

# Dealing with Uncertainty

## Lecture 4 - Management Science

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### Introduction

#### Client Briefing: TechVenture Innovation Fund

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CEO's Dilemma:

"We have €2M to invest in 2 of 4 startups. Each promises great returns, but the future is uncertain. How do we make the best choice without just gambling?"

#### Business: Valuing Uncertainty

Question: Why can't we just pick the two startups with the highest average returns?

...

- Hidden Risk: A startup with 30% average return but 50% chance of failure might be worse than 20% return with 5% failure chance
- Portfolio Effects: Two risky startups together might amplify risk beyond acceptable levels
- Tail Events: The worst-case scenario can matter as much as the average case

...

#### Warning

Common Pitfall: Optimizing on averages ignores the distribution of outcomes.

### Real-World Examples

Where uncertainty modeling is critical:

Netflix Series Decisions

- Will a show hit 10M viewers?
- Range: 500K to 50M
- Investment: €20M per season

Pharmaceutical R&D

- Will the drug pass trials?

- Success rate: 10-20%
- Investment: €1B over 10 years

...

### ! Important

When decisions are expensive and outcomes are uncertain, Monte Carlo simulation can be helpful to reduce risk and maximize value!

## Core Concepts

### Rolling the Dice 10,000 Times I

Question: If you roll two dice, what's the probability of getting exactly 7 as result?

...

Method 1: Math

- Count combinations: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)
- Total combinations: 36
- Probability:  $6/36 = 16.67\%$

### Rolling the Dice 10,000 Times II

Question: If you roll two dice, what's the probability of getting exactly 7 as result?

...

Method 2: Simulation

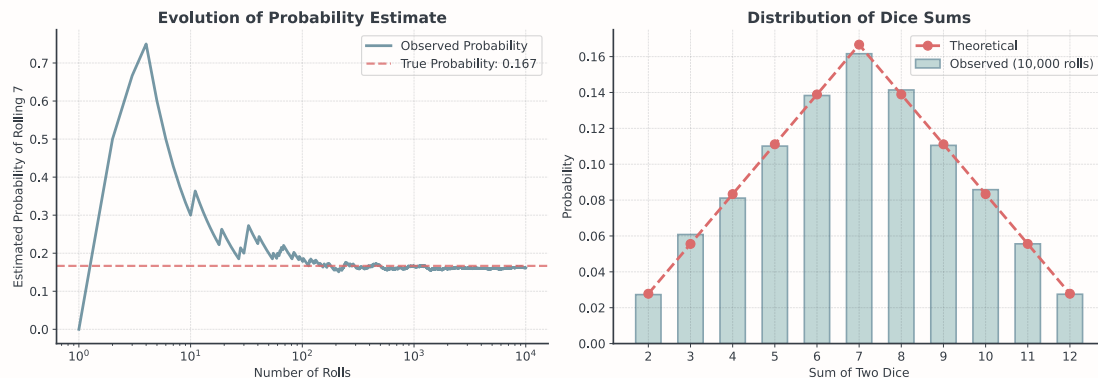
```
import numpy as np
np.random.seed(42)

# Roll two dice 10,000 times
dice1 = np.random.randint(1, 7, size=10_000)
dice2 = np.random.randint(1, 7, size=10_000)
total = dice1 + dice2

# What fraction equals 7?
probability = (total == 7).mean()
print(f"Simulated probability of rolling 7: {probability:.1%}")
```

Simulated probability of rolling 7: 16.2%

## How Probability Converges



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### Tip

As we roll more dice, the estimated probability converges to the true value (16.7%)

## The Law of Large Numbers

Fundamental Principle: As sample size increases, sample average converges to the true expected value

...

If  $X_1, X_2, \dots, X_n$  are independent random samples from the same distribution with mean  $\mu$ :

$$\text{As } n \rightarrow \infty, \quad \bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i \rightarrow \mu$$

...

### Note

This is WHY simulations works. More simulations = better estimates!

## The Central Limit Theorem

Another Fundamental Principle: The sum of many random variables tends toward a normal distribution

...

What it means:

- Even if individual returns are NOT normally distributed...
- The portfolio of many assets WILL be approximately normal
- The average of many simulations WILL be approximately normal

...

