Lecture V - Production Planning in Breweries

Applied Optimization with Julia

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Introduction

Case Study

- **Large brewery**
- Brews and sells beverages
- Production planning by hand
- Planner has a **lot of experience**
- **But** will retire soon

Challenges

- **Strong** competition
- Customer **demand is changing**
- Craft beer **gains popularity**
- **Variety** of drinks is increasing
- **Batch sizes are getting smaller**

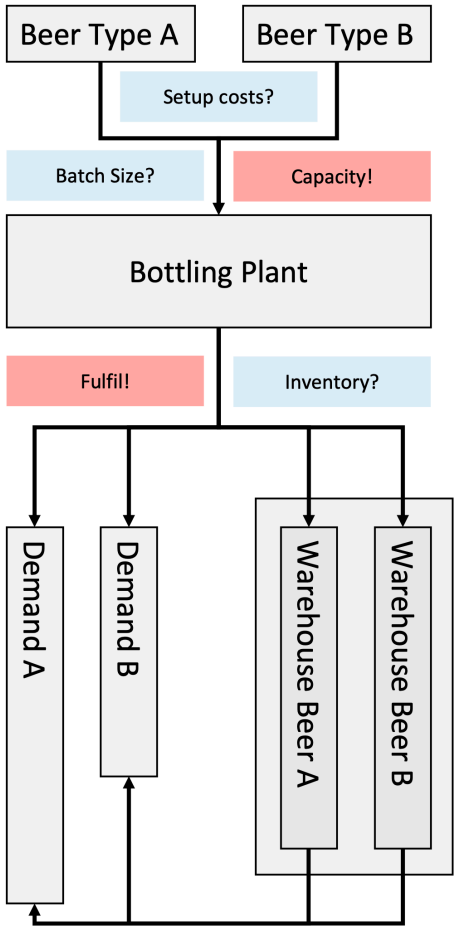
Different costs

- Plant can fill **multiple types**
- Time depends on **type and batch**
- **Changing type** leads to set-up costs for preparation and cleaning
- **Unsold beer bottles** can be **stored in a warehouse**
- This leads to **inventory costs**

Where is the challenge?

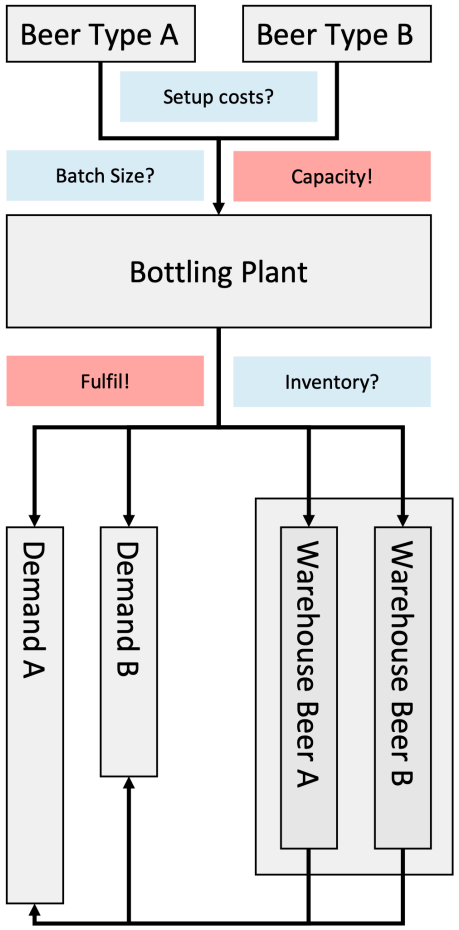
Problem Structure

Objective



Question: What could be the objective?

Trade-Off



Question: What is the trade-off?

Available Sets

Question: What are sets again?

Available Parameters

Question: What are possible parameters?

Decision Variables?

We have the following sets:

- Beer types indexed by $i \in \{1, 2, \ldots, |\mathcal{I}|\}$
- Time periods of the planning horizon indexed by $t \in \{1, 2, \ldots, |\mathcal{T}|\}$

Decision Variables

- $W_{i,t}$ - Inventory of type $i \in \mathcal{I}$ at the end of $t \in \mathcal{T}$
- $Y_{i,t}$ - 1, if type $i \in \mathcal{I}$ is bottled in $t \in \mathcal{T}$, 0 otherwise
- $X_{i,t}$ - Batch size of type $i \in \mathcal{I}$ in $t \in \mathcal{T}$

Model Formulation

Objective Function?

