

# Lecture VIII - Police Districting

## Applied Optimization with Julia

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University of Hamburg - Fall 2025

### Introduction

#### Police Service District Planning

T. Vlček, K. Haase, M. Fliedner, and T. Cors [1]

#### Challenges

Question: What makes the work of emergency services complex?

- Dynamic urban development
- Changing population patterns
- Resource constraints
- Need for rapid response
- Multiple stakeholder interests

#### Emergency Services



Emergency services address the needs of three interest groups:

- Citizens
- Service personnel
- Administrators

Question: What could be the objectives of these groups?

## Stakeholder Objectives

1. Citizens
  - Fast response times
  - Reliable service coverage
2. Service Personnel
  - Manageable workloads

- Safe working conditions
3. Administrators
- Cost efficiency
  - Resource optimization

**i Note**

Aligning the objectives of the three interest groups is challenging.

## Emergency Service Districting



Question: Why might current district layouts be suboptimal?

- Many layouts date back several decades

- Often designed along highways and regions [2]
  - Extensive data not used for data-driven improvement
- 

How can we improve  
this situation?

## The Role of Data

Question: What data can help improve emergency services?

- Historical incident patterns
- Response time analysis
- Resource utilization metrics
- Population densities and traffic patterns

...

### i Note

Extensive data collected, but often lack of tools or knowledge to leverage it.

## Optimization

- Operations research (OR) models can help!
  - Based on incident records and geographical information
  - Improve the response of emergency services
  - Help administrators in making strategic decisions
  - Locate new departments or close departments [3]
- 

## Case Studies

### Police Districting

For an efficient and effective distribution of resources, police jurisdictions are divided into precincts or command districts with separate departments. These are further divided into patrol beats [4].

### Service Priority Extremes

- High Priority
  - Life-threatening situations
  - Active crimes in progress
  - Multiple unit response needed
- Low Priority
  - Minor incidents

- Administrative tasks

## Case Studies

- Different urban contexts
  - Study of jurisdictions in
    - Germany: Large metropolitan area
    - Belgium: Large rural area
  - Focus on response time optimization
- ...

### Note

Part of the force patrols the streets, another part is stationed at the departments.

## Dispatching

- Dispatchers assign all CFS to vehicles from the corresponding districts and patrol areas
- Officers are familiar with the area and are thus better prepared to respond appropriately [5]
- To cope with high demands, dispatchers can assign vehicles from nearby districts or beats

## Potential Problem

Question: What could be the potential problem?

...

- This can lead to a domino effect
- Transferring vehicles from other districts or beats reduces coverage in those locations [6]
- This makes them vulnerable to missing resources when they need assistance themselves

## Overloaded Systems

- This can lead to overloaded systems!
  - Long dispatching delays due to staff shortages
  - Preventive patrol hardly possible [7]
  - Dispatchers constantly draw on patrol resources
  - Reduces the response time of emergency services
- ...

### Warning

This is a common problem in many emergency services.

Response Time



- Central criterion to measure the effectiveness of emergency services is the response time
- Time between a call for aid and the arrival at the incident location
- Low response time increases the likelihood of helping and improves confidence [5]

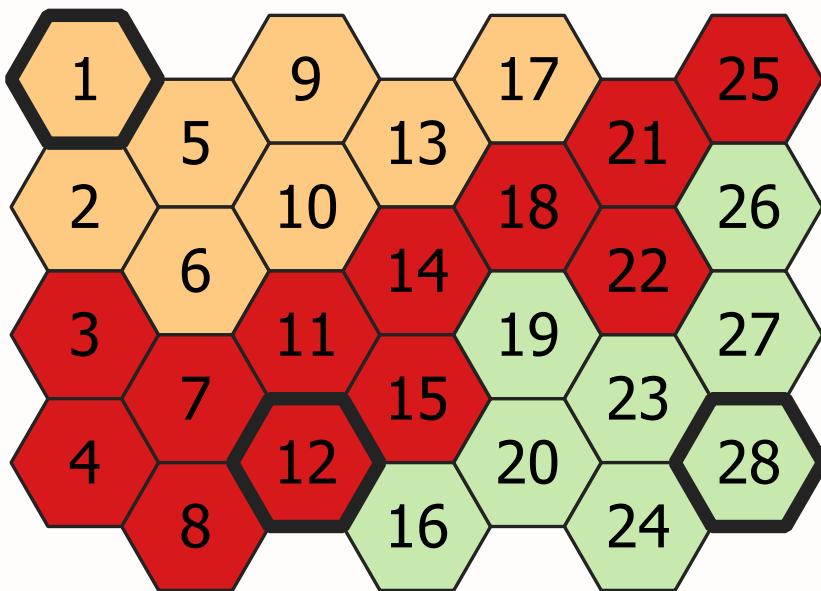
## Response Time Influencers

Question: What affects response time?

- Initial contact
  - Information gathering
  - Unit assignment
  - Resource coordination
  - Route to location
  - Traffic conditions
- 

## Territory Design Problem

Territory Design Problem



Aggregation of small geographic areas, called basic areas (BAs), into geographic clusters, called districts, so that these are acceptable according to pre-defined planning criteria<sup>1</sup>.

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## Objective

Question: What could be the objective?

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<sup>1</sup>A. A. Zoltners and P. Sinha [8]

...

- Minimize the response time to help citizens faster while increasing the confidence in the service

