GEBZE TECHNICAL UNIVERSITY

ELECTRONICS ENGINEERING DEPARTMENT

ELM 365 Fundamentals of Digital Communication

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Introduction

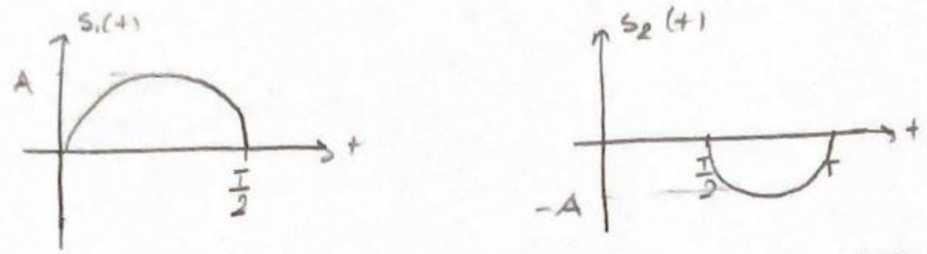
This project focuses on the transmission of binary signals in a baseband communication system, where two distinct signals represent the binary digits: ‘1’ and ‘0’. Specifically, we utilize a sinusoidal waveform for ‘1’ and its negative counterpart, delayed by half the period, for ‘0’. These signals are transmitted through an Additive White Gaussian Noise (AWGN) channel, a common theoretical model for noise in digital communication systems.

The primary objective of this project is to analyze and quantify the Bit Error Rate (BER), a critical performance metric that measures the rate at which errors occur in the received binary data compared to the transmitted data.

Analytical Solution

At first, the signal given in sinusoidal form $s_1(t)$ and $s_2(t)$ are sketched and their power is calculated by using a sinusoidal signal from equation which is the square of amplitude divided by 2 it’s multiplied with signals time interval.

Then using the bit probability of each bit and their energy, average bit energy can be found.

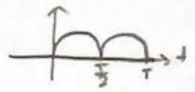


$$P_{s_1} = \frac{A^2}{2} \cdot \frac{T}{2} = \frac{A^2 T}{4} \qquad P_{s_2} = \frac{A^2}{2} \cdot \frac{T}{2} = \frac{A^2 T}{4}$$

$$E_b = \underbrace{E_{s_1}}_{\frac{A^2 T}{4}} \cdot \underbrace{P(s_1)}_{\frac{1}{2}} + \underbrace{E_{s_2}}_{\frac{A^2 T}{4}} \cdot \underbrace{P(s_2)}_{\frac{1}{2}} = \boxed{\frac{A^2 T}{4} = E_b}$$

For each $s_1(t)$ and $s_2(t)$ signal their a_1 and a_2 values are calculated like below. They are each other’s negatives and that is something expected.

$$a_1 = \int_0^T s_1(t)\left[s_1(t)-s_2(t)\right]dt = \int_0^{T/2} s_1^2(t)\,dt = \int_0^{T/2} A^2 \underbrace{\sin^2\left(\frac{2\pi t}{T}\right)}_{\frac{1}{2}\left[1-\cos\left(\frac{2\cdot 2\pi t}{T}\right)\right]}dt$$



$$= \frac{A^2 t}{2} - \frac{A^2}{2}\sin\left(\frac{4\pi t}{T}\right)\Big|_0^{T/2}$$

$$= \frac{A^2}{2}\left(t - \sin\left(\frac{4\pi t}{T}\right)\right)\Big|_0^{T/2} = \frac{A^2}{2}\left(\left[\frac{T}{2} - \underbrace{\sin(2\pi)}_{0}\right] - \left[0 - \underbrace{\sin(0)}_{0}\right]\right)$$

$$\boxed{a_1 = \frac{A^2 T}{4}}$$

$$a_2 = \int_0^T s_2(t)\left[s_1(t)-s_2(t)\right]dt = -\int_{T/2}^{T} s_2^2(t)\,dt = -\int_0^{T/2} +A^2 \sin^2\left(\frac{2\pi t}{T}\right)dt$$

