

# Lecture V - Production Planning in Breweries

Applied Optimization with Julia

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# Introduction

# Case Study



- Large brewery
- Brews and sells beverages
- Production planning by hand
- Planner has a **lot of experience**
- **But** will retire soon

# Challenges



- **Strong competition**
- **Customer demand is changing**
- **Craft beer gains popularity**
- **Variety of drinks is increasing**
- **Batch sizes are getting smaller**

# Different costs

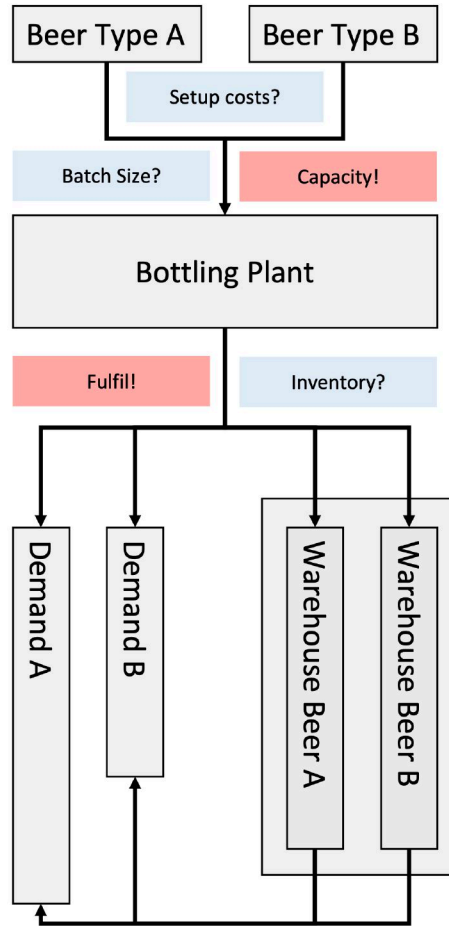


- Plant can fill **multiple types**
- Time depends on **type and batch**
- **Changing type** leads to set-up costs for preparation and cleaning
- **Unsold beer bottles** can be **stored in a warehouse**
- This leads to **inventory costs**

Where is the  
challenge?

# Problem Structure

# Objective

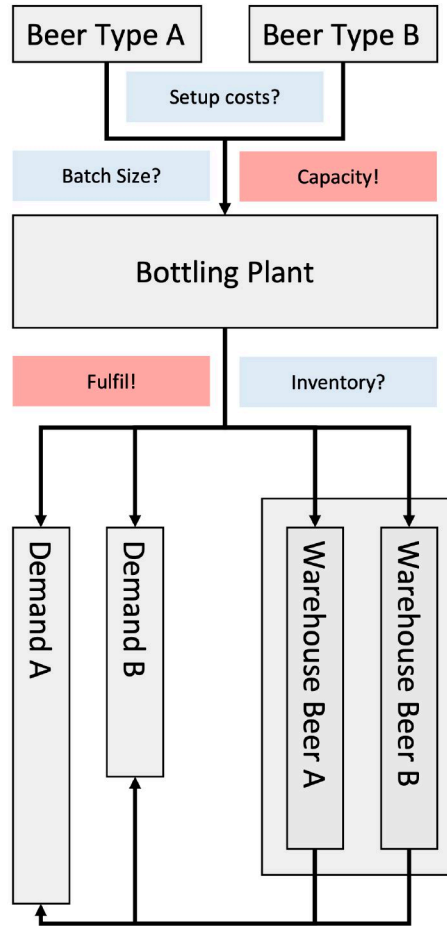


**Question:** What could be the objective?

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.



# Trade-Off



**Question:** What is the trade-off?

Larger batches require **less setup cost per bottle**, but increase the storage cost.

# Available Sets

**Question:** What are sets again?

Sets are **collections of objects**.

**Question:** What could be the sets here?

- $\mathcal{I}$  - Set of beer types indexed by  $i \in \{1, 2, \dots, |\mathcal{I}|\}$
- $\mathcal{T}$  - Set of time periods indexed by  $t \in \{1, 2, \dots, |\mathcal{T}|\}$

# Available Parameters

**Question:** What are possible parameters?

- $a_t$  - Available time on the bottling plant in period  $t \in \mathcal{T}$
- $b_i$  - Time used for bottling one unit of beer type  $i \in \mathcal{I}$
- $g_i$  - Setup time for beer type  $i \in \mathcal{I}$
- $f_i$  - Setup cost of beer type  $i \in \mathcal{I}$
- $c_i$  - Inventory holding cost for one unit of beer type  $i \in \mathcal{I}$
- $d_{i,t}$  - Demand of beer type  $i \in \mathcal{I}$  in period  $t \in \mathcal{T}$

# Decision Variables?

 We have the following sets:

- Beer types indexed by  $i \in \{1, 2, \dots, |\mathcal{I}|\}$
- Time periods of the planning horizon indexed by  $t \in \{1, 2, \dots, |\mathcal{T}|\}$

 Our objective is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

**Question:** What could be our decision variable/s?

# Decision Variables

- $W_{i,t}$  - Inventory of type  $i \in \mathcal{I}$  at the end of  $t \in \mathcal{T}$
- $Y_{i,t}$  - 1, if type  $i \in \mathcal{I}$  is bottled in  $t \in \mathcal{T}$ , 0 otherwise
- $X_{i,t}$  - Batch size of type  $i \in \mathcal{I}$  in  $t \in \mathcal{T}$

# Model Formulation

# Objective Function?

! Our objective is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

**Question:** What could be our objective function?

i We need the following variables:

- $W_{i,t}$  - Inventory of type  $i \in \mathcal{I}$  at the end of  $t \in \mathcal{T}$
- $Y_{i,t}$  - 1, if type  $i \in \mathcal{I}$  is bottled in  $t \in \mathcal{T}$ , 0 otherwise

# Objective Function

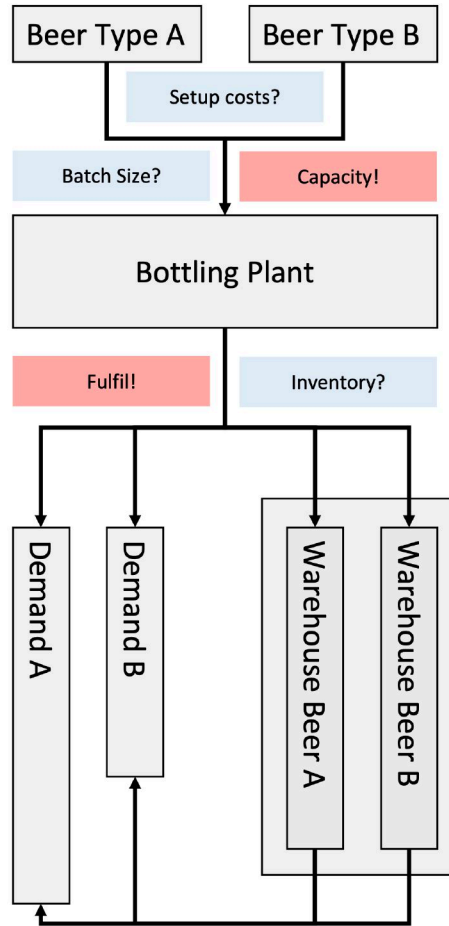
 We need the following parameters:

- $f_i$  - Setup cost of beer type  $i \in \mathcal{I}$
- $c_i$  - Inventory holding cost for one unit of beer type  $i \in \mathcal{I}$

$$\text{Minimize} \quad \sum_{i=1}^{\mathcal{I}} \sum_{t=1}^{\mathcal{T}} (c_i \times W_{i,t} + f_i \times Y_{i,t})$$



# Constraints



**Question:** What constraints?

- Transfer **unused** inventory
- **Fulfill** the customer demand
- **Set up** beer types
- Calculate the **batch size** per set-up
- Compute **remaining** inventory
- **Limit** the bottling plant

# Demand/Inventory Constraints?

! The goal of these constraints is to:

Consider the current inventory and batch sizes and compute the remaining inventory.

i We need the following variables and parameters:

- $W_{i,t}$  - Inventory of beer type  $i \in \mathcal{I}$  at the end of period  $t \in \mathcal{T}$
- $X_{i,t}$  - Batch size of beer type  $i \in \mathcal{I}$  in  $t \in \mathcal{T}$
- $d_{i,t}$  - Demand of beer type  $i \in \mathcal{I}$  in period  $t \in \mathcal{T}$

**Question:** What could the constraint look like?

# Demand/Inventory Constraints

$$W_{i,t-1} + X_{i,t} - W_{i,t} = d_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} | t > 1$$

 Remember, these are the variables and parameters:

- $W_{i,t}$  - Inventory of beer type  $i \in \mathcal{I}$  at the end of period  $t \in \mathcal{T}$
- $X_{i,t}$  - Batch size of beer type  $i \in \mathcal{I}$  in  $t \in \mathcal{T}$
- $d_{i,t}$  - Demand of beer type  $i \in \mathcal{I}$  in period  $t \in \mathcal{T}$

**Question:** What does  $|t > 1$  mean?

# Setup Constraints?

! The goal of these constraints is to:

Set up beer types where the batch size is  $\geq 0$ .

i We need the following variables and parameters:

- $Y_{i,t}$  - 1, if beer type  $i \in \mathcal{I}$  is bottled in period  $t \in \mathcal{T}$ , 0 otherwise
- $X_{i,t}$  - Batch size of beer type  $i \in \mathcal{I}$  in  $t \in \mathcal{T}$
- $d_{i,t}$  - Demand of beer type  $i \in \mathcal{I}$  in period  $t \in \mathcal{T}$

**Question:** What could the second constraint be?

# Setup Constraints

$$X_{i,t} \leq Y_{i,t} \times \sum_{\tau=1}^{\mathcal{T}} d_{i\tau} \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

**Question:** Do you know this type of constraint?

This type of constraint is called a “**Big-M**” constraint!

