

# Multi-Objective Optimization

## Lecture 8 - Management Science

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### Introduction

#### Client Briefing: EcoExpress Logistics

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Operations Director's Dilemma:

"EU regulations demand 40% emission cuts, but we can't sacrifice profitability, service quality, or reliability!"

#### The Fleet Challenge

EcoExpress operates regional last-mile delivery across 3 cities

- EU Green Deal: 40% emission reduction by 2025
- Rising fuel costs (€2.1/L diesel)
- Amazon entering our market (speed pressure)
- Driver shortage (need automation-friendly vehicles)

...

Question: How do we transform our fleet while staying competitive?

#### Today's Learning Objectives

By the end of this lecture, you will be able to:

1. Explain why most decisions involve competing objectives
2. Identify and visualize Pareto optimal solutions
3. Apply normalization techniques to make objectives comparable
4. Implement approaches to find trade-off solutions
5. Make decisions from a Pareto frontier

#### Quick Recap: Local Search

Last week we optimized routes for delivery:

- Started with greedy construction (e.g. Nearest Neighbor)
- Improved with local search (e.g. 2-opt)
- Considered time windows
- But: We only optimized distance

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Question: What if we also care about emissions, cost, AND customer satisfaction?

## The Problem

### Single vs Multi-Objective

#### Single Objective

- “Minimize total distance”
- Clear winner. Easy, right!

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#### Multiple Objectives

- “Minimize cost AND emissions AND maximize speed”
- No clear answer...

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Question: Any idea how to approach this?

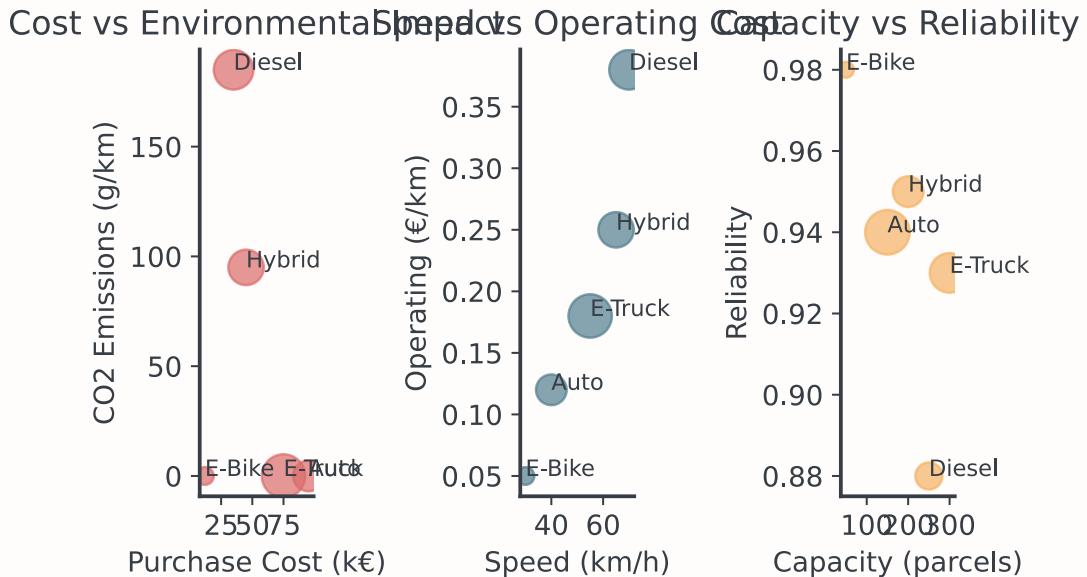
### EcoExpress Vehicle Options

Type	Purchase Cost (€)	Operat-ing (€/ km)	CO2 (g/ km)	Speed (km/h)	Capacity (parcels)	Reliability
E-Truck	75000	0.18	0	55	300	0.93
Hybrid	45000	0.25	95	65	200	0.95
Diesel	35000	0.38	185	70	250	0.88
E-Bike	12000	0.05	0	30	50	0.98
Auto	95000	0.12	0	40	150	0.94

...

Question: Which vehicle is “best” for EcoExpress?

## Trade-offs Everywhere



...

! Important

Every vehicle excels at something different!

