

# Lecture VIII - Police Districting

## Applied Optimization with Julia

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University of Hamburg - Fall 2025

### Introduction

#### Police Service District Planning

T. Vlček, K. Haase, M. Fliedner, and T. Cors [1]

#### Challenges

Question: What makes the work of emergency services complex?

- Dynamic urban development
- Changing population patterns
- Resource constraints
- Need for rapid response
- Multiple stakeholder interests

#### Emergency Services



Emergency services address the needs of three interest groups:

- Citizens
- Service personnel
- Administrators

Question: What could be the objectives of these groups?

## Stakeholder Objectives

1. Citizens
  - Fast response times
  - Reliable service coverage
2. Service Personnel
  - Manageable workloads

- Safe working conditions
3. Administrators
- Cost efficiency
  - Resource optimization

**i Note**

Aligning the objectives of the three interest groups is challenging.

## Emergency Service Districting



Question: Why might current district layouts be suboptimal?

- Many layouts date back several decades

- Often designed along highways and regions [2]
- Extensive data not used for data-driven improvement

How can we improve

this situation?

## The Role of Data

Question: What data can help improve emergency services?

- Historical incident patterns
- Response time analysis
- Resource utilization metrics
- Population densities and traffic patterns

...

### **i Note**

Extensive data collected, but often lack of tools or knowledge to leverage it.

## Optimization

- Operations research (OR) models can help!
- Based on incident records and geographical information
- Improve the response of emergency services
- Help administrators in making strategic decisions
- Locate new departments or close departments [3]

## Case Studies

### Police Districting

For an efficient and effective distribution of resources, police jurisdictions are divided into precincts or command districts with separate departments. These are further divided into patrol beats [4].

### Service Priority Extremes

- High Priority
  - Life-threatening situations
  - Active crimes in progress
  - Multiple unit response needed
- Low Priority
  - Minor incidents
  - Administrative tasks

## Case Studies

- Different urban contexts
- Study of jurisdictions in
  - Germany: Large metropolitan area
  - Belgium: Large rural area
- Focus on response time optimization

...

### i Note

Part of the force patrols the streets, another part is stationed at the departments.

## Dispatching

- Dispatchers assign all CFS to vehicles from the corresponding districts and patrol areas
- Officers are familiar with the area and are thus better prepared to respond appropriately [5]
- To cope with high demands, dispatchers can assign vehicles from nearby districts or beats

## Potential Problem

Question: What could be the potential problem?

...

- This can lead to a domino effect
- Transferring vehicles from other districts or beats reduces coverage in those locations [6]
- This makes them vulnerable to missing resources when they need assistance themselves

## Overloaded Systems

- This can lead to overloaded systems!
- Long dispatching delays due to staff shortages
- Preventive patrol hardly possible [7]
- Dispatchers constantly draw on patrol resources
- Reduces the response time of emergency services

...

### ⚠ Warning

This is a common problem in many emergency services.

## Response Time



- Central criterion to measure the effectiveness of emergency services is the response time
- Time between a call for aid and the arrival at the incident location
- Low response time increases the likelihood of helping and improves confidence [5]

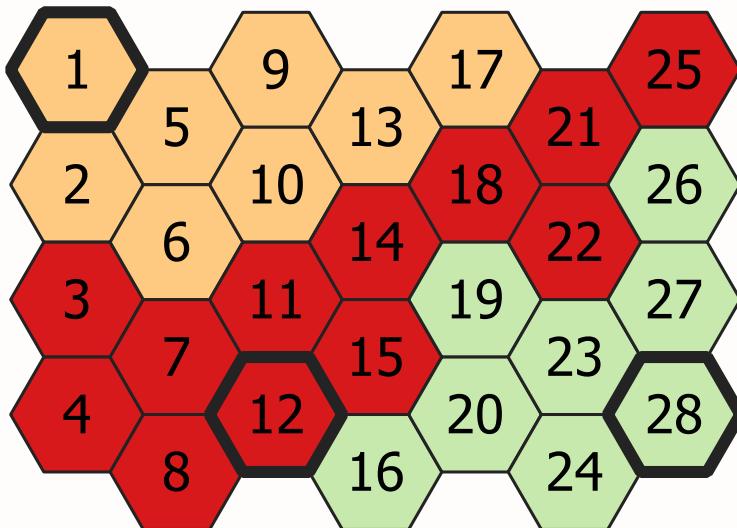
### Response Time Influencers

Question: What affects response time?

- Initial contact
- Information gathering
- Unit assignment
- Resource coordination
- Route to location
- Traffic conditions

### Territory Design Problem

What is Territory Design?



Aggregation of small geographic areas, called basic areas (BAs), into geographic clusters, called districts, so that these are acceptable according to pre-defined planning criteria<sup>1</sup>.

...

#### Note

Territory design is a general framework applicable to many domains: emergency services, sales territories, political districts, school districts, etc.

## Territory Design Components

Question: What are the key components?

- Basic Areas (BAs): Smallest indivisible geographic units
- Districts/Territories: Aggregation of BAs
- Planning Criteria: Rules and objectives for grouping
- Decision: Which BAs belong to which district?

...

#### Tip

In our police case: BAs are hexagonal cells, districts are service areas for police departments.

## Common TD Objectives

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<sup>1</sup>A. A. Zoltners and P. Sinha [8]

Balance Criteria:

- Equal workload distribution
- Equal sales potential
- Equal population coverage
- Resource balance

Geographic Criteria:

- Contiguity: No isolated basic areas
- Compactness: “Round”
- Minimize travel distances
- Respect boundaries

...

**! Important**

These criteria often conflict with each other... Optimization helps find the best trade-off!

## Applications of TD

Question: Where else is this used?

- Sales force deployment: Sales reps to territories
- Political districting: Electoral boundaries
- School districts: Student assignment zones
- Waste collection: Service route planning
- Healthcare: Hospital catchment areas

...

**i Note**

Same mathematical structure, different objectives and constraints!

## Police Districting Specifics

Question: What could be the objective?

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- Minimize the response time to help citizens faster while increasing the confidence in the service

...

Question: What could be further objectives?

...

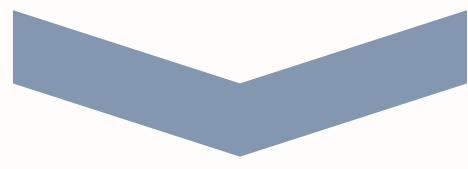
- Reallocate only part of the police department's

- Compact and contiguous territories to improve patrol
- Prevention of isolated departments

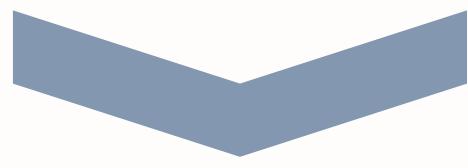
## Response Time Components



Call  
Length



Dispatch  
Time



Driving  
Time

Question: How can we model response time?

