Brian Faure RU ID: 150003563 DLD Homework #1 Section 4

* Q_1: Drill Problem 4.1: Use variables NERD, DESIGNER, FAILURE and STUDIED to write Boolean expression which is 1 for designers who never studied and for nerds who studied. successful

→ IF (DESIGNER · FAILURE' · STUDIED') then 1

→ if (NERD · STUDIED) then 1

Another way put together →

$[(DESIGNER \cdot FAILURE' \cdot STUDIED') + (NERD \cdot STUDIED) = 1]$

* Q_2: Drill Problem 4.2: Prove Theorems T2 - T5 using perfect induction

(T2) $[X + 1 = 1]$ → $X = 0$ // $0 + 1 = 1$ (+ = "or")

→ $X = 1$ // $1 + 1 = 1$

(T3) $[X + X = X]$ → $X = 0$ // $0 + 0 = 0$

→ $X = 1$ // $1 + 1 = 1$

(T4) $[(X')' = X]$ → $X = 0$ // $(0')' = 0$

→ $X = 1$ // $(1')' = 1$

(T5) $[X + X' = 1]$ → $X = 0$ // $0 + 0' = 0 + 1 = 1$

→ $X = 1$ // $1 + 1' = 1 + 0 = 1$

* Q_3: Drill Problem 4.3: Prove Theorems T1' - T3' and T5' using Perfect Induction

(T1') $[X \cdot 1 = X]$ → $X = 0$ // $0 \cdot 1 = 0$ (· = "and")

→ $X = 1$ // $1 \cdot 1 = 1$

(T2') $[X \cdot 0 = 0]$ → $X = 0$ // $0 \cdot 0 = 0$

→ $X = 1$ // $1 \cdot 0 = 0$

(T3') $[X \cdot X = X]$ → $X = 0$ // $0 \cdot 0 = 0$

→ $X = 1$ // $1 \cdot 1 = 1$

(T5') $[X \cdot X' = 0]$ → $X = 0$ // $0 \cdot 0' = 0 \cdot 1 = 0$

→ $X = 1$ // $1 \cdot 1' = 1 \cdot 0 = 0$

* Q4: Drill Problem 4.4: Prove Theorems T6 - T9 using perf. ind.

(T6) $[X+Y=Y+X] \rightarrow X=0\ Y=0$ // $0+0=0+0=0$

$\rightarrow X=0\ Y=1$ // $0+1=1+0=0+1=1$ (commutativity)

