CENG 384 - Signals and Systems for Computer Engineers
Spring 2020
Written Assignment 1

Aydın, Onur
e217126@metu.edu.tr

Bütün, Beyza
e2171411@ceng.metu.edu.tr

March 4, 2020

---

1. (a) Assume z = $x + yj$ and z+1 = j-3$\bar{z}$ is given.
   z+1 $= j - 3\bar{z}$
   z+3$\bar{z}$ $= j - 1$
   $x + yj$+$3x - 3yj = j - 1$
   $4x - 2yj = j - 1$
   From this equation x=-1/4 and y=-1/2.

   Therefore,
   z=$\frac{-1}{4} - \frac{1}{2}j$

   (i) $|z|^2 = (\sqrt{(-1/4)^2 + (-1/2)^2})^2 = 5/16$

   (ii) To plot z on complex plane, we should know the magnitude and the degree of it.
   As we calculated, the magnitude $|z| = (\sqrt{(-1/4)^2 + (-1/2)^2}) = \sqrt{5}/4$

   Let's find the angle:
   $\angle z = \theta = \arctan(y/x)$
   $\arctan((-1/2)/(-1/4)) + \pi$ (We added π because x,y<0)
   $\arctan(2) + \pi \approx 243.43°$

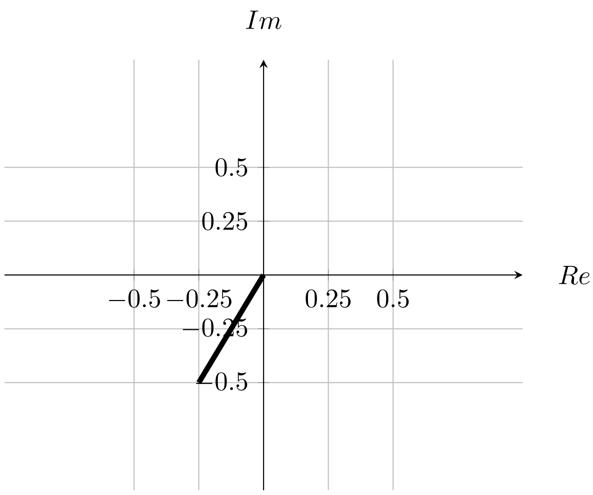

Figure 1: z=$\frac{-1}{4} - \frac{1}{2}j$.

   (b) First, we know when $z = re^{j\theta}$, $z^n = (re^{j\theta})^n = r^n e^{jn\theta}$ and $z^{1/n} = r^{1/n} e^{j\theta/n}$
   $z^2 = 25j$ is given. So we can conclude that $z^2 = 25e^{(\pi/2)j}$.

   Then, $z = (25)^{1/2} e^{\frac{(\pi/2)j}{2}} = 5e^{(\pi/4)j} = 5\cos\frac{\pi}{4} + j5\sin\frac{\pi}{4}$

   (c) $1 + j = \sqrt{2}e^{(\pi/4)j}$
   $1 - j = \sqrt{2}e^{-(\pi/4)j}$

$1-\sqrt{3}j = 2e^{-(\pi/3)j}$
Then,

$z = \frac{(\sqrt{2}e^{(\pi/4)j})(2e^{-(\pi/3)j})}{\sqrt{2}e^{-(\pi/4)j}}$
$z = 2e^{(\frac{\pi}{4}-\frac{\pi}{3}+\frac{\pi}{4})j}$
$z = 2e^{(\pi/6)j}$

So, the magnitude $|z| = 2$ and the angle is $\pi/6$.

(d) $z = je^{-j\pi/2} = e^{j\pi/2}e^{-j\pi/2} = e^{0j} = 1$

2. The signal can be represented as $x(t) \begin{cases} 0 & 0 \le t \le 1 \\ t-1 & 1 < t \le 3 \\ 2 & 3 < t \le 5 \\ 7-t & 5 < t \le 7 \\ 0 & 7 < t \le 8 \end{cases}$

To find x(2t-2), all t should be 2t-2. Therefore,

$$x(2t-2) \begin{cases} 0 & 0 \le 2t-2 \le 1 \\ 2t-3 & 1 < 2t-2 \le 3 \\ 2 & 3 < 2t-2 \le 5 \\ 9-2t & 5 < 2t-2 \le 7 \\ 0 & 7 < 2t-2 \le 8 \end{cases} == x(2t-2) \begin{cases} 0 & 1 \le t \le 3/2 \\ 2t-3 & 3/2 < t \le 5/2 \\ 2 & 5/2 < t \le 7/2 \\ 9-2t & 7/2 < t \le 9/2 \\ 0 & 9/2 < t \le 5 \end{cases}$$

Finally;

$$y(t) = \frac{1}{2}x(2t-2) \begin{cases} 0 & 1 \le t \le 3/2 \\ t-3/2 & 3/2 < t \le 5/2 \\ 1 & 5/2 < t \le 7/2 \\ 9/2-t & 7/2 < t \le 9/2 \\ 0 & 9/2 < t \le 5 \end{cases}$$

And $y(t) = \frac{1}{2}x(2t-2)$ plot will be as the following.

