3. kontrolna naloga
1. A, 14. 2. 2023

Ime in priimek: MITJA ŠEVERKAR Razred: 1.a

dosežene točke	možne točke	odstotki	ocena
38	42	90	5

ČAS PISANJA: 45 minut

1. Določi največji skupni delitelj in najmanjši skupni večkratnik izrazov $I_1 = x^4 - 13x^2 + 36$, $I_2 = 2x^5 + 54x^2$ in $I_3 = x^3 - 2x^2 - 9x + 18$. [5t] 1

$v(I_1, I_2, I_3) = 2x^2(x^2-x-6)(x^2+x-6)(x+3)(x^2-3x+9)(x-2)(x-9)$

$D(I_1, I_2, I_3) = \cancel{x}\ x+3$

$I_1 = x^4 - 13x^2 + 36 = x^4 - 12x^2 + 36 - x^2 = (x^2-6)^2 - x^2 = *$

$I_2 = 2x^5 + 54x^2 = 2x^2(x^3 + 27) = 2x^2(x+3)(x^2-3x+9)$

$I_3 = x^3 - 2x^2 - 9x + 18 = x^2(x-2) - 9(x-2) =$

$= (x-2)(x^2-9) = \cancel{x}(x-2)(x^2-9) = (x-2)(x+3)(x-2)$

$* = (x^2-6-x)(x^2-6+x) = (x^2-x-6)(x^2+x-6)$

$= (x-3)(x+2)(x+3)(x-2)$

2. Neko naravno število da pri deljenju s 76 ostanek 43. Kolikšen ostanek da to isto število pri deljenju z 19? Odgovor naj bo računsko utemeljen. [4t] 4

$a = 76k + 43$ $k \in \mathbb{N}$

$a : 19 = 4k + 2$, ost. 5

$a = 76k + 43$

3. Dokaži, da sta števili $3n+4$ in $14n+19$ tuji za vsak $n \in \mathbb{N}$. [4t] 4

$I_1 = 3n+4$ $I_2 = 14n+19$

$14n + 19 = 4(3n+4) + 2n + 3$

$3n + 4 = 2n + 3 + n + 1$

$2n + 3 = 2(n+1) + 1 \rightarrow D(I_1, I_2) | 1$

$D(I_1, I_2) = 1$

4. Določi vsa naravna števila a in b, za katera velja $a - b = 180$, $D(a, b) = 15$, $a < 260$.

[6t] 6

$a = 15k_1$ $k_1, k_2 \in \mathbb{N}$ $D(k_1, k_2) = 1$

$b = 15k_2$

$(a, b) \in \{(195, 15), (255, 75)\}$

$a - b = 180$

$15k_1 - 15k_2 = 180$

$15(k_1 - k_2) = 180$

$k_1 - k_2 = 12$

$12 < k_1 < 18$

$k_1, k_2 = 13, 1$ $D = 1$

$k_1, k_2 = 14, 2$ $D \neq 1$ X

$k_1, k_2 = 15, 3$ $D \neq 1$ X

$k_1, k_2 = 16, 4$ $D \neq 1$ X

$k_1, k_2 = 17, 5$ $D = 1$

5. a) Reši enačbo: $3AB_{(13)} = x_{(7)}$.

[4t] 4

$13^2\ 13^1\ 13^0$

$3 \cdot 13^2 + 10 \cdot 13^1 + 11 \cdot 13^0 =$

$= 507 + 130 + 11 =$

$= 648$

$648 = 7 \cdot 92 + 4$

$92 = 7 \cdot 13 + 1$

$13 = 7 \cdot 1 + 6$

$1 = 7 \cdot 0 + 1$

$x_{(7)} = 1614_{(7)}$

b) Dokaži, da lahko število $14641_{(a)}$, za vsak $a \in \mathbb{N}$, $a > 6$, zapišemo v desetiškem sistemu kot četrto potenco nekega naravnega števila. Četrta potenca je potenca z eksponentom 4.

