Lecture IV - Scheduling

Programming: Everyday Decision-Making Algorithms

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Kühne Logistics University Hamburg - Winter 2024

Introduction

Today's Topic: Scheduling Algorithms

Topic: Understanding scheduling problems and algorithmic solutions for optimal task ordering

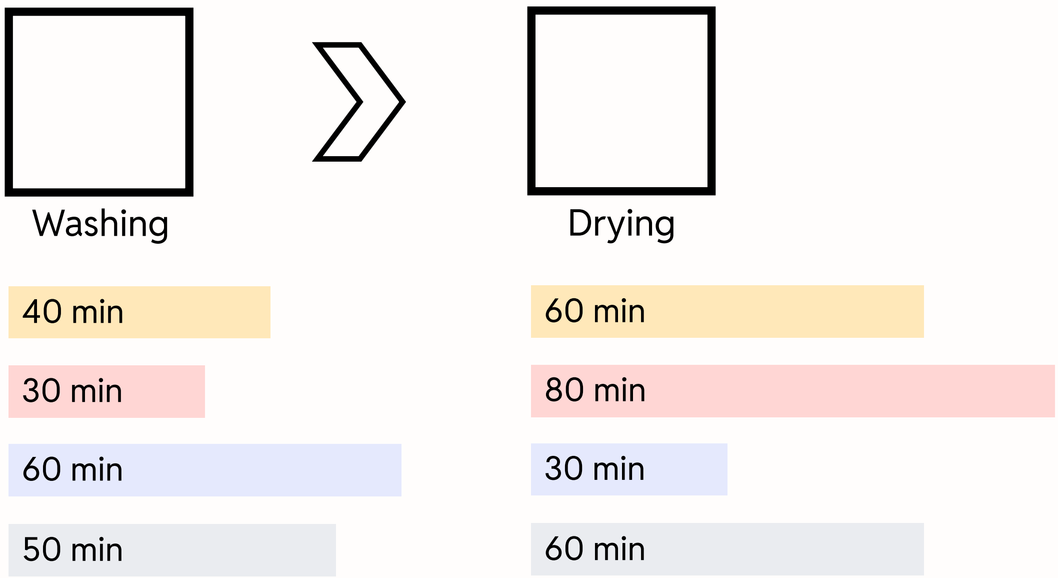
Why this matters: Every day you make scheduling decisions - from organizing your to-do list to managing projects. Today we'll learn the mathematical algorithms that can optimize these decisions and see how they connect to programming logic.

Today's Agenda

1. Scheduling Fundamentals - Johnson's Rule and two-machine problems
2. Historical Context - How scheduling theory developed
3. Core Algorithms - EDD for deadlines, SPT for efficiency
4. Advanced Topics - Dependencies, real-time scheduling, thrashing
5. Programming Connections - How this builds algorithmic problem-solving skills

Scheduling Fundamentals

A Simple Scheduling Problem



Washing Machine & Dryer

Let's solve this simple scheduling problem:

. . .

```
Task  Washing  Drying
  A    40min   60min
  B    30min   80min
  C    60min   20min
  D    50min   60min
```

. . .

Goal: Minimize total time for washing and drying all loads

. . .

Question: An idea how to solve this?

Johnson's Rule

Rule: To find the optimal solution:

1. Find the job with shortest duration:
   - If on Machine 1 ⟶ Schedule First
   - If on Machine 2 ⟶ Schedule Last
   - If equal ⟶ Choose randomly
2. Remove job from list and repeat

Applying Johnson's Rule

```
Task  Washing  Drying  Schedule
  A    40min   60min
  B    30min   80min
  C    60min   20min
  D    50min   60min
```

. . .

Question: What's the first scheduled task?

Applying Johnson's Rule

```
Task  Washing  Drying  Schedule
  A    40min   60min
  B    30min   80min
  C    60min   20min       4
  D    50min   60min
```

- In Task C, the dryer is the shortest task.
- It is on Machine 2 ⟶ Schedule Last

. . .

Question: What's the next task?

Applying Johnson's Rule

```
Task  Washing  Drying  Schedule
  A     40min   60min
  B     30min   80min        1
  C     60min   20min        4
  D     50min   60min
```

- In Task B, the washing machine is the shortest task.
- It is on Machine 1 ⟶ Schedule First

. . .

Question: What's the next task?

Applying Johnson's Rule

```
Task  Washing  Drying  Schedule
  A     40min   60min        2
  B     30min   80min        1
  C     60min   20min        4
  D     50min   60min
```

- In Task A, the washing machine is the shortest task.
- It is on Machine 1 ⟶ Schedule Second

. . .

Now, we only have one task left!

