Brian Faure Discrete Math Hw #5 BUID: 15000 3563

#1.) A boolean operator $\odot$ is defined as $1 \odot 1 = 1$, $1 \odot 0 = 0$, $0 \odot 1 = 0$ and $0 \odot 0 = 1$. Show that $X \odot Y = XY + (\bar{X})(\bar{Y})$.

X	Y	XY	$(\bar{X})(\bar{Y})$	$XY + (\bar{X})(\bar{Y})$	$X \odot Y$
0	0	0	1	1	1
0	1	0	0	0	0
1	0	0	0	0	0
1	1	1	0	1	1
A		B	C	D	E

$0 \odot 0 = 1$
$0 \odot 1 = 0$
$1 \odot 0 = 0$
$1 \odot 1 = 1$

Columns D and E are identical thus $X \odot Y = XY + \bar{X}\bar{Y}$

#2.) After proving that (complement of 1 · complement of 0) + (1 · complement of 0) = 1, translate the equation into a propositional logic equivalent. Equal sign becomes a propositional equivalence sign.

(complement of 1 · complement of 0) + (1 · complement of 0) = 1

$$(0 \cdot 1) + (1 1) = 1$$

$$0 + 1 = 1$$

$$\boxed{1 = 1} \rightarrow \text{Both sides are equivalent}$$

Translating the statement into propositional logic

Complement of $\rightarrow$ $\neg$
equals $\rightarrow$ $\equiv$
$1 \rightarrow T$
$0 \rightarrow F$
$\cdot \rightarrow \wedge$
$+ \rightarrow \vee$

$$\boxed{(\neg T \wedge \neg F) \wedge (T \wedge \neg F) \equiv T}$$

Propositional Logic Equivalent

#3.) Find the sum of products expension of a boolean function $f(x, Y, Z)$ that is 1 if and only if $x = Y = 1$ and $Z = 0$, or $x = 0$ and $y = Z = 1$, or if $X = Y = 0$ and $Z = 1$.

$$f(X, Y, Z) = (XY \cap \neg Z) + (\neg X + YZ) + (\neg X \neg Y + Z)$$

$$= \boxed{(XY\bar{Z}) + (\bar{X}YZ) + (\bar{X}\bar{Y}Z)}$$

= Sum of products in boolean logic

#4.) Prove or disprove that $X + XY + XYZ = X$ whenever X, Y and Z are boolean variables

a.) Using the truth table

X	Y	Z	XY	XYZ	$X + XY + XYZ$
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	1
1	0	1	0	0	1
1	1	0	1	0	1
1	1	1	1	1	1
A	B	C	D	E	F

Because columns A and F are equivalent, the expression has been proven.

b.) Using boolean identities

1. $X + XY + XYZ$
2. $X \cdot (1 + Y + YZ)$ — Pulled out X using distributive law
3. $X \cdot 1$ — Any variable multiplied by 1 = Original Variable
4. X — Expression Proven

$\therefore X + XY + XYZ = X$

#5.) The circuit diagrams for $\overline{\bar{X}Y} + \overline{X\bar{Y}}$ and $X + Y$ produce the same output True or False?

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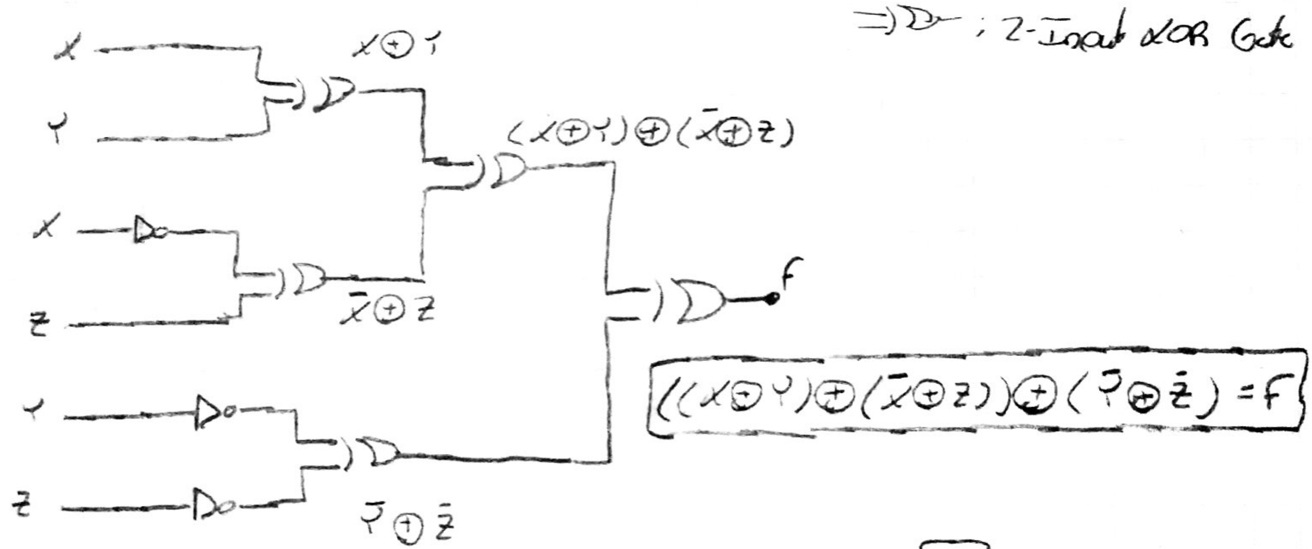
x	y	$\bar{x}\bar{y}$	$\overline{xy}$	$\bar{x}\bar{y}+\overline{xy}$	$\overline{\bar{x}\bar{y}+\overline{xy}}$	$x+y$
0	0	1	1	1	0	0
0	1	0	1	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	1
A	B	C	D	E	F	G

Because columns F and G are not equivalent, the statement is $\boxed{\text{False}}$

#6.) What is the boolean function F which is the output of the circuit below:



$\boxed{((x \oplus y) \oplus (\bar{x} \oplus z)) \oplus (\bar{y} \oplus \bar{z}) = F}$

x	y	z	(A) $x \oplus y$	(B) $\bar{x} \oplus z$	(C) (A) $\oplus$ (B)	(D) $\bar{y} \oplus \bar{z}$	$\boxed{F}$ (C) $\oplus$ (D)
0	0	0	0	1	1	0	1
0	0	1	0	0	0	1	1
0	1	0	1	1	0	1	1
0	1	1	1	0	1	0	1
1	0	0	1	0	1	0	1
1	0	1	1	1	0	1	1
1	1	0	0	0	0	1	1
1	1	1	0	1	1	0	1

[Output F is always true]