Multi-Objective Optimization

Lecture 8 - Management Science

Dr. Tobias Vlćek

Introduction

Client Briefing: EcoExpress Logistics

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Operations Director's Dilemma:

"EU regulations demand 40% emission cuts, but we can't sacrifice profitability, service quality, or reliability!"

The Fleet Challenge

EcoExpress operates regional last-mile delivery across 3 cities

- EU Green Deal: 40% emission reduction by 2025
- Rising fuel costs (€2.1/L diesel)
- Amazon entering our market (speed pressure)
- Driver shortage (need automation-friendly vehicles)

. . .

Question: How do we transform our fleet while staying competitive?

Today's Learning Objectives

By the end of this lecture, you will be able to:

1. Explain why most real business decisions involve multiple competing objectives
2. Identify and visualize Pareto optimal solutions in multi-objective problems
3. Apply normalization techniques to make objectives comparable
4. Implement weighted sum and ε-constraint methods to find trade-off solutions
5. Choose the appropriate MOO method for different problem types
6. Make data-driven decisions from a Pareto frontier
7. Analyze real-world multi-objective trade-offs (Amazon, airlines, Tesla)

Quick Recap: Local Search

Last week we optimized routes for delivery:

- Started with greedy construction (e.g. Nearest Neighbor)
- Improved with local search (e.g. 2-opt)

- Considered time windows
- But: We only optimized distance

. . .

Question: What if we also care about emissions, cost, AND customer satisfaction?

The Problem

Single vs Multi-Objective

Single Objective

- “Minimize total distance”
- Clear winner. Easy, right!

. . .

Multiple Objectives

- “Minimize cost AND emissions AND maximize speed”
- No clear answer…

. . .

Question: Any idea how to approach this?

EcoExpress Vehicle Options

Type	Purchase Cost (€)	Operating (€/km)	CO2 (g/km)	Speed (km/h)	Capacity (parcels)	Range (km)
E-Truck	75000	0.18	0	55	300	250
Hybrid Van	45000	0.25	95	65	200	600
Diesel Van	35000	0.38	185	70	250	800
E-Cargo Bike	12000	0.05	0	30	50	60
Autonomous	95000	0.12	0	40	150	180

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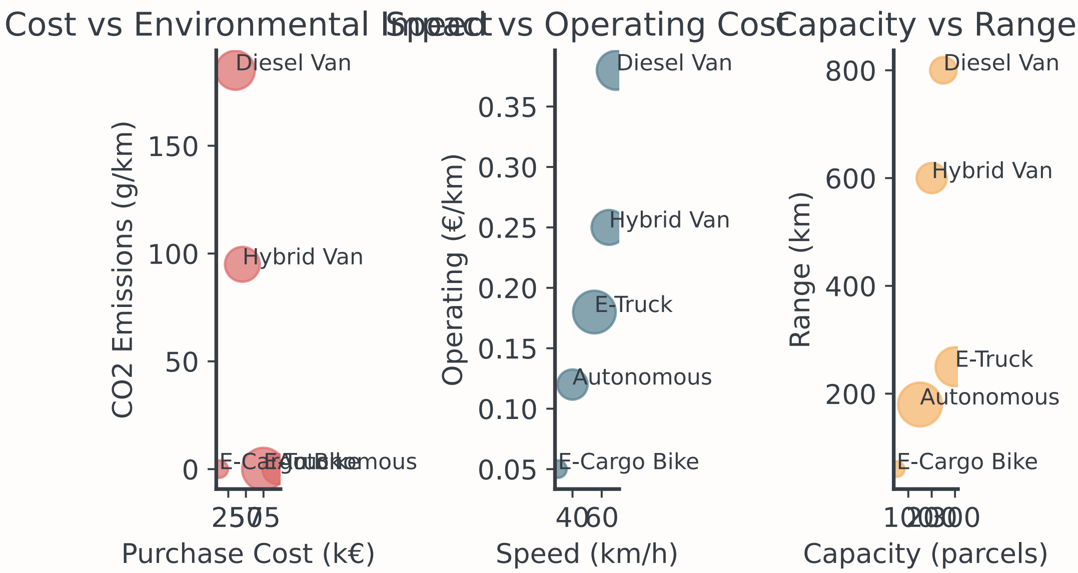
Question: Which vehicle is “best” for EcoExpress?

Trade-offs Everywhere



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> **! Important**
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> Every vehicle excels at something different!

