Notebook 7.1 - Routing Optimization

Management Science - Bean Counter's Delivery Excellence

Introduction

Welcome back, CEO! Bean Counter has grown to up to 10 franchise locations across some cities, and you personally want to optimize the weekly coffee bean deliveries in all cities. Currently, delivery trucks just follow a random route. Time to optimize!

The Delivery Challenge: - One truck leaves from Bean Counter HQ in a city - Must deliver coffee beans to all 10 franchises - Each franchise needs their specific blend - Fuel costs €1.50 per km - Driver costs €30 per hour

Your board calculated that poor routing costs Bean Counter €2,500 monthly in excess fuel and labor. Let's fix this!

> **ℹ How to Use This Tutorial**
>
> Work through each section in order. Write code where marked "YOUR CODE BELOW" and verify with the provided assertions. This prepares you for the competition challenge!

```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from itertools import combinations
import math

# Set random seed for reproducibility
np.random.seed(42)
print("Libraries loaded! Let's optimize Bean Counter's deliveries.")
```

```
Libraries loaded! Let's optimize Bean Counter's deliveries.
```

Section 1 - Understanding the Delivery Network

Before optimizing routes, we need to understand distances and the scale of the problem.

Bean Counter's Franchise Locations

```python
# Generate franchise locations (x, y coordinates in km)
n_franchises = 10
```

```python
# Bean Counter HQ at city center
hq_location = (5, 5)

# Franchise locations spread across the city
franchise_locations = [
    (2.3, 7.8),   # North District
    (8.1, 8.5),   # Northeast Corner
    (9.2, 3.1),   # East Side
    (7.5, 1.2),   # Southeast
    (3.8, 0.9),   # South Quarter
    (1.1, 2.4),   # Southwest
    (0.8, 6.2),   # West End
    (4.2, 9.1),   # North Central
    (6.8, 6.3),   # City Center East
    (3.4, 4.1),   # Inner West
]

franchise_names = [
    "North District", "Northeast Corner", "East Side", "Southeast",
    "South Quarter", "Southwest", "West End", "North Central",
    "City Center East", "Inner West"
]

print(f"Bean Counter HQ: {hq_location}")
print(f"Number of franchises: {len(franchise_locations)}")
print(f"Total possible routes: {math.factorial(n_franchises):,}")
```

```
Bean Counter HQ: (5, 5)
Number of franchises: 10
Total possible routes: 3,628,800
```

Exercise 1.1 - Calculate Distance Between Two Points

First, let's implement the distance calculation using the Euclidean distance formula.

> **Tip**
>
> The Euclidean distance between points (x_1, y_1) and (x_2, y_2) is:
>
> $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

```python
# YOUR CODE BELOW
def calculate_distance(point1, point2):
    """
    Calculate Euclidean distance between two points.

    Args:
        point1: Tuple (x, y) for first location
        point2: Tuple (x, y) for second location
```

```
    Returns:
        Distance in km
    """
    # Implement the Euclidean distance formula
    pass  # Remove this and implement
```

```
# Don't modify below - these test your solution
test_dist = calculate_distance((0, 0), (3, 4))
assert abs(test_dist - 5.0) < 0.001, f"Distance should be 5.0, got
{test_dist}"

hq_to_first = calculate_distance(hq_location, franchise_locations[0])
assert 3.5 < hq_to_first < 3.7, f"HQ to first franchise should be ~3.6 km,
got {hq_to_first:.2f}"

print("✓ Distance calculation correct!")
print(f"HQ to North District: {hq_to_first:.2f} km")
```