Applying Johnson's Rule

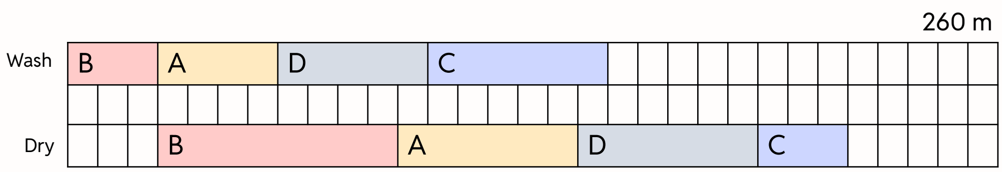
```
Task  Washing  Drying  Schedule
  A     40min   60min        2
  B     30min   80min        1
  C     60min   20min        4
  D     50min   60min        3
```

. . .

Final sequence: B A D C

Optimal Solution

Optimal Solution: B A D C



Total time: 4 hours 20 minutes

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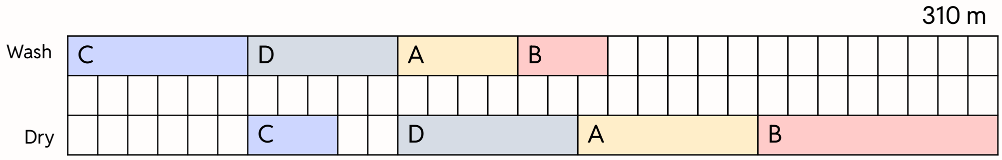
Question: Is there a worse solution?

Suboptimal Solution

Suboptimal Solution: C D A B



Total time: 5 hours 10 minutes

. . .

Question: What's the difference?

History

Industrial Revolution

. . .

- First systematic visualization by Frederick Taylor
- Henry Gantt develops the Gantt Chart around 1910
- Key tool during Industrial Revolution
- But no scheduling theory yet!

. . .

Question: Who knows what a Gantt Chart is?

Modern Scheduling Theory

. . .

- RAND Corporation founded (1948)
- Selmer Johnson publishes Johnson's Rule in 1954
- Beginning of modern scheduling theory
- Many more algorithms and methods developed since

Professional Applications

Where do we see scheduling in professional practice?

- Project Management: Task dependencies, resource allocation, critical path
- Software Development: CPU scheduling, thread management, job queues
- Operations: Production scheduling, supply chain optimization
- Healthcare: Patient scheduling, surgery planning, staff allocation
- Transportation: Route optimization, crew scheduling, maintenance planning

. . .

Connection to Programming: These are the same optimization problems that algorithms solve!

Scheduling Tasks

Task Classification

Question: What properties can scheduled tasks have?

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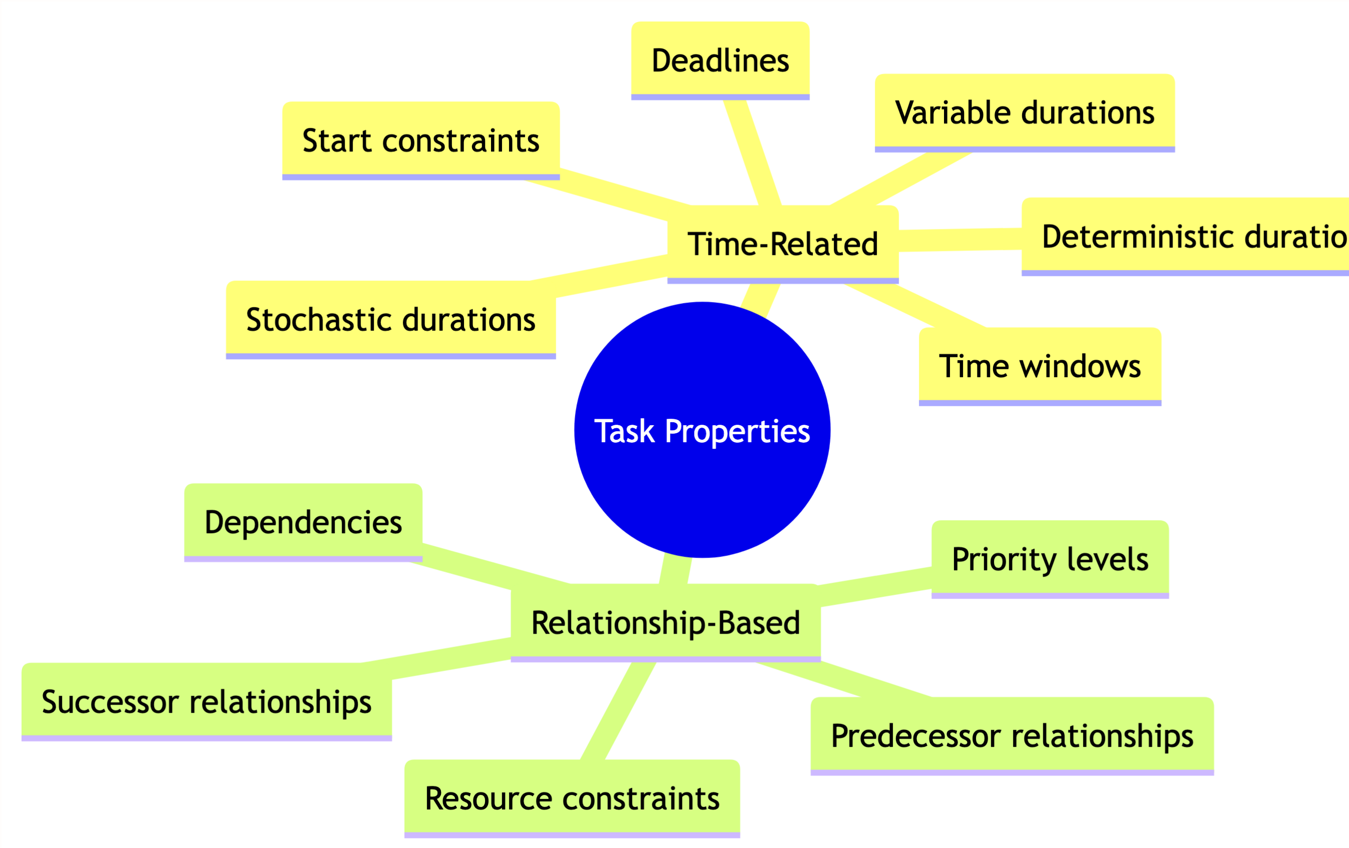