Real Business Constraints

Beyond the numbers, consider:

- EU regulations: Carbon tax of €100/ton CO_2 starting 2025
- Competition: Amazon promises 2-hour delivery
- Labor market: Autonomous vehicles reduce driver dependency
- Urban zones: Zero-emission zones in city centers
- Peak times: Black Friday = 3x normal volume

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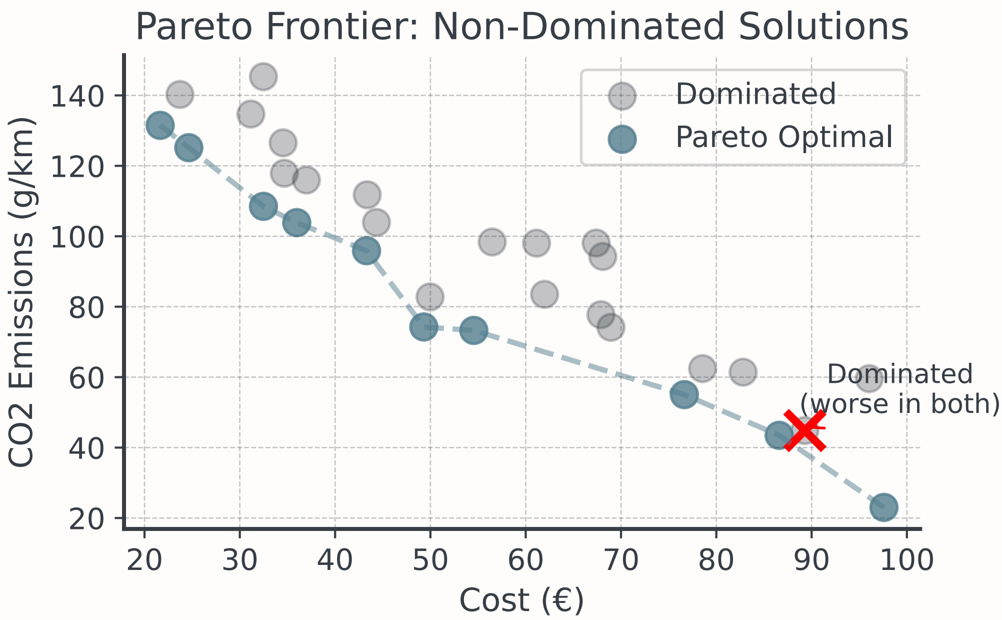
> **! Important**
>
> There is no single "optimal" solution - only trade-offs

Pareto Optimality

Dominated Solutions

A solution is dominated if another solution is:



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> **! Important**
>
> Better in at least one objective and not worse in any objective!

The Pareto Frontier

The Pareto frontier is the set of all non-dominated solutions

- No solution is objectively “better”
- Each represents a different trade-off
- Moving along frontier: gain in one objective, loss in another
- Decision makers choose based on preferences

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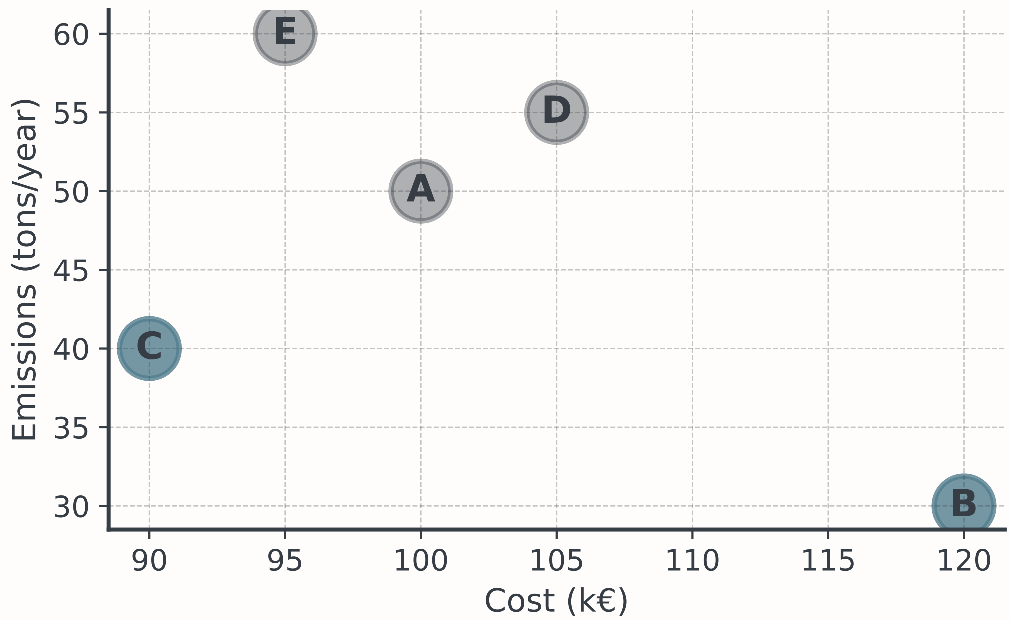
Question Do you think you get the idea?

Find the Non-Dominated



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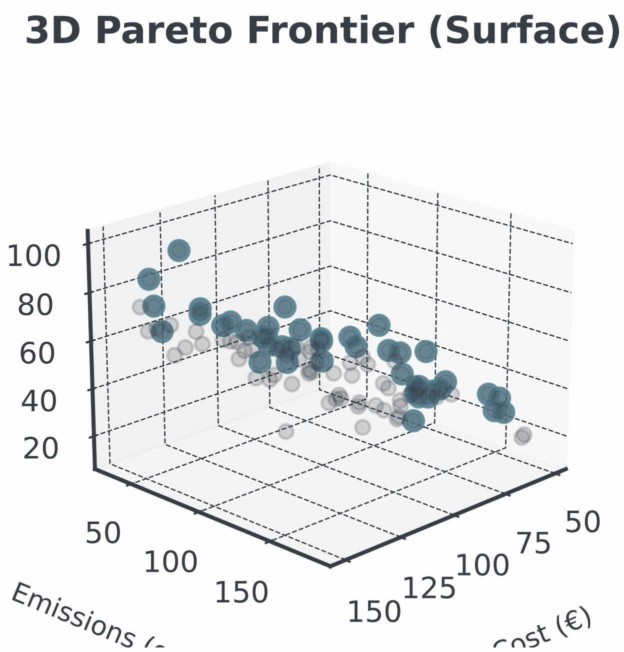
Question: Which fleets are non-dominated?

