

Multi-Objective Optimization

Lecture 8 - Management Science

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Introduction

Client Briefing: EcoExpress Logistics

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Operations Director's Dilemma:

“EU regulations demand 40% emission cuts, but we can't sacrifice profitability, service quality, or reliability!”

The Fleet Challenge

EcoExpress operates regional last-mile delivery across 3 cities

- EU Green Deal: 40% emission reduction by 2025
- Rising fuel costs (€2.1/L diesel)
- Amazon entering our market (speed pressure)
- Driver shortage (need automation-friendly vehicles)

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Question: How do we transform our fleet while staying competitive?

Today's Learning Objectives

By the end of this lecture, you will be able to:

1. Explain why most decisions involve competing objectives
2. Identify and visualize Pareto optimal solutions
3. Apply normalization techniques to make objectives comparable
4. Implement approaches to find trade-off solutions
5. Make decisions from a Pareto frontier

Quick Recap: Local Search

Last week we optimized routes for delivery:

- Started with greedy construction (e.g. Nearest Neighbor)
- Improved with local search (e.g. 2-opt)
- Considered time windows
- But: We only optimized distance

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Question: What if we also care about emissions, cost, AND customer satisfaction?

The Problem

Single vs Multi-Objective

Single Objective

- “Minimize total distance”
- Clear winner. Easy, right!

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Multiple Objectives

- “Minimize cost AND emissions AND maximize speed”
- No clear answer...

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Question: Any idea how to approach this?

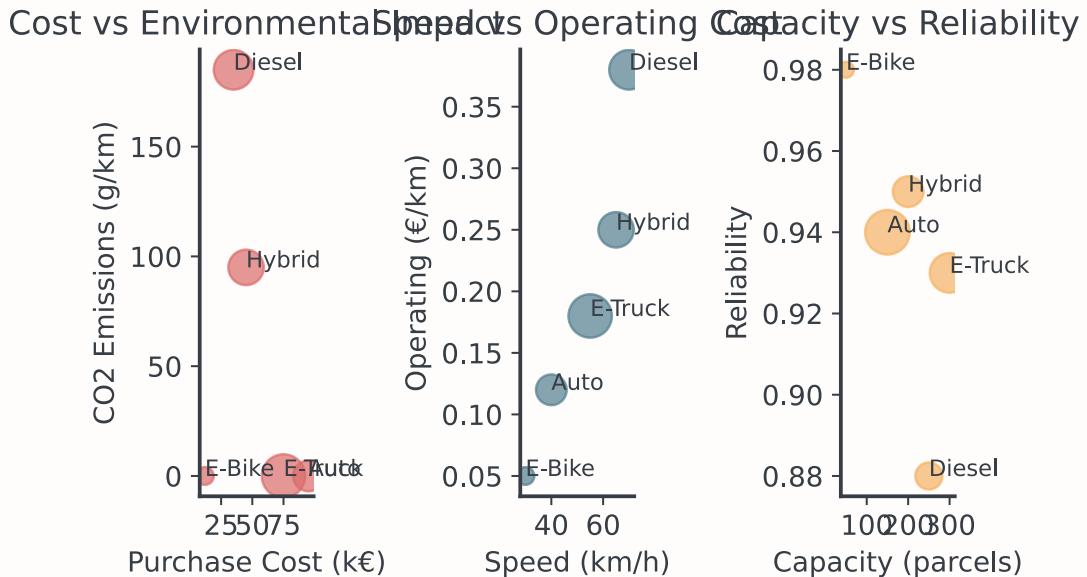
EcoExpress Vehicle Options

Type	Purchase Cost (€)	Operat-ing (€/ km)	CO2 (g/ km)	Speed (km/h)	Capacity (parcels)	Reliability
E-Truck	75000	0.18	0	55	300	0.93
Hybrid	45000	0.25	95	65	200	0.95
Diesel	35000	0.38	185	70	250	0.88
E-Bike	12000	0.05	0	30	50	0.98
Auto	95000	0.12	0	40	150	0.94

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Question: Which vehicle is “best” for EcoExpress?

Trade-offs Everywhere



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! Important

Every vehicle excels at something different!