- M (here  $\sum_{\tau=1}^{\mathcal{T}} d_{i\tau}$ ) is a large number
- It is coupled with a binary variable (here  $Y_{i,t}$ )
- **Like an if-then constraint**

# Capacity Constraints?

! The goal of these constraints is to:

Limit the capacity of the bottling plant per period.

i We need the following variables and parameters:

- $Y_{i,t}$  - 1, if beer type  $i \in \mathcal{I}$  is bottled in period  $t \in \mathcal{T}$ , 0 otherwise
- $X_{i,t}$  - Batch size of beer type  $i \in \mathcal{I}$  in  $t \in \mathcal{T}$
- $a_t$  - Available time on the bottling plant in period  $t \in \mathcal{T}$
- $b_i$  - Time used for bottling one unit of beer type  $i \in \mathcal{I}$
- $g_i$  - Setup time for beer type  $i \in \mathcal{I}$

# Capacity Constraints

**Question:** What could the third constraint be?

It has more variables and parameters when compared to the other constraints but it is easier to understand.

$$\sum_{i=1}^{\mathcal{I}} (b_i \times X_{i,t} + g_i \times Y_{i,t}) \leq a_t \quad \forall t \in \mathcal{T}$$

**And that's basically it!**

# CLSP: Objective Function

$$\text{Minimize} \quad \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} (c_i \times W_{i,t} + f_i \times Y_{i,t})$$

❗ The goal of the objective function is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.



# CLSP: Constraints

$$W_{i,t-1} + X_{i,t} - W_{i,t} = d_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} | t > 1$$

$$X_{i,t} \leq Y_{i,t} \times \sum_{\tau \in \mathcal{T}} d_{i,\tau} \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

$$\sum_{i \in \mathcal{I}} (b_i \times X_{i,t} + g_i \times Y_{i,t}) \leq a_t \quad \forall t \in \mathcal{T}$$

! Our constraints ensure:

Demand is met, inventory transferred, setup taken care of, and capacity respected.

# CLSP: Variable Domains

$$Y_{i,t} \in \{0, 1\} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$$

$$W_{i,t}, X_{i,t} \geq 0 \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$$

⚠ The variable domains make sure that:

The binary setup variable is either 0 or 1 and that the inventory and batch size are non-negative.

# Model Characteristics

# Recap on some Basics

There exist several types of optimization problems:

- **Linear (LP):** Linear constraints and objective function
- **Mixed-integer (MIP):** Linear constraints and objective function, but discrete variable domains
- **Quadratic (QP):** Quadratic constraints and/or objective
- **Non-linear (NLP):** Non-linear constraints and/or objective
- **And more!**

# Recap on Solution Algorithms

- **Simplex algorithm** to solve LPs
- **Branch & Bound** to solve MIPs
- **Outer-Approximation** for mixed-integer NLPs
- **Math-Heuristics** (e.g., Fix-and-Optimize, Tabu-Search, ...)
- **Decomposition methods** (Lagrange, Benders, ...)
- **Heuristics** (greedy, construction method, n-opt, ...)
- **Graph theoretical methods** (network flow, shortest path)

# Model Characteristics

## Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?
- What kind of solver could we use?
- Can the Big-M constraint be tightened?

# Model Assumptions

## Questions: On model assumptions

- What assumptions have we made?
- What is the problem with the planning horizon?
- Any idea how to solve it?

# Impact



Can this be  
applied?



# Scale as a Problem

Solving the problem with commercial solvers is not feasible.





# Scale of the Case Study

- 220 finished products
- 100 semi-finished products
- 13 production resources
- 8 storage resources
- 3 main production levels
- 52 weeks planning horizon

Any idea what  
could be done?

# Heuristics and Optimization

- Multi-level Capacitated Lot-Sizing Problem
- Heuristic fix and optimize approach <sup>1</sup>
- Operating cost reduction by 5% and planning effort by 40%

 And that's it for today's lecture!

We now have covered the basics of the CLSP and are ready to start solving some tasks in the upcoming tutorial.

1. Mickein, Koch, and Haase (2022)

# Questions?

# Literature

# Literature I

For more interesting literature to learn more about Julia, take a look at the [literature list](#) of this course.



# Literature II

Mickein, Markus, Matthes Koch, and Knut Haase. 2022. “A Decision Support System for Brewery Production Planning at Feldschlösschen.” *INFORMS Journal on Applied Analytics* 52 (2): 158–72.