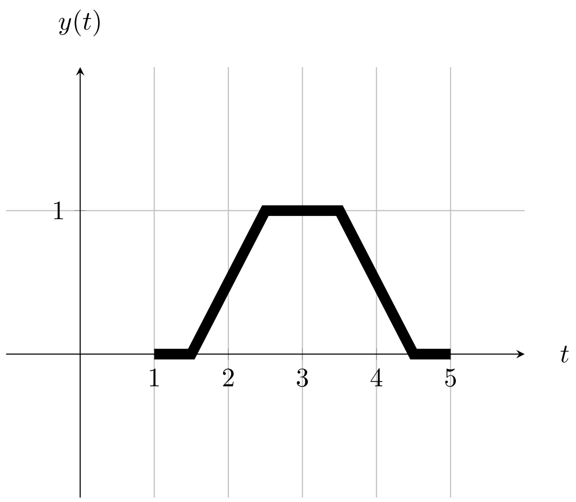

Figure 2: t vs. $y(t)$.

3. (a) $x[-n] + x[2n-1]$ can be drawn as the following:

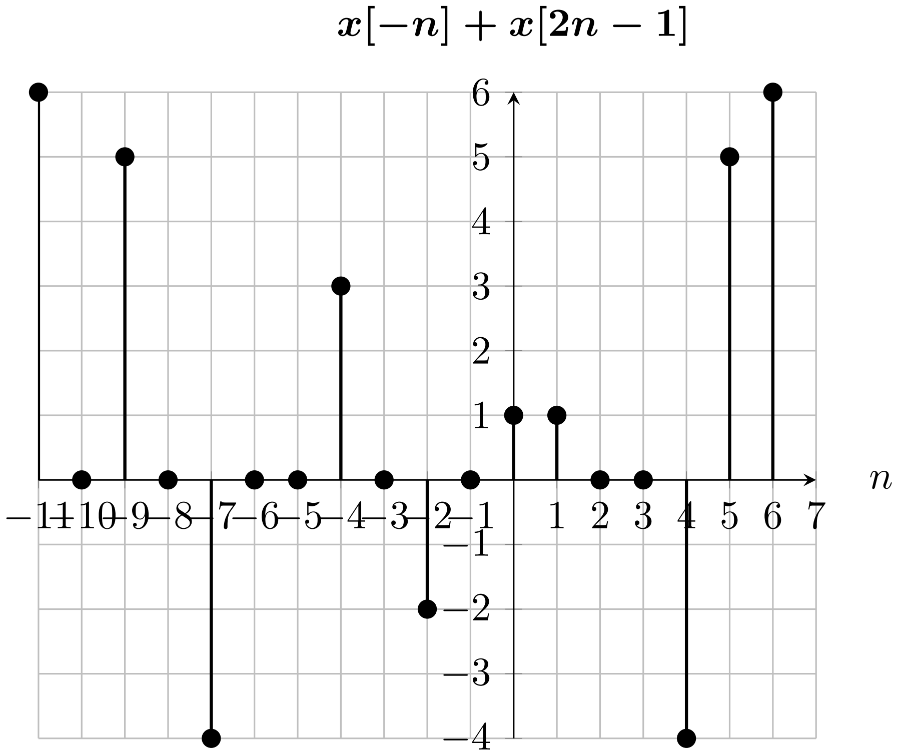

Figure 3: n vs. $x[-n] + x[2n-1]$.

(b) $x[-n]+x[2n-1] = 6\delta[n+11]+5\delta[n+9]-4\delta[n+7]+3\delta[n+4]-2\delta[n+2]+\delta[n]+\delta[n-1]-4\delta[n-4]+5\delta[n-5]+6\delta[n-6]$

4. (a) $x[n] = 7sin[\frac{5\pi}{8}n - \frac{2\pi}{3}] + 2cos[\frac{2\pi}{3}n]$

For only $sin[\frac{5\pi}{8}n - \frac{2\pi}{3}]$ part,
$N_1 = \frac{2\pi}{\frac{5\pi}{8}}k = \frac{16}{5}k$ If we choose k=5, $N_0 = 16$

For only $cos[\frac{2\pi}{3}n]$ part,
$N_2 = \frac{2\pi}{\frac{2\pi}{3}}k = 3k$ If we choose k=1, $N_0 = 3$

The signal $x[n]$ is periodic with fundamental period $N_0 =$LCM$(16, 3) = 48$.