Real Business Constraints

Beyond the numbers, consider:

- EU regulations: Carbon tax of €100/ton CO₂ starting 2025
- Competition: Amazon promises 2-hour delivery
- Labor market: Autonomous vehicles reduce driver dependency
- Urban zones: Zero-emission zones in city centers
- Peak times: Black Friday = 3x normal volume

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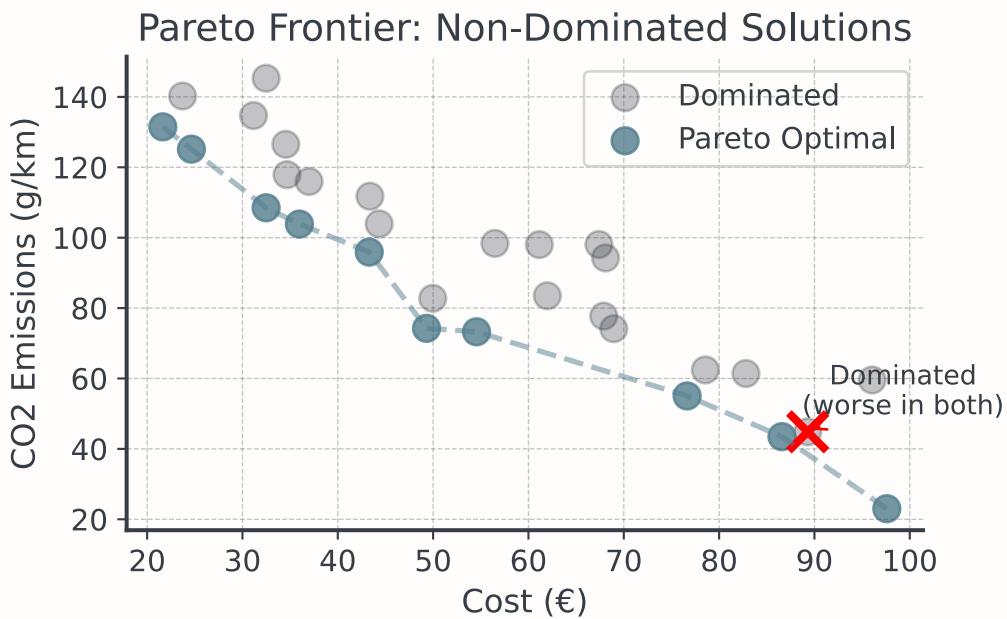
! Important

There is no single “optimal” solution - only trade-offs

Pareto Optimality

Dominated Solutions

A solution is dominated if another solution is:



! Important

Better in at least one objective and not worse in any objective!

The Pareto Frontier

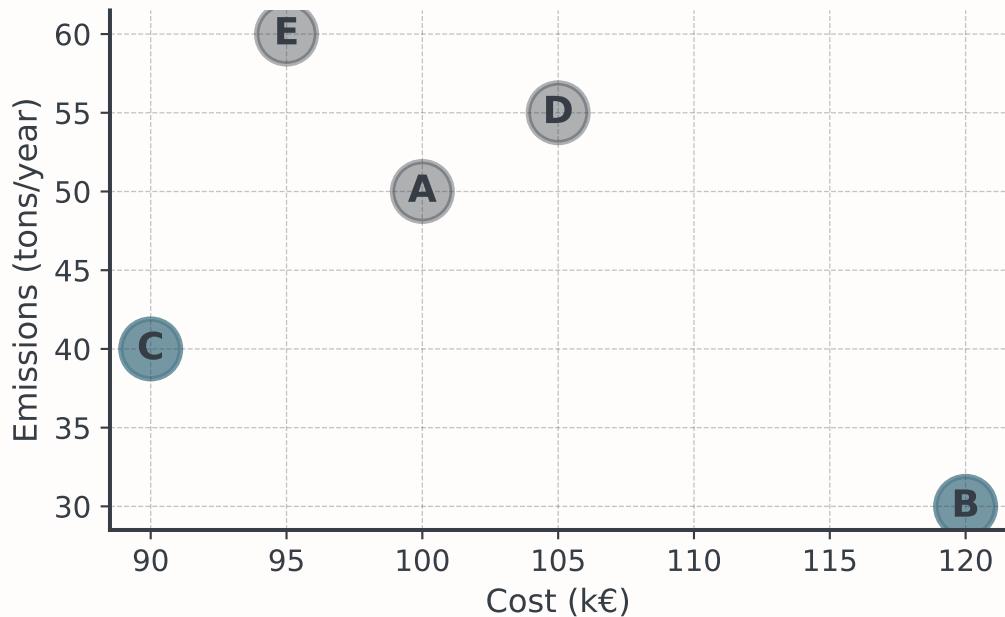
The Pareto frontier is the set of all non-dominated solutions

- No solution is objectively “better”
- Each represents a different trade-off
- Moving along frontier: gain in one objective, loss in another
- Decision makers choose based on preferences

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Question Do you think you get the idea?

