# CS301 A1

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## 1 Problem 1

(a)$\Theta(n^3)$

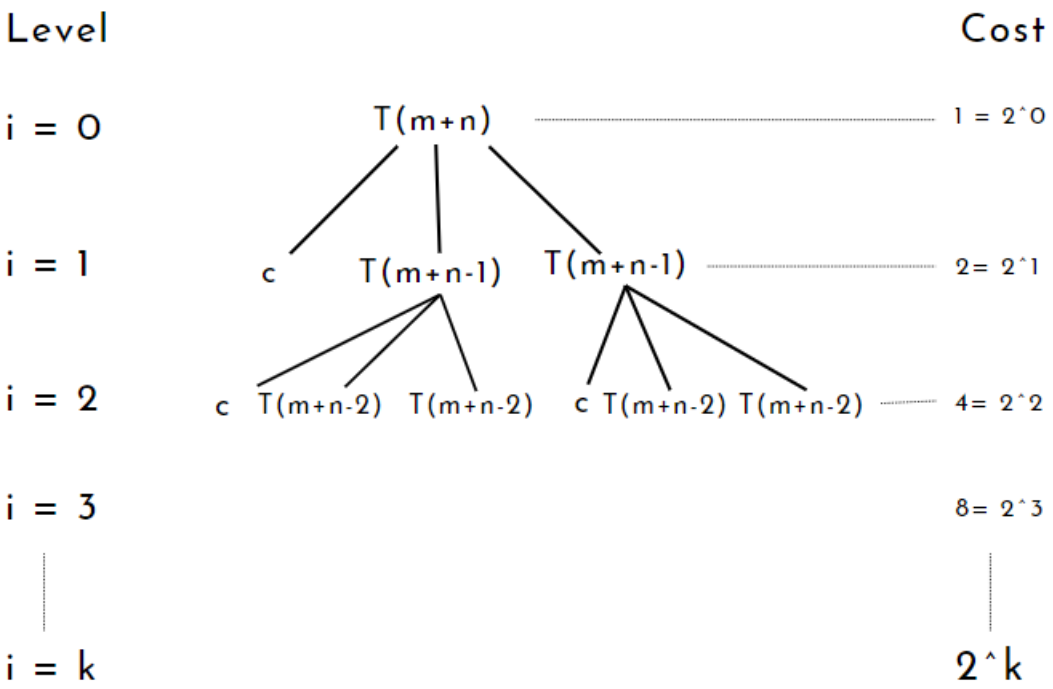
$(b)\Theta(n^{\log_2 7})$

$(c)\Theta(\sqrt{n}\log n)$

$(d)\Theta(n^2)$

## 2 Problem 2

(a)
(i) Best asymptotic worst case means there is no common sub-sequence between these 2 words. That means the algorithm will try to find common sub-sequence and will fail. In that way, it will be searching from the beginning to the end. In figure 1, which shows the naive algorithm, since length of lcs is 0, else part will be called every time until the end. Else part is calling the max of lcs(X,Y,i,j-1) and lcs(X,Y,i-1,j) recursively. We want to find T(m+n).
T(m+n) = T(m + (n-1)) + T ((m-1) + n) + $O(1)$
it is equal to:
T(m+n) = 2T(m+n-1) + $O(1)$, m+n is greater than 0
T(m+n) = 1, m+n = 0
Let's solve this equation with recursion tree. We put c instead of $O(1)$




