Lecture VIII - Police Districting

Applied Optimization with Julia

Dr. Tobias Vlćek
University of Hamburg - Fall 2025

Introduction

Police Service District Planning

T. Vlćek, K. Haase, M. Fliedner, and T. Cors [1]

Challenges

Question: What makes the work of emergency services complex?

- Dynamic urban development
- Changing population patterns
- Resource constraints
- Need for rapid response
- Multiple stakeholder interests

Emergency Services

Emergency services address the needs of three interest groups:

- Citizens
- Service personnel
- Administrators

Question: What could be the objectives of these groups?

Stakeholder Objectives

1. Citizens
    - Fast response times
    - Reliable service coverage
2. Service Personnel
    - Manageable workloads

- Safe working conditions

3. Administrators
   - Cost efficiency
   - Resource optimization

> **ℹ Note**
>
> Aligning the objectives of the three interest groups is challenging.

Emergency Service Districting

Question: Why might current district layouts be suboptimal?

- Many layouts date back several decades

- Often designed along highways and regions [2]
- Extensive data not used for data-driven improvement

How can we improve

this situation?

The Role of Data

Question: What data can help improve emergency services?

- Historical incident patterns
- Response time analysis
- Resource utilization metrics
- Population densities and traffic patterns

. . .

> ℹ Note
>
> Extensive data collected, but often lack of tools or knowledge to leverage it.

Optimization

- Operations research (OR) models can help!
- Based on incident records and geographical information
- Improve the response of emergency services
- Help administrators in making strategic decisions
- Locate new departments or close departments [3]

Case Studies

Police Districting

> For an efficient and effective distribution of resources, police jurisdictions are divided into precincts or command districts with separate departments. These are further divided into patrol beats [4].

Service Priority Extremes

- High Priority
  - Life-threatening situations
  - Active crimes in progress
  - Multiple unit response needed
- Low Priority
  - Minor incidents
  - Administrative tasks

Case Studies

- Different urban contexts
- Study of jurisdictions in
  - Germany: Large metropolitan area
  - Belgium: Large rural area
- Focus on response time optimization

. . .

> **Note**
>
> Part of the force patrols the streets, another part is stationed at the departments.

Dispatching

- Dispatchers assign all CFS to vehicles from the corresponding districts and patrol areas
- Officers are familiar with the area and are thus better prepared to respond appropriately [5]
- To cope with high demands, dispatchers can assign vehicles from nearby districts or beats

Potential Problem

Question: What could be the potential problem?

. . .

- This can lead to a domino effect
- Transferring vehicles from other districts or beats reduces coverage in those locations [6]
- This makes them vulnerable to missing resources when they need assistance themselves

Overloaded Systems

- This can lead to overloaded systems!
- Long dispatching delays due to staff shortages
- Preventive patrol hardly possible [7]
- Dispatchers constantly draw on patrol resources
- Reduces the response time of emergency services

. . .

> **Warning**
>
> This is a common problem in many emergency services.

Response Time

- Central criterion to measure the effectiveness of emergency services is the response time
- Time between a call for aid and the arrival at the incident location
- Low response time increases the likelihood of helping and improves confidence [5]

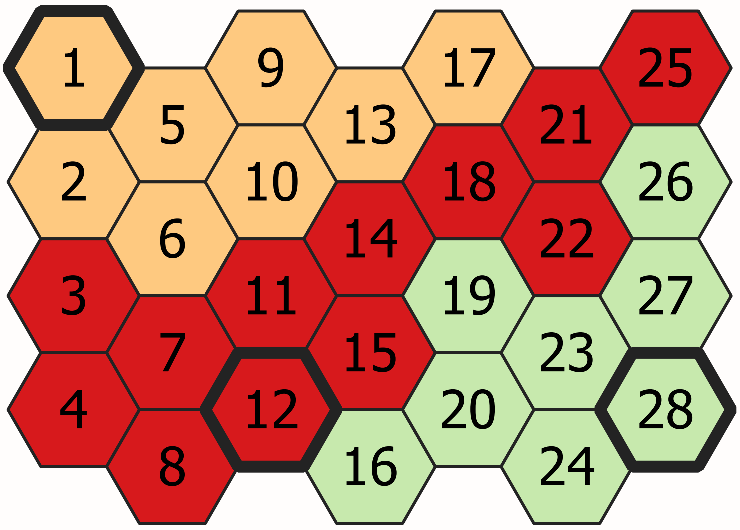
Response Time Influencers

Question: What affects response time?

- Initial contact
- Information gathering
- Unit assignment
- Resource coordination
- Route to location
- Traffic conditions

Territory Design Problem

What is Territory Design?



Aggregation of small geographic areas, called basic areas (BAs), into geographic clusters, called districts, so that these are acceptable according to pre-defined planning criteria[1].

. . .

> **Note**
>
> Territory design is a general framework applicable to many domains: emergency services, sales territories, political districts, school districts, etc.

Territory Design Components

Question: What are the key components?

- Basic Areas (BAs): Smallest indivisible geographic units
- Districts/Territories: Aggregation of BAs
- Planning Criteria: Rules and objectives for grouping
- Decision: Which BAs belong to which district?

. . .

> **Tip**
>
> In our police case: BAs are hexagonal cells, districts are service areas for police departments.

Common Territory Design Objectives

[1] A. A. Zoltners and P. Sinha [8]

Balance Criteria:

- Equal workload distribution
- Equal sales potential
- Equal population coverage
- Resource balance

Geographic Criteria:

- Contiguity: No isolated areas
- Compactness: Round shape
- Minimize travel distances
- Respect boundaries

. . .

> ! Important
>
> These criteria often conflict with each other - optimization helps find the best trade-off!

Applications of Territory Design

Question: Where else is this used?

- Sales force deployment: Assign sales representatives to territories
- Political districting: Electoral boundaries
- School districts: Student assignment zones
- Waste collection: Service route planning
- Healthcare: Hospital catchment areas

. . .

> i Note
>
> Same mathematical structure, different objectives and constraints!

Police Districting Specifics

Question: What could be the objective?

. . .

- Minimize the response time to help citizens faster while increasing the confidence in the service

. . .

Question: What could be further objectives?