$s_1^2(t)$'nin aynısı olur. (eksilisi)

$$\boxed{a_2 = \frac{-A^2 T}{4}}$$

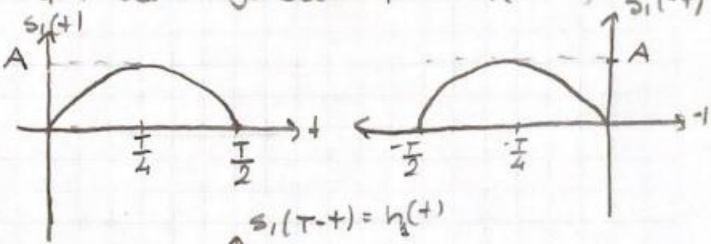
For the maximum SNR value h(t) is chosen to be $s_1(t)-s_2(t)$. Filter is plotted and its energy is calculated.

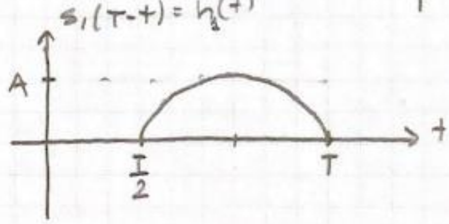
Then γ_0 is calculated with a_1 and a_2 values. Since these are same value and opposite sign their addition becomes zero so that γ_0 too. γ_0 stands between the a_1 and a_2 values in likelihood function. Also, γ_0 is important because it will be used as a comparator value in the comparator filter.

a) $P(1) = P(0) = 1/2$

$P_b = Q\left(\frac{a_1 - a_2}{2N_0}\right)$

$s(t)$ reel oldugundan $h_1(t) = s_1(T-t)$

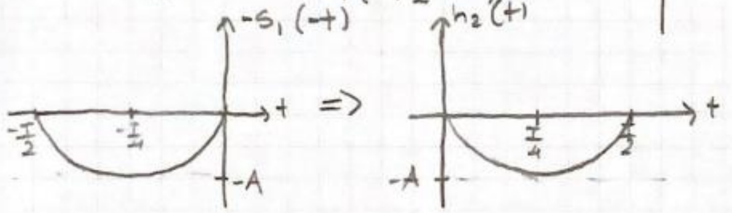




$s_2(t)$ reel oldugundan

$h_2(t) = s_2(T-t) = -s_1(T-t-T/2)$

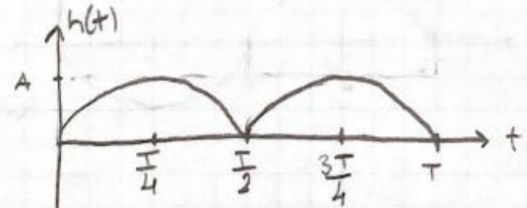
$s_2(T-t) = -s_1(T/2 - t)$



$h(t) = h_1(t) - h_2(t)$

$h(t) = s_1(T-t) - s_2(T-t)$

$$\boxed{\gamma_0 = \frac{a_1 + a_2}{2} = 0}$$



$$\boxed{E_h = \frac{A^2 T}{2}}$$

Lastly, to find bit error probability variance is calculated with the given steps below.

$E_b = \frac{A^2T}{4} = 1$ ise

$a_1 = \frac{A2T}{4}$ ve $a_2 = -\frac{A^2T}{4}$ de "1" olur.

$$\sigma_0^2 = \frac{N_0}{2}.E_h = \frac{N_0}{2}.\frac{A^2T}{2} = \boxed{\frac{N_0A^2T}{4}}$$

E_h'yi bulmuştuk

$$\boxed{\sigma_0 = \frac{A}{2}\sqrt{N_0.T}}$$

$$P_b = Q\left(\frac{a_1-a_2}{2\sigma_0}\right) = Q\left(\frac{\frac{A^2T}{4}-\left(-\frac{A^2T}{4}\right)}{2.\frac{A}{2}\sqrt{N_0T}}\right) = Q\left(\sqrt{\frac{A^2T}{4N_0}}\right)$$

$$\boxed{P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

$E_b = 1$ için $P_b = Q\left(\sqrt{\frac{1}{N_0}}\right)$

For the second part probability of bits are changed and they become ¼ for bit 1 and ¾ for bit 0.

Variance, γ_0 and bit error probability are calculated again.