## Real Business Constraints

Beyond the numbers, consider:

- EU regulations: Carbon tax of €100/ton CO<sub>2</sub> starting 2025
- Competition: Amazon promises 2-hour delivery
- Labor market: Autonomous vehicles reduce driver dependency
- Urban zones: Zero-emission zones in city centers
- Peak times: Black Friday = 3x normal volume

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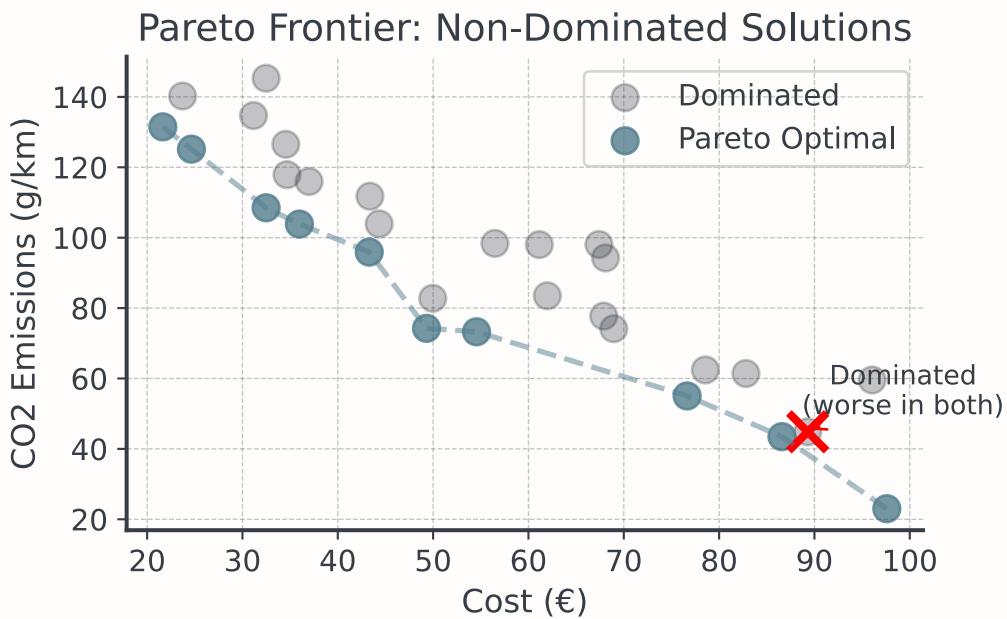
! Important

There is no single “optimal” solution - only trade-offs

## Pareto Optimality

### Dominated Solutions

A solution is dominated if another solution is:



! Important

Better in at least one objective and not worse in any objective!

## The Pareto Frontier

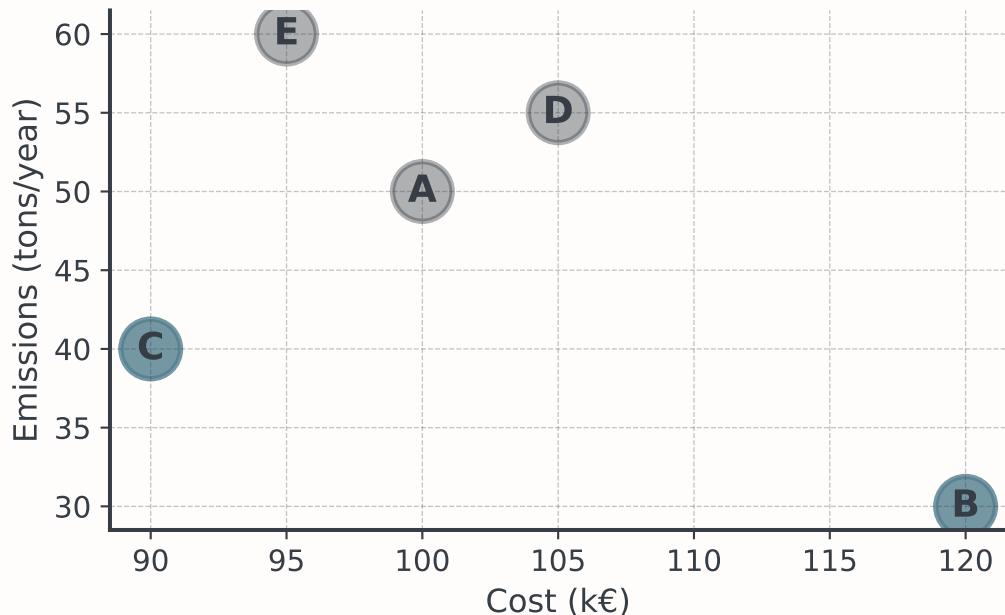
The Pareto frontier is the set of all non-dominated solutions

- No solution is objectively “better”
- Each represents a different trade-off
- Moving along frontier: gain in one objective, loss in another
- Decision makers choose based on preferences

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Question Do you think you get the idea?

