Ch9: Benchmarking Quantum Computers

Basic idea

Benchmarking is how the performance of a computing system is determined.

One must choose the appropriate benchmark and metrics to extract meaningful results.

Given that QPUs have noise, how do you characterize what QPU you have ?

How do you tell if its good enough to be better than CPUs ?

Levels of benchmarking

One can distinguish three different levels of benchmarking:

(0) – Qubit benchmarking: Benchmarking a single (or two) qubit system – how reliable the quantum gates are ?

(1) – Quantifying capabilities: How much work a quantum computer is capable of, in contrast to performance results on specific algorithms.

(2) – Program benchmarks: Establishing a set of programs and measuring the performance of a computing system performing each one.

Pure states

Fidelity is a measure of the "closeness" of two quantum states. It expresses the probability that one state will pass a test to identify as the other.

Fidelity as a distance measure between pure states is given by the transition probability from one state to another.

That is for two states described by unit vectors $|\psi\rangle$, $|\phi\rangle$:

$$F(|\phi\rangle, |\psi\rangle) = |\langle\phi|\psi\rangle|^2$$

Mixed states

For a pure state $|\psi\rangle$ and a mixed state ρ this generalizes to the averaged fidelity:

$$F(\rho, |\psi\rangle) = \langle\psi|\rho|\psi\rangle$$

And for two density matrices ρ, σ it is generalized as the largest fidelity between any two purifications of the given states.

According to Uhlmann's theorem, this leads to the expression:

$$F(\rho, \sigma) = \left[Tr\left\{\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right\}\right]^2$$

Mixed states

If $[\rho, \sigma] = 0$, the definition simplifies to:

$$F(\rho,\sigma) = \left[Tr\{\sqrt{\rho\sigma}\}\right]^2 = \left(\sum_k \sqrt{p_k q_k}\right)^2$$

where p_k, q_k are the eigenvalues of ρ, σ respectively.

To see this, remember that if two operators commute then they can be diagonalized in the same basis:

$$\rho = \sum_k p_k |k\rangle\langle k| \qquad\qquad \sigma = \sum_k q_k |k\rangle\langle k|$$

so that:

$$Tr\{\sqrt{\rho\sigma}\} = Tr\left\{\sum_k \sqrt{p_k q_k}|k\rangle\langle k|\right\} = \sum_k \sqrt{p_k q_k}$$

Properties

Other basic properties:

- Bounded: $0 \leq F(\sigma, \rho) \leq 1 \qquad (F(\sigma, \rho) = 1 \quad \Leftrightarrow \quad \sigma = \rho)$
- Symmetric: $F(\sigma, \rho) = F(\rho, \sigma)$
- Unitary invariant: $F(U\sigma U^\dagger, U\rho U^\dagger) = F(\sigma, \rho)$
- Monotonic: $F(\mathcal{E}(\sigma), \mathcal{E}(\rho)) \geq F(\sigma, \rho)$

1-qubit

So to measure fidelity we need to prepare states and measure their density matrix ρ. We must have multiple copies of ρ in order to characterize it.

Suppose we have multiple copies of ρ – or can make them on demand, we can expand:

$$\rho = \frac{Tr\{\rho\}I + Tr\{X\rho\}X + Tr\{Y\rho\}Y + Tr\{Z\rho\}Z}{2}$$

(I, X, Y, Z form a basis for 2x2 complex matrices). $Tr\{X\rho\}$ is $\langle X\rangle_\rho$, i.e. the expectation value of ρ when measured by the X observable – see Ch5.

Thus to get ρ we must calculate $\langle X\rangle_\rho, \langle Y\rangle_\rho, \langle Z\rangle_\rho$ (the 4th from normalization) $\rightarrow$ 3 sets of code – see Ex. 3 assignment #1.

N-qubits

Quantum state tomography scales really badly ! For n-qubits you need $(2^{n^2}-1)$ types of observables.

It works well but it takes time...

Quantum process tomography

Analogous to quantum state tomography except instead of learning a density matrix ρ we learn a representation of a quantum process, see Nielsen and Chuang chapter 8.4.2.

Standard QPT requires 2^{n^2} different state tomography experiments for n-qubits.