- Call length is independent of territory
- Dispatch time is difficult to model
- Driving time can be minimized directly

### Conclusion

We focus on minimizing expected driving times between departments and incident locations.

Let's build our model

step by step!

## Model Formulation

### Key Model Components

Question: What could be our key model components?

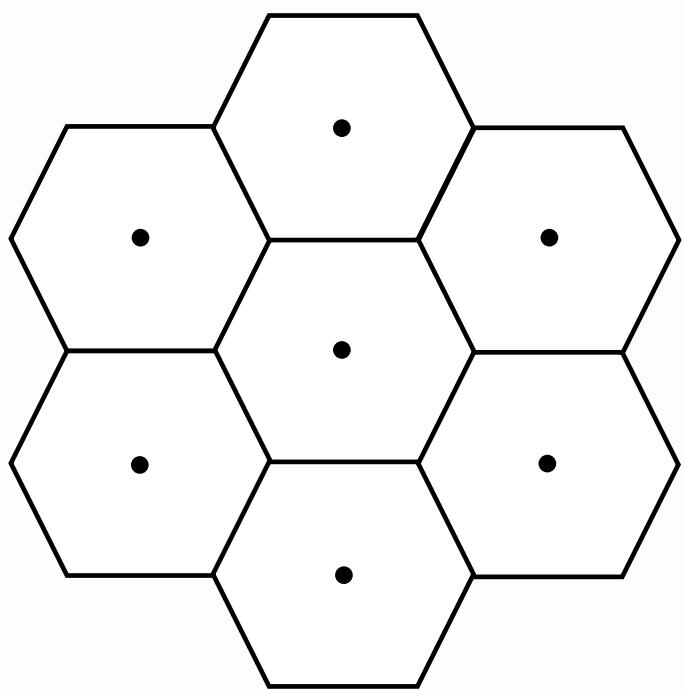
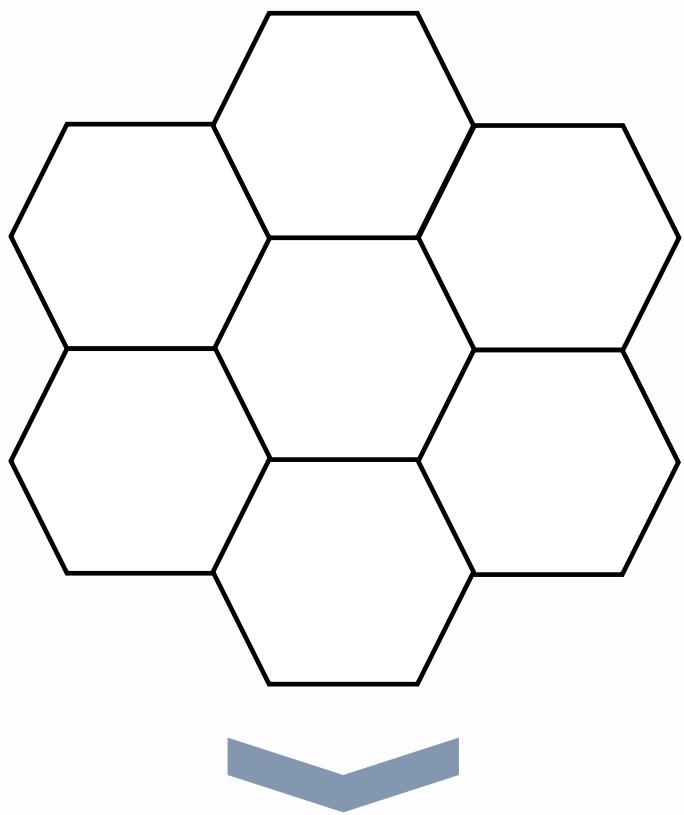
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- Basic areas (BAs) and potential department locations
- Driving times between basic areas
- Forecasted incident data
- Assignment decisions

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Question: Which are sets, parameters, and variables?

### From Geography to Graph



Question: How do we model this mathematically?

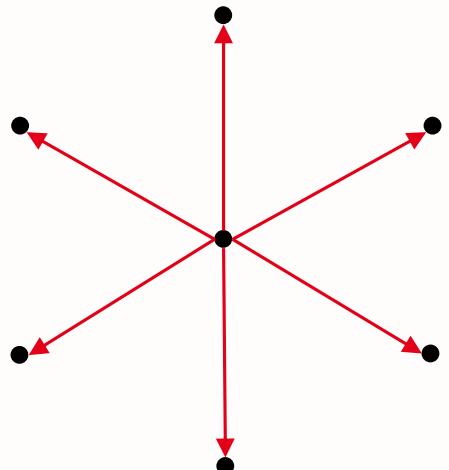
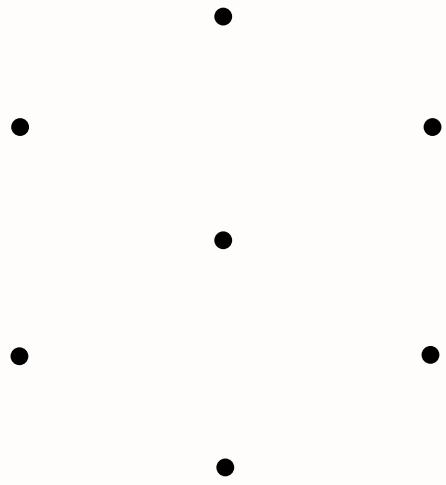
- Geographic areas → Centroids
- Centroids → Vertices in a graph
- Spatial relationships → Edges between vertices

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 Tip

This abstraction allows us to use optimization techniques from graph theory and location science!

## Mathematical Structure



Question: What sets do we need?