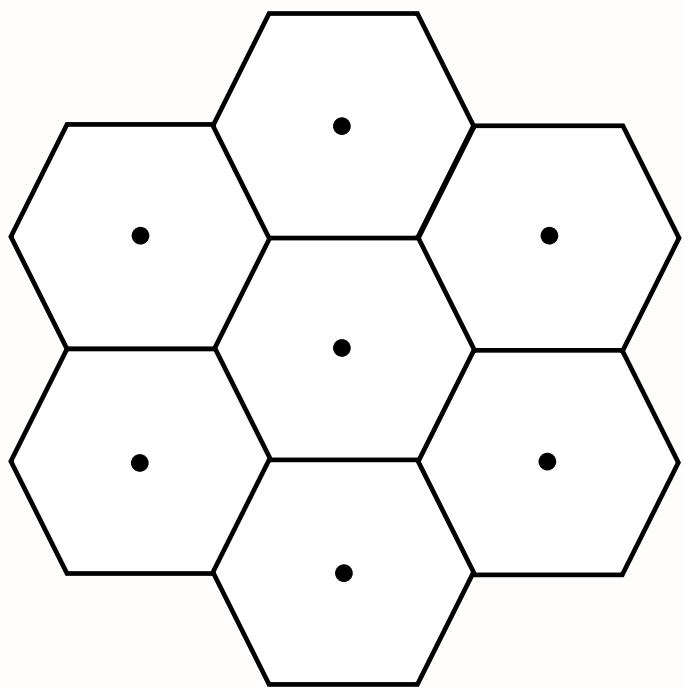
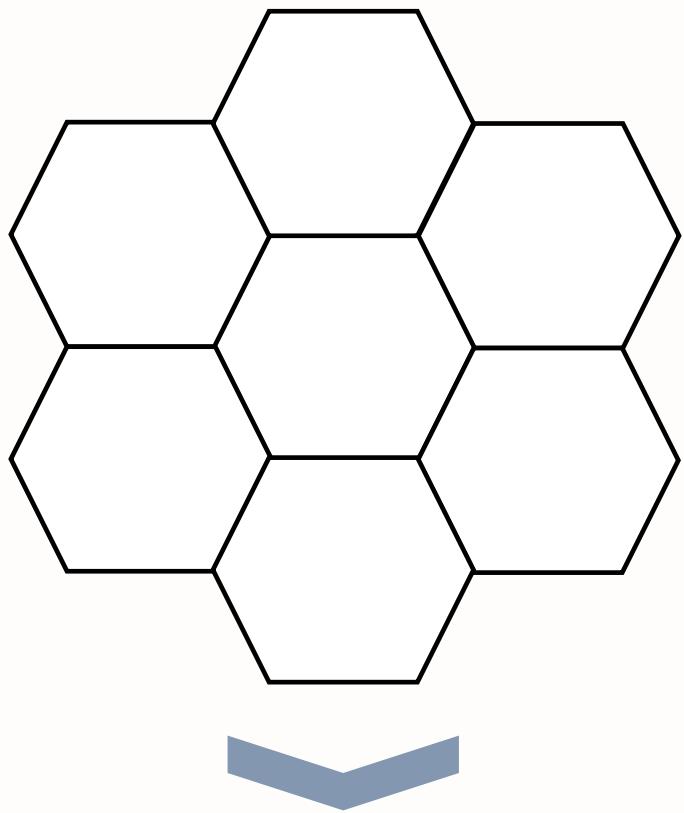
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Question: What could be further objectives?

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- Relocate only part of the police department's
- Compact and contiguous territories to improve patrol
- Prevention of isolated departments

## Basic Structure

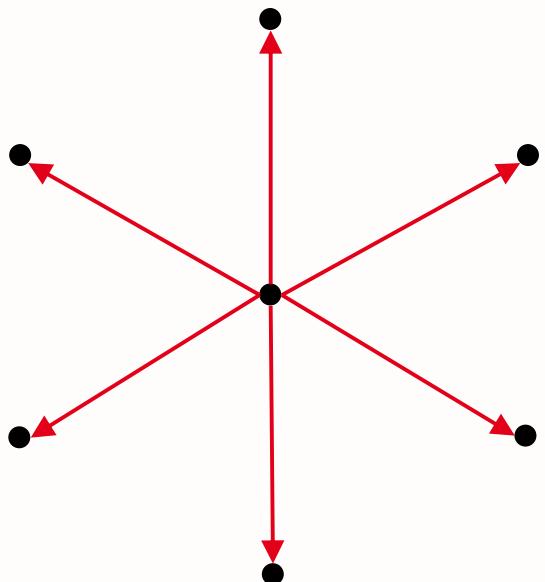
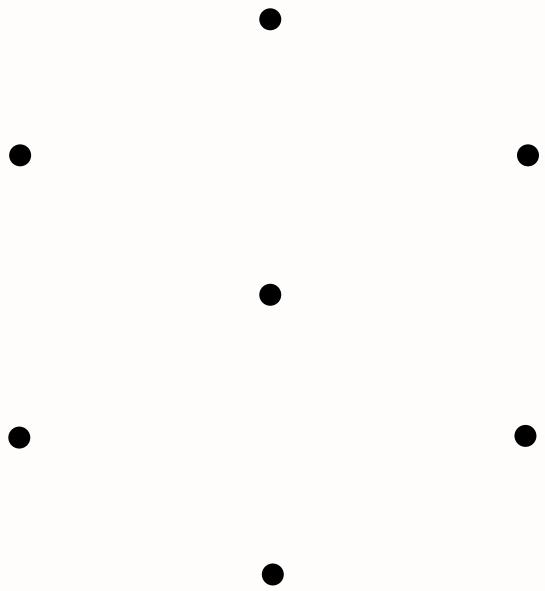


Question: How can we structure this?

- Model as a digraph with vertices and edges
- Each BA centroid becomes a vertex
- $\mathcal{J}$  : set of BAs, indexed by  $j$
- $\mathcal{I}$  : set of potential district centres ( $\mathcal{I} \subseteq \mathcal{J}$ )

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Why Hexagons?



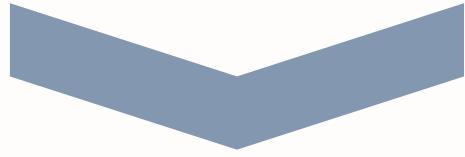
Question: Advantages of hexagons?

- Equal distances to all neighboring centroids
  - Reduces sampling bias from edge effects [9]
  - Special properties that help with the enforcement of compactness
  - Better representation of urban geography
- 

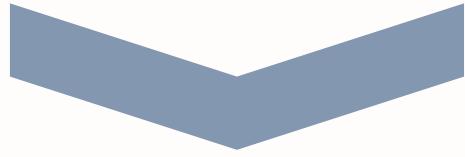
Response Time Components



Call  
Length



Dispatch  
Time



Driving  
Time

Question: How can we model response time?

- Call length is independent of territory
- Dispatch time is difficult to model
- Driving time can be minimized directly

#### Conclusion

We focus on minimizing expected driving times between departments and incident locations.

## Model Formulation

Let's build our model

step by step!

### Key Model Components

Question: What could be our key model components?

...

- Basic areas (BAs) and potential department locations
- Driving times between basic areas
- Forecasted incident data
- Assignment decisions

...

Question: Which are sets, parameters, and variables?

### Sets and Indices

- $\mathcal{J}$  - Set of BAs, indexed by  $j$
- $\mathcal{I}$  - Set of potential district centres ( $\mathcal{I} \subseteq \mathcal{J}$ ), indexed by  $i$

...

#### Note

The depot locations are a subset of the basic areas!

### Parameters

Question: What parameters do we need?

