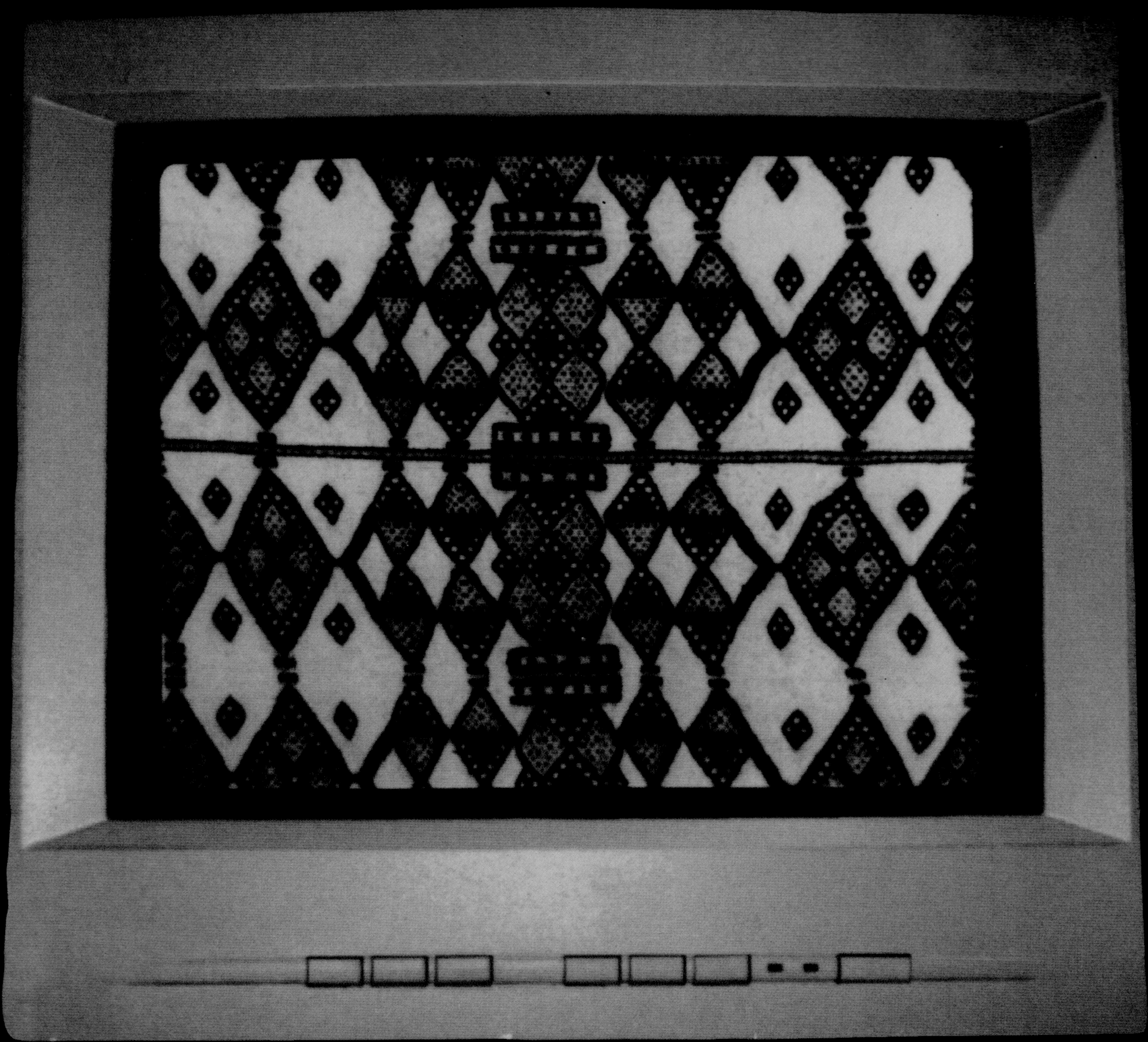


African Fractals

MODERN COMPUTING AND INDIGENOUS DESIGN

RON EGLASH

CHAPTER

2

Fractals in African settlement architecture

Architecture often provides excellent examples of cultural design themes, because anything that is going to be so much a part of our lives—a structure that makes up our built environment, one in which we will live, work or play—is likely to have its design informed by our social concepts. Take religious architecture for example. Several churches have been built using a triangular floor plan to symbolize the Christian trinity; others have used a cross shape. The Roman Pantheon was divided into three vertical levels: the bottom with seven niches representing the heavenly bodies, the middle with the 12 zodiac signs, and on top a hemisphere symbolizing the order of the cosmos as a whole.[1] But we don't need to look to grandiose monuments; even the most mundane shack will involve geometric decisions—should it be square or oblong? pitched roof or flat? face north or west?—and so culture will play a role here as well.

At first glance African architecture might seem so varied that one would conclude its structures have nothing in common. Although there is great diversity among the many cultures of Africa, examples of fractal architecture can be found in every corner of the African continent. Not all architecture in Africa is fractal—fractal geometry is not the only mathematics used in Africa—but its repeated presence among such a wide variety of shapes is quite striking.

In each case presented here we will compare the aerial photo or architectural diagram of a settlement to a computer-generated fractal model. The fractal simulation will make the self-similar aspects of the physical structure more evident, and in some cases it will even help us understand the local cultural meaning of the architecture. Since the African designers used techniques like iteration in building these structures, our virtual construction through fractal graphics will give us a chance to see how the patterns emerge through this process.

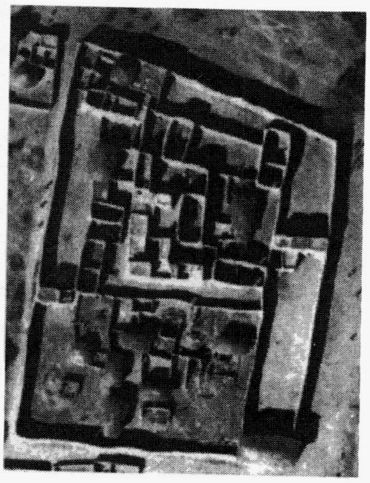
Rectangular fractals in settlement architecture

If you fly over the northern part of Cameroon, heading toward Lake Chad along the Logone River, you will see something like figure 2.1a. This aerial photo shows the city of Logone-Birni in Cameroon. The Kotoko people, who founded this city centuries ago, use the local clay to create huge rectangular building complexes. The largest of these buildings, in the upper center of the photo, is the palace of the chief, or "Miarre" (fig. 2.1b). Each complex is created by a process often called "architecture by accretion," in this case adding rectangular enclosures to preexisting rectangles. Since new enclosures often incorporate the walls of two or more of the old ones, enclosures tend to get larger and larger as you go outward from the center. The end result is the complex of rectangles within rectangles within rectangles that we see in the photo.

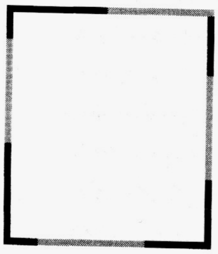
Since this architecture can be described in terms of self-similar scaling—it makes use of the same pattern at several different scales—it is easy to simulate using a computer-generated fractal, as we see in figures 2.1c–e. The seed shape of the model is a rectangle, but each side is made up of both active lines (gray) and passive lines (black). After the first iteration we see how a small version of the original rectangle is reproduced by each of the active lines. One more iteration gives a range of scales that is about the same as that of the palace; this is enlarged in figure 2.1e.

During my visit to Logone-Birni in the summer of 1993, the Miarre kindly allowed me to climb onto the palace roof and take the photo shown in figure 2.1f. I asked several of the Kotoko men about the variation in scale of their architecture. They explained it in terms of a combination of patrilocal household expansion, and the historic need for defense. "A man would like his sons to live next to him," they said, "and so we build by adding walls to the father's house." In the past, invasions by northern marauders were common, and so a larger defensive wall was also needed. Sometimes the assembly of families would outgrow this defensive enclosure, and so they would turn that wall into housing, and build an even larger enclosure around it. These scaling additions created the tradition of self-similar shapes we still see today, although the population is far below the

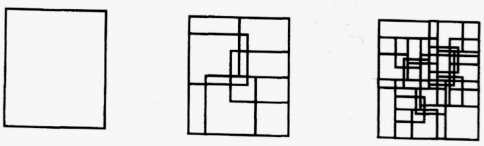
a. An aerial view of the city of Logone-Birni in Cameroon. The largest building complex, in the center, is the palace of the chief.
Photo courtesy Musée de l'Homme, Paris.



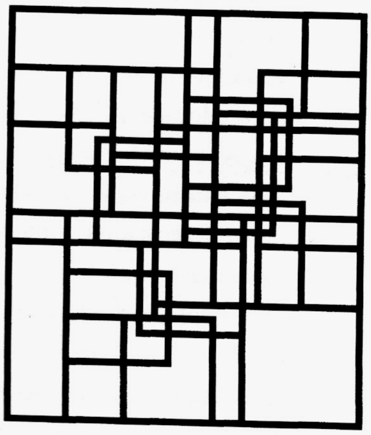
b. A closer view of the palace. The smallest rectangles, in the center, are the royal chambers.



c. Seed shape for the fractal simulation of the palace. The active lines, in gray, will be replaced by a scaled-down replica of the entire seed.



d. First three iterations of the fractal simulation.

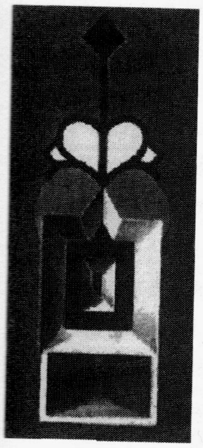


e. Enlargment of the third iteration.

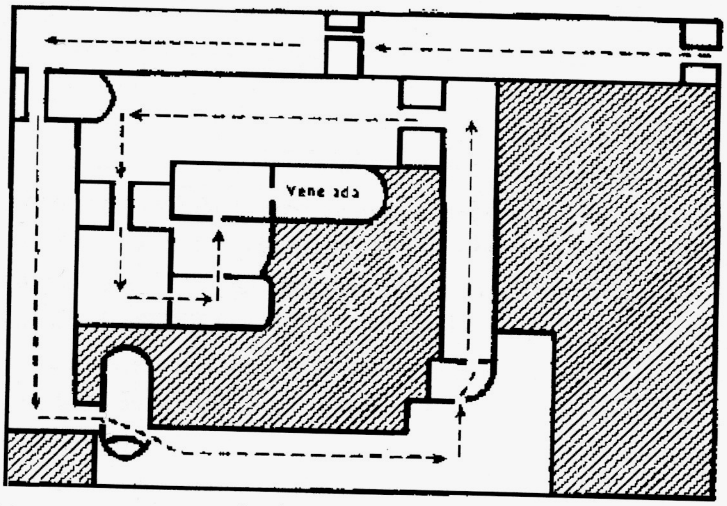
FIGURE 2.1
Logone-Birni

(figure continues)

f. Photo by the author taken from the roof of the palace.



g. The guti, the royal insignia, painted on the palace walls. *By permission of Lebeuf 1969.*



Le chemin de la lumière

h. The spiral path taken by visitors to the throne. *By permission of Lebeuf 1969.*

FIGURE 2.1 *(continued)*

Inside Logone-Birni

original 180,000 estimated for Logone-Birni's peak in the nineteenth century. At that time there was a gigantic wall, about 10 feet thick, that enclosed the perimeter of the entire settlement.

The women I spoke with were much less interested in either patrilineage or military history; their responses concerning architectural scaling were primarily about the contrast between the raw exterior walls and the stunning waterproof finish they created for courtyards and interior rooms. This began by smoothing wet walls flat with special stones, applying a resin created from a plant extract, and then adding beautifully austere decorative lines.

The most important of these decorative drawings is the *guti*, a royal insignia (fig. 2.1g). The central motif of the guti shows a rectangle inside a rectangle inside a rectangle; it is a kind of abstract model that the Kotoko themselves have created. The reason for choosing scaling rectangles as a symbol of royalty becomes clear when we look at the passage that one must take to visit the Miarre (fig. 2.1h). The passage as a whole is a rectangular spiral. Each time you enter a smaller scale, you are required to behave more politely. By the time you arrive at the throne you are shoeless and speak with a very cultured formality.[2] Thus the fractal scaling of the architecture is not simply the result of unconscious social dynamics; it is a subject of abstract representation, and even a practical technique applied to social ranking.

To the west near the Nigerian border the landscape of Cameroon becomes much greener; this is the fertile high grasslands region of the Bamileke. They too have a fractal settlement architecture based on rectangles (fig. 2.2a), but it has no cultural relation to that of the Kotoko. Rather than the thick clay of Logone-Birni, these houses and the attached enclosures are built from bamboo, which is very strong and widely available. And there was no mention of kinship, defense, or politics when I asked about the architecture; here I was told it is patterns of agricultural production that underlie the scaling. The grassland soil and climate are excellent for farming, and the gardens near the Bamileke houses typically grow a dozen different plants all in a single space, with each taking its characteristic vertical place. But this is labor intensive, and so more dispersed plantings—rows of corn and ground-nut—are used in the wider spaces farther from the house. Since the same bamboo mesh construction is used for houses, house enclosures, and enclosures of enclosures, the result is a self-similar architecture. Unlike the defensive labyrinth of Kotoko architecture, where there were only a few well-protected entryways, the farming activities require a lot of movement between enclosures, so at all scales we see good-sized openings. The fractal simulation in figures 2.2b,c shows how this scaling structure can be modeled using an open square as the seed shape.

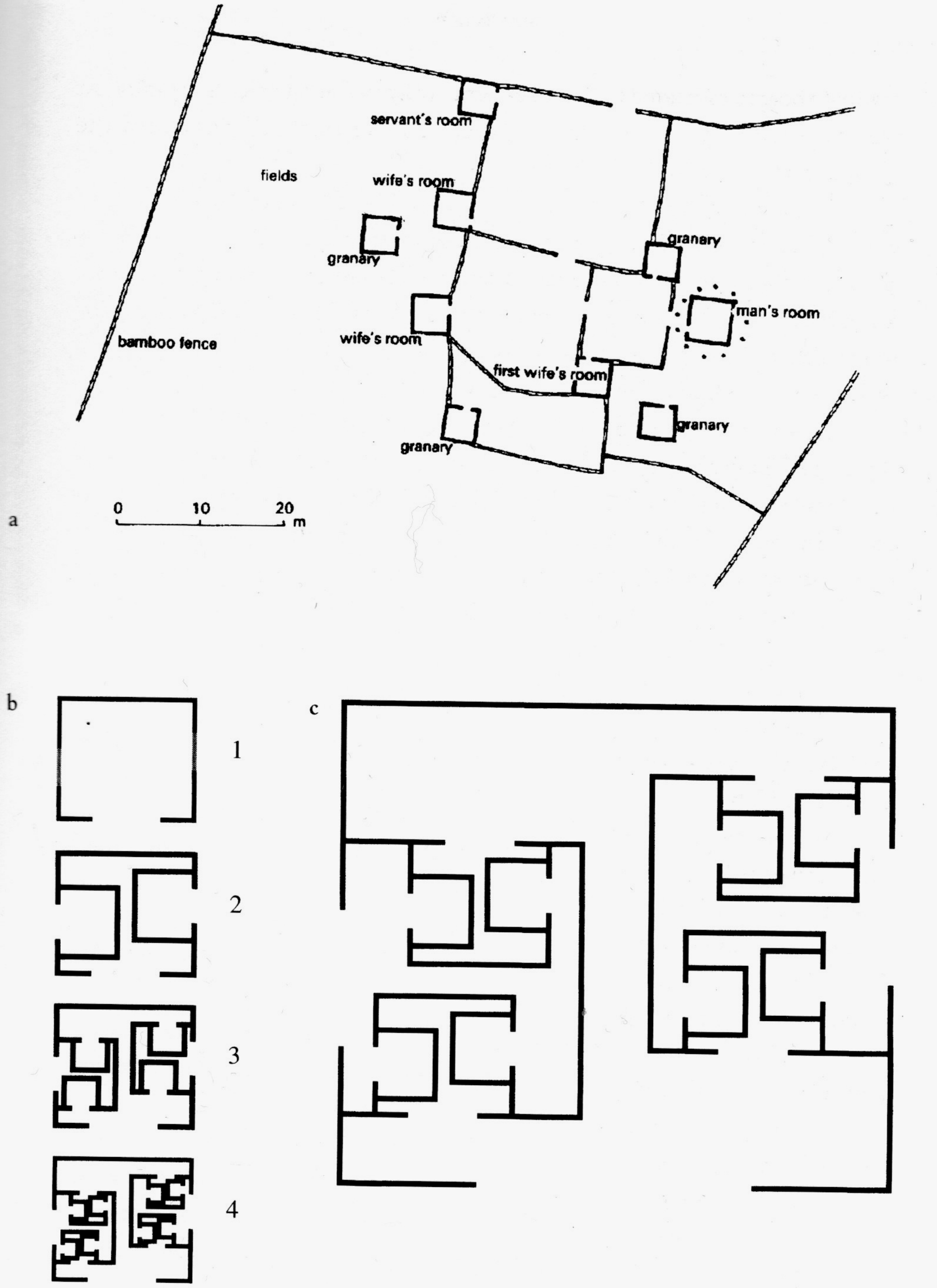


FIGURE 2.2

Bamileke settlement

(a) Plan of Bamileke settlement from about 1960. **(b)** Fractal simulation of Bamileke architecture. In the first iteration ("seed shape"), the two active lines are shown in gray. **(c)** Enlarged view of fourth iteration.

(a, *Beguin 1952; reprinted with permission from* ORSTOM).