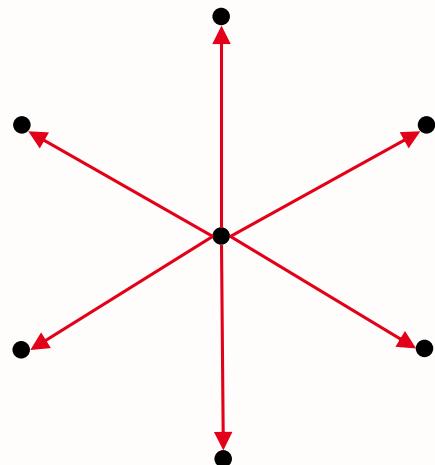
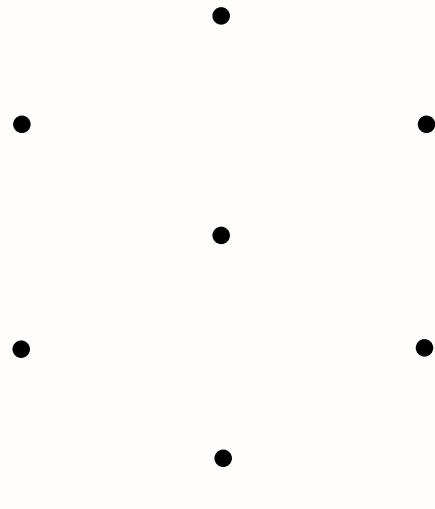
- $\mathcal{J}$  : Set of all BAs (basic areas), indexed by  $j$
- $\mathcal{I}$  : Set of potential department locations where  $\mathcal{I} \subseteq \mathcal{J}$ , indexed by  $i$

...

! Important

Department locations are a subset of all BAs as not every BA can host a department!

## Why Hexagons?



Question: Advantages of hexagons?

- Equal distances to all neighboring centroids
- Reduces sampling bias from edge effects [9]
- Special properties that help with the enforcement of compactness
- Better representation of urban geography

## Parameters

Question: What parameters do we need?

- $p$  - Number of district centres (departments)
- $t_{i,j}$  - Expected driving times between  $i$  and  $j$

...



Tip

Parameters should be carefully calibrated with real-world data!

Decision Variable(s)?

**i** We have the following sets:

- BAs, indexed by  $j \in \mathcal{J}$
- Potential department locations, indexed by  $i \in \mathcal{I}$

...

! Our objective is to:

Minimize the expected response time of the emergency services by optimizing the assignment of BAs to departments.

...

Question: What decisions do we need to model?

Decision variable/s

- $X_{i,j}$ : 1 if BA  $j$  assigned to department  $i$ , 0 otherwise

...

Question: What is the domain of our decision variable?

...

- $X_{i,j} \in \{0, 1\} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$

Let's build our

objective function!

## Objective Function?

! Our objective is to:

Minimize the expected response time of the emergency services by optimizing the assignment of BAs to departments.

...

Question: How do we minimize response time?

- We want to minimize total driving time
- Consider frequency of incidents in each BA
- Don't include fixed costs (handled by constraints)

## Objective Function

Question: What could be our objective function?

...

$$\text{minimize} \quad \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} t_{i,j} \times X_{i,j}$$

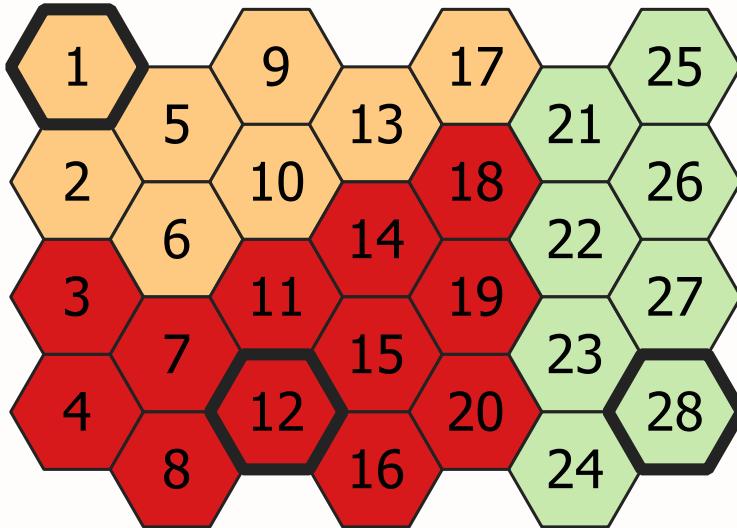
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! Expected Driving Time

- Total driving time across all assignments
- Weighted by incident frequency
- Considers all possible BA-department pairs

## Constraints

### Key Constraints



Question: Constraints needed?

1. BA must have one department
2. Limit number of departments
3. Only assign active departments
4. Ensure contiguous districts
5. Maintain district compactness

Single Assignment Constraint?

Question: Why do we need this constraint?

...

- Each BA must be assigned to exactly one department
- Prevents overlapping jurisdictions
- Ensures complete coverage

...

**i** We need the following variables:

- $X_{i,j}$  - 1 if BA  $j$  assigned to department  $i$ , 0 otherwise

Single Assignment Constraint?

Question: What could the constraint look like?

...

$$\sum_{i \in \mathcal{I}} X_{i,j} = 1 \quad \forall j \in \mathcal{J}$$

...

### i Note

Each BA must be assigned to exactly one department.

## Department Count Constraint?

! The goal of these constraints is to:

Ensure that exactly  $p$  departments are opened.

...

i We need the following sets and variables:

- $\mathcal{I}$  - Set of potential department locations, indexed by  $i$
- $\mathcal{J}$  - Set of BAs, indexed by  $j$
- $X_{i,j}$  - 1, if BA  $j$  assigned to department  $i$ , 0 otherwise
- $p$  - Number of departments

## Department Count Constraint

Question: What could the constraint look like?

...

$$\sum_{i \in \mathcal{I}} X_{i,i} = p$$

...

Question: What happens if we have more departments than potential locations?

...

- We can't open more departments than there are locations
- The model will be infeasible

## Active Department Constraint?

! The goal of these constraints is to:

Ensure that each BA is assigned to an active department, e.g. a department that is opened and that could dispatch vehicles.

...

i We need the following sets and variables:

- $X_{i,j}$  - 1, if BA  $j$  assigned to department  $i$ , 0 otherwise

Question: How do we ensure assignments only to active departments?

### Active Department Constraint

$$X_{i,j} \leq X_{i,i} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$$

...

#### i Note

This constraint creates a logical connection between department locations and BA assignments where BAs can only be assigned to opened departments.

## p-Median Problem

### Connection to Facility Location

Question: How does this relate to these problems?