- $p$  - Number of district centres (departments)
- $t_{i,j}$  - Expected driving times between  $i$  and  $j$

...

### 💡 Tip

Parameters should be carefully calibrated with real-world data!

## Decision Variable(s)?

💡 We have the following sets:

- BAs, indexed by  $j \in \mathcal{J}$
- Potential department locations, indexed by  $i \in \mathcal{I}$

...

❗ Our objective is to:

Minimize the expected response time of the emergency services by optimizing the assignment of BAs to departments.

...

Question: What decisions do we need to model?

### Decision variable/s

- $X_{i,j}$ : 1 if BA  $j$  assigned to department  $i$ , 0 otherwise

...

Question: What is the domain of our decision variable?

...

- $X_{i,j} \in \{0, 1\} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$

Let's build our

objective function!

## Objective Function?

❗ Our objective is to:

Minimize the expected response time of the emergency services by optimizing the assignment of BAs to departments.

...

Question: How do we minimize response time?

- We want to minimize total driving time
- Consider frequency of incidents in each BA
- Don't include fixed costs (handled by constraints)

## Objective Function

Question: What could be our objective function?

...

$$\text{minimize} \quad \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} t_{i,j} \times X_{i,j}$$

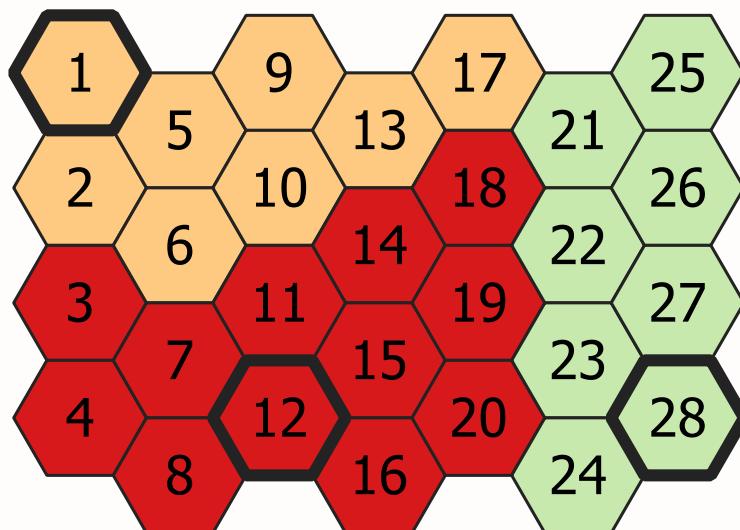
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### ! Expected Driving Time

- Total driving time across all assignments
- Weighted by incident frequency
- Considers all possible BA-department pairs

## Constraints

### Key Constraints



Question: Constraints needed?

1. BA must have one department
2. Limit number of departments

3. Only assign active departments
  4. Ensure contiguous districts
  5. Maintain district compactness
- 

### Single Assignment Constraint?

Question: Why do we need this constraint?

...

- Each BA must be assigned to exactly one department
- Prevents overlapping jurisdictions
- Ensures complete coverage

...

**i** We need the following variables:

- $X_{i,j}$  - 1 if BA  $j$  assigned to department  $i$ , 0 otherwise

### Single Assignment Constraint?

Question: What could the constraint look like?

...

$$\sum_{i \in \mathcal{I}} X_{i,j} = 1 \quad \forall j \in \mathcal{J}$$

...

**i** Note

Each BA must be assigned to exactly one department.

---

### Department Count Constraint?

**|** The goal of these constraints is to:

Ensure that exactly  $p$  departments are opened.

...

**i** We need the following sets and variables:

- $\mathcal{I}$  - Set of potential department locations, indexed by  $i$
- $\mathcal{J}$  - Set of BAs, indexed by  $j$
- $X_{i,j}$  - 1, if BA  $j$  assigned to department  $i$ , 0 otherwise
- $p$  - Number of departments

## Department Count Constraint

Question: What could the constraint look like?

...

$$\sum_{i \in \mathcal{I}} X_{i,i} = p$$

...

Question: What happens if we have more departments than potential locations?

...

- We can't open more departments than there are locations
- The model will be infeasible

## Active Department Constraint?

**!** The goal of these constraints is to:

Ensure that each BA is assigned to an active department, e.g. a department that is opened and that could dispatch vehicles.

...

**i** We need the following sets and variables:

- $X_{i,j}$  - 1, if BA  $j$  assigned to department  $i$ , 0 otherwise

Question: How do we ensure assignments only to active departments?

## Active Department Constraint

$$X_{i,j} \leq X_{i,i} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$$

...

**i Note**

This constraint creates a logical connection between department locations and BA assignments where BAs can only be assigned to opened departments.

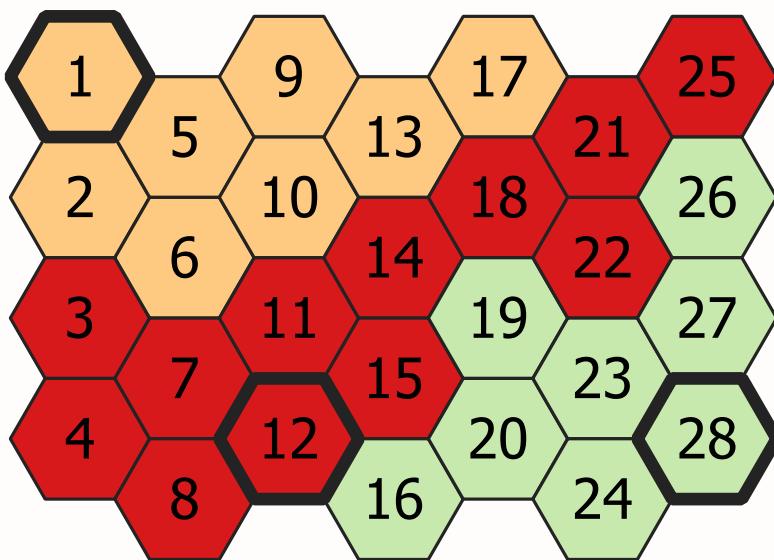
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## p-Median Problem

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### Contiguity and Compactness