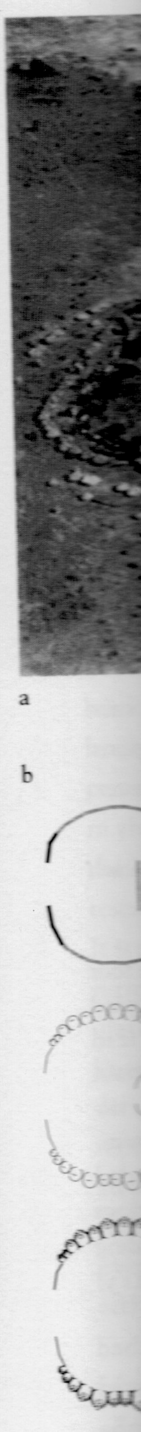
Circular fractals in settlement architecture

Much of southern Africa is made up of arid plains where herds of cattle and other livestock are raised. Ring-shaped livestock pens, one for each extended family,[3] can be seen in the aerial photo in figure 2.3a, a Ba-ila settlement in southern Zambia. A diagram of another Ba-ila settlement (fig. 2.3d) makes these livestock enclosures ("kraals") more clear. Toward the back of each pen we find the family living quarters, and toward the front is the gated entrance for letting livestock in and out. For this reason the front entrance is associated with low status (unclean, animals), and the back end with high status (clean, people).[4] This gradient of status is reflected by the size gradient in the architecture: the front is only fencing, as we go toward the back smaller buildings (for storage) appear, and toward the very back end are the larger houses. The two geometric elements of this structure—a ring shape overall, and a status gradient increasing with size from front to back—echoes throughout every scale of the Ba-ila settlement.

The settlement as a whole has the same shape: it is a ring of rings. The settlement, like the livestock pen, has a front/back social distinction: the entrance is low status, and the back end is high status. At the settlement entrance there are no family enclosures at all for the first 20 yards or so, but the farther back we go, the larger the family enclosures become.

At the back end of the interior of the settlement, we see a smaller detached ring of houses, like a settlement within the settlement. This is the chief's extended family. At the back of the interior of the chief's extended family ring, the chief has his own house. And if we were to view a single house from above, we would see that it is a ring with a special place at the back of the interior: the household altar.

Since we have a similar structure at all scales, this architecture is easy to model with fractals. Figure 2.3b shows the first three iterations. We begin with a seed shape that could be the overhead view of a single house. This is created by active lines that make up the ring-shaped walls, as well as an active line at the position of the altar at the back of the interior. The only passive lines are those adjacent to the entrance. In the next iteration, we have a shape that could be the overhead view of a family enclosure. At the entrance to the family enclosure we have only fencing, but as we go toward the back we have buildings of increasing size. Since the seed shape used only passive lines near the entrance and increasingly larger lines toward the back, this iteration of our simulation has the same size gradient that the real family enclosure shows. Finally, the third iteration provides a structure that could be the overhead view of the whole settlement. At the entrance to the settlement we have only fencing, but as we go toward



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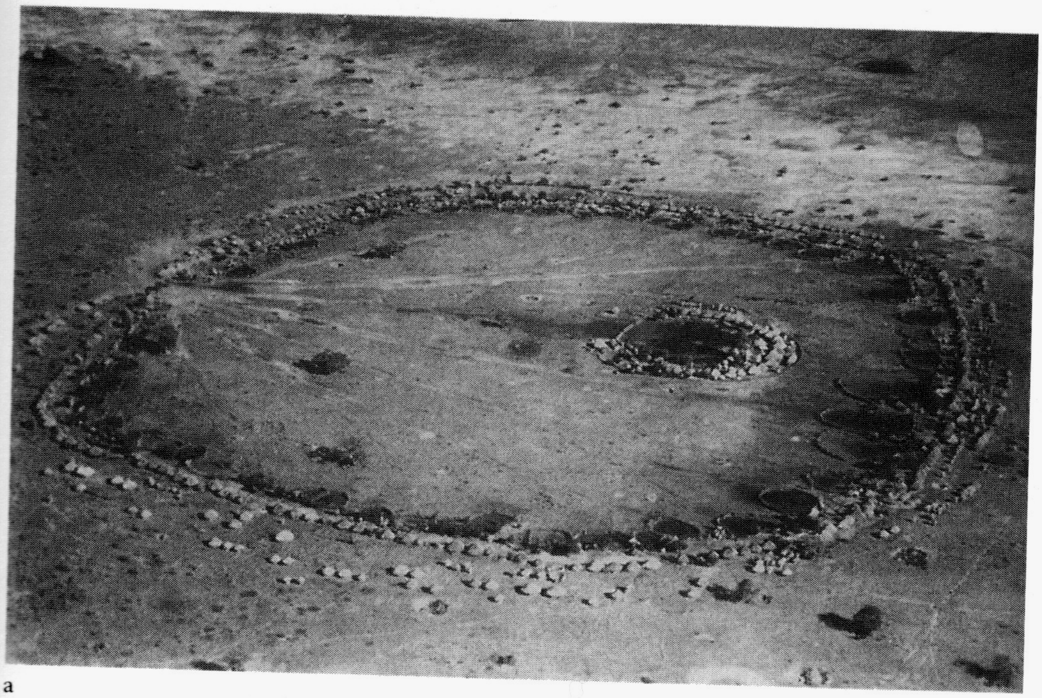

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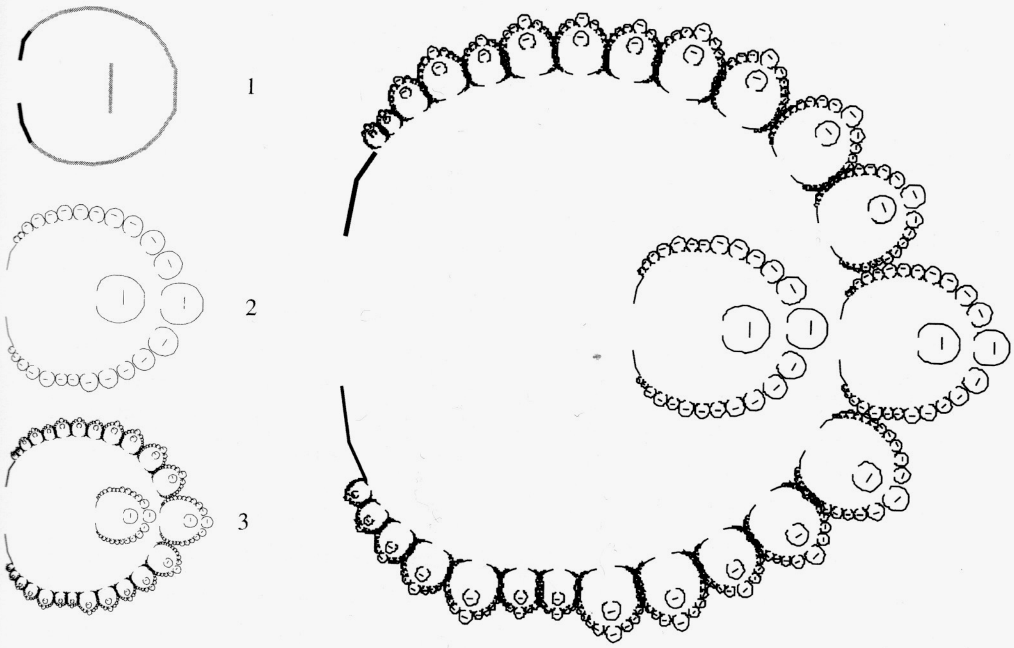


FIGURE 2.3

Ba-ila

(a) Aerial photo of Ba-ila settlement before 1944. **(b)** Fractal generation of Ba-ila simulation. Note that the seed shape has only active lines (gray) except for those near the opening (black). (**a**, *American Geographic Institute.*)

the back we have enclosures of increasing size. Again, by having the seed shape use only passive lines near the entrance and increasingly larger lines toward the back, this iteration of our simulation has the same size gradient that the real settlement shows.

I never visited the Ba-ila myself; most of my information comes from the classic ethnography by Edwin Smith and Andrew Dale, published in 1920. While their colonial and missionary motivations do not inspire much trust, they often showed a strong commitment toward understanding the Ba-ila point of view for social structure. Their analysis of Ba-ila settlement architecture points out fractal attributes. They too noted the scaling of house size, from those less than 12 feet wide near the entrance, to houses more than 40 feet wide at the back, and explained it as a social status gradient; "there being a world of difference between the small hovel of a careless nobody and the spacious dwelling of a chief" (Smith and Dale 1968, 114).

It is in Smith's discussion of religious beliefs, however, that the most striking feature of the Ba-ila's fractal architecture is illuminated. Unlike most missionaries of his time, Smith was a strong proponent of respect for local religions. He was no relativist—understanding and respect were strategies for conversion—but his delight in the indigenous spiritual strength comes across clearly in his writings and provided him with insight into the subtle relation of the social, sacred, and physical structure of the Ba-ila architectural plan.

> In this village there are about 250 huts, built mostly on the edge of a circle four hundred yards in diameter. Inside this circle there is a subsidiary one occupied by the chief, his family, and cattle. It is a village in itself, and the form of it in the plan is the form of the greater number of Ba-ila villages, which do not attain to the dimensions of Shaloba's capital. The open space in the center of the village is also broken by a second subsidiary village, in which reside important members of the chief's family, and also by three or four miniature huts surrounded by a fence: these are the *manda a mizhimo* ("the manes' huts") where offerings are made to the ancestral spirits. Thus early do we see traces of the all-pervading religious consciousness of the Ba-ila. (Smith and Dale 1968, 113)

In the first iteration of the computer-generated model there is a detached active line inside the ring, at the end opposite the entrance. This was motivated by the ring comprising the chief's family, but it also describes the location of the sacred altar within each house. As a logician would put it, the chief's family ring is to the whole settlement as the altar is to the house. It is not a status gradient, as we saw with the front-back axis, but rather a recurring functional role between different scales: "The word applied to the chief's relation to his people is *kulela*: in the extracts given above we translate it 'to rule,' but it has this only as a secondary meaning. Kulela is primarily to nurse, to cherish; it is the word applied to a woman caring for her child. The chief is the father of the community; they are his children, and what he does is *lela* them" (Smith and Dale 1968, 307).

This relationship is echoed throughout family and spiritual ties at all scales, and is structurally mapped through the self-similar architecture. The nesting of circular shapes—ancestral miniatures to chief's house ring to chief's extended family ring to the great outer ring—was not a status gradient, as we saw for the enclosure variation from front to back, but successive iterations of *lela*.

A very different circular fractal architecture can be seen in the famous stone buildings in the Mandara Mountains of Cameroon. The various ethnic groups of this area have their own separate names, but collectively are often referred to as Kirdi, the Fulani word for "pagan," because of their strong resistance against conversion to Islam. Their buildings are created from the stone rubble that commonly covers the Mandara mountain terrain. Much of the stone has natural fracture lines that tend to split into thick flat sheets, so these ready-made bricks—along with defensive needs—helped to inspire the construction of their huge castlelike complexes. But rather than being the Euclidean shapes of European castles, this African architecture is fractal.

Figure 2.4a shows the building complex of the chief of Mokoulek, one of the Mofou settlements. An architectural diagram of Mokoulek, drawn by French researchers from the ORSTOM science institute, shows its overall structure (fig. 2.4b). It is primarily composed of three stone enclosures (the large circles), each of which surrounds tightly spiraled granaries (small circles). The seed shape for the simulation requires a circle, made of passive lines, and two different sets of active lines (fig. 2.4c). Inside the circle is a scaling sequence of small active lines; these will become the granaries. Outside the circle there is a large active line; this will replicate the enclosure filled with granaries. By the fourth iteration we have created three enclosures filled with spiral clusters of granaries, plus one unfilled. The real diagram of Mokoulek shows several unfilled circles—evidence that not everything in the architectural structure can be accounted for by fractals. Nevertheless, an important feature is suggested by the simulation.

In the first iteration we see that the large external active line is to the left of the circle. But since it is at an angle, the next iteration finds this active line above and to the right. If we follow the iterations, we can see that the *dynamic construction* of the complex has a spiral pattern; the replications whorl about a central location. This spiral dynamic can be missed with just a static view—I certainly didn't see it before I tried the simulation—but our participation in the virtual construction makes the spiral quite evident.[5] The similarity between the small spirals of granaries inside the enclosures and this large-scale spiral shape of the

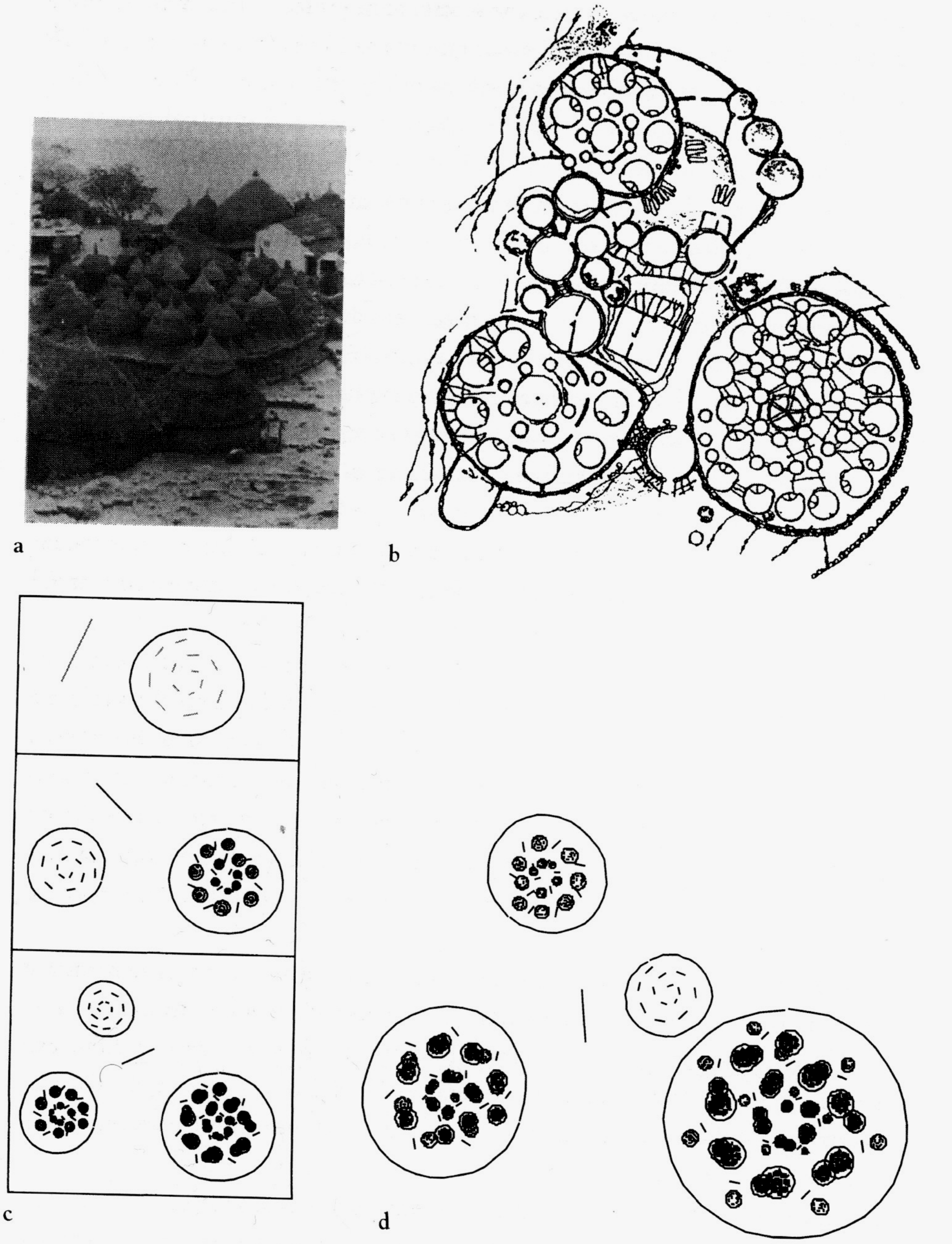


FIGURE 2.4

Mokoulek

(a) Mokoulek, Cameroon. The small buildings inside the stone wall are granaries. The rectangular building (top right) holds the sacred altar. **(b)** Architectural diagram of Mokoulek. **(c)** First three iterations of the Mokoulek simulation. The seed shape is composed of a circle drawn with passive lines (black) and with gray active lines both inside and outside the circle. **(d)** Fourth iteration of the Mokoulek simulation.

*(***a*** and* **b***, by permission from Seignobos 1982.)*

complex as a whole indicates that the fractal appearance of the architecture is not merely due to a random accumulation of various-sized circular forms. The idea of circles of increasing size, spiraling from a central point, has been applied at two different scales, and this structural coherence is confirmed by the architects' own concepts.

In our simulation the active line became located toward the center of the spiral. The Mofou also think of their architecture as spiraling from this central location, which holds their sacred altar. The altar is a kind of conceptual "active line" in their schema; it is responsible for the iterations of life. Seignobos (1982) notes that this area of the complex is the site of both religious and political authority; it is the location for rituals that generate cycles of agricultural fertility and ancestral succession. This geometric mapping between the scaling circles of the architecture and the spiritual cycles of life is represented in their divination ("fortunetelling") ritual, in which the priest creates concentric circles of stones and places himself at the center. As in the *guti* painting in Logone-Birni, in which the Kotoko had modeled their scaling rectangles, the Mofou have also created their own scaling simulation.

By the time I arrived at Mokoulek in 1994 the chief had died, and the ownership of this complex had been passed on to his widows. The new chief told me that the design of this architecture, including that of his new complex, began with a precise knowledge of the agricultural yield. This volume measure was then converted to a number of granaries, and these were arranged in spirals. The design is thus not simply a matter of adding on granaries as they are needed; in fact, it has a much more quantitative basis than my computer model, which I simply did by eyeball.