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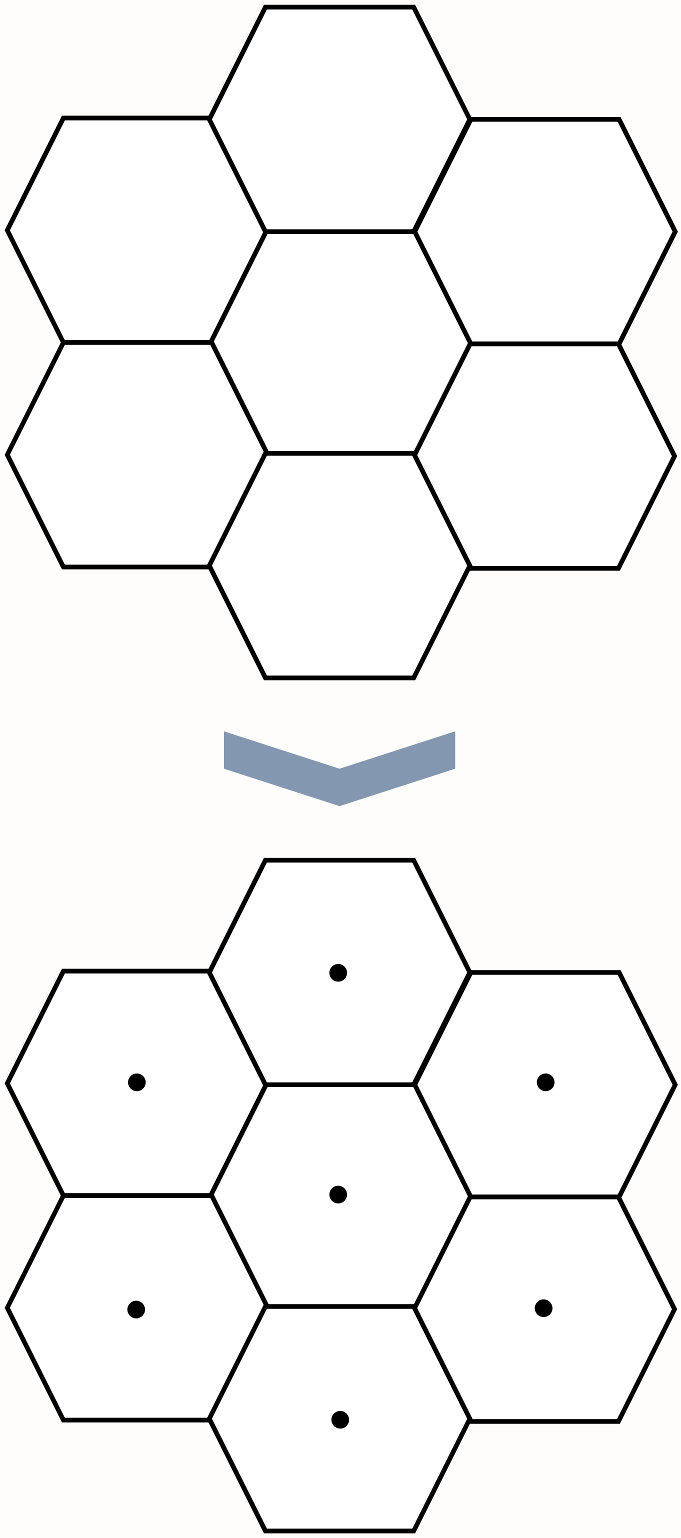
- Reallocate only part of the police department's

- Compact and contiguous territories to improve patrol
- Prevention of isolated departments

From Geography to Graph



Question: How do we model this mathematically?

- Geographic areas ⟶ Centroids (center points)
- Centroids ⟶ Vertices in a graph
- Spatial relationships ⟶ Edges between vertices

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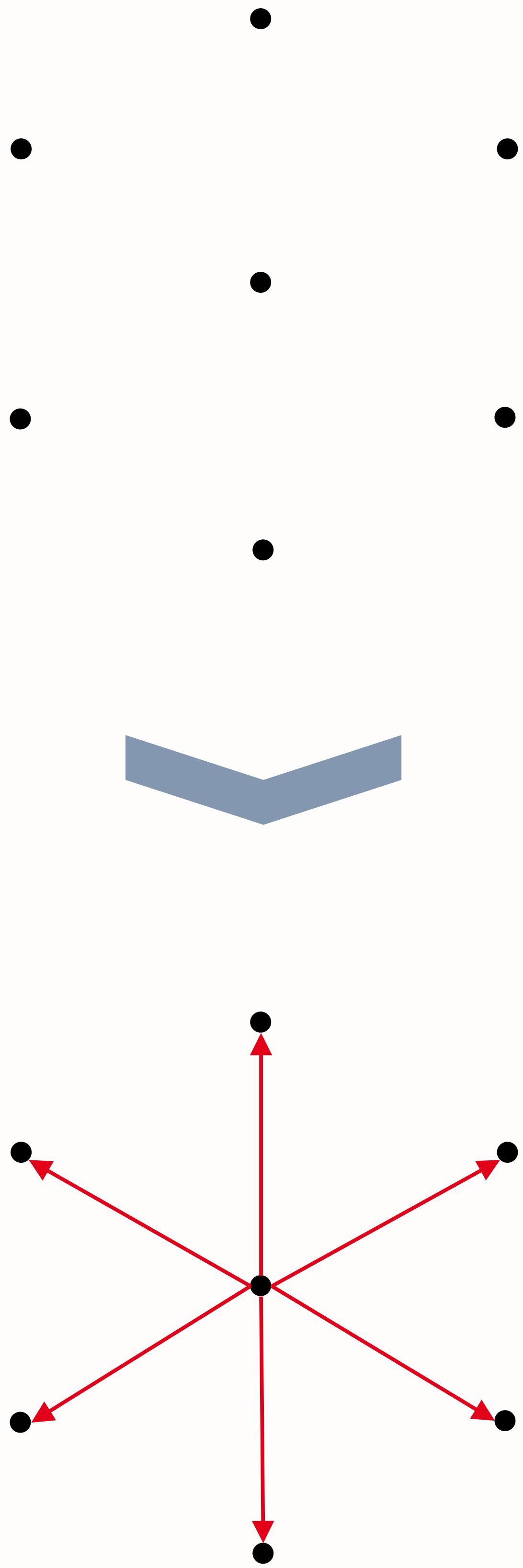
> Tip
>
> This abstraction allows us to use powerful optimization techniques from graph theory and location science!

Mathematical Structure



Question: What sets do we need?

