

# Introduction to Metaheuristics

## Lecture 9 - Management Science

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### Introduction

#### Client Briefing: La Étoile

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Restaurant Manager's Crisis:

"I need to schedule my 18 servers across 6 shifts this weekend. Shifts have different lengths (4-6 hours), and if I don't have enough experienced servers on busy shifts, we face penalties per missing experienced server from our parent company!"

#### The Staffing Challenge

A restaurant facing a weekend scheduling crisis:

La Étoile's Problem:

- 18 servers available (6 experienced @ €75/hr, 12 junior @ €25/hr)
- 6 shifts with varying lengths (4-6 hours each)
- Each shift needs 3 servers (at least 1 experienced)
- Server preferences matter (1-10 scale, affects quality)

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Question: How to balance labor costs, penalties, AND staff?

#### The Cost Impact: Why This Matters

The financial stakes are significant with these large penalties:

- Minimum Labor Cost: ~€3,500 (everyone works once)
- Experience Penalties: €0-€1,200 per missing experienced server
- Preference Penalties: €0-€180 per unhappy assignment
- Worst Case: Over €7,000 if poorly scheduled!
- Best Case: ??? with smart optimization

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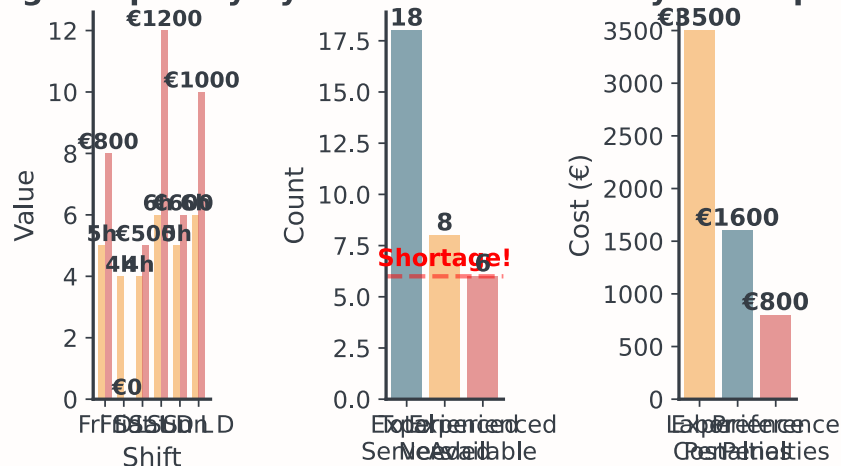
! Important

Potentially up to €3,500 difference between good and bad scheduling!

## Restaurant Staffing: The Numbers

The real-world complexity we're dealing with:

### Varying Complexity by Shift, Constraint, and Preference Cost Optimization



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⚠ Warning

With varying shifts, preferences, and penalties, this is will be a real challenge!

## Today's Objectives

What you'll understand after this lecture:

1. Why local search fails: Recap on the local optima trap
2. Escape mechanisms: How to accept worse solutions strategically
3. Four powerful metaheuristics: SA, GA, Tabu Search, ACO
4. Selection criteria: When to use which algorithm

## Hiking in Fog

Remember the metaphor with blindfolded eyes from last lecture?

- Goal: Find the highest peak in a mountain range
- Challenge: You're hiking in thick fog (can only see 10 meters)
- Position: Your X,Y coordinates = your decisions
- Altitude: Your current solution quality
- Problem: You might climb a small hill and think it's the summit!

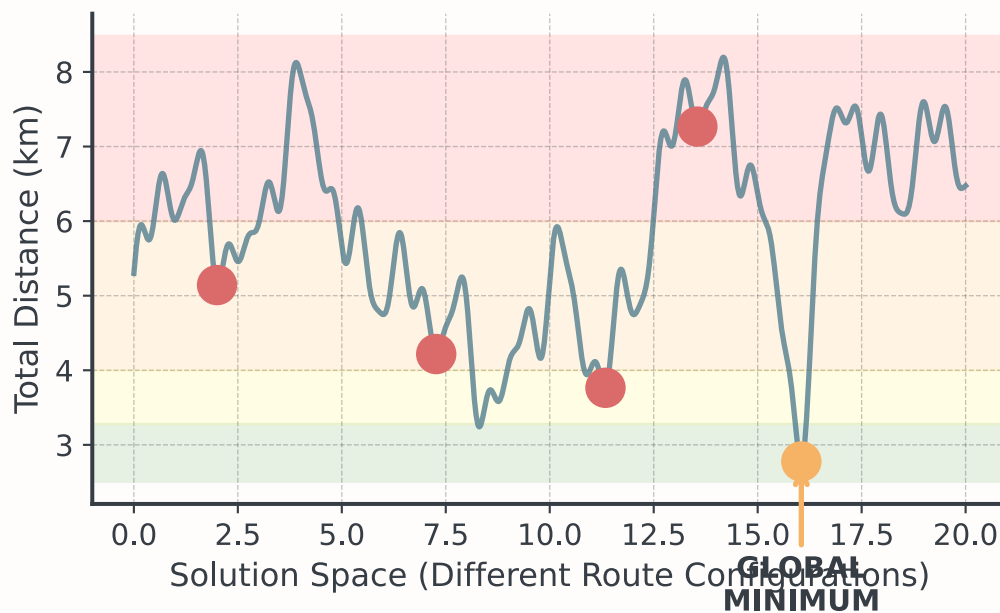
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#### 💡 Tip

This metaphor will guide us through all metaheuristics today!

