

# Lecture V - Planning in Breweries

## Applied Optimization with Julia

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Introduction

Case Study



- Large brewery
- Brews and sells beverages
- Production planning by hand
- Planner has a lot of experience
- But will retire soon

## Challenges



- Strong competition
- Customer demand is changing
- Craft beer gains popularity
- Variety of drinks is increasing
- Batch sizes are getting smaller

Different costs

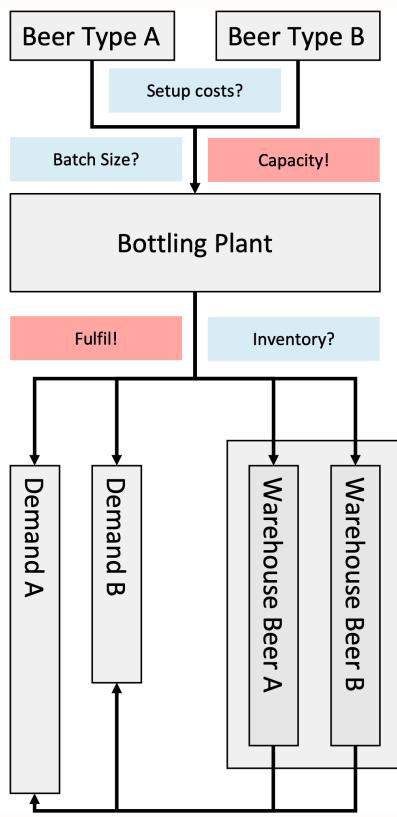


- Plant can fill multiple types
- Time depends on type and batch
- Changing type leads to set-up costs for preparation and cleaning
- Unsold beer bottles can be stored in a warehouse
- This leads to inventory costs

Where is the  
challenge?

## Problem Structure

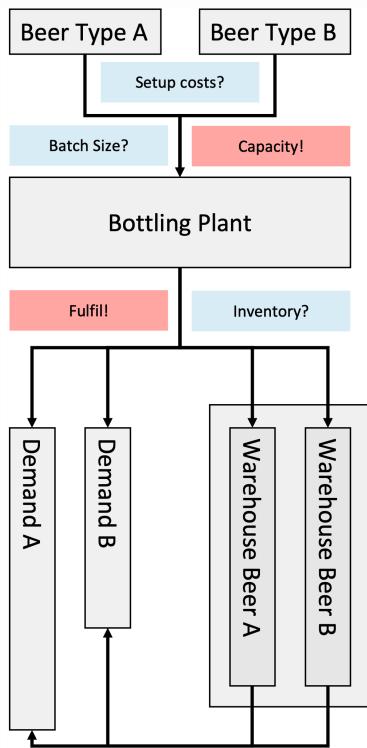
### Objective



Question: What could be the objective?

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

Trade-Off



Question: What is the trade-off?

Larger batches require less setup cost per bottle, but increase the storage cost.

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## Available Sets

Question: What are sets again?

...

Sets are collections of objects.

...

Question: What could be the sets here?

...

- $\mathcal{I}$  - Set of beer types indexed by  $i \in \{1, 2, \dots, |\mathcal{I}|\}$
- $\mathcal{T}$  - Set of time periods indexed by  $t \in \{1, 2, \dots, |\mathcal{T}|\}$

## Available Parameters

Question: What are possible parameters?

...

- $a_t$  - Available time on the bottling plant in period  $t \in \mathcal{T}$
- $b_i$  - Time used for bottling one unit of beer type  $i \in \mathcal{I}$
- $g_i$  - Setup time for beer type  $i \in \mathcal{I}$
- $f_i$  - Setup cost of beer type  $i \in \mathcal{I}$
- $c_i$  - Inventory holding cost for unit of beer type  $i \in \mathcal{I}$

- $d_{i,t}$  - Demand of beer type  $i \in \mathcal{I}$  in period  $t \in \mathcal{T}$

## Decision Variables?

**i** We have the following sets:

- Beer types indexed by  $i \in \{1, 2, \dots, |\mathcal{I}|\}$
- Time periods of the planning horizon indexed by  $t \in \{1, 2, \dots, |\mathcal{T}|\}$

...

**!** Our objective is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

...

Question: What could be our decision variable/s?

## Decision Variables

- $W_{i,t}$  - Inventory of type  $i \in \mathcal{I}$  at the end of  $t \in \mathcal{T}$
- $Y_{i,t}$  - 1, if type  $i \in \mathcal{I}$  is bottled in  $t \in \mathcal{T}$ , 0 otherwise
- $X_{i,t}$  - Batch size of type  $i \in \mathcal{I}$  in  $t \in \mathcal{T}$

## Model Formulation

### Objective Function?

**!** Our objective is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

...

Question: What could be our objective function?

...