Not all circular architectures in Africa have the kind of centralized location that we saw in Mokoulek. The Songhai village of Labbezanga in Mali (fig. 2.5a), for example, shows circular swirls of circular houses without any single focus. But comparing this to the fractal image of figure 2.5b, we see that a lack of central focus does not mean a lack of self-similarity. It is important to remember that while "symmetry" in Euclidean geometry means similarity within one scale (e.g., similarity between opposite sides in bilateral symmetry), fractal geometry is based on symmetry between different scales. Even these decentralized swirls of circular buildings show a scaling symmetry.

Paul Stoller, an accomplished ethnographer of the Songhai, tells me that the rectangular buildings that can be seen in figure 2.5a are due to Islamic influence, and that the original structure would have been completely circular. Thanks to Peter Broadwell, a computer programmer from Silicon Graphics Inc., we were able to run a quantitative test of the photo that confirmed what our eyes

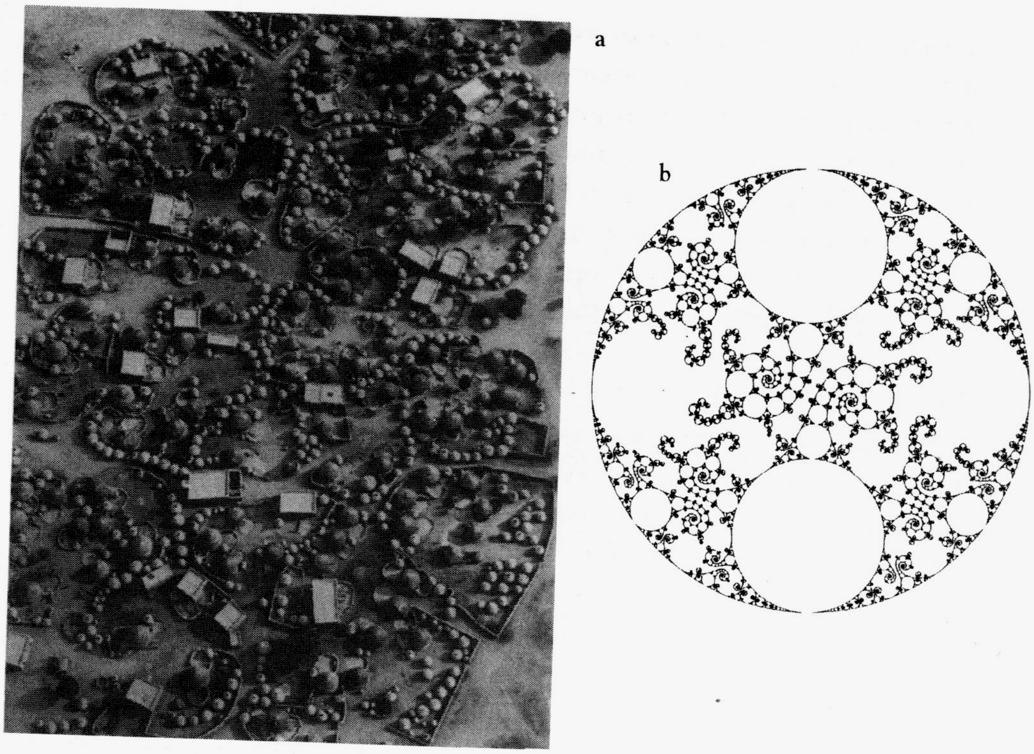
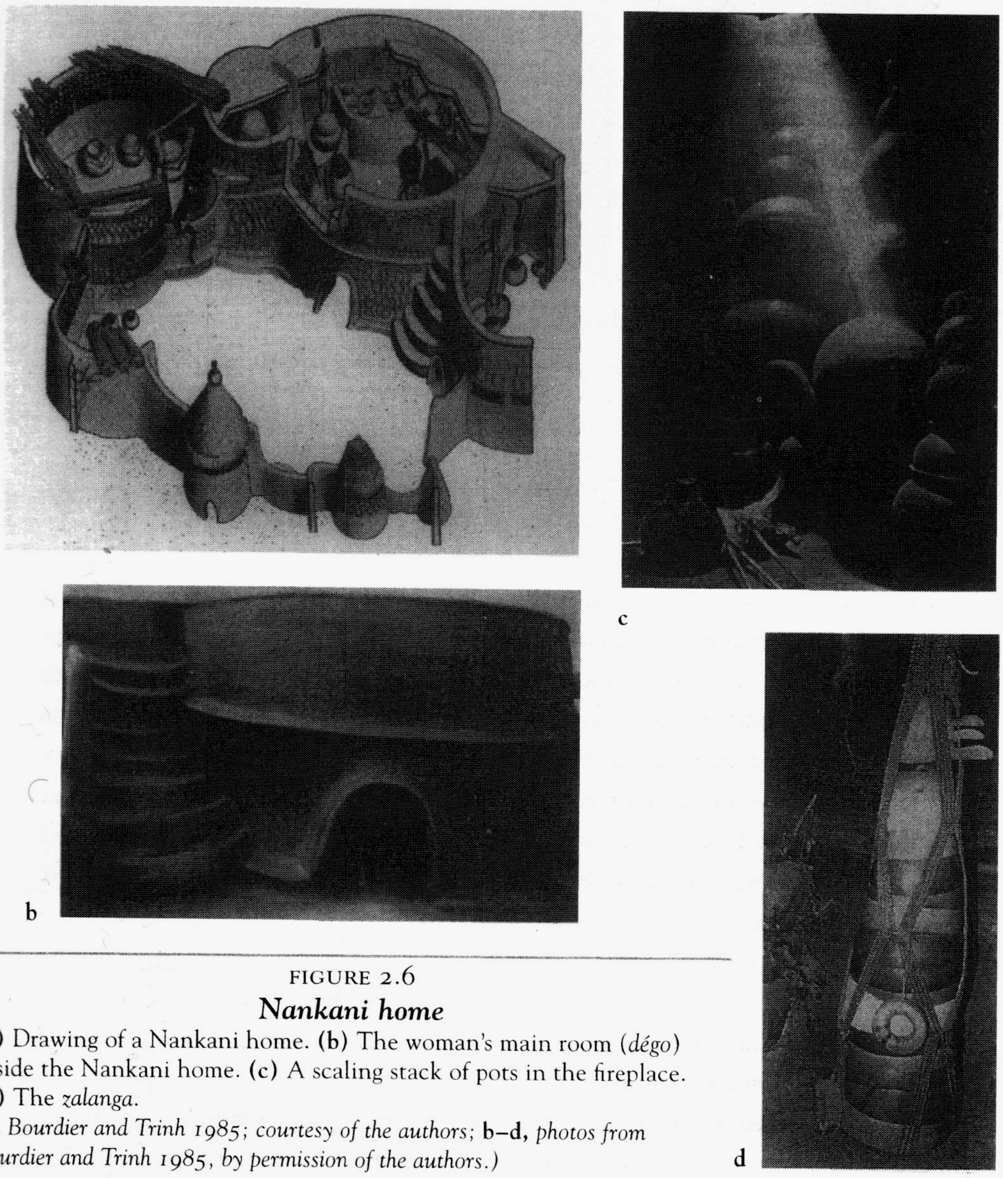


FIGURE 2.5
Labbezanga
(a) Aerial view of the village of Labbezanga in Mali. **(b)** Fractal graphic.
*(***a***, photo by Georg Gerster;* **b***, by permission of Benoit Mandelbrot.)*

were telling us: the Songhai architecture can be characterized by a fractal dimension similar to that of the computer-generated fractal of figure 2.5b.[6]

This kind of dense circular arrangement of circles, while occurring in all sorts of variations, is common throughout inland west Africa. Bourdier and Trinh (1985), for example, describe a similar circular architecture in Burkina Faso. The scaling of individual buildings is beautifully diagrammed in their cover illustration (fig. 2.6a), a portion of one of the large building complexes created by the Nankani society. As for the Songhai, foreign cultural influences have now introduced rectangular buildings as well. In the Nankani complex the outermost enclosure (the perimeter of the complex) is socially coded as male. As we move in, the successive enclosures become more female associated, down to the circular woman's *dégo* (fig. 2.6b), the circular fireplace, and finally the scaling stacks of pots (fig. 2.6c).

Using a technique quite close to that of the Kotoko women, the women of Nankani also waterproof and decorate these walls. The recurrent image of a triangle in these decorations (see walls of dégo) represents the *zalanga*, a nested stack of calabashes (circular bowls carved from gourds) that each woman keeps in her kitchen (fig. 2.6d). Since these calabashes are stacked from large to small, they (and the rope that holds them) form a triangle—thus the triangular decorations also represent scaling circles, just in a more abstract way. The smallest container in a woman's zalanga is the *kumpio*, which is a shrine for her soul. When she dies, the zalanga, along with her pots, is smashed, and her soul is released to eternity. The eternity concept, associated with well-being, is symbolically



a b c d

FIGURE 2.6
Nankani home
(**a**) Drawing of a Nankani home. (**b**) The woman's main room (*dégo*) inside the Nankani home. (**c**) A scaling stack of pots in the fireplace. (**d**) The *zalanga*.
(**a**, *Bourdier and Trinh 1985; courtesy of the authors;* **b–d**, *photos from Bourdier and Trinh 1985, by permission of the authors.)*

represented by the coils of a serpent of infinite length, sculpted into the walls of these homes.

From the 20-meter diameter of the building complex to the 0.2-meter kumpio—and not simply at one or two levels in between, but with dozens of self-similar scalings—the Nankani fractal spans three orders of magnitude, which is comparable to the resolution of most computer screens. Moreover, these scaling circles are far from unconscious accident: as in several other architectures we have examined, they have made conscious use of the scaling in their social symbolism. In this case, the most prominent symbolism is that of birthing. When a child is born, for example, it must remain in the innermost enclosure of the women's dégo until it can crawl out by itself. Each successive entrance is—spatially as well as socially—a rite of passage, starting with the biological entrance of the child from the womb. It leaves each of these nested chambers as the next iteration in life's stages is born. The zalanga models the entire structure in miniature, and its destruction in the event of death maps the journey in reverse: from the circles of the largest calabash to the tiny kumpio holding the soul—from mature adult to the eternal realm of ancestors who dwell in "the earth's womb." There is a conscious scheme to the scaling circles of the Nankani: it is a recursion which bottoms-out at infinity.

Branching fractals

While African circular buildings are typically arranged in circular clusters, the paths that lead through these settlements are typically not circular. Like the bronchial passages that oxygenate the round alveoli of the lungs, the routes that nourish circular settlements often take a branching form (e.g., figure 2.7). But despite my unavoidably organicist metaphor, these cannot be simply reduced to unconscious traces of minimum effort. For one thing, conscious design criteria are evident in communities in which there is an architectural transition from circular to rectangular buildings, since they can choose to either maintain or erase the branching forms.

Discussion concerning such decisions are apparent in the settlement of Banyo, Cameroon, where the transition has a long history (Hurault 1975). I found that few circular buildings were left, but those that were still intact served as an embodiment of cultural memory. This role was honored in the case of the chief's complex and exploited in the case of a blacksmith's shop, which was the site of occasional tourist visits. After passing approval by the government officials and the sultan, I was greeted by the official city surveyor, who—considering the fact that his raison d'être was Euclideanizing the streets—showed surprising

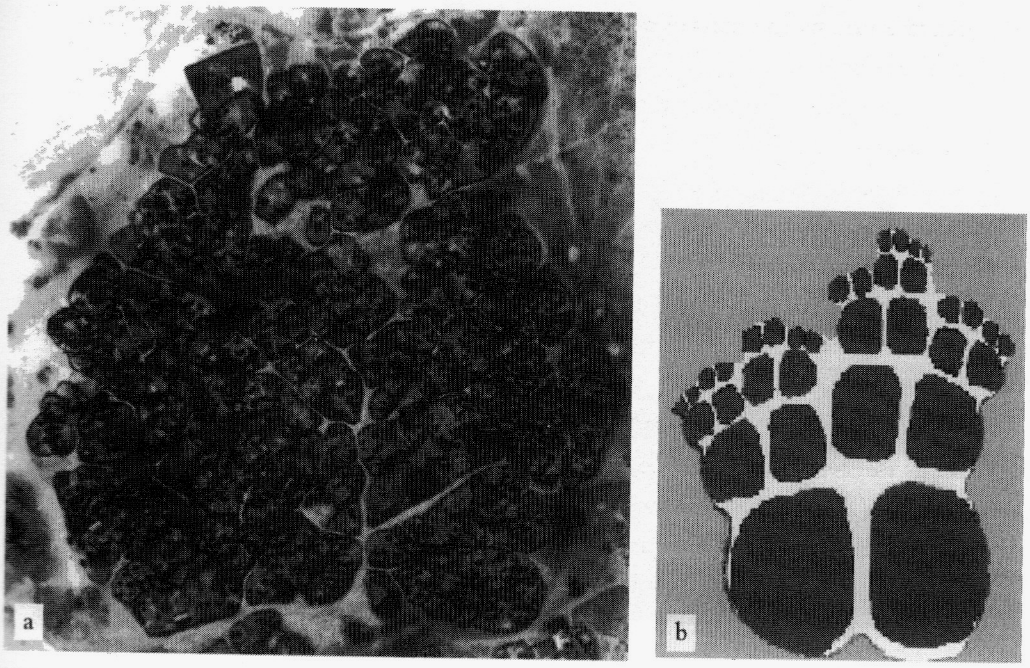
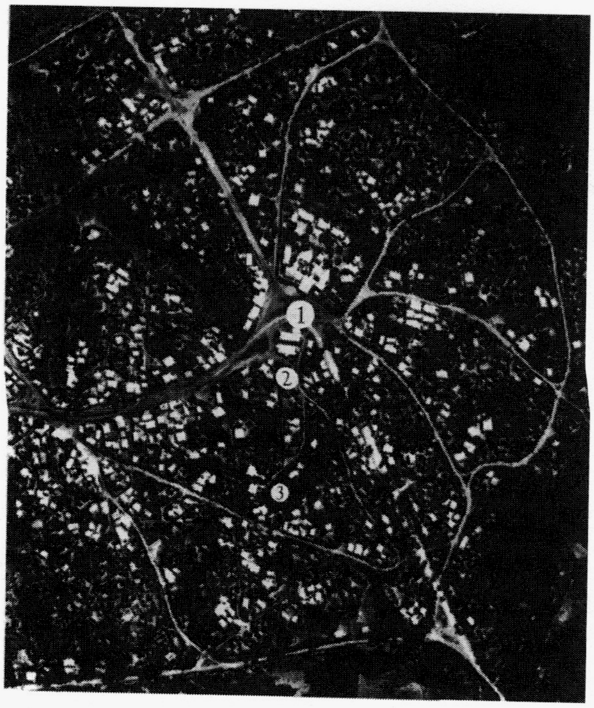


FIGURE 2.7
Branching paths in a Senegalese settlement
(a) Aerial photo of a traditional settlement in northeast Senegal. The space between enclosure walls, serving as roads and footpaths, creates a branching pattern. **(b)** A branching fractal can be created by the background of a scaling set of circular shapes.
(**a**, *courtesy Institut Geographique du Senegal.*)

appreciation for my project and helped me locate the most fractal area of the city (fig. 2.8a). At the upper left of the photo we see a portion of the Euclidean grid that covers the rest of the city, but most of this area is still fractal. The location of this carefully maintained branching—fanning out from a large plaza that is bordered by the palace of the sultan and the grand mosque—is no coincidence. By marking my position on the aerial photo as I traveled through (fig. 2.8b), I was later able to create a map by digitally altering the photo image (fig. 2.8c). This provides a stark outline—looking much like the veins in a leaf—of the fractal structure of this transportation network. I may have plunged through a wall or two in creating this map, but it certainly underestimates the fine branching of the footpaths, as I did not attempt to include their extensions into private housing enclosures.

