4. kontrolna naloga - 2. rok
2. A, 22. 4. 2024

Ime in priimek: MITJA ŠEVERKAR

dosežene točke	možne točke	odstotki	ocena
33	40	83	4

1. Za vektorje $\vec{a} = (1, -2, 0)$, $\vec{b} = (-3, 1, 2)$ in $\vec{c} = (0, 3, -1)$ velja $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} + k\vec{c}$. Izračunaj vrednost realnega števila k. [5t] 5

$$\begin{array}{cccc} 1 & -2 & 0 & 1 \\ -3 & 1 & 2 & -3 \\ 0 & 3 & -1 & 0 \end{array}$$

$\vec{d} = \vec{b}\times\vec{c} = (-7, -3, -9)$

$\vec{a}\times\vec{d} = (18, 9, -17)$

$\vec{a}\cdot\vec{c} = 1\cdot 0 + (-2)\cdot 3 + 0\cdot(-1) = -6$

$k\vec{c} = (0, 3k, -k)$

$$\begin{array}{cccc} 1 & -2 & 0 & 1 \\ -7 & -3 & -9 & -7 \end{array}$$

$\vec{a}\times\vec{d} = (\vec{a}\cdot\vec{c})\vec{b} + k\vec{c}$

$(18, 9, -17) = (18, -6, -12) + (0, 3k, -k)$

$(18, 9, -17) = (18, -6+3k, -12-k)$

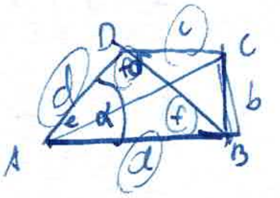
$18 = 18$

$9 = -6 + 3k \quad 3k = 15$

$-17 = -12 - k = -17 \quad k = 5$

$k = 5$

2. Dan je trapez s podatki $a = 17$ cm, $c = 4$ cm, $d = 8$ cm in $f = 15$ cm. Na eno decimalno mesto natančno izračunaj dolžino diagonale e. [5t] 5



$f^2 = a^2 + d^2 - 2ad\cdot\cos\alpha$

$15^2 = 17^2 + 8^2 - 2\cdot 17\cdot 8\cdot\cos\alpha$

$15^2 - 17^2 - 8^2 = -2\cdot 17\cdot 8\cdot\cos\alpha$

$-128 = -2\cdot 17\cdot 8\cdot\cos\alpha$

$\cos\alpha = \frac{8}{17}$

$\alpha = 61{,}9^\circ$

$\delta = 180^\circ - \alpha = 180^\circ - 61{,}9^\circ = 118{,}1^\circ$

$e^2 = c^2 + d^2 - 2cd\cdot\cos\delta =$

$= 4^2 + 8^2 - 2\cdot 4\cdot 8\cdot\cos\delta =$

$= 16 + 64 - 64\cos\delta =$

$= 80 - 64\cdot(\cos\delta) =$

$= 110{,}1476\,\text{cm}^2$

$e = 10{,}5$ cm

3. Reši enačbo: $\sqrt{x+3}+\sqrt{2x-10}-\sqrt{5x-1}=0.$ [5t] 5

$\sqrt{x+3}+\sqrt{2x-10}=\sqrt{5x-1}$ $/\cdot()^2$

$x+3+2\sqrt{(x+3)(2x-10)}+2x-10=5x-1$

$2\sqrt{2x^2-10x+6x-30}=2x+6$ $/:2$

$\sqrt{2x^2-4x-30}=x+3$ $/\cdot()^2$

$2x^2-4x-30=x^2+6x+9$

$0=-x^2+10x+39$

$0=x^2-10x-39$

$0=(x-13)(x+3)$ ✓

$x=13$ $\quad$ $x=-3$

L: $\sqrt{16}+\sqrt{26-10}-\sqrt{65-1}=4+4-\sqrt{64}=8-8=0$

D: 0

L: $\sqrt{0}+\sqrt{-6-10}-\sqrt{-15-1}=\sqrt{-16}-\sqrt{-16}$ ni R rešitev

$3\cdot 13$ //

$R=\{13\}$ ✓

$x\in\mathbb{R}$

4. Poenostavi izraz za $x>0$ in $y>0$. Rezultat zapiši s koreni, v obliki ulomka z racionaliziranim imenovalcem. [6t] 6

$$\sqrt[5]{-x\cdot\sqrt[3]{-x^2y}}\cdot\sqrt[3]{\sqrt[4]{y\cdot\sqrt[5]{x^4}}}:\sqrt{x}:\sqrt[20]{y^3}=$$

$=\left(-x\cdot\left(-x^2y\right)^{\frac{1}{3}}\right)^{\frac{1}{5}}\cdot\left(\left(y\cdot x^{\frac{4}{5}}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}:x^{\frac{1}{2}}:y^{\frac{3}{20}}$

$=\left(\cancel{-}x\cdot\left(\cancel{-}x^{\frac{2}{3}}\cdot y^{\frac{1}{3}}\right)\right)^{\frac{1}{5}}\cdot\left(y^{\frac{1}{4}}\cdot x^{\frac{1}{5}}\right)^{\frac{1}{3}}:x^{\frac{1}{2}}:y^{\frac{3}{20}}=$

$=\left(x^{\frac{5}{3}}\cdot y^{\frac{1}{3}}\right)^{\frac{1}{5}}\cdot\left(y^{\frac{1}{12}}\cdot x^{\frac{1}{15}}\right):x^{\frac{1}{2}}:y^{\frac{3}{20}}=$

$=x^{\frac{1}{3}}\cdot y^{\frac{1}{15}}\cdot y^{\frac{1}{12}}\cdot x^{\frac{1}{15}}:x^{\frac{1}{2}}:y^{\frac{3}{20}}=$

