

# Multi-Objective Optimization

## Lecture 8 - Management Science

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### Introduction

#### Client Briefing: EcoExpress Logistics

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Operations Director's Dilemma:

"EU regulations demand 40% emission cuts, but we can't sacrifice profitability, service quality, or reliability!"

#### The Fleet Challenge

EcoExpress operates regional last-mile delivery across 3 cities

- EU Green Deal: 40% emission reduction by 2025
- Rising fuel costs (€2.1/L diesel)
- Amazon entering our market (speed pressure)
- Driver shortage (need automation-friendly vehicles)

...

Question: How do we transform our fleet while staying competitive?

#### Today's Learning Objectives

By the end of this lecture, you will be able to:

1. Explain why most real business decisions involve multiple competing objectives
2. Identify and visualize Pareto optimal solutions in multi-objective problems
3. Apply normalization techniques to make objectives comparable
4. Implement weighted sum and  $\varepsilon$ -constraint methods to find trade-off solutions
5. Choose the appropriate MOO method for different problem types
6. Make data-driven decisions from a Pareto frontier
7. Analyze real-world multi-objective trade-offs (Amazon, airlines, Tesla)

#### Quick Recap: Local Search

Last week we optimized routes for delivery:

- Started with greedy construction (e.g. Nearest Neighbor)
- Improved with local search (e.g. 2-opt)

- Considered time windows
- But: We only optimized distance

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Question: What if we also care about emissions, cost, AND customer satisfaction?

## The Problem

### Single vs Multi-Objective

#### Single Objective

- “Minimize total distance”
- Clear winner. Easy, right!

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#### Multiple Objectives

- “Minimize cost AND emissions AND maximize speed”
- No clear answer...

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Question: Any idea how to approach this?

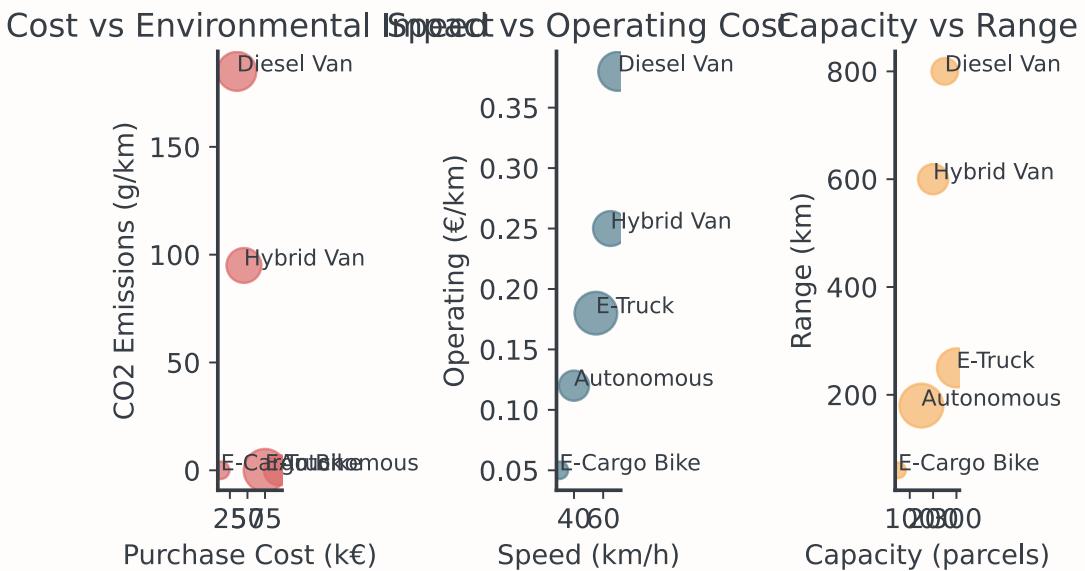
## EcoExpress Vehicle Options

Type	Purchase Cost (€)	Operat-ing (€/ km)	CO2 (g/ km)	Speed (km/h)	Capacity (parcels)	Range (km)
E-Truck	75000	0.18	0	55	300	250
Hybrid Van	45000	0.25	95	65	200	600
Diesel Van	35000	0.38	185	70	250	800
E-Cargo Bike	12000	0.05	0	30	50	60
Autonomou\$5000	95000	0.12	0	40	150	180

...

Question: Which vehicle is “best” for EcoExpress?

## Trade-offs Everywhere



! Important

Every vehicle excels at something different!

