

# Lecture IV - Scheduling

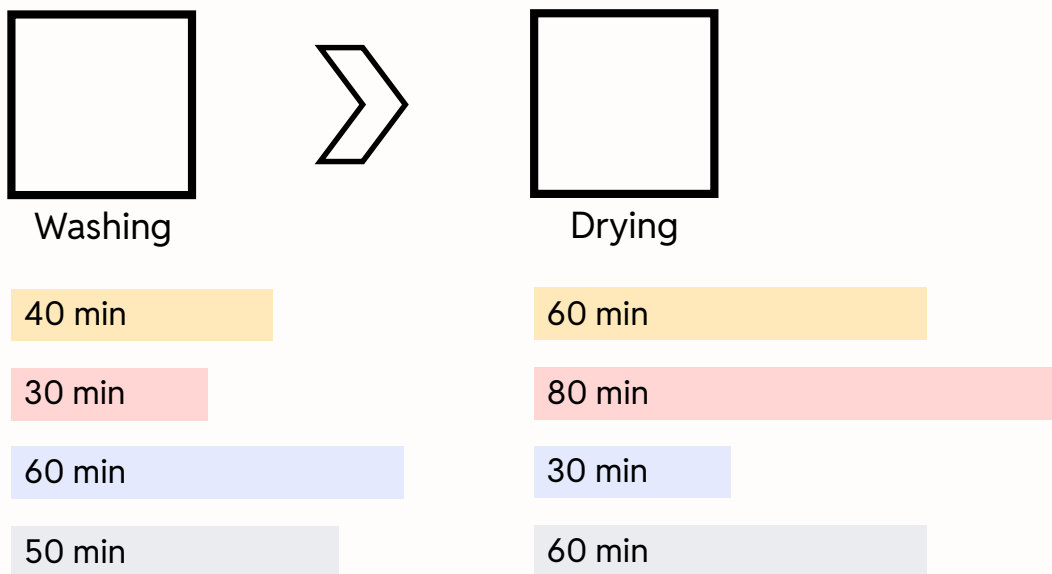
## Programming: Everyday Decision-Making Algorithms

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### Introduction

#### Introduction



### Washing Machine & Dryer

Let's solve this simple scheduling problem:

...

| Task | Washing | Drying |
|------|---------|--------|
| A    | 40min   | 60min  |
| B    | 30min   | 80min  |
| C    | 60min   | 20min  |
| D    | 50min   | 60min  |

...

Goal: Minimize total time for washing and drying all loads

...

Question: An idea how to solve this?

## Johnson's Rule

Rule: To find the optimal solution:

1. Find the job with shortest duration:
  - If on Machine 1 → Schedule First
  - If on Machine 2 → Schedule Last
  - If equal → Choose randomly
2. Remove job from list and repeat

### Applying Johnson's Rule

| Task | Washing | Drying | Schedule |
|------|---------|--------|----------|
| A    | 40min   | 60min  |          |
| B    | 30min   | 80min  |          |
| C    | 60min   | 20min  |          |
| D    | 50min   | 60min  |          |

...

Question: What's the first scheduled task?

### Applying Johnson's Rule

| Task | Washing | Drying | Schedule |
|------|---------|--------|----------|
| A    | 40min   | 60min  |          |
| B    | 30min   | 80min  |          |
| C    | 60min   | 20min  | 4        |
| D    | 50min   | 60min  |          |

- In Task C, the dryer is the shortest task.
- It is on Machine 2 → Schedule Last

...

Question: What's the next task?

### Applying Johnson's Rule

| Task | Washing | Drying | Schedule |
|------|---------|--------|----------|
| A    | 40min   | 60min  |          |
| B    | 30min   | 80min  | 1        |
| C    | 60min   | 20min  | 4        |
| D    | 50min   | 60min  |          |

- In Task B, the washing machine is the shortest task.
- It is on Machine 1 → Schedule First

...

Question: What's the next task?

## Applying Johnson's Rule

| Task | Washing | Drying | Schedule |
|------|---------|--------|----------|
| A    | 40min   | 60min  | 2        |
| B    | 30min   | 80min  | 1        |
| C    | 60min   | 20min  | 4        |
| D    | 50min   | 60min  |          |

- In Task A, the washing machine is the shortest task.
- It is on Machine 1 → Schedule Second

...

Now, we only have one task left!