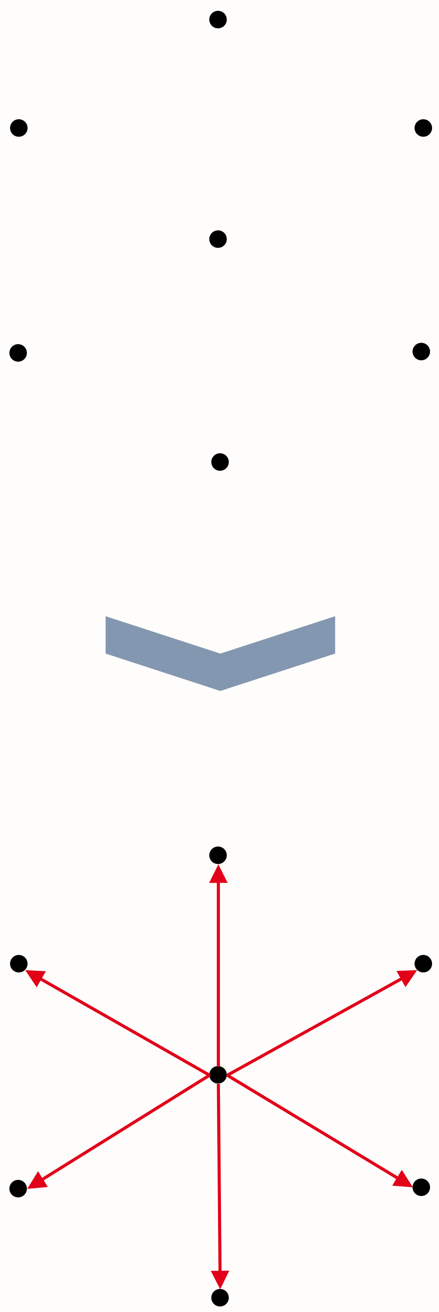
- $\mathcal{J}$: Set of all BAs (basic areas), indexed by j

- $\mathcal{I}$: Set of potential department locations where $\mathcal{I} \subseteq \mathcal{J}$, indexed by i

...

> ! Important
>
> Note: Department locations are a subset of all BAs - not every BA can host a department!

Why Hexagons?



Question: Advantages of hexagons?

- Equal distances to all neighboring centroids
- Reduces sampling bias from edge effects [9]
- Special properties that help with the enforcement of compactness
- Better representation of urban geography

Response Time Components


Call Length
Dispatch Time
Driving Time

Question: How can we model response time?

- Call length is independent of territory
- Dispatch time is difficult to model
- Driving time can be minimized directly

⚠ Conclusion

We focus on minimizing expected driving times between departments and incident locations.

Model Formulation

Let's build our model

step by step!

Key Model Components

Question: What could be our key model components?

. . .

- Basic areas (BAs) and potential department locations
- Driving times between basic areas
- Forecasted incident data
- Assignment decisions

. . .

Question: Which are sets, parameters, and variables?

Sets and Indices

- $\mathcal{J}$ - Set of BAs, indexed by j
- $\mathcal{I}$ - Set of potential district centres ($\mathcal{I} \subseteq \mathcal{J}$), indexed by i

. . .

ℹ Note

The depot locations are a subset of the basic areas!

Parameters

Question: What parameters do we need?

- p - Number of district centres (departments)
- $t_{i,j}$ - Expected driving times between i and j

. . .

Tip

Parameters should be carefully calibrated with real-world data!

Decision Variable(s)?

We have the following sets:

- BAs, indexed by $j \in \mathcal{J}$
- Potential department locations, indexed by $i \in \mathcal{I}$

...

! Our objective is to:

Minimize the expected response time of the emergency services by optimizing the assignment of BAs to departments.

...

Question: What decisions do we need to model?

Decision variable/s

- $X_{i,j}$: 1 if BA j assigned to department i, 0 otherwise

...

Question: What is the domain of our decision variable?

...

- $X_{i,j} \in \{0, 1\} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$

Let's build our

objective function!

Objective Function?

! Our objective is to:

Minimize the expected response time of the emergency services by optimizing the assignment of BAs to departments.

...

Question: How do we minimize response time?

- We want to minimize total driving time

- Consider frequency of incidents in each BA
- Don't include fixed costs (handled by constraints)

Objective Function

Question: What could be our objective function?

. . .

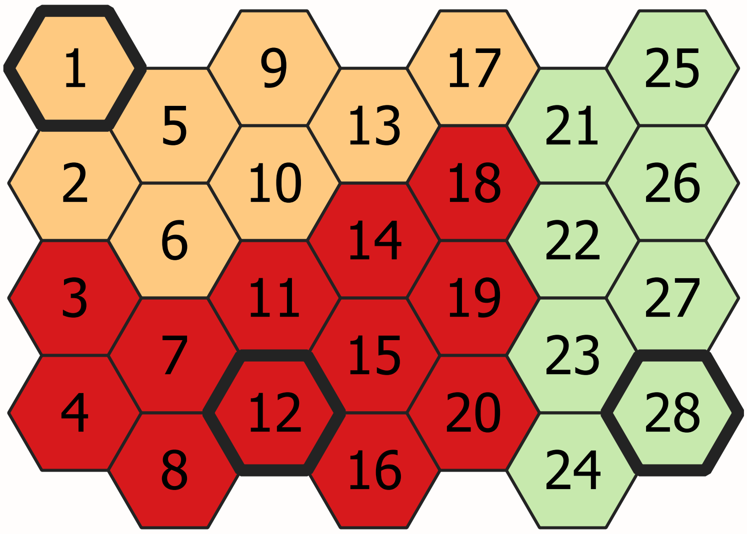
$$\text{minimize} \quad \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} t_{i,j} \times X_{i,j}$$

. . .

> ! Expected Driving Time
>
> - Total driving time across all assignments
> - Weighted by incident frequency
> - Considers all possible BA-department pairs

Constraints

Key Constraints



Question: Constraints needed?

1. BA must have one department
2. Limit number of departments
3. Only assign active departments
4. Ensure contiguous districts
5. Maintain district compactness

Single Assignment Constraint?

Question: Why do we need this constraint?

...

- Each BA must be assigned to exactly one department
- Prevents overlapping jurisdictions
- Ensures complete coverage

...

> **i** We need the following variables:
>
> - $X_{i,j}$ - 1 if BA j assigned to department i, 0 otherwise

Single Assignment Constraint?

Question: What could the constraint look like?

...

$$\sum_{i \in \mathcal{I}} X_{i,j} = 1 \quad \forall j \in \mathcal{J}$$

...

> **i** Note
>
> Each BA must be assigned to exactly one department.

Department Count Constraint?

> **!** The goal of these constraints is to:
>
> Ensure that exactly p departments are opened.

...

> **i** We need the following sets and variables:
>
> - $\mathcal{I}$ - Set of potential department locations, indexed by i
> - $\mathcal{J}$ - Set of BAs, indexed by j
> - $X_{i,j}$ - 1, if BA j assigned to department i, 0 otherwise
> - p - Number of departments

Department Count Constraint

Question: What could the constraint look like?