Equation1 and equation 2 are general formula of γ_0 and bit error probability.

$$\gamma_0 = \frac{\sigma^2}{a1-a2}.\ln\frac{P(s2)}{P(s1)} + \frac{a1+a2}{2} \quad ,(equation\ 1)$$

$$P_b = \left[1 - Q\left(\frac{\gamma_0 - a1}{\gamma_0}\right)\right].P(s1) + Q\left(\frac{\gamma_0 - a2}{\gamma_0}\right).P(s2) \quad ,(equation\ 2)$$

b) $P(1) = \frac{1}{4}$, $P(0) = \frac{3}{4}$

sinüzoidal işaretin gücü

zamanla çarpılırsa enerjiyi verir

$$\sigma_0^2 = \frac{N_0}{2} \cdot E_h \qquad E_h = \int_0^T \left| -A\sin\left(\frac{2\pi t}{T}\right)\right|^2 dt = \frac{A^2 \cdot T}{2}$$

$$\sigma_0^2 = \frac{N_0 \cdot A^2 \cdot T}{4}$$

$$\gamma_0 = \frac{\frac{N_0 \cdot A^2 \cdot T}{4}}{\underbrace{\frac{A^2T}{4} - -\frac{A^2T}{4}}_{\frac{A^2T}{2}}} \cdot \underbrace{\ln\frac{\left(\frac{3}{4}\right)}{\left(\frac{1}{4}\right)}}_{\ln(3)} + \overbrace{\frac{\frac{A^2T}{4} + -\frac{A^2T}{4}}{2}}^{0}$$

$$\boxed{\gamma_0 = \frac{N_0}{2} \cdot (1.09) \cong \frac{N_0}{2}}$$

$$P_b = \left[1 - Q\left(\frac{\gamma_0 - a_1}{\sigma_0}\right)\right] P(s_1) + Q\left(\frac{\gamma_0 - a_2}{\sigma_0}\right) P(s_2)$$

$$\boxed{P_b = \left[Q\left(\frac{\frac{A^2T}{4} - \frac{N_0}{2}}{\frac{A}{2}\sqrt{N_0 T}}\right)\right] \cdot \frac{1}{4} + Q\left(\frac{\frac{N_0}{2} - -\frac{A^2T}{4}}{\frac{A}{2}\sqrt{N_0 T}}\right) \cdot \frac{3}{4}}$$

In order to plot SNR versus P_b a relation between N_o and SNR is needed and this relation can be found using the equation below. Since its known that E_b=1 equations becomes;

$$N_0 = 10^{\frac{-SNR}{10}} \qquad => \quad P_b = Q\left(\sqrt{\frac{1}{10^{\frac{-SNR}{10}}}}\right)$$

$$\boxed{SNR = 10\log_{10}\frac{E_b}{N_0}} \quad => \frac{SNR}{10} = \log_{10}\frac{E_b}{N_0} => 10^{\frac{SNR}{10}} = \frac{E_b}{N_0}$$

$$\boxed{N_0 = E_b \cdot 10^{-\frac{SNR}{10}}}$$

MATLAB codes

a) Find the bit error rate (BER) expression of this system over additive white Gaussian channel (AWGN) for P(1)=1/2, P(0)=1/2 and plot it. Do the simulation of the system to obtain BER curve versus SNR. Compare and comment on the theoretical and simulated BER curves.

```
% P(s1)=P(s2)  a) BER curve with calculations
A=2;
T=2*10^;
Ts= 0.000001;
t=0: Ts : T-Ts;

s1 = @(t) A.*sin((2*pi*t)/T) .* (0<=t & t<=T/2); %defining s1
s2 = @(t) -s1(t-T/2) .* (T/2<=t & t<=T); %defining s2

% plotting s1 and s2
figure;
subplot(2,1,1)
plot(t,s1(t))
xlabel('t(sn)')
ylabel('s1(t)')

subplot(2,1,2)
plot(t, s2(t))
xlabel('t(sn)')
ylabel('s2(t)')



Eb_calc = (A.^2)*T/2 ; % definnig Eb
SNRdb_calc= 0:0.01:16; % giving SNR values from 0 to 16
Pb_calc= qfunc(sqrt(10.^(SNRdb_calc/10))); %defining Pb equation including SNR

% plotting SNR versus Pb
figure;
semilogy(SNRdb_calc, Pb_calc);
xlabel('SNR(db) calculated');
ylabel('Pb calculated');
grid on;
```