Three+ Objectives

With 3 objectives, the Pareto frontier becomes a surface:



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> ! Important
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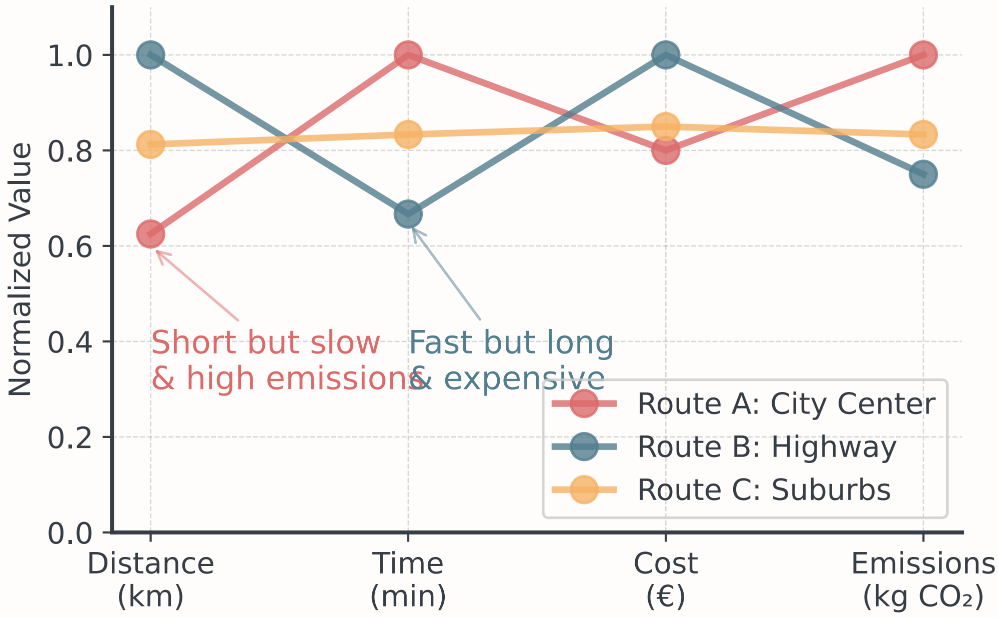
> Harder to visualize, but same principle applies!

Transportation Problem

Multi-Objective Transportation



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> 💡 Tip
>
> Logistics decisions involve trade-offs: City traffic (slow, high emissions), Night delivery? → Highway (fast, but more distance), Customer priority? → Direct route (expensive)

Classic Transportation

Let's understand the foundation

. . .

From/To	Zone A	Zone B	Zone C	Zone D	Supply
DC Berlin	8	10	11	14	300
DC Hamburg	12	9	7	8	250
DC Munich	15	13	10	9	200

From/To	Zone A	Zone B	Zone C	Zone D	Supply
Demand	200	180	220	150	750

ℹ Note

Cost per 100 parcels (€) in the middle of the table!

Objective Function

The foundation of the classical model

$$\text{Minimize} \quad Z = \sum_i \sum_j c_{ij} \cdot x_{ij}$$

Where:

- x_{ij} = parcels shipped from DC i to Zone j
- c_{ij} = cost per 100 parcels from i to j

Second Objective: Emissions

Now the real challenge - each route has different emissions:

From/To	Zone A	Zone B	Zone C	Zone D
DC Berlin	120	95	85	70
DC Hamburg	45	110	100	90
DC Munich	60	50	115	105

ℹ Note

Emissions per parcel (g CO_2) in the table

Objective Function II

Now with two objectives!

$$\text{Minimize} \quad Z_1 = \sum_i \sum_j c_{ij} \cdot x_{ij} \quad \text{(Cost)}$$

$$\text{Minimize} \quad Z_2 = \sum_i \sum_j e_{ij} \cdot x_{ij} \quad \text{(Emissions)}$$

Where e_{ij} = emissions per parcel from i to j

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Notice: Cheapest routes ≠ Greenest routes!

Data Source

Where Do These Numbers Come From?

