1. $y = -\frac{1}{2}gt^2 + ut + C$
$g = 10, u = 5, C = 1$
$y = -5t^2 + 5t + 1$
$y' = -10t + 5$
if $y' = -10t + 5 = 0$
$t = \frac{1}{2}$
$y'' = -10 < 0$
maximum height $= -\frac{1}{2} \times 10 \times (\frac{1}{2})^2 + 5 \times \frac{1}{2} + 1$
$= \frac{9}{4}$

2. $f(x) = 2x^3 - 3x^2 - 36x + 2$
$f'(x) = 6x^2 - 6x - 36$
if $f'(x) = 0$ $f'(x) = 6(x-3)(x+2) = 0$
$f''(x) = 6(x^2 - x - 6)$ $x = 3$ or $x = -2$
when $x = 3$ $f(x) = 2 \times 3^3 - 3 \times 3^2 - 36 \times 3 + 2$
$= 54 - 27 - 108 + 2$
$= -79$
when $x = -2$ $f(x) = 2 \times (-2)^3 - 3 \times (-2)^2 - 36 \times (-2) + 2$
$= 2 \times (-8) - 3 \times 4 + 72 + 2$
$= 46$
when $x = 3$ $f''(x) = 12x - 6 = 12 \times 3 - 6 > 0$
minimum $= -79$
when $x = -2$ $f''(x) = 12x - 6 = 12 \times (-2) - 6 < 0$
maximum $= 46$

3.
$f(x) = \frac{1}{1 + \exp(-m(x-x_0))}$
(a) when $m = 0$, $f(x) = \frac{1}{2}$
when $m > 0$, $\lim\limits_{x \to \infty} f(x) = 1$, $\lim\limits_{x \to -\infty} f(x) = 0$
when $m < 0$, $\lim\limits_{x \to \infty} f(x) = 0$, $\lim\limits_{x \to -\infty} f(x) = 1$

(b) $f'(x) = \frac{m \cdot \exp(-m(x-x_0))}{[1 + \exp(-m(x-x_0))]^2}$
$= m \left[\frac{1 + \exp(-m(x-x_0))}{[1 + \exp(-m(x-x_0))]^2} - \frac{1}{[1 + \exp(-m(x-x_0))]^2} \right]$
$= m \frac{1}{1 + \exp(-m(x-x_0))} \left[1 - \frac{1}{1 + \exp(-m(x-x_0))} \right]$
$= m f(x)(1 - f(x))$
$f''(x) = m \left\{ f'(x)[1 - f(x)] + f(x)[1 - f(x)]' \right\}$
$= m \left[f'(x) - f'(x) f(x) - f'(x) \cdot f(x) \right]$
$= m \left[f'(x) - 2 f'(x) f(x) \right]$
$= m f'(x) [1 - 2 f(x)]$
$= m f(x)(1 - f(x))[1 - 2f(x)]$
the stationary point $\Rightarrow f''(x) = 0 \Rightarrow$ { $f(x) = 0$, impossible
{ $f(x) = 1$, impossible
{ $f(x) = \frac{1}{2}$,
when $f(x) = \frac{1}{2}$, $\exp(-m(x-x_0)) = 1$
$f(x) = \frac{1}{2} \Rightarrow \exp(-m(x-x_0)) = 1 \to -m(x-x_0) = 0$
$x = x_0$

```
In [149]: x=np.linspace(-5,5,200)
          y = 1/(1+ np.exp(-x+1))
          plt.xlabel("x")
          plt.ylabel("y")

          plt.plot(x,y,color ='r')
          plt.title('1/(1+ np.exp(-x+1)')
```

Out[149]: Text(0.5, 1.0, '1/(1+ np.exp(-x+1)')

(c) When $m = 1$, $x_0 = 1$ $f(x) = \frac{1}{1 + \exp(-x+1)}$
From the plot, when $f(x) = 1$, $f(x) = 0$
$x \to \infty$ $x \to -\infty$

4. $E = -\sum\limits_{i=1}^{2} \sum\limits_{j=1}^{2} W_{ij} x_i x_j$
$= -(W_{11} x_1 x_1 + W_{12} x_1 x_2 + W_{22} x_2 x_2 + W_{21} x_2 x_1)$
$= -(W_{11} x_1^2 + W_{12} x_1 x_2 + W_{22} x_2^2 + W_{21} x_2 x_1)$
$\frac{\partial E}{\partial x_1} = -(2 W_{11} x_1 + W_{12} x_2 + W_{21} x_2)$
$= -2 W_{11} x_1 - (W_{12} + W_{21}) x_2$

$\frac{\partial E}{\partial W_{12}} = -(x_1 x_2)$
$= -x_1 x_2$