DISCRETE MATH HW #1 (100 points)

1. Let p be "it is cold" and let q be "it is raining." Translate in English each of the following logic statements:
(a) $\neg p$; (b) $p \wedge q$; (c) $p \vee q$; (d) $q \vee \neg p$.

2. Verify that the proposition $p \vee \neg(p \wedge q)$ is a tautology.

3. Show that the propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

4. Determine the truth value of each of the following statements where $U = \{1, 2, 3\}$ is the universal set:
(a) $\exists x \forall y, x^2 < y + 1$; (b) $\forall x \exists y, x^2 + y^2 < 12$;
(c) $\forall x \forall y, x^2 + y^2 < 12$.

5. Negate each of the following statements:
(a) $\exists x \forall y, p(x, y)$; (b) $\forall x \forall y, p(x, y)$;
(c) $\exists y \exists x \forall z, p(x, y, z)$.

6. Let $p(x)$ denote the sentence "$x + 2 > 5$". State whether or not $p(x)$ is a propositional function on each of the following sets:
(a) N, the set of positive integers;
(b) $M = \{-1, -2, -3, \ldots\}$
(c) C, the set of complex numbers.