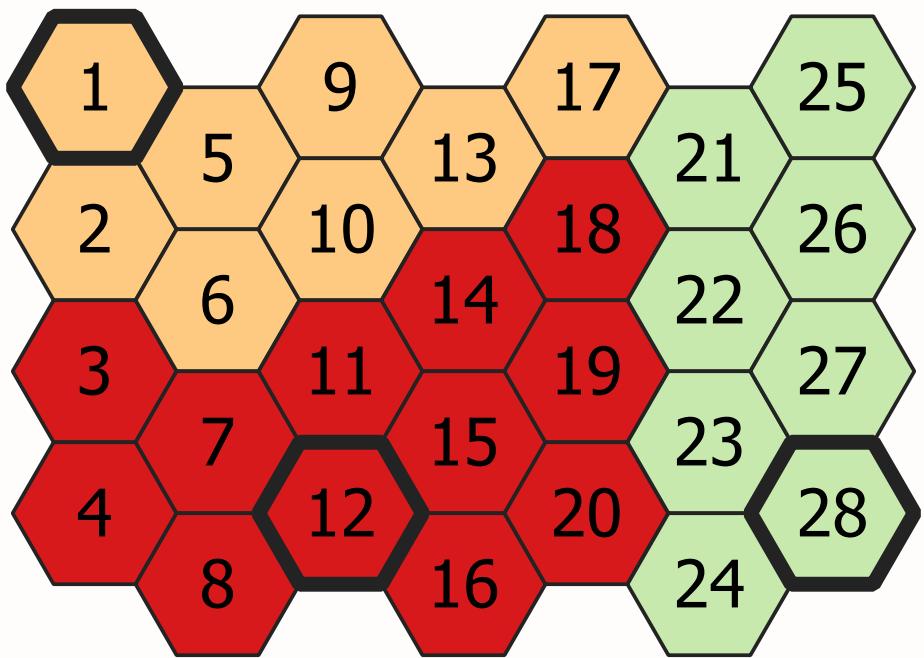
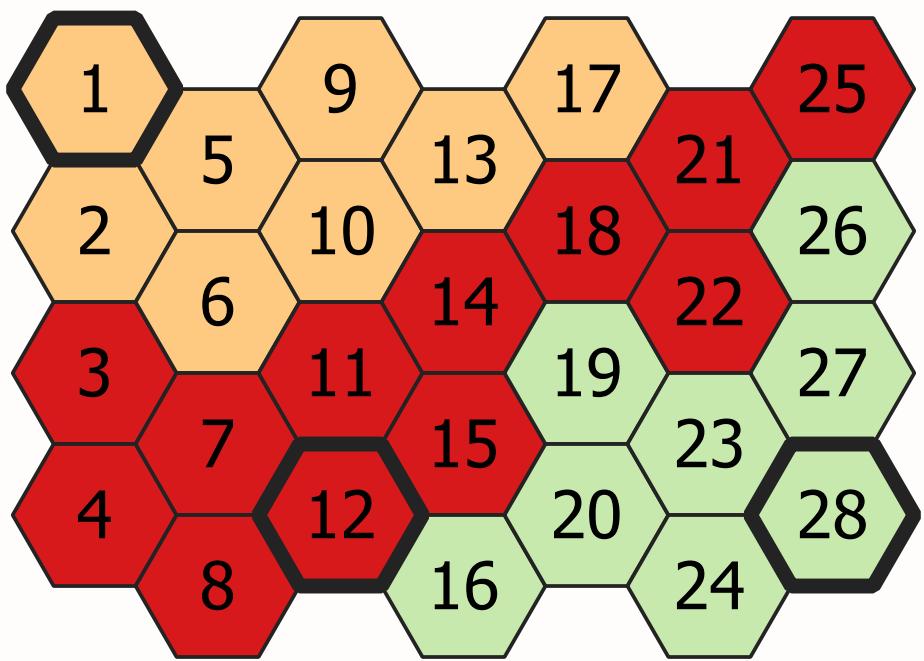
#### Contiguity Introduction

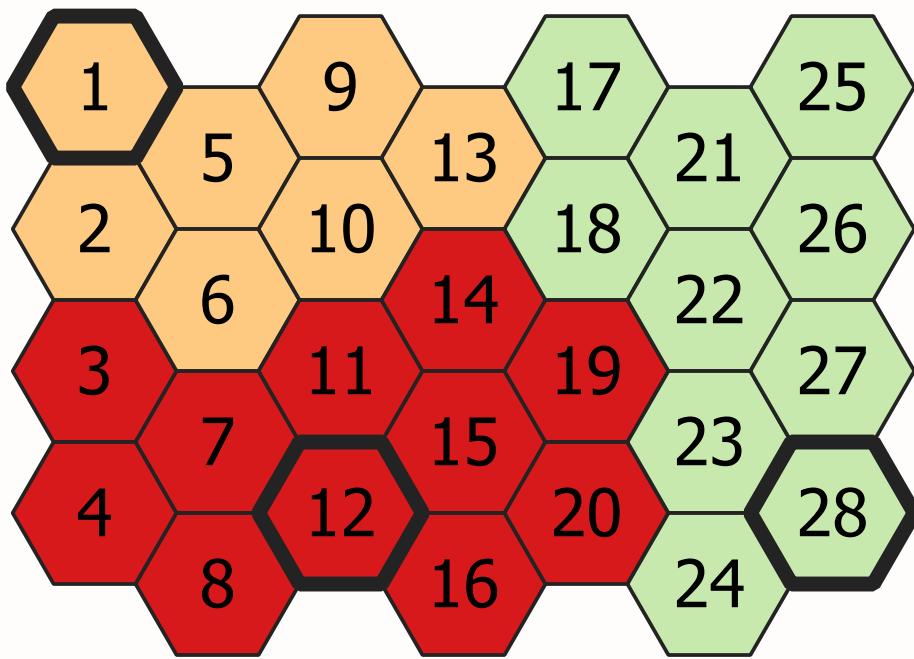


Question: Why is contiguity important?

- Prevents isolated areas
- Ensures contiguous patrol routes
- Maintains operational coherence

What is compactness?