Find the Non-Dominated



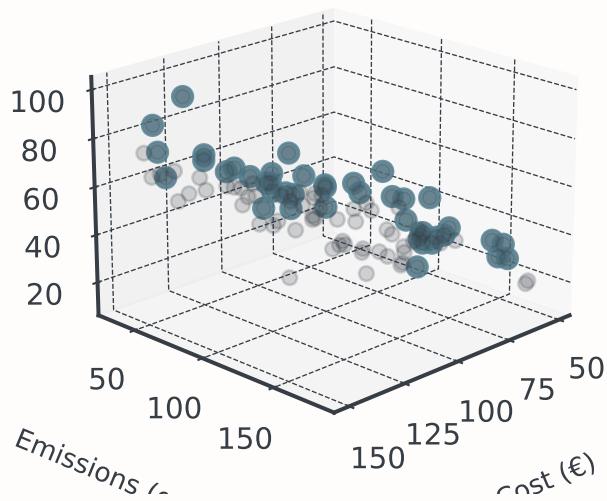
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Question: Which fleets are non-dominated?

Three+ Objectives

With 3 objectives, the Pareto frontier becomes a surface:

3D Pareto Frontier (Surface)



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! Important

Harder to visualize, but same principle applies!

Fleet Composition Problem

The Fleet Challenge

EcoExpress needs to replace their 80 diesel vans

- Must meet EU regulation: Average emissions $\leq 111 \text{ g CO}_2/\text{km}$
- Need capacity for 22,000 parcels/day
- Must balance cost vs. service quality
- 5 vehicle types available, each with trade-offs

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Question: How do we choose the right mix?

Vehicle Options Recap

Type	Purchase Cost (€)	Operat-ing (€/ km)	CO2 (g/ km)	Speed (km/h)	Capacity (parcels)	Reliability
E-Truck	75000	0.18	0	55	300	0.93
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...

Notice: No single vehicle is “best” at everything!

Fleet Composition Framework

This is a discrete selection problem, not continuous allocation

Decision Variables:

- Fleet: How many of each vehicle type? (discrete/integer)
- n_i = number of vehicles of type i (integers!)
- Example: $n_{\text{E-Truck}} = 20$, $n_{\text{Hybrid}} = 30$, etc.

Objective 1: Total Cost

Purchase cost + Operating cost over 3 years

$$\text{Total Cost} = \sum_i n_i \cdot (P_i + O_i \cdot d \cdot y)$$

- n_i = quantity of vehicle type i
- P_i = purchase cost of vehicle type i
- O_i = operating cost per km for type i
- d = daily distance \times days per year
- y = years

Objective 2: Service Score

Composite measure of fleet performance

$$\text{Service Score} = 0.5 \cdot C_{\text{score}} + 0.3 \cdot R_{\text{score}} + 0.2 \cdot S_{\text{score}}$$

- $C_{\text{score}} = \min(1.0, \frac{\text{Total Capacity}}{22000})$ (capacity adequacy)
- $R_{\text{score}} = \frac{\sum n_i \cdot r_i}{\sum n_i}$ (weighted avg. reliability)
- $S_{\text{score}} = \frac{\sum n_i \cdot s_i}{70 \cdot \sum n_i}$ (normalized speed)

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i Note

Service score captures multiple performance dimensions in one metric!

Hard Constraint: Emissions

EU regulation creates a feasibility boundary

$$\text{Average CO}_2 = \frac{\sum_i n_i \cdot e_i}{\sum_i n_i} \leq 111 \text{ g/km}$$

Where e_i = CO₂ emissions per km for vehicle type i

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This eliminates some solutions:

- All diesel vans: 185 g/km > 111
- Mix with too many diesel: Still violates
- Zero-emission + some diesel: Might work

Data Source

Where Do These Numbers Come From?

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Vehicle Specifications:

- Purchase costs: Manufacturer quotes, market research
- Operating costs: Fuel/electricity prices, maintenance records
- Capacity: Vehicle specs (cargo volume, weight limits)

- Reliability: Historical uptime data, manufacturer warranties
- EU Standards: WLTP certification for vehicles
- Electric vehicles: Grid carbon intensity (kWh → g CO₂)

Example Fleet Comparison

Three Fleet Strategies:

	name	cost	service	co2	capacity	vehicles
Cost-Focused	28.9996	0.809705	120.714286		15000	70
Balanced	19.0478	0.731840	33.928571		13250	70
Green-Focused	15.3102	0.695373	0.000000		12750	75

Cost-Focused: ✗ VIOLATES (CO₂: 120.7 g/km)

