

# Better Routing

## Lecture 7 - Management Science

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### Introduction

#### Client Briefing: Artisan Bakery

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Master Baker's Morning Dilemma:

"Every morning at 5 AM, our delivery van leaves with fresh bread for 12 cafés across the city. Our driver takes 3 hours for what should be a 2-hour route. We're burning fuel and arriving late. Can you optimize our morning delivery?"

#### The Delivery Challenge

Artisan Bakery's daily logistics puzzle:

- 12 Cafés: Each expecting fresh bread by 8 AM
- One Van: Limited capacity, must visit all locations
- Time Windows: 3 cafés open early (6:30 AM) and need priority
- Current Problem: Driver uses "gut feeling" for routing
- Cost Impact: Extra hour = €50 fuel + €30 labor + unhappy customers

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#### ! Important

The Stakes: Poor routing costs €80+ daily, or €29,200 annually. Plus reputation damage from late deliveries!

#### Quick Recap: Greedy Decisions

Last week we learned greedy algorithms for scheduling:

- SPT: Process shortest jobs first
- EDD: Process by earliest due date
- Fast & Simple: Made quick decisions, no looking back

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Question: Can we use the same greedy approach for routing?

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### Note

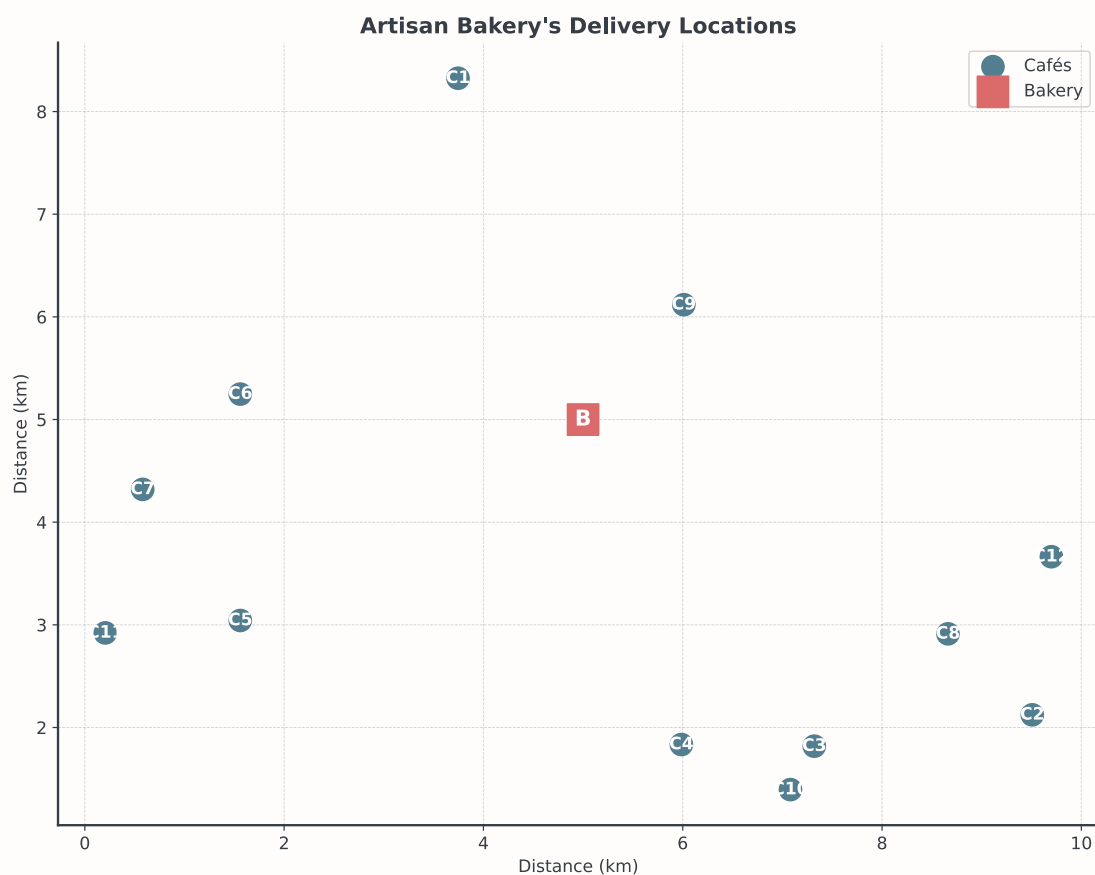
Today's Journey: We'll start greedy (nearest neighbor), then learn how to improve solutions with local search!