When we sum up the costs the total cost is $1 + 2 + 2^2 + 2^3 + ...2^k$

According to the math rules it is equal to $2^{k+1} - 1$

$We assume m + n - k = 0 then k = m + n$

$Then we get 2^{m+n} - 1 as a result$

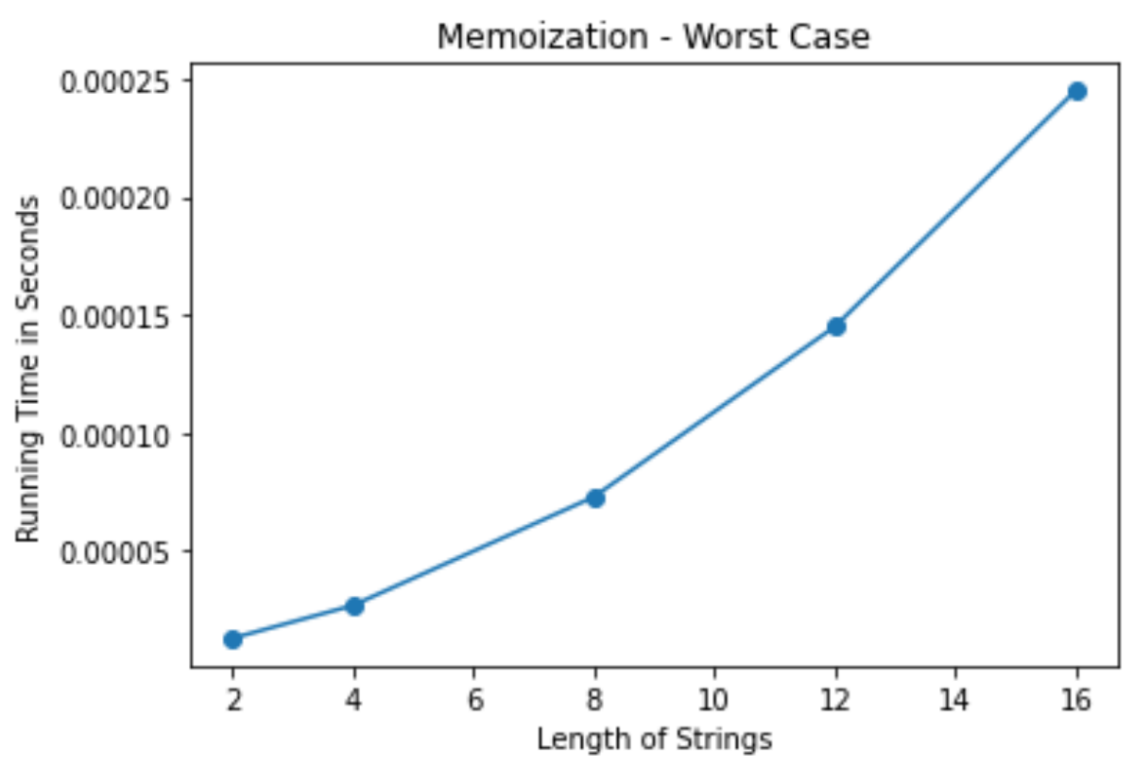
$That means our time complexity is O(2^{m+n})$

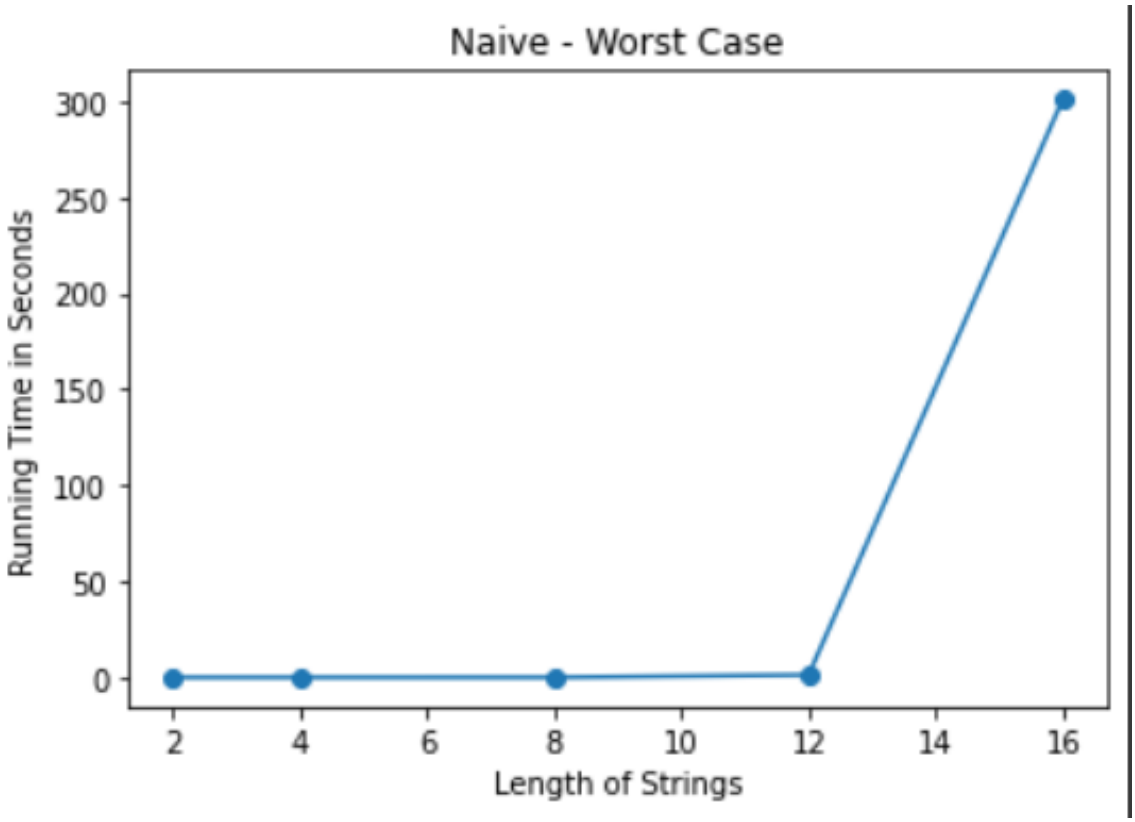
(b)