## Applying Johnson's Rule

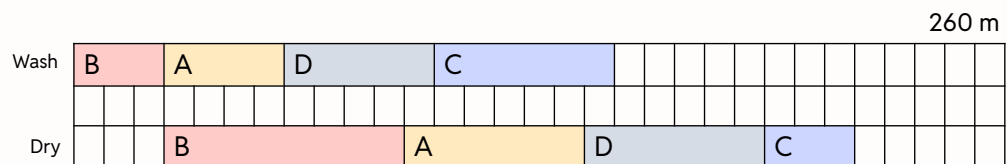
| Task | Washing | Drying | Schedule |
|------|---------|--------|----------|
| A    | 40min   | 60min  | 2        |
| B    | 30min   | 80min  | 1        |
| C    | 60min   | 20min  | 4        |
| D    | 50min   | 60min  | 3        |

...

Final sequence: B A D C

## Optimal Solution

Optimal Solution: B A D C



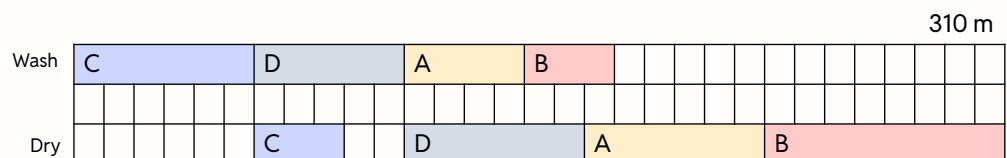
Total time: 4 hours 20 minutes

...

Question: Is there a worse solution?

## Suboptimal Solution

Suboptimal Solution: C D A B



Total time: 5 hours 10 minutes

...

Question: What's the difference?

## History

### Industrial Revolution

...

- First systematic visualization by Frederick Taylor
- Henry Gantt develops the Gantt Chart around 1910
- Key tool during Industrial Revolution
- But no scheduling theory yet!

...

Question: Who knows what a Gantt Chart is?

### Modern Scheduling Theory

...

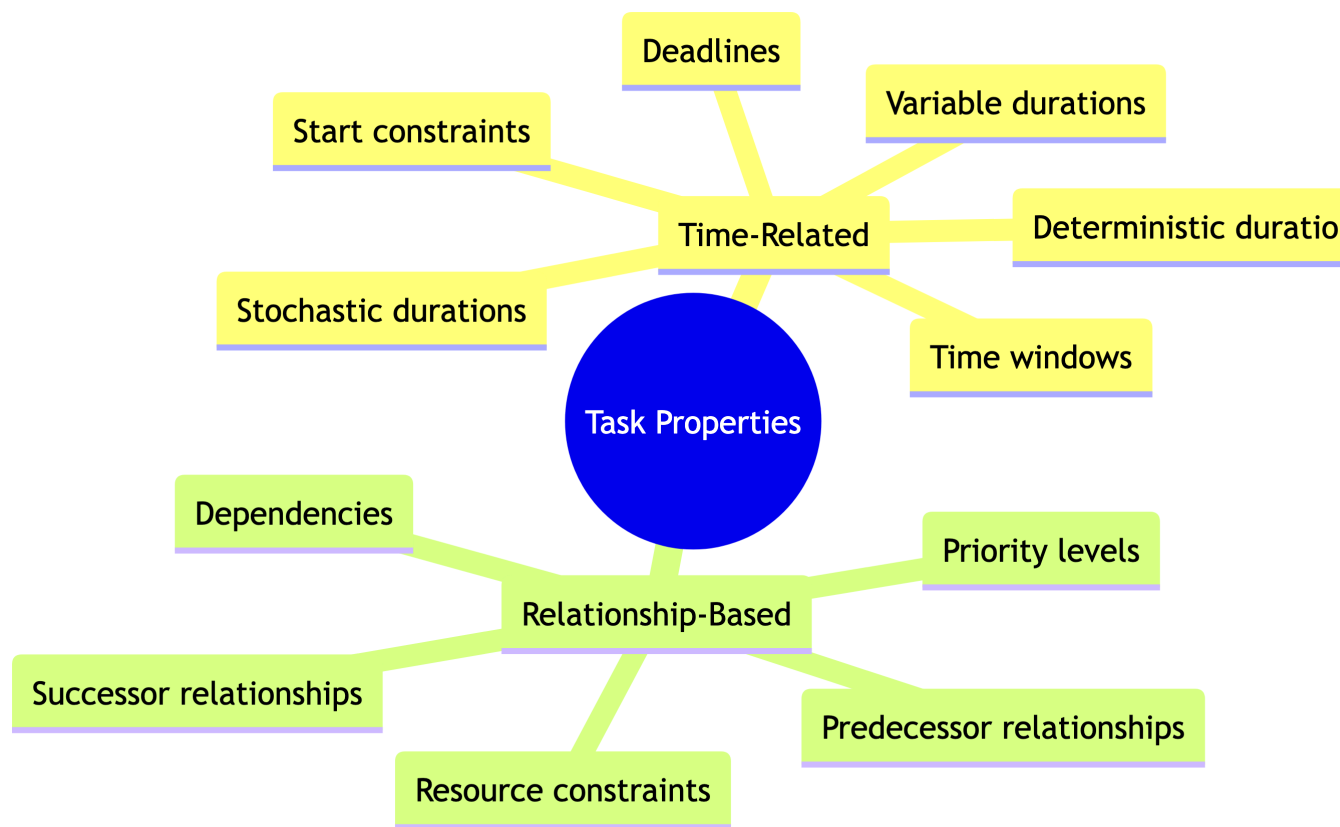
- RAND Corporation founded (1948)
- Selmer Johnson publishes Johnson's Rule in 1954
- Beginning of modern scheduling theory
- Many more algorithms and methods developed since