Our objective is to:

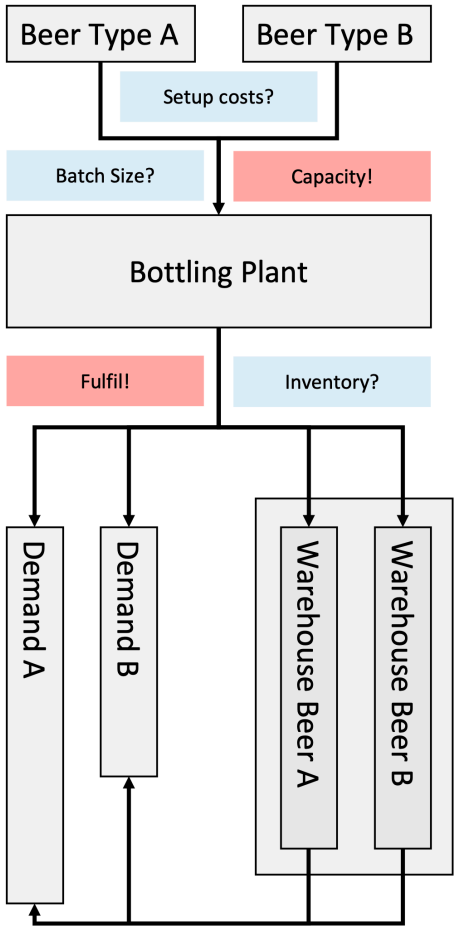
Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

Objective Function

We need the following parameters:

- f_i - Setup cost of beer type $i \in \mathcal{I}$
- c_i - Inventory holding cost for one unit of beer type $i \in \mathcal{I}$

Constraints



Question: What constraints?

Demand/Inventory Constraints?

The goal of these constraints is to:

Consider the current inventory and batch sizes and compute the remaining inventory.

Demand/Inventory Constraints

$$W_{i,t-1} + X_{i,t} - W_{i,t} = d_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} | t > 1$$

Setup Constraints?

The goal of these constraints is to:

Set up beer types where the batch size is ≥ 0.

Setup Constraints

$$X_{i,t} \leq Y_{i,t} \times \sum_{\tau=1}^{\mathcal{T}} d_{i\tau} \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

Capacity Constraints?

The goal of these constraints is to:

Limit the capacity of the bottling plant per period.

Capacity Constraints

Question: What could the third constraint be?

It has more variables and parameters when compared to the other constraints but it is easier to understand.

CLSP: Objective Function

$$\text{Minimize} \quad \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} (c_i \times W_{i,t} + f_i \times Y_{i,t})$$

The goal of the objective function is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

CLSP: Constraints

$$W_{i,t-1} + X_{i,t} - W_{i,t} = d_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} | t > 1$$

$$X_{i,t} \leq Y_{i,t} \times \sum_{\tau \in \mathcal{T}} d_{i,\tau} \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

$$\sum_{i \in \mathcal{I}} (b_i \times X_{i,t} + g_i \times Y_{i,t}) \leq a_t \quad \forall t \in \mathcal{T}$$

Our constraints ensure:

Demand is met, inventory transferred, setup taken care of, and capacity respected.

CLSP: Variable Domains

$$Y_{i,t} \in \{0,1\} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$$

$$W_{i,t}, X_{i,t} \geq 0 \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$$

The variable domains make sure that:

The binary setup variable is either 0 or 1 and that the inventory and batch size are non-negative.

Model Characteristics

Recap on some Basics

There exist several types of optimization problems:

Recap on Solution Algorithms

Model Characteristics

Questions: On model characteristics

Model Assumptions

Questions: On model assumptions

Impact

Can this be applied?

Scale as a Problem

Scale of the Case Study

- **220** finished products
- **100** semi-finished products
- **13** production resources
- **8** storage resources
- **3** main production levels
- **52** weeks planning horizon

Any idea what could be done?

Heuristics and Optimization

- Multi-level Capacitated Lot-Sizing Problem
- Heuristic fix and optimize approach [1]
- Operating cost reduction by 5% and planning effort by 40%

1. Mickein, Koch, and Haase (2022)

Questions?

Literature

Literature I

For more interesting literature to learn more about Julia, take a look at the literature list of this course.

Literature II

Mickein, Markus, Matthes Koch, and Knut Haase. 2022. “A Decision Support System for Brewery Production Planning at Feldschlösschen.” *INFORMS Journal on Applied Analytics* 52 (2): 158–72.