## Recap: Local Optima

Real problems often have thousands of local optima!



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Question: Any idea how to escape local optima?

## Why Simple Methods Fail

### The Silo Problem

Why neighborhood optimization fails:

Technical View: Local Optima

- Algorithm climbs nearest hill
- Gets stuck on “foothill”
- Can’t see the mountain beyond
- Every move looks worse
- Believes it found the best

Analogy: Department Silos

- Sales optimizes sales metrics
- Engineering optimizes quality

- Finance optimizes costs
- Each department “wins”
- Company performance loses!

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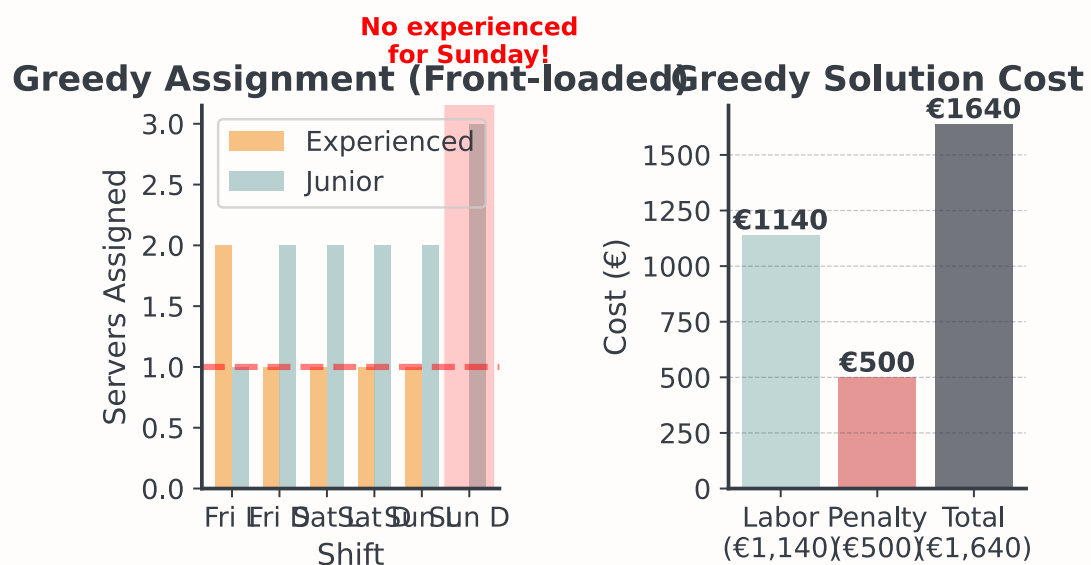
**! Important**

Sum of local bests  $\neq$  Global best

## Why Greedy Gets Stuck

Greedy algorithms can simply trap themselves:

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**! Warning**

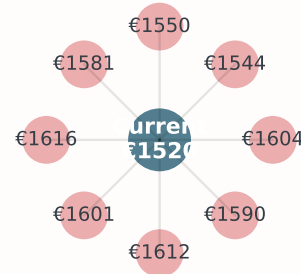
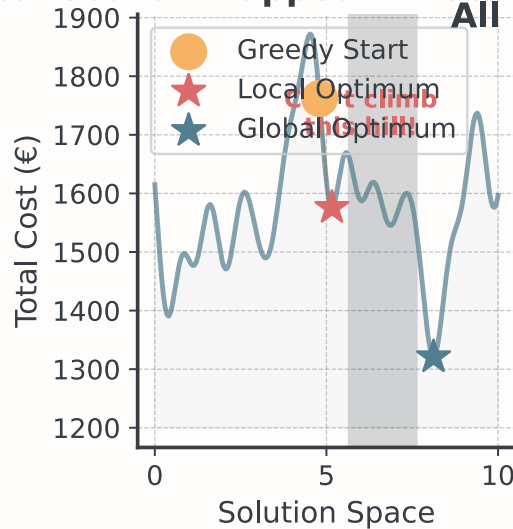
Greedy allocates resources early, creating problems later!

## Local Search Also Struggles

Because we only ever accept better solutions during search:

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## Local Search Trapped in Local Minimum



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Question: What can we do to cope with this situation?

## Metaheuristic #1: Simulated Annealing

### Core Concepts

The fundamental components:

- Solution = One complete schedule/route/plan
- Neighbor = A slightly modified version
- Cost = How good/bad the solution is

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### The Strategy

- Always accept improvements
- Sometimes accept worse solutions (the change!)

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Think of it as strategic risk-taking that decreases over time!

## The Metallurgy Metaphor

How annealing steel inspired an optimization algorithm:

Annealing Metal:

1. Heat to high temperature
2. Atoms move freely
3. Slowly cool down
4. Forms crystal structure

Optimization:

1. Start with high “temperature”
2. Accept bad moves often
3. Gradually reduce temperature
4. Converge to good solution

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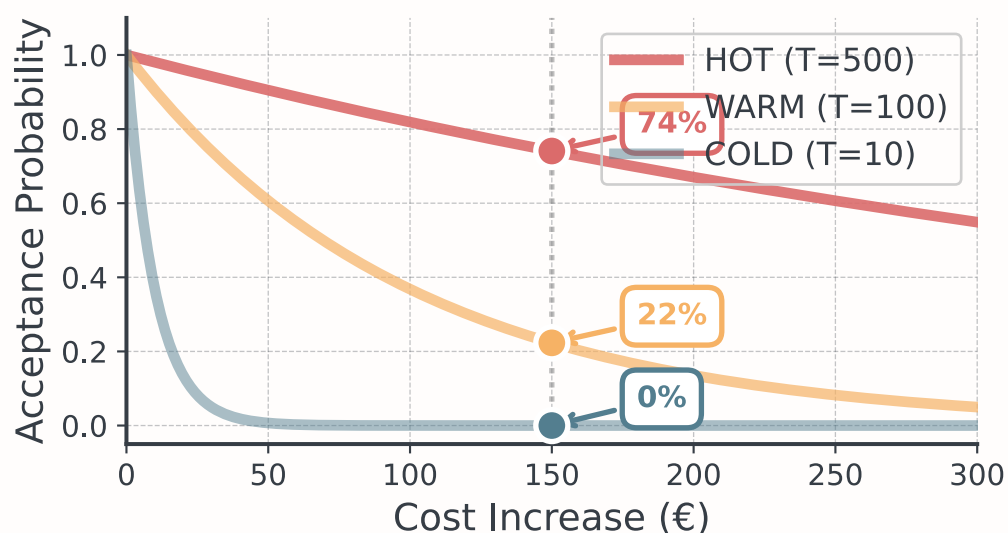
### ! Important

The willingness to temporarily accept worse solutions is what enables finding the summit!

## Temperature Controls Acceptance

Probability of accepting worse solutions lowers with temp:

$$\text{Formula: } P(\text{accept}) = \exp(-\Delta\text{cost} / T)$$



...

### 💡 Tip

We essentially compare the cost of the new schedule to the current cost and decide whether to accept the change based on the temperature and the difference in cost.

## Concept

How Simulated Annealing Works (Pseudocode)

```

def simulated_annealing_concept(current_schedule):
    temperature = 500 # Start "hot" (adventurous)
    best_schedule = current_schedule

    while temperature > 1:
        # Step 1: Try a random change (like swapping two shifts)
        new_schedule = make_random_change(current_schedule)

        # Step 2: Is it better?
        if cost(new_schedule) < cost(current_schedule):
            current_schedule = new_schedule # Always accept improvements
        else:
            # NEW: Sometimes accept worse solutions!
            # Hot temperature = more likely to accept
            # Cold temperature = less likely to accept
            if random() < acceptance_probability(temperature):
                current_schedule = new_schedule # Accept anyway!

        # Step 3: Cool down (become less adventurous)
        temperature = temperature * 0.95

        # Remember the best we've ever seen
        if cost(current_schedule) < cost(best_schedule):
            best_schedule = current_schedule

    return best_schedule

```

## SA in Action: Restaurant Staffing

A simplified weekend scheduling problem we'll use throughout:

```

# SIMPLIFIED PROBLEM (used across ALL metaheuristics today)
"""
La Étoile's Weekend Challenge:
- 12 servers: 4 experienced (E1-E4), 8 junior (J1-J8)
- 6 shifts: Need 2 servers each
- Everyone works exactly 1 shift
"""

```

...

The initial greedy schedule has the following results:

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```

Greedy Schedule Cost: €5,240
Labor: €2250, Penalties: €1700, Unhappiness: €1290

```

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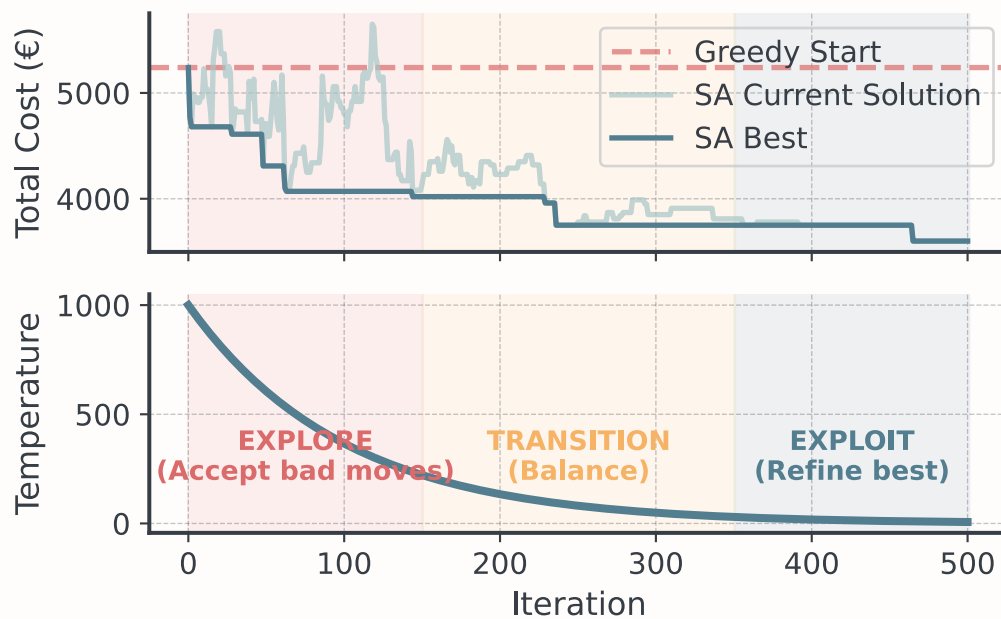


Tip

Let's see how Simulated Annealing can improve the solution!

## Visualizing SA Performance

How temperature affects the search behavior:



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### ! Important

See how SA accepts worse solutions early, enabling escape from local optima!

## Common SA Mistakes

Avoid these common implementation errors:

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### Mistake #1: Starting Too Cold

- If temperature is too low → Acts like greedy (no exploration)
- Fix: Start hot enough to accept bad moves

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### Mistake #2: Cooling Too Quickly

- If you cool fast → Get stuck early
- Fix: Cool slowly (multiply by 0.95-0.99, not 0.5)



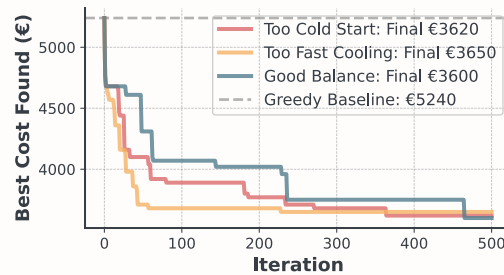
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### ⚠ Warning

Quick cooling is tempting for speed, but defeats the purpose of SA!

## Temperature Parameter Impact

### Impact of Temperature Parameters on SA Performance (Same problem, different settings)



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### 💡 Tip

The “Good Balance” explores widely early, then refines carefully. Often you need to balance exploration and exploitation by experimenting with different parameters.

## Metaheuristic #2: Genetic Algorithms

### Evolution as Optimization

How natural selection inspires computational optimization:

Natural Selection:

1. Population of individuals
2. Fittest survive & reproduce
3. Offspring inherit traits
4. Mutations create diversity
5. Evolution finds adaptation

Optimization:

1. Population of solutions
2. Best solutions selected
3. Crossover combines solutions
4. Mutation adds variation
5. Evolution finds optimum

...

### 💡 Tip

Just like successful products get more market share, better solutions get more “offspring” in the next generation. It’s survival of the fittest, but for schedules, routes, or designs!

## The Genetic Process

Four stages repeat each generation:

1. Selection: Choose parents based on fitness
2. Crossover: Combine to create children
3. Mutation: Randomly modify children
4. Replacement: New generation replaces old
5. Population Evolution: Improve population quality

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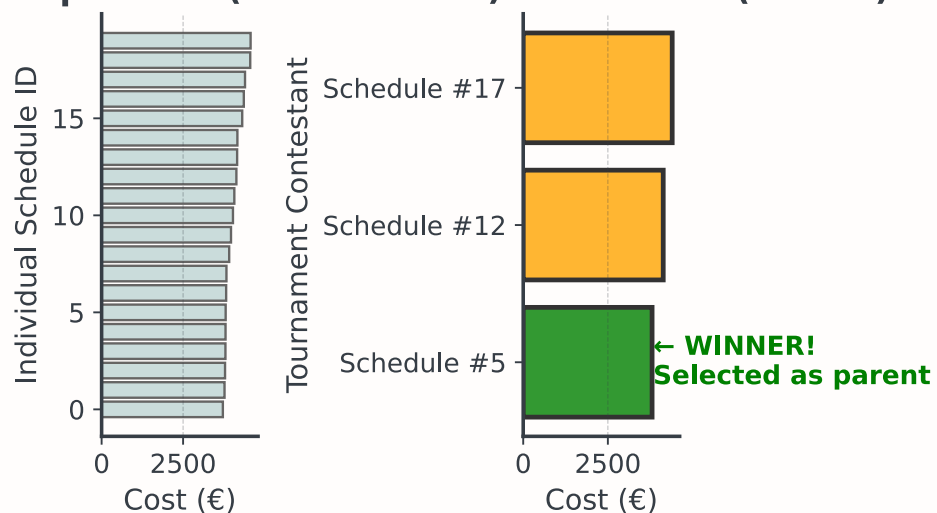
### 💡 Tip

Let’s see each stage in detail with our restaurant problem!

## Stage 1: Selection (Tournament)

How to choose which schedules get to “reproduce”:

### Current Population (20 schedules) Tournament Selection (size=3)



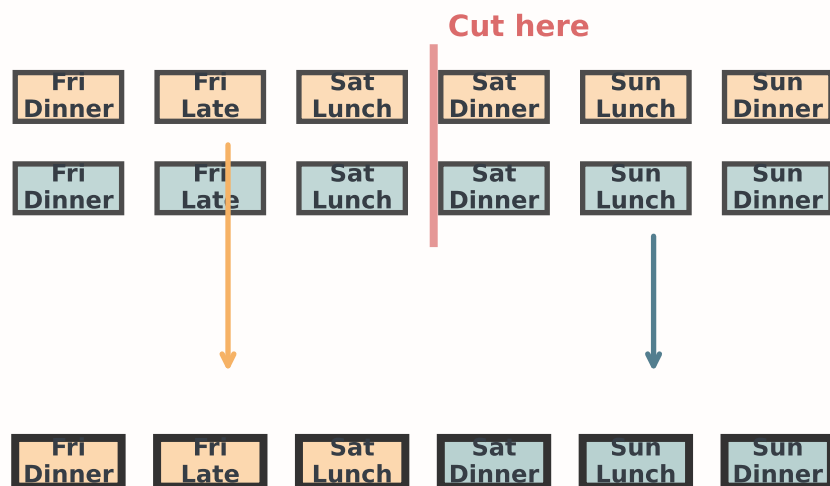
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### ! Important

Each tournament selects one parent, then we pair them up sequentially for crossover.

## Stage 2: Crossover (Recombination)

Combine two parent schedules to create offspring:



*First 3 shifts from Parent 1, last 3 from Parent 2*

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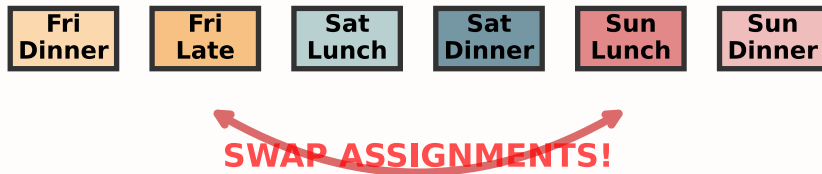
### 💡 Tip

Crossover randomly combines good building blocks from both parents!

