Dealing with Uncertainty

Lecture 4 - Management Science

Dr. Tobias Vlćek

Introduction

Client Briefing: TechVenture Innovation Fund

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CEO's Dilemma:

"We have €2M to invest in 2 of 4 startups. Each promises great returns, but the future is uncertain. How do we make the best choice without just gambling?"

Business: Valuing Uncertainty

Question: Why can't we just pick the two startups with the highest average returns?

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- Hidden Risk: A startup with 30% average return but 50% chance of failure might be worse than 20% return with 5% failure chance
- Portfolio Effects: Two risky startups together might amplify risk beyond acceptable levels
- Tail Events: The worst-case scenario can matter as much as the average case

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⚠ Warning

Common Pitfall: Optimizing on averages ignores the distribution of outcomes.

Real-World Examples

Where uncertainty modeling is critical:

Netflix Series Decisions

- Will a show hit 10M viewers?
- Range: 500K to 50M
- Investment: €20M per season

Pharmaceutical R&D

- Will the drug pass trials?

- Success rate: 10-20%
- Investment: €1B over 10 years

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> ! Important
>
> When decisions are expensive and outcomes are uncertain, Monte Carlo simulation can be helpful to reduce risk and maximize value!

Core Concepts

Rolling the Dice 10,000 Times I

Question: If you roll two dice, what's the probability of getting exactly 7 as result?

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Method 1: Math

- Count combinations: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)
- Total combinations: 36
- Probability: 6/36 = 16.67%

Rolling the Dice 10,000 Times II

Question: If you roll two dice, what's the probability of getting exactly 7 as result?

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Method 2: Simulation

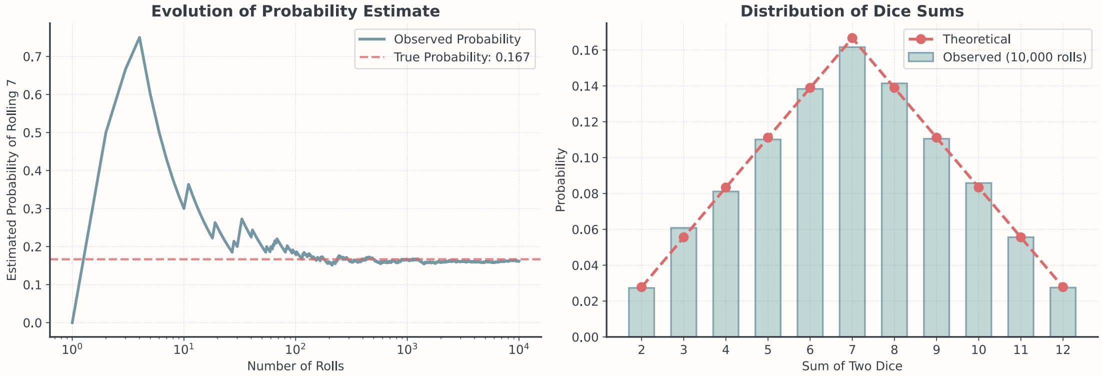
```python
import numpy as np
np.random.seed(42)

# Roll two dice 10,000 times
dice1 = np.random.randint(1, 7, size=10_000)
dice2 = np.random.randint(1, 7, size=10_000)
total = dice1 + dice2

# What fraction equals 7?
probability = (total == 7).mean()
print(f"Simulated probability of rolling 7: {probability:.1%}")
```

```
Simulated probability of rolling 7: 16.2%
```

How Probability Converges



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> 💡 Tip
>
> As we roll more dice, the estimated probability converges to the true value (16.7%)

The Law of Large Numbers

Fundamental Principle: As sample size increases, sample average converges to the true expected value

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If $X_1, X_2, ..., X_n$ are independent random samples from the same distribution with mean μ:

$$\text{As} \quad n \to \infty, \quad \bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \to \mu$$

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> ℹ Note
>
> This is WHY simulations works. More simulations = better estimates!