**i** We need the following variables:

- $W_{i,t}$  - Inventory of type  $i \in \mathcal{I}$  at the end of  $t \in \mathcal{T}$
- $Y_{i,t}$  - 1, if type  $i \in \mathcal{I}$  is bottled in  $t \in \mathcal{T}$ , 0 otherwise

## Objective Function

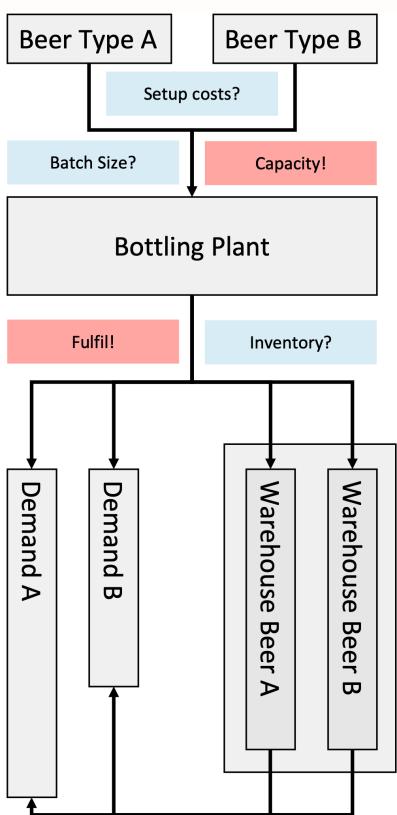
**i** We need the following parameters:

- $f_i$  - Setup cost of beer type  $i \in \mathcal{I}$
- $c_i$  - Inventory holding cost for one unit of beer type  $i \in \mathcal{I}$

...

$$\text{Minimize} \quad \sum_{i=1}^{\mathcal{I}} \sum_{t=1}^T (c_i \times W_{i,t} + f_i \times Y_{i,t})$$

## Constraints



Question: What constraints?

- Transfer unused inventory
- Fulfill the customer demand
- Set up beer types
- Calculate batch size per set-up
- Compute remaining inventory
- Limit the bottling plant

## Demand/Inventory Constraints?

! The goal of these constraints is to:

Consider the current inventory and batch sizes and compute the remaining inventory.

...

i We need the following variables and parameters:

- $W_{i,t}$  - Inventory of beer type  $i \in \mathcal{I}$  at the end of period  $t \in \mathcal{T}$
- $X_{i,t}$  - Batch size of beer type  $i \in \mathcal{I}$  in  $t \in \mathcal{T}$
- $d_{i,t}$  - Demand of beer type  $i \in \mathcal{I}$  in period  $t \in \mathcal{T}$

...

Question: What could the constraint look like?

## Demand/Inventory Constraints

$$W_{i,t-1} + X_{i,t} - W_{i,t} = d_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \mid t > 1$$

...

i Remember, these are the variables and parameters:

- $W_{i,t}$  - Inventory of beer type  $i \in \mathcal{I}$  at the end of period  $t \in \mathcal{T}$
- $X_{i,t}$  - Batch size of beer type  $i \in \mathcal{I}$  in  $t \in \mathcal{T}$
- $d_{i,t}$  - Demand of beer type  $i \in \mathcal{I}$  in period  $t \in \mathcal{T}$

...

Question: What does  $| t > 1$  mean?

## Setup Constraints?

! The goal of these constraints is to:

Set up beer types where the batch size is  $\geq 0$ .

...

i We need the following variables and parameters:

- $Y_{i,t}$  - 1, if beer type  $i \in \mathcal{I}$  is bottled in period  $t \in \mathcal{T}$ , 0 otherwise
- $X_{i,t}$  - Batch size of beer type  $i \in \mathcal{I}$  in  $t \in \mathcal{T}$
- $d_{i,t}$  - Demand of beer type  $i \in \mathcal{I}$  in period  $t \in \mathcal{T}$

...

Question: What could the second constraint be?

### Setup Constraints

$$X_{i,t} \leq Y_{i,t} \times \sum_{\tau=1}^{\mathcal{T}} d_{i\tau} \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

...

Question: Do you know this type of constraint?

...

This type of constraint is called a “Big-M” constraint!

...

- M (here  $\sum_{\tau=1}^{\mathcal{T}} d_{i\tau}$ ) is a large number
- It is coupled with a binary variable (here  $Y_{i,t}$ )
- Like an if-then constraint

### Capacity Constraints?

! The goal of these constraints is to:

Limit the capacity of the bottling plant per period.

...

i We need the following variables and parameters:

- $Y_{i,t}$  - 1, if beer type  $i \in \mathcal{I}$  is bottled in period  $t \in \mathcal{T}$ , 0 otherwise
- $X_{i,t}$  - Batch size of beer type  $i \in \mathcal{I}$  in  $t \in \mathcal{T}$
- $a_t$  - Available time on the bottling plant in period  $t \in \mathcal{T}$
- $b_i$  - Time used for bottling one unit of beer type  $i \in \mathcal{I}$
- $g_i$  - Setup time for beer type  $i \in \mathcal{I}$

### Capacity Constraints

Question: What could the third constraint be?

It has more variables and parameters when compared to the other constraints but it is easier to understand.