## Scheduling Tasks

### Task Classification

Question: What properties can scheduled tasks have?

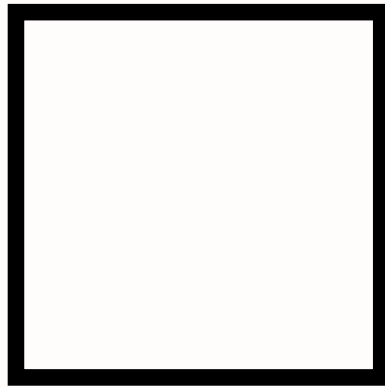
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...

Question: What types of tasks do you deal with most often?

## Single Machine Scheduling



A human

A 40 min

B 30 min

C 60 min

D 50 min

Question: What is different from before?

Order is Irrelevant

**i** Order is Irrelevant

Under simple minimization of total processing time, order doesn't matter!

...

Question: But is it that simple?

...

## ! Order Matters

Order becomes crucial when we consider, Deadlines, Priorities and Dependencies!

## Deadlines

### Earliest Due Date (EDD)

Tasks with individual deadlines:

| Task | Duration | Deadline |
|------|----------|----------|
| A    | 40min    | 110min   |
| B    | 30min    | 90min    |
| C    | 60min    | 150min   |
| D    | 50min    | 70min    |
| E    | 30min    | 210min   |

...

Goal: Minimize maximum deadline violation

...

Question: An idea how to solve this?

### EDD Solution

Rule: Sort the tasks by deadline.

| Task | Duration | Deadline |
|------|----------|----------|
| A    | 40min    | 110min   |
| B    | 30min    | 90min    |
| C    | 60min    | 150min   |
| D    | 50min    | 70min    |
| E    | 30min    | 210min   |

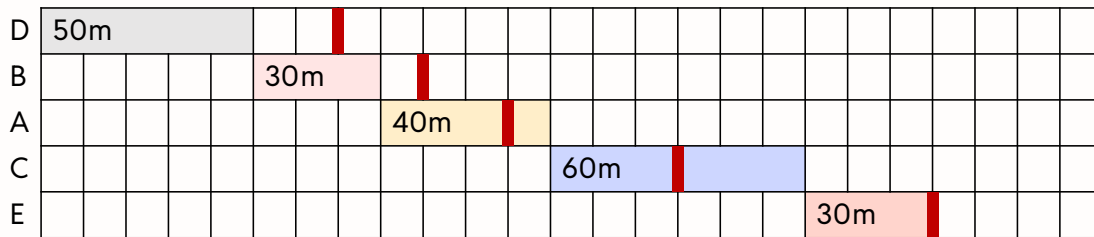
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| Task | Duration | Deadline |
|------|----------|----------|
| D    | 50min    | 70min    |
| B    | 30min    | 90min    |
| A    | 40min    | 110min   |
| C    | 60min    | 150min   |
| E    | 30min    | 210min   |

...

Let's visualize this!