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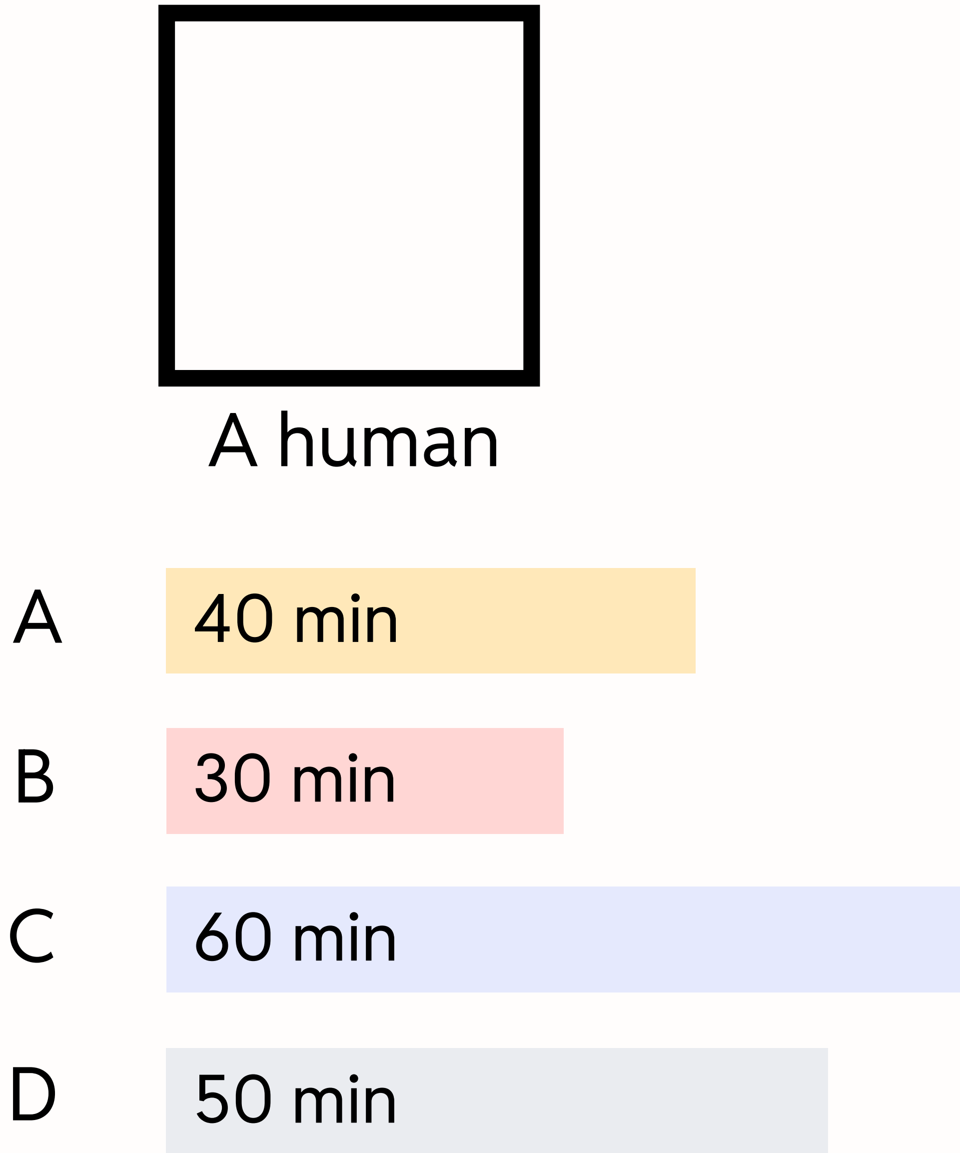
Question: What types of tasks do you deal with most often?