## The Routing Problem

### The Traveling Salesman Problem

The classic optimization challenge: Visit all locations exactly once, minimize total distance.

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### Why Can't Computers Just Try Everything?

Let's calculate: 12 cafés =  $12! = 479,001,600$  routes

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If your computer checks 1 million routes per second:

- 5 cafés: 0.0001 seconds ✓

- 10 cafés: 0.18 seconds ✓
- 12 cafés: 8 minutes ⚠️
- 15 cafés: 12 hours ⚠️
- 20 cafés: 77,000 years ☠️

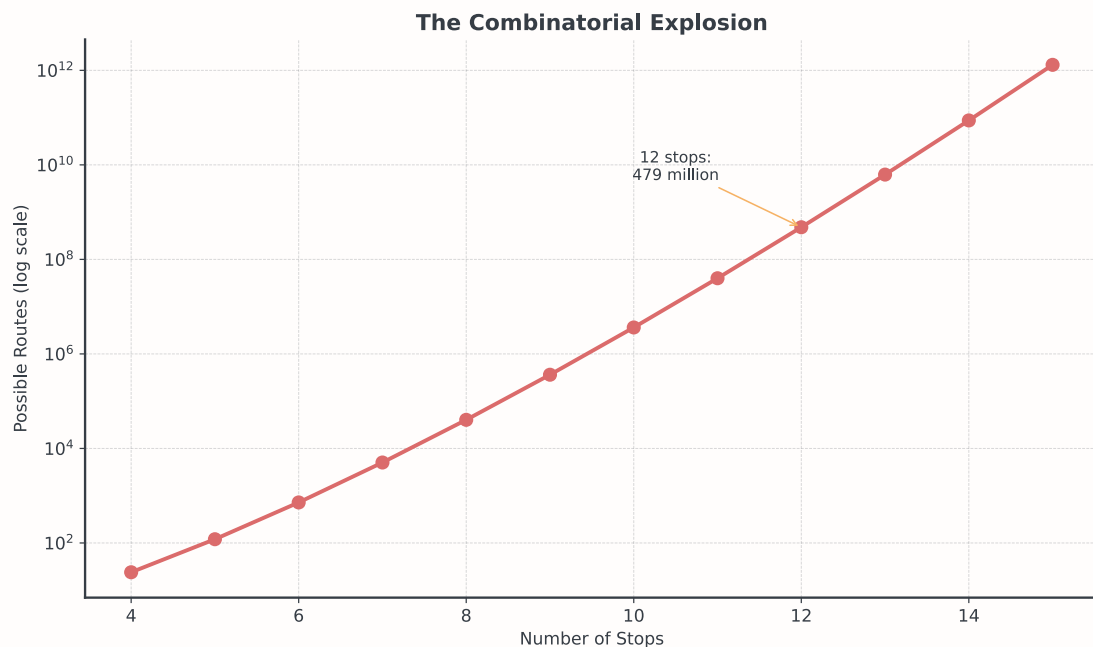
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### ! Important

The Reality: Amazon has 100,000+ delivery stops daily. Exact optimization would take longer than the universe has existed!

## The Complexity Explosion

The factorial growth makes exhaustive search impossible.