(b) $x[n] = 3cos[5n - \frac{3\pi}{4}]$

The signal $x[n]$ is not periodic because
$N = \frac{2\pi}{5}k$, where there is no integer k to make the fundamental period the smallest possible integer.

(c) $x(t) = 4sin(5\pi t - \frac{3\pi}{5})$

$T = \frac{2\pi}{5\pi}k = \frac{2}{5}k$
The signal $x(t)$ is periodic where $T = \frac{2}{5}k$, if we choose k=1, $T_0 = \frac{2}{5}$.

(d) $x(t) = x(t+T_0) = je^{j2t} = je^{j2(t+T_0)}$

$e^{j2T_0} = 1 = cos(2T_0) + jsin(2T_0)$
$2T_0 = 2\pi k$
The signal x(t) is periodic. If we choose k as 1, the fundamental period $T_0 = \pi$.

5. The signal is neither even nor odd, so we can find the even and odd compositions of the signal as the following:

x(t) = Odd{x(t)} + Ev{x(t)}
Odd{x(t)} = 1/2(x(t)-x(-t))
Ev{x(t)} = 1/2(x(t)+x(-t))

The figures of the odd and even signals are as follows:

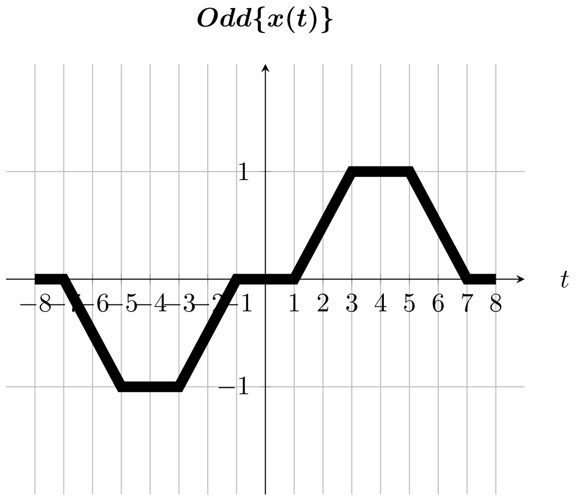

Figure 4: t vs. $Odd\{x(t)\}$.

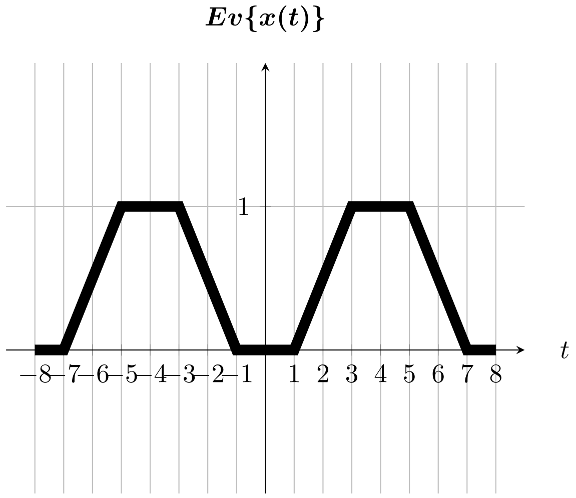

Figure 5: t vs. $Ev\{x(t)\}$.

6. (a) x(t) = u(t-1) - 3u(t-3) + 4u(t-4)

   (b) We know that the unit step function u(t) = $\int_{-\infty}^{t} \delta(\tau) d\tau$ and $\delta(t) = \frac{du(t)}{dt}$.

   Therefore;
   $\frac{dx(t)}{dt} = \delta(t-1) - 3\delta(t-3) + 4\delta(t-4)$

   The following figure shows the unit impulse function:

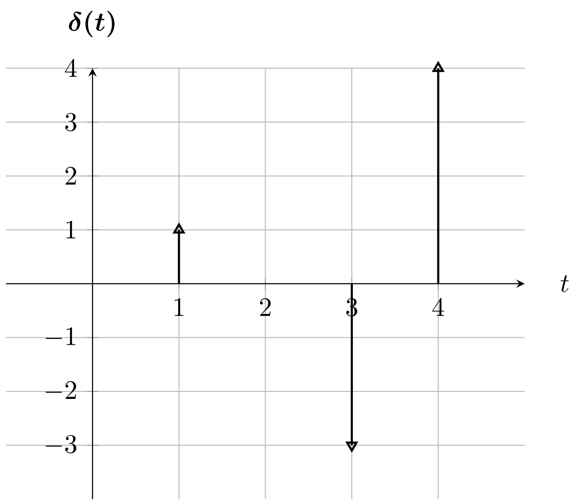

Figure 6: t vs. $\delta(t)$.