Single Machine Scheduling



Question: What is different from before?

Order is Irrelevant

> **i Order is Irrelevant**
>
> Under simple minimization of total processing time, order doesn't matter!

. . .

Question: But is it that simple?

. . .

! Order Matters

Order becomes crucial when we consider, Deadlines, Priorities and Dependencies!

Deadlines

Earliest Due Date (EDD)

Tasks with individual deadlines:

```
Task  Duration  Deadline
  A     40min    110min
  B     30min     90min
  C     60min    150min
  D     50min     70min
  E     30min    210min
```

. . .

Goal: Minimize maximum deadline violation

. . .

Question: An idea how to solve this?

EDD Solution

Rule: Sort the tasks by deadline.

```
Task  Duration  Deadline
  A     40min    110min
  B     30min     90min
  C     60min    150min
  D     50min     70min
  E     30min    210min
```

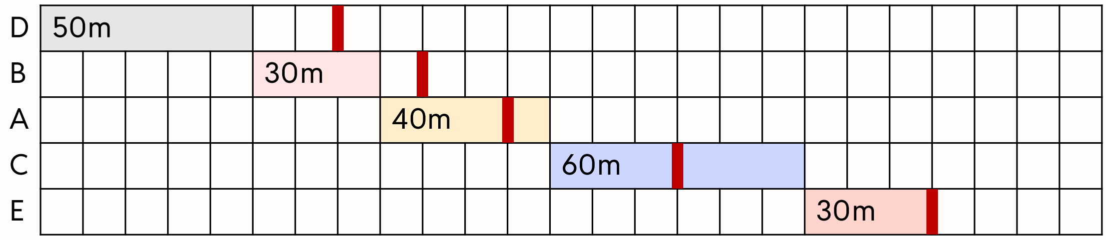
. . .

```
Task  Duration  Deadline
  D     50min     70min
  B     30min     90min
  A     40min    110min
  C     60min    150min
  E     30min    210min
```

. . .

Let’s visualize this!

EDD Schedule



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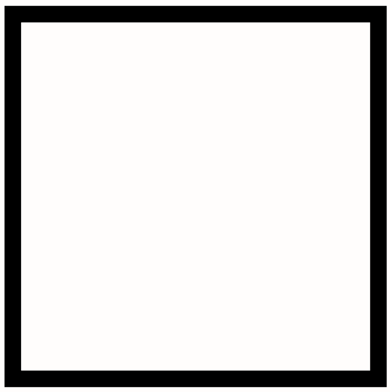
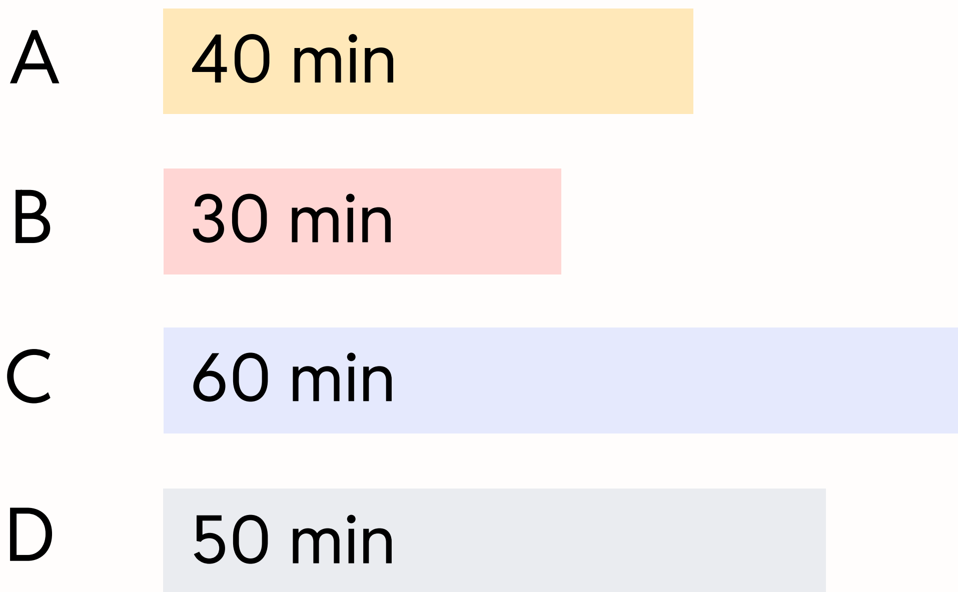
Question: What's the maximum delay here?