Find the Non-Dominated



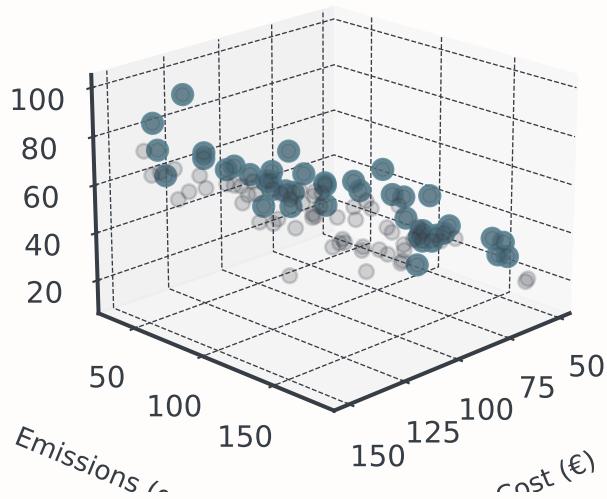
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Question: Which fleets are non-dominated?

Three+ Objectives

With 3 objectives, the Pareto frontier becomes a surface:

### 3D Pareto Frontier (Surface)



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### ! Important

Harder to visualize, but same principle applies!

## Fleet Composition Problem

### The Fleet Challenge

EcoExpress needs to replace their 80 diesel vans

- Must meet EU regulation: Average emissions  $\leq 111 \text{ g CO}_2/\text{km}$
- Need capacity for 22,000 parcels/day
- Must balance cost vs. service quality
- 5 vehicle types available, each with trade-offs

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Question: How do we choose the right mix?

### Vehicle Options Recap

Type	Purchase Cost (€)	Operat-ing (€/ km)	CO2 (g/ km)	Speed (km/h)	Capacity (parcels)	Reliability
E-Truck	75000	0.18	0	55	300	0.93
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E-Bike	12000	0.05	0	30	50	0.98
Auto	95000	0.12	0	40	150	0.94

...

Notice: No single vehicle is “best” at everything!

### Fleet Composition Framework

This is a discrete selection problem, not continuous allocation

Decision Variables:

- Fleet: How many of each vehicle type? (discrete/integer)
- $n_i$  = number of vehicles of type  $i$  (integers!)
- Example:  $n_{\text{E-Truck}} = 20$ ,  $n_{\text{Hybrid}} = 30$ , etc.

### Objective 1: Total Cost

Purchase cost + Operating cost over 3 years

$$\text{Total Cost} = \sum_i n_i \cdot (P_i + O_i \cdot d \cdot y)$$

- $n_i$  = quantity of vehicle type  $i$
- $P_i$  = purchase cost of vehicle type  $i$
- $O_i$  = operating cost per km for type  $i$
- $d$  = daily distance  $\times$  days per year
- $y$  = years

## Objective 2: Service Score

Composite measure of fleet performance

$$\text{Service Score} = 0.5 \cdot C_{\text{score}} + 0.3 \cdot R_{\text{score}} + 0.2 \cdot S_{\text{score}}$$

- $C_{\text{score}} = \min(1.0, \frac{\text{Total Capacity}}{22000})$  (capacity adequacy)
- $R_{\text{score}} = \frac{\sum n_i \cdot r_i}{\sum n_i}$  (weighted avg. reliability)
- $S_{\text{score}} = \frac{\sum n_i \cdot s_i}{70 \cdot \sum n_i}$  (normalized speed)

...

### **i** Note

Service score captures multiple performance dimensions in one metric!

## Hard Constraint: Emissions

EU regulation creates a feasibility boundary

$$\text{Average CO}_2 = \frac{\sum_i n_i \cdot e_i}{\sum_i n_i} \leq 111 \text{ g/km}$$

Where  $e_i$  = CO<sub>2</sub> emissions per km for vehicle type  $i$

...

This eliminates some solutions:

- All diesel vans: 185 g/km > 111
- Mix with too many diesel: Still violates
- Zero-emission + some diesel: Might work

## Data Source

Where Do These Numbers Come From?

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Vehicle Specifications:

- Purchase costs: Manufacturer quotes, market research
- Operating costs: Fuel/electricity prices, maintenance records
- Capacity: Vehicle specs (cargo volume, weight limits)

- Reliability: Historical uptime data, manufacturer warranties
- EU Standards: WLTP certification for vehicles
- Electric vehicles: Grid carbon intensity (kWh → g CO<sub>2</sub>)

## Example Fleet Comparison

Three Fleet Strategies:

	name	cost	service	co2	capacity	vehicles
Cost-Focused	28.9996	0.809705	120.714286		15000	70
Balanced	19.0478	0.731840	33.928571		13250	70
Green-Focused	15.3102	0.695373	0.000000		12750	75

Cost-Focused: ✗ VIOLATES (CO<sub>2</sub>: 120.7 g/km)