## Local Search Framework

### The Four Pillars of Local Search

Any problem can be solved with local search by defining:

1. Search Space: All possible solutions (3.6M routes for 10 cafés!)
2. Initial Solution: Starting point (our greedy route)
3. Objective Function: How we measure quality (total distance)
4. Neighborhood Structure: How to create “nearby” solutions (2-opt swaps)

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### ! Important

The power of local search: The same “engine” works for routing, scheduling, or any combinatorial problem - just plug in different components!

## Greedy Construction

### Nearest Neighbor Algorithm

Build a route by always visiting the closest unvisited location.

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The Algorithm: 1. Start at the bakery 2. Find the nearest unvisited café 3. Go there 4. Repeat until all visited 5. Return to bakery

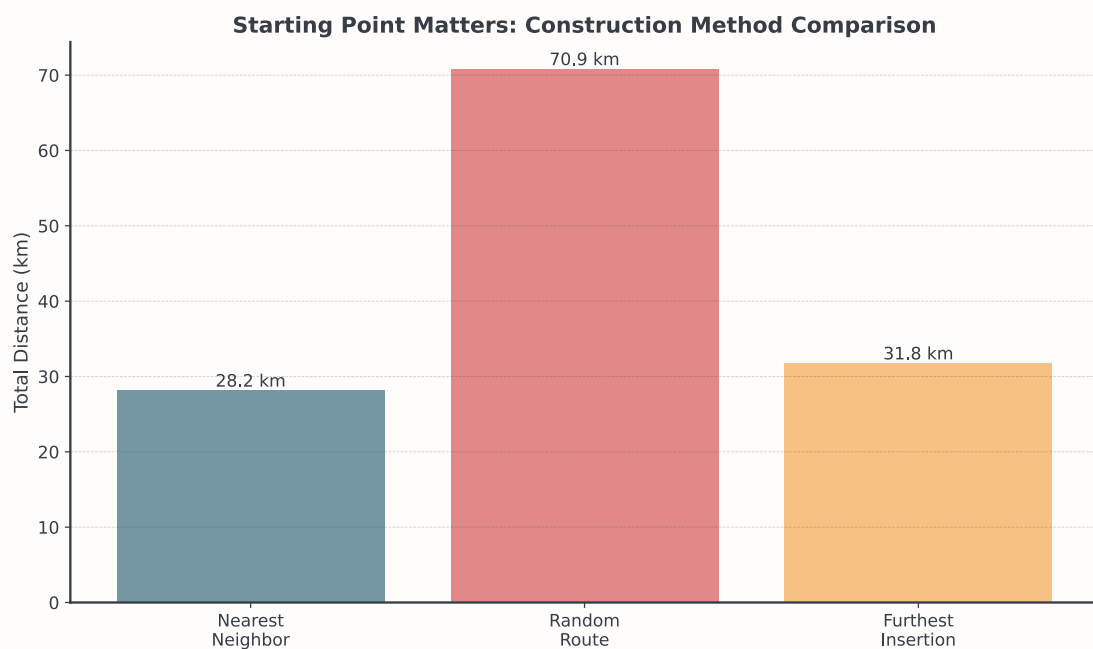
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### 💡 Tip

Intuition: Like picking low-hanging fruit - grab what’s easiest (nearest) first!

## Construction Methods Comparison

Different ways to build initial routes:

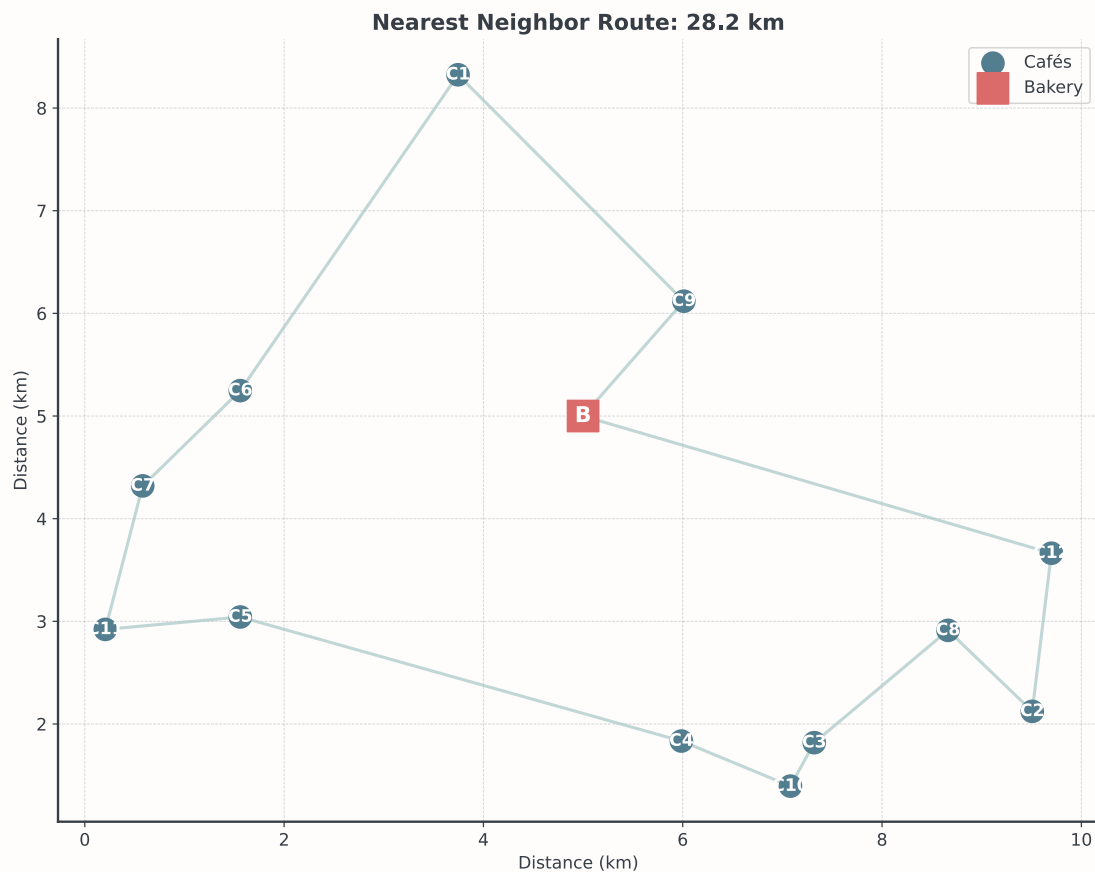


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### Note

Different construction methods give different starting points. The better your start, the better your final result after improvement!

## Nearest Neighbor in Action



## The Problem with Greedy

Question: Can you spot any obvious inefficiencies in this route?

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Common Issues: :: incremental - Crossing paths: Route crosses over itself - Long return: Far from bakery at the end - Myopic decisions: Can't see the "big picture" ::

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### Warning

Nearest neighbor typically gives solutions 15-25% worse than optimal. Can we improve it?