The Central Limit Theorem

Another Fundamental Principle: The sum of many random variables tends toward a normal distribution

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What it means:

- Even if individual returns are NOT normally distributed…
- The portfolio of many assets WILL be approximately normal
- The average of many simulations WILL be approximately normal

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> Tip

For Business: This is why we can use normal distributions to model portfolio returns, even when individual assets have skewed or unusual distributions!

Why This Matters for Business

Question: How many simulations do we need for reliable results?

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```
# Test convergence with different sample sizes
sample_sizes = [10, 100, 1000, 10000, 100000]
estimates = []

for n in sample_sizes:
    dice1 = np.random.randint(1, 7, size=n)
    dice2 = np.random.randint(1, 7, size=n)
    total = dice1 + dice2
    prob = (total == 7).mean()
    estimates.append(prob)
    print(f"n={n:6d}: Estimated probability = {prob:.4f}")
```

```
n=    10: Estimated probability = 0.2000
n=   100: Estimated probability = 0.1900
n=  1000: Estimated probability = 0.1480
n= 10000: Estimated probability = 0.1652
n=100000: Estimated probability = 0.1670
```

Practical Guidelines

How many simulations should you run?

- Quick exploration: 10,000 simulations
  - Good for initial insights, prototyping
- Critical decisions: 100,000+ simulations
  - Financial risk models, regulatory compliance
- When to stop: When more simulations don't change conclusion

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> ! Important

If your decision changes with 10x more simulations, you didn't run enough!

Monte Carlo Method

The Monte Carlo Method

Three Simple Steps:

1. Model the Uncertainty:
   - Define probability distributions for unknown variables
2. Simulate Many Scenarios:
   - Generate thousands of possible outcomes
3. Analyze the Results:
   - Calculate statistics from the simulation

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> **ℹ Note**
>
> Monte Carlo Casino in Monaco inspired the method's development in the 1940s.

Step 1: Model the Uncertainty

Key Function: `np.random.normal(loc, scale, size)`

- loc: The center (mean/average)
- scale: The spread (standard deviation)
- size: How many samples to generate

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```python
# AI-Growth: average 38% return, ±25% volatility
returns = np.random.normal(loc=0.38, scale=0.25, size=10_000)
print(f"Mean return: {returns.mean():.1%}")
print(f"Std deviation: {returns.std():.1%}")
print(f"Minimum: {returns.min():.1%}")
print(f"Maximum: {returns.max():.1%}")
```

```
Mean return: 38.8%
Std deviation: 24.8%
Minimum: -67.0%
Maximum: 125.7%
```

Expected Returns

Let's calculate percentiles with `np.percentile()`.

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Question: Do you still know what a percentile is?

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```python
print(f"\nPercentiles:")
print(f"  5th: {np.percentile(returns, 5):.1%} (worst 5% of scenarios)")
print(f" 25th: {np.percentile(returns, 25):.1%} (worst 25% of scenarios)")
print(f" 50th: {np.percentile(returns, 50):.1%} (median)")
print(f" 75th: {np.percentile(returns, 75):.1%} (best 25% of scenarios)")
print(f" 95th: {np.percentile(returns, 95):.1%} (best 5% of scenarios)")
```