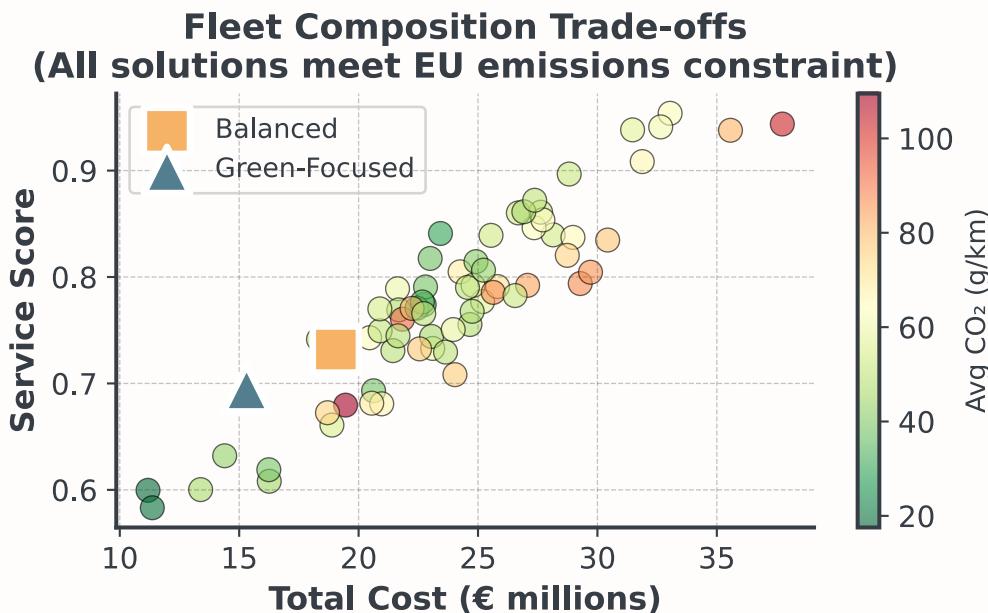
Balanced: ✓ Compliant (CO<sub>2</sub>: 33.9 g/km)

Green-Focused: ✓ Compliant (CO<sub>2</sub>: 0.0 g/km)

...

Question: Which strategy would you choose?

## Visualizing Fleet Trade-offs



Generated 68 feasible fleet compositions

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! Important

Each point is a different fleet mix, all meeting emissions constraint!

## Solution Approaches

### Multi-Objective Optimization

You can use optimization solvers or heuristics!

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#### With Optimization Solvers

- Weighted Sum Method
- $\varepsilon$ -Constraint Method
- Goal Programming
- Optimal solutions
- Need mathematical model

#### With Heuristics

- Weighted Greedy Construction
- Multi-Objective Local Search
- Metaheuristics
- Good solutions, fast
- No optimality proof

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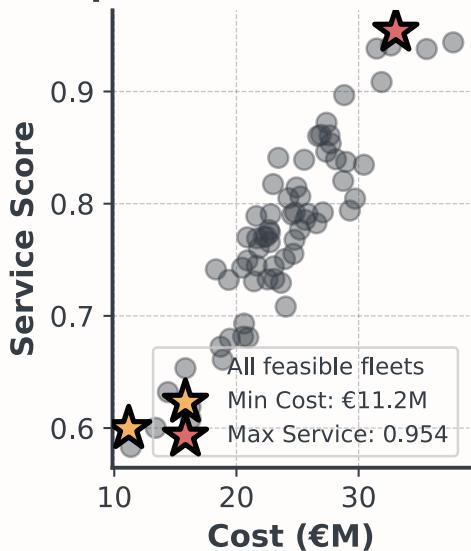
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In this lecture we use heuristic approaches!

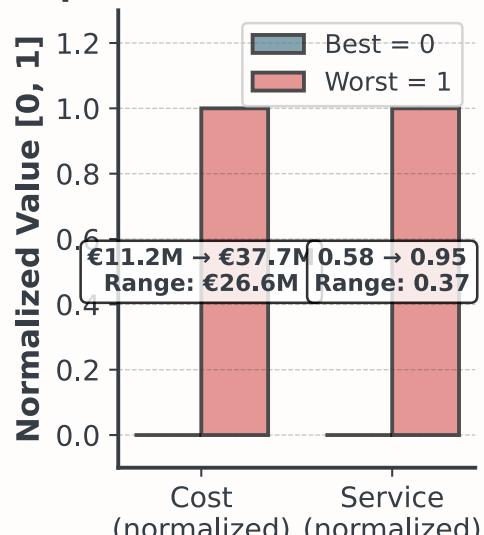
## Foundation: Extreme Points

First step for BOTH approaches - find the boundaries:

## Step 1: Find Extreme Points



## Step 2: Normalize to [0,1] Scale

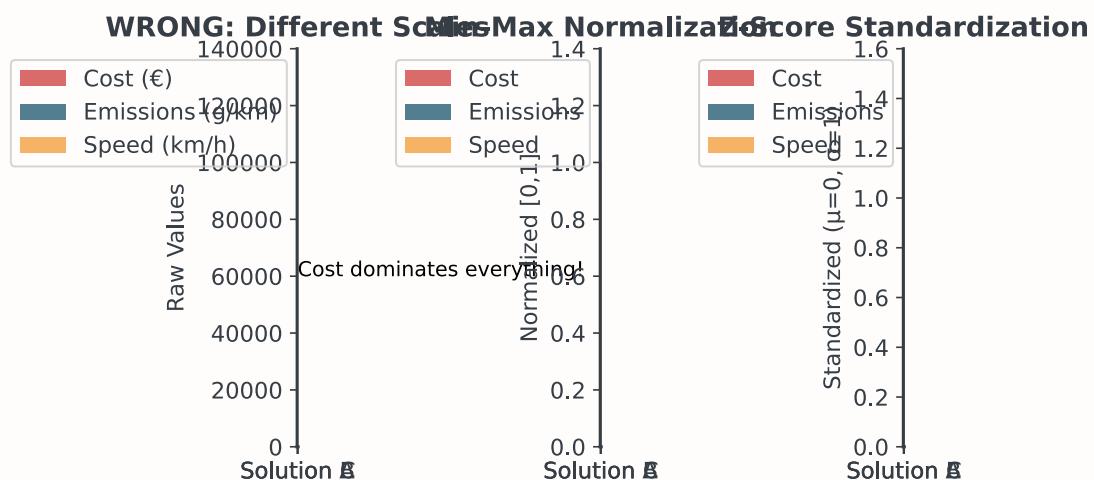


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Question: Why is normalization essential?

Critical: Normalization

Without it, your analysis is meaningless



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Question: Any intuition on how to do [0,1] normalization?

How to Normalize

The Normalization Formula for [0,1]

$$\text{Normalized}_i = \frac{x_i - x_{min}}{x_{max} - x_{min}}$$

...

In Python, this is rather simple!

...

```
def normalize_objectives(data):
    return (data - data.min()) / (data.max() - data.min())

# Now weights actually mean something
weighted_score = w1 * normalize(cost) + w2 * normalize(emissions)
```

...



Tip

Easy, right?

## Extreme Points

There are several reasons why extreme points matter:

1. Trade-off Space: Min/max values bound your Pareto frontier
2. Enable Proper Normalization: Need ranges for scaling to [0,1]
3. Feasibility: If single objectives not achievable, problem infeasible
4. Stakeholder: “Best cost is €50k, best emissions is 40kg”

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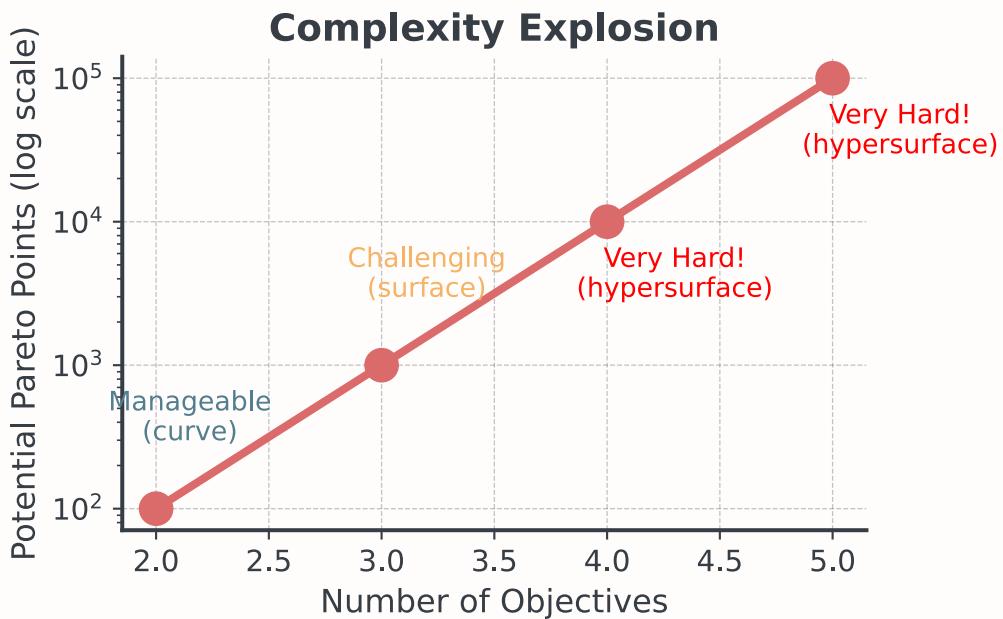
Implementation Pattern:

```
def find_extreme_points(problem):
    # Solve for minimum cost (ignore emissions)
    min_cost_solution = minimize(cost_objective, constraints)
    # Solve for minimum emissions (ignore cost)
    min_emissions_solution = minimize(emissions_objective, constraints)
```

## Computational Complexity

How hard does it get with more objectives?

