

3. kontrolna naloga - 2. rok
2. A, 5. 3. 2024

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dosežene točke	možne točke	odstotki	ocena
34	34	100	5

	0	30	45	60	90
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	

1. Izračunaj natančno vrednost izraza. Viden naj bo postopek.

[4t] 4

$$\begin{aligned}
 & \frac{\cos 240^\circ - \tan^2 300^\circ}{\sin^2 135^\circ} = \frac{\cos(180^\circ + 60^\circ) - \tan^2(360^\circ - 60^\circ)}{\sin^2(180^\circ - 45^\circ)} = \\
 & = \frac{-\cos 60^\circ - \tan^2 60^\circ}{\sin^2 45^\circ} = \frac{-\frac{1}{2} - (\sqrt{3})^2}{\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{-\frac{1}{2} - 3}{\frac{2}{4}} = \\
 & = -\frac{7}{2} \cdot \frac{2}{1} = -7
 \end{aligned}$$

2. Izračunaj in v enotskem krogu predstavi vse kote $\alpha \in [0^\circ, 360^\circ]$, za katere je $\tan \alpha = -\frac{\sqrt{3}}{3}$.

[5t] 5

$$\tan \alpha = \frac{\sqrt{3}}{3}$$

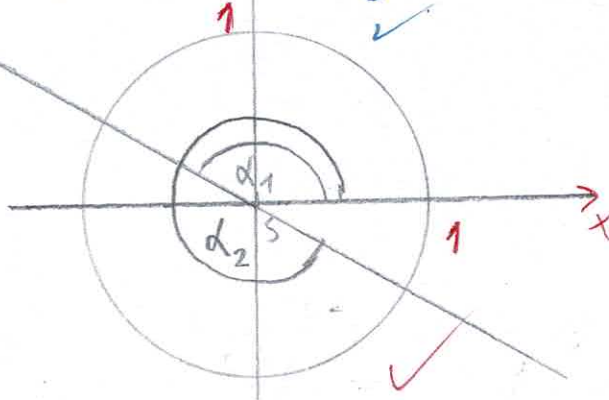
$$\tan(180^\circ - 30^\circ) = \tan(150^\circ) = -\frac{\sqrt{3}}{3} \quad \checkmark$$

$$\tan(180^\circ + 30^\circ) = \tan(210^\circ) = \tan 30^\circ = \frac{\sqrt{3}}{3} \quad \checkmark$$

$$\tan(360^\circ - 30^\circ) = \tan(330^\circ) = -\tan 30^\circ = -\frac{\sqrt{3}}{3} \quad \checkmark$$

$$\mathcal{R} = \{150^\circ, 330^\circ\}$$

$$\begin{aligned}
 \alpha_1 &= 150^\circ \\
 \alpha_2 &= 330^\circ
 \end{aligned}$$



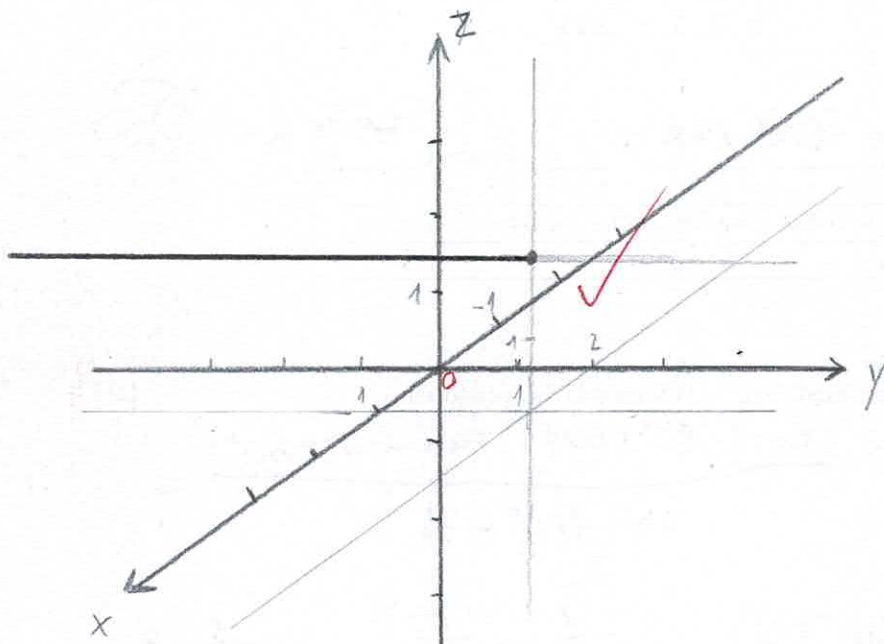
$$|\vec{a}| = \sqrt{a_1^2 + \dots}$$

$$|\vec{a}|^2 = \vec{a} \cdot \vec{a}$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + \dots$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos \alpha$$

