Lecture: Applied Optimization I

Introduction

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UHH, Institute of Logistics, Transport and Production @Helmut Schmidt University

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Course Structure

1. **Lectures**
    - Live lectures every Wednesday between 8.00 AM and 9.30 AM
    - First two lectures repeat mathematical modeling and programming basics
    - Afterward lectures discuss practical problems and their implementation
    - Lectures are interactive $\rightarrow$ We discuss approaches during the lectures!
    - Communication takes place via E-Mail

Course Structure

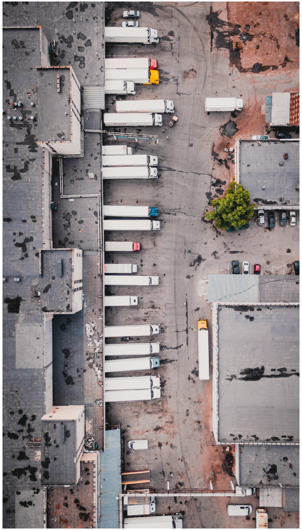
2. Tutorials

- Live tutorials every Wednesday between 9.45 AM and 11.15 AM
- In these tutorials we are working on assignments
- Most assignments are based on applied problems of the lecture
- If available, please bring a laptop with Windows, macOS, or Linux!
- You can form groups of up to 3 students to solve assignments together
- Groups can submit their solution to earn bonus points for the exam
- Max. 0.5 point per tutorial, points only count if the mark is at least 4.0

Course Objective

Applied Optimization

- Many real-world problems can be addressed thanks to mathematical models
- Our objective is to foster your interest in the topic and to enable you to recognize and solve problem structures on your own
- This includes problem understanding and implementation in software
- Great seminar and thesis preparation



Societies and Journals

National and international societies

GOR	Society for Operations Research in Germany
INFORMS	Institute for Operations Research and the Management Sciences
IFORS	International Federation of OR-Societies

National and international journals

- European Journal of OR
- Journal of the Operational Research Society
- Journal on Applied Analytics (before Interfaces)
- Management Science
- Operations Research
- OR Spectrum

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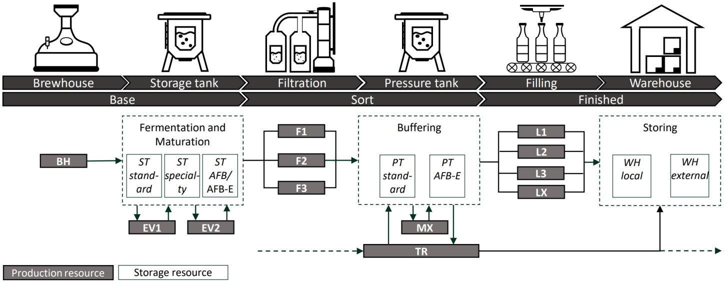
Case: Brewery Production Planning

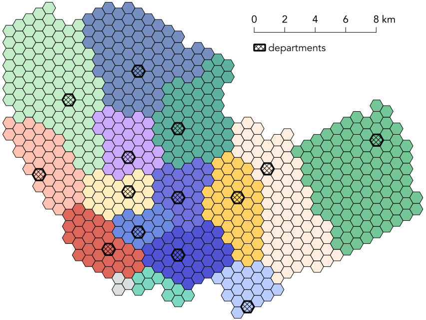


Storage resource types: storage tank (ST), pressure tank (PT), and warehouse (WH)
Production resource types: brewhouse (BH), filtration line (F), filling line/station (L), evaporation station (EV), mixing station (MX), and transfer station (TR)

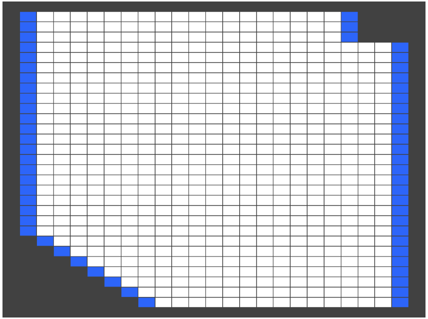
- Lot size planning and resource utilization analysis
- Markus Mickein, Matthes Koch, and Knut Haase (2022). "A Decision Support System for Brewery Production Planning at Feldschlösschen". In: *INFORMS Journal on Applied Analytics* 52.2, pp. 158–172