How does the creation of these scaling branches interact with the kinds of iterative construction and social meaning we have seen associated with other examples of fractal architecture? A good illustration can be found in the



a

b

Position 1—outside palace

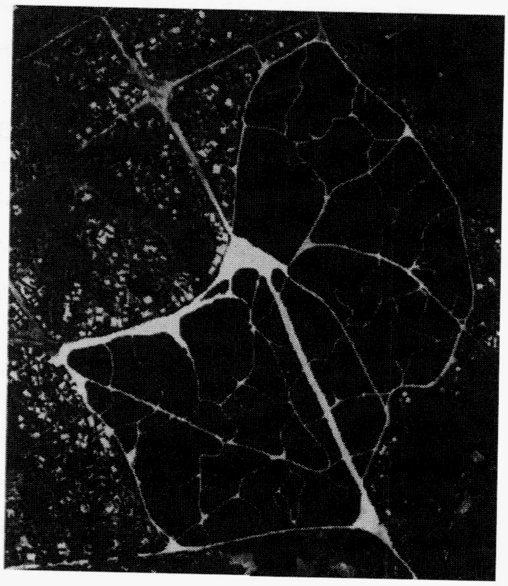
Position 2—road below mosque

Position 3—narrow walkway

FIGURE 2.8

Branching paths in Banyo

(a) Aerial photo of the city of Banyo, Cameroon. **(b)** Successive views of the branching paths, as marked on the photo above. The clay walls require their own roof, which comes in both thatched and metal versions along the walkway in the last photo. **(c)** Aerial photo of Banyo with only public paths showing. (**a,** *courtesy National Institute of Cartography, Cameroon.*)



c

a

b

c

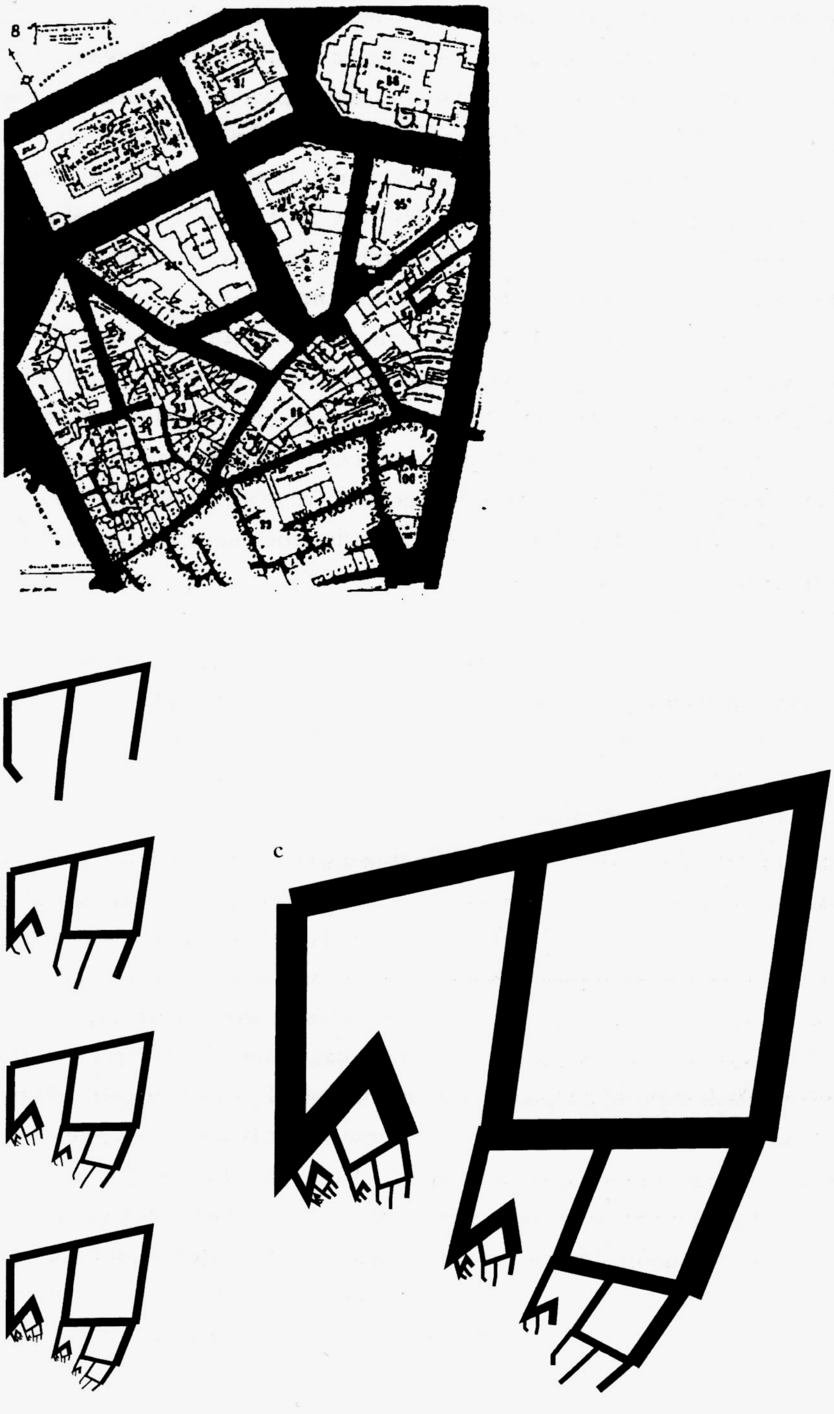


FIGURE 2.9

Streets of Cairo

(a) Map of streets of Cairo, 1898. **(b)** Fractal simulation for Cairo streets. **(c)** Enlarged view of fourth iteration.

branching streets of North African cities. Figure 2.9a shows a map of Cairo, Egypt, in 1898. The map was created by an insurance company, and I have colored the streets black to make the scaling branches more apparent. Figure 2.9b shows its computer simulation. Delaval (1974) has described the morphogenesis of Saharan cities in terms of successive additions similar to the line replacement in the fractal algorithms we have used here. The first "seed shape" consists of a mosque connected by a wide avenue to the marketplace, and successive iterations of construction add successive contractions of this form.

Since these fractal Saharan settlement architectures predate Islam (see Devisse 1983), it would be misleading to see them as an entirely Muslim invention; but given the previous observations about the introduction of Islamic architecture as an interruption of circular fractals in sub-Saharan Africa, it is important to note that Islamic cultural influences have made strong contributions to African fractals as well. Heaver (1987) describes the "arabesque" artistic form in North African architecture and design in terms that recall several fractal concepts (e.g., "cyclical rhythms" producing an "indefinitely expandable" structure). He discussed these patterns as visual analogues to certain Islamic social concepts, and we will examine his ideas in greater detail in chapter 12 of this book.

Conclusion

Throughout this chapter, we have seen that a wide variety of African settlement architectures can be characterized as fractals. Their physical construction makes use of scaling and iteration, and their self-similarity is clearly evident from comparison to fractal graphic simulations. Chapter 3 will show that fractal architecture is not simply a typical characteristic of non-Western settlements. This alone does not allow us to conclude an indigenous African knowledge of fractal geometry; in fact, I will argue in chapter 4 that certain fractal patterns in African decorative arts are merely the result of an intuitive esthetic. But as we have already seen, the fractals in African architecture are much more than that. Their design is linked to conscious knowledge systems that suggest some of the basic concepts of fractal geometry, and in later chapters we will find more explicit expressions of this indigenous mathematics in astonishing variety and form.

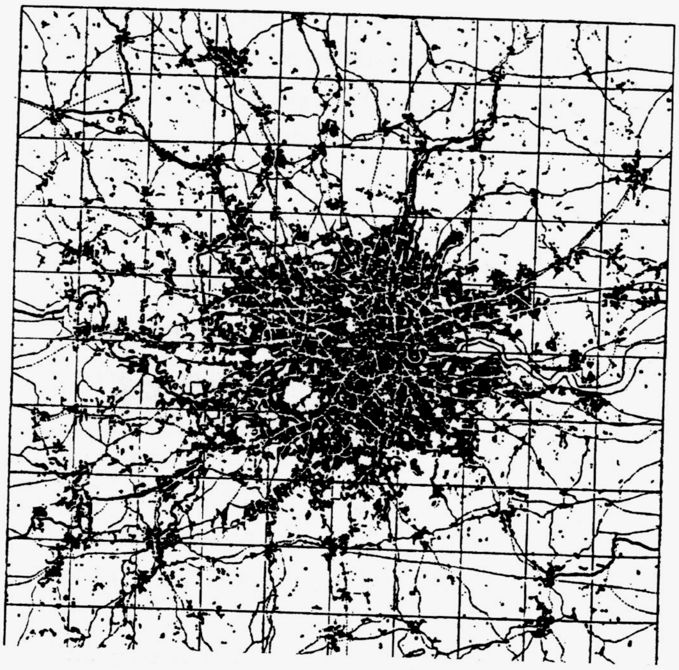


FIGURE 4.1
Urban sprawl in London
Large-scale urban sprawl generally has a fractal structure. The urban sprawl fractals only exist at very large scales—about 100 sq. miles—and result from the unconscious accumulation of urban population dynamics. At levels of conscious intent (e.g., the grid of city blocks), European cities are typically Euclidean. Area is 10 × 10 kilometers.
(Reprinted with permission from Batty et al. 1989.)

occurs in chemical systems when particles in a solution are attracted to an electrode. Fractal urban sprawl is clearly the result of unconscious social dynamics, not conscious design. At the smaller scales in which there is conscious planning, European and American settlement architectures are typically Euclidean.

Fractals from nature: mimesis versus modeling

It might be tempting to think that the contrast between the Euclidean designs of Europe and the fractal designs of Africa can be explained by the important role of the natural environment in African societies. But this assumption turns out to be wrong; if anything, there is a tendency for indigenous societies to favor Euclidean shapes. Physicist Kh. S. Mamedov observed such a contrast in his reflections on his youth in a nomadic culture:

> My parents and countrymen . . . up to the second world war had been nomads. . . . Outside our nomad tents we were living in a wonderful kingdom of various curved lines and forms. So why were the aesthetic signs not formed from them, having instead preserved geometric patterns . . . ? [I]n the cities where the straight-line geometry was predominant the aesthetic signs were formed . . . with nature playing the dominating role. . . . [T]he nomad did not need the "portrait" of an oak to be carried with him elsewhere because he could view all sorts of oaks every day and every hour . . . while for the townsfolk their inclination to nature was more a result of nostalgia. (Mamedov 1986, 512–513)

Contrary to romantic portraits of the "noble savage" living as one with nature, most indigenous societies seem quite interested in differentiating themselves from their surroundings. It is the inhabitants of large state societies, such as those of modern Europe, who yearn to mimic the natural. When European designs are fractal, it is usually due to an effort to mimic nature. African fractals based on mimicry of natural form are relatively rare; their inspiration tends to come from the realm of culture.

How should we place such nature-based designs in our intentionality spectrum? That depends on the difference between mimesis and modeling. Mimesis is an attempt to mirror the image of a particular object, a goal explicitly stated by Plato and Aristotle as the essence of art, one that was subsequently followed in Europe for many centuries (see Auerbach 1953). A photograph is a good example of mimesis. A photo might capture the fractal image of a tree, but it would be foolish to conclude that the photographer knows fractal geometry. If artisans are simply trying to copy a particular natural object, then the scaling is an unintended by-product.

The most important attributes that separate mimesis from modeling are abstraction and generalization. Abstraction is an attempt to leave out many of the concrete details of the subject by creating a simpler figure whose structure is still roughly analogous to the original—often called a "stylized" representation in the arts. Generalization means selecting an analogous structure that is common to all examples of the subject; what is often referred to as an "underlying" form or law.[1] For example, Mandelbrot (1981) points to the European Beaux Arts style as an attempt not merely to imitate nature, but to "guess its laws." He notes that the interior of the Paris opera house makes use of scaling arches-within-arches, a pattern that generalizes some of the scaling characteristics of nature, but is not a copy of any one particular natural object.

Since the ultimate generalization is a mathematical model, why didn't design practices such as the Beaux Arts style result in an early development of fractal geometry? For Europeans, such lush ornamentation was presented—and generally accepted—as embodying the *opposite* of mathematics; it was an effort to create designs that could only be understood in irrational, emotional, or intuitive terms. Even some movements against this attempt, such as the use of distinctly Euclidean forms in the high modern arts style, simply reinforced the association because it only offered a reversal, suggesting that "mathematical" shapes (cubes, spheres, etc.) could be esthetically appreciated. With rare exceptions (e.g., Thompson 1917), mimesis of nature in pre-WW II European art traditions merely furthered the assumption that Euclidean geometry was the only true geometry.[2]

The difference between mimesis and modeling provides two different positions along the intentionality spectrum. The least intentional would be merely holding a mirror to nature—for example, if someone was just shooting a camera or painting a realistic picture outdoors and happened to include a fractal object (cloud, tree, etc.). This mimesis does not count as mathematical thinking. More intentional is a stylized representation of nature. If the artist has reduced the natural image to a structurally analogous collection of more simple elements, she has created an abstract model. Whether or not such abstractions move toward more mathematical models is a matter of local preference.

The two examples of African representations of nature we observed in the previous chapter certainly show that the artisans have gone beyond mere mimesis. The Mandiack cobra pattern we saw in figure 3.2. shows a strictly systematic scaling pattern. This textile design conveys the scaling property of the natural cobra skin pattern—diamonds at many scales—in a stylized or abstract way. We can take this idea a step further by examining another Bwami bat sculpture (fig. 4.2). This spiral pattern is also a stylized representation of the natural scaling of the bat's wing, but it is a different geometric design than the expanding zigzag pattern we saw in figure 3.4c. It is more styl-

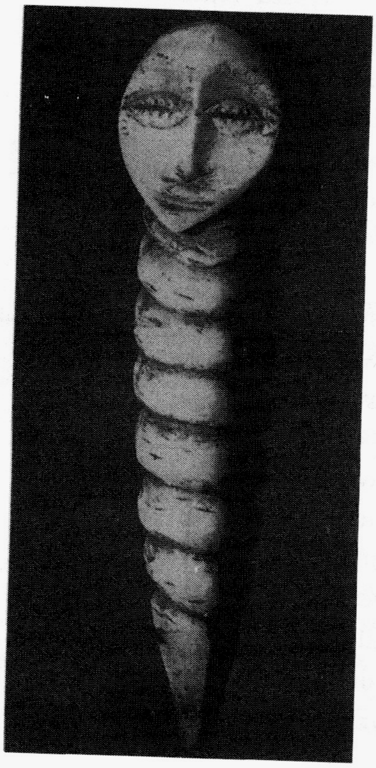


FIGURE 4.2
Stylized sculpture of a bat
Another Lega bat sculpture, but unlike the zigzag design we saw in figure 3.4c, here the scaling of the wing folds is represented by a spiral.
(By permission of the Museum of African Art, N.Y.)

ized in the sense of being further abstracted from the original natural bat's wing. This provides further evidence that the sculptors were focused on the scaling properties—the generalized underlying feature—and not particular concrete details.

The greatest danger of this book is that readers might misinterpret its meaning in terms of primitivism. The fact that African fractals are rarely the result of imitating natural forms helps remind us that they are *not* due to "primitives living close to nature." But even for those rare cases in which African fractals are representations of nature, it is clearly a self-conscious abstraction, not a mimetic reflection. The geometric thinking that goes into these examples may be simple, but it is quite intentional.

The fractal esthetic

Just as we saw how designs based on nature range from unconscious to intentional, artificial designs also vary along a range of intention, with some simply the result of an intuitive inspiration, and others a more self-conscious application of rules or principles. The examples of African fractals in figure 4.3 did not appear to be related to anything other than the artisan's esthetic intuition or sense of beauty. As far as I could determine from descriptions in the literature and my own fieldwork, there were no explicit rules about how to construct these designs, and no meaning was attached to the particular geometric structure of the figures other than looking good. In particular, I spent several weeks in Dakar wandering the streets asking about certain fractal fabric patterns and jewelry designs, and the insistence that these patterns were "just for looks" was so adamant that if someone finally had offered an explanation, I would have viewed it with suspicion.

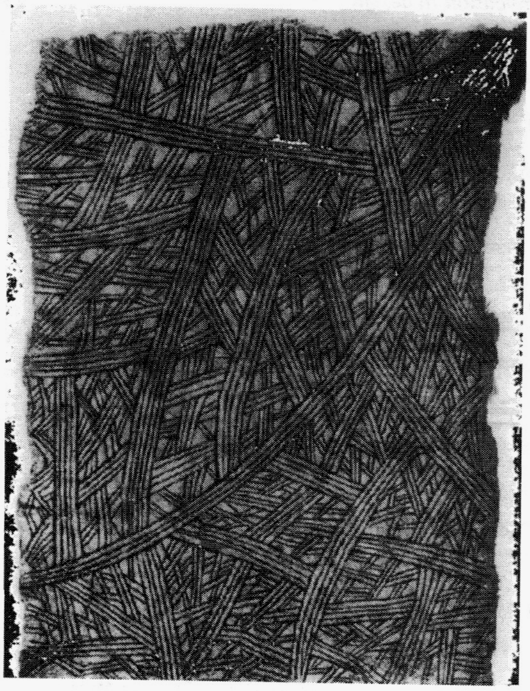
Since some professional mathematicians report that their ideas were pure intuition—a sudden flash of insight, or "Aha!" as Martin Gardner puts it—we shouldn't discount the geometric thinking of an artisan who reports "I can't tell you how I created that, it just came to me." Esthetic patterns clearly qualify as intentional designs. On the other hand, there isn't much we can say about the mathematical ideas behind these patterns; they will have to remain a mystery unless something more is revealed about their meaning or the artisan's construction techniques. It is worth noting, however, that they do contribute to the fractal design theme in Africa. Esthetic patterns help inspire practical designs, and vice versa. Since ancient trade networks were well established, the diffusion of esthetic patterns is probably one part of the explanation for how fractals came to be so widespread across the African continent.

FIGURE 4.3

Esthetic fractals

(a) Meurant (quoted in Reif 1996) reports that the Mbuti women who created this fractal design, a bark-cloth painting, told him the design was not "telling stories," nor was it "representing any particular object." **(b)** Scaling patterns can be found in many African decorative designs that are reported to be "just for beauty." *Upper left*, Shoowa Raffia cloth; *lower left*, Senegalese tie dye; *right*, Senegalese pendant.