#### 💡 Tip

For Business: This is why we can use normal distributions to model portfolio returns, even when individual assets have skewed or unusual distributions!

## Why This Matters for Business

Question: How many simulations do we need for reliable results?

...

```
# Test convergence with different sample sizes
sample_sizes = [10, 100, 1000, 10000, 100000]
estimates = []

for n in sample_sizes:
    dice1 = np.random.randint(1, 7, size=n)
    dice2 = np.random.randint(1, 7, size=n)
    total = dice1 + dice2
    prob = (total == 7).mean()
    estimates.append(prob)
    print(f"n={n:6d}: Estimated probability = {prob:.4f}")
```

```
n=   10: Estimated probability = 0.2000
n=  100: Estimated probability = 0.1900
n= 1000: Estimated probability = 0.1480
n=10000: Estimated probability = 0.1652
n=100000: Estimated probability = 0.1670
```

## Practical Guidelines

How many simulations should you run?

- Quick exploration: 10,000 simulations
  - Good for initial insights, prototyping
- Critical decisions: 100,000+ simulations
  - Financial risk models, regulatory compliance
- When to stop: When more simulations don't change conclusion

...

#### ! Important

If your decision changes with 10x more simulations, you didn't run enough!

# Monte Carlo Method

## The Monte Carlo Method

Three Simple Steps:

1. Model the Uncertainty:
  - Define probability distributions for unknown variables
2. Simulate Many Scenarios:
  - Generate thousands of possible outcomes
3. Analyze the Results:
  - Calculate statistics from the simulation

...

### Note

Monte Carlo Casino in Monaco inspired the method's development in the 1940s.

## Step 1: Model the Uncertainty

Key Function: `np.random.normal(loc, scale, size)`

- loc: The center (mean/average)
- scale: The spread (standard deviation)
- size: How many samples to generate

...

```
# AI-Growth: average 38% return, ±25% volatility
returns = np.random.normal(loc=0.38, scale=0.25, size=10_000)
print(f"Mean return: {returns.mean():.1%}")
print(f"Std deviation: {returns.std():.1%}")
print(f"Minimum: {returns.min():.1%}")
print(f"Maximum: {returns.max():.1%}")
```

```
Mean return: 38.8%
Std deviation: 24.8%
Minimum: -67.0%
Maximum: 125.7%
```

## Expected Returns

Let's calculate percentiles with `np.percentile()`.

...

Question: Do you still know what a percentile is?

...

```
print(f"\nPercentiles:")
print(f" 5th: {np.percentile(returns, 5):.1%} (worst 5% of scenarios)")
print(f" 25th: {np.percentile(returns, 25):.1%} (worst 25% of scenarios)")
print(f" 50th: {np.percentile(returns, 50):.1%} (median)")
print(f" 75th: {np.percentile(returns, 75):.1%} (best 25% of scenarios)")
print(f" 95th: {np.percentile(returns, 95):.1%} (best 5% of scenarios)")
```

```
Percentiles:
 5th: -2.7% (worst 5% of scenarios)
 25th: 22.1% (worst 25% of scenarios)
 50th: 38.9% (median)
 75th: 55.7% (best 25% of scenarios)
 95th: 78.6% (best 5% of scenarios)
```

## Understanding the Distribution

Question: Before we plot, what shape do you expect for `np.random.normal()`?

...



## Risk Analysis

Question: What's the probability that AI-Growth loses money?

...

```
# Calculate risk metrics
prob_loss = (returns < 0).mean() # proportion of returns that are less than zero
prob_double = (returns > 1.0).mean() # proportion greater than 100%
```

```
print(f"Probability of loss: {prob_loss:.1%}")
print(f"Probability of doubling money: {prob_double:.1%}")
```

Probability of loss: 6.0%  
Probability of doubling money: 0.8%

...

### ! Important

With 6 % chance of loss, AI-Growth is relatively safe. Easy for one startup, right?

## Different Distributions

Attention: Not everything follows a normal distribution!