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Tip

Why? Because there are just way more potential solutions to check!

## Solver-Based Methods

Quick overview - you won't implement these in assignments

1. Weighted Sum: Minimize  $w_1 \times \text{cost} + w_2 \times \text{emissions}$ 
  - Simple, fast for convex problems
2.  $\varepsilon$ -Constraint: Minimize cost subject to emissions  $\leq \varepsilon$ 
  - Systematically vary  $\varepsilon$  to find complete frontier
3. Goal Programming: Minimize deviations from targets
  - Set target for each objective, minimize weighted deviations

...

Note

For your fleet optimization: You'll use heuristic approaches instead!

## Heuristic Approach

### The Heuristic Strategy

For problems without mathematical models

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1. Construction: Build initial solutions with weighted greedy
  2. Improvement: Multi-objective local search
  3. Selection: Filter dominated solutions to find Pareto frontier
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### ! Important

Key difference from solvers:

- Solvers: Need mathematical model, guarantee optimality
- Heuristics: Work with any evaluation function, find good solutions fast

## Why Heuristics?

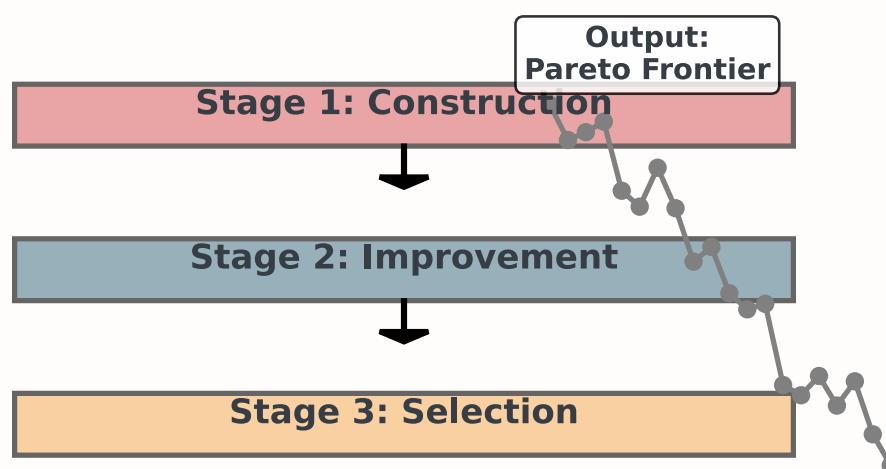
Depending on the problem:

- Combinatorial explosion
  - Huge solution space even for one problem
  - Evaluating one solution might thus take too long
  - Need diverse Pareto frontier, not just one “optimal” solution
  - Open Source Solvers too slow
  - Commercial solvers too expensive
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Question: How do we build good solutions without a solver?

## The Three-Stage Heuristic Process

### **Heuristic Multi-Objective Optimization Workflow**



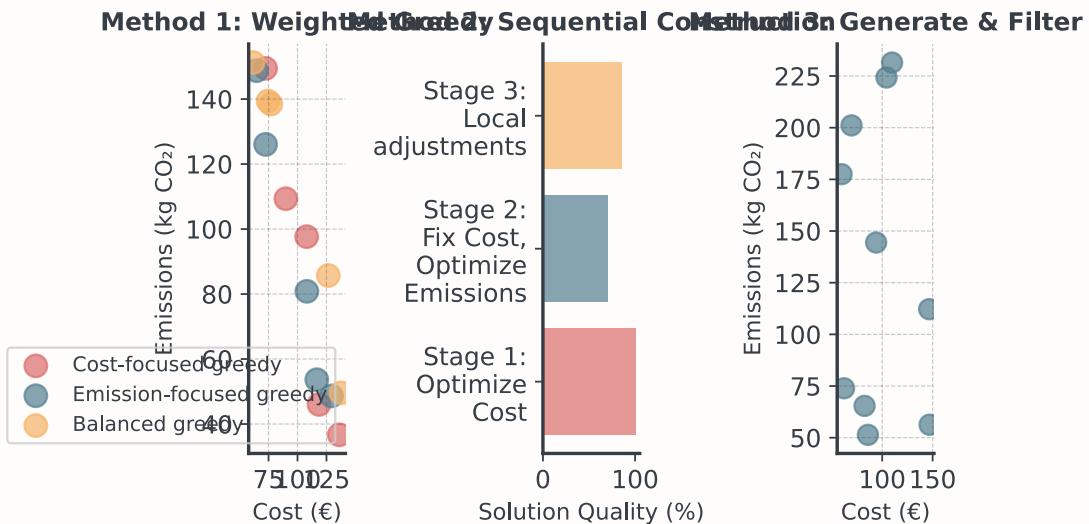
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This is what you'll implement in your assignments!