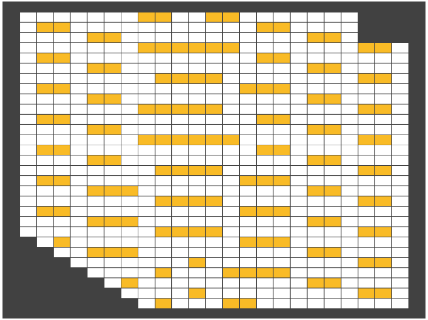
Case: Police Service District Planning (Germany and Belgium)

- Improving locations and district layouts to lower the response time of emergency services
- Tobias Vlćek et al. (2023b). "Police Service District Planning". In: *Accepted at OR Spectrum*



Case: Venue seating under COVID-19 distancing rules (Germany)

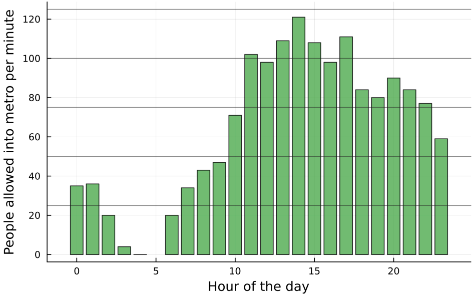




- Maximizing ticket sales under COVID-19 distancing rules for a German football club
- Usama Dkaidik and Matthes Koch; Current research with a likely paper submission in 2024

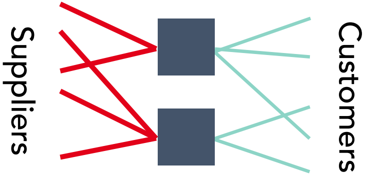
Case: Metro Inflow Management (Qatar)

- Scheduling the inflow into metro stations during the world cup in Doha 2022 to prevent crowd disasters at central stations
- Tobias Vlćek et al. (2023a). "Controlling Passenger Flows into Metro Systems to Mitigate Overcrowding during large-scale Events". In: *Working Paper*

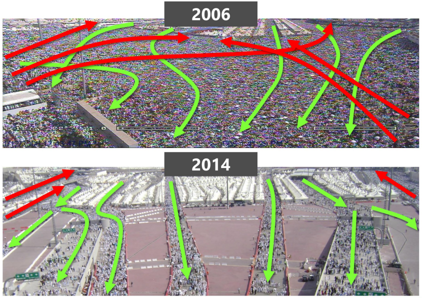


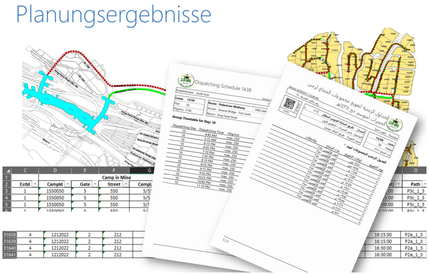
Case: Split-Order Minimization in E-Commerce (Germany)

- Lowering split-orders of e-commerce retailers through improved product-warehouse allocations
- Tobias Vlćek and Guido Voigt (2023). “Optimizing SKU-Warehouse Allocations to minimize Split Parcels in E-Commerce Environments”. In: *Working Paper*



Case: Crowd Management (Mecca)





- Scheduling for the Islamic pilgrimage to Mecca
- Knut Haase et al. (2016). "Improving pilgrim safety during the hajj: an analytical and operational research approach". In: *Interfaces* 46.1, pp. 74–90

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Lecture Topics this semester

1. Introduction
2. Julia and Mathematical Modeling in JuMP
3. Capacitated Lot Sizing Problem
4. Vehicle Routing for Central Libraries
5. Improving District Layouts of Police Departments
6. Sales Force Deployment for Medical Companies
7. Lowering split-orders for e-commerce retailers
8. Safety planning for the Islamic Pilgrimage to Mecca
9. Allocating International Student to Universities
10. Maximizing Venue Sales under COVID-19 Restrictions
11. Controlling Passenger Flows into Metro Systems
12. Recap Session

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Julia Programming Language

In this lecture we are going to work in the Julia Programming Language.

Here are the main reasons we will use it:

Julia Programming Language

- **Speed:** Julia was designed to be as general as Python, as statistics-friendly as R, but as fast as C++. This focus on speed allows for fast data workflows, particularly in scientific computing.

Julia Programming Language

- **Syntax:** Despite its high-performance, Julia has a dynamically-typed syntax that is more similar to R, Matlab and Python, making it accessible and easy to learn compared to more complex programming languages.