...

p-Median Problem:

- Fixed number of facilities (exactly  $p$ )
- Minimize total distance
- All customers served
- No capacity constraints

Similar problems:

- UFLP: Uncapacitated Facility Location
- p-Center: Minimize maximum distance
- Capacitated: Add limits

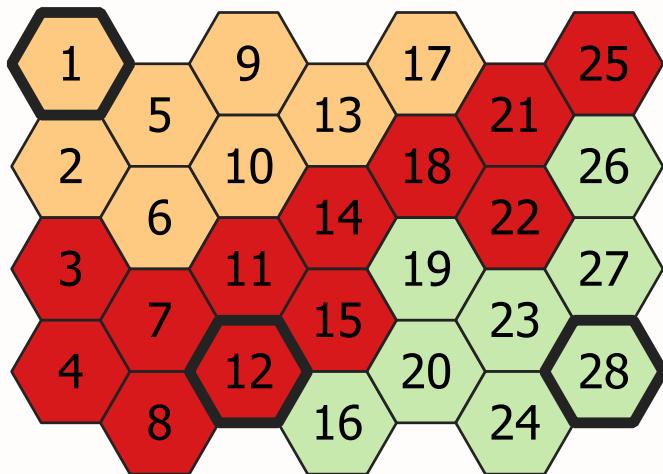
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#### i Note

Our police districting is a p-Median variant with additional geographic constraints (contiguity, compactness)!

## Contiguity and Compactness

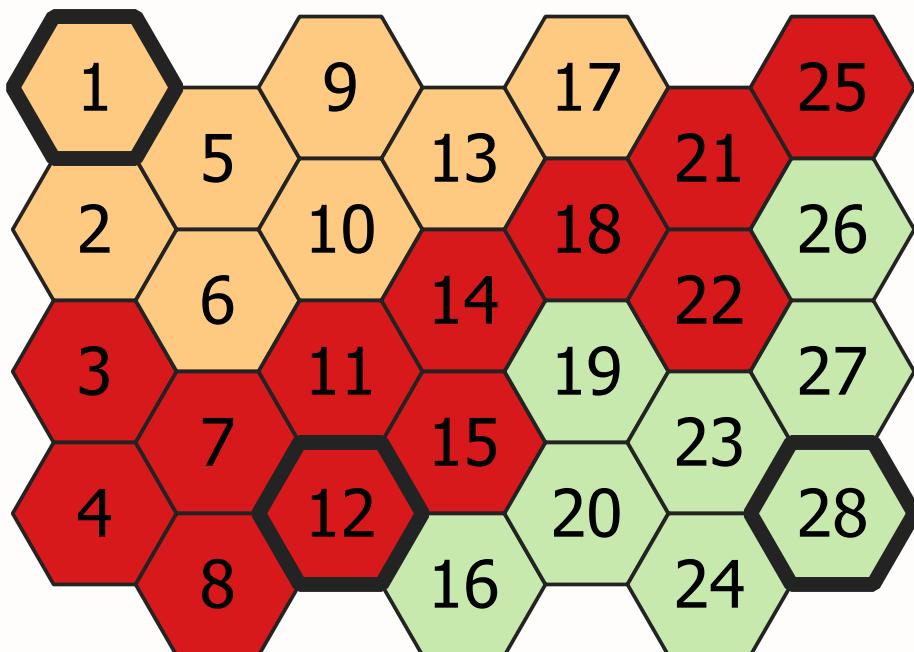
### Contiguity Introduction

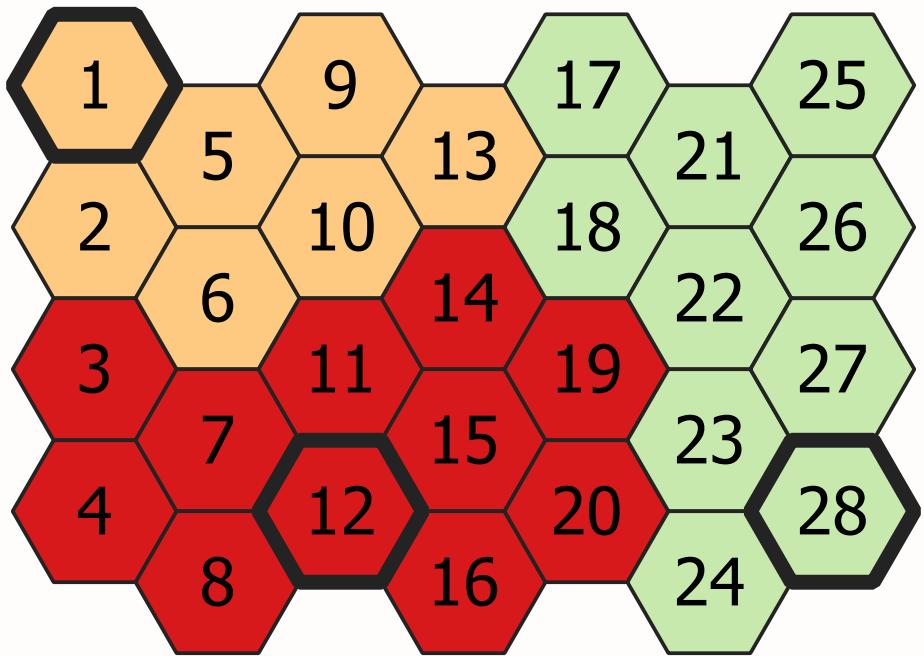
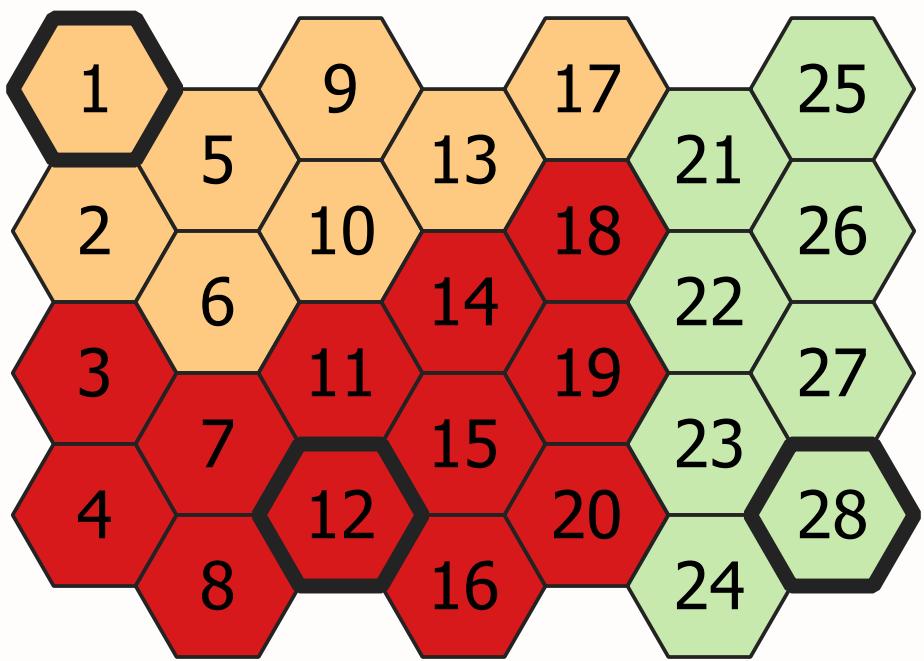


Question: Why is contiguity important?

