Arena Seat Planning under Distancing Rules

Dkaidik, Usama and Koch, Matthes

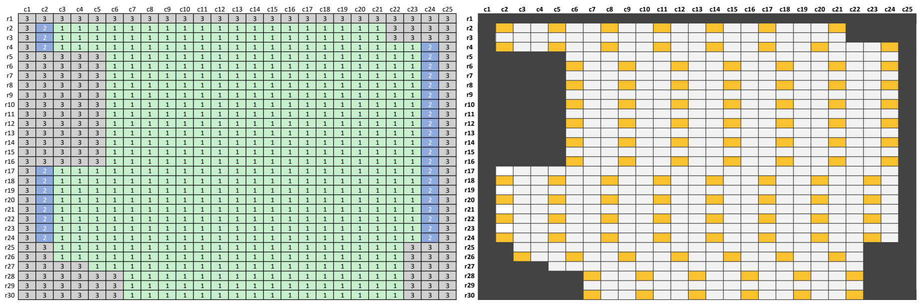
University of Hamburg, Institute of Logistics, Transport and Production,
Moorweidenstraße 18, 20148 Hamburg, Germany
usama.dkaidik@uni-hamburg.de
matthes.koch@uni-hamburg.de

Abstract. The COVID-19 pandemic has significantly impacted the organization and attendance of live events, especially in confined spaces like public sports and entertainment arenas. We have developed models for optimal arena seating that adhere to social distancing protocols to address these challenges. Our approach involves creating modified 2D knapsack-type model instances for event arenas, allowing for the quick generation of seating layouts while considering different objective criteria. We have explored solution approaches on operational/tactical and strategic levels, tailored to different event types and priorities such as maximizing overall attendance or accommodating VIP and sponsor groups. Our collaboration with a German football club has demonstrated the effectiveness and adaptability of our model, and the potential for substantial gains, 12% in seating capacity utilization and event revenue compared to manual plan generation approaches. Our framework offers a robust decision-making tool that event planners can quickly adapt to infrastructures requiring similar social distancing protocols.

1 Introduction

The COVID-19 pandemic has drastically altered the landscape of public events, requiring stringent safety measures to prevent the spread of the virus [3]. Social distancing, a critical measure, significantly affects seating in venues like sports arenas and theatres. During the 2020 peak, distancing and lockdown rules caused a sharp drop in attendance revenue. For instance, the German football league's top 18 clubs saw match day revenues fall from 520 million EUR in 2018/2019 to 22 million EUR in 2020/2021.[5].

Figure 1 shows the grid layout of an artificial stadium block, with available seats are labelled 1, seats on the aisles are labelled 2, and category 3 represents infrastructure unavailable for seating, such as stairs or media coverage zones. Figure 1 (b) shows a social distancing implementation with two empty seats between each spectator and every second row empty, a common post-lockdown practice in German stadiums. However, such layouts drastically reduce usable space, as shown by only 108 of 581 seats being allocated, or 18.6% capacity. Venue managers face the challenge to find concepts that enable higher capacity utilization while respecting social distancing rule sets at the same time.



(a) Grid of an arena block (b) Manual solution

Fig. 1: Layout of an example block with manually constructed 1-seat layout

This paper explores the problem of arena seat planning under distancing rules, focusing on formulating and implementing an optimization model that allocates groups into the layout grid. Unlike single-seat layouts, grouping households who accept close seating can use space more efficiently. In [1], the authors tackle the problem by implementing an optimization model based on trapezoid packings. In contrast, this study presents a two-dimensional knapsack-type formulation [4] and a decomposition approach to provide a robust framework for optimizing seating layouts.

The proposed model considers group sizes, seating preferences, and safety distances to generate optimal seating plans. This research contributes to event management and offers practical solutions for safely hosting large gatherings during and after the pandemic.

2 Problem Structure

The problem of arena seat planning under distancing rules involves several key components: the physical layout of the venue, the demand, and the distancing requirements. Each component presents unique challenges that must be addressed to develop an effective seating plan.

Venues such as sports arenas, concert halls, and theaters have fixed seating layouts, often with varying row lengths, obstacles, and aisles. The seating plan must account for these structural elements to ensure feasibility. For instance, specific seats may be blocked due to structural features or reserved for particular purposes, such as VIP seating or media coverage. Depending on the seats installed in a venue, switching easily from one row to another may be physically possible. If that is not possible, a group may be unable to avoid crossing other groups at close distance when moving along the row to leave the auditorium. In that case, a hard constraint must limit the number of groups allowed to be allocated to a row.

Attendees are a key factor in the arena seat planning problem. We assume, that they may arrive in groups of varying sizes, from individuals to families or friend groups. Each group has specific seating preferences, often preferring to sit together. To simplify implementation, we assume that the group members need to sit next to each other. However, extending the problem to multi-row group assignments is straight-forward. We further assume a group based demand for allocating the seats in the blocks. As shown in figure 1, we model the seating plan as a grid $\mathcal{A}$ of rows $r \in \mathcal{R}$ and columns $c \in \mathcal{C}$. We can usually decompose the venue into several independent blocks, each with its own entrances, stairs, etc. Figure 2 shows two blocks modeled for the football arena in Osnabrück, Germany. As such, the planning problem can be block-wise decomposed into sub problems and be solved in parallel.