(i) macOS Monterey, 2,6 GHz 6-Core Intel Core i7, 16 GB RAM

| Algorithm | $m = n = 2$ | $m = n = 4$ | $m = n = 8$ | $m = n = 12$ | $m = n = 16$ |
|---|---|---|---|---|---|
| Naive | 0.0000081 | 0.000052 | 0.007712 | 1.315021 | 300.984532 |
| Memoziation | 0.0000132 | 0.000027 | 0.0000731 | 0.000148 | 0.000246 |

(ii)


Memoization - Worst Case
Running Time in Seconds
0.00025
0.00020
0.00015
0.00010
0.00005
2
4
6
8
10
12
14
16
Length of Strings



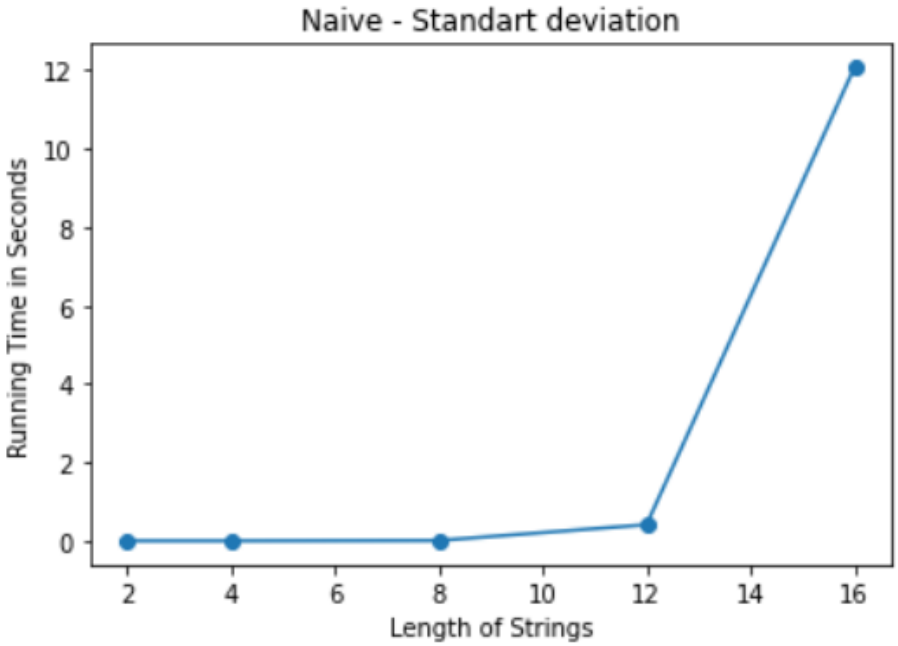

(c)