- Prevents isolated areas
- Ensures contiguous patrol routes
- Maintains operational coherence

What is compactness?



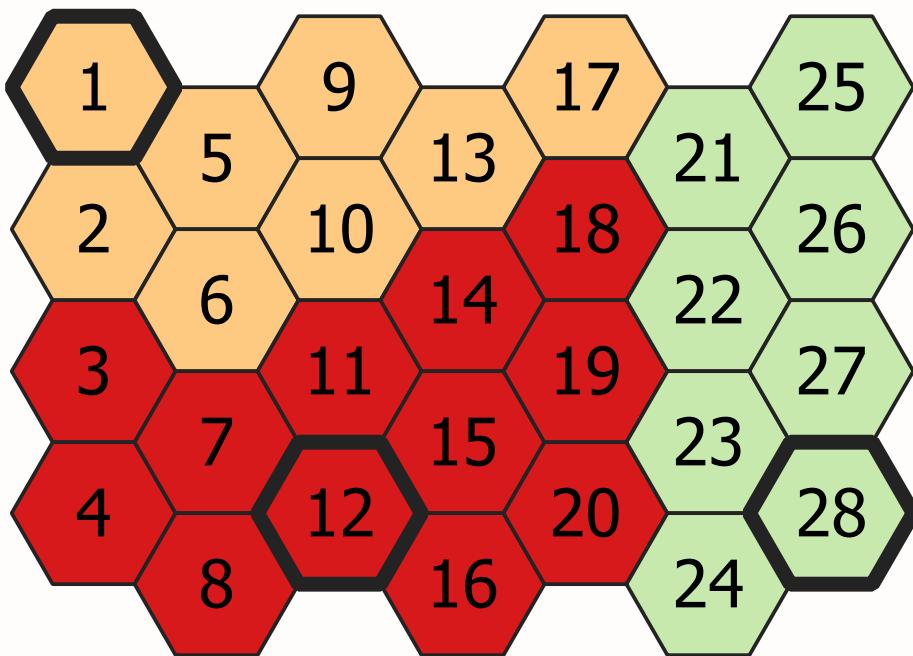


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### i Compactness

Compactness has no univocal definition; a district is commonly declared compact if it is ‘somehow round-shaped and undistorted’ [10].

## Contiguity and Compactness



Question: Are our resulting districts based on the model contiguous and compact?

- This depends on  $t_{i,j}$
- If Euclidean distance
  - Districts will be contiguous
  - Likely of compact shape

## Compactness p-Median

Question: Is this likely for police service districting?

- No, as we minimize the driving time within a city
- Highways, Tunnels, etc.
- Multiplied by the differing number of requested cars
- This can contribute to distorted district shapes

## Contiguity Sets

Additional Set and Parameter

- $e_{i,j}$  - Euclidean distance between centroids
- $\mathcal{A}_j$  - Sets of BAs adjacent to BA  $j$

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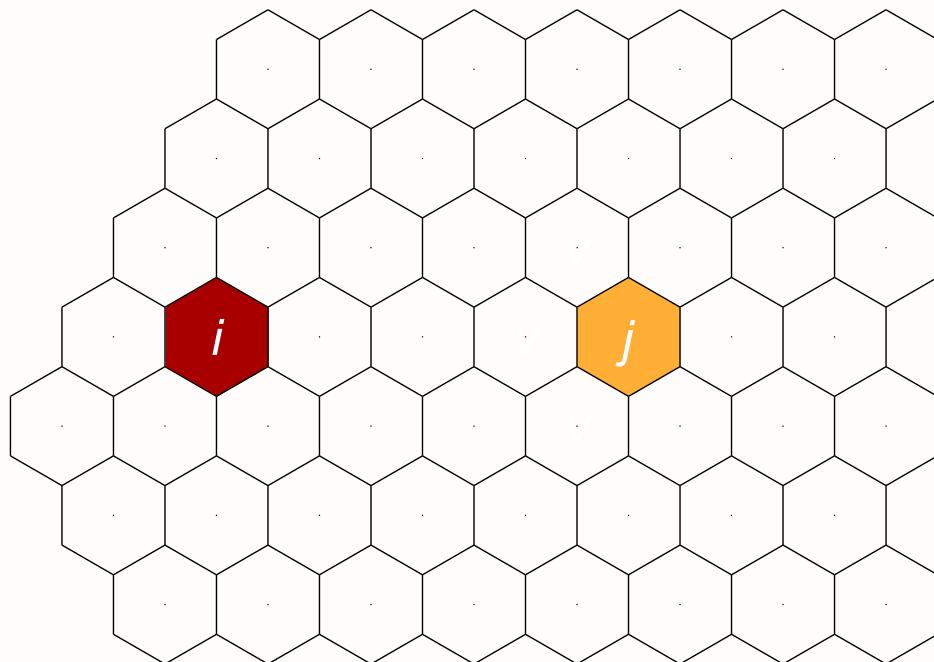
$$\mathcal{N}_{i,j} = \{v \in \mathcal{A}_j \mid e_{i,v} < e_{i,j}\} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$$

...