```
Percentiles:
  5th: -2.7% (worst 5% of scenarios)
 25th: 22.1% (worst 25% of scenarios)
 50th: 38.9% (median)
 75th: 55.7% (best 25% of scenarios)
 95th: 78.6% (best 5% of scenarios)
```

Understanding the Distribution

Question: Before we plot, what shape do you expect for `np.random.normal()`?

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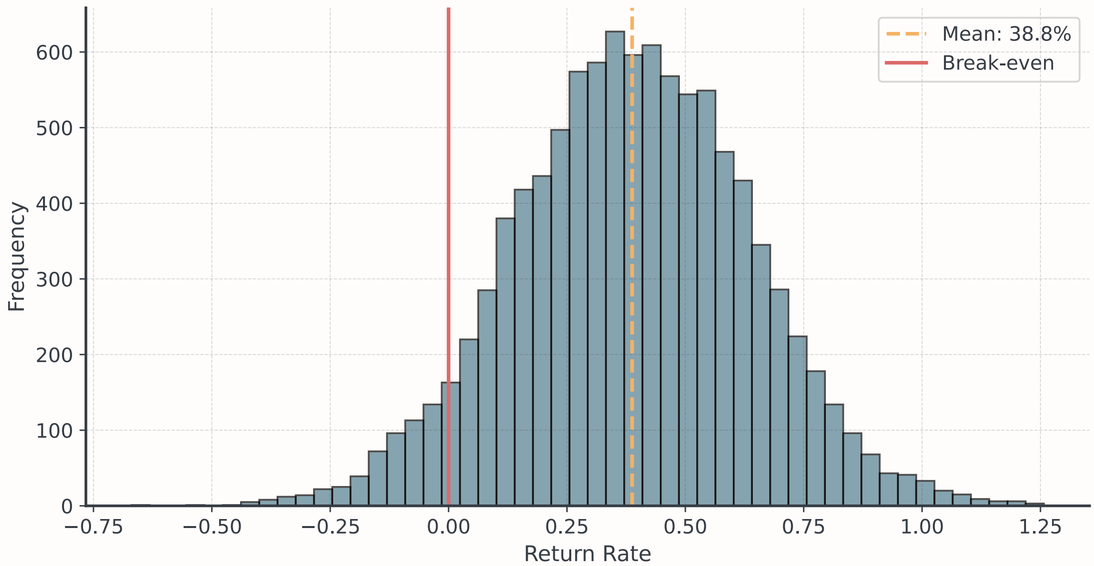

Risk Analysis

Question: What's the probability that AI-Growth loses money?

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```python
# Calculate risk metrics
prob_loss = (returns < 0).mean() # proportion of returns that are less than
zero
prob_double = (returns > 1.0).mean()  # proportion greater than 100%
```

```python
print(f"Probability of loss: {prob_loss:.1%}")
print(f"Probability of doubling money: {prob_double:.1%}")
```

```
Probability of loss: 6.0%
Probability of doubling money: 0.8%
```

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> **! Important**
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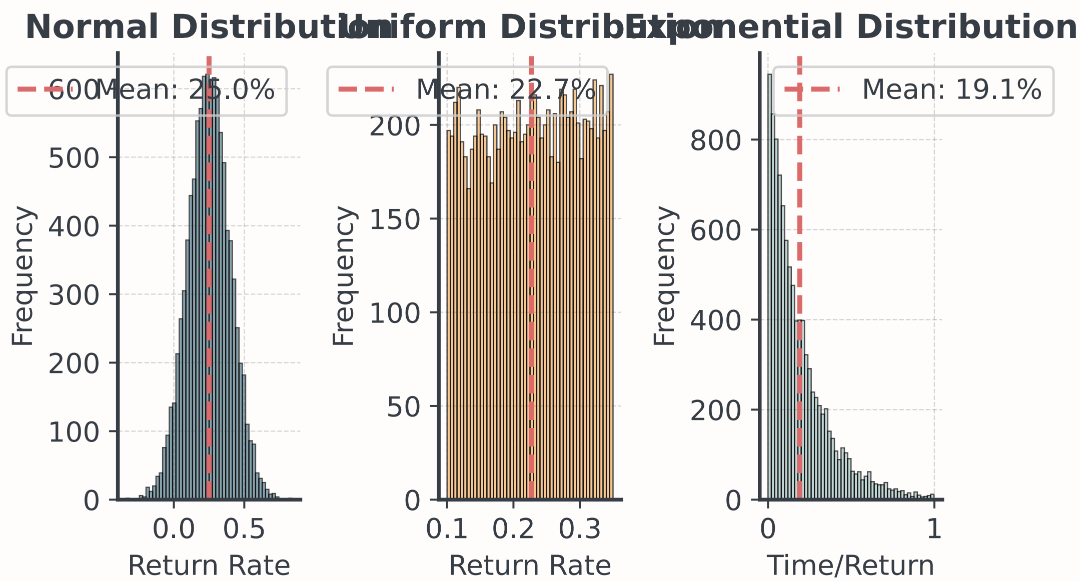
> With 6 % chance of loss, AI-Growth is relatively safe. Easy for one startup, right?