## Construction & Improvement

### Construction Methods for MOO

How to build initial solutions when you have multiple objectives?



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#### Note

Three choices (for starters). Let's check them out!

### Weighted Greedy Construction

Making greedy choices on a weighted objective

1. Choose weight vector  $w = (w_1, w_2)$
2. At each step, pick the choice that minimizes:

$$w_1 \cdot \text{cost } (x) + w_2 \cdot \text{emissions } (x)$$

3. Build complete solution greedily
4. Repeat with different weights to explore frontier

...

#### Tip

Different weights explore different trade-offs! Easy, right?

## Sequential Greedy (Lexicographic)

Optimize one objective at a time, in priority order

1. Rank objectives by priority
  - E.g. cost (most important) and then emissions (tie-breaker)
2. At each step:
  - Find choices that minimize primary objective
  - If tie → use secondary objective
3. Build one working solution

...

### 💡 Tip

We could also accept primary values within 10% of best so secondary has more influence!

## Diverse Starting Pool

Generate many random solutions, keep the non-dominated ones

1. Generate N random solutions (e.g., N=100)
2. Evaluate all solutions on both objectives
3. Filter to keep only non-dominated solutions
4. Result: A diverse set of Pareto-optimal solutions

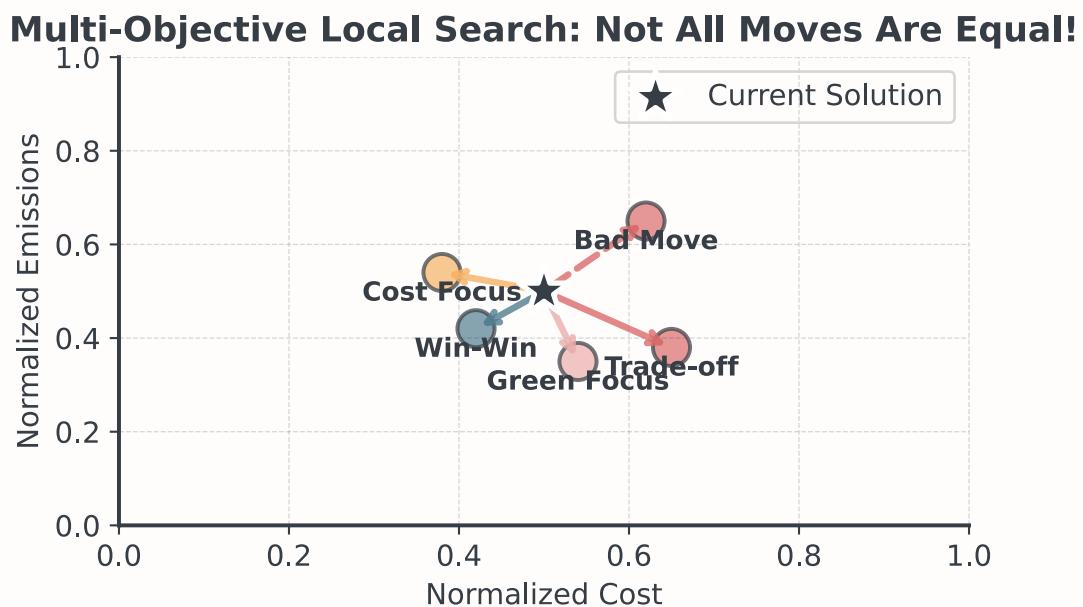
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### 💡 Tip

- Explores entire solution space
- No bias toward specific weights
- Great for warm-starting local search

## Local Search for Multi-Objective

Special moves that improve multiple objectives:



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Question: Which moves are acceptable?

#### MOO Local Search Rules

Accept a move if:

1. Dominance: New solution dominates current (win-win!)
2. Trade-off: Improves primary, acceptable loss in secondary
3. Probabilistic: Use temperature (like simulated annealing)

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**! Important**

Always keep all your objectives in mind when making decisions.

