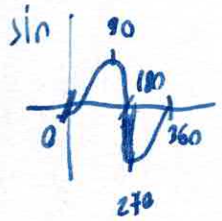
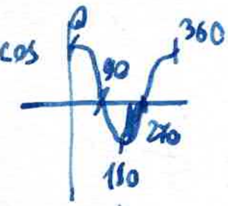
	0	30	45	60	90
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan			1	$\sqrt{3}$	
cot			1	~~1~~	

3. kontrolna naloga
2. A, 7. 2. 2024

Ime in priimek: MITJA ŠEVERKAR Razred: 2. A

dosežene točke	možne točke	odstotki	ocena
25	38	66	3





$\tan = \frac{\sin}{\cos}$

1. Izračunaj natančno vrednost izraza. Viden naj bo postopek. [5t] 5

$$\frac{\cos 210^\circ + \tan^2 240^\circ}{\sin^2 150^\circ \cdot \cot 315^\circ} = \frac{\cos(180^\circ + 30^\circ) + \tan^2(180^\circ + 60^\circ)}{\sin^2(180^\circ - 30^\circ) \cdot \cot(360^\circ - 45^\circ)} =$$

$$= \frac{-\cos 30^\circ + \tan^2 60^\circ}{\sin^2 30^\circ \cdot (-\cot 45^\circ)} = \frac{-\frac{\sqrt{3}}{2} + (\sqrt{3})^2}{\left(\frac{1}{2}\right)^2 \cdot (-1)} = \frac{-\frac{\sqrt{3}}{2} + 3}{-\frac{1}{4}} =$$

$$= \frac{\frac{6 - \sqrt{3}}{2}}{-\frac{1}{4}} = \frac{\cancel{24} - 4(6 - \sqrt{3}) \cdot 2}{\cancel{2}} = 2\sqrt{3} - 12$$

2. Naj bo $\cot x = 2\sqrt{6}$ in $\pi < x < \frac{3\pi}{2}$. Natančno izračunaj vrednost izraza $\sin x + 5\cos^2 x$. [5t] 0

$180^\circ < x < 270^\circ$

$\cot x = 2\sqrt{6}$

$\cot x = \frac{\cos x}{\sin x} =$

tan

$\cot x = \frac{\cos x}{\sin x}$

$\cot^2 x = \frac{\cos^2 x}{\sin^2 x}$

$\sin^2 + \cos^2 = 1$

$1 - \sin^2 x = \cos^2 x$

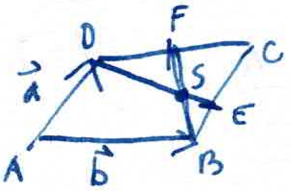
$\cot^2 x = \frac{1 - \sin^2 x}{\sin^2 x}$

$\cot^2 x = \frac{1}{\sin^2 x} - 1$

$\cot^2 x + 1 = \frac{1}{\sin^2 x}$

$\sin x + 5(1 - \sin^2 x)$

3. V paralelogramu $ABCD$ na stranici BC leži točka E, ki deli stranico tako, da je $|BE| : |EC| = 2 : 3$. Točka F je razpolovišče stranice CD. Izračunaj, v kakšnem razmerju daljica FB razdeli daljico ED. [7t] 7



$\overrightarrow{BE} = \frac{2}{5}\vec{a}$

$\overrightarrow{ED} = \frac{3}{5}\vec{a} - \vec{b}$

$\overrightarrow{BF} = \vec{a} - \frac{1}{2}\vec{b}$

$\overrightarrow{ES} = l \cdot (\overrightarrow{ED}) = \frac{3}{5}l\vec{a} - l\vec{b}$ ✓

$\overrightarrow{ES} = -\frac{2}{5}\vec{a} + k \cdot \overrightarrow{BF} = -\frac{2}{5}\vec{a} + k\vec{a} - \frac{1}{2}k\vec{b}$

$\frac{3}{5}l\vec{a} - l\vec{b} = -\frac{2}{5}\vec{a} + k\vec{a} - \frac{1}{2}k\vec{b}$

$\frac{3}{5}l\vec{a} + \frac{2}{5}\vec{a} - k\vec{a} + \frac{1}{2}k\vec{b} - l\vec{b} = 0$

$|ES| : |SD| = 2 : 5$

$\left(\frac{3}{5}l + \frac{2}{5} - k\right)\vec{a} + \left(\frac{1}{2}k - l\right)\vec{b} = 0$ ✓

$\frac{3}{5}l + \frac{2}{5} - k = 0 \quad /\cdot 5 \qquad 3l + 2 - 5k = 0$

$\frac{1}{2}k - l = 0 \quad /\cdot 10 \qquad 5k - 10l = 0$ +

$-7l + 2 = 0$

$-7l = -2$

$l = \frac{2}{7}$ ✓

4. Kot med vektorjema $\vec{a}$ in $\vec{b}$ meri $60°$. Skalarni produkt vektorjev $\vec{a}$ in $\vec{b}$ je enak 15, skalarni produkt vektorjev $\vec{a}$ in $\vec{a}+\vec{b}$ pa 51. Izračunaj dolžine vektorjev $\vec{a}$, $\vec{b}$ in $\vec{a}-2\vec{b}$. [6t] 6