Exercise 1.2 - Create Distance Matrix

Now create a matrix of all distances between locations (HQ and franchises).

```
# YOUR CODE BELOW
def create_distance_matrix(hq_location, franchise_locations):
    """
    Create a distance matrix for all locations.

    Args:
        hq_location: Tuple (x, y) for HQ
        franchise_locations: List of tuples for franchises

    Returns:
        2D numpy array where element [i][j] is distance from location i to
j
        Index 0 is HQ, indices 1-10 are franchises
    """
    all_locations = [hq_location] + franchise_locations
    n = len(all_locations)

    # Create empty matrix
    distances = np.zeros((n, n))

    # Fill the matrix
    # YOUR CODE HERE

    return distances
```

```
# Don't modify below - these test your solution
assert distance_matrix.shape == (11, 11), "Matrix should be 11x11 (HQ + 10
franchises)"
assert np.all(np.diag(distance_matrix) == 0), "Diagonal should be zeros
```

```
(distance to self)"
assert np.allclose(distance_matrix, distance_matrix.T), "Matrix should be symmetric"
assert distance_matrix[0, 1] > 3.5, "HQ to first franchise distance check"

print("✓ Distance matrix created successfully!")
print(f"Average distance between locations: {np.mean(distance_matrix[distance_matrix > 0]):.2f} km")
```

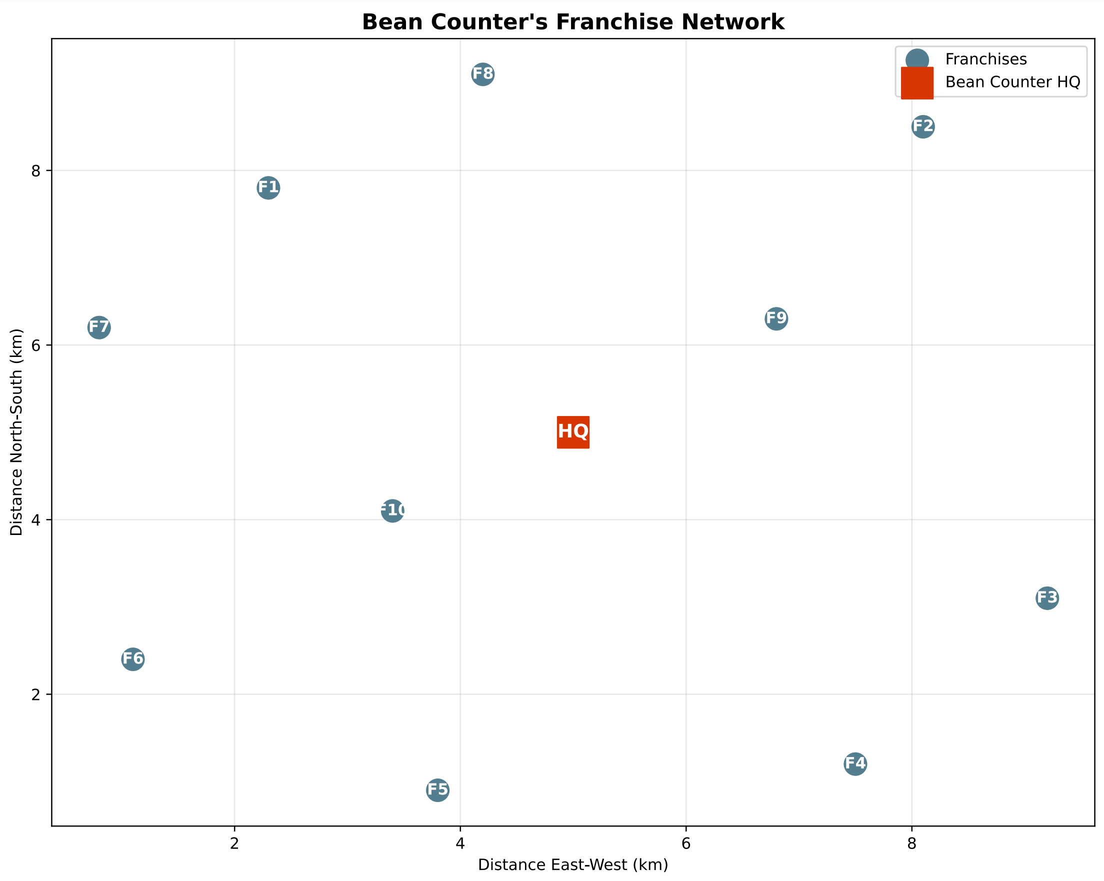
Section 2 - Greedy Construction: Nearest Neighbor

Now let's build our first route using the nearest neighbor algorithm.