## Overview

### Normal

```
# Most common in nature/business
# Bell-shaped, symmetric
returns = np.random.normal(mean, std, size)

# Example: CloudAI startup returns
cloudai = np.random.normal(0.25, 0.15, 10000) # 25% ± 15%
```

#### Main Characteristics:

- Symmetric bell curve
- Most values cluster around mean

- Common in nature and business

## Uniform

```
# Equal probability across range
# Example: FinFlow returns between 10-35%
returns = np.random.uniform(0.10, 0.35, size)

# Example: FinFlow startup returns
finflow = np.random.uniform(0.10, 0.35, 10000) # 10-35% equally likely
```

Main Characteristics:

- All values equally likely
- Hard boundaries (min/max)
- Good for modeling complete uncertainty within range

## Exponential

```
# Time between events
# Example: Customer arrivals, equipment failure
times = np.random.exponential(scale, size)

# Example: Time between customer arrivals (minutes)
arrivals = np.random.exponential(5, 10000) # Average 5 minutes
```

Main Characteristics:

- Many small values, few large ones
- Always positive
- Common for waiting times and rare events

## Portfolios

### Combining Investments

Suppose we have the following startups:

CloudAI, GreenGrid, HealthTrack, FinFlow

...

Question: If we must pick 2 of 4, how many unique pairs exist?

...

The Math:

$$\binom{4}{2} = \frac{4!}{2! \times 2!} = \frac{4 \times 3 \times 2 \times 1}{(2 \times 1) \times (2 \times 1)} = \frac{24}{4} = 6$$

...





Tip

Each combination has different risk-return characteristics!

## Four Startup Profiles



...

Question: Which startup is the best choice?

## Key Metrics for Decision Making

Question: Which metrics matter most for investment decisions?

- Expected Return: Average outcome across all scenarios
- Volatility (Risk): Standard deviation of returns
- Probability of Loss: How often do we lose money?
- Upside Potential: Chance of exceptional returns (>50%)
- Tail Risk: What happens in the worst 10% of cases?

...

### ! Important

No metric tells the whole story. Investors consider multiple dimensions of risk and return.

## Understanding Tail Risk

Tail Risk: The danger lurking in worst-case scenarios

### Expected Shortfall (ES)

- Average loss in worst X% of cases
- Goes beyond simple probability
- Measures depth of potential losses
- Critical for risk management

...

#### Warning

A portfolio with higher average returns might have catastrophic tail risk. Always look at the extremes!

## Correlation & Dependence

### The Independence Assumption

So far, we've assumed startups succeed or fail independently.

...

Independent Events:

- CloudAI's success doesn't affect GreenGrid's success
- Each startup faces separate, unrelated risks
- Portfolio risk = Average of individual risks

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Question: Is this realistic in the real world?

...

#### Warning

Reality Check: Many business risks are correlated! Economic downturns, market trends, and technology shifts affect multiple companies simultaneously.

### What is Correlation?

Correlation measures how two variables move together.

$$\rho_{X,Y} = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y} \quad \text{where } -1 \leq \rho \leq 1$$

...

Interpreting Correlation:

- $\rho = +1$ : Perfect positive correlation (move together)
- $\rho = 0$ : No correlation (independent)

- $\rho = -1$ : Perfect negative correlation (move opposite)

## Correlation in Practice



...

### 💡 Tip

In Python: `np.corrcoef(returns1, returns2)` calculates correlation

## Why Correlation Matters

Two AI startups in your portfolio:

Scenario 1: Independent ( $\rho = 0$ )

- One fails due to technical issues, other succeeds
- Risk is averaged out

Scenario 2: Positively Correlated ( $\rho = 0.8$ )

- Both rely on same AI infrastructure provider - risk is amplified!

...

### ⚠ Warning

Diversification only reduces risk when investments are not highly correlated!

## Impact on Portfolio Risk



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### ! Important

Higher correlation = Wider distribution = More risk!

## Real-World Correlation Examples

Common sources of correlation in business:

- Industry-specific: All tech startups affected by downturn
- Geographic: All European companies affected by EU regulations
- Supply chain: Multiple companies relying on same supplier
- Macroeconomic: Interest rates, inflation affect most businesses

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### 💡 Tip

Diversification: Choose investments with LOW correlation to reduce portfolio risk!