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Cost Data:

- Historical records: Your accounting system
- Quotes: Request from carriers/suppliers
- APIs: Google Maps Distance Matrix (distance → cost)

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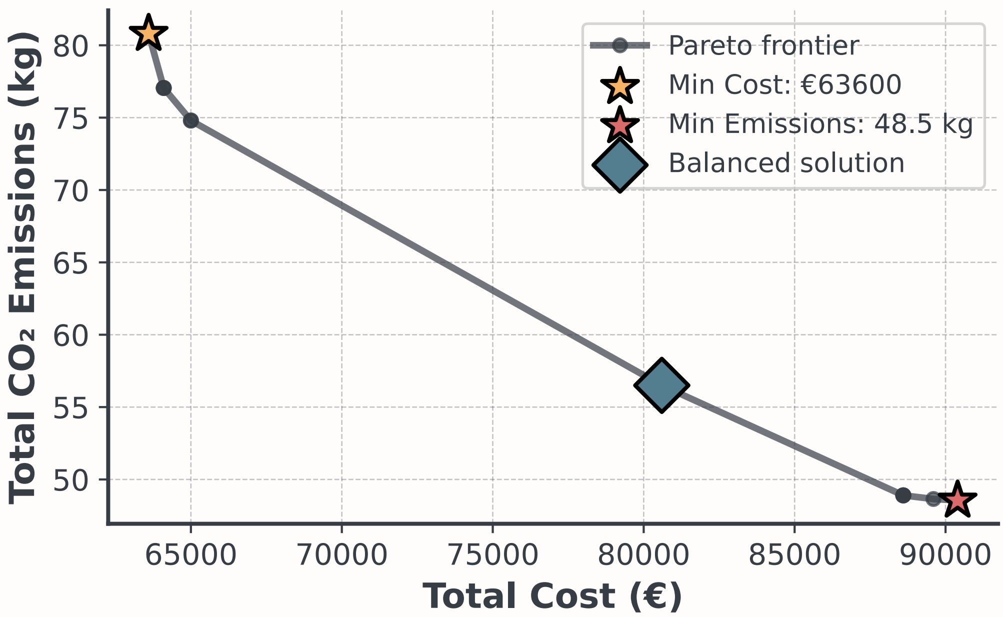
Emissions Data:

- EU Standards or Carrier data
- Formula: `Emissions = Distance × Weight × EmissionFactor`

The Transportation Trade-off




```
Cost increase for greenest solution: +€26800 (42.1%)
Emissions reduction from cheapest: -32.2 kg (39.9%)
```

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> ! Important
>
> Each point represents a different allocation strategy!

Solution Approaches

Multi-Objective Optimization

You can use optimization solvers or heuristics!

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With Optimization Solvers

- Weighted Sum Method
- ε-Constraint Method
- Goal Programming
- Optimal solutions
- Need mathematical model

With Heuristics

- Weighted Greedy Construction
- Multi-Objective Local Search
- Metaheuristics
- Good solutions, fast
- No optimality proof

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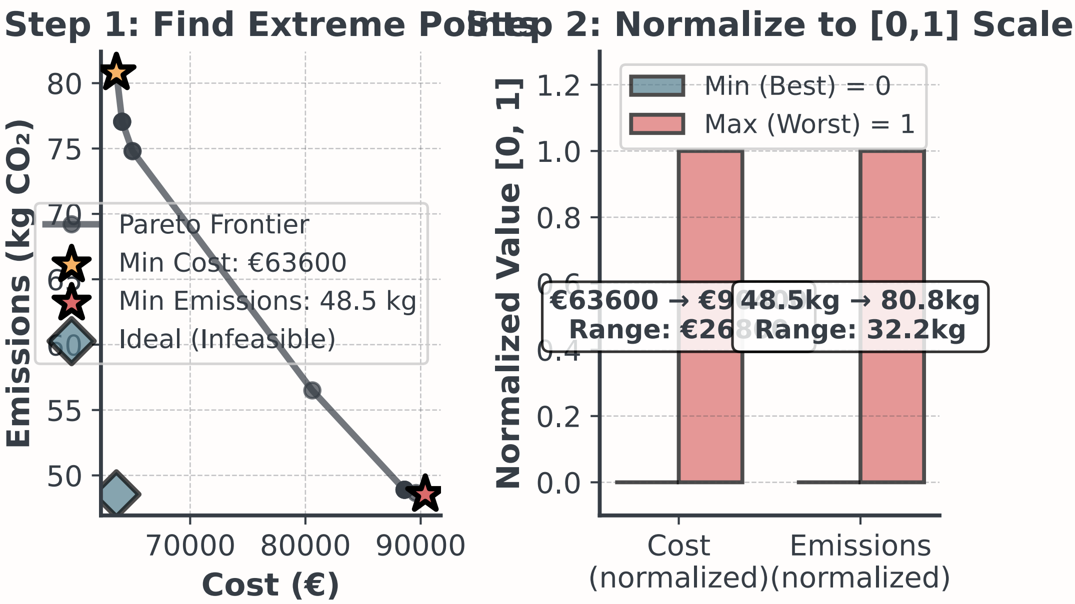
! Important

In this lecture we use heuristic approaches!