(**a,** *courtesy Georges Meruant.* **b:** *Upper left, British Museum; lower left, from Musée Royal de l'Afrique Central, Belgium; right, photo courtesy IFAN, Dakar.)*



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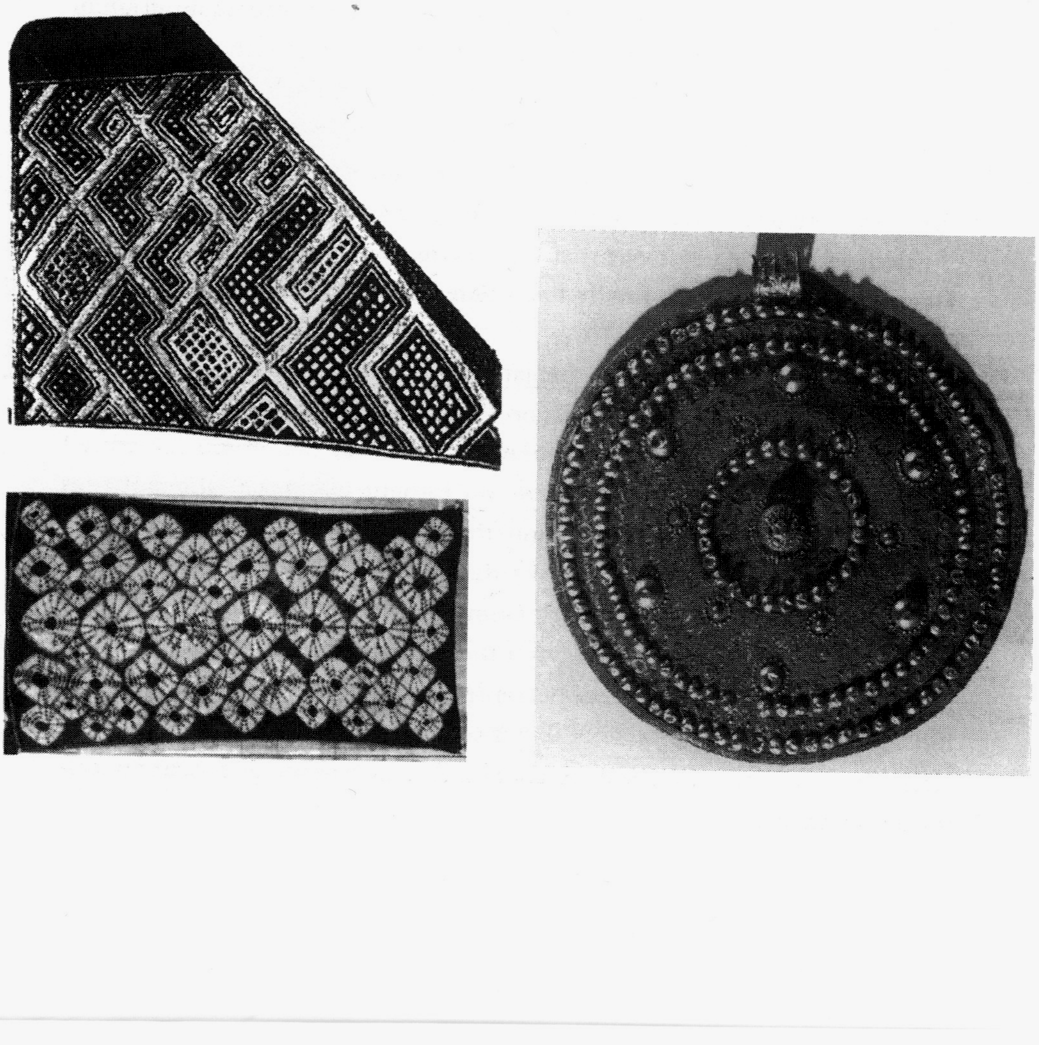


FIGURE 4.4
The quincunx fractal
A customer in Touba, Senegal, selects a fractal quincunx pattern for his leather neck bag. The quincunx is historically important because of its use by early African American "man of science" Benjamin Banneker.

Of course, there are plenty of African designs that are strictly Euclidean, but even these can occur in "fractalized" versions. One particularly interesting example is the *quincunx* (fig. 4.4). The basic quincunx is a pattern of five squares, with one at the center and one at each corner. The design is common in Senegal, where it is said to represent the "light of Allah." The quincunx is historically important because the image was recorded as being of religious significance to the early African American "man of science" Benjamin Banneker. Since evidence shows that Banneker's grandfather (Bannaka) came from Senegal, the quincunx is a fascinating possibility for geometry in the African diaspora (see Eglash 1997c for details). Because of the fractal esthetic, this religious symbol is often arranged in a recursive pattern—five squares of five squares—as shown in figure 4.4 in the design for a leather neck bag.

Finally, there are also examples of the fractal esthetic in common household furnishings. Euro-American furniture is differentiated by form and function—stools are structured differently from chairs, which are structured differently from couches. But in African homes one often sees different sizes of the same shape (fig. 4.5). A similar difference has been noted in cross-cultural comparisons of housing. Whereas Euro-Americans would never think to have a governer's mansion shaped like a peasant's shack (or vice versa), precolonial African architecture typically used the same form at different sizes (as we saw for the status distinctions in the Ba-ila settlement in chapter 2). It is unfortunate that this African structural characteristic is typically described in terms of a lack—as the absence of shape distinctions rather than as the presence of a scaling design theme.

FIGURE 4.5
The fractal esthetic in household objects
African stools, chairs, and benches are often created in a scaling series.
(Photo courtesy of Africa Place, Inc.)

Conclusion

We now have some guidelines to help determine which fractal designs should count as mathematics, which should not, and which are in between. Figure 4.6 summarizes this spectrum. Fractals produced by unconscious activity, or as the unintentional by-product from some other purpose, cannot be attributed to indigenous concepts. But some artistic activities, such as the creation of stylized represen-

Unintentional	***Intentional but implicit***	***Intentional and explicit***
Unconscious activity •urban sprawl	*Conscious use of natural scaling* •stylistic abstraction of natural scaling	*Construction techniques*
Accidental fractals •"mirror" portrait of nature (*mimesis*; e.g., photography)	*Esthetic design* •intuitive fractal design theme	*Knowledge systems*

FIGURE 4.6
From unconscious accident to explicit design

tations of nature or purely esthetic designs, do show intentional activity focused on fractals. Such examples may be restricted in terms of geometric thinking—the artisans may only report that the design suddenly came to them in a flash of intuition—but these are clearly distinguished from those which are unconscious or accidental. The following chapters will consider examples that are not only intentional, but also include enough explicit information about design techniques and knowledge systems to be easily identifiable as mathematical practice and ideas.

CHAPTER

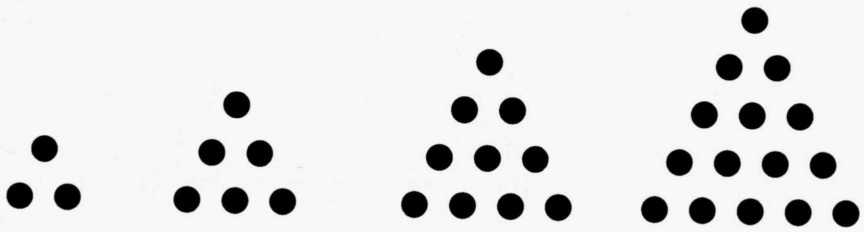
7

Numeric systems

So far we have focused on geometric structures rather than numeric systems. The only exception was in the windscreen, where the nonlinear scaling was created by counting a specific sequence of diagonal straw rows. But there are many other instances in which the African approach to fractal geometry makes use of numbers.

Nonlinear additive series in Africa

The counting numbers (1,2,3 . . .) can be thought of as a kind of iteration, but only in the most trivial way.[1] It is true that we could produce the counting numbers from a recursive loop, that is, a function in which the output at one stage becomes the input for the next: $X_{n+1} = X_n + 1$. But this is a strictly linear series, increasing by the same amount each time—the numeric equivalent of what we saw in the linear concentric circle and linear spiral. Addition can, however, produce nonlinear series,[2] and there are at least two examples of nonlinear additive series in African cultures. The triangular numbers (1,3,6,10,15 . . .) are used in a game called "tarumbeta" in east Africa (Zaslavsky 1973, 111). Figure 7.1 shows how these numbers are derived from the shape of triangles of increasing size, and how the numeric series can be created by a recursive loop. As in the case of cer-



number of stones:	3	6	10	15
number of iterations:	2	3	4	5

A game called "tarumbeta" in East Africa makes use of the triangular numbers, starting with 3 (3, 6, 10, 15 . . .). In this game, one player calls out a count as he removes stones consecutively, left to right and bottom to top, while the other player, with his back turned, must signal whenever the first stone in a row has been removed.

The stones in each triangular array can be built up in an iterative fashion, that is, the next triangle can be created by adding another layer to any side of the previous triangle. The number to be added in each additional layer is simply the number of iterations. For each iteration **i**, and total number of stones **N**, we have:

$N_{i+1} = N_i + i$ (starting with $N_0 = 0$)

$1 = 0 + 1$ (a trivial array, not used in the game)

$3 = 1 + 2$

$6 = 3 + 3$

$10 = 6 + 4$

$15 = 10 + 5$

.

.

.

In other words, the next number will be given by the last number plus the iteration count:

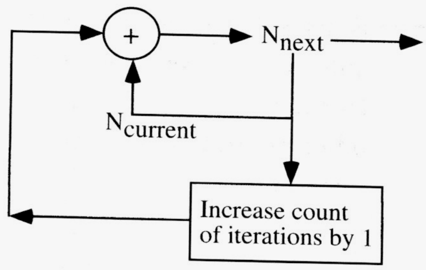


FIGURE 7.1

The triangular numbers in an East African game

tain formal age-grade initiation practices (see chapters 5 and 8), the simple versions are used by smaller children, and the higher iterations are picked up with increasing age. While there is no indication of a formal relationship in this instance, there is still an underlying parallel between the iterative concept of aging common to many African cultures—each individual passing through multiple turns of the "life cycle"—and the iterative nature of the triangular number series.

Another nonlinear additive series was found in archaeological evidence from North Africa. Badawy (1965) noted what appears to be use of the Fibonacci series in the layout of the temples of ancient Egypt. Using a slightly different approach, The Fibonacci series (1, 1, 2, 3, 5, 8, 13 . . .) was found by Badawy (1965) in his study of the layout of the temples of Egypt. His analysis was quite complex, but it is not difficult to create a simple visualization. Here we see the series in the successive chambers of the temple of Karnak.

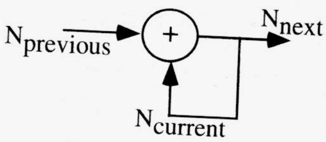
The Fibonacci series is produced by adding the previous number to the current number to get the next number, starting with 1 + 1 = 2. For each iteration **i**, the number **N** in the series is given by:

$$N_{i+1} = N_i + N_{i-1}$$

that is,

$$N_{\text{next}} = N_{\text{current}} + N_{\text{previous}}$$

$$\begin{array}{ccccc} 1 & + & 1 & = & 2 \\ 1 & + & 2 & = & 3 \\ 2 & + & 3 & = & 5 \\ 3 & + & 5 & = & 8 \\ 5 & + & 8 & = & 13 \\ & & \vdots & & \end{array}$$



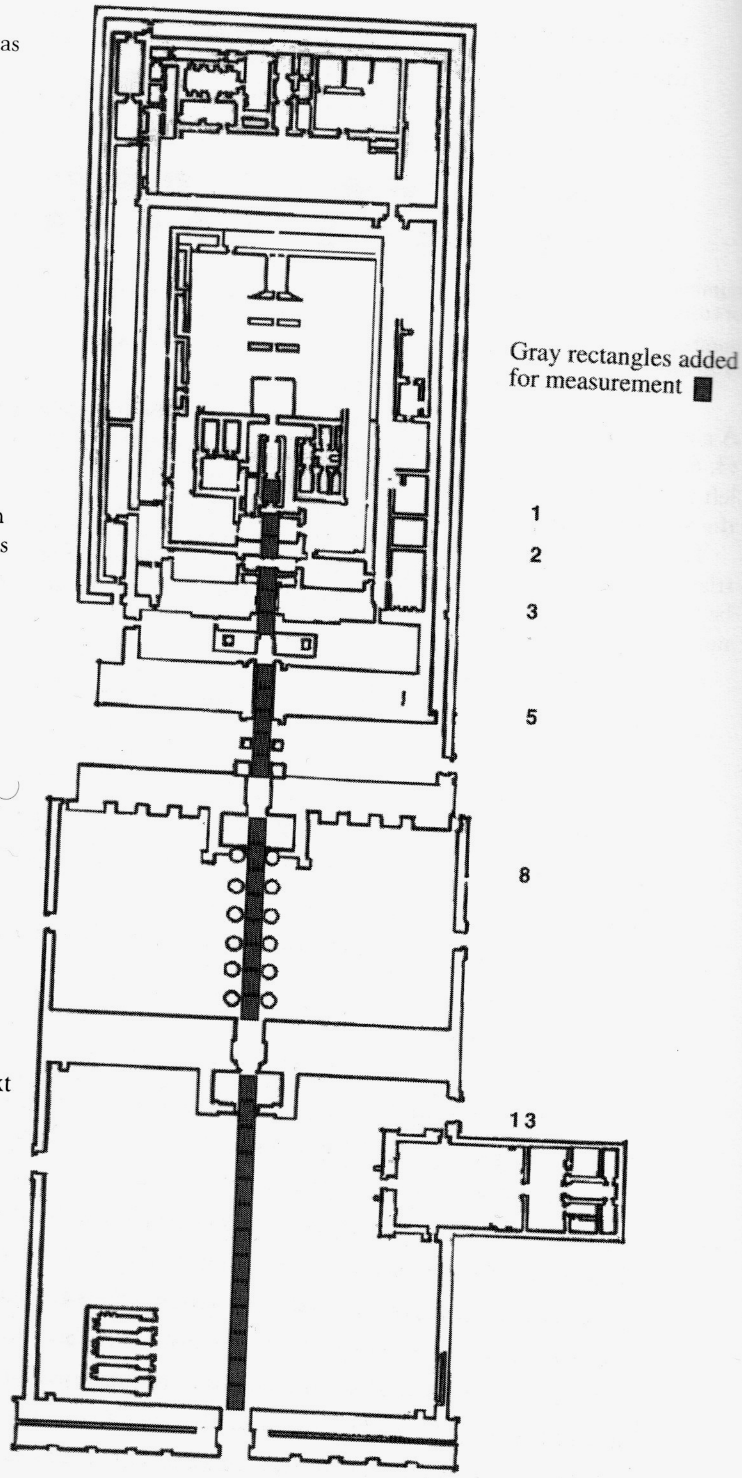


FIGURE 7.2
The Fibonacci series in ancient Egypt

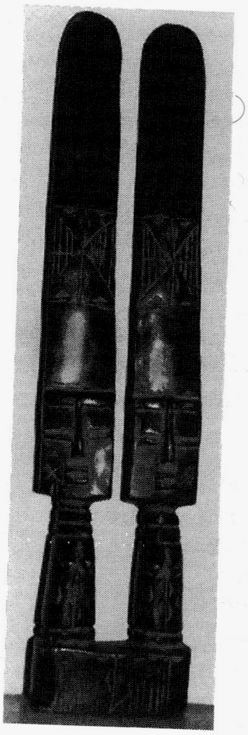
I found a visually distinct example of this series in the successive chambers of the temple of Karnak, as shown in figure 7.2a. Figure 7.2b shows how these numbers can be generated using a recursive loop. This formal scaling plan may have been derived from the nonnumeric versions of scaling architecture we see throughout Africa. An ancient set of balance weights, apparently used in Egypt, Syria, and Palestine circa 1200 B.C.E., also appear to employ a Fibonacci sequence (Petruso 1985). This is a particularly interesting use, since one of the striking mathematical properties of the series is that one can create any positive integer through addition of selected members—a property that makes it ideal for application to balance measurements (Hoggatt 1969, 76). There is no evidence that ancient Greek mathematicians knew of the Fibonacci series. There was use of the Fibonacci series in Minoan design, but Preziosi (1968) cites evidence indicating that it could have been brought from Egypt by Minoan architectural workers employed at Kahun.

Doubling series in Africa

Some accounts report that Africans use a "primitive" number system in which they count by multiples of two. It is true that many cases of African arithmetic are based on multiples of two, but as we will see, base-2 systems are not crude artifacts from a forgotten past. They have surprising mathematical significance, not only in relation to African fractals, but to the Western history of mathematics and computing as well.

The presence of doubling as a cultural theme occurs in many different African societies and in many different social domains, connecting the sacredness of twins, spirit doubles, and double vision with material objects, such as the blacksmith's twin bellows and the double iron hoe given in bridewealth (fig. 7.3). Figure 7.4a shows the Ishango bone, which is around 8,000 years old and appears to show a doubling sequence. Doubling is fundamental to many of the counting systems of Africa in modern times as well. It is common, for example, to have the word for an even number 2N mean "N plus N" (e.g., the number 8 in the Shambaa language of Tanzania is "ne na ne," literally "four and four"). A similar doubling takes place for the precisely articulated system of number hand gestures (fig. 7.4b), for example, "four" represented by two groups of two fingers, and "eight" by two groups of four. Petitto (1982) found that doubling was used in multiplication and division techniques in West Africa (fig. 7.4c). Gillings (1972) details the persistent use of powers of two in ancient Egyptian mathematics as well, and Zaslavsky (1973) shows archaeological evidence suggesting that ancient Egypt's use of base-2 calculations derived from the use of base-2 in sub-Saharan Africa.