## Real Business Constraints

Beyond the numbers, consider:

- EU regulations: Carbon tax of €100/ton CO<sub>2</sub> starting 2025
- Competition: Amazon promises 2-hour delivery
- Labor market: Autonomous vehicles reduce driver dependency
- Urban zones: Zero-emission zones in city centers
- Peak times: Black Friday = 3x normal volume

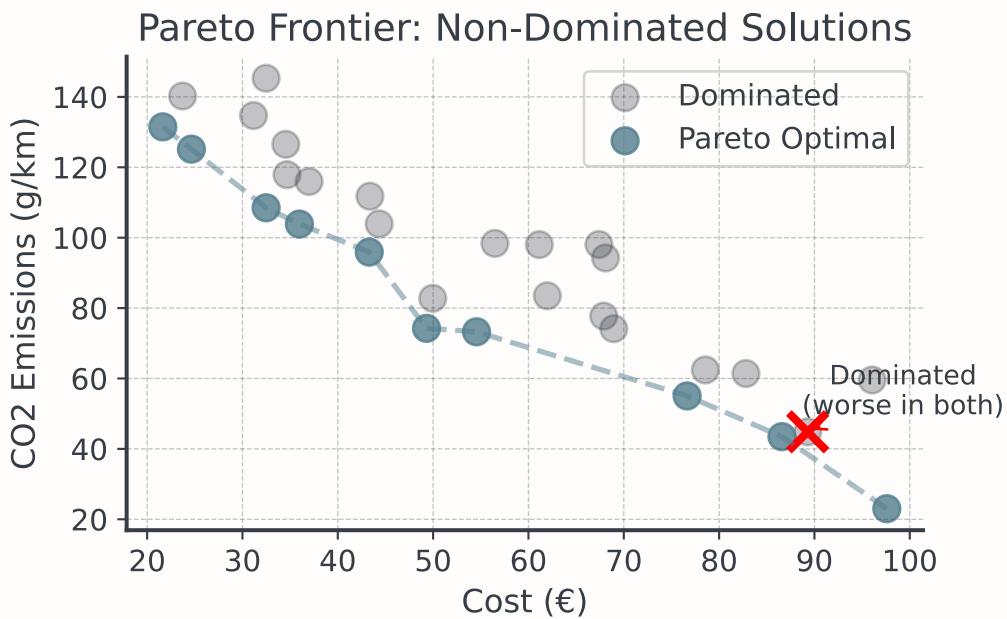
! Important

There is no single “optimal” solution - only trade-offs

## Pareto Optimality

### Dominated Solutions

A solution is dominated if another solution is:



! Important

Better in at least one objective and not worse in any objective!

## The Pareto Frontier

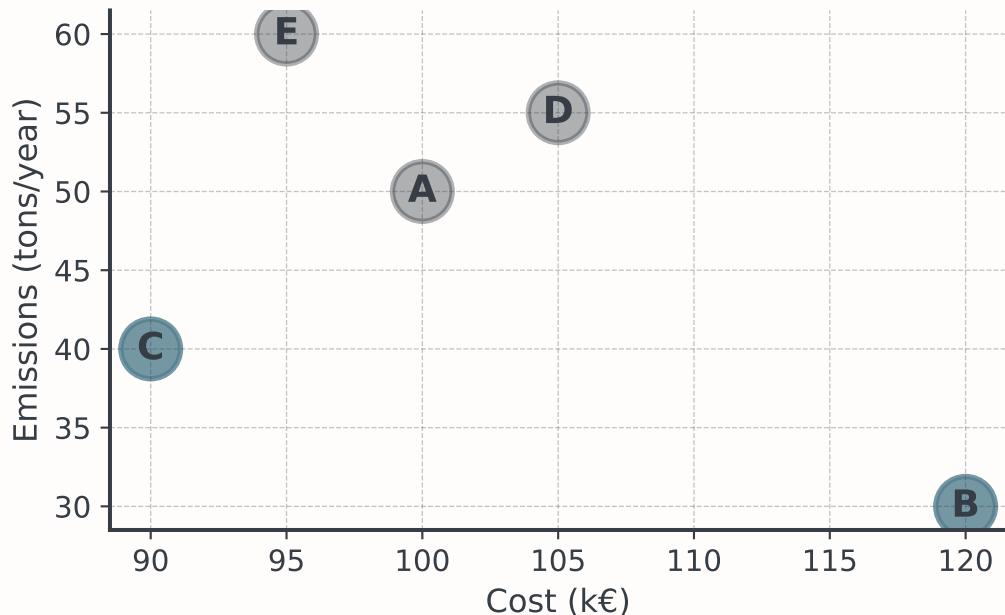
The Pareto frontier is the set of all non-dominated solutions

- No solution is objectively “better”
- Each represents a different trade-off
- Moving along frontier: gain in one objective, loss in another
- Decision makers choose based on preferences

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Question Do you think you get the idea?

Find the Non-Dominated



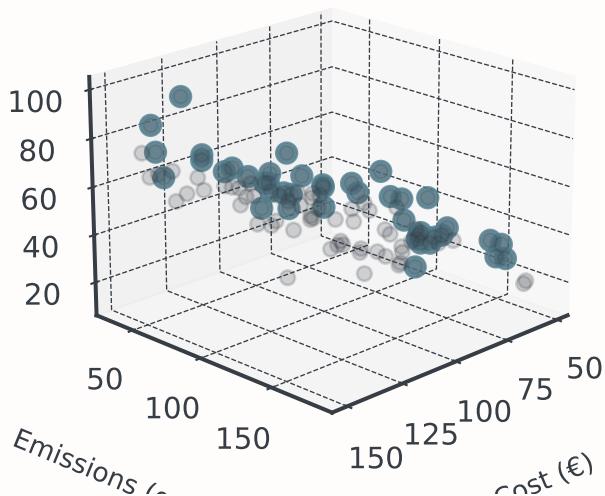
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Question: Which fleets are non-dominated?