...

$$\sum_{i \in \mathcal{I}} X_{i,i} = p$$

. . .

Question: What happens if we have more departments than potential locations?

. . .

- We can't open more departments than there are locations
- The model will be infeasible

Active Department Constraint?

! The goal of these constraints is to:

Ensure that each BA is assigned to an active department, e.g. a department that is opened and that could dispatch vehicles.

. . .

i We need the following sets and variables:

- $X_{i,j}$ - 1, if BA j assigned to department i, 0 otherwise

Question: How do we ensure assignments only to active departments?

Active Department Constraint

$$X_{i,j} \leq X_{i,i} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$$

. . .

i Note

This constraint creates a logical connection between department locations and BA assignments where BAs can only be assigned to opened departments.

p-Median Problem

Connection to Facility Location

Question: How does this relate to facility location problems?

. . .

p-Median Problem:

- Fixed number of facilities (p)
- Minimize total distance
- All customers served
- No capacity constraints

Similar problems:

- UFLP: Uncapacitated Facility Location
- p-Center: Minimize maximum distance
- Capacitated: Add capacity limits

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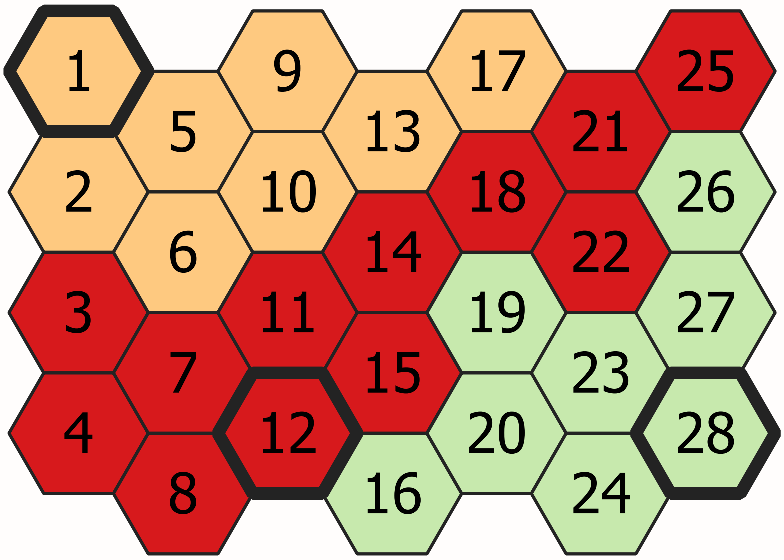
> ℹ Note
>
> Our police districting is a p-Median variant with additional geographic constraints (contiguity, compactness)!

Contiguity and Compactness

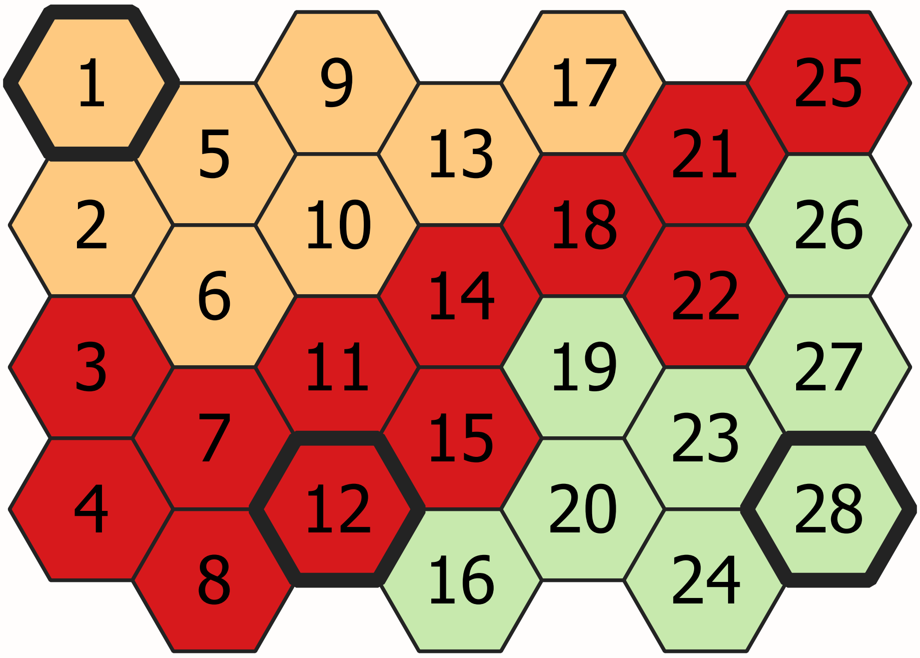
Contiguity Introduction



Question: Why is contiguity important?

- Prevents isolated areas
- Ensures contiguous patrol routes
- Maintains operational coherence