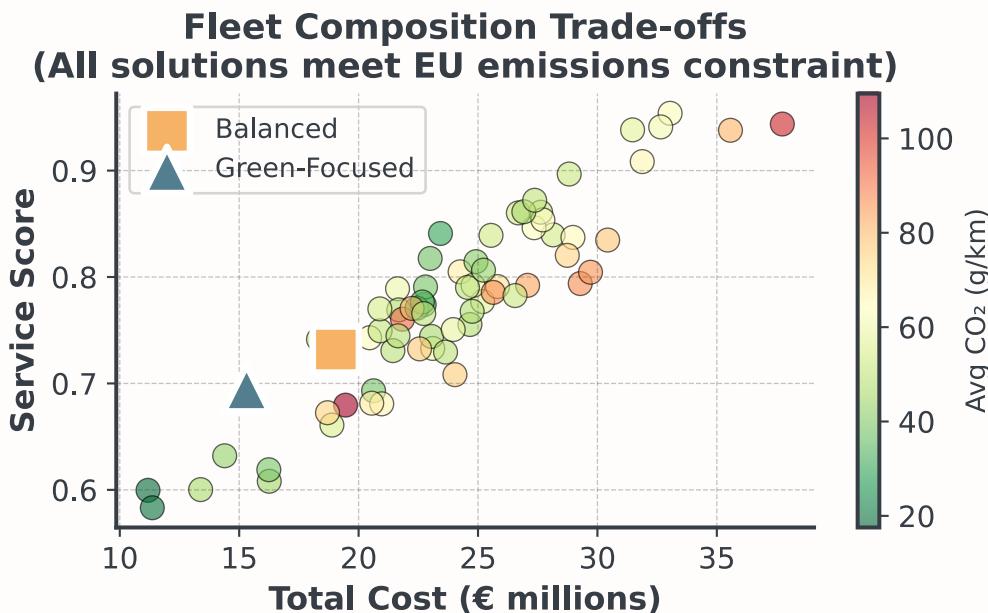
Balanced: ✓ Compliant (CO₂: 33.9 g/km)

Green-Focused: ✓ Compliant (CO₂: 0.0 g/km)

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Question: Which strategy would you choose?

Visualizing Fleet Trade-offs



Generated 68 feasible fleet compositions

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! Important

Each point is a different fleet mix, all meeting emissions constraint!

Solution Approaches

Multi-Objective Optimization

You can use optimization solvers or heuristics!

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With Optimization Solvers

- Weighted Sum Method
- ε -Constraint Method
- Goal Programming
- Optimal solutions
- Need mathematical model

With Heuristics

- Weighted Greedy Construction
- Multi-Objective Local Search
- Metaheuristics
- Good solutions, fast
- No optimality proof

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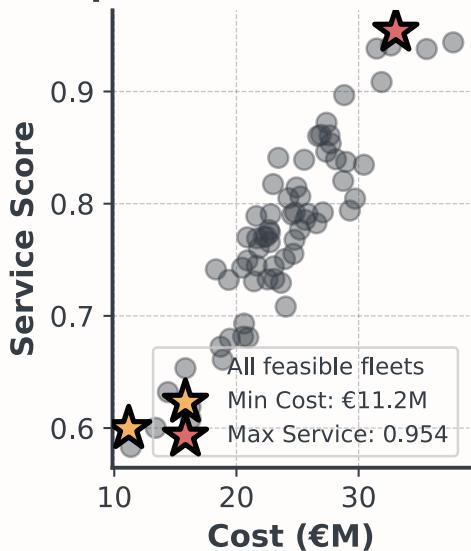
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In this lecture we use heuristic approaches!

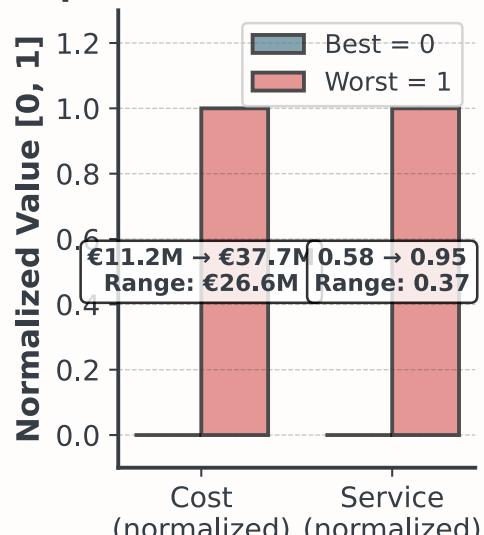
Foundation: Extreme Points

First step for BOTH approaches - find the boundaries:

Step 1: Find Extreme Points



Step 2: Normalize to [0,1] Scale

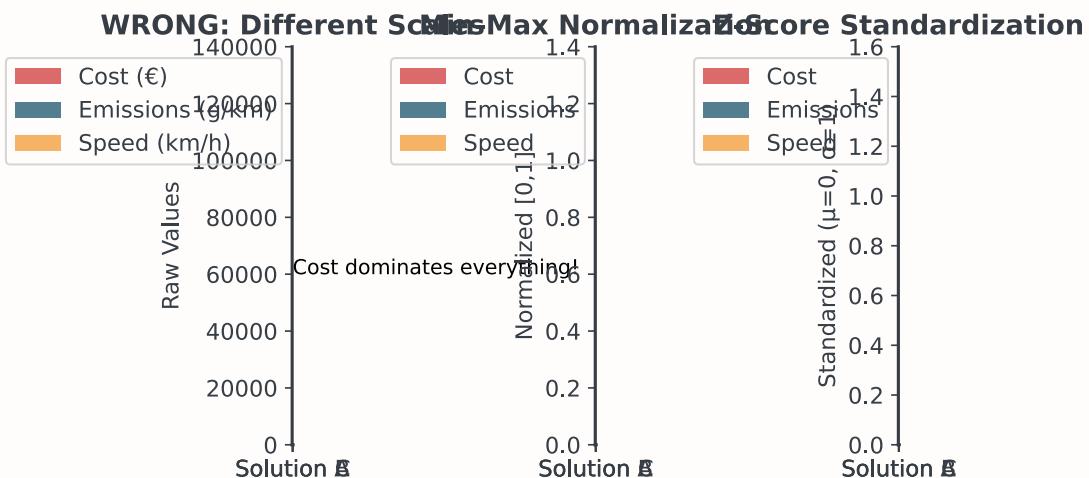


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Question: Why is normalization essential?

Critical: Normalization

Without it, your analysis is meaningless



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Question: Any intuition on how to do [0,1] normalization?

How to Normalize

The Normalization Formula for [0,1]

$$\text{Normalized}_i = \frac{x_i - x_{min}}{x_{max} - x_{min}}$$

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In Python, this is rather simple!

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```
def normalize_objectives(data):
    return (data - data.min()) / (data.max() - data.min())

# Now weights actually mean something
weighted_score = w1 * normalize(cost) + w2 * normalize(emissions)
```

...



Tip

Easy, right?

Extreme Points

There are several reasons why extreme points matter:

1. Trade-off Space: Min/max values bound your Pareto frontier
2. Enable Proper Normalization: Need ranges for scaling to [0,1]
3. Feasibility: If single objectives not achievable, problem infeasible
4. Stakeholder: “Best cost is €50k, best emissions is 40kg”

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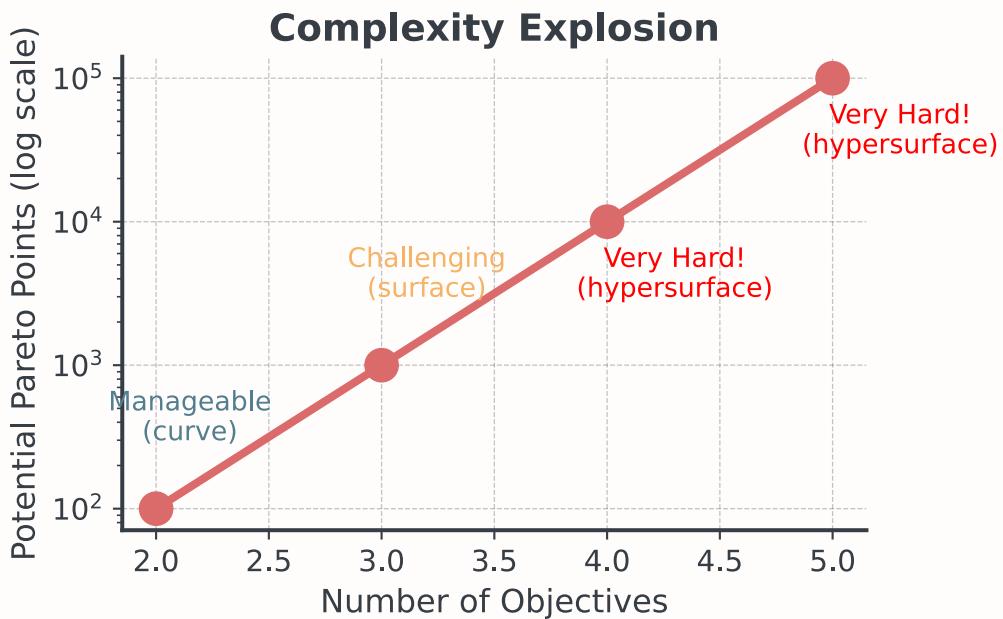
Implementation Pattern:

```
def find_extreme_points(problem):
    # Solve for minimum cost (ignore emissions)
    min_cost_solution = minimize(cost_objective, constraints)
    # Solve for minimum emissions (ignore cost)
    min_emissions_solution = minimize(emissions_objective, constraints)
```

Computational Complexity

How hard does it get with more objectives?

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Tip

Why? Because there are just way more potential solutions to check!

Solver-Based Methods

Quick overview - you won't implement these in assignments

1. Weighted Sum: Minimize $w_1 \times \text{cost} + w_2 \times \text{emissions}$
 - Simple, fast for convex problems
2. ε -Constraint: Minimize cost subject to emissions $\leq \varepsilon$
 - Systematically vary ε to find complete frontier
3. Goal Programming: Minimize deviations from targets
 - Set target for each objective, minimize weighted deviations

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Note

For your fleet optimization: You'll use heuristic approaches instead!

Heuristic Approach

The Heuristic Strategy

For problems without mathematical models

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1. Construction: Build initial solutions with weighted greedy
 2. Improvement: Multi-objective local search
 3. Selection: Filter dominated solutions to find Pareto frontier
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! Important

Key difference from solvers:

- Solvers: Need mathematical model, guarantee optimality
- Heuristics: Work with any evaluation function, find good solutions fast

Why Heuristics?

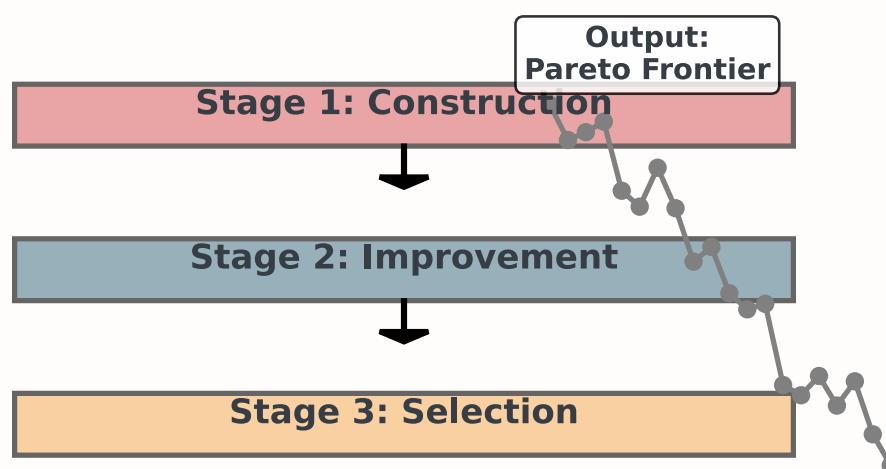
Depending on the problem:

- Combinatorial explosion
 - Huge solution space even for one problem
 - Evaluating one solution might thus take too long
 - Need diverse Pareto frontier, not just one “optimal” solution
 - Open Source Solvers too slow
 - Commercial solvers too expensive
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Question: How do we build good solutions without a solver?

The Three-Stage Heuristic Process

Heuristic Multi-Objective Optimization Workflow



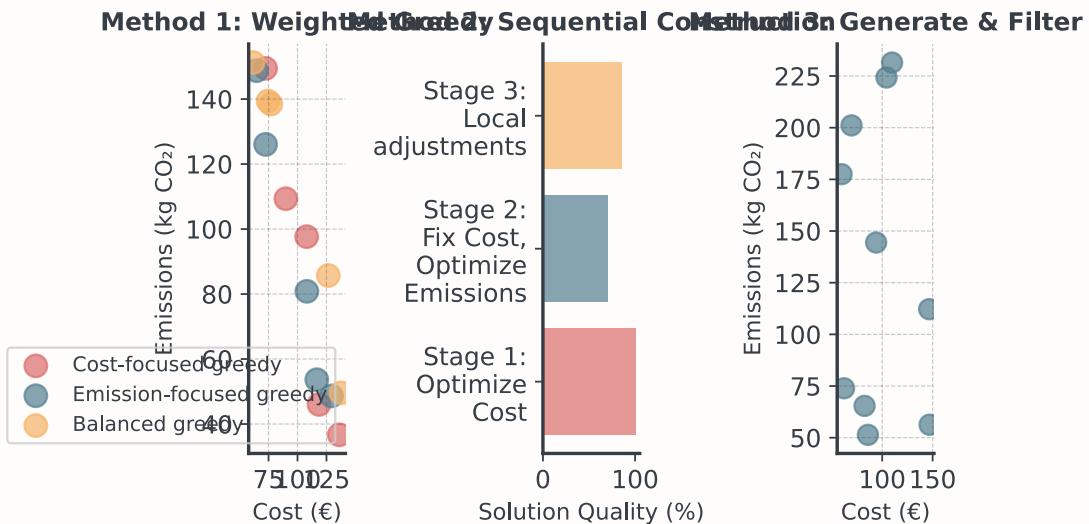
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This is what you'll implement in your assignments!

Construction & Improvement

Construction Methods for MOO

How to build initial solutions when you have multiple objectives?



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Note

Three choices (for starters). Let's check them out!