## When Diversification Fails



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### Warning

2008 Financial Crisis: Many “diversified” portfolios collapsed due to correlations!

## When Monte Carlo?

### When to Use Monte Carlo

Question: For our simple startup examples so far, do we really NEED Monte Carlo?

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Short answer: No! For basic mean/variance calculations, we can use analytical formulas.

...

So when is Monte Carlo truly necessary?

### Necessary vs. Convenient

You can use math (analytical solutions):

- Simple distributions: Mean and variance of normal distributions
- Linear combinations: Portfolio of independent assets

...

You NEED Monte Carlo when:

- Complex dependencies: Nonlinear relationships
- Path-dependent problems: Outcome depends on a sequence
- No closed-form solution: The math is intractable

## Real Monte Carlo Applications

Where Monte Carlo is ESSENTIAL, not just convenient:

### 1. Option Pricing with Path Dependencies

```
# Payoff depends on AVERAGE price over time
n_simulations = 10000; strike_price = 105; payoffs = []
for sim in range(n_simulations):
    prices = [100] # Starting price
    for day in range(365):
        prices.append(prices[-1] * (1 + np.random.normal(0.001, 0.02)))
    payoff = max(0, np.mean(prices) - strike_price)
    payoffs.append(payoff)
payoffs = np.array(payoffs)
print(f"Average option value: ${payoffs.mean():.2f}")
print(f"Probability of profit: {(payoffs > 0).mean():.1%}")
```

Average option value: \$19.58  
Probability of profit: 68.9%

## Real Monte Carlo Applications II



## Real Monte Carlo Applications III

### 2. Supply Chain with Cascading Effects

```
# Each stage affects the next (nonlinear dependencies)
n_simulations = 10000; factory_capacity = 150; demand = 120; price = 5
penalty = 10; revenues = []
```

```

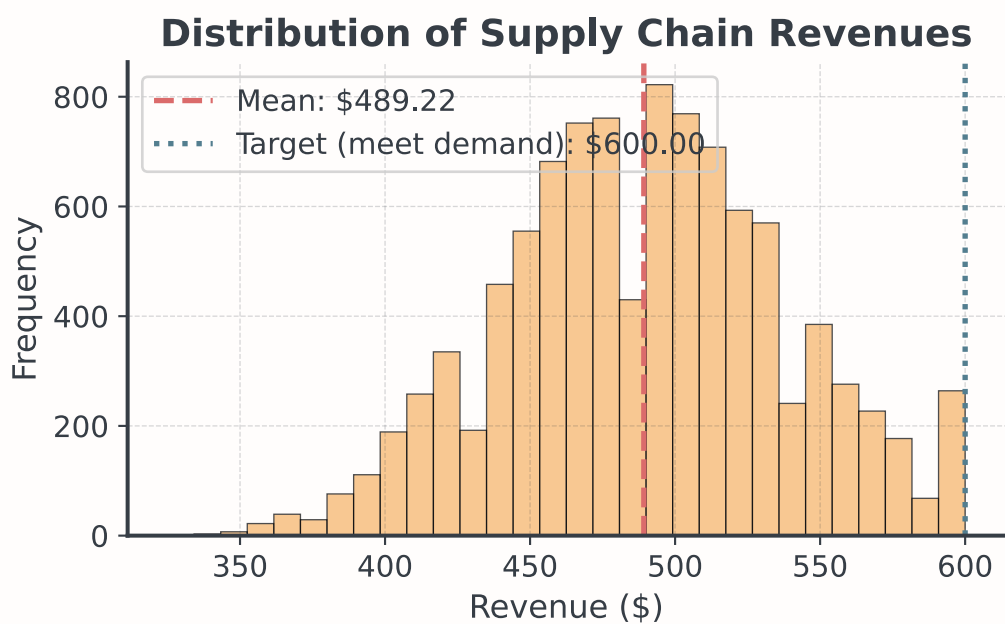
for sim in range(n_simulations):
    parts_delivered = np.random.poisson(100)
    production = min(parts_delivered, factory_capacity)
    if production >= demand:
        revenue = demand * price
    else:
        revenue = production * price - penalty
    revenues.append(revenue)

revenues = np.array(revenues)
print(f"Average revenue: ${revenues.mean():.2f}")
print(f"Worst-case revenue: ${revenues.min():.2f}")