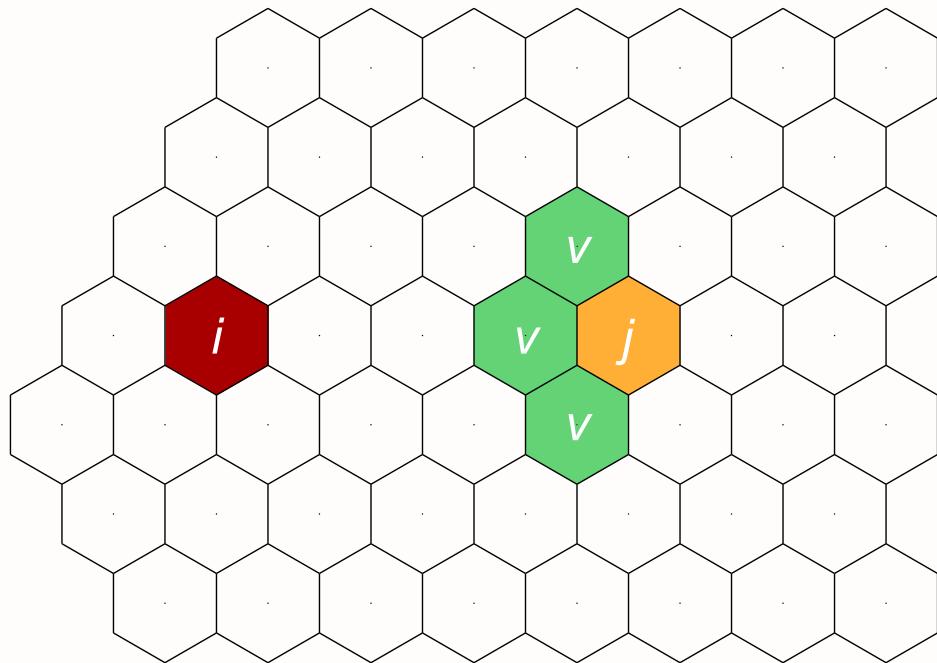
### i The idea

BAs closer to department  $i$  than BA  $j$  on euclidian distance and adjacent to  $j$ !

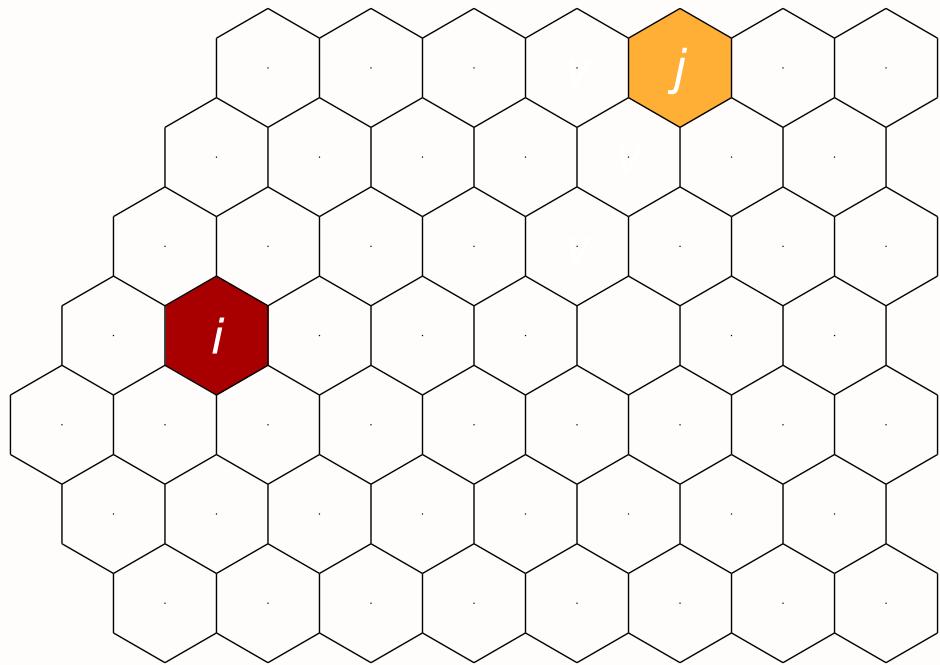
### Example A



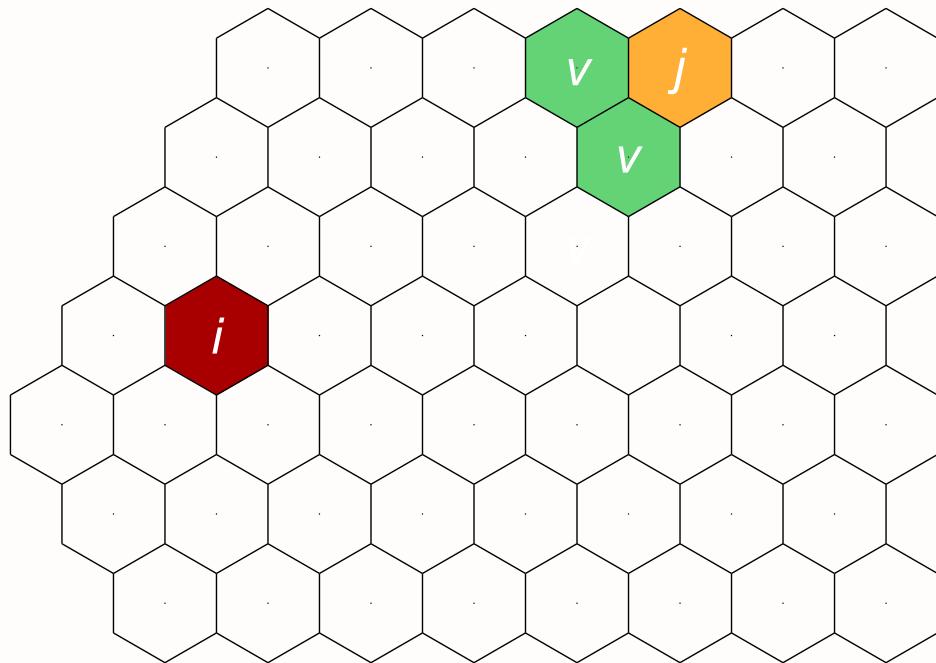
Example A



Example B



## Example B



### Enforcing Contiguity

All districts have to be contiguous

$$X_{i,j} \leq \sum_{v \in \mathcal{N}_{i,j}} X_{i,v} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \setminus \mathcal{A}_i : i \neq j$$

...

! The idea

At least one department has to be assigned to a BA that is adjacent to BA  $j$  and closer to department  $i$ !

### Contiguity and Compactness

All districts have to be contiguous and compact

...