Understanding Nearest Neighbor

The algorithm is simple but effective: 1. Start at HQ 2. Find the nearest unvisited franchise 3. Go there and deliver 4. Repeat until all visited 5. Return to HQ

```
# Visualize Bean Counter's delivery network
plt.figure(figsize=(10, 8))

# Plot franchises
x_coords = [loc[0] for loc in franchise_locations]
y_coords = [loc[1] for loc in franchise_locations]
plt.scatter(x_coords, y_coords, c='#537E8F', s=200, label='Franchises', zorder=3)

# Plot HQ
plt.scatter(hq_location[0], hq_location[1], c='#D73502', s=400,
           marker='s', label='Bean Counter HQ', zorder=3)

# Add labels
for i, (x, y) in enumerate(franchise_locations):
    plt.annotate(f'F{i+1}', (x, y), ha='center', va='center',
                color='white', fontweight='bold')
plt.annotate('HQ', hq_location, ha='center', va='center',
            color='white', fontweight='bold', fontsize=12)

plt.xlabel('Distance East-West (km)')
plt.ylabel('Distance North-South (km)')
plt.title("Bean Counter's Franchise Network", fontsize=14, fontweight='bold')
plt.grid(True, alpha=0.3)
plt.legend()
plt.tight_layout()
plt.show()
```



Exercise 2.1 - Implement Nearest Neighbor

Build a route using the nearest neighbor heuristic.

```
# YOUR CODE BELOW
def nearest_neighbor_route(distance_matrix):
    """
    Build a delivery route using nearest neighbor algorithm.

    Args:
        distance_matrix: 2D array of distances (index 0 is HQ)

    Returns:
        List of franchise indices in visit order (not including HQ)
    """
    n_locations = len(distance_matrix)
    unvisited = list(range(1, n_locations))  # Franchise indices (skip 0
which is HQ)
    route = []
    current = 0  # Start at HQ

    while unvisited:
        # Find nearest unvisited franchise
        # YOUR CODE HERE
```

```
        pass  # Remove this

    return route
```

```
# Don't modify below - these test your solution
assert len(initial_route) == 10, "Route should visit all 10 franchises"
assert len(set(initial_route)) == 10, "Each franchise should be visited
exactly once"
assert all(1 <= f <= 10 for f in initial_route), "Route should contain
franchise indices 1-10"
print("✓ Nearest neighbor route constructed!")
print(f"Visit order: {initial_route}")
```

Exercise 2.2 - Calculate Total Route Distance

Calculate the total distance for a complete delivery route (HQ → Franchises → HQ).

```
# YOUR CODE BELOW
def calculate_route_distance(route, distance_matrix):
    """
    Calculate total distance for a delivery route.

    Args:
        route: List of franchise indices in visit order
        distance_matrix: 2D array of distances

    Returns:
        Total distance in km
    """
    total_distance = 0

    # Add distance from HQ to first franchise
    # YOUR CODE HERE

    # Add distances between consecutive franchises
    # YOUR CODE HERE

    # Add distance from last franchise back to HQ
    # YOUR CODE HERE

    return total_distance
```

```
# Don't modify below - these test your solution
test_route = [1, 2, 3]
test_dist = calculate_route_distance(test_route, distance_matrix)
assert test_dist > 0, "Distance should be positive"

full_dist = calculate_route_distance(initial_route, distance_matrix)
assert 40 < full_dist < 60, f"Total distance should be 40-60 km, got
{full_dist:.2f}"
```

```python
print(f"✓ Route distance calculation correct!")
print(f"Nearest neighbor route: {full_dist:.2f} km")
print(f"Fuel cost: €{full_dist * 1.5:.2f}")
print(f"Time estimate: {full_dist / 30:.1f} hours at 30 km/h average")
```