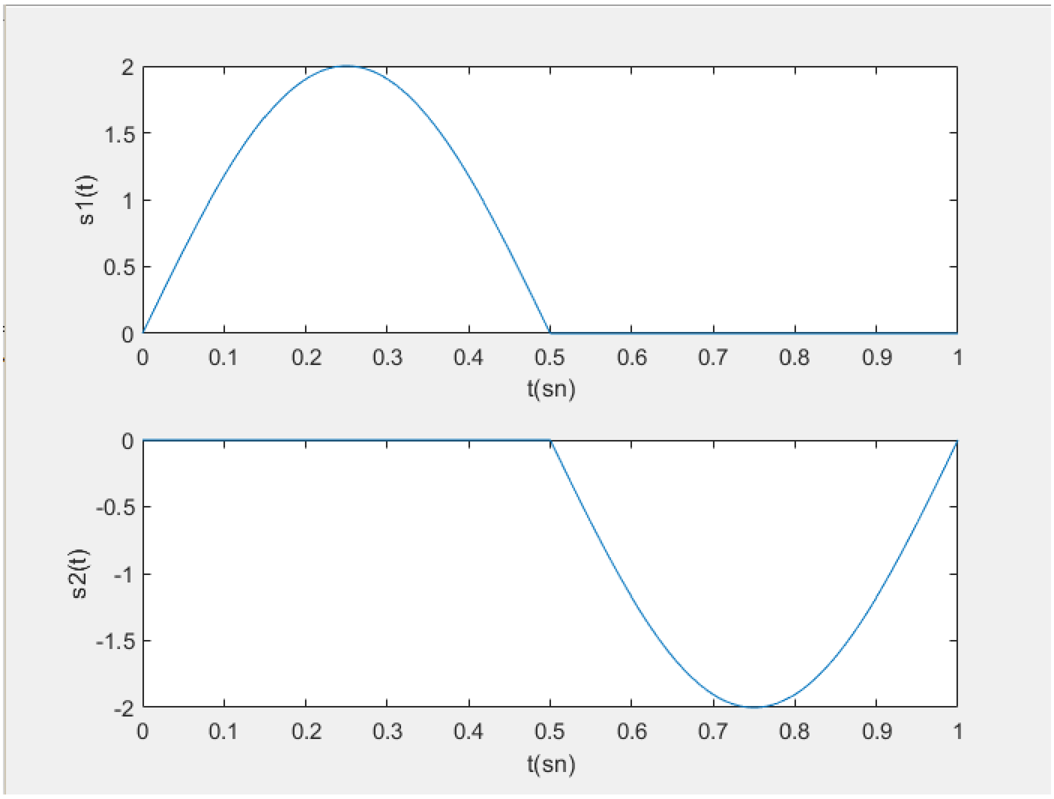


Figure 1 s1(t) and s2(t).