! The idea

At least one department has to be assigned to two BAs that are adjacent to BA  $j$  and closer to department  $i$

Comparison

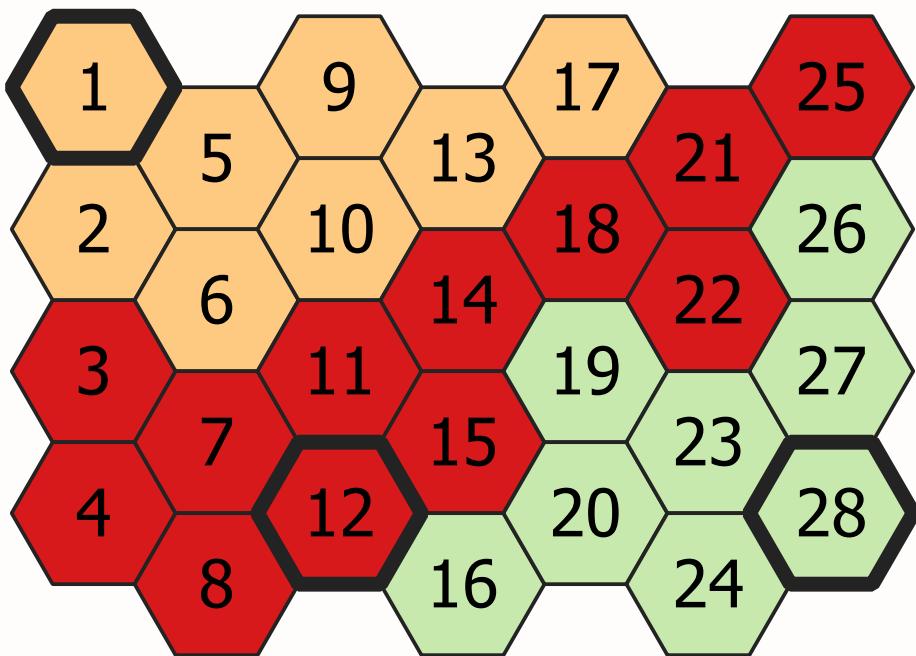


Figure 1: One department

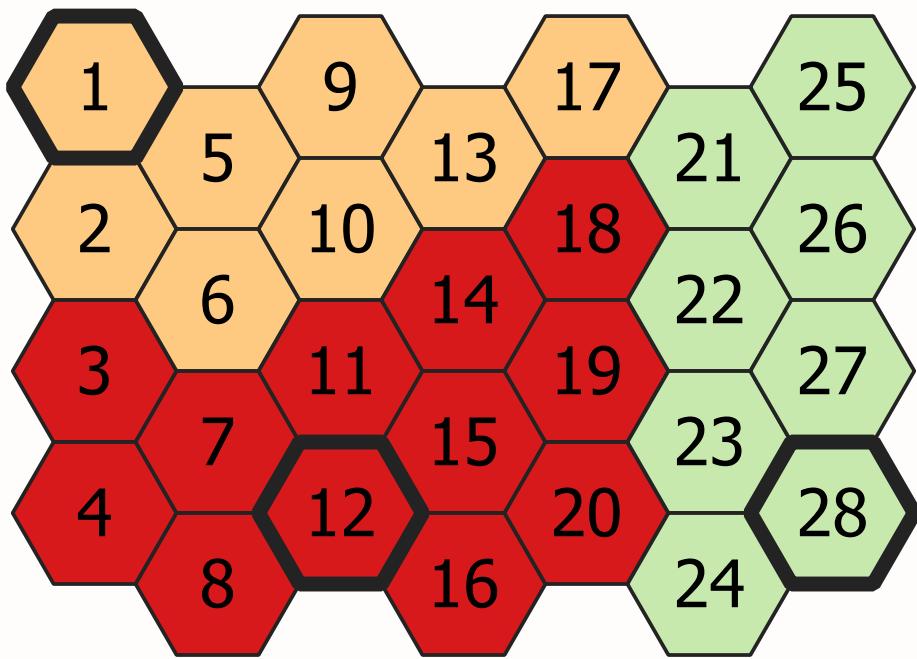


Figure 2: Two departments

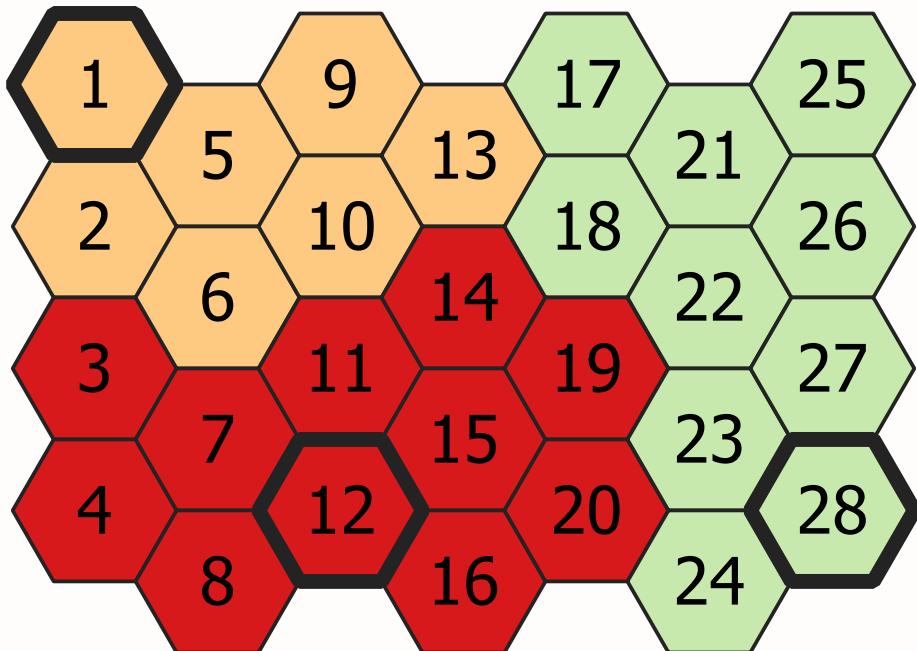


Figure 3: Up to three departments

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### **i** Why does this work?

Due to the constraints, there is always a path back to the department if a BA is assigned to a department!

## Model Characteristics

### Characteristics

Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?
- What do you think, can the model be solved quickly?
- Have we prevented isolated districts?

### Model Assumptions

Questions: On model assumptions

- What assumptions have we made?
- Use Euclidean distances to approximate driving time?
- Can we rely on incident data collected by the police?

## Implementation and Impact

### Overview of Studies

Question: Where did we apply our model?

