Appendix - EM-SEC: Efficient Multi-head Set-valued Evidential Classification

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1 Proof of Equation (16)

The binomial belief co-multiplication [1] is defined for two independent binomial beliefs. However, in our proposed approach we need to combine multiple binomial beliefs. Hence, in this section, we present and prove the generalization of binomial belief co-multiplication for multiple states. independent events in subjective logic:

Lemma 1. *Let $C = \{1, 2, \ldots, n\}$ be a finite set of events, and let b_i denote the* belief mass *associated with event i. Under the assumption of independence, the combined belief mass for the disjunction $1 \vee 2 \vee \cdots \vee n$ is given by*

$$b_C \;=\; 1 \;-\; \prod_{i \in C} \bigl(1 - b_i\bigr). \tag{1}$$

The proof proceeds by induction on the number of events.

Base Case: $n = 2$

For two events i and j with belief masses b_i and b_j, the *binomial co-multiplication* rule for their disjunction $i \vee j$ is:

$$b_{i \vee j} \;=\; b_i + b_j - b_i\, b_j \;=\; 1 - \bigl(1 - b_i\bigr)\bigl(1 - b_j\bigr). \tag{2}$$

Thus, the theorem holds for $n = 2$:

$$b_{1 \vee 2} \;=\; 1 \;-\; \prod_{k \in \{1,2\}} \bigl(1 - b_k\bigr).$$

Inductive Step

Assume the statement holds for any set of n events. That is, for $\{1, 2, \ldots, n\}$, we have:

$$b_{1 \vee 2 \vee \cdots \vee n} \;=\; 1 \;-\; \prod_{i=1}^{n} \bigl(1 - b_i\bigr).$$

We now add one more event, $n + 1$, and examine the disjunction $(1 \vee 2 \vee \cdots \vee n) \vee (n + 1)$. Denote

$$b_n^* \;=\; b_{1 \vee 2 \vee \cdots \vee n}.$$

By the inductive hypothesis,

$$b_n^* \;=\; 1 \;-\; \prod_{i=1}^{n}\bigl(1-b_i\bigr).$$

Applying the two-event co-multiplication (2) to $\bigl\{\, b_n^*,\, b_{n+1}\bigr\}$, we get:

$$\begin{aligned}
b_{(1\vee\cdots\vee n)\vee(n+1)} &= \; b_n^* \;+\; b_{n+1} \;-\; b_n^*\, b_{n+1} \\
&= \; \bigl(1-\prod_{i=1}^{n}(1-b_i)\bigr) \;+\; b_{n+1} \;-\; \bigl(1-\prod_{i=1}^{n}(1-b_i)\bigr)\, b_{n+1} \\
&= \; 1 \;-\; \prod_{i=1}^{n}(1-b_i) \;-\; \bigl(1-\prod_{i=1}^{n}(1-b_i)\bigr)\, b_{n+1} \;+\; b_{n+1}.
\end{aligned}$$

Notice that $-\bigl(1-\prod_{i=1}^{n}(1-b_i)\bigr)\, b_{n+1} + b_{n+1} = -\, b_{n+1}\, \prod_{i=1}^{n}(1-b_i)$. Hence the expression simplifies to

$$1-\prod_{i=1}^{n}(1-b_i) \,-\, b_{n+1}\prod_{i=1}^{n}(1-b_i) \;=\; 1-\bigl(1-b_{n+1}\bigr)\prod_{i=1}^{n}(1-b_i) \;=\; 1-\prod_{i=1}^{n+1}(1-b_i).$$

This completes the inductive step, showing that the result holds for $n+1$ events whenever it holds for n events.

Conclusion

By induction, for any finite set $C = \{1, 2, \ldots, n\}$, we conclude

$$b_{1\vee 2\vee\cdots\vee n} \;=\; 1 \;-\; \prod_{i=1}^{n}\bigl(1-b_i\bigr).$$

Equivalently,

$$b_C \;=\; 1 \;-\; \prod_{i\in C}\bigl(1-b_i\bigr),$$

which proves Equation (1).

References

1. Jøsang, A.: Subjective logic, vol. 3. Springer (2016)