...

$$\sum_{i=1}^{\mathcal{I}} (b_i \times X_{i,t} + g_i \times Y_{i,t}) \leq a_t \quad \forall t \in \mathcal{T}$$

...

And that's basically it!

### CLSP: Objective Function

$$\text{Minimize} \quad \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} (c_i \times W_{i,t} + f_i \times Y_{i,t})$$

! The goal of the objective function is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

### CLSP: Constraints

$$W_{i,t-1} + X_{i,t} - W_{i,t} = d_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \mid t > 1$$

$$X_{i,t} \leq Y_{i,t} \times \sum_{\tau \in \mathcal{T}} d_{i,\tau} \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

$$\sum_{i \in \mathcal{I}} (b_i \times X_{i,t} + g_i \times Y_{i,t}) \leq a_t \quad \forall t \in \mathcal{T}$$

! Our constraints ensure:

Demand is met, inventory transferred, setup taken care of, and capacity respected.

### CLSP: Variable Domains

$$Y_{i,t} \in \{0, 1\} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$$

$$W_{i,t}, X_{i,t} \geq 0 \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$$

! The variable domains make sure that:

The binary setup variable is either 0 or 1 and that the inventory and batch size are non-negative.

## Julia Implementation

### Variables in Julia

Question: How could we formulate the variables in Julia?

...

```
using JuMP, HiGHS
beer_types = ["IPA", "Lager", "Stout"] # Beer types
periods = ["week_1", "week_2", "week_3"] # Time periods
clsp_model = Model(HiGHS.Optimizer)
```

...

```
@variable(clsp_model, Y[i in beer_types, t in periods], Bin)
@variable(clsp_model, X[i in beer_types, t in periods] >= 0)
@variable(clsp_model, W[i in beer_types, t in periods] >= 0)
```

```
2-dimensional DenseAxisArray{VariableRef,2,...} with index sets:
    Dimension 1, ["IPA", "Lager", "Stout"]
    Dimension 2, ["week_1", "week_2", "week_3"]
And data, a 3x3 Matrix{VariableRef}:
W[IPA,week_1]    W[IPA,week_2]    W[IPA,week_3]
W[Lager,week_1]   W[Lager,week_2]   W[Lager,week_3]
W[Stout,week_1]   W[Stout,week_2]   W[Stout,week_3]
```

## Objective Function in Julia

$$\text{Minimize } \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} (c_i \times W_{i,t} + f_i \times Y_{i,t})$$

...

```
c = Dict("IPA" => 0.1, "Lager" => 0.1, "Stout" => 0.1) # Holding costs
f = Dict("IPA" => 5000, "Lager" => 4000, "Stout" => 6000) # Setup costs
```

...

```
@objective(clsp_model, Min,
    sum(c[i] * W[i,t] + f[i] * Y[i,t]
        for i in beer_types, t in periods)
)
```

## Model Characteristics

### Recap on some Basics

There exist several types of optimization problems:

- Linear (LP): Linear constraints and objective function
- Mixed-integer (MIP): Linear constraints and objective function, but discrete variable domains
- Quadratic (QP): Quadratic constraints and/or objective
- Non-linear (NLP): Non-linear constraints and/or objective
- And more!

### Recap on Solution Algorithms

- Simplex algorithm to solve LPs
- Branch & Bound to solve MIPs
- Outer-Approximation for mixed-integer NLPs
- Math-Heuristics (e.g., Fix-and-Optimize, Tabu-Search, ...)
- Decomposition methods (Lagrange, Benders, ...)

- Heuristics (greedy, construction method, n-opt, ...)
- Graph theoretical methods (network flow, shortest path)

## Model Characteristics

Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?
- What kind of solver could we use?
- Can the Big-M constraint be tightened?

## Model Assumptions

Questions: On model assumptions

- What assumptions have we made?
- What is the problem with the planning horizon?
- Any idea how to solve it?

## Impact

Can this be  
applied?

## Scale as a Problem

Solving the problem with commercial solvers is not feasible.

## Scale of the Case Study

- 220 finished products
- 100 semi-finished products
- 13 production resources
- 8 storage resources
- 3 main production levels
- 52 weeks planning horizon

Any idea what  
could be done?

## Heuristics and Optimization

- Multi-level Capacitated Lot-Sizing Problem
- Heuristic fix and optimize approach<sup>1</sup>
- Operating cost reduction by 5% and planning effort by 40%

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<sup>1</sup>M. Mickein, M. Koch, and K. Haase [1]

...

**i** And that's it for todays lecture!

We now have covered the basics of the CLSP and are ready to start solving some tasks in the upcoming tutorial.

Questions?

## Literature

### Literature I

For more interesting literature to learn more about Julia, take a look at the [literature list](#) of this course.

### Literature II

### Bibliography

- [1] M. Mickein, M. Koch, and K. Haase, “A Decision Support System for Brewery Production Planning at Feldschlösschen,” INFORMS Journal on Applied Analytics, vol. 52, no. 2, pp. 158–172, 2022.