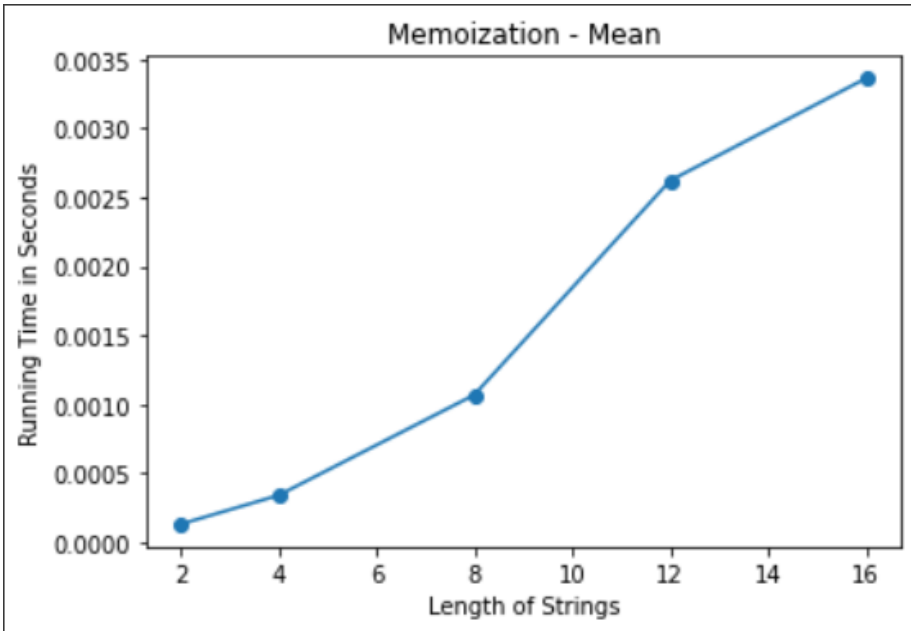
(i)

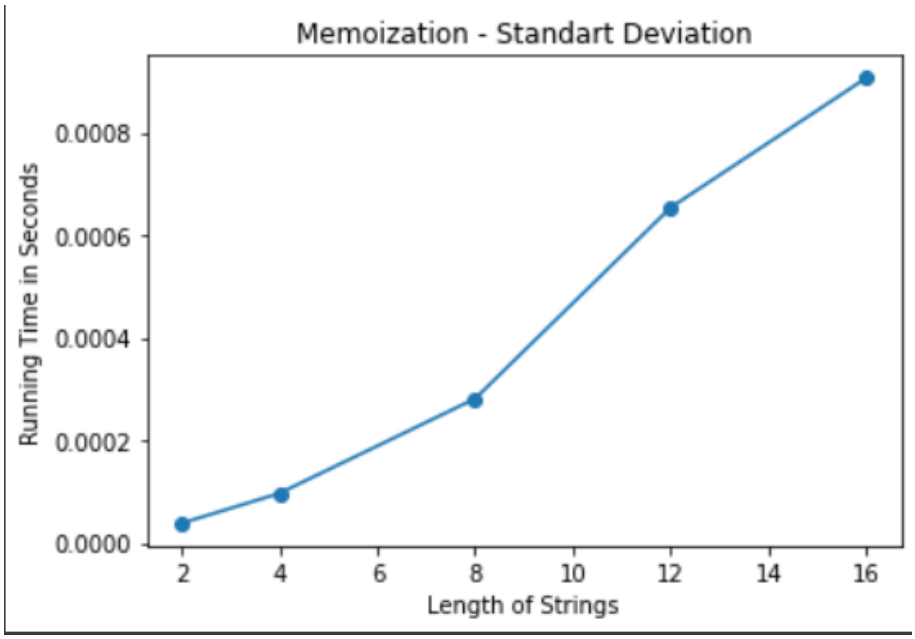
| Algorithm | m = n = 2 | | m = n = 4 | | m = n = 8 | |
|---|---|---|---|---|---|---|
| | μ | σ | μ | σ | μ | σ |
| Naive | 0.0000078 | 0.0000062 | 0.000043 | 0.00018 | 0.00200 | 0.00829 |
| Memoization | 0.000134 | 0.000038 | 0.0003439 | 0.000096 | 0.001073 | 0.0002810 |

| Algorithm | m = n = 12 | | m = n = 16 | |
|---|---|---|---|---|
| | μ | σ | μ | σ |
| Naive | 0.09420 | 0.410899 | 3.365674 | 12.0742688 |
| Memoization | 0.002621 | 0.000654 | 0.003368 | 0.000906 |

(ii)


Naive - Standart deviation
Running Time in Seconds
Length of Strings
0
2
4
6
8
10
12
2
4
6
8
10
12
14
16


Memoization - Mean
Running Time in Seconds
Length of Strings
0.0000
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
0.0035
2
4
6
8
10
12
14
16




For this homework I have studied with Ebrar Guler's laptop since my laptop is in service.