Foundation: Extreme Points

First step for BOTH approaches - find the boundaries:



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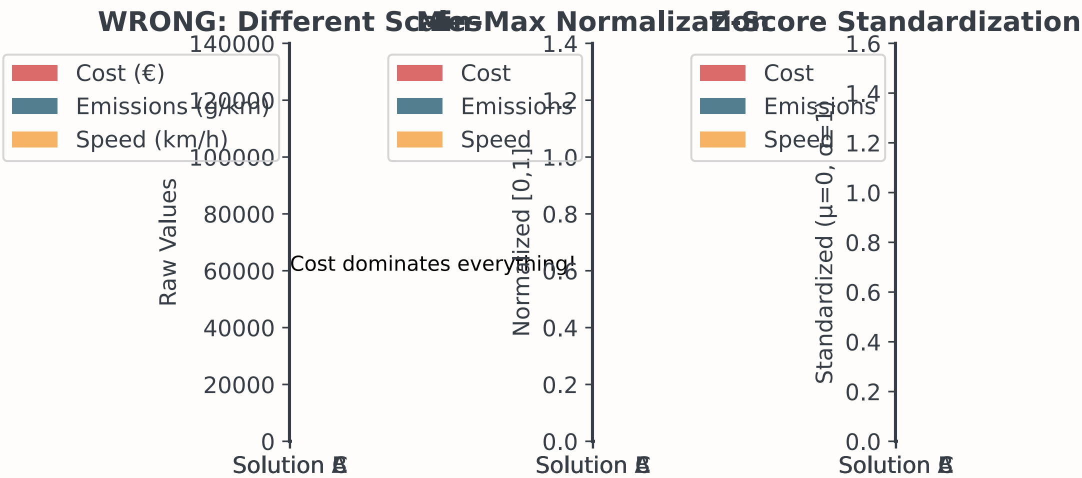
Question: Why is normalization essential?

Critical: Normalization

Without it, your analysis is meaningless



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Question: Any intuition on how to do [0,1] normalization?

How to Normalize

The Normalization Formula for [0,1]

$$\text{Normalized}_i = \frac{x_i - x_{min}}{x_{max} - x_{min}}$$

. . .

In Python, this is rather simple!

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```python
def normalize_objectives(data):
    return (data - data.min()) / (data.max() - data.min())

# Now weights actually mean something
weighted_score = w1 * normalize(cost) + w2 * normalize(emissions)
```

. . .

Tip

Easy, right?

Extreme Points

There are several reasons why extreme points matter:

1. Trade-off Space: Min/max values bound your Pareto frontier
2. Enable Proper Normalization: Need ranges for scaling to [0,1]
3. Feasibility: If single objectives not achievable, problem infeasible
4. Stakeholder: "Best cost is €50k, best emissions is 40kg"

. . .

Implementation Pattern:

```python
def find_extreme_points(problem):
    # Solve for minimum cost (ignore emissions)
    min_cost_solution = minimize(cost_objective, constraints)
    # Solve for minimum emissions (ignore cost)
    min_emissions_solution = minimize(emissions_objective, constraints)
```

Computational Complexity

How hard does it get with more objectives?

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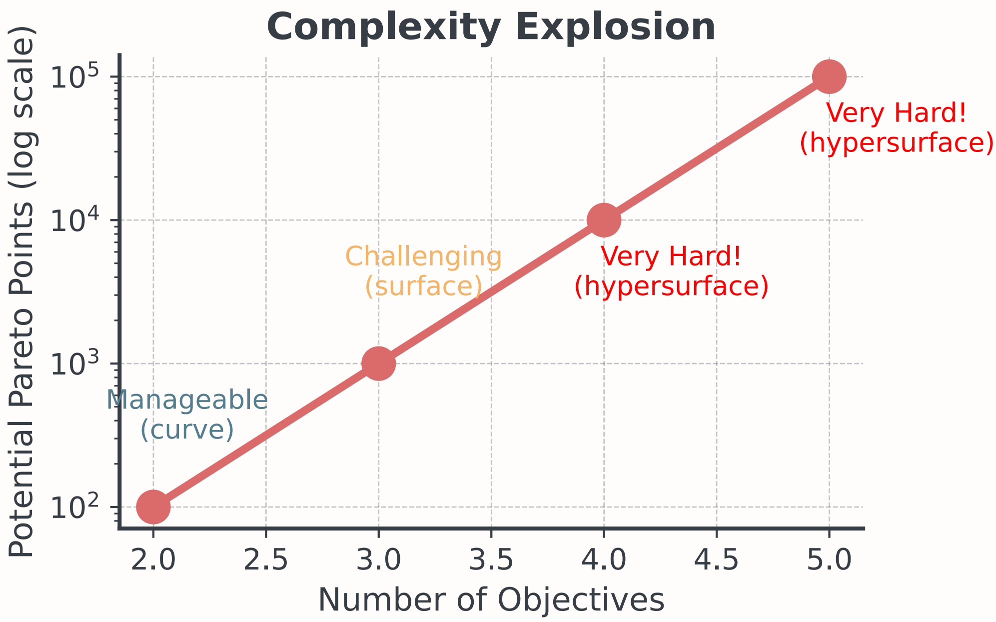

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Tip

Why? Because there are just way more potential solutions to check!

Solver-Based Methods

Quick overview - you won't implement these in assignments

1. Weighted Sum: Minimize $w_1 \times \text{cost} + w_2 \times \text{emissions}$
   - Simple, fast for convex problems
2. ε-Constraint: Minimize cost subject to emissions $\leq \varepsilon$
   - Systematically vary ε to find complete frontier
3. Goal Programming: Minimize deviations from targets
   - Set target for each objective, minimize weighted deviations

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Note

For your fleet optimization: You'll use heuristic approaches instead!

Heuristic Approach

The Heuristic Strategy

For problems without mathematical models

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1. Construction: Build initial solutions with weighted greedy
2. Improvement: Multi-objective local search
3. Selection: Filter dominated solutions to find Pareto frontier

. . .

> ! Important
>
> Key difference from solvers:
>
> - Solvers: Need mathematical model, guarantee optimality
> - Heuristics: Work with any evaluation function, find good solutions fast

Why Heuristics?

Depending on the problem:

- Combinatorial explosion
- Huge solution space even for one problem
- Evaluating one solution might thus take too long
- Need diverse Pareto frontier, not just one "optimal" solution
- Open Source Solvers too slow
- Commercial solvers too expensive

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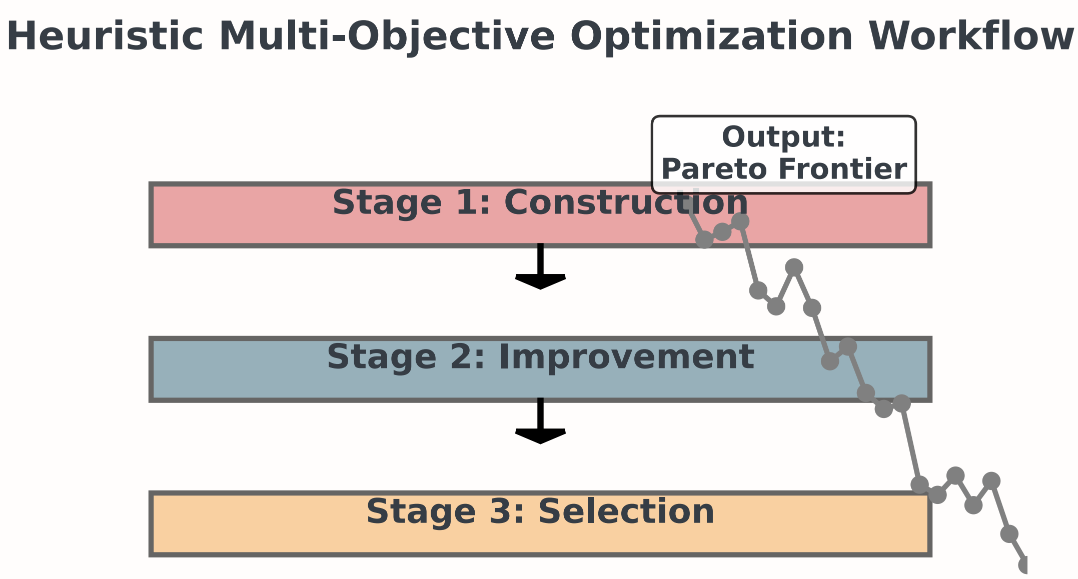
Question: How do we build good solutions without a solver?

The Three-Stage Heuristic Process



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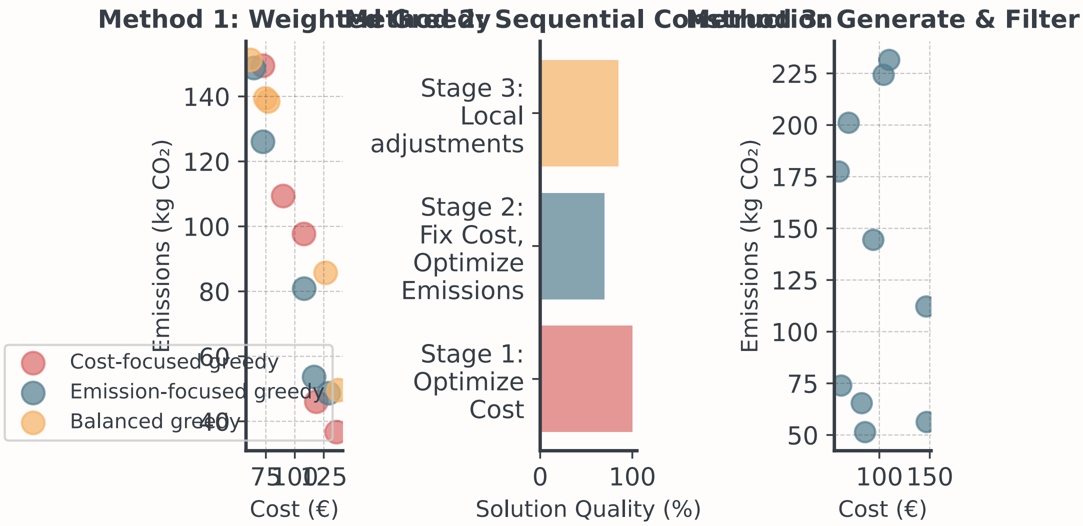
This is what you'll implement in your assignments!

Construction & Improvement

Construction Methods for MOO

How to build initial solutions when you have multiple objectives?



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> **Note**
>
> Three choices (for starters). Let's check them out!

Weighted Greedy Construction

Making greedy choices on a weighted objective

1. Choose weight vector $w = (w_1, w_2)$
2. At each step, pick the choice that minimizes:

$$w_1 \cdot \text{cost}\ (x) + w_2 \cdot \text{emissions}\ (x)$$

3. Build complete solution greedily
4. Repeat with different weights to explore frontier

. . .

> **Tip**
>
> Different weights explore different trade-offs! Easy, right?

Sequential Greedy (Lexicographic)

Optimize one objective at a time, in priority order

1. Rank objectives by priority
   - E.g. cost (most important) and then emissions (tie-breaker)
2. At each step:
   - Find choices that minimize primary objective
   - If tie $\longrightarrow$ use secondary objective
3. Build one working solution

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> Tip
>
> We could also accept primary values within 10% of best so secondary has more influence!

Diverse Starting Pool

Generate many random solutions, keep the non-dominated ones

1. Generate N random solutions (e.g., N=100)
2. Evaluate all solutions on both objectives
3. Filter to keep only non-dominated solutions
4. Result: A diverse set of Pareto-optimal solutions

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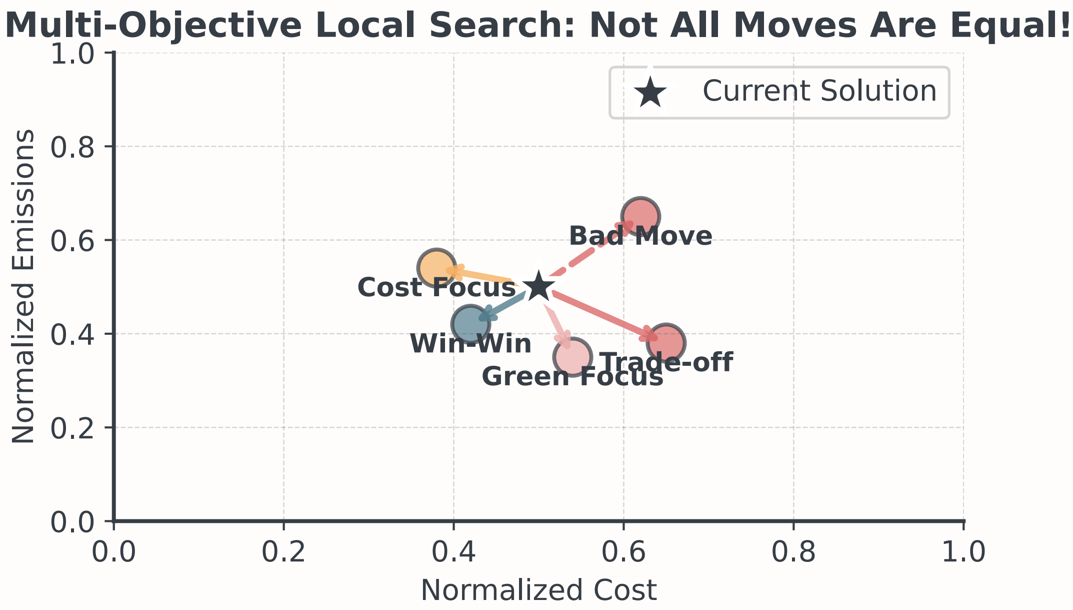
> Tip
>
> - Explores entire solution space
> - No bias toward specific weights
> - Great for warm-starting local search

Local Search for Multi-Objective

Special moves that improve multiple objectives:



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Question: Which moves are acceptable?

MOO Local Search Rules

Accept a move if:

1. Dominance: New solution dominates current (win-win!)
2. Trade-off: Improves primary, acceptable loss in secondary
3. Diversity: Fills gap in current Pareto front
4. Probabilistic: Use temperature (like simulated annealing)

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> ! Important
>
> Always keep all your objectives in mind when making decisions.

From Pareto Front to Decision

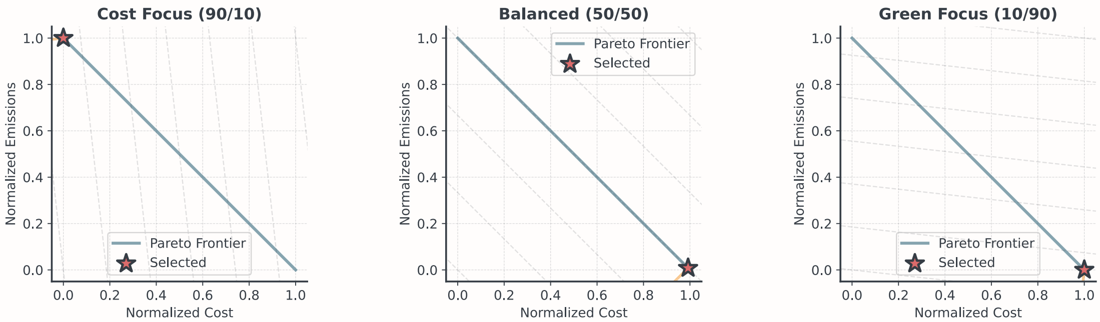
How to Choose!

1. The Knee Point: Find the "elbow" where improvement slows
2. Satisficing Levels: Set minimum acceptable thresholds
   - Cost must be < €100k (budget constraint)
   - Emissions must be < 100 kg (regulatory limit)
   - Service level must be > 90% (customer requirement)
3. Stakeholder Preferences: Let business priorities guide
   - Sustainability: Minimum emissions that meets constraints

- Operations: Maximum service level within budget

Weighted Sum Along the Frontier

The weight influences the final choice:



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> **i Note**
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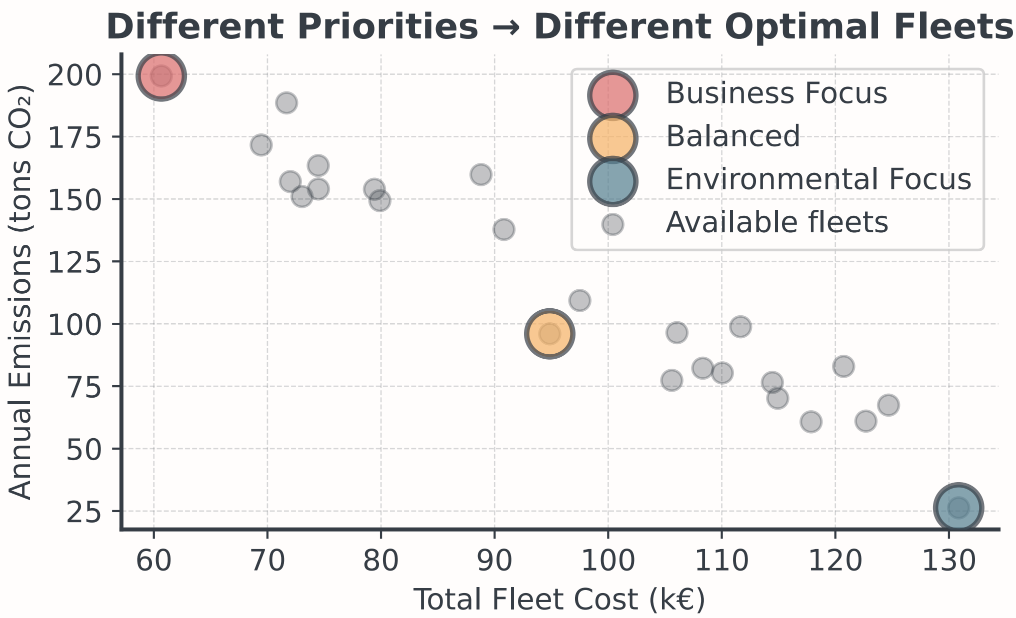
> The iso-cost lines show the trade-offs between cost and emissions.

Weighting has an Impact

The weights thus reflect your values!