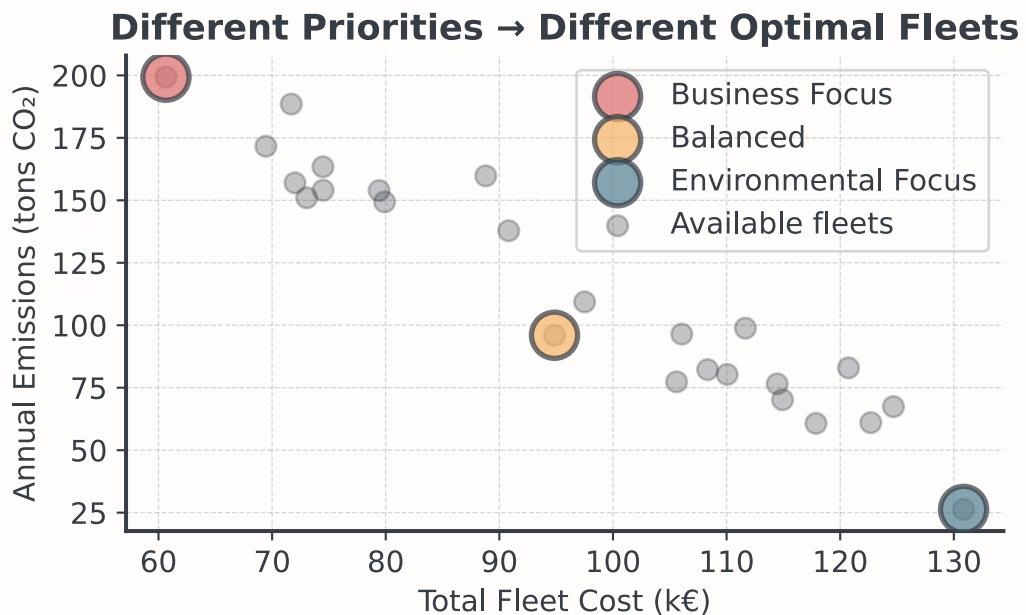
### From Pareto Front to Decision

#### How to Choose!

1. The Knee Point: Find the “elbow” where improvement slows
2. Satisficing Levels: Set minimum acceptable thresholds
  - Cost must be < €100k (budget constraint)
  - Emissions must be < 100 kg (regulatory limit)
  - Service level must be > 90% (customer requirement)
3. Stakeholder Preferences: Let business priorities guide
  - Sustainability: Minimum emissions that meets constraints
  - Operations: Maximum service level within budget

## Weighting has an Impact

The weights thus reflect your values!



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#### 💡 Tip

Depending on your weight, the choice will vary.

## Advanced

### Speed vs Sustainability Dilemma

The Three-Way Trade-off in E-Commerce

1. Minimize Delivery Time (1-day/2-hour promise)
2. Minimize Cost (fuel, labor, fulfillment)
3. Minimize Environmental Impact (carbon footprint)

...

Faster delivery = More vehicles less full = Higher emissions

...

Question: What could retailers do?

### Moving the Frontier

Instead of point on the frontier, move the entire frontier:

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Question: Any idea of examples?

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 Tip

R&D can fundamentally change what's possible!

## Briefing

### Today

#### Hour 2: This Lecture

- Multi-objective
- Pareto optimality
- Weighted greedy
- Local search MOO

#### Hour 3: Notebook

- Bean Counter CEO
- Find Pareto frontier
- Apply weighted greedy
- Normalize objectives

#### Hour 4: Competition

- Fleet composition
- Vehicle selection
- Cost vs service
- Justify choice!

## The Competition Challenge

### EcoExpress Sustainable Fleet Design

...

1. Select optimal fleet mix (5 vehicle types)
2. Balance cost vs. service score
3. Meet EU emission constraint ( $\leq 111 \text{ g CO}_2/\text{km}$ )
4. Ensure sufficient capacity (22,000 parcels/day)

...

 Important

Find the best trade-off for your business priorities!

## Choosing Your MOO Approach

Different situations call for different methods:

Situation	Best	Why
Clear priorities	Sequential greedy	Fast, hierarchy
Exploring	Weighted greedy	Different solutions
Many solutions	Diverse pool	Builds frontier
Quick solution	Single weighted	One good compromise
Improve existing	Multi-objective local	Refines trade-offs

...



Competition? Generate diverse pool or weighted, then improve with local search.

## Implementation Pitfalls to Avoid

Common bugs that cost you time:

1. Forgetting to Normalize
  - Always normalize to [0,1] first!
2. Optimizing Too Many Objectives
  - 2-3: Manageable, 4+: Exponentially harder
  - Combine related objectives or use constraints
3. Not Checking Solution Feasibility
  - Always verify constraints after optimization

## Summary

Key Takeaways:

- Real decisions have multiple conflicting objectives
- Pareto frontier shows all rational trade-offs
- Normalization is essential for fair comparison
- Weights reflect values, make them explicit
- Visualization crucial for decision-making

## Break!

Take 20 minutes, then we start the practice notebook

Next up: You'll become Bean Counter's expert

Then: The Sustainability competition

## Bibliography