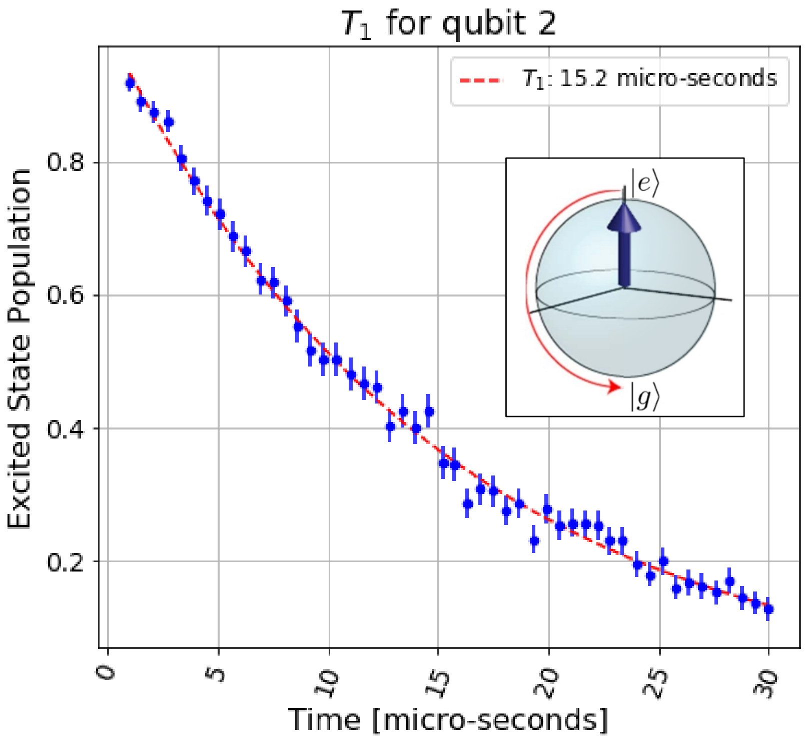
That is 2^{n^2}x 2^{n^2} types of programs to benchmark n-qubits...

T1 coherence

T1 measures the impact of amplitude damping – the loss of energy from the system.

Basic idea:

- Initialize the qubit to state 0
- Apply X
- Wait for time t
- Measure the probability of being in state 1



We expect an exponential decay:

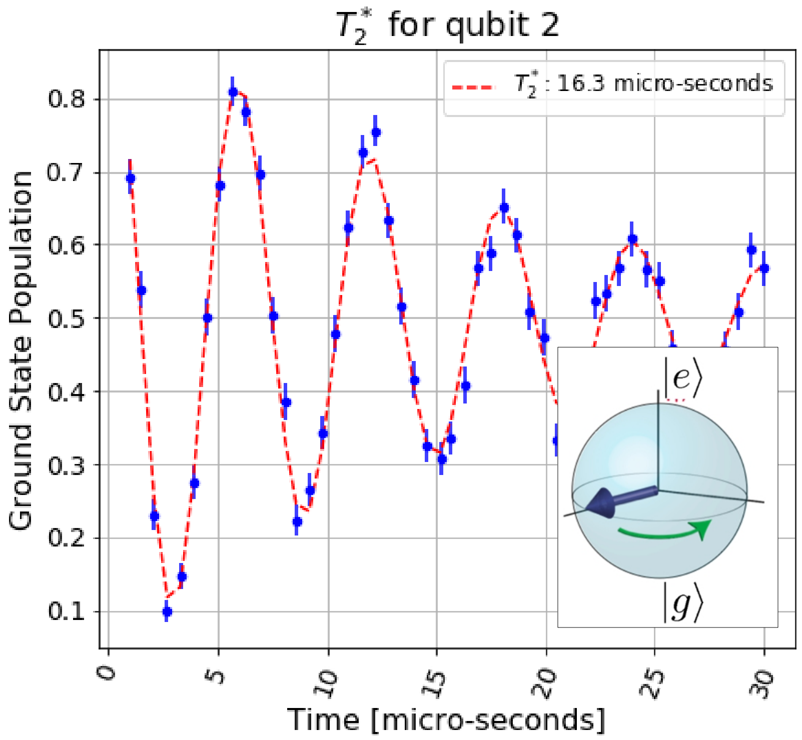
$$P_1(t) = Ae^{-t/T_1} + C$$

T2 coherence

T2 measures the impact of phase damping – phase shifted version of T1.