Three+ Objectives

With 3 objectives, the Pareto frontier becomes a surface:

### 3D Pareto Frontier (Surface)



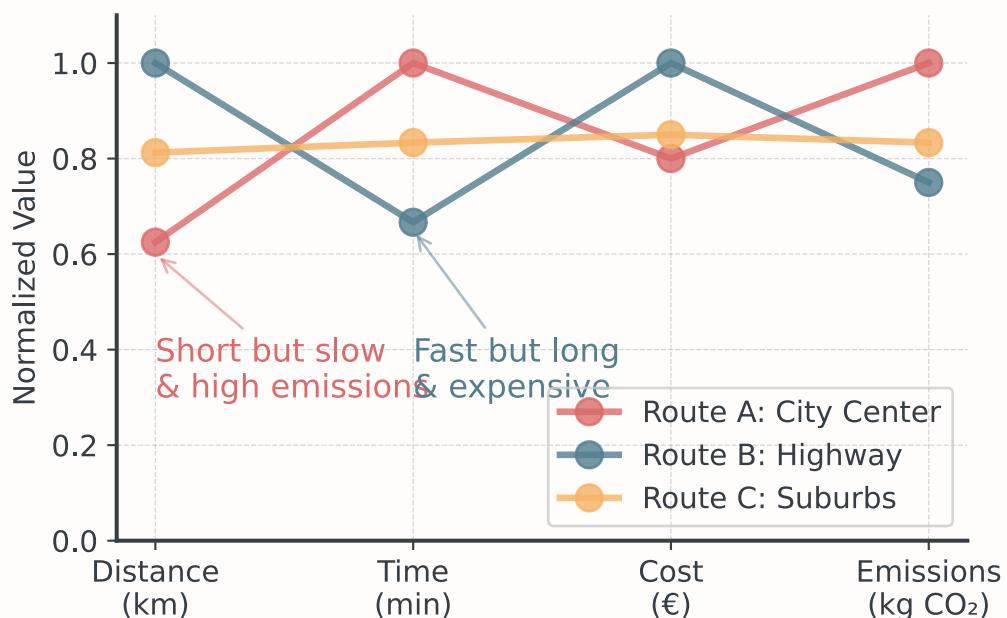
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! Important

Harder to visualize, but same principle applies!

## Transportation Problem

### Multi-Objective Transportation



💡 Tip

Logistics decisions involve trade-offs: City traffic (slow, high emissions), Night delivery? → Highway (fast, but more distance), Customer priority? → Direct route (expensive)

## Classic Transportation

Let's understand the foundation

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From/To	Zone A	Zone B	Zone C	Zone D	Supply
DC Berlin	8	10	11	14	300
DC Hamburg	12	9	7	8	250
DC Munich	15	13	10	9	200

From/To	Zone A	Zone B	Zone C	Zone D	Supply
Demand	200	180	220	150	750

**i Note**

Cost per 100 parcels (€) in the middle of the table!

## Objective Function

The foundation of the classical model

$$\text{Minimize } Z = \sum_i \sum_j c_{ij} \cdot x_{ij}$$

Where:

- $x_{ij}$  = parcels shipped from DC  $i$  to Zone  $j$
- $c_{ij}$  = cost per 100 parcels from  $i$  to  $j$

## Second Objective: Emissions

Now the real challenge - each route has different emissions:

From/To	Zone A	Zone B	Zone C	Zone D
DC Berlin	120	95	85	70
DC Hamburg	45	110	100	90
DC Munich	60	50	115	105

**i Note**

Emissions per parcel (g CO<sub>2</sub>) in the table

## Objective Function II

Now with two objectives!

$$\text{Minimize } Z_1 = \sum_i \sum_j c_{ij} \cdot x_{ij} \quad (\text{Cost})$$

$$\text{Minimize } Z_2 = \sum_i \sum_j e_{ij} \cdot x_{ij} \quad (\text{Emissions})$$

Where  $e_{ij}$  = emissions per parcel from  $i$  to  $j$

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Notice: Cheapest routes ≠ Greenest routes!

## Data Source

Where Do These Numbers Come From?