Processing Time

Shortest Processing Time (SPT)



A human



Instead of deadlines, we now have processing times.

Goal: Min. total waiting time

Question: Any ideas?

Rule: Always schedule the shortest remaining task

Shortest Processing Time Applied

Rule: Always schedule the shortest remaining task. Choose random if multiple tasks are tied.

```
Task  Duration  Schedule
  A     40min
  B     30min
  C     60min
  D     50min
  E     30min
```

...

Question: What’s the order of scheduled tasks?

Shortest Processing Time Applied

Rule: Always schedule the shortest remaining task. Choose random if multiple tasks are tied.

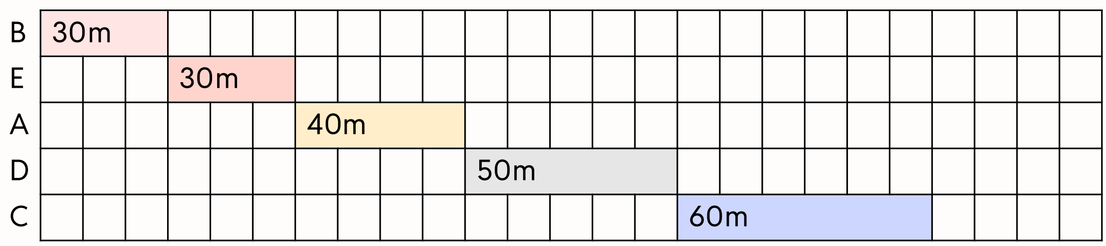
```
Task  Duration  Schedule
  A     40min       3
  B     30min       1
  C     60min       5
  D     50min       4
  E     30min       2
```

...

Final sequence: B E A D C or E B A D C

SPT Solution

Optimal sequence:



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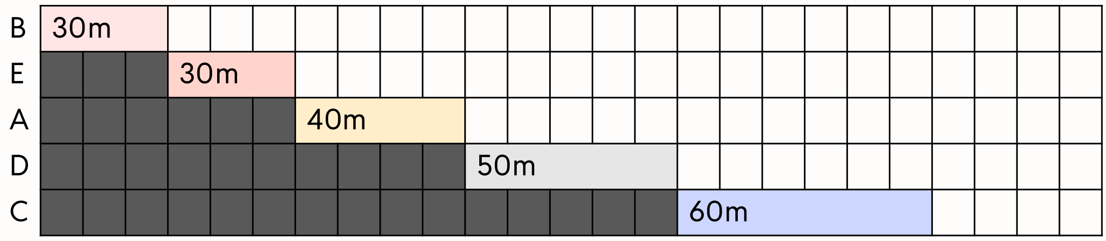
Question: Where can we see the waiting time?

SPT Waiting Time

Total waiting time: 340 minutes



. . .

Question: Would this be applicable for your work?

Weighted SPT

- Change: Tasks with additional priorities
- Priorities could be, e.g., revenue if we are consultants.

Task	Duration	Revenue
A	20min	€240
B	30min	€200
C	60min	€120
D	50min	€70
E	30min	€130
F	40min	€120
G	20min	€100
H	70min	€110
I	50min	€90

. . .

Question: Any ideas how to approach this?

Gain/Revenue Per Minute

Rule: Schedule by revenue per minute (descending)

Task	Duration	Revenue	Revenue/Min	Schedule
A	20min	€240	12.0	
B	30min	€200	6.7	
C	60min	€120	2.0	
D	50min	€70	1.4	
E	30min	€130	4.3	
F	40min	€120	3.0	
G	20min	€100	5.0	
H	70min	€110	1.6	
I	50min	€90	1.8	

. . .

Question: What’s the order of scheduled tasks?

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E	30min	€130	4.3	4
F	40min	€120	3.0	5
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F	40min	€120	3.0	5
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C	60min	€120	2.0	6
D	50min	€70	1.4	9
E	30min	€130	4.3	4
F	40min	€120	3.0	5
G	20min	€100	5.0	3
H	70min	€110	1.6	8
I	50min	€90	1.8	7

...

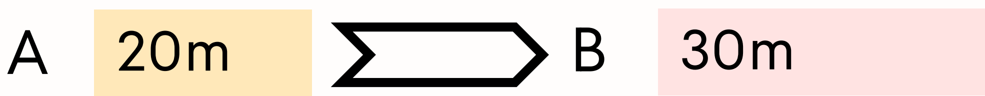
> **Metric Priorities**
>
> Without revenues, we can use the same approach with metric priorities!