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Tip

Depending on your weight, the choice will vary.

Advanced

Speed vs Sustainability Dilemma

The Three-Way Trade-off in E-Commerce

1. Minimize Delivery Time (1-day/2-hour promise)
2. Minimize Cost (fuel, labor, fulfillment)
3. Minimize Environmental Impact (carbon footprint)

. . .

Faster delivery = More vehicles less full = Higher emissions

. . .

Question: What could retailers do?

Moving the Frontier

Instead of point on the frontier, move the entire frontier:

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Question: Any idea of examples?

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Tip

R&D can fundamentally change what's possible!

Briefing

Today

Hour 2: This Lecture

- Multi-objective
- Pareto optimality
- Weighted greedy
- Local search MOO

Hour 3: Notebook

- Bean Counter CEO
- Find Pareto frontier
- Apply weighted greedy

- Normalize objectives

Hour 4: Competition

- Transportation problem
- Fleet selection
- Cost vs emissions
- Justify choice!

The Competition Challenge

EcoExpress Sustainable Fleet Design

. . .

1. Solve multi-objective transportation (DCs → Zones)
2. Select optimal fleet mix (5 vehicle types)
3. Balance cost, emissions, service quality
4. Meet EU emission targets (40% reduction)

. . .

> ! Important
>
> Find the best trade-off for your business priorities!

Choosing Your MOO Approach

Different situations call for different methods:

Situation	Best	Why
Clear priorities	Sequential greedy	Fast, hierarchy
Exploring	Weighted greedy	Different solutions
Many solutions	Diverse pool	Builds frontier
Quick solution	Single weighted	One good compromise
Improve existing	Multi-objective local	Refines trade-offs

. . .

> Tip
>
> Competition? Generate diverse pool or weigted, then improve with local search.

Implementation Pitfalls to Avoid

Common bugs that cost you time:

1. Forgetting to Normalize

    - Always normalize to [0,1] first!
2. Optimizing Too Many Objectives
    - 2-3: Manageable, 4+: Exponentially harder
    - Combine related objectives or use constraints
3. Not Checking Solution Feasibility
    - Always verify constraints after optimization

Summary

Key Takeaways:

- Real decisions have multiple conflicting objectives
- Pareto frontier shows all rational trade-offs
- Normalization is essential for fair comparison
- Weighted sum works for convex frontiers
- ε-constraint handles non-convex cases
- Weights reflect values, make them explicit
- Visualization crucial for decision-making

Break!

Take 20 minutes, then we start the practice notebook

Next up: You'll become Bean Counter's expert

Then: The Sustainability competition

Bibliography