Julia Programming Language

- **JuMP:** Julia's seamless integration with JuMP simplifies the process of solving complex optimization problems. It provides a high-level, user-friendly interface for modeling and solving linear and nonlinear optimization problems, making it a valuable tool for operations research, data science, and engineering applications.

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Do you have experience with algebraic modeling?

What is an algebraic model?

Algebraic Modeling

How to learn algebraic modeling?

- Practice, practice, and practice!
- Understand standard models and apply their approach to other problems
- Develop an understanding that the constraints of the algebraic model describe the admissible solution space
- Use an available standard algorithm to determine an (optimal) solution

Algebraic Modeling

Central Questions

- What is to be decided?
- What is relevant to the decision?
- What information is given and relevant?
- What parameters (data) are needed?
- Which variables and of which type are needed?

Model Components

1. Objective function
2. Constraints
3. Variables

Linear Optimization Model

Basic Model Formulation

$$\text{maximize} \quad F = \sum_{j \in \mathcal{J}} c_j \times X_j \tag{1}$$

subject to

$$\sum_{j \in \mathcal{J}} a_{i,j} \times X_j \leq b_i \qquad \forall i \in \mathcal{I} \tag{2}$$

$$X_j \geq 0 \qquad \forall j \in \mathcal{J} \tag{3}$$

F : Objective function variable,
X_j : decision variables,
$\mathcal{I}$: set of $i \in \mathcal{I}$,
$\mathcal{J}$: set of $j \in \mathcal{J}$,
c_j : objective function coefficients,
$a_{i,j}$: parameters,
b_i : parameters

We can model and illustrate many problems with graphs (networks).

Undirected Graph

System (N, E) with non-empty node set $N = \{1, ..., n\}$ and an **edge** set $E = \{e_1, e_2, ..., e_m\}$.

- The graph $G = (N, E)$ consists of 2 sets N and E
- The edge set consists of 2-element subsets of the node set
- $e_k = \{i_k, j_k\}, k = 1, ...m$ with $i_k, j_k \in N$
- Designation of edge elements in models: $e \in E$ or $\{i, j\} \in E$.
- Nodes i and j in $e = \{i, j\}$ are called endpoints of the edge

Graphs

Directed Graph

System (N, A) with non-empty node set $N = \{1, ..., n\}$ and an **arc** set $A = \{a_1, a_2, ..., a_m\}$

- The graph $G = (N, A)$ consists of 2 sets N and A
- Edges have an orientation (start and end nodes)
- The edge $a = (i, j)$ is different from $a' = (j, i)$
- Very common in logistics
- Directed edge: also ‘arrow’ or ‘arc’ (hence the set is called A)

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Solar Panel Transport

A company is producing solar panels in Dresden and Laupheim and has to transport them to new solar farms near Hamburg, Munich, and Berlin. The quantities offered and demanded (truckloads) and the transport costs per truckload in Euro are as follows:

Origin/Destination	Hamburg	Munich	Berlin	Available
Dresden	5010	4640	1980	34
Laupheim	7120	1710	6430	41
Demand	21	17	29	

Example: a truckload from Dresden ($i = 1$) to Munich ($j = 2$) costs $c_{12} = 4640$ Euro. Moreover, it is necessary to fulfil all customer demands, as the contract has already been signed.

Solar Panel Transport

What could be the objective?

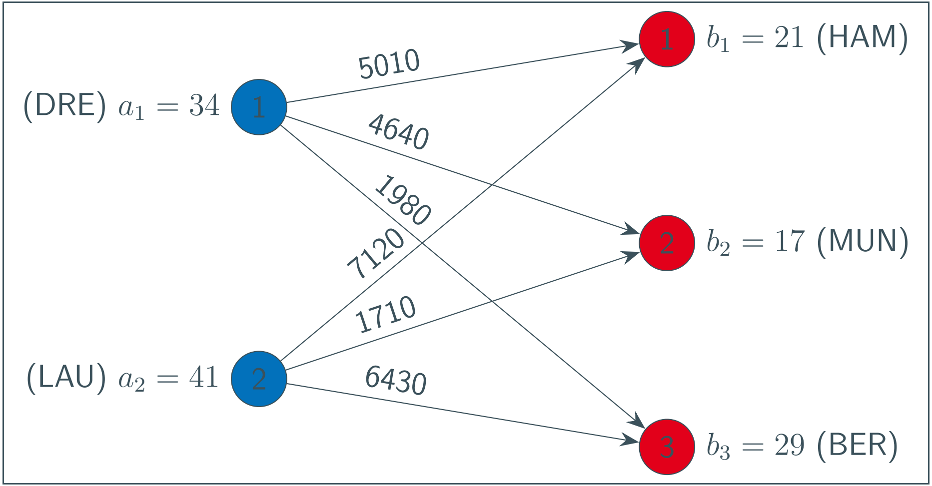
Solar Panel Transport

What could be the objective?