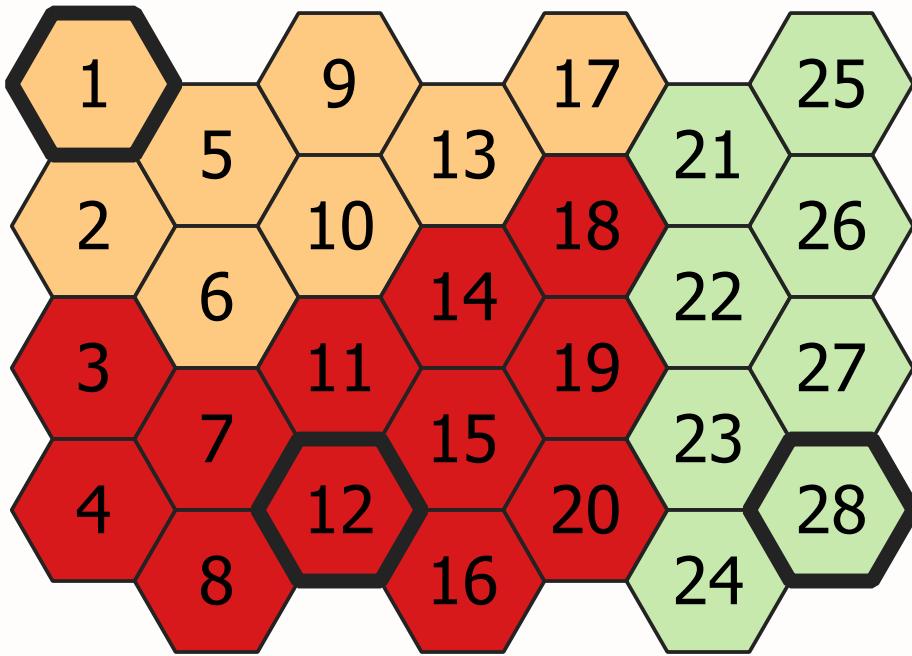


...

#### i Compactness

Compactness has no univocal definition; a district is commonly declared compact if it is ‘somehow round-shaped and undistorted’ [10].

### Contiguity and Compactness



Question: Are our resulting districts based on the model contiguous and compact?

- This depends on  $t_{i,j}$
- If Euclidean distance
  - Districts will be contiguous
  - Likely of compact shape

### Compactness p-Median

Question: Is this likely for police service districting?

- No, as we minimize the driving time within a city
- Highways, Tunnels, etc.
- Multiplied by the differing number of requested cars
- This can contribute to distorted district shapes

### Contiguity Sets

Additional Set and Parameter

- $e_{i,j}$  - Euclidean distance between centroids
- $\mathcal{A}_j$  - Sets of BAs adjacent to BA  $j$

...

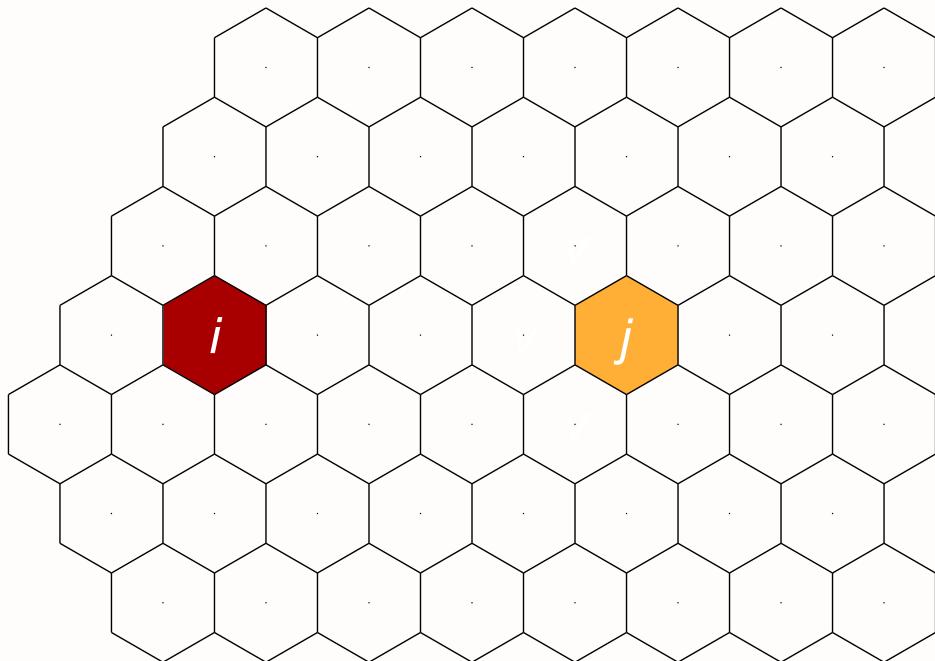
$$\mathcal{N}_{i,j} = \{v \in \mathcal{A}_j \mid e_{i,v} < e_{i,j}\} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$$

...

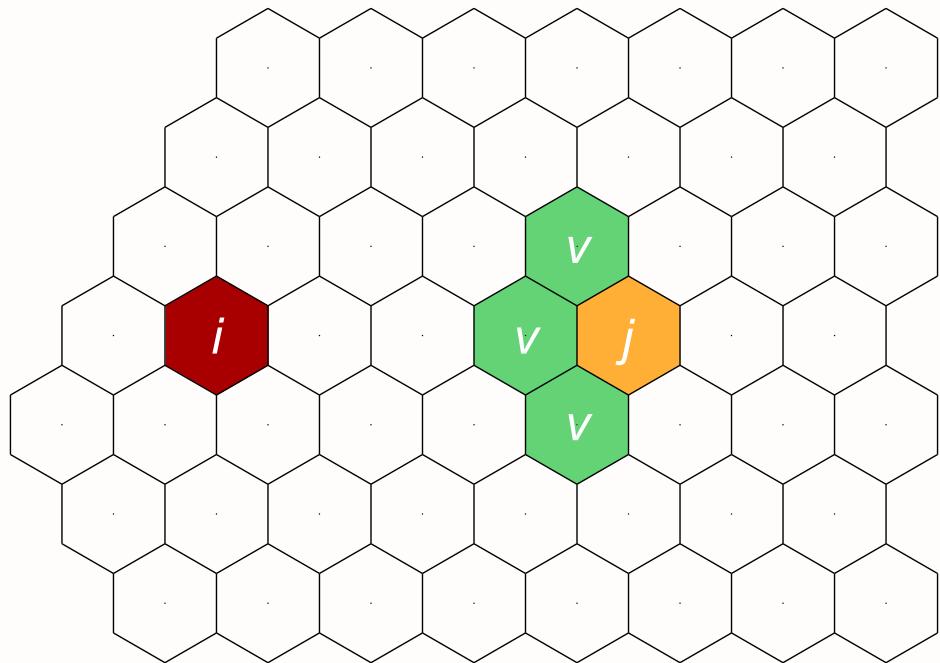
**i** The idea

BAs closer to department  $i$  than BA  $j$  on euclidian distance and adjacent to  $j$ !