Basic idea:

- Initialize the qubit to state 0
- Apply H
- Wait for time t
- Apply H
- Measure the probability of being in state 1



We expect a modulated exponential decay:

$$P_1(t) = Ae^{-2t/T_2}\sin\left(\omega_d(t-\phi)\right) + C$$

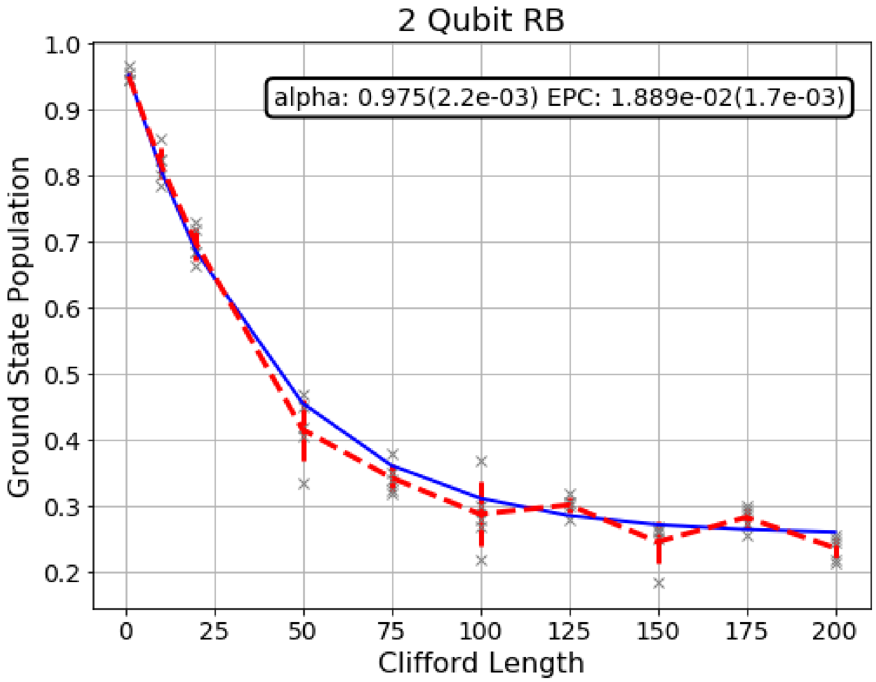
Basic idea

Fully characterizing all the quantum operations of our system is resource intensive. Instead we would like a general metric that tells us how our operations perform on average – for some definition of average.

Randomized benchmarking is widely used to measure an error rate of a set of quantum gates, by performing random circuits that would do nothing if the gates were perfect.

Exponential decay

In the limit of no finite-sampling error, the exponential decay rate of the observable survival probabilities, versus circuit length, yields a single error metric *r*.



For Clifford gates with arbitrary small errors described by process matrices, *r* corresponds to the mean, over all Cliffords, of the average gate infidelity between the imperfect gates and their ideal counterparts.

Fitting function: $Ar^m + B$

Error per Clifford: $EPC = \frac{2^n - 1}{2^n}(1 - r)$

Interpretation

For Clifford gates with arbitrary small errors described by process matrices, r corresponds to the mean, over all Cliffords, of the average gate infidelity between the imperfect gates and their ideal counterparts.

Protocol for single-qubit randomized benchmarking

(1) Choose a series of lengths for $L_1 < L_2 < \ldots < L_m$ for your circuit

Do N_r times:

(2) For each L_i, generate a random sequence of $L_i - 3$ gates taken from $RB_{1Q} = \{I, RX(+/- 180°), RX(+/- 90°), RY(+/- 180°), RY(+/- 90°)\}$ and Calculate their completion A and its representation as a combination of 3 elements of RB_{1Q}

$\rightarrow$ This produces a sequence of L_i elements of RB_{1Q} that compose to the identity operation

Do N_e times:

(3) Run the sequence of gates on your qubit and measure if you remain in state 0

$\rightarrow$ You now have $N_e \times N_r \times m$ points of data

(4) Calculate $P_{Li} = \{$probability of measuring 0 at sequence length $L_i\}$

(5) Fit $P_{Li} = A + (B + CL_i)p^{Li}$, where A , B, C and p are fit parameters

(6) Calculate $r = (1 - p)/2$ as the RB number