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Cost Data:

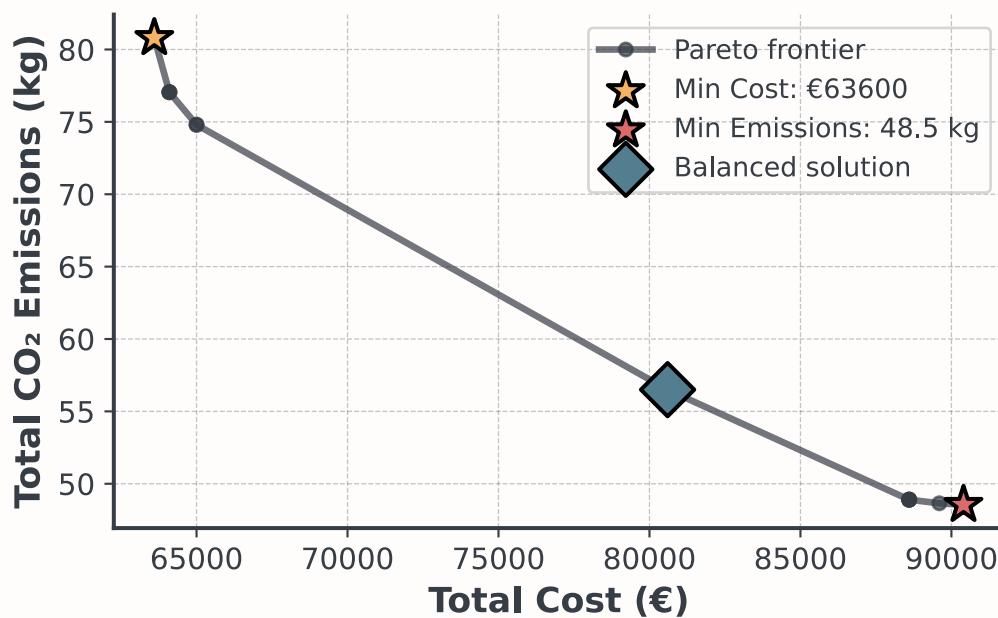
- Historical records: Your accounting system
- Quotes: Request from carriers/suppliers
- APIs: Google Maps Distance Matrix (distance → cost)

...

Emissions Data:

- EU Standards or Carrier data
- Formula:  $\text{Emissions} = \text{Distance} \times \text{Weight} \times \text{EmissionFactor}$

## The Transportation Trade-off



Cost increase for greenest solution: +€26800 (42.1%)  
Emissions reduction from cheapest: -32.2 kg (39.9%)

...

**! Important**

Each point represents a different allocation strategy!

# Solution Approaches

## Multi-Objective Optimization

You can use optimization solvers or heuristics!

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### With Optimization Solvers

- Weighted Sum Method
- $\varepsilon$ -Constraint Method
- Goal Programming
- Optimal solutions
- Need mathematical model

### With Heuristics

- Weighted Greedy Construction
- Multi-Objective Local Search
- Metaheuristics
- Good solutions, fast
- No optimality proof

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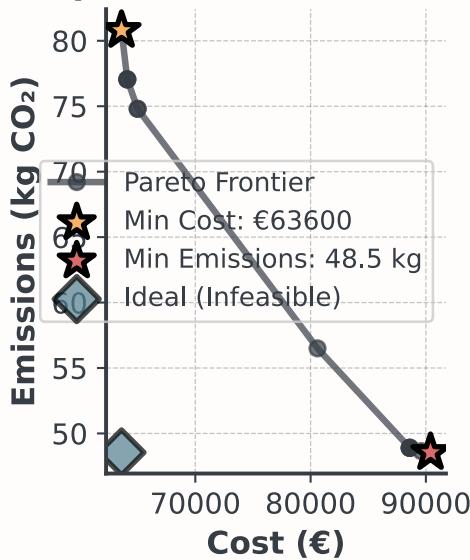
### ! Important

In this lecture we use heuristic approaches!

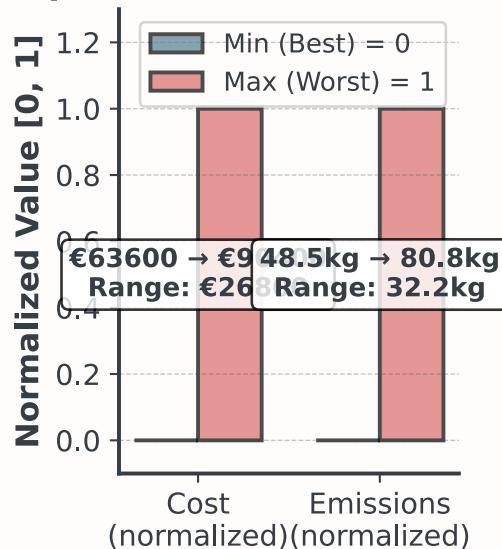
## Foundation: Extreme Points

First step for BOTH approaches - find the boundaries:

## Step 1: Find Extreme Points



## Step 2: Normalize to [0,1] Scale

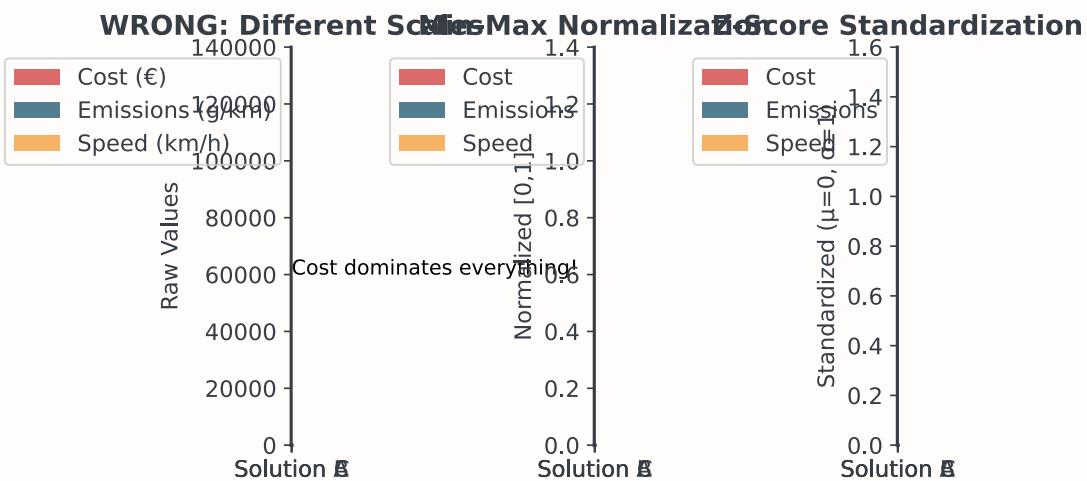


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Question: Why is normalization essential?

Critical: Normalization

Without it, your analysis is meaningless



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Question: Any intuition on how to do [0,1] normalization?

How to Normalize

The Normalization Formula for [0,1]

$$\text{Normalized}_i = \frac{x_i - x_{min}}{x_{max} - x_{min}}$$

...

In Python, this is rather simple!

...

```
def normalize_objectives(data):
    return (data - data.min()) / (data.max() - data.min())

# Now weights actually mean something
weighted_score = w1 * normalize(cost) + w2 * normalize(emissions)
```

...



Tip

Easy, right?

## Extreme Points

There are several reasons why extreme points matter:

1. Trade-off Space: Min/max values bound your Pareto frontier
2. Enable Proper Normalization: Need ranges for scaling to [0,1]
3. Feasibility: If single objectives not achievable, problem infeasible
4. Stakeholder: “Best cost is €50k, best emissions is 40kg”

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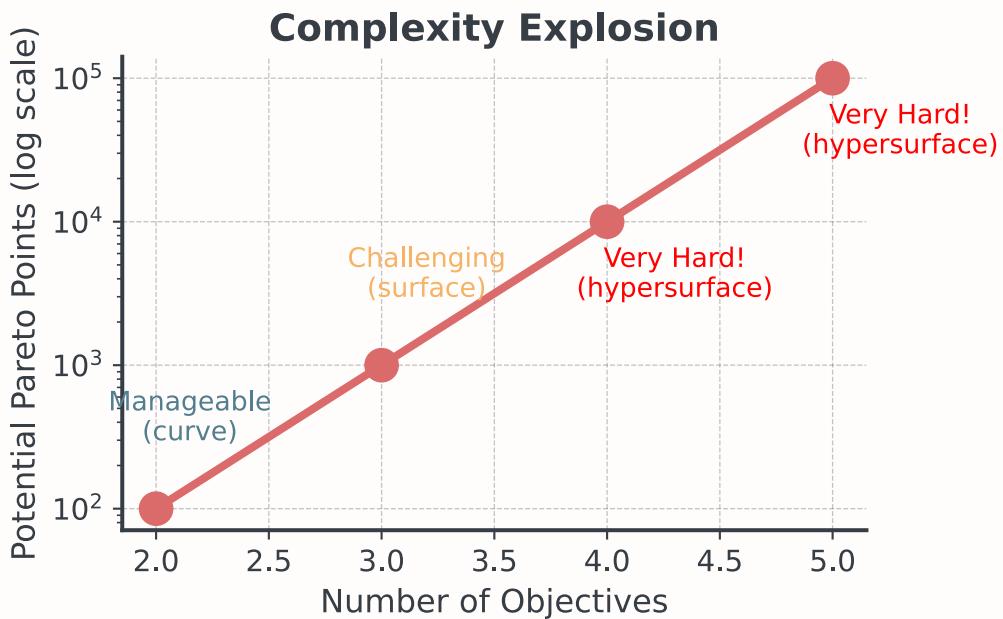
Implementation Pattern:

```
def find_extreme_points(problem):
    # Solve for minimum cost (ignore emissions)
    min_cost_solution = minimize(cost_objective, constraints)
    # Solve for minimum emissions (ignore cost)
    min_emissions_solution = minimize(emissions_objective, constraints)
```

## Computational Complexity

How hard does it get with more objectives?

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Tip

Why? Because there are just way more potential solutions to check!