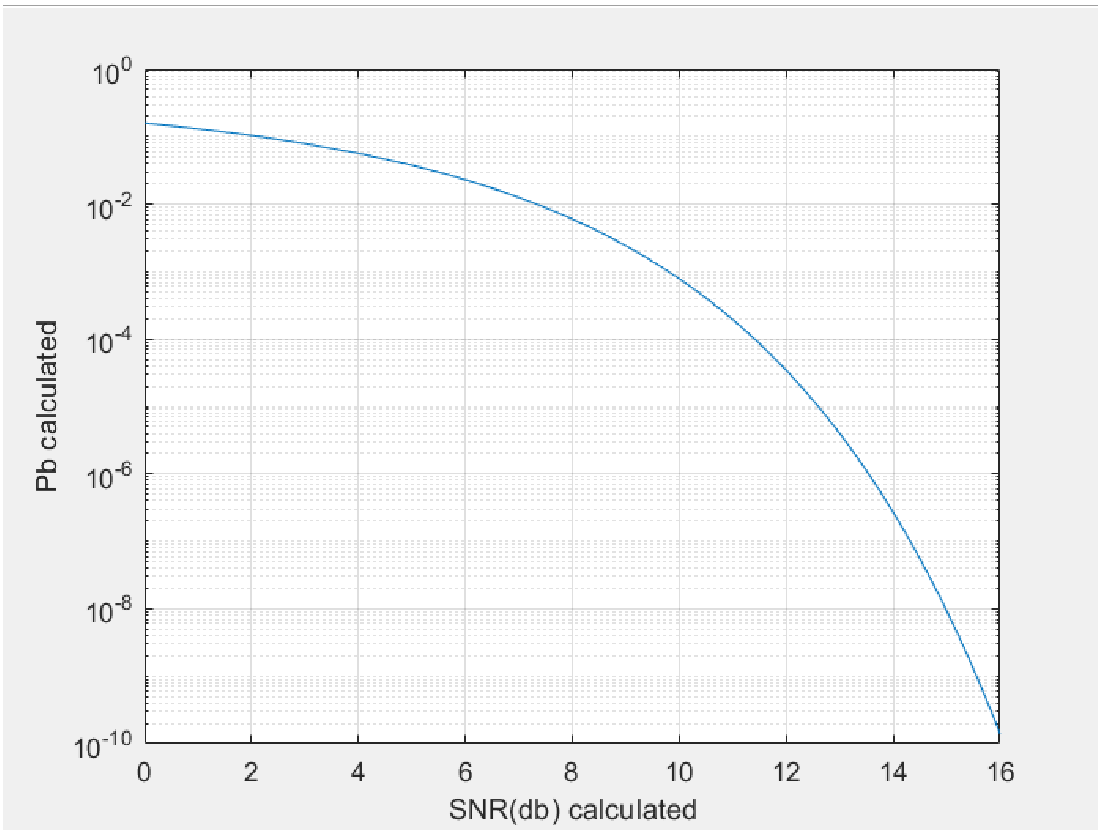
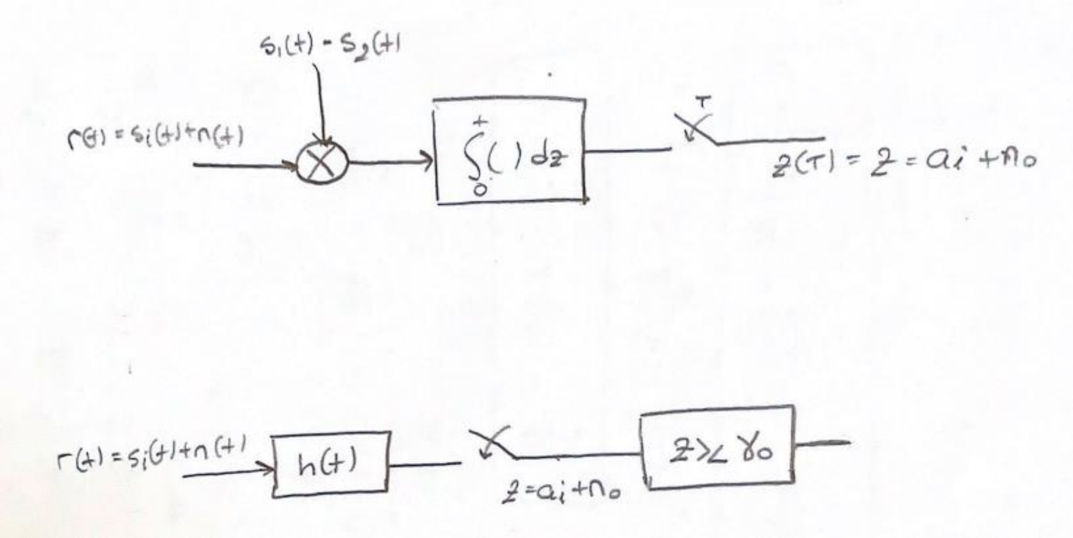