What is compactness?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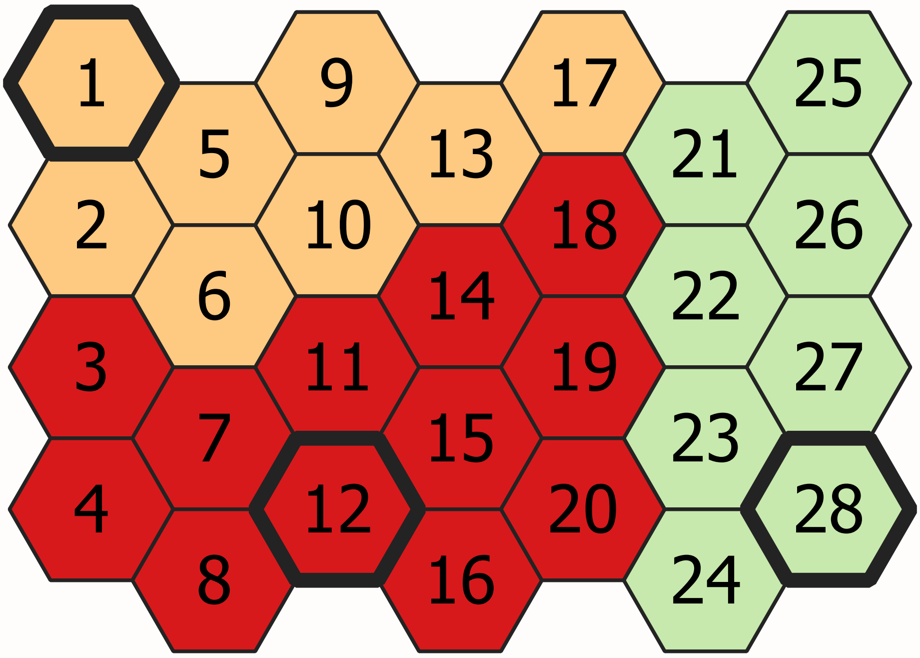
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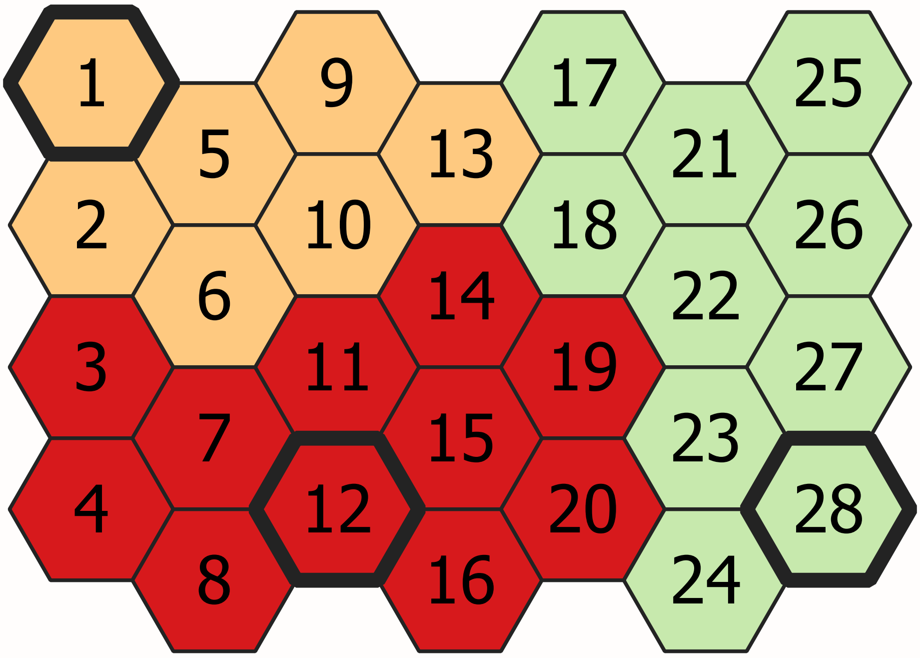

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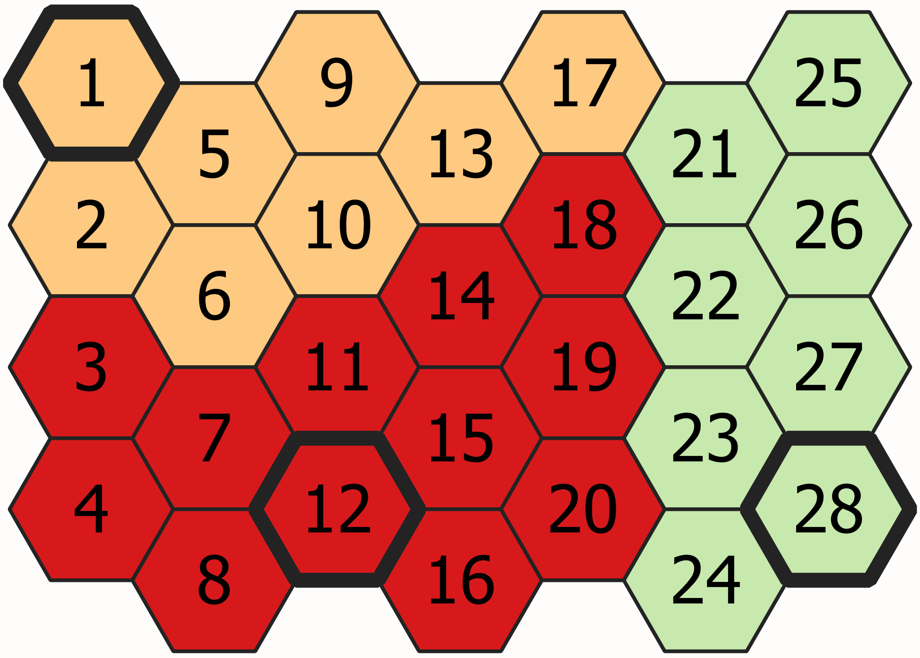
ℹ Compactness

Compactness has no univocal definition; a district is commonly declared compact if it is ‘somehow round-shaped and undistorted’ [10].

Contiguity and Compactness



Question: Are our resulting districts based on the model contiguous and compact?

- This depends on $t_{i,j}$
- If Euclidean distance
  - Districts will be contiguous
  - Likely of compact shape

Compactness p-Median

Question: Is this likely for police service districting?

- No, as we minimize the driving time within a city
- Highways, Tunnels, etc.
- Multiplied by the differing number of requested cars
- This can contribute to distorted district shapes

Contiguity Sets

Additional Set and Parameter

- $e_{i,j}$ - Euclidean distance between centroids
- $\mathcal{A}_j$ - Sets of BAs adjacent to BA j

...

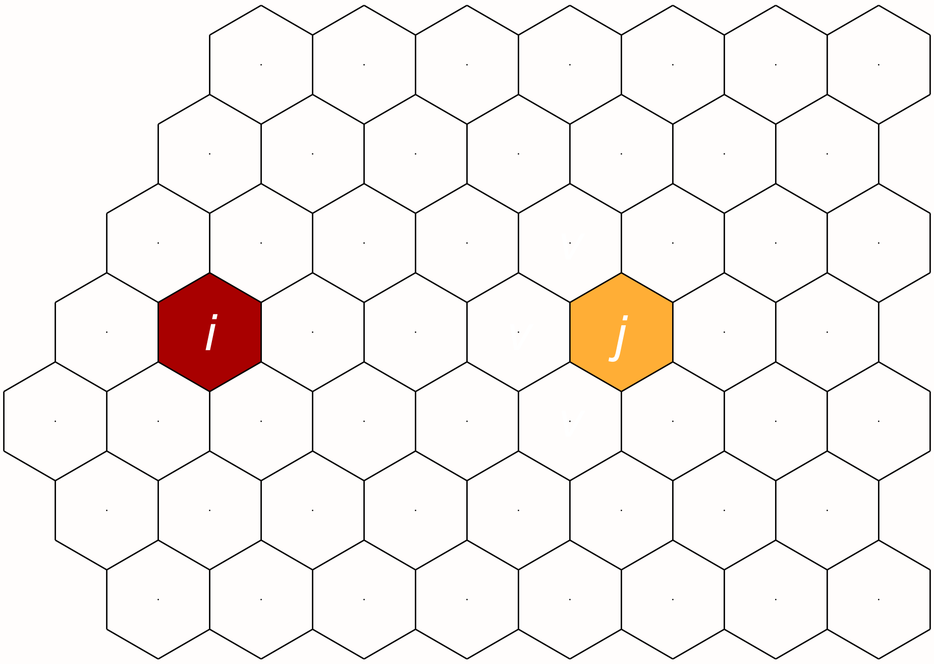
$$\mathcal{N}_{i,j} = \{v \in \mathcal{A}_j \mid e_{i,v} < e_{i,j}\} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$$

...