$\rightarrow X=1\ Y=0$ // $1+0=0+1=1+0=1$

$\rightarrow X=1\ Y=1$ // $1+1=1+1=1$

(T7) $[(X+Y)+Z=X+(Y+Z)]$

$X=0, Y=0, Z=0 \rightarrow (0+0)+0=0+(0+0)=0+0=0$

$X=0, Y=0, Z=1 \rightarrow (0+0)+1=0+(0+1)$

$0+1=0+1$

$1=1$

$X=0, Y=1, Z=0 \rightarrow (0+1)+0=0+(1+0)$

$1+0=0+1$

$1=1$

$X=1, Y=0, Z=0 \rightarrow (1+0)+0=1+(0+0)$

$1+0=1+0$

$1=1$

$X=0, Y=1, Z=1 \rightarrow (0+1)+1=0+(1+1)$

$1+1=0+1$

$1=1$

$X=1, Y=0, Z=1 \rightarrow (1+0)+1=1+(0+1)$

$1+1=1+0$

$1=1$

$X=1, Y=1, Z=0 \rightarrow (1+1)+0=1+(1+0)$

$1+0=1+1$

$1=1$

$X=1, Y=1, Z=1 \rightarrow (1+1)+1=1+(1+1)$

$1+1=1+1$

$1=1$

(T8) $[X \cdot Y + X \cdot Z = X \cdot (Y+Z)]$

$X=0, Y=0, Z=0 \rightarrow 0 \cdot 0 + 0 \cdot 0 = 0 \cdot (0+0)$

$0 = 0$

$X=0, Y=0, Z=1 \rightarrow 0 \cdot 0 + 0 \cdot 1 = 0 \cdot (0+1)$

$0 + 0 = 0 \cdot 1$

$0 = 0$

$X=0, Y=1, Z=0 \rightarrow 0 \cdot 1 + 0 \cdot 0 = 0 \cdot (1+0)$

$0 + 0 = 0 \cdot 1$

$0 = 0$

$X=1, Y=0, Z=0 \rightarrow 1 \cdot 0 + 1 \cdot 0 = 1 \cdot (0+0)$

$0 + 0 = 1 \cdot 0$

$0 = 0$

$X=0, Y=1, Z=1 \rightarrow 0 \cdot 1 + 0 \cdot 1 = 0 \cdot (1+1)$

$0 + 0 = 0 \cdot 1$

$0 = 0$

$X=1, Y=0, Z=1 \rightarrow 1 \cdot 0 + 1 \cdot 1 = 1 \cdot (0+1)$

$0 + 1 = 1 \cdot 1$

$1 = 1$

$X=1, Y=1, Z=0 \rightarrow 1 \cdot 1 + 1 \cdot 0 = 1 \cdot (1+0)$

$1 + 0 = 1 \cdot 1$

$1 = 1$

$X=1, Y=1, Z=1 \rightarrow 1 \cdot 1 + 1 \cdot 1 = 1 \cdot (1+1)$

$1 = 1$

(T9) $[X + X \cdot Y = X]$ $X=0, Y=0$ // $0 + 0 \cdot 0 = 0 = 0$

$X=0, Y=1$ // $0 + 0 \cdot 1 = 0$

$0 + 0 = 0$

$X=1, Y=0$ // $1 + 1 \cdot 0 = 1$

$1 + 0 = 1$

$X=1, Y=1$ // $1 + 1 \cdot 1 = 1$

$1 + 1 = 1$

$1 = 1$

* Q5: Drill Problem 4.5: According to DeMorgan's theorem, the complement of $W \cdot X + Y \cdot Z$ is $W' + X' \cdot Y' + Z'$. Unfortunately both equate to one with the input $WXYZ = 1110$. What's wrong?

- The problem with the complement expression is that in order to maintain the meaning of the original statement, parentheses must be added.

Incorrect complement → $W' + X' \cdot Y' + Z'$

correct → $(W' + X') \cdot (Y' + Z')$

To check the result $[WXYZ = 1110]$ // $(1' + 1') \cdot (1' + 0') = 0 \cdot 1 = \boxed{0}$

* Q6: Drill Problem 4.6: Use switching-algebra to simplify the following

a.) $F = W \cdot X \cdot Y \cdot Z \cdot (W \cdot X \cdot Y \cdot Z' + W \cdot X' \cdot Y \cdot Z + W' \cdot X \cdot Y \cdot Z + W \cdot X \cdot Y \cdot Z')$

→ first use (T8) to distribute

$(WXYZ)(WXYZ') + (WXYZ)(WXY'Z) + (WXYZ)(WX'YZ) + (WXYZ)(W'XYZ)$

$Z \cdot Z' = 0$ ↗ $Y \cdot Y' = 0$ ↗ $X \cdot X' = 0$ ↗ $W \cdot W' = 0$

$0 + 0 + 0 + 0 = \boxed{0} = F$

b.) $F = A \cdot B + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A' \cdot B \cdot C' \cdot E + A' \cdot B' \cdot C' \cdot E$

Pull out AB → $AB(1 + C'D + DE') + A'BC'E + A'B'C'E$

→ $AB(1) + A'BC'E + A'B'C'E$

→ $AB + A'BC'E + A'B'C'E = AB + A'(BC'E + B'C'E)$

$= AB + A'(E(BC' + B'C'))$

$= AB + A'(E(C'(B + B')))$

$= AB + A'(E(C'(1)))$

$= \boxed{AB + A'EC'} = F$

c.) $F = M \cdot B \cdot P + Q \cdot O' \cdot B' + M \cdot N + O \cdot N \cdot M + Q \cdot P \cdot M \cdot O'$

* Q7: Drill Problem 2.1: Perform number system conversions

a.) $1101011_2 = ?_{16}$ → (0110)(1011) → (6)(11) → $6B_{16}$

b.) $174003_8 = ?_2$ 1 7 4 0 0 3

(001)(111)(100)(000)(000)(011)

= 001111100000000011_2

c.) $10110111_2 = ?_{16}$ → (1011)(0111) → (11)(7) → $B7_{16}$

d.) $67.248 = ?_2$ → 6 7 . 2 4

→ (110) (111) . (010) (100)

→ 110111.0101_2

e.) $10100.1101_2 = ?_{16}$ → (0001)(0100). (1101) → (1)(4). (13)

→ $14.D_{16}$

f.) $F3A5_{16} = ?_2$ F 3 A 5

(15) (3) (10) (5) → (1111)(0011)(1010)(0101)

→ 1111001110100101_2

g.) $1011001_2 = ?_8$ → (011)(011)(001)

3 3 1 → 331_8

h.) $AB3D_{16} = ?_2$ (A) (B) (3) (D) → (10)(11)(3)(13)

→ (1010)(1011)(0011)(1101)

→ 1010101100111101_2

i.) $101111.0111_2 = ?_8$ (101)(111).(011)(100)

5 7 . 3 4 → 57.34_8

j.) $15C.38_{16} = ?_2$ → 1 5 C . 3 8

(0001) (0101) (1100). (0011) (1000)

= 000101011100.0011_2

* Q8 Drill Problem 2.2: Convert following octal into Binary/Hexadecimal

a.) 12348 Binary → (1) (2) (3) (4)

001 010 011 100 = 1010011100_2

Hex → (0010)(1001)(1100) → (2)(9)(12) → $29C_{16}$

b.) 1746378 Binary → 1 7 4 6 3 7

(001)(111)(100)(110)(011)(111) = 1111001100111111_2

Hex → (1111)(1001)(1001)(1111) → (15)(9)(9)(15)