Weighted Greedy Construction

Making greedy choices on a weighted objective

1. Choose weight vector $w = (w_1, w_2)$
2. At each step, pick the choice that minimizes:

$$w_1 \cdot \text{cost } (x) + w_2 \cdot \text{emissions } (x)$$

3. Build complete solution greedily
4. Repeat with different weights to explore frontier

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Tip

Different weights explore different trade-offs! Easy, right?

Sequential Greedy (Lexicographic)

Optimize one objective at a time, in priority order

1. Rank objectives by priority
 - E.g. cost (most important) and then emissions (tie-breaker)
2. At each step:
 - Find choices that minimize primary objective
 - If tie → use secondary objective
3. Build one working solution

...

💡 Tip

We could also accept primary values within 10% of best so secondary has more influence!

Diverse Starting Pool

Generate many random solutions, keep the non-dominated ones

1. Generate N random solutions (e.g., N=100)
2. Evaluate all solutions on both objectives
3. Filter to keep only non-dominated solutions
4. Result: A diverse set of Pareto-optimal solutions

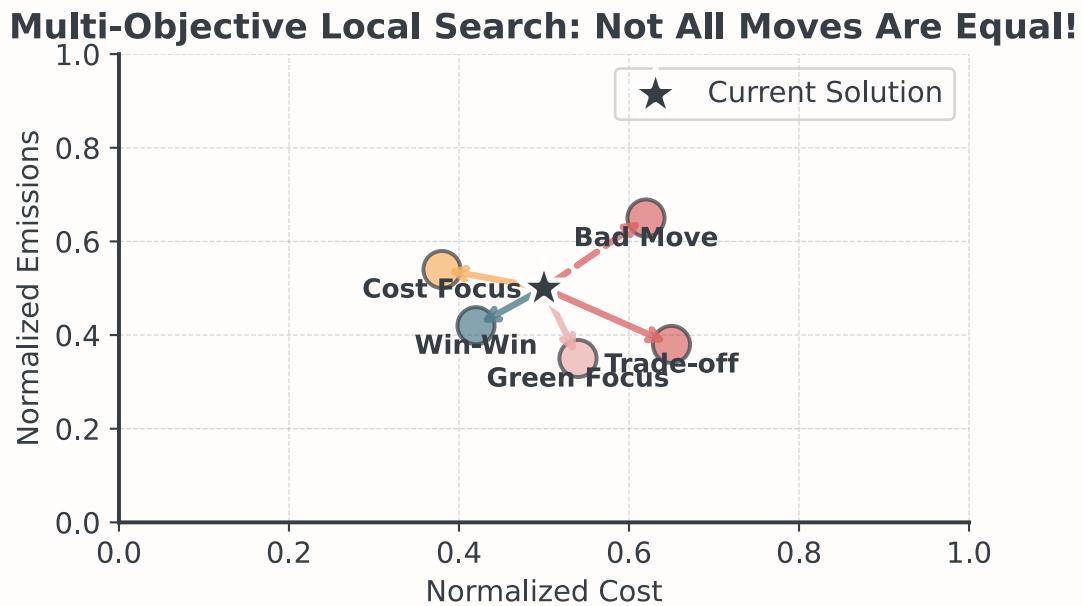
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💡 Tip

- Explores entire solution space
- No bias toward specific weights
- Great for warm-starting local search

Local Search for Multi-Objective

Special moves that improve multiple objectives:



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Question: Which moves are acceptable?

MOO Local Search Rules

Accept a move if:

1. Dominance: New solution dominates current (win-win!)
2. Trade-off: Improves primary, acceptable loss in secondary
3. Probabilistic: Use temperature (like simulated annealing)

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! Important

Always keep all your objectives in mind when making decisions.

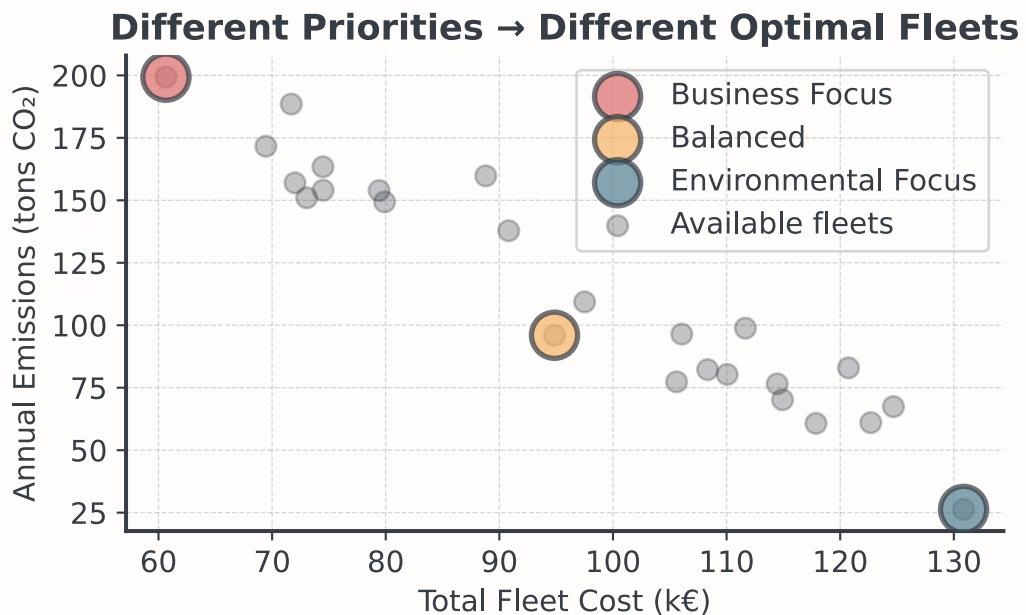
From Pareto Front to Decision

How to Choose!

1. The Knee Point: Find the “elbow” where improvement slows
2. Satisficing Levels: Set minimum acceptable thresholds
 - Cost must be < €100k (budget constraint)
 - Emissions must be < 100 kg (regulatory limit)
 - Service level must be > 90% (customer requirement)
3. Stakeholder Preferences: Let business priorities guide
 - Sustainability: Minimum emissions that meets constraints
 - Operations: Maximum service level within budget

Weighting has an Impact

The weights thus reflect your values!



Tip