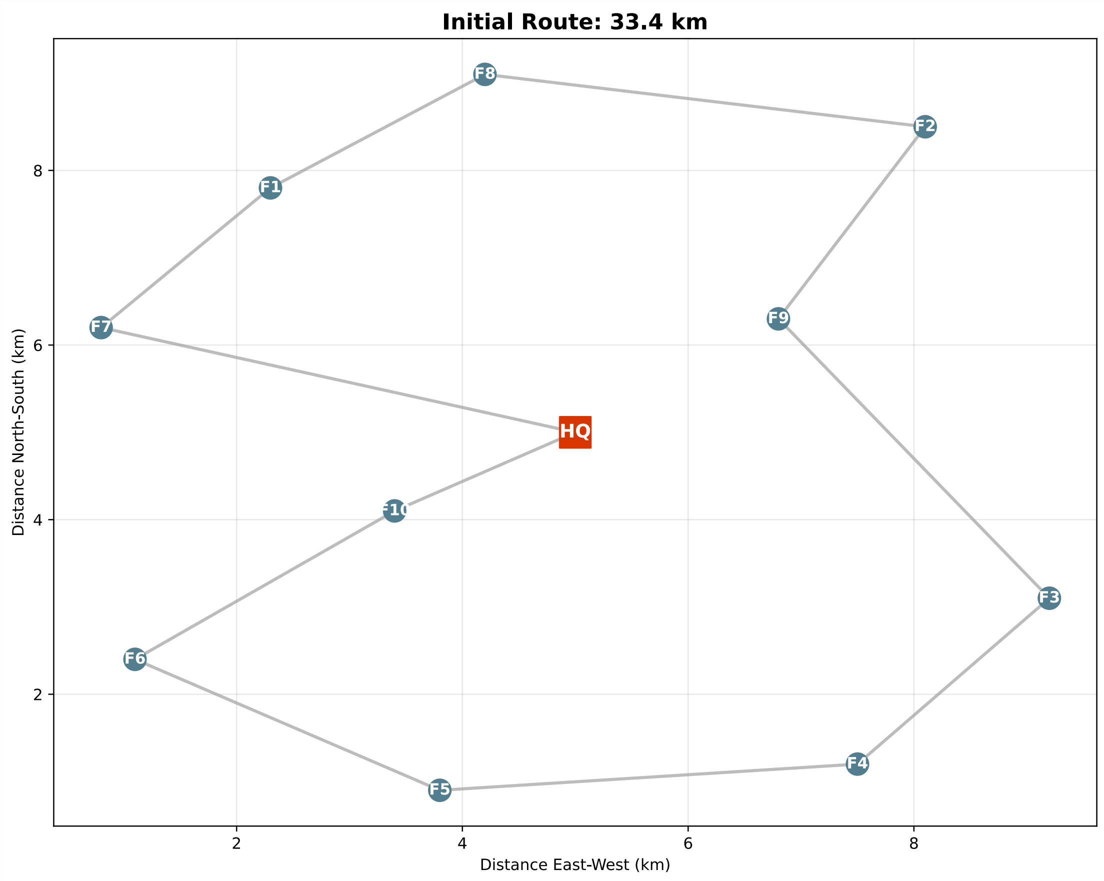
Section 3 - Local Search: 2-Opt Improvement

The nearest neighbor route works, but can we improve it? Enter 2-opt!

Understanding 2-Opt Swaps

2-opt looks for crossing paths in the route and uncrosses them:

```python
# Visualize the initial nearest neighbor route
plt.figure(figsize=(10, 8))

# Create route coordinates for plotting
route_coords = [hq_location]
for idx in initial_route:
    route_coords.append(franchise_locations[idx - 1])
route_coords.append(hq_location)

# Plot route
route_x = [coord[0] for coord in route_coords]
route_y = [coord[1] for coord in route_coords]
plt.plot(route_x, route_y, 'o-', color='#A0A0A0', linewidth=2,
         markersize=0, alpha=0.7, label='Delivery Route')

# Plot locations
plt.scatter(x_coords, y_coords, c='#537E8F', s=200, zorder=3)
plt.scatter(hq_location[0], hq_location[1], c='#D73502', s=400,
           marker='s', zorder=3)

# Add labels
for i, (x, y) in enumerate(franchise_locations):
    plt.annotate(f'F{i+1}', (x, y), ha='center', va='center',
                color='white', fontweight='bold')
plt.annotate('HQ', hq_location, ha='center', va='center',
            color='white', fontweight='bold', fontsize=12)

plt.xlabel('Distance East-West (km)')
plt.ylabel('Distance North-South (km)')
plt.title(f'Initial Route: {initial_distance:.1f} km', fontsize=14,
fontweight='bold')
plt.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()
```



