Understanding P-values

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Test of Significance Procedure

1. Define the null and the alternative Hypothesis
2. Compute the test statistics
3. Compute the p-value
4. Decide whether reject or fail to reject a null hypothesis

State Hypothesis

- Testing a single value of a parameter
- Is a coin fair?
- Is a true coefficient β of linear regression equals zero?
- Does the population have mean (μ) equal to some value (μ_0)?
- Is there a difference in corneal thickness between the Glaucoma and Normal eyes?
- Does the BMI for the covid-19 patients equal to the expected normal BMI for a given age.

$$H_0 : \theta = \theta_0, \qquad H_0 : \theta \leqslant \theta_0, \qquad H_0 : \theta \geqslant \theta_0,$$
$$H_A : \theta \neq \theta_0. \qquad H_A : \theta > \theta_0. \qquad H_A : \theta < \theta_0.$$

Test Statistics

- If the parameter is population mean, and population variance σ^2 is known, $H_0 : \mu = \mu_0$,and $H_A : \mu \neq \mu_0$:

$$z = \frac{\sqrt{n}(\bar{X} - \mu)}{\sigma}.$$

- If σ^2 is not known we use the *t*-statistics:

$$t = \frac{\sqrt{n}(\bar{X} - \mu)}{S}.$$

- If the parameter is population proportion P:

$$z = \frac{(\hat{P} - P)}{\sqrt{\frac{PQ}{n}}}.$$

The *p*-value

- The P-value is the probability value of observing a test statistics as or more extreme than the one calculated from the sample under assumption that null is true.
- A small *p*-value is an evidence against null, because it is a probability of seeing such an extreme (big or small) value of test statistics, if the null actually holds.

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For a Right-tailed rejection region:

$$p\text{-value} = P(t_{df} \geqslant t).$$

For a Left-tailed rejection region:

$$p\text{-value} = P(t_{df} \leqslant -t).$$

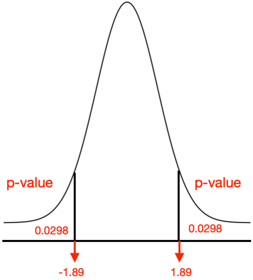
For a Two-tailed rejection region:

$$p\text{-value} = P(|t_{df}| \geqslant t).$$

p-value examples

- t-statistics = -1.89, p-value = $P(|t_{310}| \geqslant 1.89) = 0.0597$, two-tailed t-test,

  `R command: 2*pt(-1.89, df=310)`

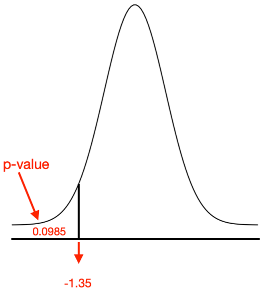


- The p-value is 0.06, meaning if H_0 is true we will only see $|t|$ this large 6% of the time, fails to reject null hypothesis, one possibility null hypothesis is actually true, other is we did not get evidence from this sample against null.

p-value examples contd..

- t-statistics = -1.35, p-value = $P(t_{15} \leqslant -1.35)$ = 0.0985, left-tailed t-test,

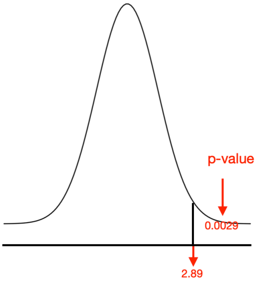
  `R command: pt(-1.35, df=15)`



p-value examples contd..

- t-statistics = 2.89, p-value = $P(t_{46} \geqslant 2.89) = 0.0029$, right-tailed t-test,

  R command: `pt(2.89, df=46, lower.tail = FALSE)`



Conclusion

A small p-value provides an evidence against null.

- If p-value less than 0.01, strong evidence against null hypothesis.
- If p-value is between 0.01 and 0.05, some evidence against null hypothesis.
- If p-value greater than 0.05, insufficient evidence against null hypothesis.

p-value miss-interpretations

- Small *p*-value does not equate to practical or scientific significance
- *p*-value is not probability of null hypothesis being true
- Significant *p*-value does not implies cause and effect