Figure 2 BER curve with calculated values in equal probability.

```
% P(s1) = P(s2)  b) BER curve with simulation

Eb = 1; % defining Eb giving 1 is prefered
SNRdb = 0:16; % defining SNR between 0 and 16


a1= 1; % since Eb=1 a1's value can be found
a2= -1; % also this can be found too
gama= (a1+a2)/2; % this gives gama as zero also this value was found in analytical solutions too


bit_sayim=10^7; %this parameter gives the number of bits that will be send
bitler = randi([0,1], 1, bit_sayim); %here with the randi function for a number of bit_sayim an array with
random 1's and 0's created with equal probability.

bit= zeros(1, length(bitler)); % an array with zeros and lenght of 10^7 is created

bit(bitler==1) = a1; %if a bit is equal to 1 then it becomes a1 in 'bit' array
bit(bitler==0) = a2; % same thing for a2


Pb= zeros(1, length(SNRdb)); % an array with zeros is created for bit error rate lenght of SNRdb since
they will be plotted they must be in same length
N0 = 10.^(-SNRdb./10); % noise is defined like before

for i= 1:length(SNRdb) %for each value of SNR same process repeted

   n0 = sqrt(N0(i)).*randn(1,bit_sayim); %varyansı N0'ın karekökü olcak onla çarpılması lazım
   z = bit + n0; %noise is added to bits

   compare = z>=gama; %z values compared with gama
   bit_hata=xor(bitler,compare); % original bit array is xored with compared if there is a difference xor
will give an 1 and if sum of that is calculated it will give the number of wrong bits
   Pb(i) = sum(bit_hata)/bit_sayim; %number of wrong bits divided to total number of bits gives the bit
error probability
end

% BER is plotted
figure;
semilogy(SNRdb, Pb);
xlabel('SNR(db)');
ylabel('Pb');
grid on;
```



Schematics above are used to create logic behind the code.

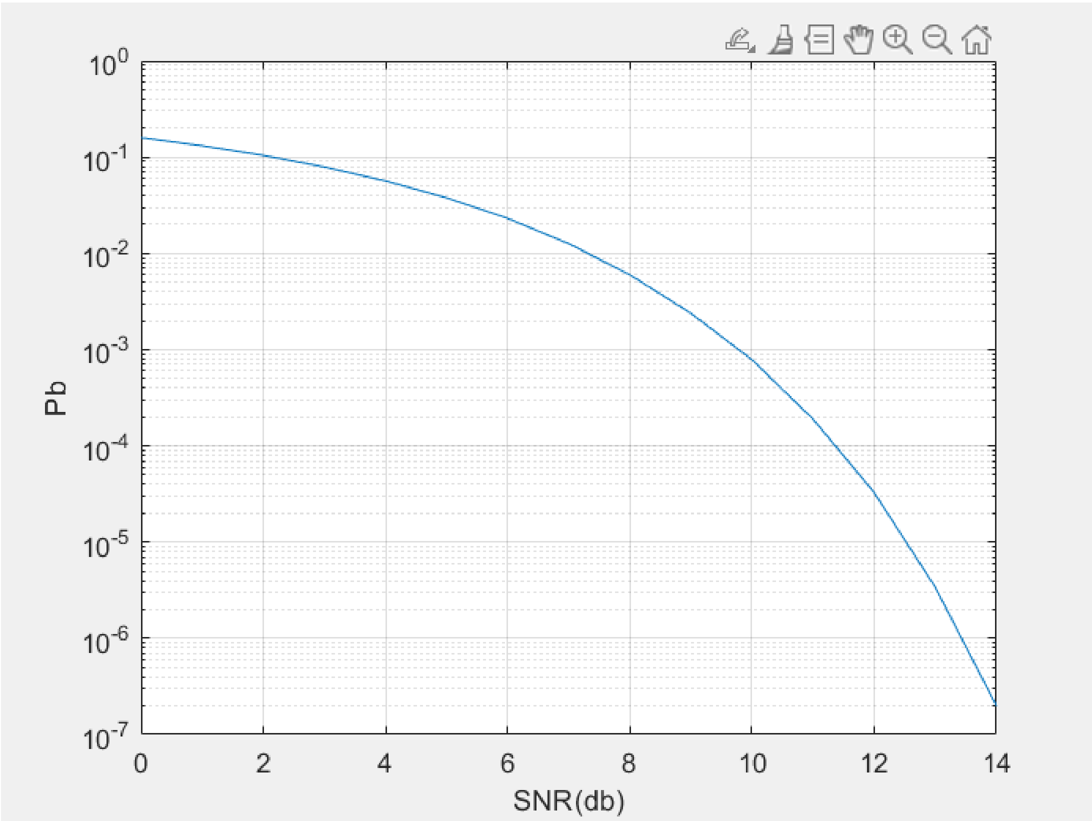


Figure 3 BER curve with Simulation a).