Minimizing the transport costs over all truckloads while meeting the demand based on the available solar panels adhering to the available panels.

Graphical Illustration

Graphically the problem can be illustrated as follows:



Available data

Sets

$\mathcal{I}$ set of production sites, indexed by i with $i \in \{1, \ldots, |\mathcal{I}|\}$ and
$\mathcal{J}$ set of customers, indexed j with $j \in \{1, \ldots, |\mathcal{J}|\}$.

Parameters

$c_{i,j}$ costs per truck load for transport from i to j,
a_i Available truck loads at i and
b_j Demand of customer at j.

Decision variable/s

We have the following sets:

- All the production sites, $i \in \mathcal{I}$
- All customers, $j \in \mathcal{J}$

Objective

Minimizing the transport costs over all truckloads while meeting the demand based on the available solar panels adhering to the available panels.

What could be our decision variable/s?

Decision variable/s

We have the following sets:

- All the production sites, $i \in \mathcal{I}$
- All customers, $j \in \mathcal{J}$

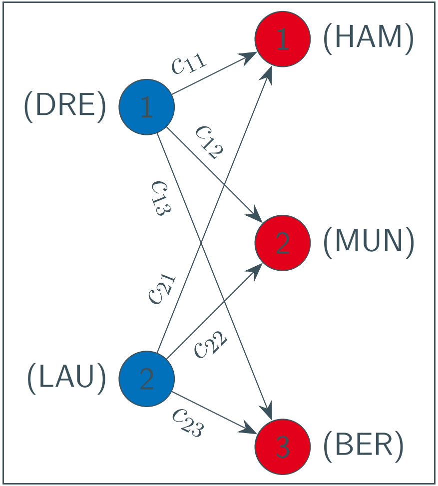
Objective

Minimizing the transport costs over all truckloads while meeting the demand based on the available solar panels adhering to the available panels.

What could be our decision variable/s?

$X_{i,j}$ the number of trucks that deliver panels from site i to customer j

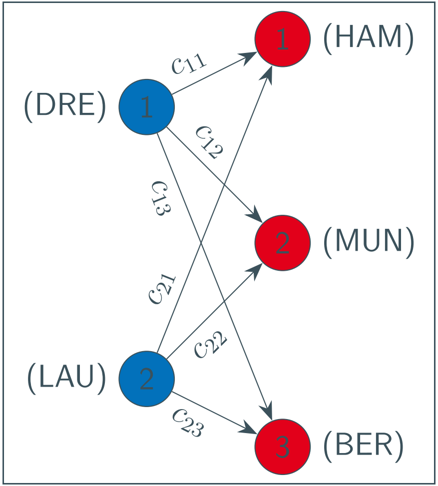
Objective Function



Objective Function Transport Problem

Minimizing the transport costs of truckloads while meeting the demand based on the available products.

Objective Function

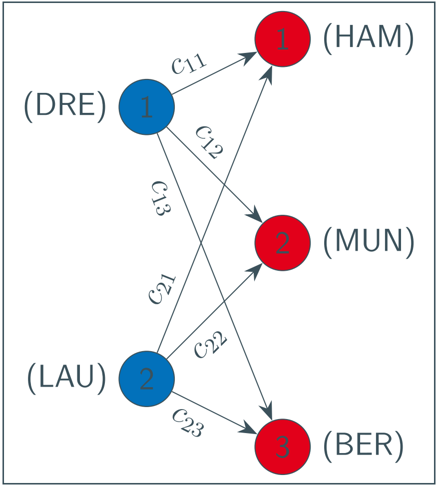


Objective Function Transport Problem

Minimizing the transport costs of truckloads while meeting the demand based on the available products.

$X_{i,j}$	number of trucks from i to j and
$c_{i,j}$	costs for transport from i to j.