## Solver-Based Methods

Quick overview - you won't implement these in assignments

1. Weighted Sum: Minimize  $w_1 \times \text{cost} + w_2 \times \text{emissions}$ 
  - Simple, fast for convex problems
2.  $\varepsilon$ -Constraint: Minimize cost subject to emissions  $\leq \varepsilon$ 
  - Systematically vary  $\varepsilon$  to find complete frontier
3. Goal Programming: Minimize deviations from targets
  - Set target for each objective, minimize weighted deviations

...

Note

For your fleet optimization: You'll use heuristic approaches instead!

## Heuristic Approach

### The Heuristic Strategy

For problems without mathematical models

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1. Construction: Build initial solutions with weighted greedy
  2. Improvement: Multi-objective local search
  3. Selection: Filter dominated solutions to find Pareto frontier
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### ! Important

Key difference from solvers:

- Solvers: Need mathematical model, guarantee optimality
- Heuristics: Work with any evaluation function, find good solutions fast

## Why Heuristics?

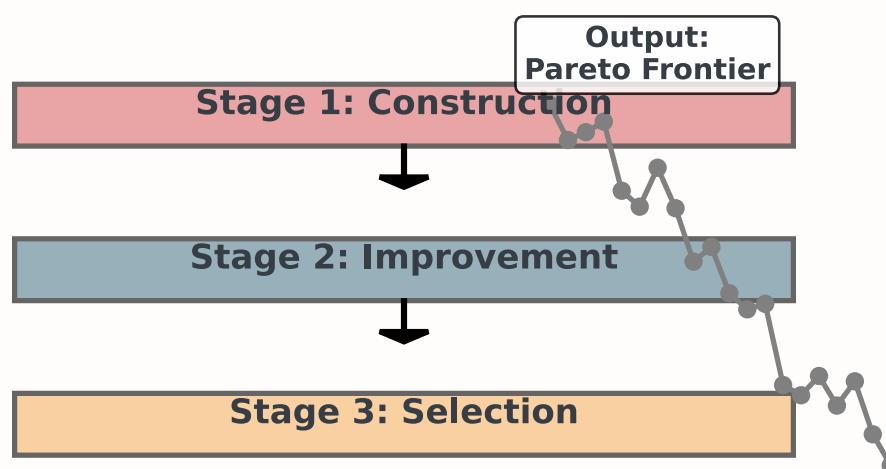
Depending on the problem:

- Combinatorial explosion
  - Huge solution space even for one problem
  - Evaluating one solution might thus take too long
  - Need diverse Pareto frontier, not just one “optimal” solution
  - Open Source Solvers too slow
  - Commercial solvers too expensive
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Question: How do we build good solutions without a solver?

## The Three-Stage Heuristic Process

### **Heuristic Multi-Objective Optimization Workflow**



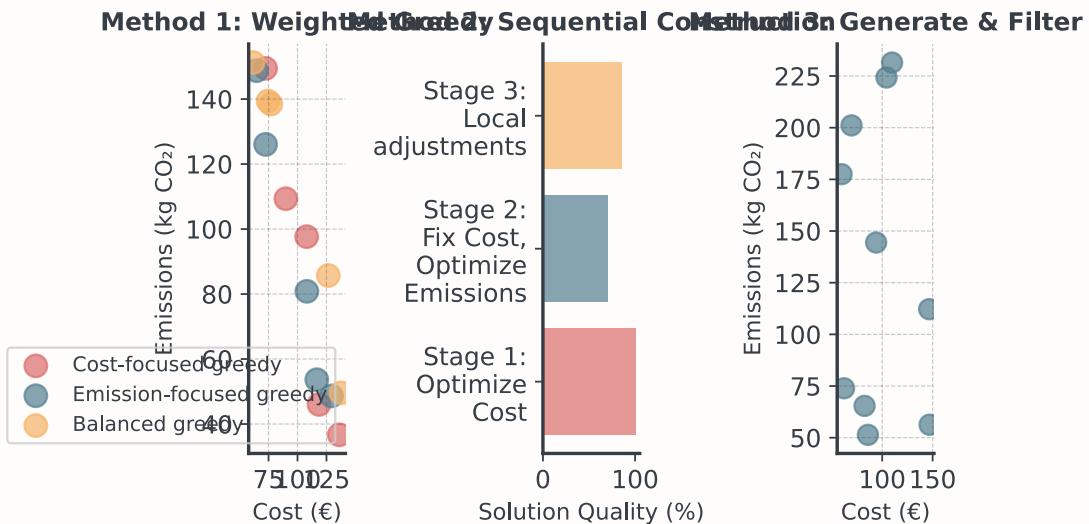
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This is what you'll implement in your assignments!

## Construction & Improvement

### Construction Methods for MOO

How to build initial solutions when you have multiple objectives?



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#### Note

Three choices (for starters). Let's check them out!

### Weighted Greedy Construction

Making greedy choices on a weighted objective

1. Choose weight vector  $w = (w_1, w_2)$
2. At each step, pick the choice that minimizes:

$$w_1 \cdot \text{cost } (x) + w_2 \cdot \text{emissions } (x)$$

3. Build complete solution greedily
4. Repeat with different weights to explore frontier

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#### Tip

Different weights explore different trade-offs! Easy, right?