Depending on your weight, the choice will vary.

Advanced

Speed vs Sustainability Dilemma

The Three-Way Trade-off in E-Commerce

1. Minimize Delivery Time (1-day/2-hour promise)
2. Minimize Cost (fuel, labor, fulfillment)
3. Minimize Environmental Impact (carbon footprint)

Faster delivery = More vehicles less full = Higher emissions

Question: What could retailers do?

Moving the Frontier

Instead of point on the frontier, move the entire frontier:

Question: Any idea of examples?

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 Tip

R&D can fundamentally change what's possible!

Briefing

Today

Hour 2: This Lecture

- Multi-objective
- Pareto optimality
- Weighted greedy
- Local search MOO

Hour 3: Notebook

- Bean Counter CEO
- Find Pareto frontier
- Apply weighted greedy
- Normalize objectives

Hour 4: Competition

- Fleet composition
- Vehicle selection
- Cost vs service
- Justify choice!

The Competition Challenge

EcoExpress Sustainable Fleet Design

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1. Select optimal fleet mix (5 vehicle types)
2. Balance cost vs. service score
3. Meet EU emission constraint ($\leq 111 \text{ g CO}_2/\text{km}$)
4. Ensure sufficient capacity (22,000 parcels/day)

...

 Important

Find the best trade-off for your business priorities!

Choosing Your MOO Approach

Different situations call for different methods:

Situation	Best	Why
Clear priorities	Sequential greedy	Fast, hierarchy
Exploring	Weighted greedy	Different solutions
Many solutions	Diverse pool	Builds frontier
Quick solution	Single weighted	One good compromise
Improve existing	Multi-objective local	Refines trade-offs

...



Competition? Generate diverse pool or weighted, then improve with local search.

Implementation Pitfalls to Avoid

Common bugs that cost you time:

1. Forgetting to Normalize
 - Always normalize to [0,1] first!
2. Optimizing Too Many Objectives
 - 2-3: Manageable, 4+: Exponentially harder
 - Combine related objectives or use constraints
3. Not Checking Solution Feasibility
 - Always verify constraints after optimization

Summary

Key Takeaways:

- Real decisions have multiple conflicting objectives
- Pareto frontier shows all rational trade-offs
- Normalization is essential for fair comparison
- Weights reflect values, make them explicit
- Visualization crucial for decision-making

Break!

Take 20 minutes, then we start the practice notebook

Next up: You'll become Bean Counter's expert

Then: The Sustainability competition

Bibliography