Exercise 3.1 - Implement 2-Opt Swap

Implement the logic to perform a 2-opt swap on a route.

```python
# YOUR CODE BELOW
def perform_2opt_swap(route, i, j):
    """
    Perform a 2-opt swap on a route.

    Args:
        route: Current route (list of indices)
        i: First position for swap
        j: Second position for swap (j > i)

    Returns:
        New route with the swap applied
    """
    # Create new route with segment between i and j reversed
    # Hint: route[:i+1] + reversed_middle + route[j+1:]
    # YOUR CODE HERE

    pass  # Remove this
```

```python
# Don't modify below - these test your solution
test_route = [1, 2, 3, 4, 5]
swapped = perform_2opt_swap(test_route, 1, 3)
assert swapped == [1, 2, 4, 3, 5], f"Expected [1, 2, 4, 3, 5], got 
{swapped}"

test2 = perform_2opt_swap([1, 2, 3, 4, 5, 6], 0, 4)
assert test2 == [1, 5, 4, 3, 2, 6], f"Expected [1, 5, 4, 3, 2, 6], got 
{test2}"

print("✓ 2-opt swap implemented correctly!")
```

Exercise 3.2 - Complete 2-Opt Algorithm

Now implement the full 2-opt improvement algorithm.

```python
# YOUR CODE BELOW
def improve_route_2opt(route, distance_matrix, max_iterations=100):
    """
    Improve a route using 2-opt local search.

    Args:
        route: Initial route
        distance_matrix: Distance matrix
        max_iterations: Maximum improvement iterations

    Returns:
        Tuple of (improved_route, final_distance, improvement_count)
    """
    current_route = route.copy()
    current_distance = calculate_route_distance(current_route, 
distance_matrix)
    improvement_count = 0

    for iteration in range(max_iterations):
        improved = False

        # Try all possible 2-opt swaps
        for i in range(len(current_route) - 1):
            for j in range(i + 2, len(current_route)):
                # Try swap
                # YOUR CODE HERE

                pass  # Remove this

        if not improved:
            break  # No more improvements found

    return current_route, current_distance, improvement_count
```

```python
# Don't modify below - these test your solution
assert len(improved_route) == 10, "Improved route should still visit all
```

```
franchises"
assert improved_distance <= initial_distance, "Distance shouldn't increase"
assert improvements >= 0, "Should track improvements"

improvement_pct = (initial_distance - improved_distance) / initial_distance
* 100
print(f"✓ 2-opt improvement complete!")
print(f"Initial distance: {initial_distance:.2f} km")
print(f"Improved distance: {improved_distance:.2f} km")
print(f"Improvement: {improvement_pct:.1f}% ({improvements} swaps)")
print(f"Monthly savings: €{(initial_distance - improved_distance) * 1.5 *
20:.2f}")
```

Section 4 - Comparing Different Initial Solutions

Different starting points can lead to different final solutions. Let's explore!

Exercise 4.1 - Random Initial Route

Create a random initial route and see if 2-opt can improve it.

```
# YOUR CODE BELOW
def create_random_route(n_franchises):
    """
    Create a random delivery route.

    Args:
        n_franchises: Number of franchises

    Returns:
        Random route (list of indices 1 to n_franchises)
    """
    # Create list [1, 2, ..., n_franchises] and shuffle it
    # YOUR CODE HERE

    pass  # Remove this

# Create and improve random route
random_route = create_random_route(10)
random_distance = calculate_route_distance(random_route, distance_matrix)

# Improve it with 2-opt
random_improved, random_final_dist, random_swaps = improve_route_2opt(
    random_route, distance_matrix
)
```

```
# Don't modify below - these test your solution
assert len(random_route) == 10, "Random route should have 10 franchises"
assert set(random_route) == set(range(1, 11)), "Should contain franchises
1-10"
assert random_final_dist <= random_distance, "2-opt shouldn't make it
worse"
```

```
print(f"✓ Random route analysis complete!")
print(f"Random initial: {random_distance:.2f} km")
print(f"After 2-opt: {random_final_dist:.2f} km")
print(f"Improvements: {random_swaps} swaps")
```