Dependencies

Priority Inversion

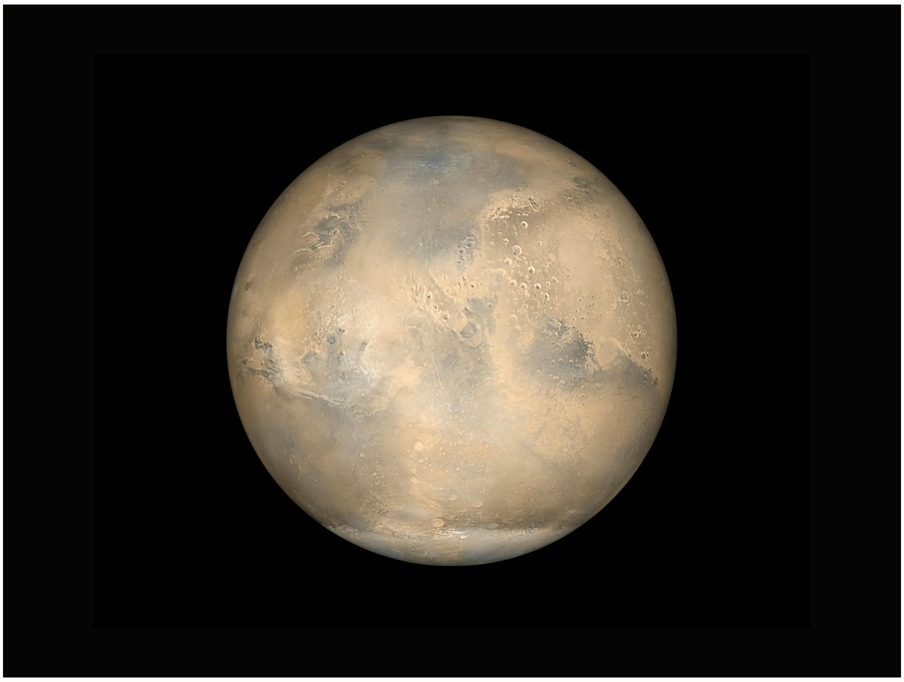
Setup:

Task	Duration	Priority
A	20min	3
B	30min	1
C	30min	2
D	30min	2
E	30min	2



Challenge: High-priority tasks depend on low-priority tasks.

Risk: Priority inversion can lead to significant delays!



Priority Inheritance

Question: How to handle with shortest processing time?

- Rule: Tasks inherit priority from their dependents.
- A gets the highest priority from B
- This ensures the critical path completion

. . .

```
Task  Duration  Priority
  A     20min     3
  B     30min     3
  C     30min     2
  D     30min     2
  E     30min     2
```

EDD and Dependencies

Question: What’s was earliest due date again?

. . .

- Sort the tasks by deadline, schedule equal tasks randomly
- Things get more complex when we add dependencies

. . .

```
Task  Duration  Deadline  Predecessor
  A     40min   110min    None
  B     30min    90min    D
  C     60min   150min    A
```

```
  D     50min     70min     None
  E     30min    210min     C
```

Question: Any ideas how to solve this?

Lawler's Algorithm

Rule: We can use Lawler's Algorithm (1968)

1. Consider all tasks without successors
2. Choose the one with latest deadline
3. Schedule the task last
4. Remove it from the network and start again

Lawler's Applied

```
Task  Duration  Deadline  Predecessor Schedule
  A     40min    110min     None
  B     30min     90min     D
  C     60min    150min     A
  D     50min     70min     None
  E     30min    210min     C
```

. . .

Question: What's the schedule?

Lawler's Applied

```
Task  Duration  Deadline  Predecessor Schedule
  A     40min    110min     None
  B     30min     90min     D
  C     60min    150min     A
  D     50min     70min     None
  E     30min    210min     C              5
```

Lawler's Applied

```
Task  Duration  Deadline  Predecessor Schedule
  A     40min    110min     None
  B     30min     90min     D
  C     60min    150min     A              4
  D     50min     70min     None
  E     30min    210min     C              5
```

Lawler's Applied

```
Task  Duration  Deadline  Predecessor Schedule
  A     40min    110min     None
  B     30min     90min     D              3
  C     60min    150min     A              4
```

```
  D     50min     70min    None
  E     30min    210min    C              5
```

Lawler’s Applied

```
Task  Duration  Deadline  Predecessor Schedule
  A     40min    110min    None
  B     30min     90min    D              3
  C     60min    150min    A              4
  D     50min     70min    None           2
  E     30min    210min    C              5
```

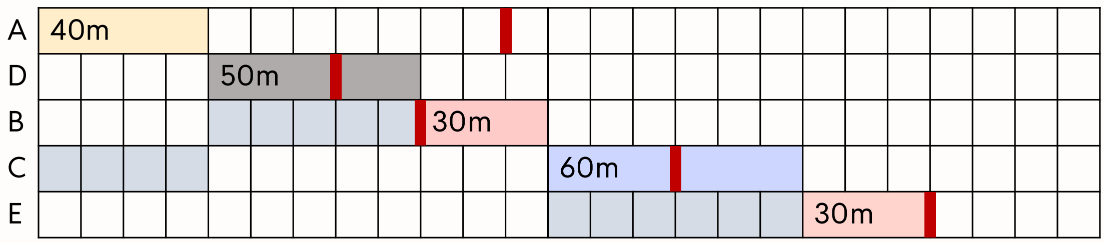
Lawler’s Applied

```
Task  Duration  Deadline  Predecessor Schedule
  A     40min    110min    None           1
  B     30min     90min    D              3
  C     60min    150min    A              4
  D     50min     70min    None           2
  E     30min    210min    C              5
```

. . .