$=x^{\frac{5}{15}}\cdot x^{\frac{1}{15}}\cdot y^{\frac{4}{60}}\cdot y^{\frac{5}{60}}:x^{\frac{1}{2}}:y^{\frac{3}{20}}=$

$=\dfrac{x^{\frac{6}{15}}\cdot y^{\frac{9}{60}}}{x^{\frac{1}{2}}}:y^{\frac{3}{20}}=\dfrac{x^{\frac{12}{30}}\cdot \cancel{y^{\frac{9}{60}}}}{x^{\frac{15}{30}}\cdot \cancel{y^{\frac{9}{60}}}}=x^{-\frac{3}{30}}=x^{-\frac{1}{10}}$

$=\dfrac{1}{\sqrt[10]{x}}=\dfrac{\sqrt[10]{x^9}}{x}=\dfrac{\sqrt[10]{x^9}}{x}$ ✓

$/\cdot\sqrt[10]{x^9}$

5. Izračunaj vrednost izraza $(A-B)^{-1}$, če je $A = \frac{3x^{-\frac{1}{3}}}{x^{\frac{2}{3}}-2x^{-\frac{1}{3}}}$ in $B = \frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}}-x^{\frac{1}{3}}}$. [5t] 2

$$\left(\frac{3x^{-\frac{1}{3}}}{x^{\frac{2}{3}}-2x^{-\frac{1}{3}}} - \frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}}-x^{\frac{1}{3}}}\right)^{-1} =$$

$$= \left(\frac{3x^{-\frac{1}{3}}\left(x^{\frac{4}{3}}-x^{\frac{1}{3}}\right) - x^{\frac{1}{3}}\left(x^{\frac{2}{3}}-2x^{-\frac{1}{3}}\right)}{(x^{2/3}-2x^{-1/3})(x^{4/3}-x^{1/3})}\right)^{-1} =$$

$$= (3x - 3 - x + 2)^{-1} = (2x-1)^{-1} =$$

$$= \frac{1}{2x-1} \qquad x \neq \frac{1}{2}$$

6. Dana je funkcija $f(x) = \sqrt[5]{-x^3} + 2x$.

a) Ali je funkcija f soda, liha ali nič od tega? Odgovor naj bo računsko utemeljen. [3t] 3

sodost: $f(-x) = f(x)$

lihost: $f(-x) = -f(x)$

$$f(-x) = \sqrt[5]{x^3} - 2x$$
$$-f(x) = \sqrt[5]{x^3} - 2x$$
$$-f(x) = -\sqrt[5]{x^3} - 2x$$

Funkcija f je liha, ni pa ~~soda~~.

b) Naj bo $g(x) = f(2x-1) - 4x + 2$. Zapiši inverzno funkcijo g^{-1} funkcije g. [5t] 2

$$f(2x-1) = \sqrt[5]{-(2x-1)^3} + 4x - 2 - 4x + 2$$

$$y = g(x) = -\sqrt[5]{(2x-1)^3} + 4x - 2$$

$$\cancel{y - 4x + 2 = -\sqrt[5]{8x^3 - 3\cdot 4x^2 + 3\cdot 2x - 1}}$$

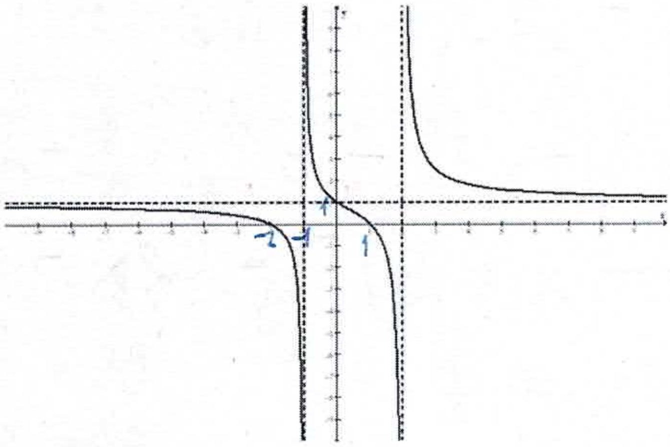
$$\cancel{(-y + 4x - 2)^5 = 8x^3 - 12x^2 + 6x - 1}$$

$$-y + 4x - 2 = \sqrt[5]{(2x-1)^3}$$

$$-y + 4x - 2 = \sqrt{8x^3 - 12x^2 + 6x - 1} \qquad /()^5$$

$$(-y + 4x - 2)^5 = 8x^3 - 12x^2 + 6x - 1$$

7. Na sliki je graf funkcije $f : \mathcal{D}_f \to \mathbb{R}$. Asimptote grafa so narisane črtkano, graf se jim v neskončnosti (izven prikazanega območja) približuje, se jih nikoli ne dotakne in jih nikoli ne seka.



a) Določi definicijsko območje, zalogo vrednosti, ničle in intervale padanja funkcije. [4t] 3

$\mathcal{D}_f = \mathbb{R} - \{-1, 2\}$ ✓

$\mathcal{Z}_f = \mathbb{R} - \{1\}$

ničle: $-2, 1$ ✓

intervali padanja: $(-\infty, -1), (-1, 2), (2, \infty)$ ✓

b) Določi množici $\mathcal{A}$ in $\mathcal{B}$ (eno od možnosti) tako, da bo funkcija $f : \mathcal{A} \to \mathcal{B}$ (z enakim predpisom, kot jo ima funkcija, katere graf je na sliki) bijektivna. [2t] 2

$A = (-1, 2)$

$B = \mathbb{R}$ ✓

$f : A \to B$

$f : (-1, 2) \to \mathbb{R}$