Objective Function



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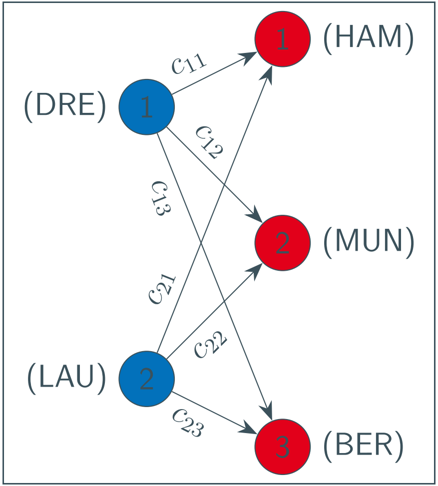
Objective Function



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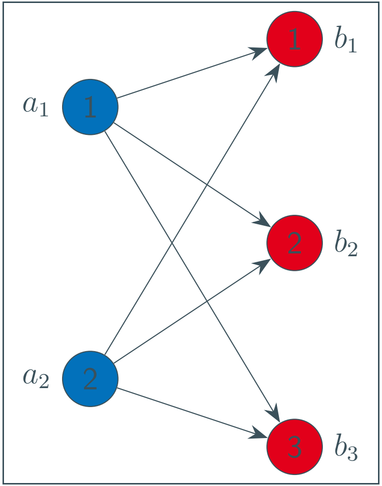
Minimizing the transport costs of truckloads while meeting the demand based on the available products.

$X_{i,j}$	number of trucks from i to j and
$c_{i,j}$	costs for transport from i to j.

What could be the objective function?

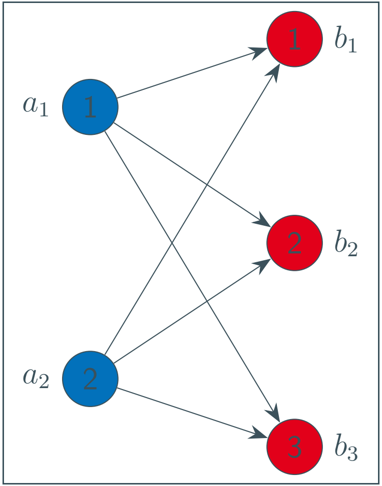
$$\text{Minimize} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} c_{i,j} \times X_{i,j}$$

Constraints



What kind of constraints do we need?

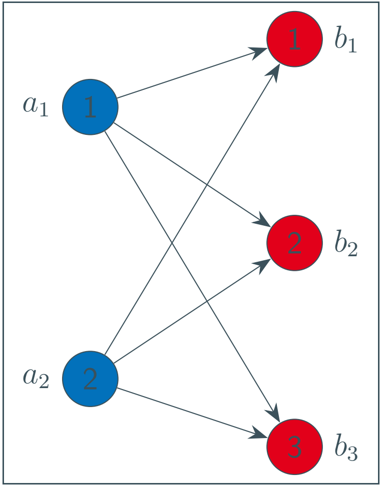
Constraints



What kind of constraints do we need?

- Ensure that the number of panels transported from a location does not exceed the available panels
- We have to ensure that the demand of each customer is covered

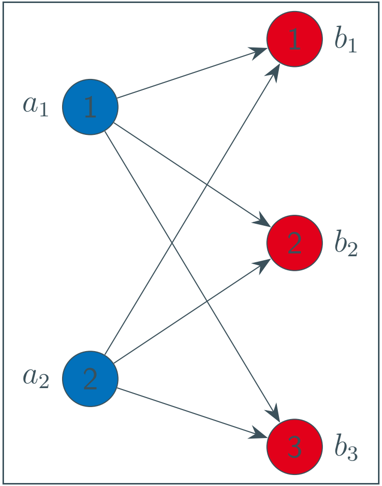
Constraints



Number of panels transported from a site does not exceed the available panels

$X_{i,j}$	number of trucks from i to j and
a_i	available panels at site i.

Constraints

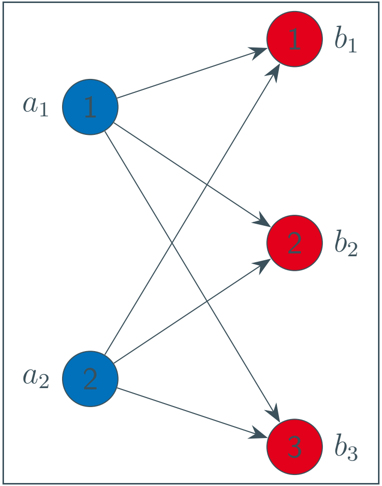


Number of panels transported from a site does not exceed the available panels

$X_{i,j}$	number of trucks from i to j and
a_i	available panels at site i.