b) Find the bit error rate (BER) expression of this system over additive white Gaussian channel (AWGN) for P(1)=1/4, P(0)=3/4 and plot it. Do the simulation of the system to obtain BER curve versus SNR. Compare and comment on the theoretical and simulated BER curves.

```
% P(1)=1/4 ,P(0)=3/4  b) BER curve with calculations

gama_b= N0/2; % gama is defined from calculated equation

P1=0.75; % probability of 1 bit
P2=0.25; % probability of 0 bit

sigma_b=sqrt(N0); % sigma is defined from calculated equation

SNRdb_calc_b= 0:16; % defining SNR between 0 and 16
Pb_calc_b= (1-qfunc((gama_b-a1)./sigma_b)).*P2  + (qfunc((gama_b-a2)./sigma_b)).*P1  ; % bit error
rate is defined

% BER curve is plotted
figure;
semilogy(SNRdb_calc_b, Pb_calc_b); % semilog is recommended
xlabel('SNR(db) calculated b');
ylabel('Pb calculated b');
grid on;
```

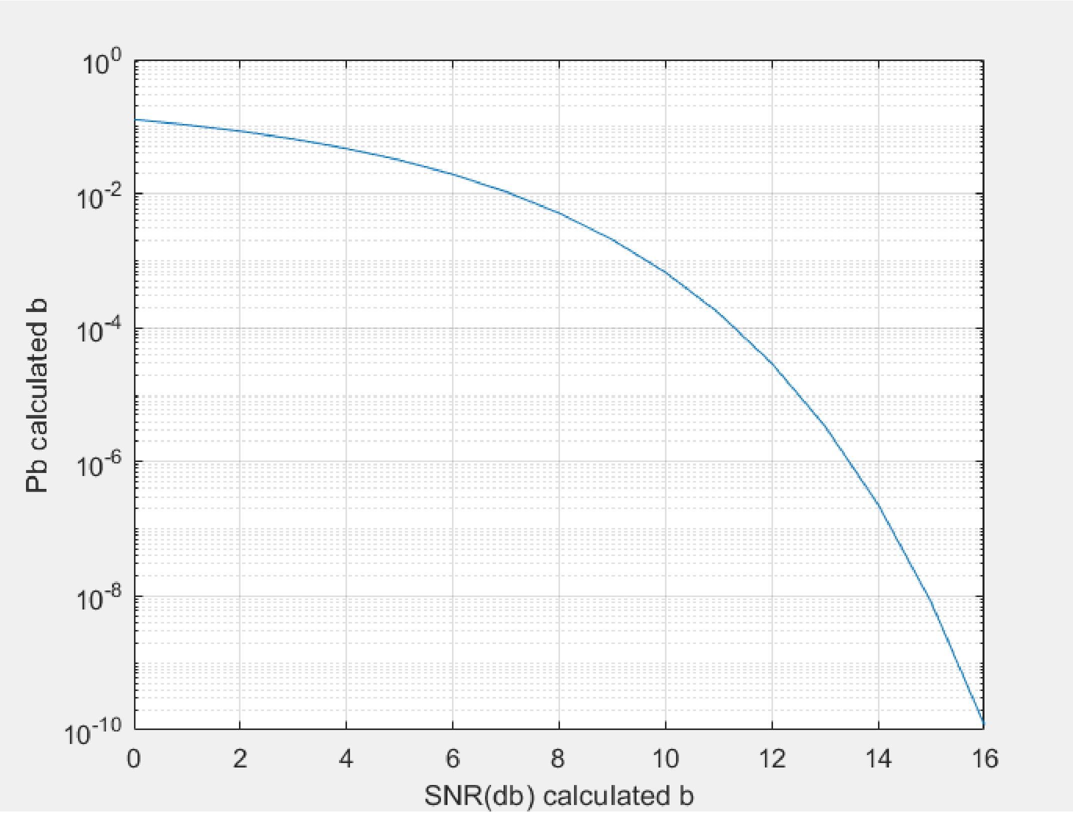


Figure 4 BER curve with calculated values in different probability.

```
% P(1)=1/4 ,P(0)=3/4  BER curve with simulation

bitler_b = randsrc(1,bit_sayim,[0 1; P1 P2]); % random bit array with 1's and 0's but this time array is
created with bits probability so when array is created there will be more 0 bits than 1

bit_b= zeros(1, length(bitler_b));

bit_b(bitler_b==1) = a1; %if a bit is equal to 1 then it becomes a1 in 'bit_b' array
bit_b(bitler_b==0) = a2; % same thing for a2

Pb= zeros(1, length(SNRdb_calc_b)); % an array with zeros is created