## Stage 3: Mutation

Random changes maintain diversity and explore new solutions:

## Before Mutation



## After Mutation



*Mutation rate: 20% (happens to 1 in 5 offspring)*

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### 💡 Tip

Mutation adds random exploration, like trying something completely new occasionally!

## Stage 4: Replacement I

How do offspring join the population?

- Generational: Replace entire population with offspring
- Steady-State: Replace only worst individuals
- Elitism: Always keep the best solutions

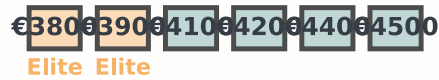
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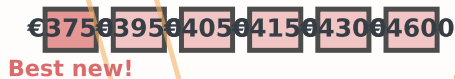
Our approach: Generational with Elitism, we create 20 offspring via repeated selection/crossover/mutation, but preserve the 2 best from current generation.

## Stage 4: Replacement II

### Current Generation (Population = 6)



### Offspring Created (6 new solutions)



### Next Generation (Elitism: keep 2 best)



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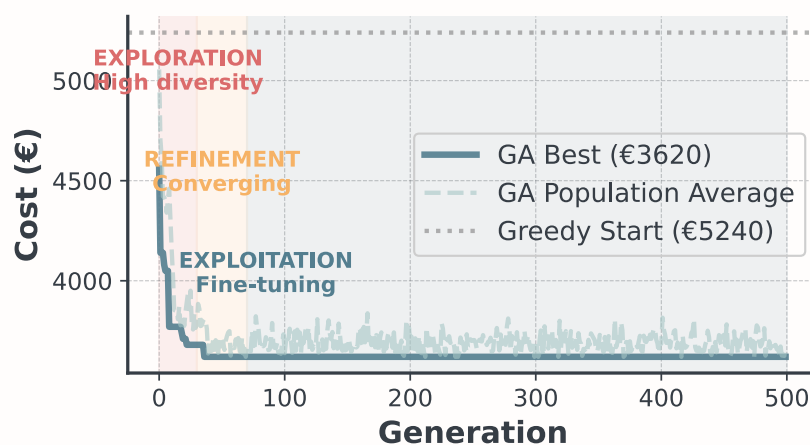
#### Note

Elitism ensures we never lose our best solutions while exploring new ones!

## Stage 5. Population Evolution

How the population improves over generations:

### Genetic Algorithm: Population Improvement Over Time



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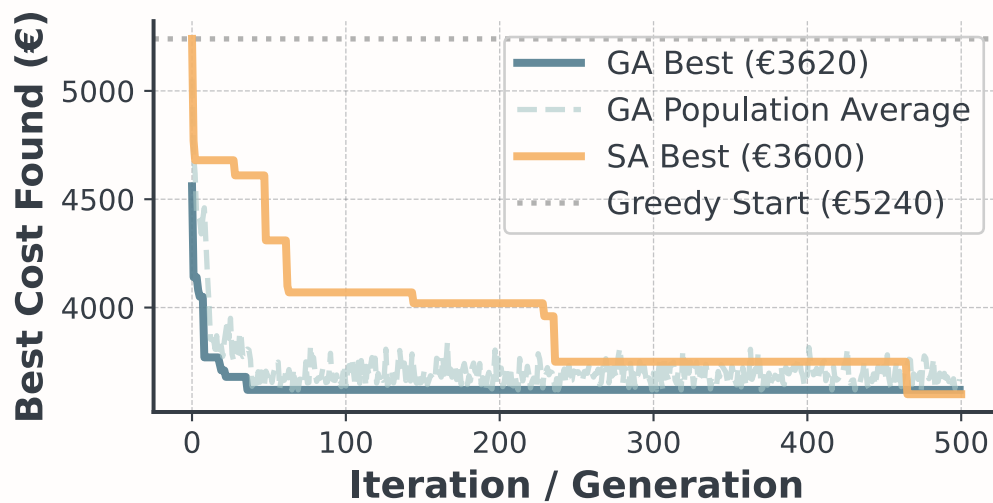
#### Note

Notice how population average also improves (not just the best)!

## GA vs SA: Head-to-Head

Comparing exploration strategies on the restaurant problem:

### Genetic Algorithm vs Simulated Annealing (Same Restaurant Problem)



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#### Tip

GA maintains population diversity, SA explores single solution path!

## GA Mistakes: Population Issues

Avoid these population-related errors:

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Mistake #1: Everyone Becomes Identical

- If all solutions look the same → Lost diversity
- Fix: More mutation, bigger population, tournament selection

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Mistake #2: Too Greedy in Selection

- Only keeping the very best → Premature convergence
- Fix: Keep some variety, even if not perfect (elitism of 10-20%)

## GA Mistakes: Implementation

Technical pitfalls to watch out for:

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### Mistake #3: Breaking the Rules

- Crossover might create invalid schedules
- Fix: Always check and repair after crossover/mutation

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### Mistake #4: Evolution Too Slow

- Population too large or too many generations
- Fix: Start small (50-100), tune based on convergence

## Metaheuristic #3: Tabu Search

### Core Idea

Using memory to avoid cycling through bad solutions:

Analogy:

- Keep a list of recent dates
- “Not going back there!”
- Forces you to meet new people
- After time, memory fades

In Optimization:

- Recent solutions = “tabu”
- Don’t revisit same schedules
- Forces exploration of new areas
- Tabu list has limited size

...

#### Tip

Like keeping “lessons learned”, you remember not to use them again, but after a while, you might reconsider!

### Concept

How Tabu Search Works (Pseudocode)

