $t \in \mathcal{T}$
- $d_{i,t}$ - Demand of beer type $i \in \mathcal{I}$ in period $t \in \mathcal{T}$

...

Question: What does $\mid t > 1$ mean?

Setup Constraints?

! The goal of these constraints is to:

Set up beer types where the batch size is $\geq$ 0.

...

i We need the following variables and parameters:

- $Y_{i,t}$ - 1, if beer type $i \in \mathcal{I}$ is bottled in period $t \in \mathcal{T}$, 0 otherwise
- $X_{i,t}$ - Batch size of beer type $i \in \mathcal{I}$ in $t \in \mathcal{T}$
- $d_{i,t}$ - Demand of beer type $i \in \mathcal{I}$ in period $t \in \mathcal{T}$

. . .

Question: What could the second constraint be?

Setup Constraints

$$X_{i,t} \leq Y_{i,t} \times \sum_{\tau=1}^{\mathcal{T}} d_{i\tau} \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

. . .

Question: Do you know this type of constraint?

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This type of constraint is called a "Big-M" constraint!

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- M (here $\sum_{\tau=1}^{\mathcal{T}} d_{i\tau}$) is a large number
- It is coupled with a binary variable (here $Y_{i,t}$)
- Like an if-then constraint

Capacity Constraints?

! The goal of these constraints is to:

Limit the capacity of the bottling plant per period.

. . .

i We need the following variables and parameters:

- $Y_{i,t}$ - 1, if beer type $i \in \mathcal{I}$ is bottled in period $t \in \mathcal{T}$, 0 otherwise
- $X_{i,t}$ - Batch size of beer type $i \in \mathcal{I}$ in $t \in \mathcal{T}$
- a_t - Available time on the bottling plant in period $t \in \mathcal{T}$
- b_i - Time used for bottling one unit of beer type $i \in \mathcal{I}$
- g_i - Setup time for beer type $i \in \mathcal{I}$

Capacity Constraints

Question: What could the third constraint be?

It has more variables and parameters when compared to the other constraints but it is easier to understand.

. . .

$$\sum_{i=1}^{\mathcal{J}} (b_i \times X_{i,t} + g_i \times Y_{i,t}) \leq a_t \quad \forall t \in \mathcal{T}$$

. . .

And that's basically it!

CLSP: Objective Function

$$\text{Minimize} \quad \sum_{i \in \mathcal{J}} \sum_{t \in \mathcal{T}} (c_i \times W_{i,t} + f_i \times Y_{i,t})$$

! The goal of the objective function is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

CLSP: Constraints

$$W_{i,t-1} + X_{i,t} - W_{i,t} = d_{i,t} \quad \forall i \in \mathcal{J}, t \in \mathcal{T} \mid t > 1$$

$$X_{i,t} \leq Y_{i,t} \times \sum_{\tau \in \mathcal{T}} d_{i,\tau} \quad \forall i \in \mathcal{J}, \forall t \in \mathcal{T}$$

$$\sum_{i \in \mathcal{J}} (b_i \times X_{i,t} + g_i \times Y_{i,t}) \leq a_t \quad \forall t \in \mathcal{T}$$

! Our constraints ensure:

Demand is met, inventory transferred, setup taken care of, and capacity respected.

CLSP: Variable Domains

$$Y_{i,t} \in \{0, 1\} \quad \forall i \in \mathcal{J}, t \in \mathcal{T}$$

$$W_{i,t}, X_{i,t} \geq 0 \quad \forall i \in \mathcal{J}, t \in \mathcal{T}$$

! The variable domains make sure that:

The binary setup variable is either 0 or 1 and that the inventory and batch size are non-negative.

Model Characteristics

Recap on some Basics

There exist several types of optimization problems:

- Linear (LP): Linear constraints and objective function
- Mixed-integer (MIP): Linear constraints and objective function, but discrete variable domains
- Quadratic (QP): Quadratic constraints and/or objective
- Non-linear (NLP): Non-linear constraints and/or objective
- And more!

Recap on Solution Algorithms

- Simplex algorithm to solve LPs
- Branch & Bound to solve MIPs
- Outer-Approximation for mixed-integer NLPs
- Math-Heuristics (e.g., Fix-and-Optimize, Tabu-Search, …)
- Decomposition methods (Lagrange, Benders, …)
- Heuristics (greedy, construction method, n-opt, …)
- Graph theoretical methods (network flow, shortest path)

Model Characteristics

Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?
- What kind of solver could we use?
- Can the Big-M constraint be tightened?

Model Assumptions

Questions: On model assumptions

- What assumptions have we made?
- What is the problem with the planning horizon?
- Any idea how to solve it?

Impact

Can this be

applied?

Scale as a Problem

Solving the problem with commercial solvers is not feasible.

Scale of the Case Study

- 220 finished products
- 100 semi-finished products
- 13 production resources
- 8 storage resources
- 3 main production levels
- 52 weeks planning horizon

Any idea what

could be done?

Heuristics and Optimization

- Multi-level Capacitated Lot-Sizing Problem
- Heuristic fix and optimize approach[1]
- Operating cost reduction by 5% and planning effort by 40%

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> **i And that's it for todays lecture!**
>
> We now have covered the basics of the CLSP and are ready to start solving some tasks in the upcoming tutorial.

Questions?

Literature

Literature I

For more interesting literature to learn more about Julia, take a look at the literature list of this course.

Literature II

Bibliography

[1] M. Mickein, M. Koch, and K. Haase, "A Decision Support System for Brewery Production Planning at Feldschlösschen," INFORMS Journal on Applied Analytics, vol. 52, no. 2, pp. 158–172, 2022.

[1] M. Mickein, M. Koch, and K. Haase [1]