Doubling practices were also used by African descendants in the Americas. Benjamin Banneker, for example, made unusual use of doubling in his calculations, which may have derived from the teachings of his African father and grandfather (Eglash 1997c). Gates (1988) examined the cultural significance of doubling in West African religions such as vodun and its transfer to "voodoo" in the Americas. In the religion of Shango, for example, the vodun god of thunder and lightning is represented by a double-bladed axe (fig. 7.5a), used by Shango devotees in the new world as well (R. Thompson 1983). Figure 7.5b shows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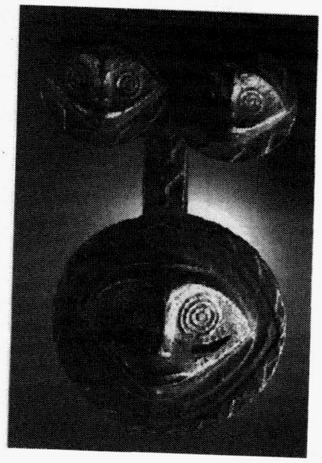
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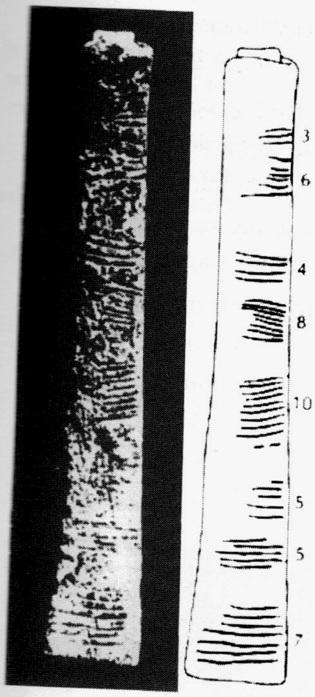
FIGURE 7.3
Doubling in African social practices
(a) This figure is used by women in Ghana to encourage the birth of twins. **(b)** A double iron hoe is sometimes used as part of the bride price ceremony. **(c)** The double bellows of the blacksmith. **(d)** Double vision; a common theme in several African spiritual practices, often implying that one can see both the material world and the spirit world.
(**b,** *Marc and Evelyn Bernheim from Rapho Guillumette; courtesy of Uganda National Museum.* **c,** *photo courtesy IFAN, Dakar.* **d,** *from Berjonneau and Sonnery 1987.*)



a

(a) The Ishango bone, estimated to be over 8,000 years old, shows what appears to be use of doubling: 3 + 3 = 6, 4 + 4 = 8, 10 = 5 + 5.

b

(b) Even numbers are typically represented by doubling in the precisely articulated system of African hand gestures.

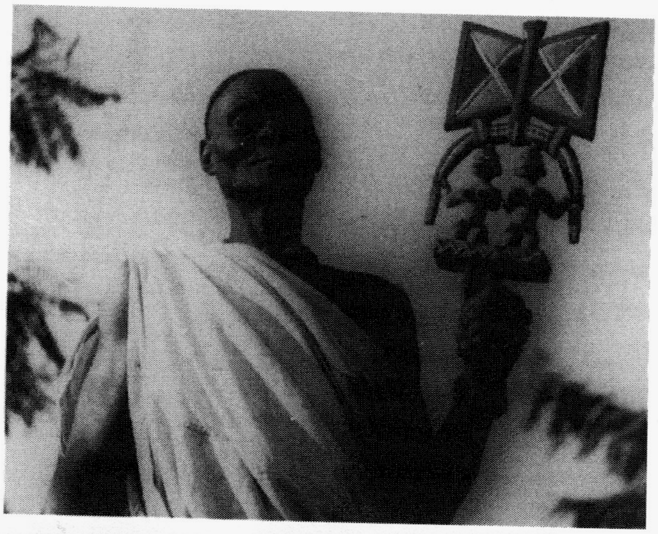
(c) Doubling was traditionally used by tailors in West Africa when doing large mental multiplications; it is essentially based on what we would call factoring.

For example, 3 × 273 ("3 taken 273 times") would be calculated by successively doubling 3 (6, 12, 24 . . .) while keeping track of the counterpart in powers of two (2, 4, 8 . . .). When the next power of two would overshoot 273, he then has to memorize the number reached so far through doublings of 3 (768), while subtracting the power of two that was reached (273 – 256 = 17). Then he starts again, doubling 3, and keeping track of the powers of two. When the next power of two would overshoot 17, he again memorizes the number reached through doublings of 3 (48) and subtracts the power of two (17 – 16 = 1). Since one is left over, he just needs to add an additional 3. The answer is then given by the sum of the underlined terms: 768 + 48 + 3 = 819.

Despite the complexity of the method, the tailors were quite fast at performing these silent mental operations.

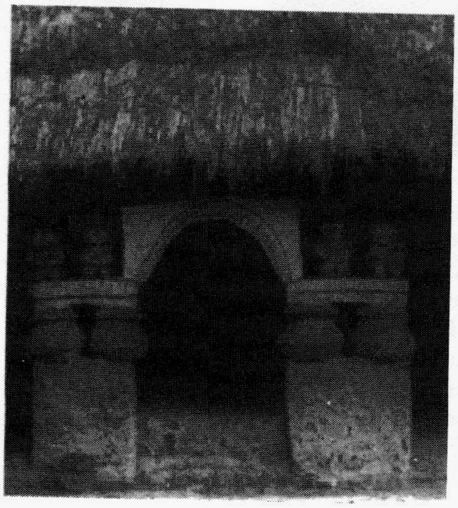
FIGURE 7.4
Doubling in African arithmetic
(**a,** *photo: Heinzelin; schematic: Lawrence Hill Books;* **b,** *Sam Zaslavsky.)*

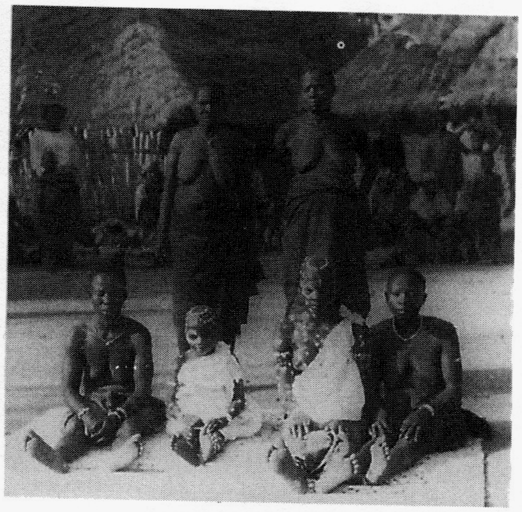
the use of a doubling sequence in the structure of a Shango temple and in religious ceremonies (ritual choreography aligning two priests, four children, eight legs). A curator at the Musée Ethnographique in Porto Novo, Benin, who specialized in Shango explained to me that these doubling structures were used because the god of lightning required a portrait of the forked structure of a lightning bolt. The model is particularly interesting in that the lengths of each iteration are shortened, so that one could have infinite doublings in a finite



(a) Shango, the god of lightning, is part of the vodun religion of Benin and was one of the important components in the creation of the voodoo religion in the New World. Here we see the double-bladed "thunder axe," with another double blade within each side.

a





b

(b) Shango temple and initiation. Here we see the doubling sequence carried out further, using the bilateral symmetry of the human body itself in the last iteration. This is used to symbolize the bifurcating pattern of the lightning bolt.

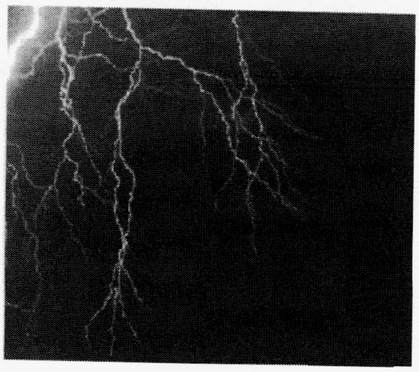


FIGURE 7.5

Doubling in the religion of Shango

(**a,** *courtesy IFAN, Dakar.* **b:** *both center photos, courtesy IFAN, Dakar; lower right, courtesy Dave Crowley, www.stormguy.com.)*

space—a true fractal. The self-similar structure of lightning has been a favorite example for fractal geometry texts (see Mandelbrot 1977). The doubling sequence used to model the fractal structure of lightning in Shango would not give an accurate value for the empirical fractal dimension—real lightning tends to branch much more than doubling allows for—but it's enough to know that the vodun representation offers a testable quantitative model.

The most mathematically significant aspect of doubling in African religion occurs in the divination ("fortunetelling") techniques of vodun and its religious relatives (Eglash 1997b). The famous Ifa divination system (fig. 7.6) is based on tossing pairs of flat shells or seeds split in two. Each lands open-side or closed-side (like "heads or tails" in a coin toss). They are connected by a doubled chain to make four pairs. Each group of four pairs gives one of the 16 divination symbols, which tell the future of the diviner's client. The Ifa system is what a mathematician would call "stochastic," that is, it operates by pure chance. But a closely related divination system, Cedena, has a nonstochastic element—it is closer to what mathematicians call "deterministic chaos."

My introduction to *cedena*, or sand divination, took place in Dakar, Senegal, where the local Islamic culture credits the Bamana (also known as Bambara) with a potent pagan mysticism. Almost all diviners had some kind of physical deformity—"the price paid for their power."[3] One diviner seemed quite willing to teach me about the system, suggesting that it "would be just like school." The first few sessions went smoothly, with the diviner showing me a symbolic code in which each symbol, represented by a set of four vertical dashed lines drawn in the sand, stood for some archetypical concept (travel, desire, health, etc.) with which he assembled narratives about the future. But when I finally asked how he derived the symbols—in particular, the meaning of some of the patterns drawn prior to the symbol writing—they all laughed at me and shook their heads. "That's the secret!" My offers of increasingly high payments were met with disinterest. Finally, I tried to explain the social significance of cross-cultural mathematics. I happened to have a copy of Linda Garcia's *Fractal Explorer* with me and began by showing a graph of the Cantor set, explaining its recursive construction. The head diviner, with an expression of excitement, suddenly stopped me, snapped the book shut, and said "show him what he wants!"

As it turns out, the recursive construction of the Cantor set was just the right thing to show, because the Bamana divination is also based on recursion (fig. 7.7). The divination begins with four horizontal dashed lines, drawn rapidly, so that there is some random variation in the number of dashes in each. The dashes are then connected in pairs, such that each of the four lines is left with either one single dash (in the case of an odd number) or no dashes (all pairs, the case

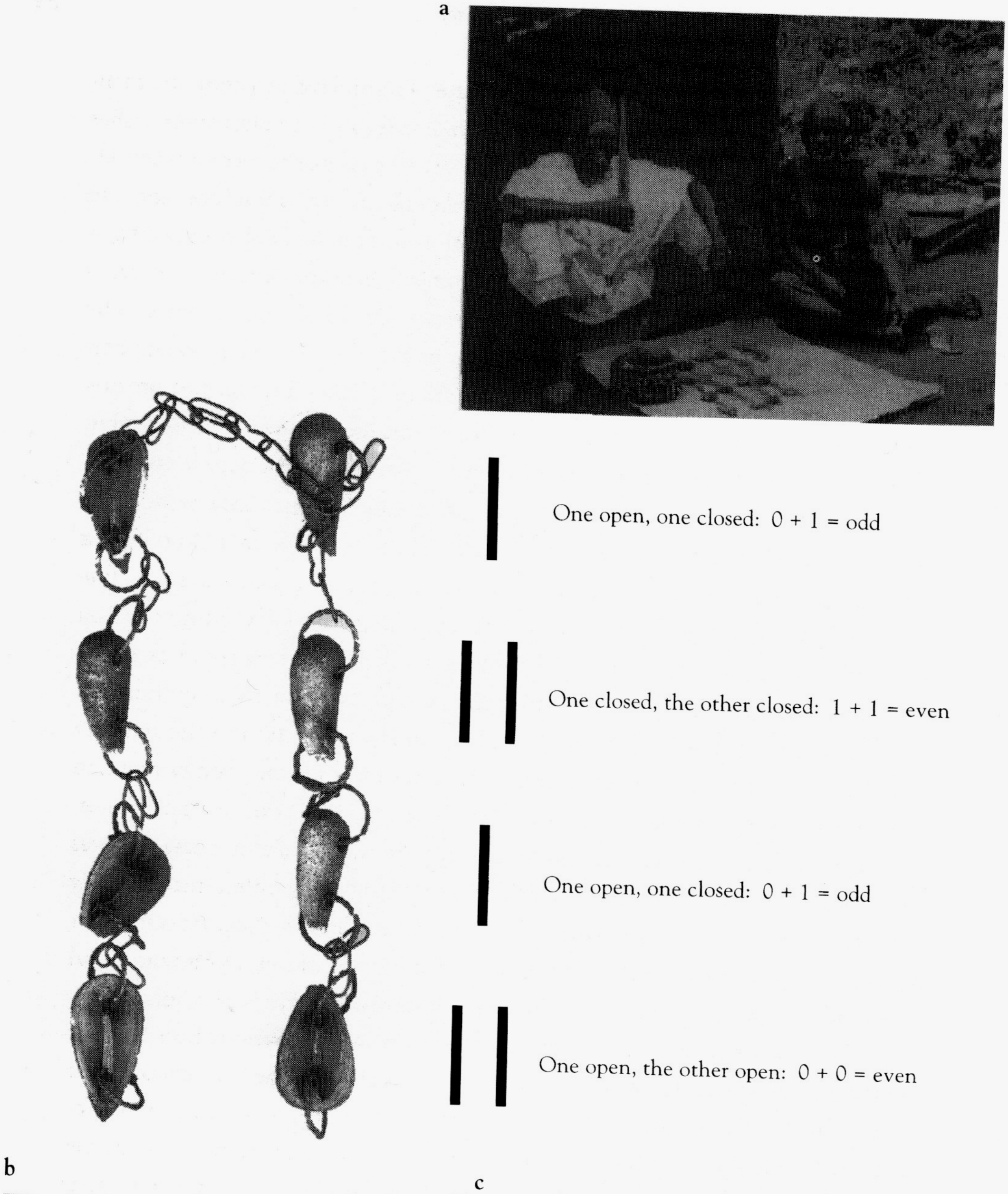


FIGURE 7.6
Binary codes in divination

(a) This Nigerian priest is telling the future by Ifa divination, in which pairs of flat shells or seeds split in two are tossed with each landing open-side or closed-side. They are connected by a doubled chain to make four pairs, giving a total of 16 divination symbols. In this version of Ifa (used in the Abigba region of Nigeria) they use two doubled chains and consider the cast more accurate if there is a correlation between the two sets. **(b)** Here we see a chain using split seeds. Each half lands either "closed" (meaning we see the rounded outside) or "open" (meaning we see the interior). By using open to represent 0 (double lines), and closed to represent 1 (single line), we can see how the divination symbol is obtained. **(c)** The divination chain is interpreted as pairs summing to odd (one stroke) or even (two strokes).

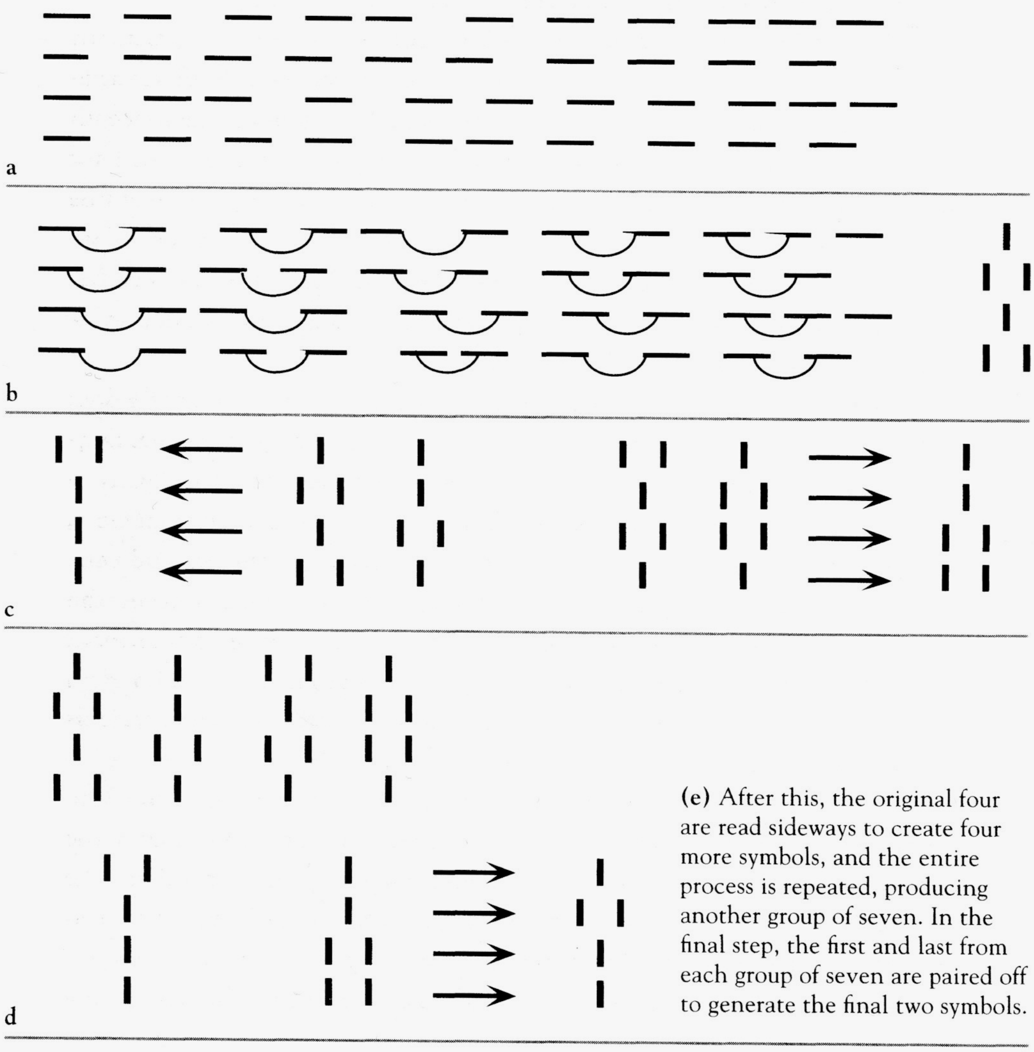
(**a,** *photo by* E. M. McClelland, *courtesy* Royal Anthropological Institute.)

of an even number). The narrative symbol is then constructed as a column of four vertical marks, with double vertical lines representing an even number of dashes and single lines representing an odd number. At this point the system is similar to the famous Ifa divination: there are two possible marks in four positions, so 16 possible symbols. Unlike Ifa, however, the random symbol production is repeated four times rather than two. The difference is quite significant. Each of the Ifa symbol pairs are interpreted as one of 256 possible Odu, or verses. The Ifa diviner must memorize the Odu; hence, four symbols would be too cumbersome (65,536 possible verses). But the Bamana divination does not require any verse memorization; as we will see, its use of recursion allows for verse self-assembly.