```
def tabu_search_concept():  
    tabu_list = [] # Our "never again" list  
    current_solution = initial_schedule
```

```

best_solution = current_solution

while not done:
    # Look at all possible moves
    possible_moves = get_all_neighbor_moves(current_solution)

    # Filter out the "forbidden" moves
    allowed_moves = []
    for move in possible_moves:
        if move not in tabu_list: # Not forbidden
            allowed_moves.append(move)

    # Pick the best allowed move (even if worse!)
    best_move = select_best(allowed_moves)
    current_solution = apply(best_move)

    # Update best if improved
    if cost(current_solution) < cost(best_solution):
        best_solution = current_solution

    # Remember this move (add to tabu list)
    tabu_list.append(best_move)
    if len(tabu_list) > 10: # Keep list size manageable
        tabu_list.pop(0) # Forget oldest

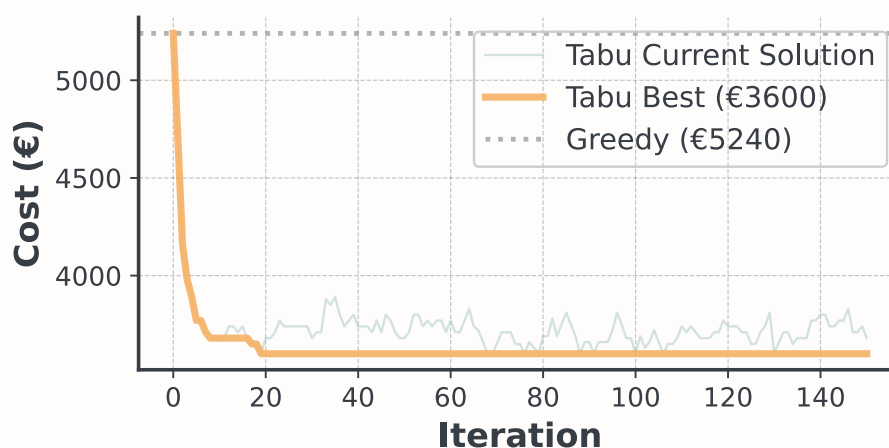
return best_solution

```

## Tabu Search on Restaurant Problem

Real implementation with memory-based exploration:

### Tabu Search on Restaurant Staffing Problem (Memory prevents cycling back to poor solutions)



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Tip

Tabu Search's memory prevents revisiting bad solutions!

## Metaheuristic #4: Ant Colony Optimization

### The Core Idea

Collective intelligence through chemical signals:

Reviews:

- Popular choices get more 5-star reviews
- More reviews → more visibility
- But old reviews fade over time
- New patterns can emerge

In Optimization:

- Good assignments get “pheromones”
- Stronger chosen more likely
- Evaporation against stagnation
- Colony finds patterns



Tip

Imagine each server-shift pairing has a “rating” that increases when it works well in a schedule. Over time, the best pairings naturally get chosen more often!

### How ACO Works on Scheduling

Four key stages in each iteration:

1. Construction: Each ant builds a schedule probabilistically
2. Evaluation & Evaporation: Measure quality, then fade all pheromones
3. Reinforcement: Good schedules deposit pheromones
4. Evolution: Repeat until colony converges

...



Tip

Let's see each stage visually!

### ACO: Key Parameters

Two critical parameters control the balance:

Evaporation Rate ( $\rho$ )

- Higher  $\rho \rightarrow$  More forgetting
- Lower  $\rho \rightarrow$  Stronger memory
- Typical: 0.1 - 0.5
- Too high: Colony never learns
- Too low: Gets stuck early

Number of Ants

- More ants  $\rightarrow$  More exploration
- Fewer ants  $\rightarrow$  Faster iterations
- Typical: 10 - 50
- Too many: Slow, redundant
- Too few: Miss good patterns

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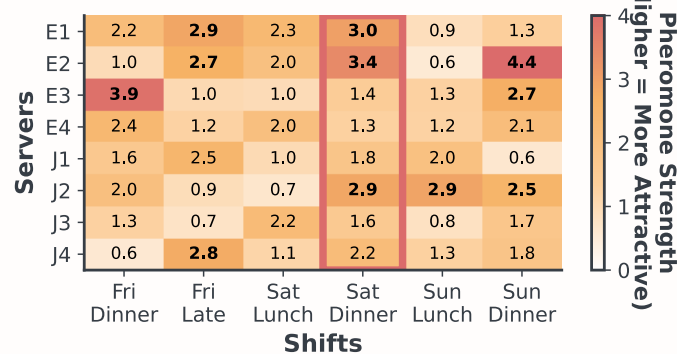
💡 Tip