Different Distributions

Attention: Not everything follows a normal distribution!



Overview

Normal

```python
# Most common in nature/business
# Bell-shaped, symmetric
returns = np.random.normal(mean, std, size)

# Example: CloudAI startup returns
cloudai = np.random.normal(0.25, 0.15, 10000)  # 25% ± 15%
```

Main Characteristics:

- Symmetric bell curve
- Most values cluster around mean

- Common in nature and business

Uniform

```python
# Equal probability across range
# Example: FinFlow returns between 10-35%
returns = np.random.uniform(0.10, 0.35, size)

# Example: FinFlow startup returns
finflow = np.random.uniform(0.10, 0.35, 10000)  # 10-35% equally likely
```

Main Characteristics:

- All values equally likely
- Hard boundaries (min/max)
- Good for modeling complete uncertainty within range

Exponential

```python
# Time between events
# Example: Customer arrivals, equipment failure
times = np.random.exponential(scale, size)

# Example: Time between customer arrivals (minutes)
arrivals = np.random.exponential(5, 10000)  # Average 5 minutes
```

Main Characteristics:

- Many small values, few large ones
- Always positive
- Common for waiting times and rare events

When NOT to Use Monte Carlo

Monte Carlo is powerful, but not always the right tool:

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- You have a simple analytical solution
  - Use math directly: no need for 10,000 simulations!
- You can’t reasonably estimate input distributions
  - Garbage in = garbage out, need basis for assumptions
- The problem is deterministic (no uncertainty)
  - Simulation adds complexity without value

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> ⚠ Warning
>
> Simulation is a tool for managing uncertainty, not creating false precision!

Portfolios

Combining Investments

Suppose we have the following startups:

CloudAI, GreenGrid, HealthTrack, FinFlow

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Question: If we must pick 2 of 4, how many unique pairs exist?

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The Math:

$$\binom{4}{2} = \frac{4!}{2! \times 2!} = \frac{4 \times 3 \times 2 \times 1}{(2 \times 1) \times (2 \times 1)} = \frac{24}{4} = 6$$

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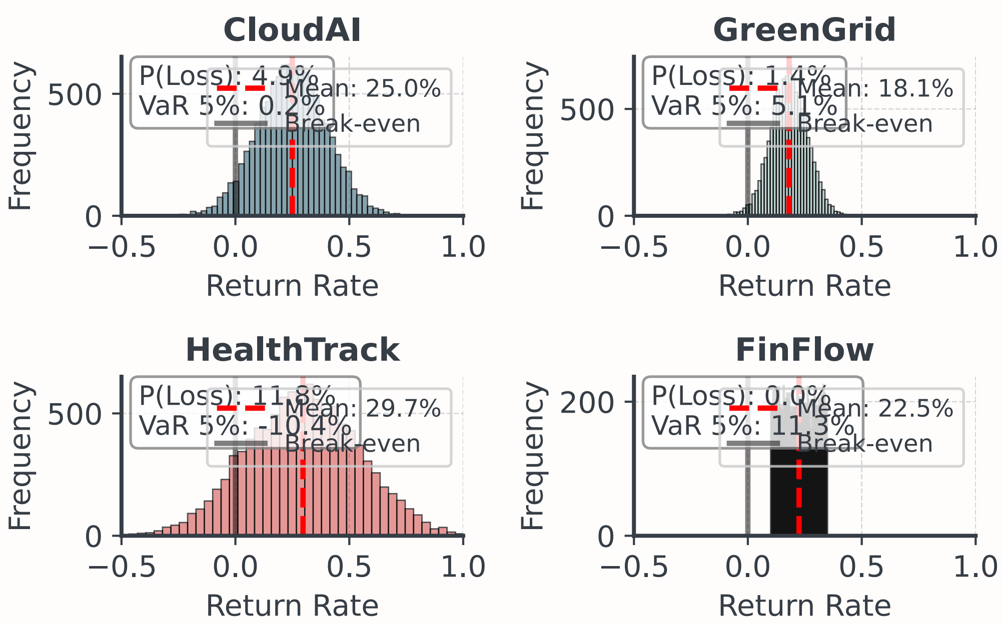
Tip