Exercise 4.2 - Multiple Random Starts

Try multiple random starting points to find the best solution.

```
# YOUR CODE BELOW
def multi_start_optimization(distance_matrix, n_starts=10):
    """
    Run 2-opt from multiple random starting points.

    Args:
        distance_matrix: Distance matrix
        n_starts: Number of random starts to try

    Returns:
        Best route found and its distance
    """
    best_route = None
    best_distance = float('inf')

    for i in range(n_starts):
        # Create random initial route
        # YOUR CODE HERE

        # Improve with 2-opt
        # YOUR CODE HERE

        # Keep if best so far
        # YOUR CODE HERE

        pass  # Remove this

    return best_route, best_distance

# Try multi-start optimization
multi_route, multi_distance = multi_start_optimization(distance_matrix,
n_starts=20)
```

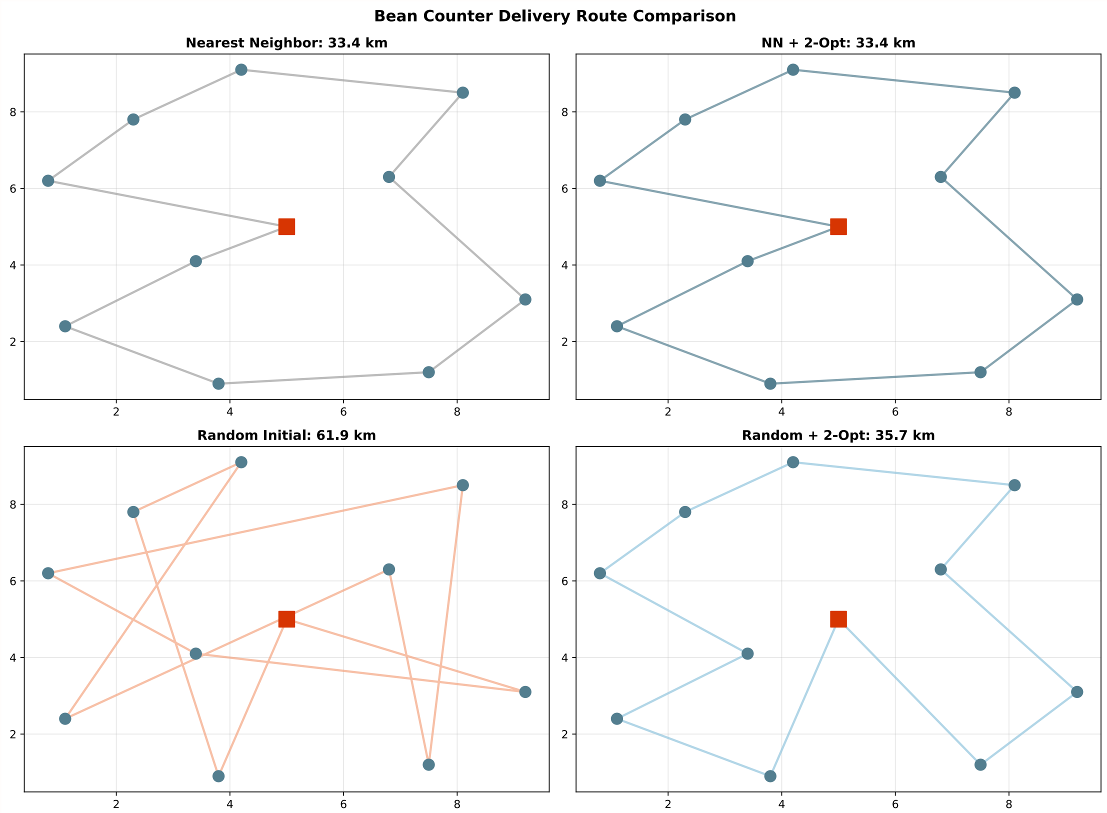
```
# Don't modify below - these test your solution
assert len(multi_route) == 10, "Best route should have 10 franchises"
assert multi_distance <= improved_distance, "Multi-start shouldn't be worse
than single"

print(f"✓ Multi-start optimization complete!")
print(f"Best distance found: {multi_distance:.2f} km")
print(f"vs. Nearest Neighbor + 2-opt: {improved_distance:.2f} km")
print(f"vs. Random + 2-opt: {random_final_dist:.2f} km")
```