## Sequential Greedy (Lexicographic)

Optimize one objective at a time, in priority order

1. Rank objectives by priority
  - E.g. cost (most important) and then emissions (tie-breaker)
2. At each step:
  - Find choices that minimize primary objective
  - If tie → use secondary objective
3. Build one working solution

...

### 💡 Tip

We could also accept primary values within 10% of best so secondary has more influence!

## Diverse Starting Pool

Generate many random solutions, keep the non-dominated ones

1. Generate N random solutions (e.g., N=100)
2. Evaluate all solutions on both objectives
3. Filter to keep only non-dominated solutions
4. Result: A diverse set of Pareto-optimal solutions

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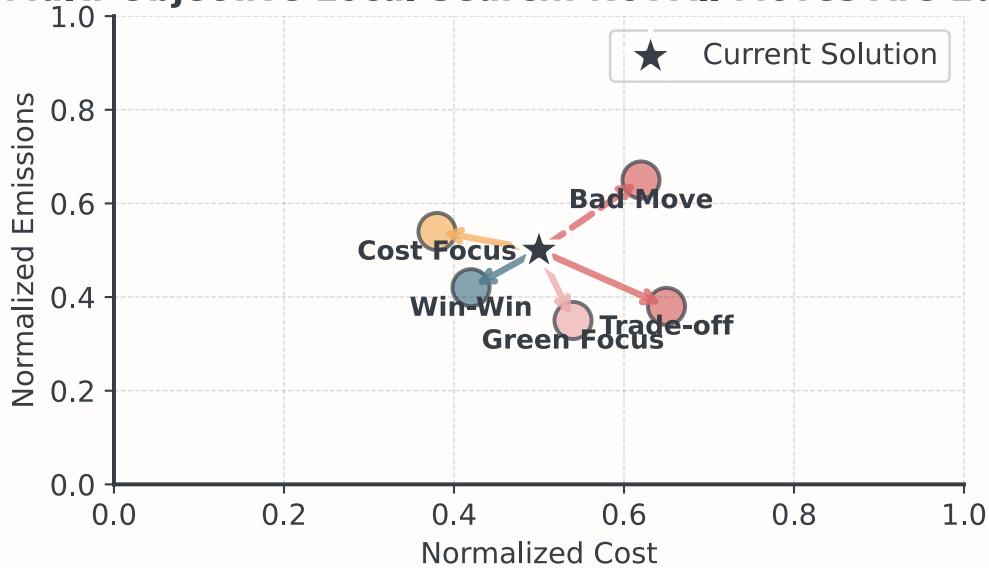
### 💡 Tip

- Explores entire solution space
- No bias toward specific weights
- Great for warm-starting local search

## Local Search for Multi-Objective

Special moves that improve multiple objectives:

## Multi-Objective Local Search: Not All Moves Are Equal!



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Question: Which moves are acceptable?

### MOO Local Search Rules

Accept a move if:

1. Dominance: New solution dominates current (win-win!)
2. Trade-off: Improves primary, acceptable loss in secondary
3. Diversity: Fills gap in current Pareto front
4. Probabilistic: Use temperature (like simulated annealing)

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**! Important**

Always keep all your objectives in mind when making decisions.

## From Pareto Front to Decision

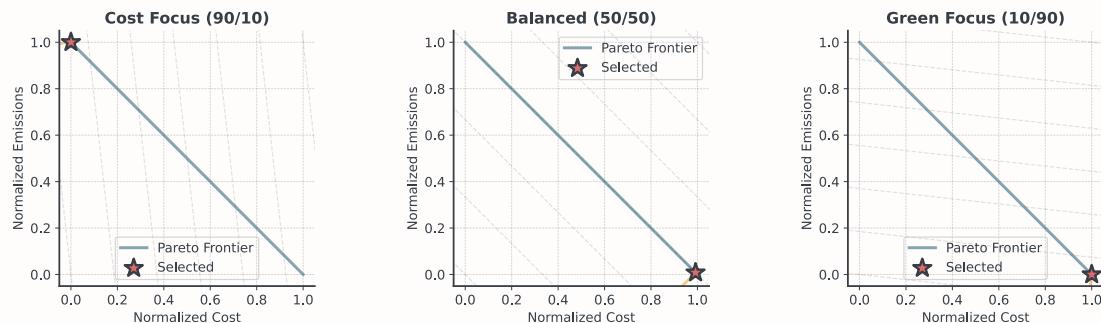
### How to Choose!