$\vec{a} \cdot \vec{b} = 15$

$\vec{a} \cdot (\vec{a}+\vec{b}) = 51$

$\alpha = 60°$

$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 + \ldots$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cdot \cos\alpha$

$6 \cdot \frac{1}{2} \cdot |\vec{b}| = 15$

$|\vec{b}| = 5$

$|\vec{a}| \cdot |\vec{b}| \cdot \cos\alpha = 15$

$\vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} = 51$

$|\vec{a}|^2 + |\vec{a}| \cdot |\vec{b}| \cdot \cos\alpha = 51$

$|\vec{a}|^2 = 36$

$|\vec{a}| = 6$ ✓

$|\vec{b}| = 5$ ✓

$|\vec{a} - 2\vec{b}| = \sqrt{(\vec{a}-2\vec{b})(\vec{a}-2\vec{b})} =$

$= \sqrt{|\vec{a}|^2 - 4\vec{a}\cdot\vec{b} + 4|\vec{b}|^2} =$

$= \sqrt{36 - 4 \cdot |\vec{a}||\vec{b}| \cdot \frac{1}{2} + 4 \cdot 25} =$

$= \sqrt{36 - 2 \cdot 30 + 4 \cdot 25} =$

$= \sqrt{136 - 60} = \sqrt{76}$ ✓

136
−60
76

$\vec a \cdot \vec b = a_1 \cdot b_1 + \dots$

$\vec a \cdot \vec b = |\vec a| |\vec b| \cdot \cos\alpha$

$|\vec a|^2 = \vec a \cdot \vec a$

$|\vec a| = \sqrt{\vec a \cdot \vec a}$

5. Dan je trikotnik ABC z oglišči v točkah $A(2, 4, -3)$, $B(4, -2, 1)$ in $C(8, 6, -6)$. Krajevni vektorji oglišč trikotnika so $\vec{r}_A$, $\vec{r}_B$ in $\vec{r}_C$.

a) Izračunaj $x \in \mathbb{R}$ tako, da bo vektor $\vec{u} = \vec{r}_A - x\vec{r}_B$ pravokoten na vektor $\vec{v} = (-2, 0, -1)$. [5t] 2

$\vec r_B = (4, -2, 1)$

$\vec r_A = (2, 4, -3)$

$\vec u \cdot \vec v = 0$

$(\vec r_A - x\vec r_B) \cdot (-2, 0, -1) = 0$

$(\vec r_A - (4x, -2x, x)) \cdot (-2, 0, -1) = 0$

$(2 - 4x, 4 + 2x, -3 - x) \cdot (-2, 0, -1) = 0$

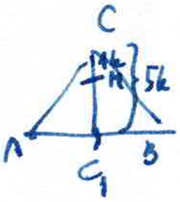
$(2-4x) \cdot (-2) = 0 \quad -4 + 8x = 0$

$(-3-x) \cdot (-1) = 0 \quad 3 - x = 0$

$-1 - 7x = 0$

$-7x = 1 \qquad x = -\frac{1}{7}$

b) Daljica CC_1 je težiščnica na stranico AB. Zapiši s koordinatami točko M, ki dano težiščnico deli v razmerju $|CM| : |CC_1| = 1 : 5$. [5t] 2



$\vec{AM} = (m_1 - 2, m_2 - 4, m_3 + 3)$

$\vec{AM} = \frac{1}{2}\vec{AB} + \frac{4}{5}\vec{C_1C} =$

$C_1\left(\frac{2+4}{2}, \frac{4-2}{2}, \frac{-3+1}{2}\right) = C_1(3, 1, -1)$

$\vec{CM} = \frac{1}{5} \cdot \vec{CC_1}$

$\vec{C_1C} = (5, 5, -5)$

$\vec{MM} = (m_1 - 8, m_2 - 6, m_3 + 6)$

$\vec{CM} = \left(\frac{4 \cdot 5}{5}, \frac{4 \cdot 5}{5}, -\frac{4 \cdot 5}{5}\right) = (4, 4, -4)$ $(-1, -1, 1)$

$m_1 - 8 = 4 \quad m_2 - 6 = 4 \quad m_3 + 6 = -4$

$m_1 = 12 \quad m_2 = 10 \quad m_3 = -10$

$M(12, 10, -10)$

c) Zapiši vektor $\vec{d}$, ki je vzporeden vektorju $\vec{AC}$ in velja $|\vec{d}| = 14$. Zapiši vse možne rešitve. [5t] 3

$\vec d = k\vec{AC}$

$|\vec d| = 14$

$|\vec{AC}| = \sqrt{16 + 24 + 11} = \sqrt{\ }$

$\vec{AC} = (6, 2, -3)$

$|\vec{AC}|^2 = \vec{AC} \cdot \vec{AC} =$

$|\vec{AC}|^2 = \sqrt{(6, 2, -3) \cdot (6, 2, -3)} =$

$= \sqrt{\ }$

$\vec{AC} \cdot \vec{AC} = 36 + 4 + 9 = 49$

$|\vec{AC}| = \sqrt{\vec{AC} \cdot \vec{AC}} = \sqrt{49} = 7$

$|\vec d| = k|\vec{AC}|$

$14 = k \cdot 7$

$k = 2$

± 2

$\vec d = ?$

DODATNA NALOGA:

Dan je trikotnik ABC. S krajevnimi vektorji oglišč trikotnika izrazi krajevni vektor nožišča N višine na stranico c. [3t]