```

Average revenue: \$489.22  
Worst-case revenue: \$325.00

## Real Monte Carlo Applications IV



## Real Monte Carlo Applications V

### 3. Project Management with Sequential Risks

```

# Project phases must happen in order, later phases only if earlier succeed
n_simulations = 10000; shape = 2; scale = 10; project_times = []

for sim in range(n_simulations):
    total_time = 0
    for phase in ['design', 'build', 'test', 'deploy']:
        # Each phase has uncertain duration
        phase_time = np.random.gamma(shape, scale)

```

```

total_time += phase_time
if np.random.random() < 0.1:
    total_time += phase_time * 0.5 # Rework time
project_times.append(total_time)

project_times = np.array(project_times)
print(f"Average project duration: {project_times.mean():.1f} days")
print(f"90th percentile: {np.percentile(project_times, 90):.1f} days")

```

Average project duration: 84.3 days  
90th percentile: 125.2 days

## Real Monte Carlo Applications VI



### Example: Why Still Useful?

Question: If we can calculate variance analytically, why simulate?

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- Visualization: Seeing the full distribution is intuitive
- Flexibility: When problems become complex, simulation adapts
- Extensions: Practice for complex problem

...

#### 💡 Tip

Think of our examples as learning the tool on simple problems so you can solve complex ones where Monte Carlo is required!



## When NOT to Use Monte Carlo

Even when you CAN use Monte Carlo, sometimes you shouldn't:

...

- Simple analytical solution exists and suffices
  - Use math directly: faster, more precise, easier to understand
- Can't reasonably estimate input distributions
  - Garbage in = garbage out; need solid basis for assumptions
- Problem is deterministic (no uncertainty)
  - Simulation adds complexity without value

...

### Warning

Simulation is a tool for managing uncertainty, not creating false precision!

## Making Smart Decisions

### Decision Framework

1. Define Your Risk Tolerance
  - Can you afford to lose money and what's your time horizon?
  - Are you risk-averse or risk-seeking?
2. Evaluate Multiple Metrics
  - Don't just maximize returns, consider volatility and risk
  - Look at probability of achieving goals
3. Scenario Test
  - What if distributions change or a company fails?

### The Plan for the Day

Hour 1:

Lecture

- Concepts
- Examples
- Visualization

Hour 2:

Practice Notebook

- Simulation
- Hands-on coding
- Build your skills

Hours 3-4:

## Competition

- TechVenture
- Team collaboration
- €2M investment

...

Remember: The lecture gives you concepts. The notebook gives you practice. The competition tests your skills!

## Hour 2: Simulation

### Your Practice Case: Bean Counter Expansion

- Model uncertain variables (customers, spending)
- Combine multiple uncertainties
- Calculate business metrics (VaR, profit probability)
- Make data-driven recommendations

## Hours 3-4: The Challenge

### TechVenture Investment Competition

- Your Budget: €2 million
- Your Choice: Pick 2 of 4 startups
- Your Goal: Maximize risk-adjusted returns
- Your Deliverable: One-slide recommendation + 3-minute pitch

...

Consider multiple risk metrics and prepare a clear justification!

...

Prizes: 10 / 6 / 3 bonus points for top three teams!

## Key Takeaways

### What You've Learned Today

#### Concepts

- Monte Carlo simulation
- Probability distributions
- Risk has multiple dimensions
- Expected Value vs. Variance
- Correlation and dependence

#### Skills

- Using `np.random` for simulation
- Calculating risk metrics
- Visualizing uncertainty

- Comparing portfolios
- Understanding correlation

...

#### Warning

Monte Carlo doesn't predict THE future - it shows possible futures! And correlation can amplify or reduce risk!

## Next Week

### Forecasting the Future

- Moving from simulation to prediction
- Time series analysis
- Trend and seasonality detection
- Measuring forecast accuracy

...

Now, short break and then we start coding!

## Bibliography