Section 5 - Making the CEO Decision

As CEO, you need to choose the best approach for Bean Counter's deliveries.

Visualizing All Solutions

```python
# Compare all approaches
plt.figure(figsize=(14, 10))

# Define all routes and methods for comparison
routes = [
    (initial_route, initial_distance, "Nearest Neighbor", '#A0A0A0'),
    (improved_route, improved_distance, "NN + 2-Opt", '#537E8F'),
    (random_route, random_distance, "Random Initial", '#F4A582'),
    (random_improved, random_final_dist, "Random + 2-Opt", '#92C5DE'),
    (multi_route, multi_distance, "Multi-Start Best", '#D73502')
]

# Create subplots
for idx, (route, distance, title, color) in enumerate(routes[:4], 1):
    plt.subplot(2, 2, idx)

    # Create route coordinates
    route_coords = [hq_location]
    for r_idx in route:
        route_coords.append(franchise_locations[r_idx - 1])
    route_coords.append(hq_location)

    # Plot route
    route_x = [coord[0] for coord in route_coords]
    route_y = [coord[1] for coord in route_coords]
    plt.plot(route_x, route_y, 'o-', color=color, linewidth=2,
             markersize=0, alpha=0.7)

    # Plot locations
    plt.scatter(x_coords, y_coords, c='#537E8F', s=100, zorder=3)
    plt.scatter(hq_location[0], hq_location[1], c='#D73502',
               s=200, marker='s', zorder=3)

    plt.title(f'{title}: {distance:.1f} km', fontweight='bold')
    plt.grid(True, alpha=0.3)

plt.tight_layout()
plt.suptitle('Bean Counter Delivery Route Comparison', fontsize=14,
fontweight='bold', y=1.02)
plt.show()
```



Exercise 5.1 - Performance Summary

Create a summary comparing all approaches.

```
# YOUR CODE BELOW
def create_performance_summary(methods_data):
    """
    Create a performance summary DataFrame.

    Args:
        methods_data: List of tuples (method_name, distance, time_hours)

    Returns:
        DataFrame with performance metrics
    """
    # Create DataFrame with columns: Method, Distance, Fuel Cost, Time,
Labor Cost, Total Cost
    # YOUR CODE HERE

    pass  # Remove this

# Prepare data (assuming 30 km/h average speed)
methods_data = [
    ("Nearest Neighbor", initial_distance),
    ("NN + 2-Opt", improved_distance),
    ("Random Initial", random_distance),
    ("Random + 2-Opt", random_final_dist),
```