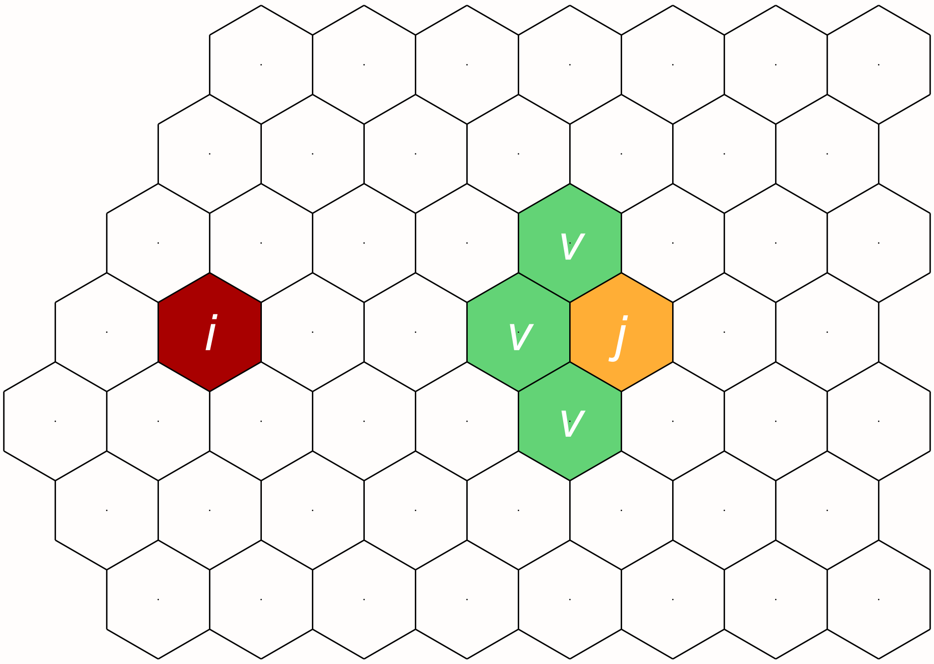
ℹ The idea

BAs closer to department i than BA j on euclidian distance and adjacent to j!

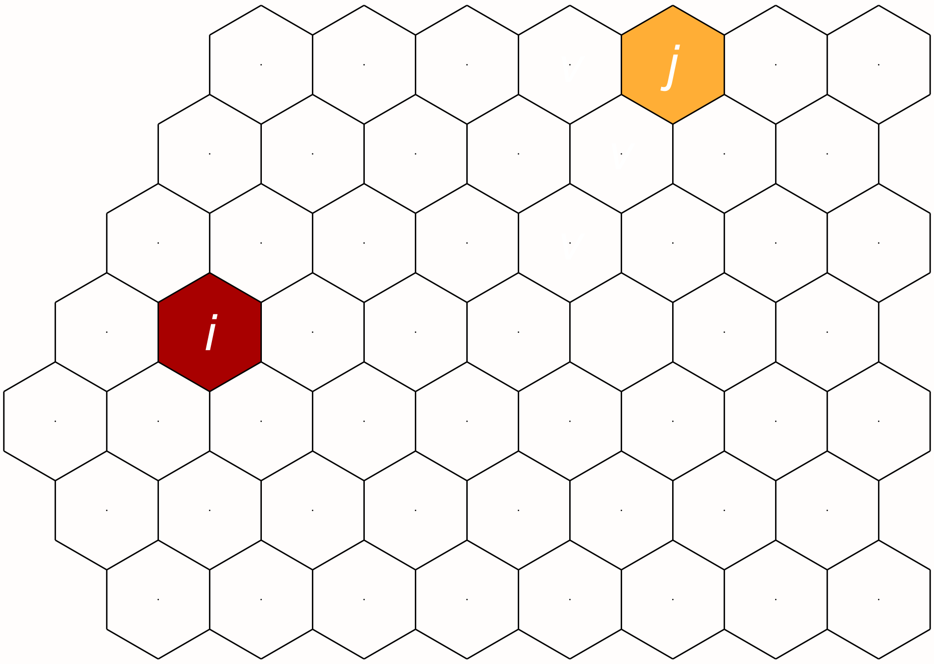
Example A



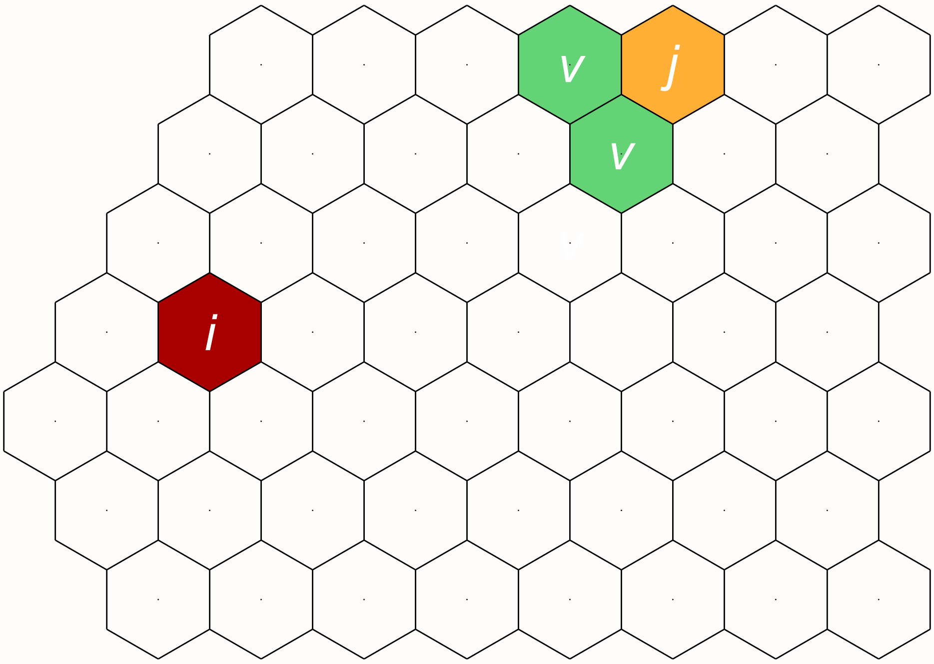
Example A



Example B



Example B



Enforcing Contiguity

All districts have to be contiguous

$$X_{i,j} \leq \sum_{v \in \mathcal{N}_{i,j}} X_{i,v} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \setminus \mathcal{A}_i : i \neq j$$

...