1. The Knee Point: Find the “elbow” where improvement slows
2. Satisficing Levels: Set minimum acceptable thresholds
  - Cost must be < €100k (budget constraint)
  - Emissions must be < 100 kg (regulatory limit)
  - Service level must be > 90% (customer requirement)
3. Stakeholder Preferences: Let business priorities guide
  - Sustainability: Minimum emissions that meets constraints

- Operations: Maximum service level within budget

## Weighted Sum Along the Frontier

The weight influences the final choice:



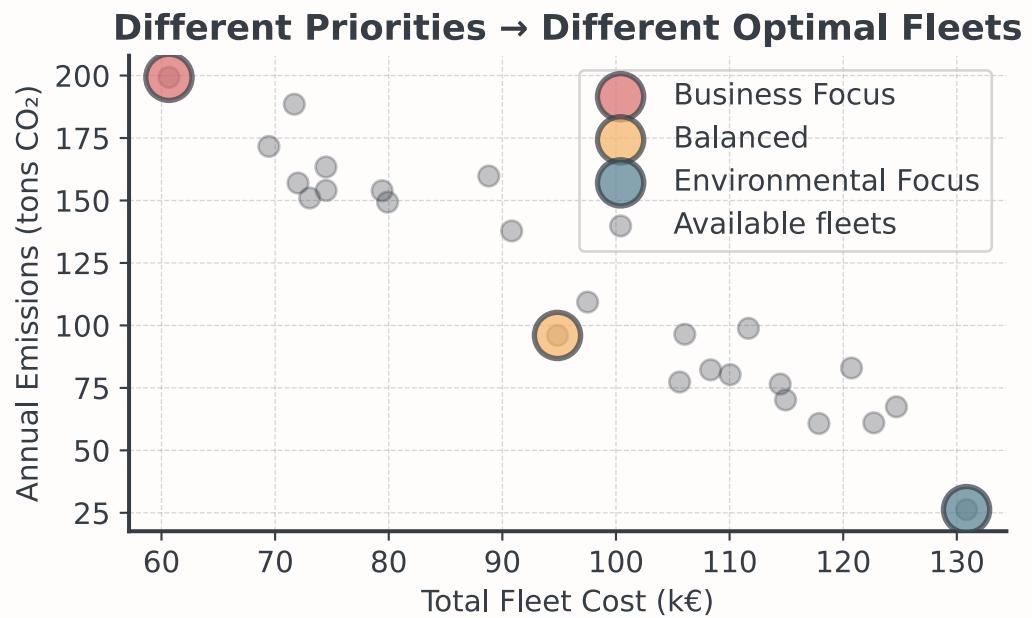
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### Note

The iso-cost lines show the trade-offs between cost and emissions.

## Weighting has an Impact

The weights thus reflect your values!



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### 💡 Tip

Depending on your weight, the choice will vary.

## Advanced

### Speed vs Sustainability Dilemma

The Three-Way Trade-off in E-Commerce

1. Minimize Delivery Time (1-day/2-hour promise)
2. Minimize Cost (fuel, labor, fulfillment)
3. Minimize Environmental Impact (carbon footprint)

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Faster delivery = More vehicles less full = Higher emissions

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Question: What could retailers do?

### Moving the Frontier

Instead of point on the frontier, move the entire frontier:

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Question: Any idea of examples?

...

### 💡 Tip

R&D can fundamentally change what's possible!

## Briefing

### Today

Hour 2: This Lecture

- Multi-objective
- Pareto optimality
- Weighted greedy
- Local search MOO

Hour 3: Notebook

- Bean Counter CEO
- Find Pareto frontier
- Apply weighted greedy

- Normalize objectives

Hour 4: Competition

- Transportation problem
- Fleet selection
- Cost vs emissions
- Justify choice!

## The Competition Challenge

EcoExpress Sustainable Fleet Design

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1. Solve multi-objective transportation (DCs → Zones)
2. Select optimal fleet mix (5 vehicle types)
3. Balance cost, emissions, service quality
4. Meet EU emission targets (40% reduction)

...

**! Important**

Find the best trade-off for your business priorities!

## Choosing Your MOO Approach

Different situations call for different methods:

Situation	Best	Why
Clear priorities	Sequential greedy	Fast, hierarchy
Exploring	Weighted greedy	Different solutions
Many solutions	Diverse pool	Builds frontier
Quick solution	Single weighted	One good compromise
Improve existing	Multi-objective local	Refines trade-offs

...

**💡 Tip**

Competition? Generate diverse pool or weighted, then improve with local search.

## Implementation Pitfalls to Avoid

Common bugs that cost you time:

1. Forgetting to Normalize

- Always normalize to [0,1] first!
2. Optimizing Too Many Objectives
    - 2-3: Manageable, 4+: Exponentially harder
    - Combine related objectives or use constraints
  3. Not Checking Solution Feasibility
    - Always verify constraints after optimization

## Summary

Key Takeaways:

- Real decisions have multiple conflicting objectives
- Pareto frontier shows all rational trade-offs
- Normalization is essential for fair comparison
- Weighted sum works for convex frontiers
- $\varepsilon$ -constraint handles non-convex cases
- Weights reflect values, make them explicit
- Visualization crucial for decision-making

## Break!

Take 20 minutes, then we start the practice notebook

Next up: You'll become Bean Counter's expert

Then: The Sustainability competition

## Bibliography