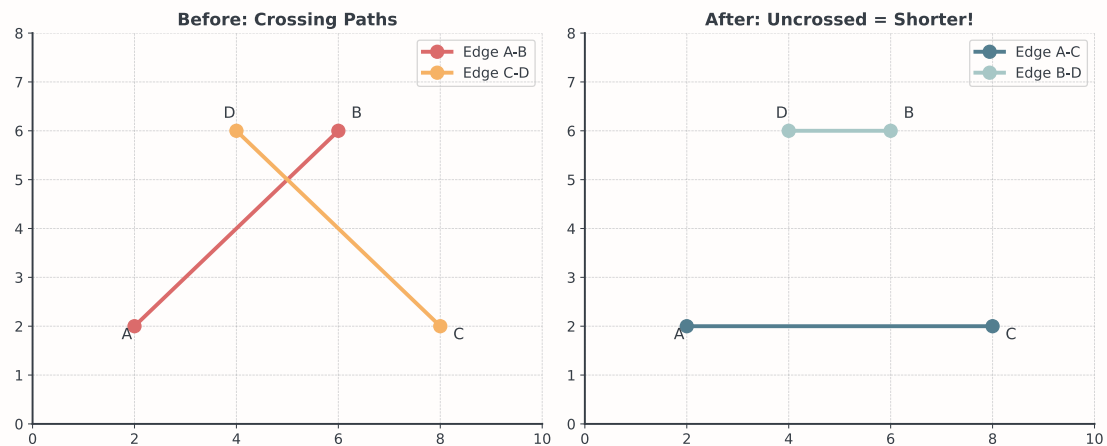
## Local Search Improvements

### The 2-Opt Algorithm

Systematically improve routes by removing crossing paths.

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The Idea: Take two edges and swap them to uncross the route



### How 2-Opt Works

The systematic improvement process:

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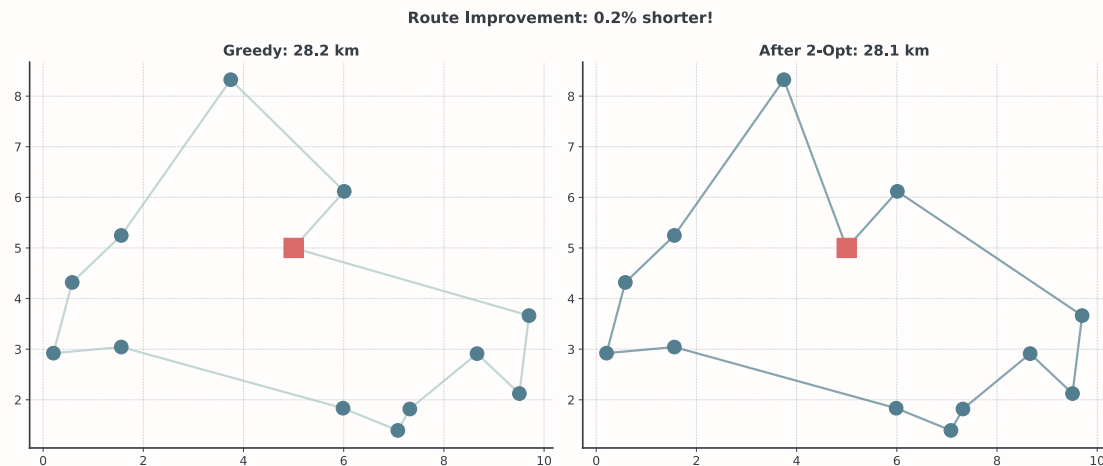
```
# Simplified 2-opt logic (conceptual)
for i in range(route_length):
    for j in range(i+2, route_length):
        # Try swapping edges (i,i+1) and (j,j+1)
        new_distance = calculate_with_swap(i, j)
        if new_distance < current_distance:
            perform_swap(i, j)
            improvement_found = True
```

...

#### **i** Note

2-opt examines all possible pairs of edges and swaps those that reduce total distance. Keep iterating until no improvement found!

## 2-Opt Applied to Our Route



## Beyond 2-Opt: More Powerful Moves

The k-opt family of improvements:

2-opt - Removes 2 edges - 1 way to reconnect -  $O(n^2)$  combinations - Fast, good results

3-opt - Removes 3 edges - 7 ways to reconnect -  $O(n^3)$  combinations - Better but slower

Or-opt - Moves 1-3 nodes - Different philosophy - Good for time windows - Preserves orientation

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### Tip

Industry practice: Start with 2-opt (fast), use 3-opt if you have time!

## Local Search Philosophy

Start with any solution, then iteratively improve it.