How does the constraint look like?

Constraints



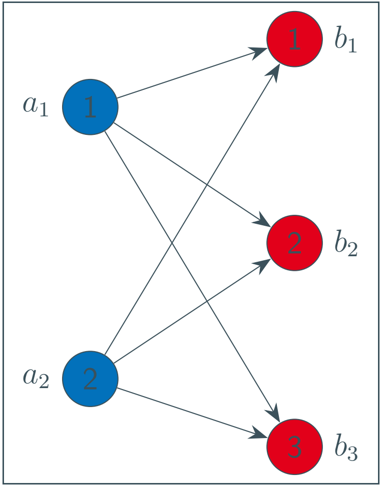
Number of panels transported from a site does not exceed the available panels

$X_{i,j}$	number of trucks from i to j and
a_i	available panels at site i.

How does the constraint look like?

$$\sum_{j \in \mathcal{J}} X_{i,j} \leq a_i \qquad i \in \mathcal{I}$$

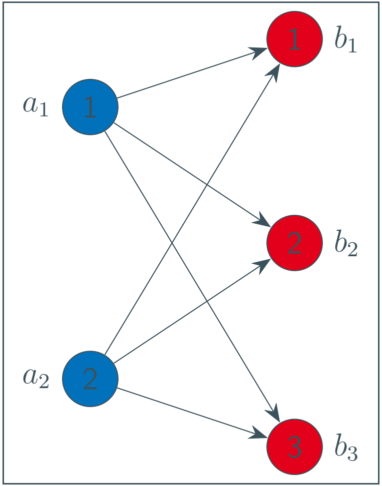
Constraints



Ensure that the demand of each customer is covered

$X_{i,j}$	number of trucks from i to j and
b_j	demand at customer i.

Constraints



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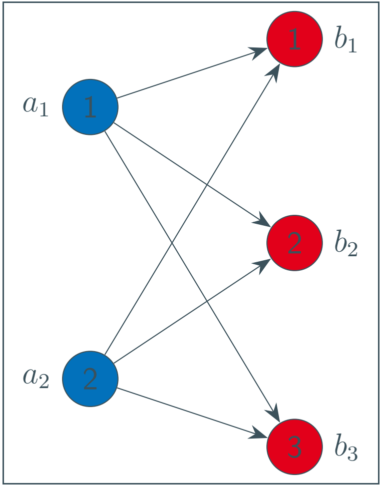

Ensure that the demand of each customer is covered

$X_{i,j}$	number of trucks from i to j and
b_j	demand at customer i.

How does the constraint look like?

$$\sum_{i \in \mathcal{I}} X_{i,j} = b_j \qquad j \in \mathcal{J}$$

Transport Problem

$$\text{Minimize} \quad F = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} c_{i,j} \times X_{ij} \tag{4}$$

subject to:

$$\sum_{j \in \mathcal{J}} X_{i,j} \leq a_i \qquad i \in \mathcal{I} \tag{5}$$

$$\sum_{i \in \mathcal{I}} X_{i,j} = b_j \qquad j \in \mathcal{J} \tag{6}$$

$$X_{i,j} \geq 0 \qquad i \in \mathcal{I}, j \in \mathcal{J} \tag{7}$$

Transport Problem

$$\text{Minimize} \quad F = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} c_{i,j} \times X_{ij} \tag{4}$$

subject to:

$$\sum_{j \in \mathcal{J}} X_{i,j} \leq a_i \qquad i \in \mathcal{I} \tag{5}$$

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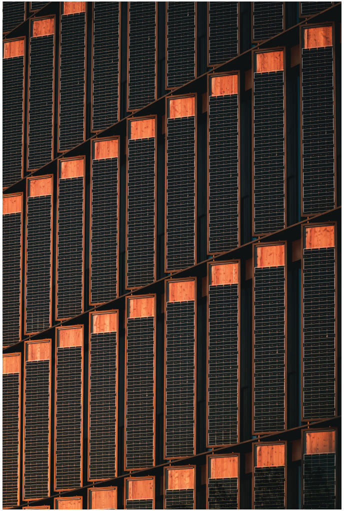
$$X_{i,j} \geq 0 \qquad i \in \mathcal{I}, j \in \mathcal{J} \tag{7}$$

Could we replace $=$ by $\geq$ in equation (6)?