## EDD Schedule

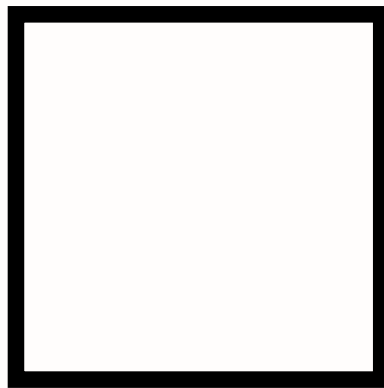


...

Question: What's the maximum delay here?

## Processing Time

Shortest Processing Time (SPT)



A human

A

40 min

B

30 min

C

60 min

D

50 min



Instead of deadlines, we now have processing times.

Goal: Min. total waiting time

Question: Any ideas?

Rule: Always schedule the shortest remaining task

### Shortest Processing Time Applied

Rule: Always schedule the shortest remaining task. Choose random if multiple tasks are tied.

| Task | Duration | Schedule |
|------|----------|----------|
| A    | 40min    |          |
| B    | 30min    |          |
| C    | 60min    |          |
| D    | 50min    |          |
| E    | 30min    |          |

...

Question: What's the order of scheduled tasks?

### Shortest Processing Time Applied

Rule: Always schedule the shortest remaining task. Choose random if multiple tasks are tied.

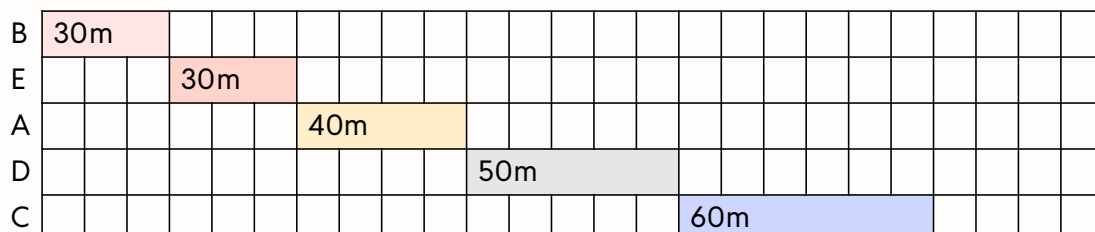
| Task | Duration | Schedule |
|------|----------|----------|
| A    | 40min    | 3        |
| B    | 30min    | 1        |
| C    | 60min    | 5        |
| D    | 50min    | 4        |
| E    | 30min    | 2        |

...

Final sequence: B E A D C or E B A D C

### SPT Solution

Optimal sequence:

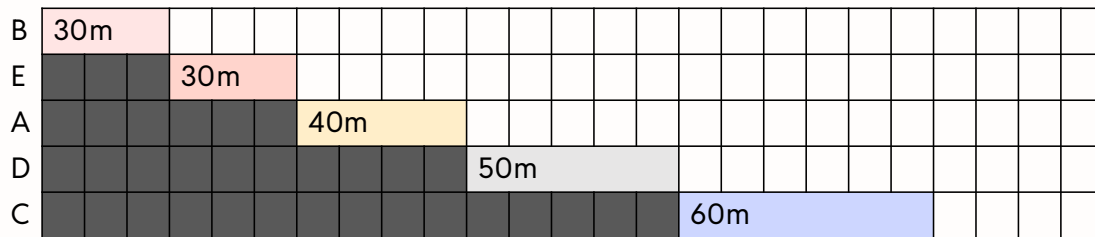


...

Question: Where can we see the waiting time?

## SPT Waiting Time

Total waiting time: 340 minutes



...

Question: Would this be applicable for your work?

## Weighted SPT

- Change: Tasks with additional priorities
- Priorities could be, e.g., revenue if we are consultants.

| Task | Duration | Revenue |
|------|----------|---------|
| A    | 20min    | €240    |
| B    | 30min    | €200    |
| C    | 60min    | €120    |
| D    | 50min    | €70     |
| E    | 30min    | €130    |
| F    | 40min    | €120    |
| G    | 20min    | €100    |
| H    | 70min    | €110    |
| I    | 50min    | €90     |

...

Question: Any ideas how to approach this?

## Gain/Revenue Per Minute

Rule: Schedule by revenue per minute (descending)

| Task | Duration | Revenue | Revenue/Min | Schedule |
|------|----------|---------|-------------|----------|
| A    | 20min    | €240    | 12.0        |          |
| B    | 30min    | €200    | 6.7         |          |
| C    | 60min    | €120    | 2.0         |          |
| D    | 50min    | €70     | 1.4         |          |
| E    | 30min    | €130    | 4.3         |          |
| F    | 40min    | €120    | 3.0         |          |
| G    | 20min    | €100    | 5.0         |          |
| H    | 70min    | €110    | 1.6         |          |
| I    | 50min    | €90     | 1.8         |          |

...

