**CSC 231 Programming Languages**

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**Turing Machine**

(from <https://en.wikipedia.org/wiki/Turing_machine> )

Watch the Turing machine animation at <https://vimeo.com/46913004>

A Turing machine is a [mathematical model of computation](https://en.wikipedia.org/wiki/Mathematical_model_of_computation) that defines an [abstract machine](https://en.wikipedia.org/wiki/Abstract_machine) which manipulates symbols on a strip of tape according to a table of rules. Despite the model's simplicity, given any [computer algorithm](https://en.wikipedia.org/wiki/Computer_algorithm), a Turing machine capable of simulating that algorithm's logic can be constructed.

The Turing machine mathematically models a machine that mechanically operates on a tape. On this tape are symbols, which the machine can read and write, one at a time, using a tape head. Operation is fully determined by a finite set of elementary instructions such as "in state 42, if the symbol seen is 0, write a 1; if the symbol seen is 1, change into state 17; in state 17, if the symbol seen is 0, write a 1 and change to state 6;" etc. In the original article ("On computable numbers, with an application to the [Entscheidungsproblem](https://en.wikipedia.org/wiki/Entscheidungsproblem)", see also [references below](https://en.wikipedia.org/wiki/Turing_machine#The_Entscheidungsproblem_(the_%22decision_problem%22):_Hilbert's_tenth_question_of_1900)), Turing imagines not a mechanism, but a person whom he calls the "computer", who executes these deterministic mechanical rules slavishly (or as Turing puts it, "in a desultory manner").

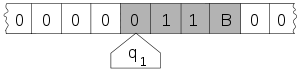
More precisely, a Turing machine consists of:

* A **tape** divided into cells, one next to the other. Each cell contains a symbol from some finite alphabet. The alphabet contains a special *blank* symbol (here written as '0') and one or more other symbols. The tape is assumed to be arbitrarily extendable to the left and to the right, i.e., the Turing machine is always supplied with as much tape as it needs for its computation. Cells that have not been written before are assumed to be filled with the blank symbol. In some models the tape has a left end marked with a special symbol; the tape extends or is indefinitely extensible to the right.
* A **head** that can read and write symbols on the tape and move the tape left and right one (and only one) cell at a time. In some models the head moves and the tape is stationary.
* A **state register** that stores the state of the Turing machine, one of finitely many. Among these is the special *start state* with which the state register is initialized. These states, writes Turing, replace the "state of mind" a person performing computations would ordinarily be in.
* A finite **table**[[](https://en.wikipedia.org/wiki/Turing_machine#cite_note-19) of instructionsthat, given the *state*(qi) the machine is currently in *and* the *symbol*(aj) it is reading on the tape (symbol currently under the head), tells the machine to do the following *in sequence* (for the 5-tuple models):

1. Either erase or write a symbol (replacing aj with aj1).
2. Move the head (which is described by dk and can have values: 'L' for one step left *or* 'R' for one step right *or* 'N' for staying in the same place).
3. Assume the same or a *new state* as prescribed (go to state qi1).

[https://upload.wikimedia.org/wikipedia/en/thumb/0/09/Turing_machine_2a.svg/300px-Turing_machine_2a.svg.png](https://en.wikipedia.org/wiki/File:Turing_machine_2a.svg)

The head is always over a particular square of the tape; only a finite stretch of squares is shown. The instruction to be performed (q4) is shown over the scanned square. (Drawing after Kleene (1952) p. 375.)

[](https://en.wikipedia.org/wiki/File:Turing_machine_2b.svg)

Here, the internal state (q1) is shown inside the head, and the illustration describes the tape as being infinite and pre-filled with "0", the symbol serving as blank. The system's full state (its *complete configuration*) consists of the internal state, any non-blank symbols on the tape (in this illustration "11B"), and the position of the head relative to those symbols including blanks, i.e. "011B". (Drawing after Minsky (1967) p. 121.)