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The Process: ::: incremental 1. Initial Solution: Use greedy (fast but suboptimal) 2. Define Neighborhood: What changes can we make? 3. Search: Try neighboring solutions 4. Accept: Move if better 5. Repeat: Until no improvements found :::

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### Tip

Local search transforms “quick and dirty” solutions into “pretty good” ones!

## The Local Optimum Trap: A Hiker's Dilemma

Imagine you're a hiker dropped in foggy mountains at night...

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Your Mission: Find the highest peak (global optimum) Your Tool: An altimeter (objective function) Your Vision: Only the ground at your feet (local neighborhood)

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The Greedy Strategy: Always step uphill

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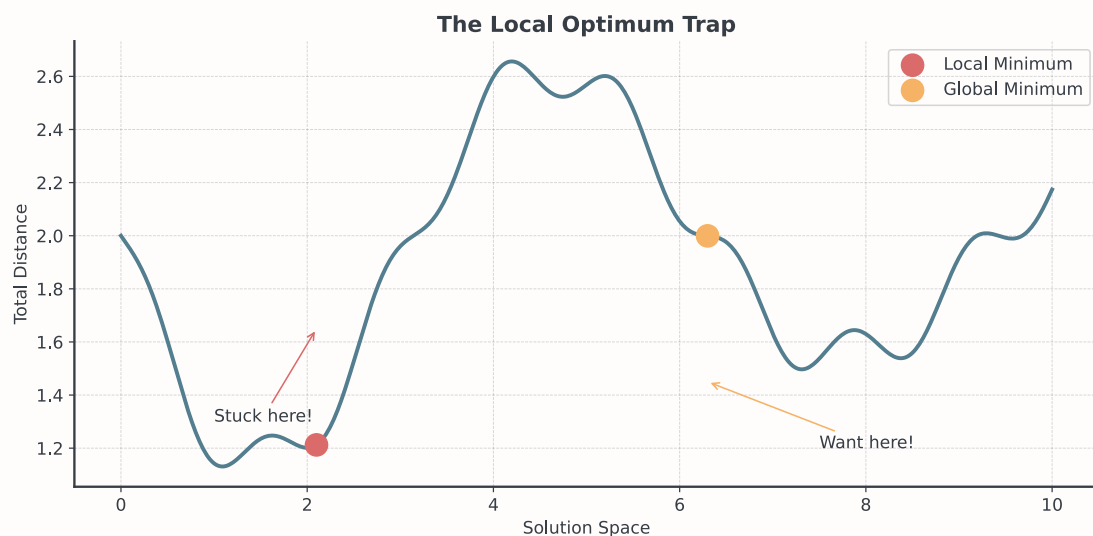
Question: What happens when you reach the top of a small hill?

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### ⚠ Warning

You're stuck! Every step is downhill, but you might be on a tiny hill while Mount Everest is nearby. This is the local optimum trap!

## Visualizing Local Optima



## Escaping Local Optima: Multi-Start Strategy

Simple fix: Don't put all eggs in one basket!

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```
def multi_start_optimization(n_starts=10):  
    """Try multiple random starts, keep the best"""  
    best_route = None  
    for i in range(n_starts):
```