Question: What's the order of scheduled tasks?

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| C    | 60min    | €120    | 2.0         |          |

|   |       |      |     |   |
|---|-------|------|-----|---|
| D | 50min | €70  | 1.4 |   |
| E | 30min | €130 | 4.3 | 4 |
| F | 40min | €120 | 3.0 | 5 |
| G | 20min | €100 | 5.0 | 3 |
| H | 70min | €110 | 1.6 |   |
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| E    | 30min    | €130    | 4.3         | 4        |
| F    | 40min    | €120    | 3.0         | 5        |
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| E    | 30min    | €130    | 4.3         | 4        |
| F    | 40min    | €120    | 3.0         | 5        |
| G    | 20min    | €100    | 5.0         | 3        |
| H    | 70min    | €110    | 1.6         |          |
| I    | 50min    | €90     | 1.8         | 7        |

### Gain/Revenue Per Minute

Rule: Schedule by revenue per minute (descending)

| Task | Duration | Revenue | Revenue/Min | Schedule |
|------|----------|---------|-------------|----------|
| A    | 20min    | €240    | 12.0        | 1        |
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| C    | 60min    | €120    | 2.0         | 6        |
| D    | 50min    | €70     | 1.4         |          |
| E    | 30min    | €130    | 4.3         | 4        |
| F    | 40min    | €120    | 3.0         | 5        |
| G    | 20min    | €100    | 5.0         | 3        |
| H    | 70min    | €110    | 1.6         | 8        |
| I    | 50min    | €90     | 1.8         | 7        |

## Gain/Revenue Per Minute

Rule: Schedule by revenue per minute (descending)

| Task | Duration | Revenue | Revenue/Min | Schedule |
|------|----------|---------|-------------|----------|
| A    | 20min    | €240    | 12.0        | 1        |
| B    | 30min    | €200    | 6.7         | 2        |
| C    | 60min    | €120    | 2.0         | 6        |
| D    | 50min    | €70     | 1.4         | 9        |
| E    | 30min    | €130    | 4.3         | 4        |
| F    | 40min    | €120    | 3.0         | 5        |
| G    | 20min    | €100    | 5.0         | 3        |
| H    | 70min    | €110    | 1.6         | 8        |
| I    | 50min    | €90     | 1.8         | 7        |

...

### 💡 Metric Priorities

Without revenues, we can use the same approach with metric priorities!

## Dependencies

### Priority Inversion

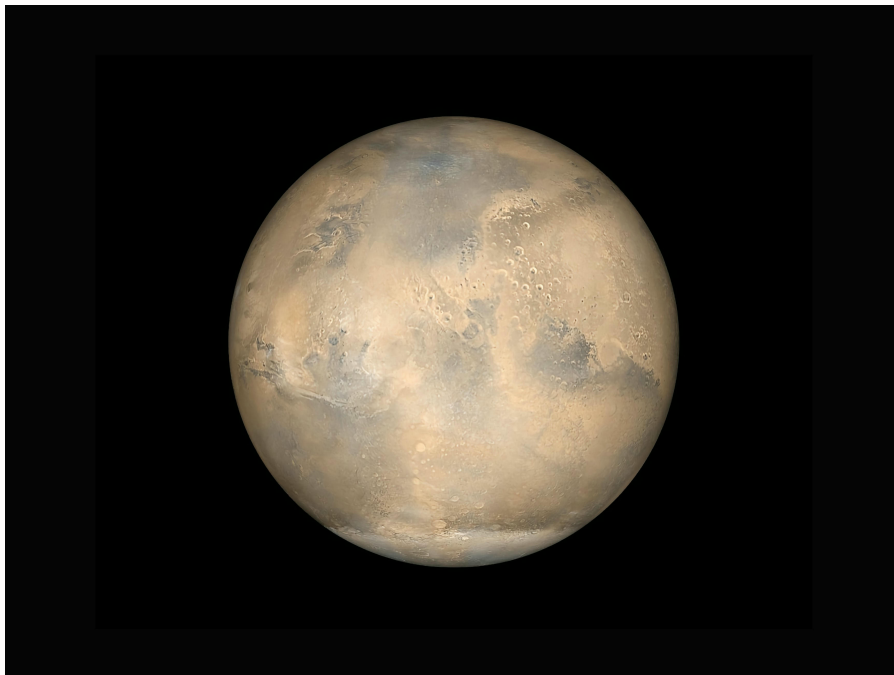
Setup:

| Task | Duration | Priority |
|------|----------|----------|
| A    | 20min    | 3        |
| B    | 30min    | 1        |
| C    | 30min    | 2        |
| D    | 30min    | 2        |
| E    | 30min    | 2        |



Challenge: High-priority tasks depend on low-priority tasks.