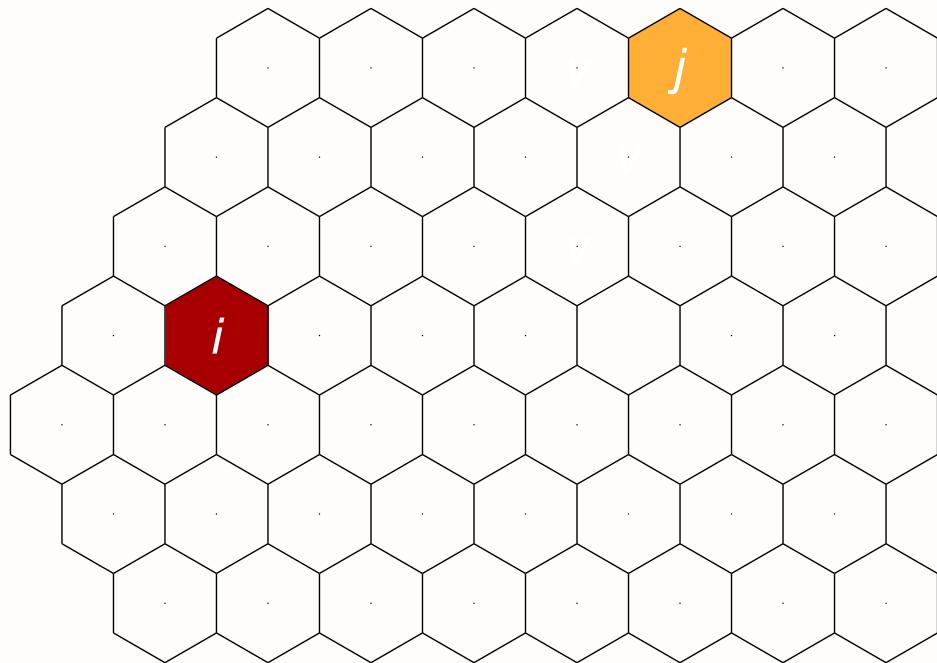
Example A



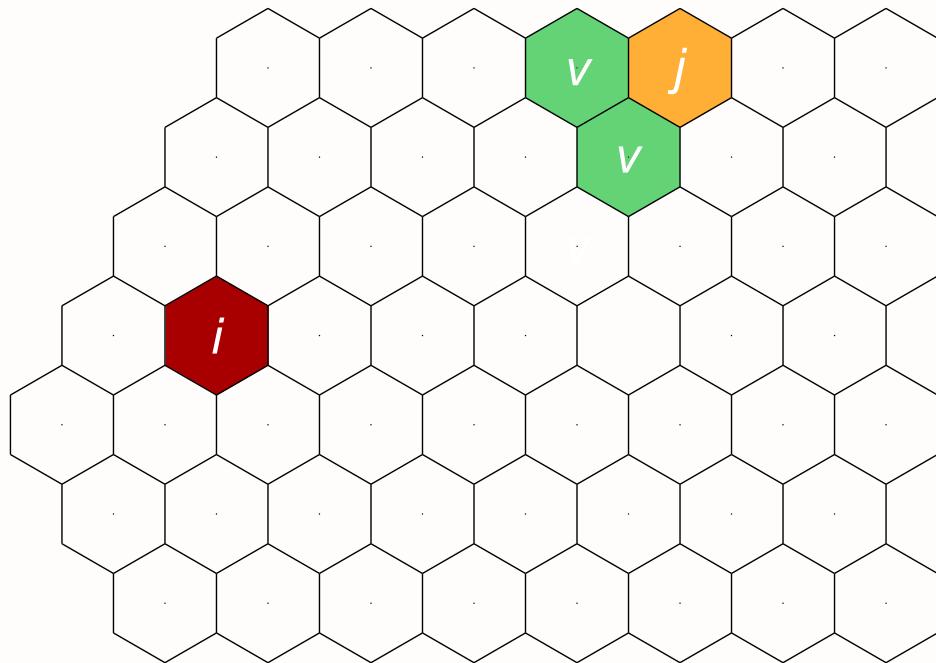
Example A



Example B



## Example B



### Enforcing Contiguity

All districts have to be contiguous

$$X_{i,j} \leq \sum_{v \in \mathcal{N}_{i,j}} X_{i,v} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \setminus \mathcal{A}_i : i \neq j$$

...

! The idea

At least one department has to be assigned to a BA that is adjacent to BA  $j$  and closer to department  $i$ !

### Contiguity and Compactness

All districts have to be contiguous and compact

...

! The idea

At least one department has to be assigned to two BAs that are adjacent to BA  $j$  and closer to department  $i$ !