```

# New random start
route = random_initial_route()
# Improve with 2-opt
route = improve_with_2opt(route)
# Keep if best so far
if better_than(route, best_route):
    best_route = route
return best_route

```

...

#### Note

Like hiring 10 drivers, letting each plan their own route, then picking the best! Simple but effective.

## Real-World Impact: How Good is Good Enough?

Industry benchmarks for delivery optimization:

Method	vs Optimal	Industry Use
Human intuition	+40-60%	Still common!
Nearest Neighbor	+20-25%	Quick dispatch
NN + 2-opt	+5-15%	Standard practice
Advanced Meta	+2-5%	Premium logistics
Exact (if possible)	0%	Research only

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#### Important

Business Reality: A 10% improvement = millions in savings for large logistics companies. Your 2-opt implementation could literally pay for itself in one day!

## Time Windows Add Complexity

Some cafés open early and need priority delivery:

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Artisan Bakery's Constraints: - Café Europa: Opens 6:30 AM (early birds) - Sunrise Bistro: Opens 6:30 AM (breakfast rush) - Morning Glory: Opens 6:45 AM (commuter stop) - Others: Open 7:00 AM or later

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### ! Important

With time windows, we must balance distance minimization with deadline satisfaction. Pure distance optimization might arrive too late!

## Mission Briefing

### Choosing Your Weapon: Algorithm Selection

Different situations call for different approaches:

Situation	Best Approach	Why
Need solution NOW (< 1 sec)	Nearest Neighbor	Lightning fast
Have 1 minute	NN + 2-opt	Good balance
Have 5 minutes	Multi-start + 2-opt	Explore more options
Time windows critical	NN (prioritize early) + Or-opt	Preserves time feasibility
Academic benchmark	3-opt or SA	Maximum quality

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### 💡 Tip

Today's Competition: You have 60 minutes - use multi-start with 2-opt!

## Implementation Pitfalls to Avoid

Common bugs that cost you 30 minutes:

❌ Forgetting return to bakery: Your distance calculation must include the trip back!

```
# WRONG
total = sum(distances between consecutive stops)
# RIGHT
total = sum(distances) + distance(last_stop, bakery)
```

❌ Index confusion in 2-opt: Remember Python slicing!

```
# The 2-opt swap reverses route[i+1:j+1], not route[i:j]
```

✗ Modifying while iterating: Make a copy!

```
new_route = current_route.copy() # Don't modify original
```

## Your Toolkit for Today

Construction + Improvement = Better Routes

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Construction (Greedy) - Nearest Neighbor - Cheapest Insertion - Savings Algorithm

Improvement (Local Search) - 2-opt swaps - 3-opt (advanced) - Or-opt (move sequences)

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### Note

Most commercial routing software uses this two-phase approach: Build quickly, then polish!

## Next: Bean Counter Practice

In the notebook, you'll help Bean Counter optimize coffee bean deliveries to 10 franchises.

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You'll implement: :: incremental - Distance calculations between locations - Nearest neighbor construction - Route distance measurement - 2-opt improvement - Multi-start optimization ::

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Then: Apply these skills to Artisan Bakery's 12-café challenge!

## The Competition Challenge

Artisan Bakery needs your help with their morning route!

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Your Mission: - Optimize delivery to 12 cafés - Handle 3 early time windows - Minimize total distance - Beat the current 3-hour route

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### 💡 Tip

Hint: Start with nearest neighbor, but don't forget about those time windows! Sometimes a slightly longer route that meets deadlines is better than the shortest route that arrives late.

## Next Week: When Local Search Isn't Enough

Today we learned to climb hills. Next week: How to jump mountains!

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Coming Attractions: - Simulated Annealing: Accept worse solutions probabilistically (like heating metal) - Tabu Search: Remember where you've been (search with memory) - Genetic Algorithms: Evolve solutions (survival of the fittest routes)

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### i Note

All use the same local search foundation - just with clever tricks to escape local optima!

## Literature

### Resources

Essential Reading: - Applegate et al. (2011): The Traveling Salesman Problem - The definitive TSP reference - Laporte (1992): The Vehicle Routing Problem - Overview of routing algorithms - Lin & Kernighan (1973): Classic paper on k-opt improvements

Python Libraries: - `scipy.spatial.distance` - Fast distance calculations - `networkx` - Graph algorithms including TSP approximations - `ortools` - Google's optimization tools with routing

### i Summary

- TSP is computationally hard (factorial growth)
- Local search is a universal framework (4 pillars)
- Greedy construction gives fast initial solutions
- 2-opt improves solutions iteratively
- Multi-start helps escape local optima
- Real constraints (time windows) add complexity
- Two-phase approach: Build then improve!

## Bibliography