[3t] 3

$a^4\ a^3\ a^2\ a^1\ a^0$

$1\ 4\ 6\ 4\ 1$

izpeljano iz pascalovega trikotnika

$a^4 + 4a^3 + 6a^2 + 4a + 1 = (a+1)^4$

6. Poenostavi izraz: $\left(\frac{a^{n-1}b^5}{c^{-4}a^{3n}}\right)^{n+1} \cdot \left(2a^{-3}b^{2n+5}c^4\right)^{-n} =$ [5t] 5

$$= \left(\frac{b^5c^4}{a^{3n-n+1}}\right)^{n+1} \cdot \left(\frac{2b^{2n+5}c^4}{a^3}\right)^{-n} =$$

$$= \left(\frac{b^5c^4}{a^{2n+1}}\right)^{n+1} \cdot \left(\frac{a^3}{2b^{2n+5}c^4}\right)^{n} =$$

$$= \frac{b^{5n+5}c^{4n+4}}{a^{(2n+1)(n+1)}} \cdot \frac{a^{3n}}{2^n b^{2n^2+5n}\cancel{c^{4n}}} =$$

$$= \frac{\cancel{b^{5n+5}}\, c^4 \cdot a^{3n}}{a^{2n^2+3n+1}\, 2^n \cdot b^{2n^2+\cancel{5n}-\cancel{5n}-5}} = \frac{c^4}{a^{2n^2+\cancel{3n}+1-\cancel{3n}} \cdot 2^n \cdot b^{2n^2-5}} =$$

$$= \frac{c^4}{2^n a^{2n^2+1} b^{2n^2-5}}$$ ✓

7. Poenostavi: $\frac{3b+3}{b^2-1} : \left(\frac{2b}{b+1} - 1\right)^{-1} + \frac{b+3}{b+5} \cdot \left(\frac{b+4}{b+1} - \frac{2}{b^2+4b+3}\right) =$ [6t] 6

$$\frac{3b+3}{b^2-1} : \left(\frac{2b}{b+1} - 1\right)^{-1} + \frac{b+3}{b+5} \cdot \left(\frac{b+4}{b+1} - \frac{2}{b^2+4b+3}\right) =$$

$$= \frac{3\cancel{(b+1)}}{(b-1)\cancel{(b+1)}} \cdot \frac{2b-b-1}{b+1} + \frac{b+3}{b+5} \cdot \left(\frac{b+4}{b+1} - \frac{2}{(b+3)(b+1)}\right) =$$

$$= \frac{3}{\cancel{b-1}} \cdot \frac{\cancel{b-1}}{b+1} + \frac{b+3}{b+5} \cdot \left(\frac{(b+4)(b+3)-2}{(b+1)(b+3)}\right) =$$

$$= \frac{3}{b+1} + \frac{\cancel{b+3}}{b+5} \cdot \frac{(b+4)(b+3)-2}{(b+1)\cancel{(b+3)}} =$$

$$= \frac{3}{b+1} + \frac{b^2+7b+12-2}{(b+5)(b+1)} =$$

$$= \frac{3}{b+1} + \frac{b^2+7b+10}{(b+5)(b+1)} = \frac{3}{b+1} + \frac{\cancel{(b+5)}(b+2)}{\cancel{(b+5)}(b+1)} =$$

$$= \frac{3}{b+1} + \frac{b+2}{b+1} = \frac{3+b+2}{b+1} = \frac{b+5}{b+1}$$ ✓

8. Poenostavi: $\frac{5x}{x+\frac{1}{x+\frac{1}{x}}} \cdot \frac{x+2x^{-1}}{2x+2x^{-1}} =$ [5t] 5

$$\frac{5x}{x+\frac{1}{x+\frac{1}{\cancel{x}}}} \cdot \frac{x+2x^{-1}}{2x+2x^{-1}} = \frac{5x}{x+\frac{1}{\frac{x^2+1}{x}}} \cdot \frac{x^2+2}{2x^2+2} =$$

$$= \frac{5x}{x+1\cdot\frac{x}{x^2+1}} \cdot \frac{x^2+2}{2(x^2+1)} =$$

$$= \cancel{\frac{5x}{x+\frac{x^2+1}{x^2+1}\cdot x}} \quad \frac{5x}{x+\frac{x}{x^2+1}} \cdot \frac{x^2+2}{2(x^2+1)} =$$

$$= \frac{5x}{\frac{x(x^2+1)+x}{x^2+1}} \cdot \frac{x^2+2}{2(x^2+1)} = \frac{5\cancel{x}\cdot\cancel{(x^2+1)}}{\cancel{x}(x^2+1+1)} \cdot \frac{x^2+2}{2\cancel{(x^2+1)}} =$$

$$= \frac{5}{\cancel{x^2+2}} \cdot \frac{\cancel{x^2+2}}{2} = \frac{5}{2} = 2\frac{1}{2}$$

DODATNA NALOGA:
Dokaži, da je ostanek pri deljenju poljubnega praštevila s 30 praštevilo. [3t] 0

Praštevilo nima ~~drugih~~ deliteljev, razen sebe in 1
Recimo, da mi je zmanjkalo časa in ne morem napisati te naloge :)