! The idea

At least one department has to be assigned to a BA that is adjacent to BA j and closer to department i!

Contiguity and Compactness

All districts have to be contiguous and compact

...

! The idea

At least one department has to be assigned to two BAs that are adjacent to BA j and closer to department i!

Comparison

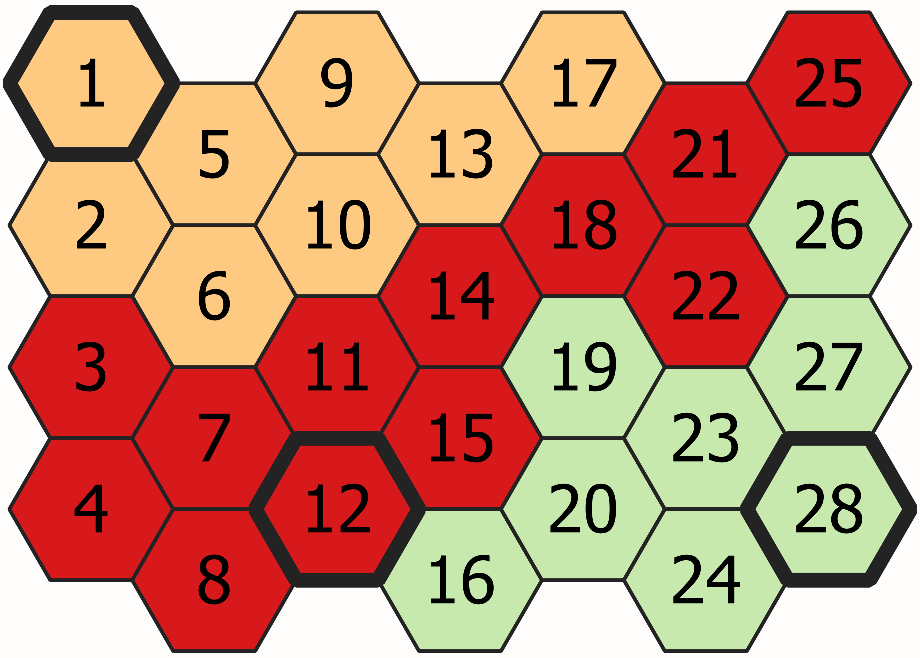


Figure 1: One department

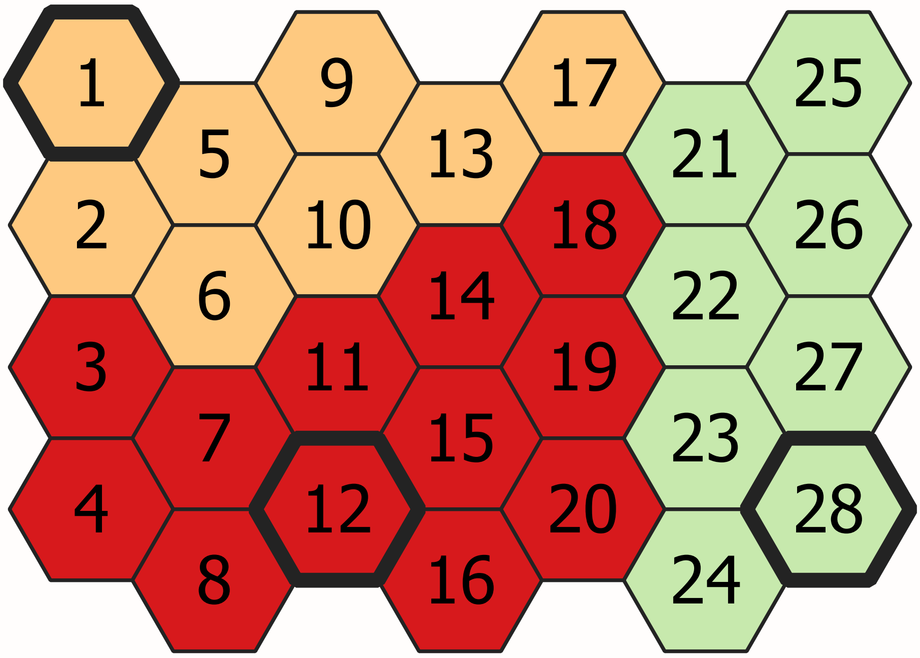


Figure 2: Two departments

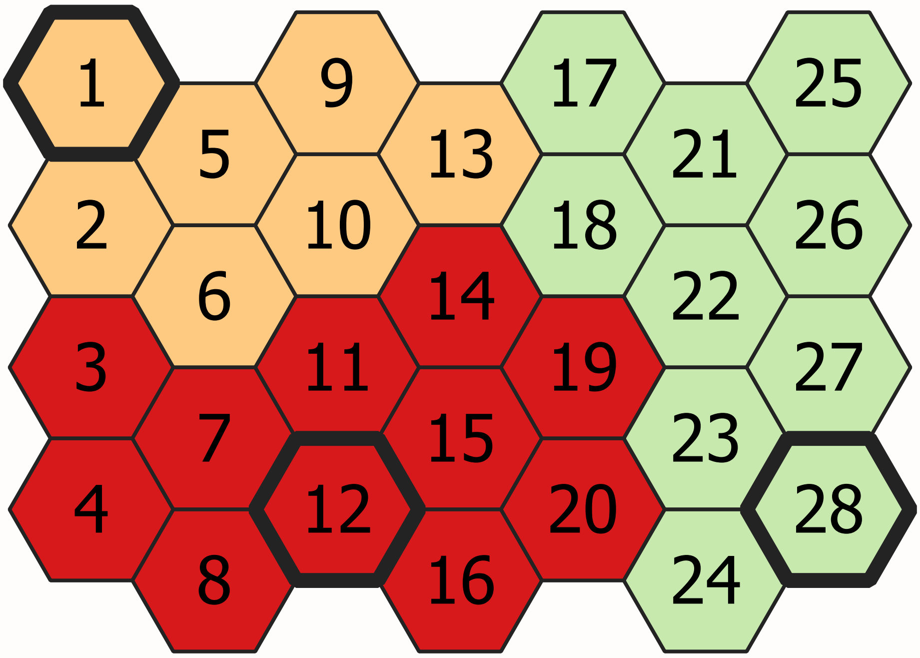


Figure 3: Up to three departments

...