Each combination has different risk-return characteristics!

Four Startup Profiles



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Question: Which startup is the best choice?

Key Metrics for Decision Making

Question: Which metrics matter most for investment decisions?

- Expected Return: Average outcome across all scenarios
- Volatility (Risk): Standard deviation of returns
- Probability of Loss: How often do we lose money?
- Upside Potential: Chance of exceptional returns (>50%)
- Tail Risk: What happens in the worst 10% of cases?

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> ! Important
>
> No metric tells the whole story. Investors consider multiple dimensions of risk and return.

Understanding Tail Risk

Tail Risk: The danger lurking in worst-case scenarios

Expected Shortfall (ES)

- Average loss in worst X% of cases
- Goes beyond simple probability
- Measures depth of potential losses
- Critical for risk management

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> ⚠ Warning
>
> A portfolio with higher average returns might have catastrophic tail risk. Always look at the extremes!

Correlation & Dependence

The Independence Assumption

So far, we've assumed startups succeed or fail independently.

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Independent Events:

- CloudAI's success doesn't affect GreenGrid's success
- Each startup faces separate, unrelated risks
- Portfolio risk = Average of individual risks

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Question: Is this realistic in the real world?

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⚠ Warning

Reality Check: Many business risks are correlated! Economic downturns, market trends, and technology shifts affect multiple companies simultaneously.

What is Correlation?

Correlation measures how two variables move together.

$$\rho_{X,Y} = \frac{\text{Cov }(X,Y)}{\sigma_X \sigma_Y} \quad \text{where } -1 \leq \rho \leq 1$$

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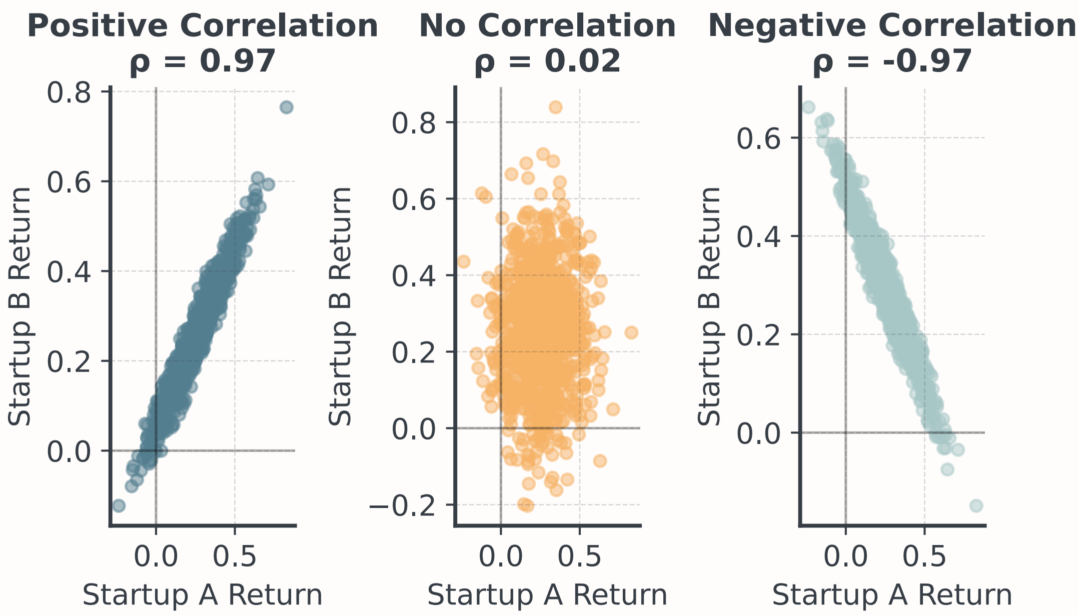
Interpreting Correlation:

- ρ = +1: Perfect positive correlation (move together)
- ρ = 0: No correlation (independent)
- ρ = −1: Perfect negative correlation (move opposite)

Correlation in Practice



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💡 Tip

In Python: `np.corrcoef(returns1, returns2)` calculates correlation

Why Correlation Matters

Two AI startups in your portfolio:

Scenario 1: Independent ($\rho = 0$)

- One fails due to technical issues, other succeeds
- Risk is averaged out

Scenario 2: Positively Correlated ($\rho = 0.8$)

- Both rely on same AI infrastructure provider - risk is amplified!

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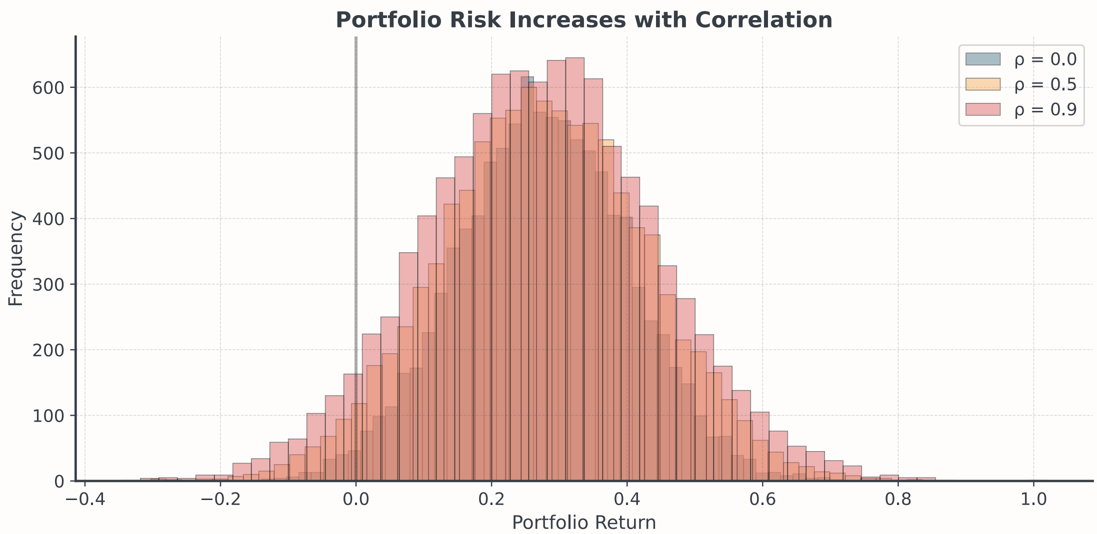
⚠ Warning

Diversification only reduces risk when investments are not highly correlated!

Impact on Portfolio Risk



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! Important

Higher correlation = Wider distribution = More risk!

Real-World Correlation Examples

Common sources of correlation in business:

- Industry-specific: All tech startups affected by downturn
- Geographic: All European companies affected by EU regulations
- Supply chain: Multiple companies relying on same supplier
- Macroeconomic: Interest rates, inflation affect most businesses