Risk: Priority inversion can lead to significant delays!



## Priority Inheritance

Question: How to handle with shortest processing time?

- Rule: Tasks inherit priority from their dependents.
- A gets the highest priority from B
- This ensures the critical path completion

...

| Task | Duration | Priority |
|------|----------|----------|
| A    | 20min    | 3        |
| B    | 30min    | 3        |
| C    | 30min    | 2        |
| D    | 30min    | 2        |
| E    | 30min    | 2        |

## EDD and Dependencies

Question: What's was earliest due date again?

...

- Sort the tasks by deadline, schedule equal tasks randomly
- Things get more complex when we add dependencies

...

| Task | Duration | Deadline | Predecessor |
|------|----------|----------|-------------|
| A    | 40min    | 110min   | None        |
| B    | 30min    | 90min    | D           |
| C    | 60min    | 150min   | A           |

|   |       |        |      |
|---|-------|--------|------|
| D | 50min | 70min  | None |
| E | 30min | 210min | C    |

Question: Any ideas how to solve this?

## Lawler's Algorithm

Rule: We can use Lawler's Algorithm (1968)

1. Consider all tasks without successors
2. Choose the one with latest deadline
3. Schedule the task last
4. Remove it from the network and start again

## Lawler's Applied

| Task | Duration | Deadline | Predecessor | Schedule |
|------|----------|----------|-------------|----------|
| A    | 40min    | 110min   | None        |          |
| B    | 30min    | 90min    | D           |          |
| C    | 60min    | 150min   | A           |          |
| D    | 50min    | 70min    | None        |          |
| E    | 30min    | 210min   | C           |          |

...

Question: What's the schedule?

## Lawler's Applied

| Task | Duration | Deadline | Predecessor | Schedule |
|------|----------|----------|-------------|----------|
| A    | 40min    | 110min   | None        |          |
| B    | 30min    | 90min    | D           |          |
| C    | 60min    | 150min   | A           |          |
| D    | 50min    | 70min    | None        |          |
| E    | 30min    | 210min   | C           | 5        |

## Lawler's Applied

| Task | Duration | Deadline | Predecessor | Schedule |
|------|----------|----------|-------------|----------|
| A    | 40min    | 110min   | None        |          |
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| C    | 60min    | 150min   | A           | 4        |
| D    | 50min    | 70min    | None        |          |
| E    | 30min    | 210min   | C           | 5        |

## Lawler's Applied

| Task | Duration | Deadline | Predecessor | Schedule |
|------|----------|----------|-------------|----------|
| A    | 40min    | 110min   | None        |          |
| B    | 30min    | 90min    | D           | 3        |
| C    | 60min    | 150min   | A           | 4        |

|   |       |        |      |   |
|---|-------|--------|------|---|
| D | 50min | 70min  | None |   |
| E | 30min | 210min | C    | 5 |

## Lawler's Applied

| Task | Duration | Deadline | Predecessor | Schedule |
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| A    | 40min    | 110min   | None        |          |
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## Lawler's Applied

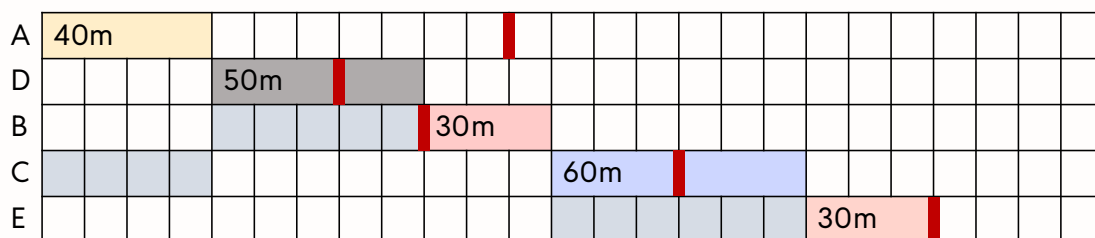
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|------|----------|----------|-------------|----------|
| A    | 40min    | 110min   | None        | 1        |
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| D    | 50min    | 70min    | None        | 2        |
| E    | 30min    | 210min   | C           | 5        |

...