Start with  $\rho=0.3$  and  $n\_ants=20$ , then tune based on problem size.

## Stage 1: Pheromone Construction

Ants don't pick randomly, they follow the chemical trails

**ACO Stage 1: Pheromone Matrix Guides Construction**  
(Each ant probabilistically selects servers based on pheromone strength)



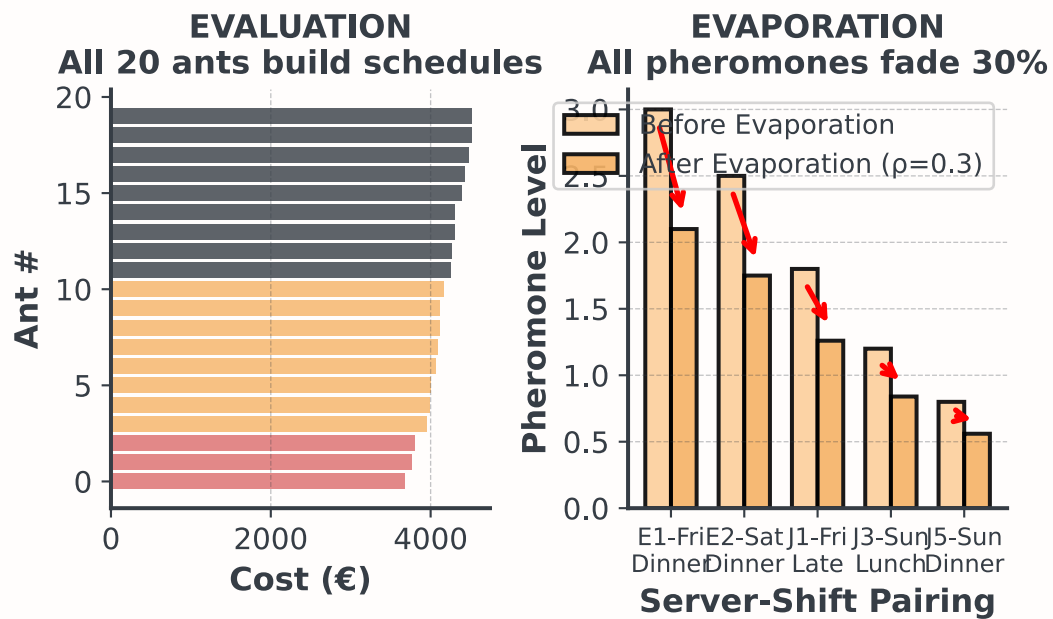
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💡 Tip

To build the initial pheromone matrix, each cell is initialized with a small positive value.

## Stage 2: Evaluation & Evaporation

After all ants build schedules:



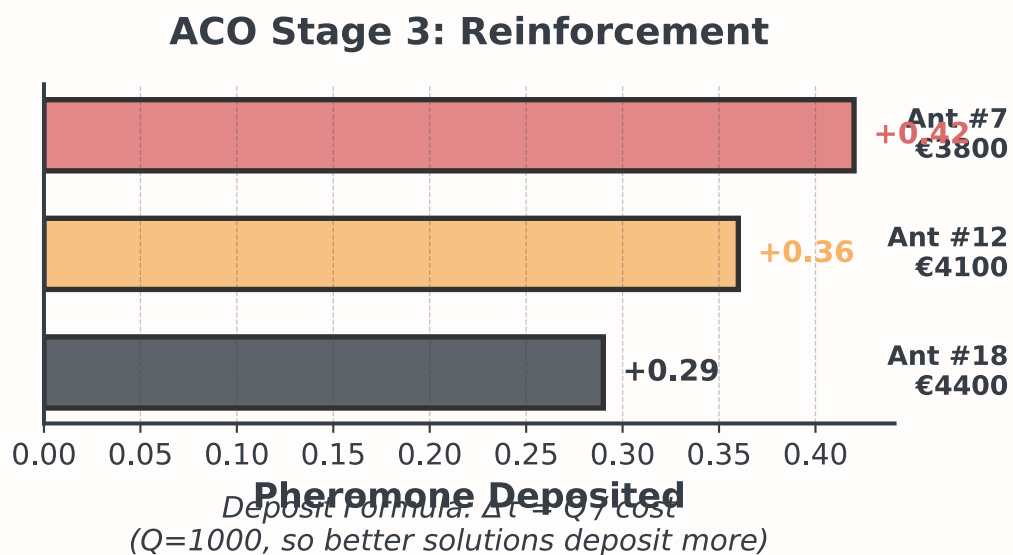
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#### Tip

Evaporation prevents premature convergence as old patterns can fade away!

### Stage 3: Reinforcement

Good ants deposit more pheromones:

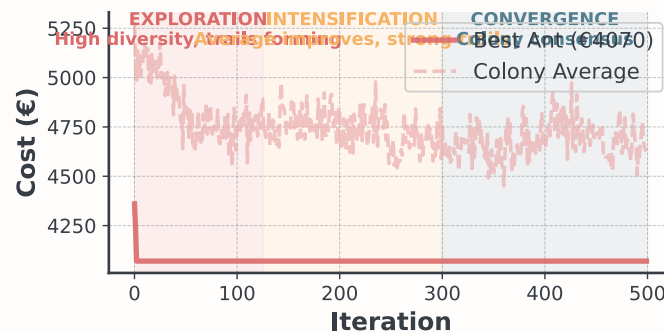


The best solutions leave the strongest trails for future iterations!

## Stage 4: Evolution Over Iterations

Full ACO implementation on restaurant staffing:

### ACO Stage 4: Evolution Through Collective Learning (Average colony performance improves as pheromones strengthen)



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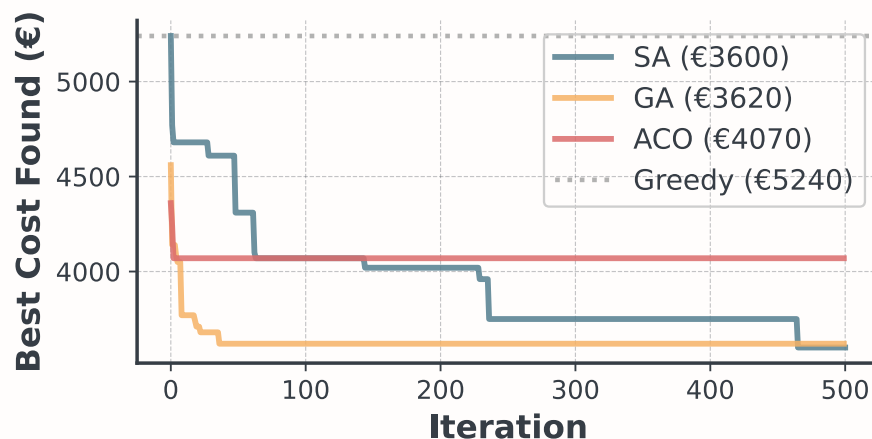
#### Tip

The colony learns collectively step by step.

## ACO vs Other Algorithms

How does the ant colony compare?

### ACO vs SA vs GA on Restaurant Staffing (Three different search philosophies, one problem)



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#### Tip

Any idea why ACO fares worse?

## Why did ACO struggle?

The Right Tool for the Wrong Job

ACO is designed for Sequential Path-Finding (Graph Traversal).

- Best For: “I am at City A, where should I go next?” (TSP)
- Our Problem: “Who should work Friday Dinner?” (Assignment)

...

### ! Important

We forced a “Graph” algorithm onto a “Bin Packing” problem. SA and GA don’t care about geometry, so they adapted better!

## Decision Framework

### When to Use Which Metaheuristic?

A decision guide for algorithm selection:

Method	Time	Quality	Complexity	Best For
Random	xxxx	x	Trivial	Baseline
Greedy	xxx	xx	Simple	Quick decisions
LS	xx	xxx	Medium	Improvement
SA	xx	xxxx	Medium	Single solution
GA	x	xxxx	High	Population
TS	xx	xxx	Medium	Avoid cycles
ACO	x	xxxx	High	Changing Paths

## The No Free Lunch Theorem

Why there’s no universal best algorithm:

“No Free Lunch Theorem”: No single algorithm is best for all problems. Your choice must match your problem structure:

- Path/Network Problems → ACO (pheromones for paths)
- Scheduling Problems → SA or Tabu (neighborhood swaps)
- Complex Design → GA (population diversity)

## Implementation Strategy

Guidelines for successful implementation:

1. Start Simple: Always try greedy first as baseline