→ $F99F_{16}$

c.) 3655178 Binary → 3 6 5 5 1 7

(011)(110)(101)(101)(001)(111) = 111010110100111_2

Hex → (0001)(1110)(1011)(0100)(1111) → (1)(14)(11)(4)(15)

→ $1EB4F_{16}$

d.) 25353218 Binary → 2 5 3 5 3 2 1

→ (010) (101) (011)(101)(011)(010) (001)

= 010101011101011010001_2

Hex → (1010)(1011)(1010)(1101)(0001) → (10)(11)(10)(13)(1)

→ $ABAD1_{16}$

e.) 7436.118 Binary → 7 4 3 6 . 1 1

→ (111) (100)(011)(110). (001)(001)

= 111100011110.001001_2

Hex → (1111)(0001)(1110).(0010)(0100) → (15)(1)(14).(2)(4)

= $F1E.24_{16}$

f.) 45316.7414_8 Binary → 4 5 3 1 6 . 7 4 1 4

→ (100)(101)(011)(001)(110).(111)(100)(001)(100)

= $100101011001110.111100001100_2$

Hex → (0100)(1010)(1100)(1110).(1111)(0000)(1100)

→ (4)(10)(12)(14).(15)(0)(12)

= $4ACE.F0C_{16}$

* 29 Drill Problem 2.3: Convert Hex into Binary and Octal

a.) 1023_{16} Binary → 1 0 2 3

→ (0001)(0000)(0010)(0011) = 1000000100011_2

Octal → (001)(000)(000)(100)(011)

= 10043_8

b.) $7E6A_{16}$ Binary → 7 E (14) 6 A (10)

→ (0111)(1110)(0110)(1010)

= 0111111001101010_2

Octal → (111)(111)(001)(101)(010) = 77152_8

c.) $ABCD_{16}$ Binary → A (10) B (11) C (12) D (13)

→ (1010)(1011)(1100)(1101) = 1010101111001101_2

Octal → (001)(010)(101)(111)(001)(101)

= 125715_8

d.) $C350_{16}$ Binary → C 3 5 0 → (1100)(0011)(0101)(000

= 1100001101010000_2

Octal → (001)(100)(001)(101)(010)(000) = 141520_8

e.) $9E36.7A_{16}$ Binary → 9 E (14) 3 6 . 7 A (10)

→ (1001)(1110)(0011)(0110).(0111)(1010)

= $1001111000110110.01111010_2$

Octal → (001)(001)(111)(000)(110)(110).(011)(110)(100)

= 117066.364_8

(13) (14) (10) (13) (11) (14) (14) (15)

f.) $DEAD.BEEF_{16}$ Binary → D E A D . B E E F

→ (1101)(1110)(1010)(1101).(1011)(1110)(1110)(1111)

= $1101111010101101.1011111011101111_2$

Octal → (001)(101)(111)(010)(101)(101).(101)(111)(101)(110)(111)(100)

= 157255.575674_8

* Q10: Drill Problem 2.5: Convert to Decimal

64 32 16 8 4 x2 x1

a.) 1101011_2 1 1 0 1 0 1 1

= 64+32+8+2+1 = **107**

32768 4096 512 64 8 1

b.) 174003_8 = 1 7 4 0 0 3

= 32768 + 28672 + 2048 + 0 + 0 + 3 = **63491**

128 64 32 16 8 4 2 1

c.) 10110111_2 1 0 1 1 0 1 1 1

= 128 + 0 + 32 + 16 + 0 + 4 + 2 + 1 = **183**

8 1 1/8 1/64

d.) 67.24_8 → 6 7 . 2 4

= 48 + 7 + 1/4 + 1/16 = **55.3125**

16 8 4 2 1 1/2 1/4 1/8 1/16

e.) 10100.1101_2 → 1 0 1 0 0 . 1 1 0 1

= 16 + 4 + 1/2 + 1/4 + 1/16 = **20.8125**

x4096 x256 x16 x1

f.) $F3A5_{16}$ → F 3 A 5

= (15)(4096) + (3)(256) + (16)(10) + 5(1) = **62373**

81 27 9 3 1

g.) 12010_3 1 2 0 1 0

= 81 + 54 + 0 + 3 = **138**

x4096 x256 x16 x1

h.) $AB3D_{16}$ → A B 3 0

= 10(4096) + (11)(256) + 3(16) + 13 = **43837**

1.) $7156_8 \rightarrow$ (×512, ×64, ×8, ×1) 7 1 5 6 $= (7)(512) + 64 + 8(5) + 6 =$ **3694**

;) $15C.38_{16} \rightarrow$ (×256, ×16, ×1, ×1/16, ×1/256) 1 5 C . 3 8

$= 256 + 16(5) + 12 + {}^{3}/_{16} + {}^{8}/_{256} =$ **348.21875**

Q11: What is the value of "money" in a base-26 system?

→ (×456976, ×17576, ×676, ×26, ×1)

→ money$_{26}$ = m o n e y

→ (×12, ×14, ×13, ×4, ×24)

$= 456976(12) + 17576(14) + 676(13) + 26(4) + 24$

$=$ **$5,738,692_{10}$**