Solar Panel Transport with Profit Maximization



Unfortunately, the margins on solar panels are low. After the previous contract has been fulfilled, the company produced the same number of panels as before. In addition, all three customers want to order the same number of truckloads with solar panels again. The sales volume per truckload of panels is 11,000 Euros. The complete production of a truckload of solar panels, including materials, costs 6,300 Euros. In the new contract, the company wants to maximize its profits while the demand has not necessarily have to be fulfilled. What changes?

Available data

Sets

$\mathcal{I}$ set of production sites, indexed by i with $i \in \{1, \ldots, |\mathcal{I}|\}$ and

$\mathcal{J}$ set of customers, indexed j with $j \in \{1, \ldots, |\mathcal{J}|\}$.

Parameters

$c_{i,j}$ costs per truck load for transport from i to j,

a_i Available truck loads at i,

b_j Demand of customer at j,

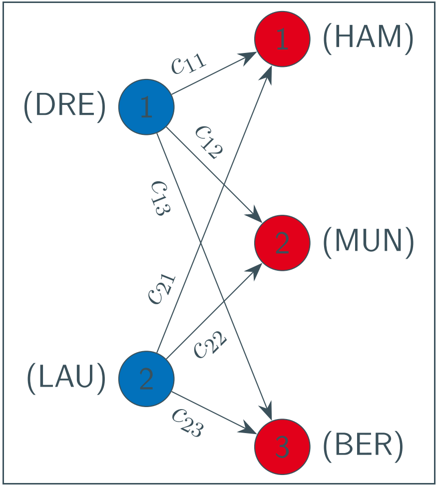
$\boldsymbol{p}$ Sales volume per truckload of solar panels and

$\boldsymbol{c}$ Production costs per truckload of solar panels.

Variable

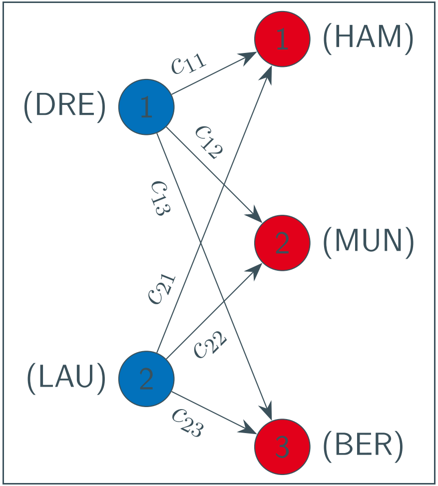
$X_{i,j}$ number of trucks from i to j.

Transport Problem



What do we change in our model?

Transport Problem



What do we change in our model?

We will talk about this in more detail in the first tutorial, after we have installed the Julia Programming Language.

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1.a. Installation of the Julia Programming Language

To prepare for the upcoming lectures, we start by installing the Julia Programming Language and an Integrated Development Environment to work with Julia.

1.a. Installation of the Julia Programming Language

If you are a windows user, head to `https://julialang.org` and download and install the current stable release of Julia for your operating system. Make sure to check the box "add to path" during the installation.

If your are on a unix based operating system, we would recommend to pass the following code into your terminal and follow the instructions to install the Julia Up Installation Manager:

$ curl -fsSL https://install.julialang.org — sh

1.a. Installation of the IDE

Next, we are going to install VS Code. Alternatively, you can install VS Codium which is essentially VS Code by Microsoft but without any tracking by Microsoft in the background.

Head to the website `https://code.visualstudio.com` or `https://vscodium.com` and download and install the latest release for your operating system.

1.a. Installation of the IDE

Great, afterwards start the IDE and check whether everything is working correctly.

Everything up and running? Search for the field ”Extensions” on the left side bar, click it and search for ”Julia”. Download and install ”Julia (Julia Language Support)”.

Now, create a new file with a ”.jl” ending and the content ”print(”Hello World!”)” and save it somewhere on your computer, e.g. in a folder that you will use in the course of the lecture.

1.a. Installation of the IDE

Run the file by clicking the "run" button in the upper right corner or by pressing "Control+Enter" or "STRG+Enter".

Does the terminal open with a "Hello World!"? If yes, perfect. If not, it is likely that the IDE cannot find the path to Julia. Determine the path and and save it, then try to run the file again.

Now we have everything set up for the next lecture. Thus, we are ready to look at the first modeling tasks in today's tutorial.