### Comparison

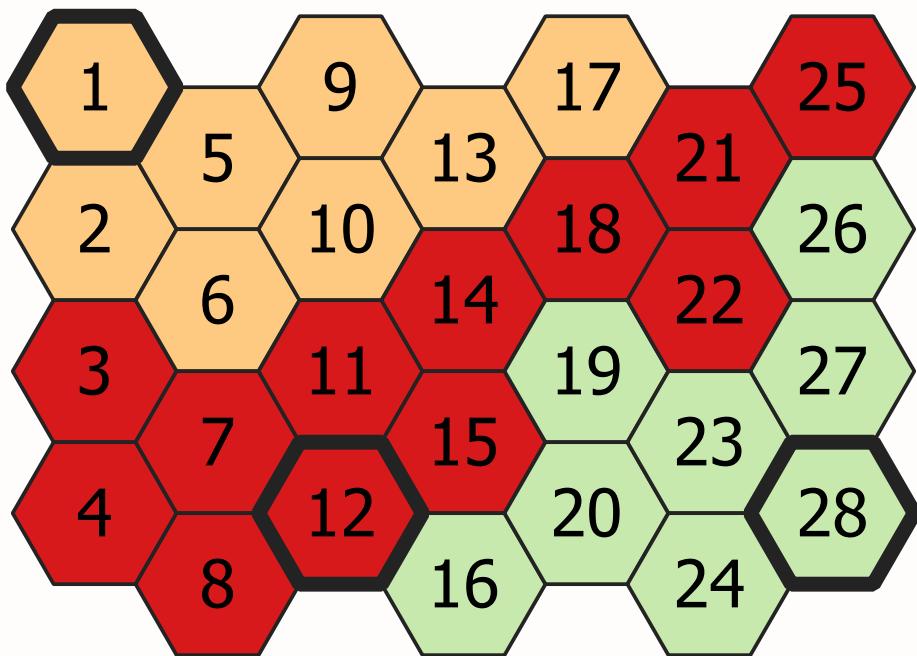


Figure 1: One department

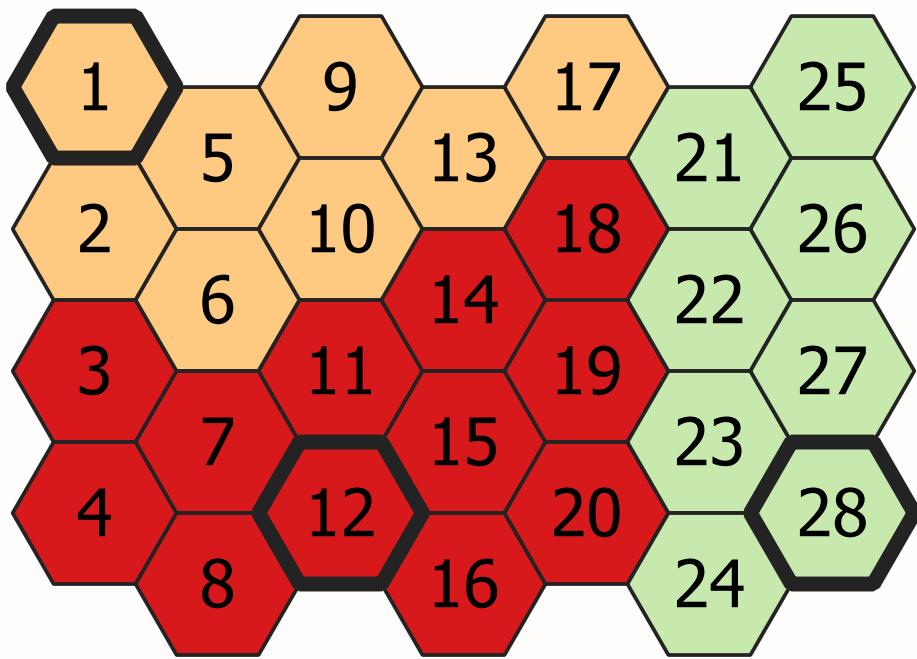


Figure 2: Two departments

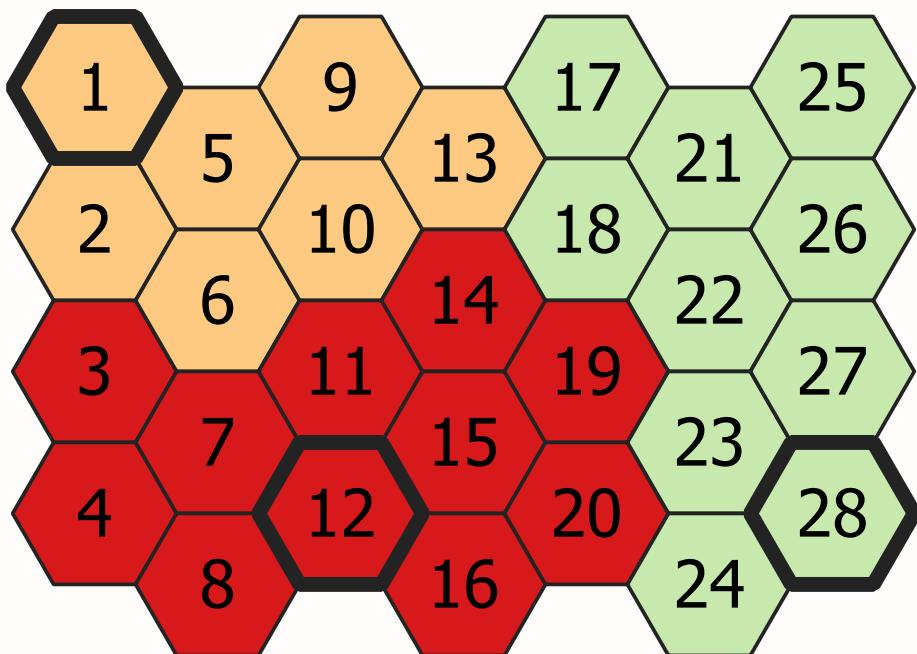


Figure 3: Up to three departments

...

### **i** Why does this work?

Due to the constraints, there is always a path back to the department if a BA is assigned to a department!

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## Model Characteristics

### Characteristics

Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?
- What do you think, can the model be solved quickly?
- Have we prevented isolated districts?

### Model Assumptions

Questions: On model assumptions

- What assumptions have we made?
  - Use Euclidean distances to approximate driving time?
  - Can we rely on incident data collected by the police?
- 

## Implementation and Impact

### Overview of Studies

Question: Where did we apply our model?

- Two distinct environments:
  1. Large metropolitan area (Germany)
  2. Rural region (Belgium)
- Different challenges and requirements
- Focus on response time optimization

### German Metropolitan Case

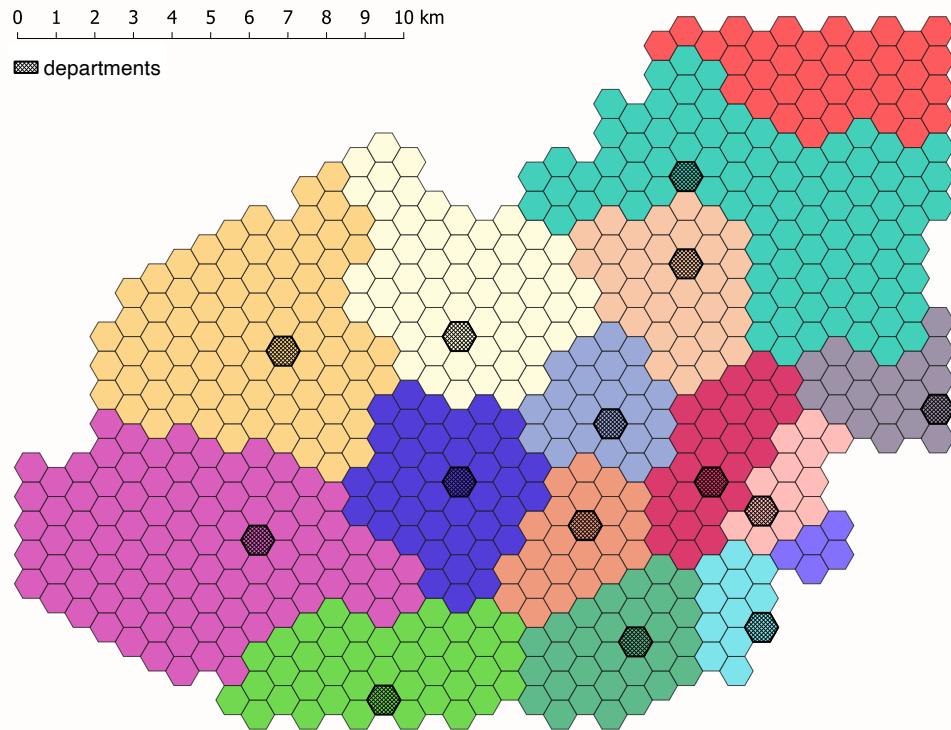
...