- Two distinct environments:
  1. Large metropolitan area (Germany)
  2. Rural region (Belgium)
- Different challenges and requirements
- Focus on response time optimization

### German Metropolitan Case

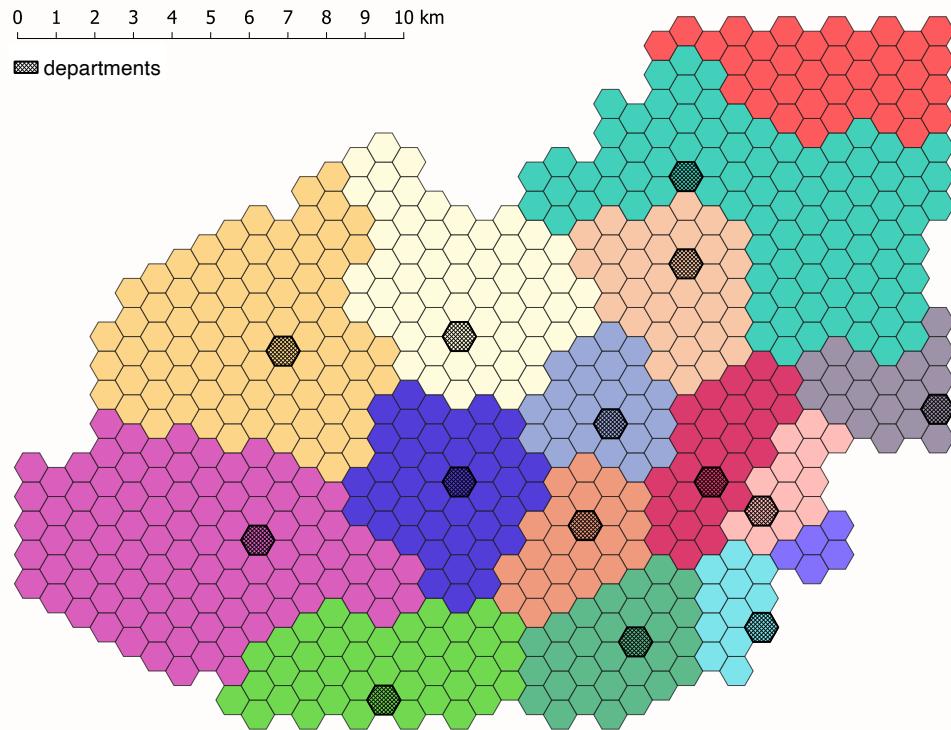
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- 1.8 mio incidents (2015-2019)
- ~20 department locations
- 1,596 basic areas
- Dense urban environment

...

Goal: Redesign districts to improve response time.

## German Metropolitan Results



## Belgian Rural Case

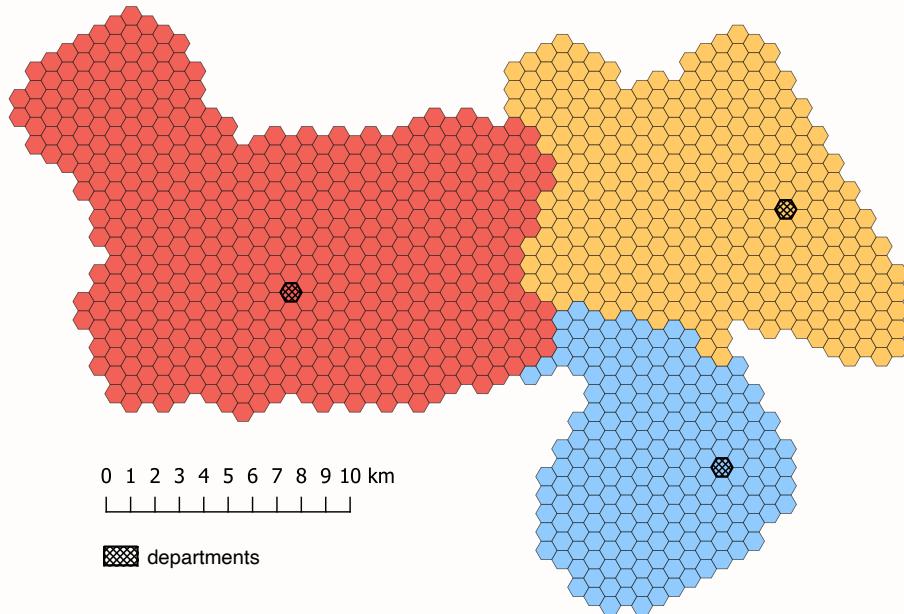
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- 50,000 incidents (2019-2020)
- 2 existing + 1 planned location
- 1,233 basic areas
- Dispersed rural setting

...

Goal: Optimize coverage with limited resources.

## Belgian Rural Results



## Simulation Framework

How did we validate the results?

...

- Spatial and temporal patterns
- Shift schedules
- Priority handling
- Rush hours
- Inter-district support
- Variable driving times

## Results

- Response time reduction up to 14.52%
- Better workload distribution
- Improved coverage equity
- More efficient resource utilization

...

**! Important**

All improvements are without additional staff!

## Conclusions

1. Model adaptability crucial
2. Local context matters
3. Stakeholder buy-in essential

#### 4. Data quality critical

...

##### Tip

Success requires balancing theoretical optimization with practical constraints!

### Future Applications

Question: Where else could this approach be useful?

- Other emergency services
- Different urban contexts
- Resource allocation problems
- Service territory design

...

##### Note

The methodology is adaptable to various public service optimization scenarios.

### Wrap Up

##### And that's it for todays lecture!

We now have covered districting problems and are ready to start solving some tasks in the upcoming tutorial.

Questions?

### Literature

#### Literature I

For more interesting literature to learn more about Julia, take a look at the [literature list](#) of this course.

#### Literature II

#### Bibliography

- [1] T. Vlček, K. Haase, M. Fliedner, and T. Cors, “Police service district planning,” OR Spectrum, Feb. 2024, doi: [10.1007/s00291-024-00745-3](https://doi.org/10.1007/s00291-024-00745-3).
- [2] C. Bruce, “Districting and Resource Allocation: A Question of Balance,” A Quarterly Bulletin of Applied Geography for the Study of Crime & Public Safety, vol. 1, no. 4, pp. 1–3, 2009.

- [3] F. Liberatore, M. Camacho-Collados, and B. Vitoriano, “Police Districting Problem: Literature Review and Annotated Bibliography,” International Series in Operations Research & Management Science: Optimal Districting and Territory Design. Springer International Publishing, Cham, pp. 9–29, 2020. doi: [10.1007/978-3-030-34312-5\\_2](https://doi.org/10.1007/978-3-030-34312-5_2).
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- [10] J. Kalcsics, S. Nickel, and M. Schröder, “Towards a unified territorial design approach — Applications, algorithms and GIS integration,” Top, vol. 13, no. 1, pp. 1–56, 2005, doi: [10.1007/BF02578982](https://doi.org/10.1007/BF02578982).