DISCRETE MATH HW # 2

100 points. Due 2/23

1. Problem 1

Let x and y domains be all integers and $Q(x, y)$ be $x + y = x - y$. What is the truth value of

a) $\exists x \exists y\, Q(x, y)$; b) $\forall x \exists y\, Q(x, y)$

c) $\forall y\, Q(2, y)$; d) $\forall x \forall y\, Q(x, y)$.

2. Problem 2

Use rules of inference to show that if $\forall x (P(x) \vee Q(x))$ and $\forall x ((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x (\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

3. Problem 3

"If n is a positive number, then $n^2 \geq n$" constitutes the propositional function $P(n)$. Prove the proposition $P(1)$ and explain what kind of proof you used.

4. Prove that given a real number y there exist unique numbers K and t, such that $y = K + t$ where K is an integer and $1 > t \geq 0$

5. Prove that given integer K which is ≥ 0, there exist a unique integer s which is nonnegative such that $s^2 \leq K < (s+1)^2$