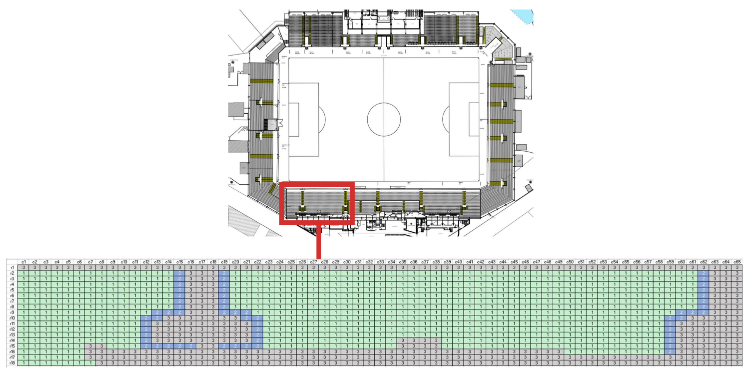


Fig. 2: Two blocks of a football arena

The primary constraint in this problem is maintaining safe distances between groups to prevent the spread of COVID-19. Because the width of the seats is known to the planner, the regulated distance between groups, usually given in meters, can be expressed as the number of empty seats required between the groups. The specific distancing requirements can vary based on local regulations and venue policies.

Individual seats can be excluded from the set of available seats. This can be useful to prohibit the use of seats at the aisles (see seats labeled 2 in Figure 1a and Figure 2) to ensure distancing between groups moving on the aisles and groups sitting close by.

3 Model Formulation

In its most basic form, the arena seat planning problem (ASPP) formulation is similar to the two-dimensional knapsack problem [4]. In it, the spectator groups $g \in \mathcal{G}$ are the items to be allocated. Each allocation results in the realization of the group's score v_g. Using this data parameter, we model priorities for groups. A high priority may be given to long-term season holders, VIPs, sponsors, and others. Alternatively, the priority may be calculated based on the group's number of past ticket assignments. A simple scheme would be to set v_g equal to the number of requested seats of group g. This approach would maximize seat capacity utilization as it favors the assignment of larger groups. We maximize the realized group values v_g by filling the seating area given distancing regulations between groups and venue-specific constraints. We define the decision variable:

$X_{g,r,c} = 1$ if the first seat of group g is assigned to row r in column c, otherwise 0.

To implement social distancing constraints, we define knapsack-type cover sets, including the relevant distance buffers. Specifically, for each allocation of group g to position (r, c) in the grid, we define the set $\mathcal{C}_{g,r,c}$, which contains all seats (r', c') that together represent the reserved space for this group allocation. We define value n to restrict the number of groups in each row. The reserved space includes safety buffers and, if realized, is not allowed to overlap the reserved space of another realized group allocation in a feasible solution. The set of feasible seat allocations of a group g is denoted $\mathcal{K}_g$. Now, we can maximize the total group values F of our seating area with the ASPP:

Maximize

$$F = \sum_{g \in \mathcal{G}} \sum_{(r,c) \in \mathcal{K}_g} v_g X_{g,r,c} \tag{1}$$

subject to

$$\sum_{(r,c) \in \mathcal{K}_g} X_{g,r,c} \le 1 \qquad \forall g \in \mathcal{G} \tag{2}$$

$$\sum_{g \in \mathcal{G}} \sum_{\substack{c \in \mathcal{S} | \\ (r,c) \in \mathcal{K}_g}} X_{g,r,c} \le n \qquad \forall r \in \mathcal{Z} \tag{3}$$

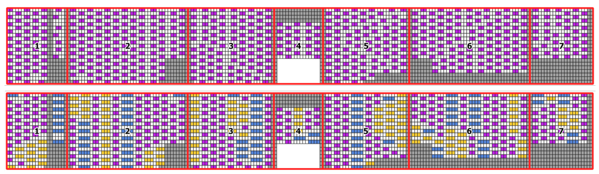
$$\sum_{g \in \mathcal{G}} \sum_{\substack{(r,c) \in \mathcal{K}_g | \\ (r',c') \in \mathcal{C}_{g,r,c}}} X_{g,r,c} \le 1 \qquad \forall (r', c') \in \mathcal{A} \tag{4}$$

$$X_{g,r,c} \in \{0, 1\} \qquad \forall g \in \mathcal{G},\ (r, c) \in \mathcal{K}_g \tag{5}$$

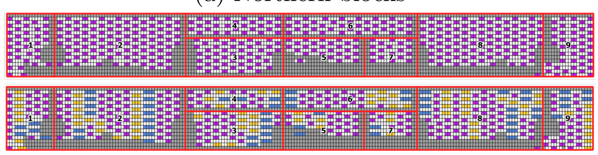
Binary variable X captures the seating assignment for each group, assigning groups to a specific row $r \in \mathcal{Z}$ and column $c \in \mathcal{S}$. Objective function (1) maximizes the total value of the allocation. Constraint (2) ensures that no group is assigned to multiple locations. We limit the number of groups assigned to a single row r to the scalar value n in (3). A typical block with exits on both sides of the row would require $n = 2$ to avoid situations where two groups pass each other at close distance. In a less strict regimen, n might be set to higher values or the constraint may be dropped entirely if no such requirements exist. The social distancing rules are implemented by constraints (4), and the definition of the cover sets $\mathcal{C}_{g,r,c}$ in accordance with the required distances.