2. Profile Your Problem: Understand constraints before choosing

3. Tune Incrementally: Don't optimize all parameters at once
4. Track Progress: Monitor convergence to know when to stop
5. Hybrid Approaches: Combine methods (e.g., GA + Local Search)
6. Use AI Assistance: Bridge the "knowledge gap" with GenAI

## Today's Briefing

### Today

#### Hour 2: This Lecture

- Metaheuristics
- Simulated annealing
- Genetic algorithms
- Tabu search & ACO

#### Hour 3: Notebook

- Bean Counter delivery
- Implement SA
- Tune parameters
- Compare performance

#### Hour 4: Competition

- Restaurant staffing
- Multi-objective
- Schedule optimization
- Justify choice!

## The Competition Challenge

### La Étoile Restaurant Weekend Staffing

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1. Schedule 18 servers across 6 shifts (different lengths)
2. Minimize labor + experience penalties + preference costs
3. Ensure experienced coverage (strategic placement)
4. Respect 2 servers per shift requirement

...

#### ! Important

Choose your metaheuristic wisely as this is a tough problem!

## Implementation Pitfalls to Avoid

Common bugs that cost you time:

1. Wrong Cooling Schedule (SA)
  - Too fast = stuck in local optimum
  - Too slow = wastes computation time
2. Poor Population Management (GA)
  - Too small = no diversity
  - Too large = slow evolution
3. Not Tracking Best
  - Current solution  $\neq$  best ever found

## Parameter Tuning Strategies

How to find good parameters without wasting time:

1. Start with Rules of Thumb
  - SA: Initial temp = max cost difference you'd accept
  - GA: Population = 10-20 $\times$  number of decision variables
  - Tabu: Tenure (limit) =  $\sqrt{\text{problem size}}$
2. Grid Search on Small Instance
  - Test 3-4 values per parameter
  - Use 10% of full dataset
  - Example: `temps = [100, 500], cooling = [0.95, 0.99]`

## Real-World Considerations

Technical realities when putting metaheuristics into production:

Factor	Questions to Ask
Time Budget	How long can optimization run?
Solution Quality	Need optimal or “good enough”?
Explainability	Must justify decisions?
Problem Changes	Static or dynamic data?
Team Skills	Who maintains this code?

...

### ! Important

Use the simplest method that meets your quality target. Complex metaheuristics are great but more costly to maintain!

## Summary

Key Takeaways:

- Metaheuristics escape local optima when exact methods fail
- SA uses temperature, GA uses evolution, Tabu uses memory

- Parameter tuning is critical for performance
- No universal best - problem structure matters
- Start simple, add complexity only if needed

Break!

Take 20 minutes, then we start the practice notebook

Next up: You'll implement metaheuristics for Bean Counter

## Bibliography