> **i Why does this work?**
>
> Due to the constraints, there is always a path back to the department if a BA is assigned to a department!

Model Characteristics

Characteristics

Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?
- What do you think, can the model be solved quickly?
- Have we prevented isolated districts?

Model Assumptions

Questions: On model assumptions

- What assumptions have we made?
- Use Euclidean distances to approximate driving time?
- Can we rely on incident data collected by the police?

Implementation and Impact

Overview of Studies

Question: Where did we apply our model?

- Two distinct environments:
  1. Large metropolitan area (Germany)
  2. Rural region (Belgium)
- Different challenges and requirements
- Focus on response time optimization

German Metropolitan Case

. . .

- 1.8 mio incidents (2015-2019)
- ~20 department locations
- 1,596 basic areas
- Dense urban environment

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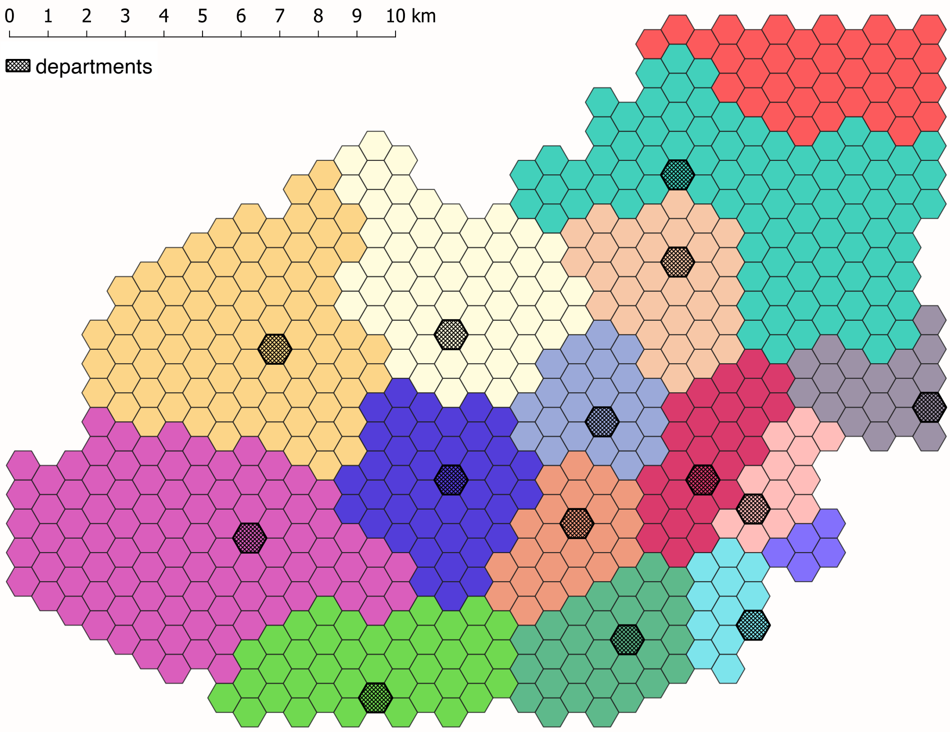
Goal: Redesign districts to improve response time.

German Metropolitan Results



Belgian Rural Case

. . .

- 50,000 incidents (2019-2020)
- 2 existing + 1 planned location
- 1,233 basic areas
- Dispersed rural setting

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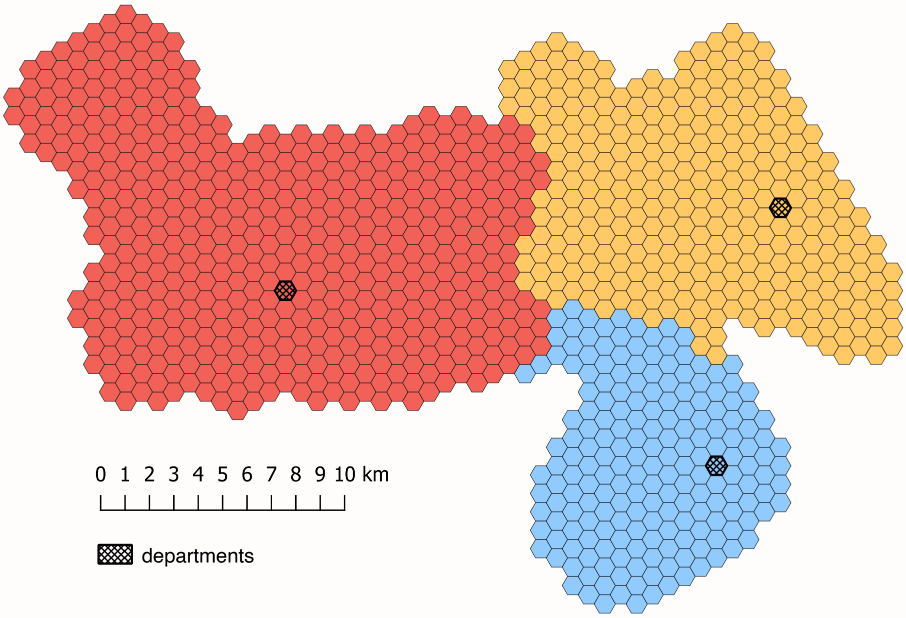
Goal: Optimize coverage with limited resources.

Belgian Rural Results



Simulation Framework

Question: How did we validate the results?

. . .

- Spatial and temporal patterns
- Shift schedules
- Priority handling
- Rush hours
- Inter-district support
- Variable driving times

Results

- Response time reduction up to 14.52%
- Better workload distribution
- Improved coverage equity
- More efficient resource utilization

. . .

> ! Important
>
> All improvements are without additional staff!

Conclusions

1. Model adaptability crucial
2. Local context matters
3. Stakeholder buy-in essential

4. Data quality critical

. . .

> Tip
>
> Success requires balancing theoretical optimization with practical constraints!

Future Applications

Question: Where else could this approach be useful?

- Other emergency services
- Different urban contexts
- Resource allocation problems
- Service territory design

. . .

> Note
>
> The methodology is adaptable to various public service optimization scenarios.

Wrap Up

> And that's it for todays lecture!
>
> We now have covered districting problems and are ready to start solving some tasks in the upcoming tutorial.

Questions?

Literature

Literature I

For more interesting literature to learn more about Julia, take a look at the literature list of this course.

Literature II

Bibliography

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