> **Successor Tasks**
>
> Note, how all tasks become tasks without successors at some point.

Lawler’s Solution



> **Predecessor Tasks**
>
> Predecessor tasks are tasks that must be completed before the current task can start. They are marked in grey in the chart.

. . .

Question: What’s the maximum delay?

Difficulty of Scheduling

SPT with Predecessors

- No solution in polynomial time
- NP-hard problem (no efficient algorithm)
- True for most scheduling problems!
- We can use heuristics, though

. . .

Question: What have we missed so far?

Real-time Scheduling

Interruptions

- In reality, we cannot predict the future
- We need to react to new tasks as they happen
- If we have a deadline, we might need to meet it
- Let's look at this for the earliest due date objective

. . .

> **i Quick reminder**
>
> An earliest due date is a specific point in time by which a task must be completed. Under this objective, we want to minimize the maximum delay.

Real-time EDD

8:00-12:00 Schedule:

```
Task           Duration  Deadline
Email A        20min      9:00
Create PPT     60min     10:50
Investor call  10min      9:00
Email B        30min     10:20
Liquidity      40min     11:00
Email C        20min     11:20
Email D        40min     10:00
```

. . .

Question: Any ideas how to start with under the objective of the earliest due date?

EDD Rule for Real-time

Rule:

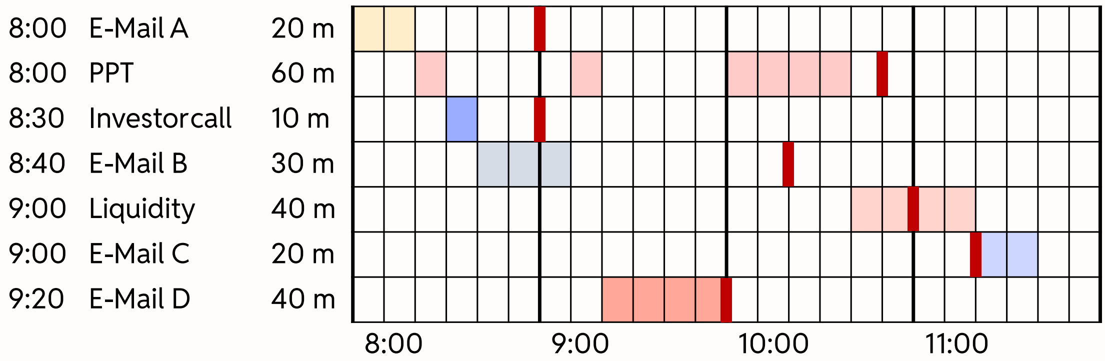
- Always schedule the task with the earliest deadline
- If a new task with an earlier deadline comes in, re-schedule
- Otherwise, stick to the original schedule.

. . .

💡 Equal Deadline

If a new task has the same deadline as the current task, you can choose either. But due to the cost of context switching, you might want to stick with the current task.

EDD Solution for Real-time



. . .

Question: What's the maximum delay with this schedule?

SPT for Real-time

💡 Quick reminder

A shortest processing time is the task with the shortest duration. Under this objective, we want to minimize the total waiting time.

. . .

Question: Any ideas how to start here?

SPT Rule for Real-time

Rule:

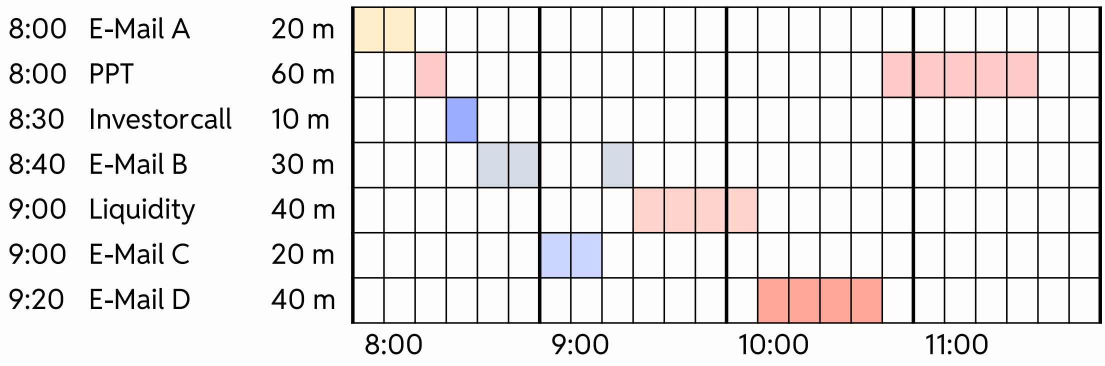
- Always schedule the task with the shortest duration
- If a new task with a shorter duration comes in, re-schedule
- Otherwise, stick to the original schedule.