1.b. Solar Panel Transport

A company is producing solar panels in Dresden and Laupheim and has to transport them to new solar farms near Hamburg, Munich, and Berlin. The quantities offered and demanded (truckloads) and the transport costs per truckload in Euro are as follows:

Origin/Destination	Hamburg	Munich	Berlin	Available
Dresden	5010	4640	1980	34
Laupheim	7120	1710	6430	41
Demand	21	17	29	

Example: a truckload from Dresden ($i = 1$) to Munich ($j = 2$) costs $c_{12} = 4640$ Euro. Moreover, it is necessary to fulfill all customer demands, as the contract has already been signed.

1.b. Solar Panel Transport

$$\text{Minimize} \quad F = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} c_{i,j} \times X_{i,j} \tag{8}$$

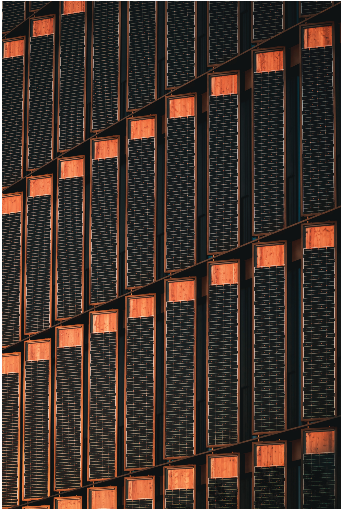
subject to:

$$\sum_{j \in \mathcal{J}} X_{i,j} \leq a_i \qquad i \in \mathcal{I} \tag{9}$$

$$\sum_{i \in \mathcal{I}} X_{i,j} \geq b_j \qquad j \in \mathcal{J} \tag{10}$$

$$X_{i,j} \geq 0 \qquad i \in \mathcal{I}, j \in \mathcal{J} \tag{11}$$

1.b. Solar Panel Transport



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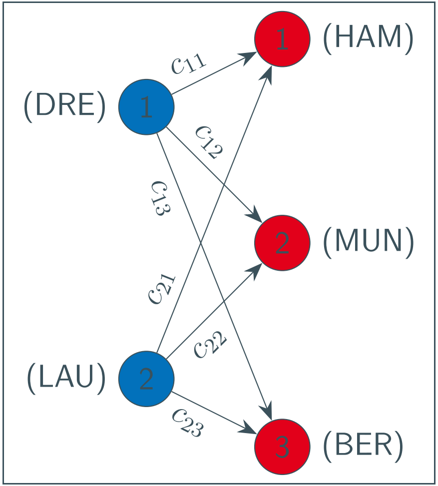
Variable

$X_{i,j}$ number of trucks from i to j.

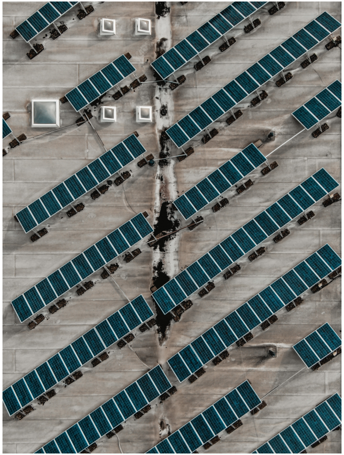
1.b. Solar Panel Transport



What could be the new objective of the company? What might we need to change in our model?

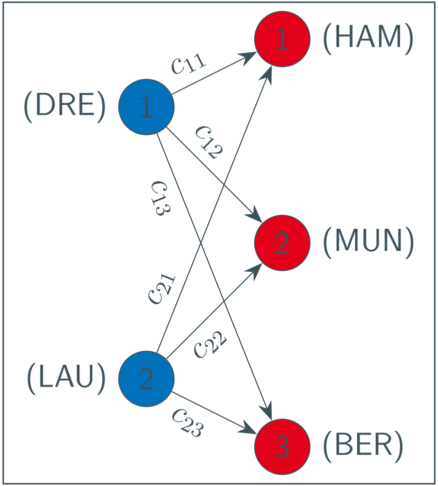
Answer both questions. Based on the changed specification, set up an algebraic model that incorporates the novel changes.

1.c. Solar Panel Transport



Due to new manufacturing restrictions, the company will have to produce and sell the same number of solar panels from each production site in the future.

1.c. Solar Panel Transport



What might we need to change in our model?

Based on the changed specification, set up an algebraic model that incorporates the novel changes.

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- Haase, Knut et al. (2016). "Improving pilgrim safety during the hajj: an analytical and operational research approach". In: *Interfaces* 46.1, pp. 74–90.
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