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> 💡 Tip
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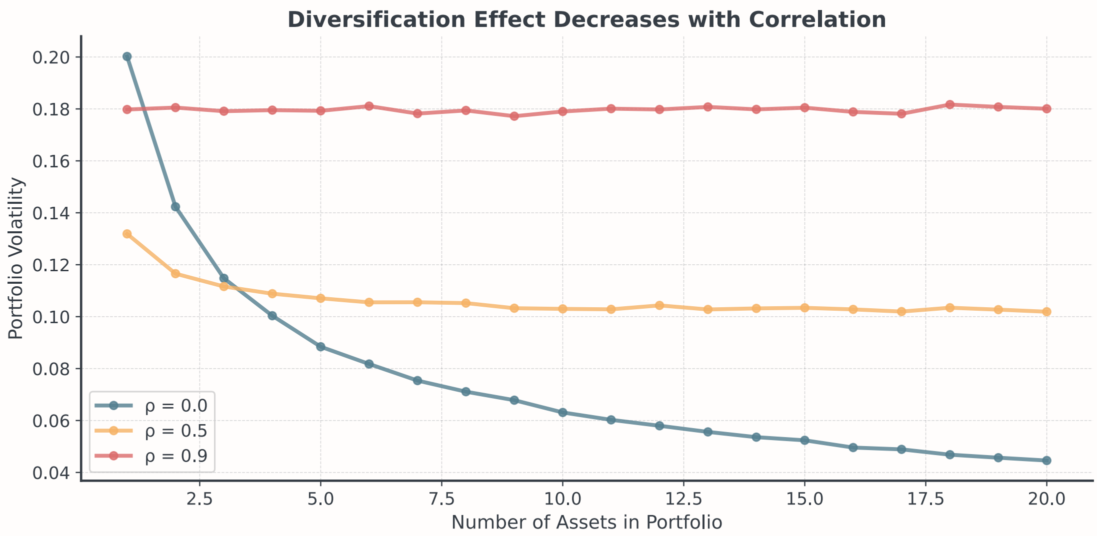
> Diversification: Choose investments with LOW correlation to reduce portfolio risk!

When Diversification Fails



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> ⚠ Warning
>
> 2008 Financial Crisis: Many "diversified" portfolios collapsed due to correlations!

Making Smart Decisions

Decision Framework

1. Define Your Risk Tolerance
   - Can you afford to lose money and what's your time horizon?
   - Are you risk-averse or risk-seeking?
2. Evaluate Multiple Metrics
   - Don't just maximize returns, consider volatility and risk
   - Look at probability of achieving goals
3. Scenario Test
   - What if distributions change or a company fails?

The Plan for the Day

Hour 1:

Lecture

- Concepts

- Examples
- Visualization

Hour 2:

Practice Notebook

- Simulation
- Hands-on coding
- Build your skills

Hours 3-4:

Competition

- TechVenture
- Team collaboration
- €2M investment

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Remember: The lecture gives you concepts. The notebook gives you practice. The competition tests your skills!

Hour 2: Simulation

Your Practice Case: Bean Counter Expansion

- Model uncertain variables (customers, spending)
- Combine multiple uncertainties
- Calculate business metrics (VaR, profit probability)
- Make data-driven recommendations

Hours 3-4: The Challenge

TechVenture Investment Competition

- Your Budget: €2 million
- Your Choice: Pick 2 of 4 startups
- Your Goal: Maximize risk-adjusted returns
- Your Deliverable: One-slide recommendation + 3-minute pitch

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Consider multiple risk metrics and prepare a clear justification!

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Prizes: 10 / 6 / 3 bonus points for top three teams!

Key Takeaways

What You've Learned Today

Concepts

- Monte Carlo simulation
- Probability distributions
- Risk has multiple dimensions
- Expected Value vs. Variance
- Correlation and dependence

Skills

- Using `np.random` for simulation
- Calculating risk metrics
- Visualizing uncertainty
- Comparing portfolios
- Understanding correlation

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> ⚠ Warning
>
> Monte Carlo doesn't predict THE future - it shows possible futures! And correlation can amplify or reduce risk!

Next Week

Forecasting the Future

- Moving from simulation to prediction
- Time series analysis
- Trend and seasonality detection
- Measuring forecast accuracy

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Now, short break and then we start coding!

Bibliography