As in the additive sequences we examined, the divination code is generated by an iterative loop in which the output of the operation is used as the input for the next stage. In this case, the operation is addition modulo 2 ("mod 2" for short), which simply gives the remainder after division by two. This is the same even/odd distinction used in the parity bit operation that checks for errors on contemporary computer systems. There is nothing particularly complex about mod 2; in fact, I was quite disappointed at first because its reapplication destroyed the potential for a binary placeholder representation in the Bamana divination. Rather than interpret each position in the column as having some meaning (as would our binary number 1011, which means one 1, one 2, zero 4s, and one 8), the diviners reapplied mod 2 to each row of the first two symbols and to each row of the last two. The results were then assembled into two new symbols, and mod 2 was applied again to generate a third symbol. Another four symbols were created by reading the rows of the original four as columns, and mod 2 was again recursively applied to generate another three symbols.

The use of an iterative loop, passing outputs of an operation back as inputs for the next stage, was a shock to me; I was at least as taken aback by the sand symbols as the diviners had been by the Cantor set. It would be naive to claim that this was somehow a leap outside of our cultural barriers and power differences—in fact, that's just the sort of pretension that the last two decades of reflexive anthropology has been dedicated against—but it would also be ethnocentric to rule out those aspects that would be attributed to mathematical collaboration elsewhere in the world: the mutual delight in two recursion fanatics discovering each other. And the appearance of the symbols laid out in two groups of seven—the Rosicrucian's mystic number—added some numerological icing on the cake.

The following day I found that the presentation had not been complete: an additional two symbols were left out. These were also generated by mod 2 recursion using the two bottom symbols to create a fifteenth, and using that last symbol with the first symbol to create a sixteenth (bringing the total depth of recursion to five iterations). The fifteenth symbol is called "this world," and the sixteenth is "the next world," so there was good reason to separate them from the others. The final part of the system—creating a narrative from the symbols—was still unclear, but I was assured that it could be learned if I carefully followed their instructions. I was to give seven coins to seven lepers, place a kola nut on



(e) After this, the original four are read sideways to create four more symbols, and the entire process is repeated, producing another group of seven. In the final step, the first and last from each group of seven are paired off to generate the final two symbols.

FIGURE 7.7
Bamana sand divination

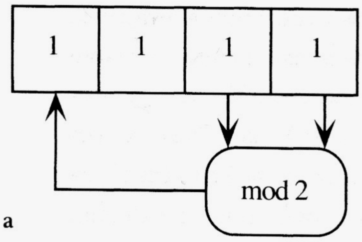
(a) Four sets of random dashes are drawn. **(b)** Each of the dashes is paired, and the odd/even results are recorded. **(c)** The process is repeated four times, resulting in four symbols. Each row of the first two symbols and the last two symbols are paired off to generate two new symbols. **(d)** The two newly generated symbols, now placed below the original four, are again paired off to generate a seventh symbol.

a pile of sand next to my bed at night, and in the morning bring a white cock, which would have to be sacrificed to compensate for the harmful energy released in the telling of the secret. I followed all the instructions, and the next morning bought a large white cock at the market. They held the chicken over the divination sand, and I was told to eat the bitter kola nut as they marked divination symbols on its feet with an ink pen. A little sand was thrown in its mouth, and then I was told to hold it down as prayers were chanted. There was no action on the part of the diviner; the chicken simply died in my hands.

While still a bit shaken by the chicken's demise (as well as experiencing a respectable buzz from the kola nut), I was told the remaining mystery. Each symbol has a "house" in which it belongs—for example, the position of the sixteenth symbol is "the next world"—but in any given divination most symbols will not be located in their own house. Thus the sixteenth symbol generated might be "desire," so we would have desire in the house of the next world, and so on. Obviously this still leaves room for creative narration on the part of the diviner, but the beauty of the system is that no verses need to be memorized or books consulted; the system creates its own complex variety.

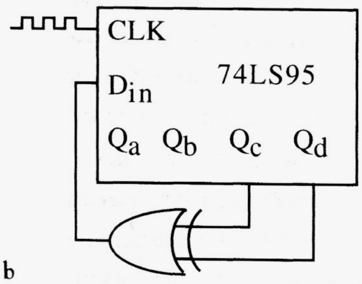
The most elegant part of the method is that it requires only four random drawings; after that the entire symbolic array is quickly self-generated. Self-generated variety is important in modern computing, where it is called "pseudo-random number generation" (fig. 7.8). These algorithms take little memory, but can generate very long lists of what appear to be random numbers, although the list will eventually start over again (this length is called the "period" of the algorithm). Although the Bamana only require an additional 12 symbols to be generated in this fashion, a maximum-length pseudorandom number generator using their initial four symbols will produce 65,535 symbols before it begins to repeat.

A similar system for self-generated variety was developed as a model for the "chaos" of nonlinear dynamics by Marston Morse (1892–1977). Before the 1970s, mathematicians had assumed that, besides a few esoteric exceptions (the algorithms for producing irrational numbers such as $\sqrt{2}$), the output of an equation would eventually start repeating. That assumption was partly based on European cultural ideas about free will: complex behavior could not be the result of predetermined systems (see Porter 1986). It was not until the 1960s–'70s that mathematicians realized that even simple, common equations describing things like population growth or fluid flow could result in what they called "deterministic chaos"—an output that never repeats, giving the appearance of random numbers from a nonrandom (deterministic) equation. Morse developed the minimal case for such behavior.



a

1111
0111
0011
0001
1000
0100
0010
1001
1100
0110
1011
0101
1010
1101
1110



b

FIGURE 7.8

Pseudorandom number generation from shift register circuits

(a) If we think of the two-strokes as zero and single stroke as one, the Bamana divination system is almost identical to the process of pseudorandom number generation used by digital circuits called "shift registers." Here the circuit takes mod 2 of the last two bits in the register and places the result in the first position. The other bits are shifted to the right, with the last discarded.

This four-bit shift register will only produce 15 binary words before the cycle starts over, but the period of the cycle increases with more bits ($2^n - 1$). For the entire 16 bits (four symbols of four bits each) that begin the Bamana divination, 65,535 binary words can be produced before repeating the cycle.

(b) Electrical circuit representation of a four-bit shift register combined with exclusive-or to perform the mod 2 operation. While schoolteachers are making increasing use of African culture in the mathematics classroom, few have explored the potential applications to technology education.

The construction of the Morse sequence begins by counting from zero in binary notation: 000, 001, 010, 011 . . . . It then takes the sum of the digits in each number—0 + 0 + 0 = 0, 0 + 0 + 1 = 1, etc.—and finally mod 2 of each sum. The result is a sequence with many recursive properties,[4] but of endless variety. Morse did the same "misreading" of the binary number as did the Bamana—although he did not have an anthropologist scowling at him for ignoring place value—and he did it for the same reason: combined with the mod 2 operation, it maximizes variety.

In my reading of divination literature I eventually came across the duplicate of the Bamana technique 5,000 miles to the east in Malagasy *sikidy* (Sussman and Sussman 1977), which inspired a study of the history of its diffusion. The strong similarity of both symbolic technique and semantic categories to what Europeans termed "geomancy" was first noted by Flacourt (1661), but it was not until Trautmann (1939) that a serious claim was made for a common source for this Arabic, European, West African, and East African divination technique. The commonality was confirmed in a detailed formal analysis by Jaulin (1966). But where did it originate?

Skinner (1980) provides a well-documented history of the diffusion evidence, from the first specific written record—a ninth-century Jewish commentary by Aran ben Joseph—to its modern use in Aleister Crowley's *Liber 777*. The oldest Arabic documents (those of az-Zanti in the thirteenth century) claim the origin of geomancy (*ilm al-raml*, "the science of sand") through the Egyptian god Idris (Hermes Trismegistus); while we need not take that as anything more than a claim to antiquity, a Nilotic influence is not unreasonable. Budge (1961) attempts to connect the use of sand in ancient Egyptian rituals to African geomancy, but it is hard to see this as unique. Mathematically, however, geomancy is strikingly out of place in non-African systems.

Like other linguistic codes, number bases tend to have an extremely long historical persistence. Even under Platonic rationalism, the ancient Greeks held 10 to be the most sacred of all numbers; the Kabbalah's Ayin Sof emanates by 10 Sefirot, and the Christian West counts on its "Hindu-Arabic" decimal notation. In Africa, on the other hand, base-2 calculation was ubiquitous, even for multiplication and division. And it is here that we find the cultural connotations of doubling that ground the divination practice in its religious significance.

The implications of this trajectory—from sub-Saharan Africa to North Africa to Europe—are quite significant for the history of mathematics. Following the introduction of geomancy to Europe by Hugo of Santalla in twelfth-century Spain, it was taken up with great interest by the pre-science mystics of those times—alchemists, hermeticists, and Rosicrucians (fig. 7.9). But these European geomancers—Raymond Lull, Robert Fludd, de Peruchio, Henry de Pisis, and others—persistently replaced the deterministic aspects of the system with chance. By mounting the 16 figures on a wheel and spinning it, they maintained their society's exclusion of any connections between determinism and unpredictability. The Africans, on the other hand, seem to have emphasized such connections. In chapter 10 we will explore one source of this difference: the African concept of a "trickster" god, one who is both deterministic and unpredictable.

On a video recording I made of the Bamana divination, I noticed that the practitioners had used a shortcut method in some demonstrations (this may have been a parting gift, as the video was shot on my last day). As they first taught me, when they count off the pairs of random dashes, they link them by drawing short curves. The shortcut method then links those curves with larger curves, and those below with even larger curves. This upside-down Cantor set shows that they are not simply applying mod 2 again and again in a mindless fashion. The self-similar physical structure of the shortcut method vividly illustrates a recursive process, and as a nontraditional invention (there is no record of its use elsewhere) it shows active mathematical practice. Other African divination practices

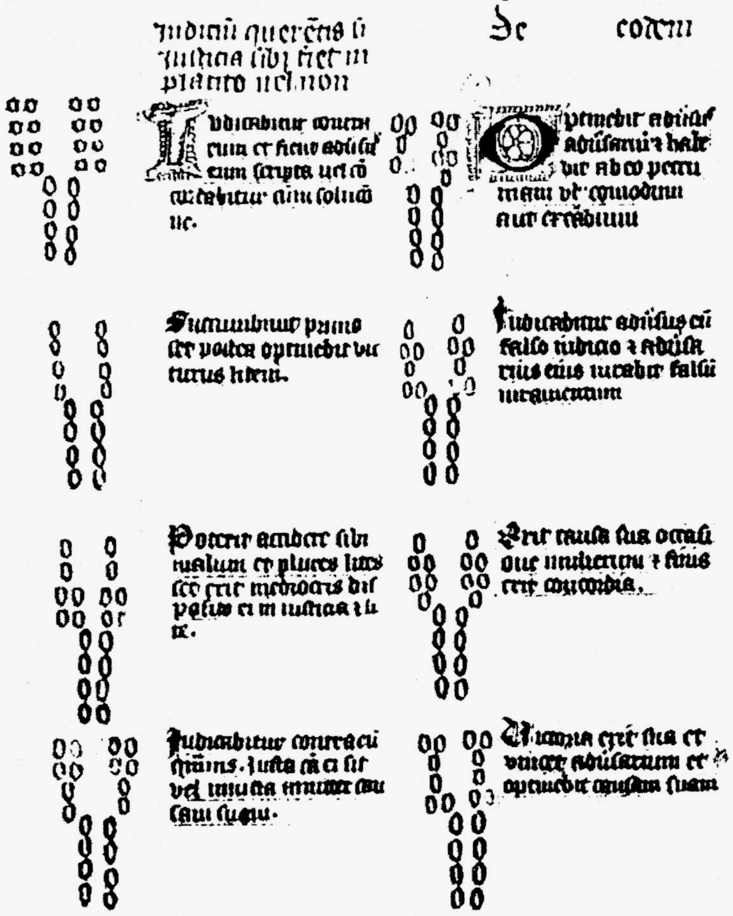


FIGURE 7.9
Geomancy

African divination was taken up under the name "geomancy" by European mystics. This chart was drawn for King Richard II in 1391.
(From Skinner 1980.)

can be linked to recursion as well; for example Devisch (1991) describes the Yaka diviners' "self-generative" initiation and uterine symbolism.

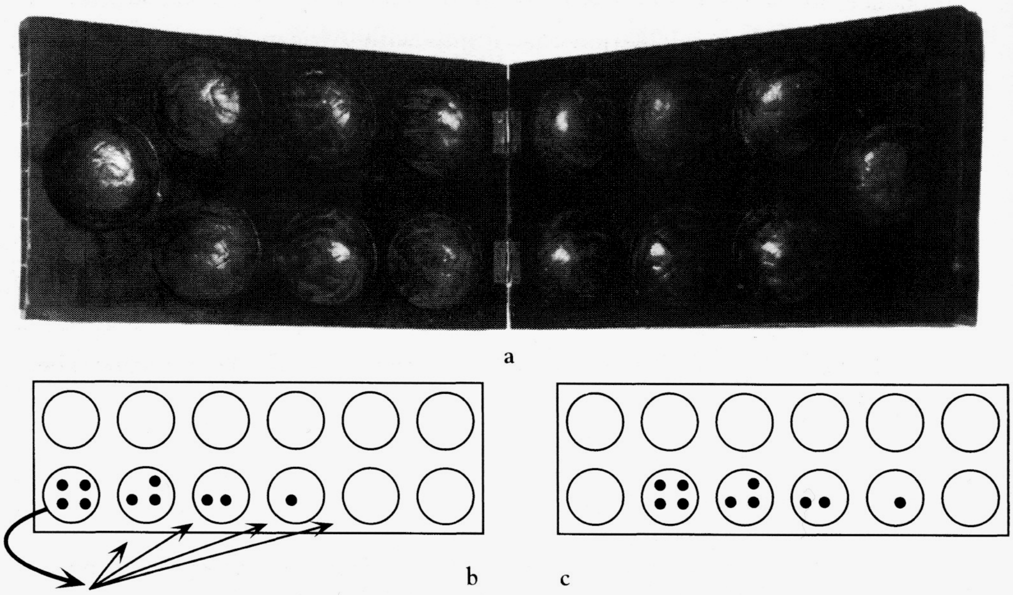
Before leaving divination, there is one more important connection to mathematical history. While Raymond Lull, like other European alchemists, created wheels with sixteen divination figures, his primary interest was in the combinatorial possibilities offered by base-2 divisions. Lull's work was closely examined by German mathematician Gottfried Leibniz, whose *Dissertatio de arte combinatoria*, published in 1666 when he was twenty, acknowledges Lull's work as a precursor. Further exploration led Leibniz to introduce a base-2 counting system, creating what we now call the binary code. While there were many other influences in the lives of Lull and Leibniz, it is not far-fetched to see a historical path for base-2 calculation that begins with African divination, runs through the geomancy of European alchemists, and is finally translated into binary calculation, where it is now applied in every digital circuit from alarm clocks to supercomputers.

In a 1995 interview in *Wired* magazine, techno-pop musician Brian Eno claimed that the problem with computers is that "they don't have enough African in them." Eno was, no doubt, trying to be complimentary, saying that there is some intuitive quality that is a valuable attribute of African culture. But in doing so he obscured the cultural origins of digital computing and did an injustice to the very concept he was trying to convey.

Discrete self-organization in Owari

Figure 7.10a shows a board game that is played throughout Africa in many different versions variously termed *ayo*, *bao*, *giuthi*, *lela*, *mancala*, *omweso*, *owari*, *tei*, and *songo* (among many other names). Boards that were cut into stones, some of extreme antiquity, have been found from Zimbabwe to Ethiopia (see Zaslavsky 1973, fig. 11-6). The game is played by scooping pebble or seed counters from one cup, and placing one of those counters into each cup, starting with the cup to the right of the scoop. The goal is to have the last counter land in a cup that has only one or two counters already in it, which allows the player to capture these counters. In the Ghanaian game of owari, players are known for utilizing a series of moves they call a "marching group." They note that if the number of counters in a series of cups each decreases by one (e.g., 4-3-2-1), the entire pattern can be replicated with a right-shift by scooping from the largest cup, and that if the pattern is left uninterrupted it can propagate in this way as far as needed for a winning move (fig. 7.10b). As simple as it seems, this concept of a self-replicating pattern is at the heart of some sophisticated mathematical concepts.

John von Neumann, who played a pivotal role in the development of the modern digital computer, was also a founder of the mathematical theory of self-organizing systems. Initially, von Neumann's theory was to be based on self-reproducing physical robots. Why work on a theory of self-reproducing machines? I believe the answer can be found in von Neumann's social outlook. Heims's (1984) biography emphasizes how the disorder of von Neumann's precarious youth as a Hungarian Jew was reflected in his adult efforts to impose a strict mathematical order on various aspects of the world. In von Neumann's application of game theory to social science, for example, Heims writes that his "Hobbesian" assumptions were "conditioned by the harsh political realities of



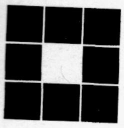
a b c

FIGURE 7.10
Owari
(a) The owari board has 12 cups, plus one cup on each side for captured counters. This board is hinged in the center, with a beautifully carved cover (see fig. 7.14). **(b)** Scoop from the first cup, and plant one counter in each succeeding cup. **(c)** The Marching Group is replicated with a right-shift. Repeated application will allow it to propagate around the board.