4 Implementation and Impact

To illustrate the implementation, consider the football arena layout introduced in Figure 2 above. We modeled the arena by defining 16 block grids. To define a realistic scenario, we compare the basic two-seat layout (*Alloc-Basic*) used in a regular match with an optimized layout (*Alloc-Opt*). For the final season 2020/2021 match, social distancing regulations required two empty seats between each group and one empty seat in front and behind each allocated seat. We allow seat allocation near the aisles and multiple groups per row as this would be consistent with the pattern used in the past. This specific set of rules represents a less strict version that was applied in the later stages of the pandemic.



(a) Northern blocks



(b) Southern blocks

Fig. 3: Seating blocks in arena 'Bremer Brücke' in Osnabrück, Germany

Using past ticket sales data from the football club, we estimated the group demand, i.e., the number of groups and their respective sizes. We modeled the optimization problem in the algebraic language *GAMS*[2] and used the *CPLEX* mixed-integer solver to calculate optimal layouts for the arena. Figure 3 depicts the seven northern and nine southern blocks for the two compared solutions. Table 1 shows that seating capacity utilization was improved by more than 12%. While the smaller blocks, in particular, yielded significant improvements in utilization, the improvement varies between 8,3, and 17,9% over the different blocks with a mixture of groups between two and six people.

5 Conclusion

While the numerical study aims to realistically represent implementation impacts, capacity utilization depends heavily on assumed demand and distancing rules.

The model can generally be used on an operational or strategic level. Operationally, the approach can produce ideal layouts for a given demand. However,

Table 1: Seat allocations

South					North				
Block	Seats	Alloc-Basic	Alloc-Opt	Improvement	Block	Seats	Alloc-Basic	Alloc-Opt	Improvement
South-01	214	72	83	15.3%	North-01	349	118	135	14.4%
South-02	587	200	218	9.0%	North-02	788	258	290	12.4%
South-03	281	94	105	11.7%	North-03	586	190	217	14.2%
South-04	165	56	61	8.9%	North-04	150	50	56	12.0%
South-05	153	54	60	11.1%	North-05	549	178	204	14.6%
South-06	225	76	83	9.2%	North-06	692	226	254	12.4%
South-07	105	36	39	8.3%	North-07	266	84	99	17.9%
South-08	576	196	213	8.7%					
South-09	251	84	96	14.3%	Total	5937	1972	2213	12.2%

the collection and real-time processing of demand data has proven challenging in practice, mainly since many venue operators rely on standard booking webshops, which may not provide the capability to collect demand first and allocate it in a second step. However, the model can also be used on a strategic/tactical level to determine which seats should be made available for ticket selling, given the desired mix of group sizes for an event. In that case, the model can be modified to approximate the desired mix as closely as possible when allocating groups in the blocks. In this context, venue managers can experiment with the different setups of group mixes and distancing rules to develop safe seating concepts. In discussions with the regulatory and health authorities, these unbiased model-based results form a solid basis for decision-making that combines safety considerations, commercial interests, and audience satisfaction

By adopting the proposed optimization framework, venues can better navigate the complexities of hosting events during and beyond the pandemic, ensuring a safe and enjoyable experience for all attendees.

References

1. Blom, D., Pendavingh, R., Spieksma, F.: Filling a theater during the covid-19 pandemic. INFORMS Journal on Applied Analytics **52**(6), 473–484 (11 2022). https://doi.org/10.1287/inte.2021.1104
2. Bussieck, M., Meeraus, A.: General Algebraic Modeling System (GAMS), pp. 137–157. Springer US (2004). https://doi.org/10.1007/978-1-4613-0215-5˙8
3. Courtemanche, C., Garuccio, J., Le, A., Pinkston, J., Yelowitz, A.: Strong social distancing measures in the united states reduced the covid-19 growth rate. Health Affairs **39**(7), 1237–1246 (7 2020). https://doi.org/10.1377/hlthaff.2020.00608
4. Hadjiconstantinou, E., Christofides, N.: An exact algorithm for general, orthogonal, two-dimensional knapsack problems. European Journal of Operational Research **83**(1), 39–56 (5 1995). https://doi.org/10.1016/0377-2217(93)e0278-6
5. Statista: Erlöse der 1. Fußball-Bundesliga von der Saison 2010/2011 bis zur Saison 2022/2023 nach Einnahmequelle (in Millionen Euro). Tech. rep., DFL (Mar 2024), https://de.statista.com/statistik/daten/studie/6530/umfrage/erloese-der-1-fussballbundesliga/