3. V koordinatnem sistemu predstavi množico točk $\{(x, y, z); x = 1, y \leq 2, z = 2\}$. [3t] **3**



4. Izračunaj skalarni produkt $(\vec{a} + 2\vec{b}) \cdot (\vec{a} - \vec{b})$, če je $|\vec{a}| = 4$, $|2\vec{a} - \vec{b}| = 7$ in $|\vec{a}| < |\vec{b}|$. Kot med vektorjema \vec{a} in \vec{b} meri 60° . [6t] **6**

$$\begin{aligned} & (\vec{a} + 2\vec{b}) \cdot (\vec{a} - \vec{b}) = \\ & = |\vec{a}|^2 + \vec{a} \cdot \vec{b} - 2|\vec{b}|^2 = \\ & = 4^2 + |\vec{a}| \cdot |\vec{b}| \cdot \cos \alpha - 2 \cdot 5^2 = \\ & = 16 + 2 \cdot 5 \cdot \cos 60^\circ - 2 \cdot 25 = \\ & = 16 + 2 \cdot 5 \cdot \frac{1}{2} - 2 \cdot 25 = \\ & = 16 + 5 - 50 = \\ & = -24 \end{aligned}$$

$$|\vec{a}| = 4$$

$$|2\vec{a} - \vec{b}| = 7$$

$$\alpha = 60^\circ$$

$$\begin{array}{r} -64 \\ +49 \\ \hline -15 \end{array}$$

$$\vec{a} \cdot \vec{a} = |\vec{a}|^2 = 4^2 = 16$$

$$(2\vec{a} - \vec{b}) \cdot (2\vec{a} - \vec{b}) = 7^2$$

$$4\vec{a} \cdot \vec{a} - 4\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} = 49$$

$$4 \cdot 16 - 4 \cdot |\vec{a}| \cdot |\vec{b}| \cdot \cos \alpha + |\vec{b}|^2 = 49$$

$$64 - 4 \cdot \frac{1}{2} \cdot |\vec{a}| \cdot |\vec{b}| + |\vec{b}|^2 = 49$$

$$64 - 8|\vec{b}| + |\vec{b}|^2 = -15$$

$$|\vec{b}|^2 - 8|\vec{b}| + 79 = 0$$

$$(|\vec{b}| - 5)(|\vec{b}| - 15) = 0$$

$$|\vec{b}| = 5 > 4$$

$$|\vec{b}| = 5$$



5. Točke $A(3, 6, -4)$, $B(-2, 5, 1)$ in $C(5, 1, 3)$ so oglišča trikotnika ABC .

a) Zapiši koordinate težišča T trikotnika ABC .

[2t] 2

$$T\left(\frac{3+(-2)+5}{3}, \frac{6+5+1}{3}, \frac{-4+1+3}{3}\right) \Rightarrow T(2, 4, 0) \checkmark$$

b) Izračunaj kosinus kota $\angle BTC$.

[5t] 5

$$\vec{TB} = (-2-2, 5-4, 1-0) = (-4, 1, 1) \checkmark$$

$$\vec{TC} = (5-2, 1-4, 3-0) = (3, -3, 3) \checkmark$$

$$|\vec{TC}| = \sqrt{3^2 + (-3)^2 + 3^2} = \sqrt{27} = 3\sqrt{3} \checkmark$$

$$|\vec{TB}| = \sqrt{(-4)^2 + 1^2 + 1^2} = \sqrt{16+2} = \sqrt{18} = 3\sqrt{2} \checkmark$$

$$\vec{TB} \cdot \vec{TC} = -4 \cdot 3 + 1 \cdot (-3) + 1 \cdot 3 = -12 - 3 + 3 = -12$$

$$\vec{TB} \cdot \vec{TC} = |\vec{TB}| \cdot |\vec{TC}| \cdot \cos \alpha \quad \cos \alpha = \frac{\vec{TB} \cdot \vec{TC}}{|\vec{TB}| \cdot |\vec{TC}|} = \frac{-12}{3\sqrt{2} \cdot 3\sqrt{3}} = \frac{-12}{9\sqrt{6}} = -\frac{4}{3\sqrt{6}}$$

$\vec{a} = k \vec{b} + \text{lin. odv.}$
lin. neodv. \parallel

c) Za katere realne vrednosti y sta krajevni vektor točke B in vektor $\vec{u} = (4, y, -2)$ linearno neodvisna?

[4t] 4

$$\vec{u}_B = (-2, 5, 1)$$

$$1 \cdot -2 \rightarrow \vec{u}_B = (4, -10, -2)$$

Za vse \mathbb{R} vrednosti, razen -10 .

č) Točko Z dobimo tako, da težišče T prezrcalimo čez razpolovišče stranice BC . Izračunaj koordinate točke Z .

[5t] 5

$$S\left(\frac{-2+5}{2}, \frac{5+1}{2}, \frac{1+3}{2}\right) \Rightarrow S\left(\frac{3}{2}, 3, 2\right) \checkmark$$

$$\vec{TS} = \left(\frac{3}{2}-2, 3-4, 2-0\right) = \left(-\frac{1}{2}, -1, 2\right)$$

$$Z\left(2 - \frac{1}{2} \cdot 2, 4 - 1 \cdot 2, 0 + 2 \cdot 2\right) \Rightarrow Z(1, 2, 4) \checkmark$$