- 1.8 mio incidents (2015-2019)
- ~20 department locations
- 1,596 basic areas
- Dense urban environment

...

Goal: Redesign districts to improve response time.

## German Metropolitan Results



## Belgian Rural Case

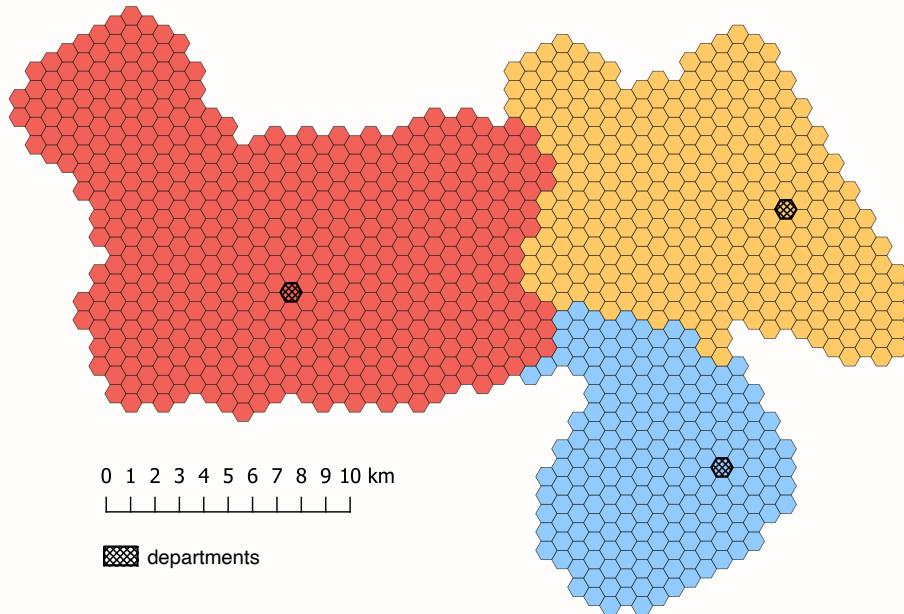
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- 50,000 incidents (2019-2020)
- 2 existing + 1 planned location
- 1,233 basic areas
- Dispersed rural setting

...

Goal: Optimize coverage with limited resources.

## Belgian Rural Results



## Simulation Framework

Question: How did we validate the results?

...

- Spatial and temporal patterns
- Shift schedules
- Priority handling
- Rush hours
- Inter-district support
- Variable driving times

## Results

- Response time reduction up to 14.52%
- Better workload distribution
- Improved coverage equity
- More efficient resource utilization

...

! Important

All improvements are without additional staff!

## Conclusions

1. Model adaptability crucial
2. Local context matters

3. Stakeholder buy-in essential
  4. Data quality critical
- ...

 Tip

Success requires balancing theoretical optimization with practical constraints!

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## Future Applications

Question: Where else could this approach be useful?

- Other emergency services
  - Different urban contexts
  - Resource allocation problems
  - Service territory design
- ...

 Note

The methodology is adaptable to various public service optimization scenarios.

## Wrap Up

 And that's it for todays lecture!

We now have covered districting problems and are ready to start solving some tasks in the upcoming tutorial.

Questions?

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## Literature

### Literature I

For more interesting literature to learn more about Julia, take a look at the [literature list](#) of this course.

### Literature II

### Bibliography

- [1] T. Vlček, K. Haase, M. Fliedner, and T. Cors, “Police service district planning,” OR Spectrum, Feb. 2024, doi: [10.1007/s00291-024-00745-3](https://doi.org/10.1007/s00291-024-00745-3).

- [2] C. Bruce, “Districting and Resource Allocation: A Question of Balance,” *A Quarterly Bulletin of Applied Geography for the Study of Crime & Public Safety*, vol. 1, no. 4, pp. 1–3, 2009.
- [3] F. Liberatore, M. Camacho-Collados, and B. Vitoriano, “Police Districting Problem: Literature Review and Annotated Bibliography,” *International Series in Operations Research & Management Science: Optimal Districting and Territory Design*. Springer International Publishing, Cham, pp. 9–29, 2020. doi: [10.1007/978-3-030-34312-5\\_2](https://doi.org/10.1007/978-3-030-34312-5_2).
- [4] S. J. D’Amico, S.-J. Wang, R. Batta, and C. M. Rump, “A simulated annealing approach to police district design,” *Computers & Operations Research*, vol. 29, no. 6, pp. 667–684, 2002, doi: [10.1016/s0305-0548\(01\)00056-9](https://doi.org/10.1016/s0305-0548(01)00056-9).
- [5] S. E. Bodily, “Police Sector Design Incorporating Preferences of Interest Groups for Equality and Efficiency,” *Management Science*, vol. 24, no. 12, pp. 1301–1313, 1978, doi: [10.1287/mnsc.24.12.1301](https://doi.org/10.1287/mnsc.24.12.1301).
- [6] A. Mayer, “Geospatial Technology Helps East Orange Crack Down on Crime,” *A Quarterly Bulletin of Applied Geography for the Study of Crime & Public Safety*, vol. 1, no. 4, pp. 8–10, 2009.
- [7] H. F. Miller and B. A. Knoppers, “Computer Simulation of Police Dispatching and Patrol Functions,” *International Symposium on Criminal Justice Information and Statistics Systems Proceedings*, vol. 0. National Institute of Justice, pp. 167–179, 1972.
- [8] A. A. Zoltners and P. Sinha, “Sales Territory Alignment: A Review and Model,” *Management Science*, vol. 29, no. 11, pp. 1237–1256, 1983, doi: [10.1287/mnsc.29.11.1237](https://doi.org/10.1287/mnsc.29.11.1237).
- [9] J. Wang and M.-P. Kwan, “Hexagon-Based Adaptive Crystal Growth Voronoi Diagrams Based on Weighted Planes for Service Area Delimitation,” *ISPRS International Journal of Geo-Information*, vol. 7, no. , p. 257, 2018, doi: [10.3390/ijgi7070257](https://doi.org/10.3390/ijgi7070257).
- [10] J. Kalcsics, S. Nickel, and M. Schröder, “Towards a unified territorial design approach — Applications, algorithms and GIS integration,” *Top*, vol. 13, no. 1, pp. 1–56, 2005, doi: [10.1007/BF02578982](https://doi.org/10.1007/BF02578982).