. . .

ℹ Equal Duration

If a new task has the same duration as the current task, you can choose either. But due to the cost of context switching, you might again want to stick with the current task.

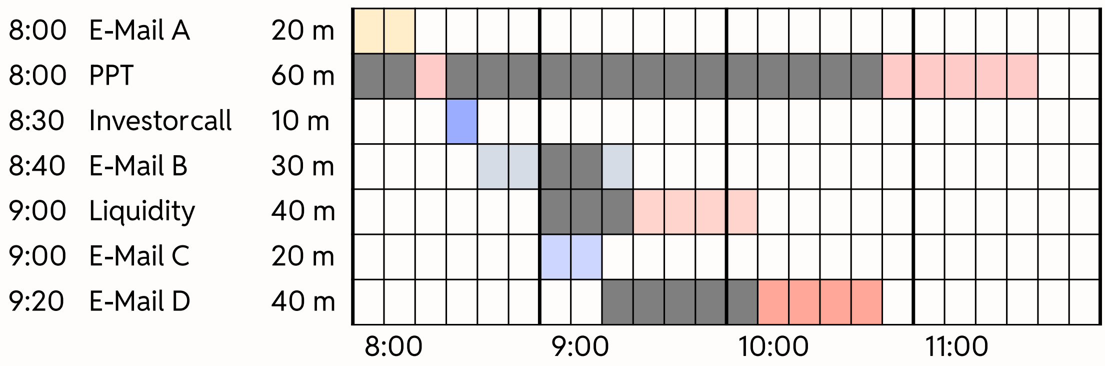
SPT Solution for Real-time



. . .

Question: Where can we see the waiting time?

SPT Solution for Real-time



Total waiting time: 260 minutes

Thrashing

What is Thrashing?

- Excessive context switching
- Organization overhead exceeds productivity
- Maximum activity, minimum output

. . .

Question: Have you ever experienced this?

Thrashing Warning Signs

- Constant task switching
- Nothing getting completed
- Increasing stress levels
- Declining quality

. . .

Question: Any ideas how to prevent this?

Preventing Thrashing Strategic

> Strategic
>
> Strategic solutions focus on long-term changes to prevent thrashing.

. . .

1. Task rejection/delegation threshold
2. Simplified organization systems
3. Minimum work period rules
4. Reduced reactivity requirements

Preventing Thrashing Tactical

1. Time blocking
2. Focus periods
3. Task batching
4. Priority freezes

. . .

Question: What strategies have worked for you?

Applying This Knowledge

Immediate Applications

How can you use these algorithms starting today?

- Daily planning: Use SPT for your to-do list to minimize waiting time
- Project management: Apply EDD when managing deadlines
- Team coordination: Use Johnson's Rule for two-stage processes
- Time blocking: Prevent thrashing with strategic scheduling

Key Takeaways

What we learned today:

1. Scheduling algorithms provide optimal solutions to time management problems
2. Different objectives require different algorithms (EDD, SPT, Johnson's Rule)
3. Dependencies and constraints add complexity but have algorithmic solutions
4. Thrashing prevention is crucial for maintaining productivity
5. These principles directly translate to programming and system design

Any questions

so far?

After the break — Scheduling

- Programming session in our new notebooks
- How to translate the idea into code and experiments
- Different scheduling algorithms applied to problems

...

> **ℹ Note**
>
> That's it for scheduling!
> Let's have a short break and then continue with our fourth Python programming session.

Literature

Interesting literature to start

- Christian, B., & Griffiths, T. (2016). Algorithms to live by: the computer science of human decisions. First international edition. New York, Henry Holt and Company.[1]

Books on Programming

- Downey, A. B. (2024). Think Python: How to think like a computer scientist (Third edition). O'Reilly. Here
- Elter, S. (2021). Schrödinger programmiert Python: Das etwas andere Fachbuch (1. Auflage). Rheinwerk Verlag.

Questions & Discussion

> **ℹ Note**
>
> Think Python is a great book to start with. It's available online for free. Schrödinger Programmiert Python is a great alternative for German students, as it is a very playful introduction to programming with lots of examples.

More Literature

For more interesting literature, take a look at the literature list of this course.

[1] The main inspiration for this lecture. Nils and I have read it and discussed it in depth, always wanting to translate it into a course.