### Successor Tasks

Note, how all tasks become tasks without successors at some point.

## Lawler's Solution



### Predecessor Tasks

Predecessor tasks are tasks that must be completed before the current task can start. They are marked in grey in the chart.

...

Question: What's the maximum delay?



## Difficulty of Scheduling

### SPT with Predecessors

- No solution in polynomial time
- NP-hard problem (no efficient algorithm)
- True for most scheduling problems!
- We can use heuristics, though

...

Question: What have we missed so far?

## Real-time Scheduling

### Interruptions

- In reality, we cannot predict the future
- We need to react to new tasks as they happen
- If we have a deadline, we might need to meet it
- Let's look at this for the earliest due date objective

...

#### Quick reminder

An earliest due date is a specific point in time by which a task must be completed. Under this objective, we want to minimize the maximum delay.

### Real-time EDD

8:00-12:00 Schedule:

| Task          | Duration | Deadline |
|---------------|----------|----------|
| Email A       | 20min    | 9:00     |
| Create PPT    | 60min    | 10:50    |
| Investor call | 10min    | 9:00     |
| Email B       | 30min    | 10:20    |
| Liquidity     | 40min    | 11:00    |
| Email C       | 20min    | 11:20    |
| Email D       | 40min    | 10:00    |

...

Question: Any ideas how to start with under the objective of the earliest due date?

### EDD Rule for Real-time

Rule:

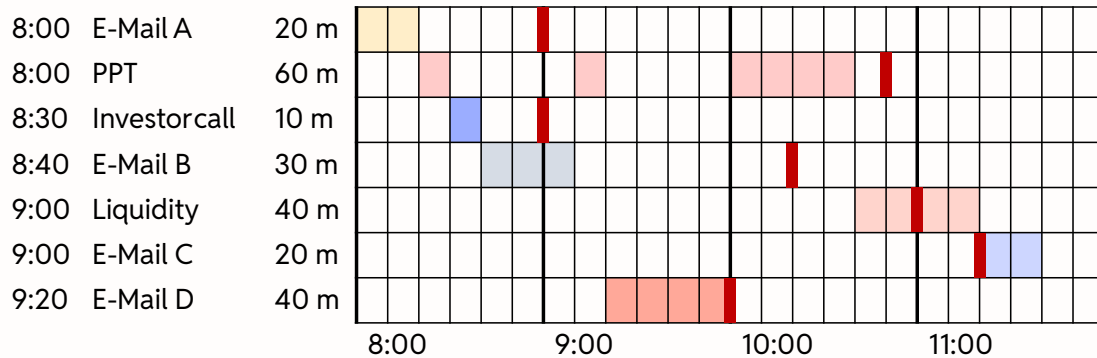
- Always schedule the task with the earliest deadline
- If a new task with an earlier deadline comes in, re-schedule
- Otherwise, stick to the original schedule.

...

#### 💡 Equal Deadline

If a new task has the same deadline as the current task, you can choose either. But due to the cost of context switching, you might want to stick with the current task.

### EDD Solution for Real-time



...

Question: What's the maximum delay with this schedule?

### SPT for Real-time

#### 💡 Quick reminder

A shortest processing time is the task with the shortest duration. Under this objective, we want to minimize the total waiting time.

...

Question: Any ideas how to start here?

### SPT Rule for Real-time

Rule:

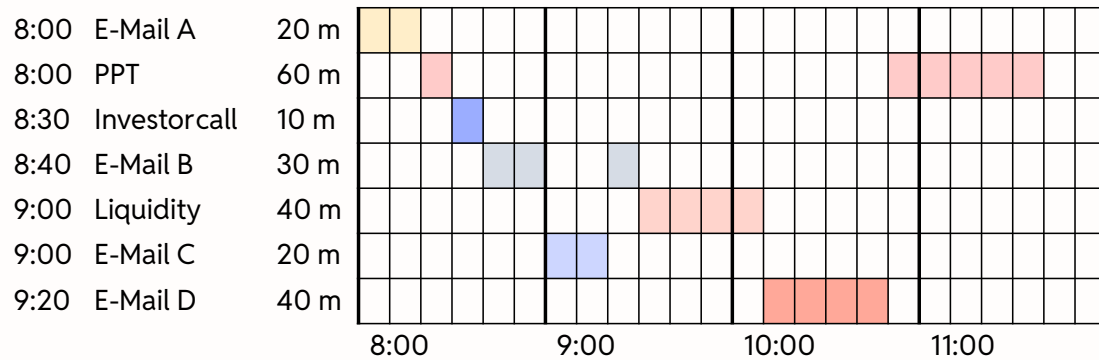
- Always schedule the task with the shortest duration
- If a new task with a shorter duration comes in, re-schedule
- Otherwise, stick to the original schedule.

...

#### i Equal Duration

If a new task has the same duration as the current task, you can choose either. But due to the cost of context switching, you might again want to stick with the current task.

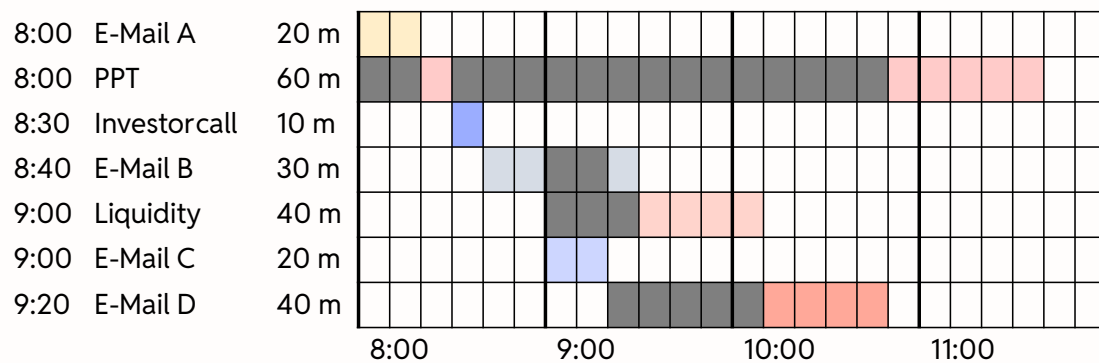
## SPT Solution for Real-time



...

Question: Where can we see the waiting time?

## SPT Solution for Real-time



Total waiting time: 260 minutes

## Thrashing

### What is Thrashing?

- Excessive context switching
- Organization overhead exceeds productivity
- Maximum activity, minimum output

...

Question: Have you ever experienced this?

### Thrashing Warning Signs

- Constant task switching
- Nothing getting completed
- Increasing stress levels
- Declining quality

...

Question: Any ideas how to prevent this?

## Preventing Thrashing Strategic

### Strategic

Strategic solutions focus on long-term changes to prevent thrashing.

...

1. Task rejection/delegation threshold
2. Simplified organization systems
3. Minimum work period rules
4. Reduced reactivity requirements

## Preventing Thrashing Tactical

1. Time blocking
2. Focus periods
3. Task batching
4. Priority freezes

...

Question: What strategies have worked for you?

## Key Takeaways

1. Scheduling is crucial for effective time management
2. Different objectives need different algorithms
  - EDD for deadline management
  - SPT for waiting time reduction
3. Address thrashing early
4. Define your reactivity goals
5. Use appropriate algorithms as foundations

## The End

### Note

That's it for today's lecture!

We now have covered a brief introduction into scheduling. Now, we can start with the tutorials! As we have now a good foundation, we can start to apply some algorithms on data sets in Python.

## Literature

### Interesting literature to start

- Christian, B., & Griffiths, T. (2016). Algorithms to live by: the computer science of human decisions. First international edition. New York, Henry Holt and Company.<sup>1</sup>

## Books on Programming

- Downey, A. B. (2024). Think Python: How to think like a computer scientist (Third edition). O'Reilly. [Here](#)
- Elter, S. (2021). Schrödinger programmiert Python: Das etwas andere Fachbuch (1. Auflage). Rheinwerk Verlag.

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### Note

Think Python is a great book to start with. It's available online for free. Schrödinger Programmiert Python is a great alternative for German students, as it is a very playful introduction to programming with lots of examples.

## More Literature

For more interesting literature, take a look at the [literature list](#) of this course.

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<sup>1</sup>The main inspiration for this lecture. Nils and I have read it and discussed it in depth, always wanting to translate it into a course.