N0 = 10.^(-SNRdb_calc_b./10); % noise is defined

for i= 1:length(SNRdb_calc_b) %for each SNR value same process repeated

  n0 = sqrt(N0(i)).*randn(1,bit_sayim); %varyansı N0'ın karekökü olcak onla çarpılması lazım N0 is
changing
  z_b = bit_b + n0; % noise added to bits

  compare_b = z_b>=gama_b(i); % same things done in section a)
  bit_hata_b=xor(bitler_b,compare_b);
  Pb(i) = sum(bit_hata_b)/bit_sayim;
end

% Plotting BER curve
figure;
semilogy(SNRdb, Pb);
xlabel('SNR(db) b');
ylabel('Pb b');
grid on;
```

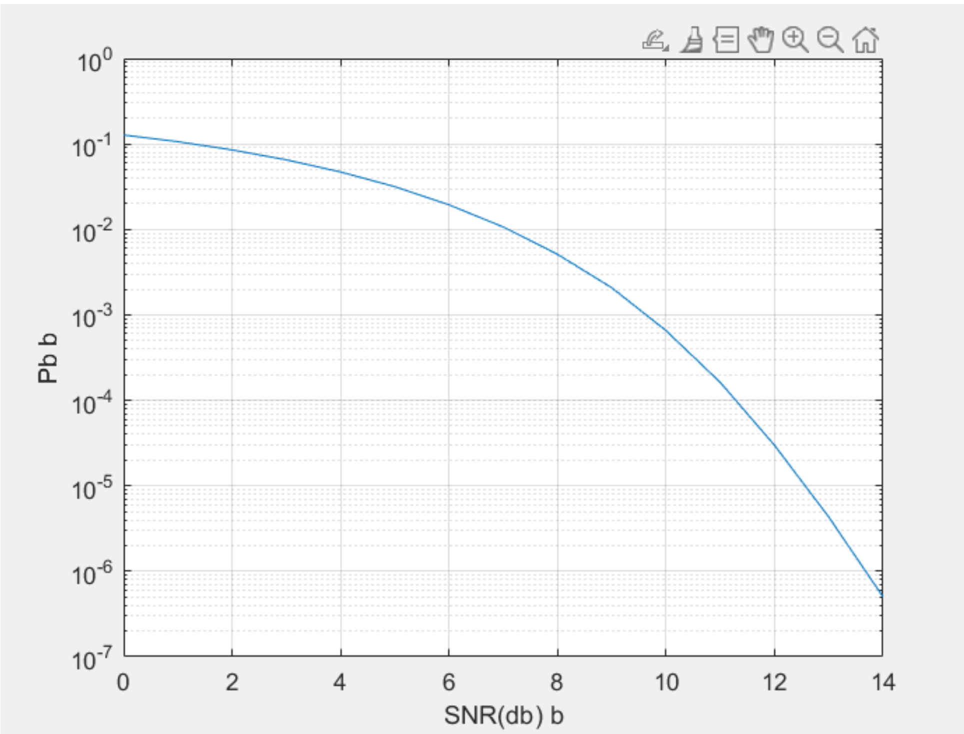


Figure 5 BER curve with Simulation b).

Conclusion

In conclusion for each questions calculated and simulated curves are same. This confirms the accuracy of calculations. As the SNR increases bit error probability decreases, which is expected. Because when noise is low change of comparator to make mistake lowers to. That's why curve decreases also the reason for it to be a curve is SNR's representation cause its represented with decibel.

For different probabilities of bits BER curves are almost identical reason for this is γ_0 and noise isn't changed that much. If the probability difference were much higher such as %5 and %95 there will be a bigger difference in BER curves.

SNR means signal to noise ratio so in communication system this ratio wanted to be greater as it can be. The greater the ratio, noise becomes less effective. And SNR is often represented with decibel (db)($10\log_{10}$).