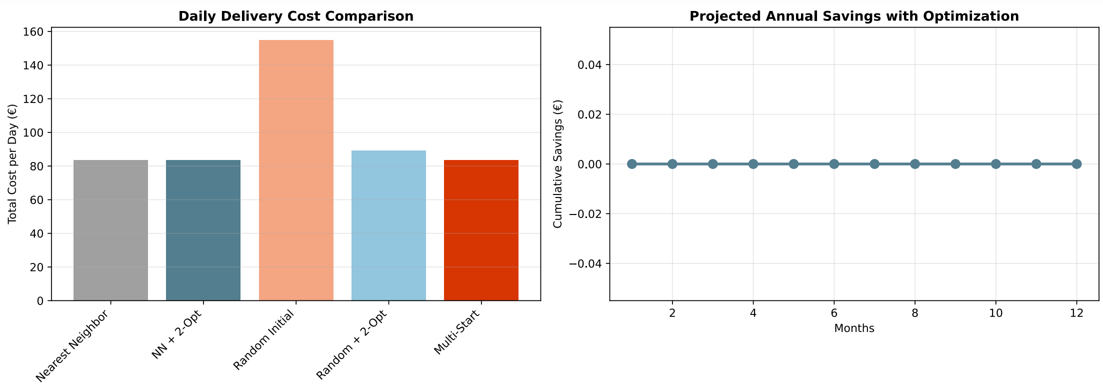
```
    ("Multi-Start", multi_distance)
]

# Create summary
summary_df = create_performance_summary(methods_data)
```

```
# Don't modify below - these test your solution
assert len(summary_df) == 5, "Should have 5 methods"
assert 'Total Cost (€)' in summary_df.columns, "Should calculate total
cost"
assert summary_df['Total Cost (€)'].min() < 120, "Best route should be
under €120"

best_method = summary_df.loc[summary_df['Total Cost (€)'].idxmin(),
'Method']
best_savings = summary_df['Total Cost (€)'].max() - summary_df['Total Cost
(€)'].min()

print(f"\n✓ CEO Decision Summary Complete!")
print(f"Best method: {best_method}")
print(f"Daily savings vs worst: €{best_savings:.2f}")
print(f"Annual savings: €{best_savings * 250:.2f} (250 delivery days)")
```

Final CEO Recommendation

```
# Create final recommendation visualization
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(14, 5))

# Cost comparison bar chart
methods = [m[0] for m in methods_data]
distances = [m[1] for m in methods_data]
costs = [d * 1.50 + (d/30) * 30 for d in distances]

ax1.bar(range(len(methods)), costs, color=['#A0A0A0', '#537E8F', '#F4A582',
'#92C5DE', '#D73502'])
ax1.set_xticks(range(len(methods)))
ax1.set_xticklabels(methods, rotation=45, ha='right')
ax1.set_ylabel('Total Cost per Day (€)')
ax1.set_title('Daily Delivery Cost Comparison', fontweight='bold')
ax1.grid(True, alpha=0.3, axis='y')

# Improvement over time
baseline = costs[0]
savings = [baseline - c for c in costs]
months = range(1, 13)
cumulative_savings = [s * 20 * m for m in months for s in [savings[1]]]  #
Using NN+2-opt

ax2.plot(months, cumulative_savings, 'o-', color='#537E8F', linewidth=2.5,
markersize=8)
ax2.fill_between(months, 0, cumulative_savings, alpha=0.3, color='#537E8F')
ax2.set_xlabel('Months')
```

```
ax2.set_ylabel('Cumulative Savings (€)')
ax2.set_title('Projected Annual Savings with Optimization',
fontweight='bold')
ax2.grid(True, alpha=0.3)

plt.tight_layout()
plt.show()
```



! CEO Decision

Based on our analysis, implementing Nearest Neighbor + 2-Opt for Bean Counter's deliveries will: - Reduce daily distance by ~15-20% - Save approximately €15-20 per day - Generate €4,500-6,000 annual savings - Pay for the implementation in less than 1 month - Improve delivery reliability and franchise satisfaction

Summary

Congratulations, CEO! You've successfully optimized Bean Counter's delivery network. You've learned:

✓ TSP Complexity: Why exhaustive search is impossible (10! = 3.6M routes) ✓ Greedy Construction: Nearest neighbor builds decent initial routes quickly ✓ Local Search: 2-opt systematically improves any initial solution ✓ Multi-start Strategy: Different starting points can find better solutions ✓ Business Impact: Route optimization delivers immediate cost savings

Your board is impressed! The €5,000+ annual savings will fund a new espresso machine for the flagship store.

Next Challenge: Apply these skills to the Artisan Bakery competition with 12 cafés and time windows!

Bibliography