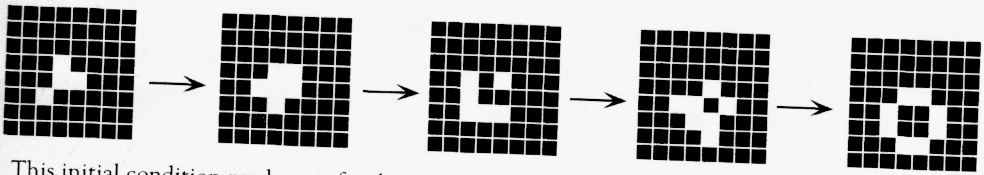
his Hungarian existence." His enthusiasm for the use of nuclear weapons against the Soviet Union is also attributed to this experience.

During the Hixon Symposium (von Neumann 1951) he was asked if computing machines could be built such that they could repair themselves if "damaged in air raids," and he replied that "there is no doubt that one can design machines which, under suitable circumstances, will repair themselves." His work on nuclear radiation tolerance for the Atomic Energy Commission in 1954–1955 included biological effects as well as machine operation. Putting these facts together, I cannot escape the creepy conclusion that von Neumann's interest in self-reproducing automata originated in fantasies about having a more perfect mechanical progeny survive the nuclear purging of organic life on this planet.

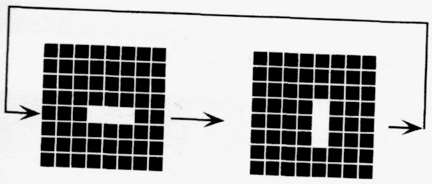
Models for physical robots turned out to be too complex, and at the suggestion of his colleague Stanislaw Ulam, von Neumann settled for a graphic abstraction: "cellular automata," as they came to be called. In this model (fig. 7.11a), each square in a grid is said to be either alive or dead (that is, in one of two possible states). The iterative rules for changing the state of any one square are based



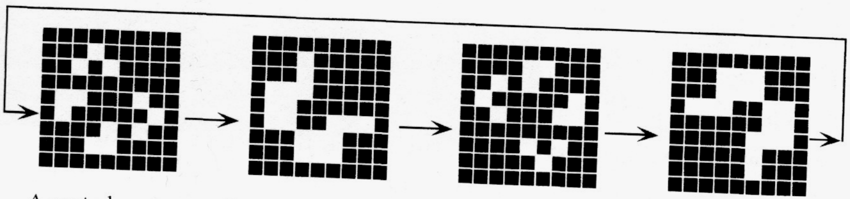
In the cellular automaton called "the game of life," each cell in the grid is in one of two states: live or dead. Here we see a live cell in the center, surrounded by dead cells in its eight nearest neighbors. The state of each cell in the next iteration is determined by a set of rules. In "classic" life (the rules first proposed by John Horton Conway), a dead cell becomes a live cell if it has three live nearest neighbors, and a cell dies unless it has two or three live neighbors.



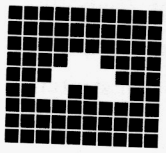
This initial condition produces a fixed pattern after four iterations. The patterns occurring before it settles down to stability are called the "transient."

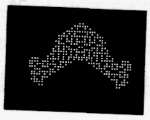


This stable pattern flips back and forth between these two states. This is called a "period-2" pattern.

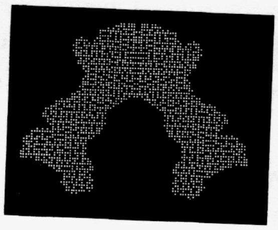


A period-4 pattern. Periods of any length can be produced, as we saw in the previous examples of pseudorandom number generation. Deterministic chaos, in which the pattern never repeats (i.e., a period-infinity pattern, like the Morse sequence), is also possible.

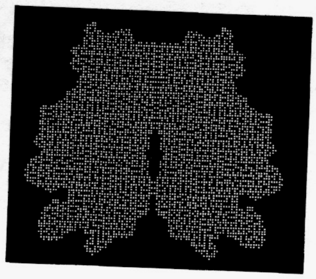




Iteration 49



Iteration 133



Iteration 182

A constant-growth pattern, shown in high resolution, looks similar to the cross-section of an internal organ. The rules: a dead cell becomes a live cell if it has three live nearest neighbors, and a cell dies only if it has seven or eight live neighbors.

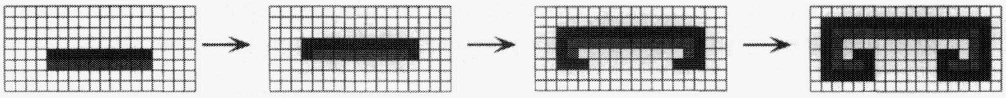
FIGURE 7.11

Cellular automata

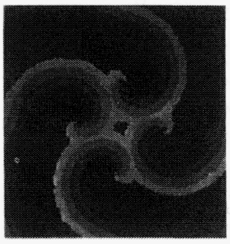
on the eight nearest neighbors (e.g., if three or more nearest-neighbors are full, the cell becomes full in the next iteration). At first, researchers carried out on these cellular automata experiments on checkered tablecloths with poker chips and dozens of human helpers (Mayer-Kress, pers. comm.), but by 1970 it had been developed into a simple computer program (Conway's "game of life"), which was described by Martin Gardner in his famous "Mathematical Games" column in *Scientific American*. The "game of life" story was an instant hit, and computer screens all over the world began to pulsate with a bizarre array of patterns (fig. 7.11b). As these activities drew increasing professional attention, a wide range of mathematically oriented scientists began to realize that the spontaneous emergence of self-sustaining patterns created in certain cellular automata were excellent models for the kinds of self-organizing patterns that had been so elusive in studies of fluid flow and biological growth.

Since scaling structures are one of the hallmarks of both fluid turbulence and biological growth, the occurrence of fractal patterns in cellular automata attracted a great deal of interest. But a more simple scaling structure, the logarithmic spiral (fig. 7.12), has garnered much of the attention. Even back in the 1950s mathematician Alan Turing, whose theory of computation provided von Neumann with the inspiration for the first digital computer, began his research on "biological morphogenesis" with an analysis of logarithmic spirals in growth patterns. Markus (1991) notes that the application areas for cellular automata models of spiral waves include nerve axons, the retina, the surface of fertilized eggs, the cerebral cortex, heart tissue, and aggregating slime molds. In the text for CALAB, the first comprehensive software for experimenting with cellular automata, mathematician Rudy Rucker (1989, 168) refers to systems that produce paired log spirals as "Zhabotinsky CAs," after the chemist who first observed such self-organizing patterns in artificial media: "When you look at Zhabotinsky CAs, you are seeing very striking three dimensional structures; things like paired vortex sheets in the surface of a river below a dam, the scroll pair stretching all the way down to the river bottom. . . . In three dimensions, a Zhabotinsky reaction would be like two paired nautilus shells, facing each other with their lips blending. The successive layers of such a growing pattern would build up very like a fetus!"

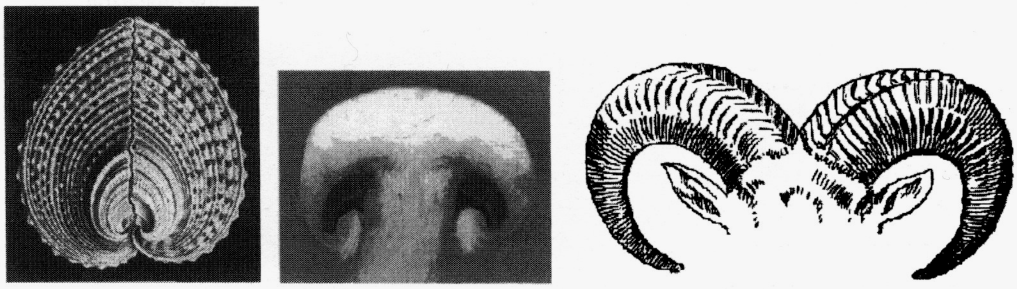
Figure 7.13 shows how the owari marching-group system can be used as a one-dimensional cellular automaton to demonstrate many of the dynamic phenomena produced on two-dimensional systems.[5] Earlier we noted that the Akan and other Ghanaian societies had a remarkable precolonial use of logarithmic spirals in iconic representations for living systems. The Ghanaian fourfold spiral (fig. 6.4a) and the four-armed computer graphic in figure 7.12b are



(a) Paired spirals emerge from a three-state cellular automation. Black cells are live, white cells are dead, and gray cells are in a refractory or "ghost" state. The rules: Any dead nearest neighbors of a live cell become live in the next iteration, and any live cell goes into the ghost state in the next iteration. The refractory layer acts as a memory, providing the directed growth (i.e., the breaking of symmetry) needed to create a spiral pattern.



(b) This four-armed logarithmic spiral from Markus (1991) was produced by a six-state cellular automaton in which a sequence of ghost states corresponds to increasingly dark shades of gray. The system makes use of a very high-resolution grid as well as some random noise to prevent the tendency for the patterns to follow the grid shape (as in the square contours of the spiral above). Compare with the Ghanaian fourfold spiral in figure 6.4a.

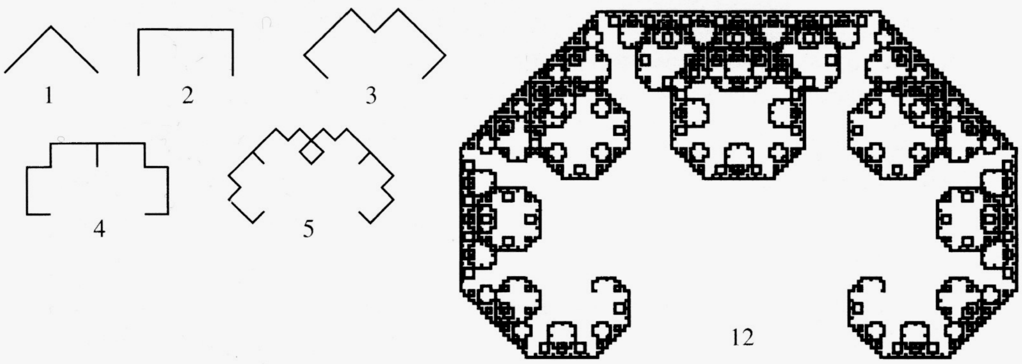


Bivalve shell.
(From Haeckel 1904.)

Mushroom cut in half.

North African sheep.
(From Cook 1914.)

(c) Paired logarithmic spirals often occur in natural growth forms.

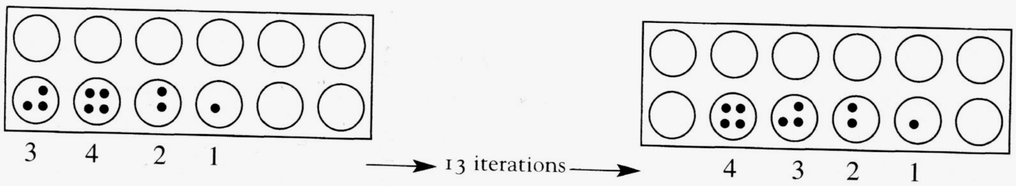


(d) Recursive line replacement, as we saw for other fractal generations, can also produce such paired spirals.

FIGURE 7.12
Spirals in cellular automata

We can view the owari board as a one-dimensional cellular automaton. One dimension is not necessarily a disadvantage; in fact, most of the professional mathematics on cellular automata (see Wolfram 1984, 1986) have been done on one-dimensional versions, because it is easier to keep track of the results. They can show all the dynamics of two dimensions.

The patterns noted by traditional owari players offer a great deal of insight into self-organizing behavior. Their observation of a class of self-propagating patterns, the "marching group," provides an excellent starting point.



3421→532→43111→4222→3331→442→5311→42211→3322→433→4411→4552→33211→4321

The marching group is an example of a constant pattern. Here we see counters in the initial sequence 3421 converge on their marching formation simply by repeating the "scoop from the left cup" rule through 13 iterations.

Just as we saw in two-dimensional cellular automata, transients of many different lengths can be produced. Transients of maximum length are used as an endgame tactic by indigenous Ghanaian players, who call it "slow motion"—accumulating pieces on your side to prevent your opponent from capturing them. In nonlinear dynamics, the constant pattern is called a "point attractor," and the transients would be said to lie in the "basin of attraction."

The marching group rule can also produce periodic behavior (a "limit cycle" or "periodic attractor" in nonlinear dynamics terms). Here is a period-3 system using only four counters:

211→22→31→211

Which leads to marching groups, and which ones lead to periodic cycles?

The numbers which lead to marching groups—1, 3, 6, 10, 15 . . . —should look familiar to readers: it's the triangular numbers we saw in tarumbeta!

The period of cycles in between each marching group is given by one plus the iteration level of the previous triangular number reached.

(*Note:* Some sequences will be truncated for 13, 14, and 15 since there are more counters than holes.)

Total number of counters	Behavior (after transients)
1	Marching
2	Period 2
3	Marching
4	Period 3
5	Period 3
6	Marching
7	Period 4
8	Period 4
9	Period 4
10	Marching
11	Period 5
12	Period 5
13	Period 5
14	Period 5
15	Marching

FIGURE 7.13
Owari as one-dimensional cellular automaton

quite distant in terms of the technologies that produced them, but there may well be some subtle connections between the two. Since cellular automata model the emergence of such patterns in modern scientific studies of living systems, and certain Ghanaian log spiral icons were also intended as generalized models for organic growth, it is not unreasonable to consider the possibility that the self-organizing dynamics observable in owari were also linked to concepts of biological morphogenesis in traditional Ghanaian knowledge systems.

Rattray's classic volume on the Asante culture of Ghana includes a chapter on owari, but unfortunately it only covers the rules and strategies of the game. Recently Kofi Agudoawu (1991) of Ghana has written a booklet on owari "dedicated to Africans who are engaged in the formidable task of reclaiming their heritage," and he does note its association with reproduction: *wari* in the Ghanaian language Twi means "he/she marries." Herskovits (1930), noting that the "awari"

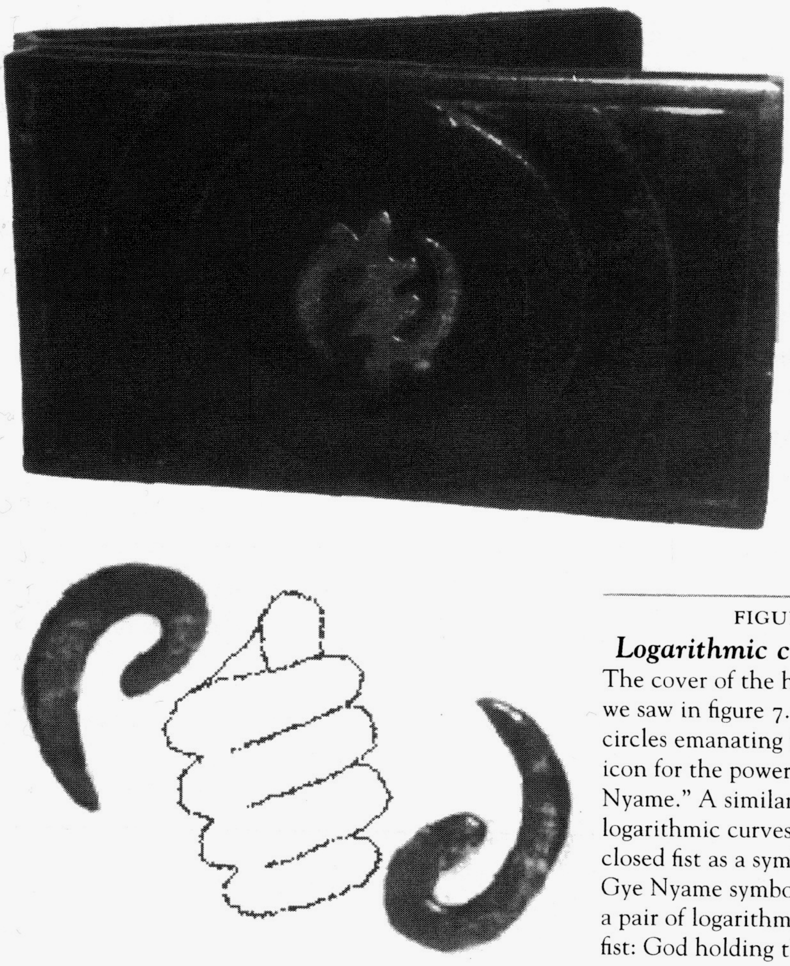


FIGURE 7.14
Logarithmic curves and owari
The cover of the hinged owari board we saw in figure 7.10 shows concentric circles emanating from the Adinkra icon for the power of god, "Gye Nyame." A similar icon, without the logarithmic curves, is attributed to a closed fist as a symbol of power. The Gye Nyame symbol thus appears to be a pair of logarithmic curves held in a fist: God holding the power of life.

game played by the descendants of African slaves in the New World had retained some of the precolonial cultural associations from Africa, reports that awari had a distinct "sacred character" to it, particularly involving the carving of the board. Owari boards with carvings of logarithmic spirals (fig. 7.14) can be commonly found in Ghana today, suggesting that Western scientists may not be the only ones who developed an association between discrete self-organizing patterns and biological reproduction. It is a bit vindictive, but I can't help but enjoy the thought of von Neumann, apostle of a mechanistic New World Order that would wipe out the irrational cacophony of living systems, spinning in his grave every time we watch a cellular automaton—whether in pixels or owari cups—bring forth chaos in the games of life.

Conclusion

Both tarumbeta and owari's marching-group dynamics are governed by the triangular numbers. There is nothing special about the triangular number series—similar nonlinear growth properties can be found in the numbers that form successively larger rectangles, pentagons, or other shapes. Nor is there anything special about the powers of two we found in divination—similar aperiodic properties can be produced by applications of mod 3, mod 4, etc. What is special is the underlying concept of recursion—the ways in which a kind of mathematical feedback loop can generate new structures in space and new dynamics in time. In the next chapter, we will see how this underlying process is found in both practical applications and abstract symbolics of African cultures.