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QUANTUM DISCOVERY

Digital quantum computing

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Digital versus analog processing

Implementation of digital and analog computing with neutral atoms

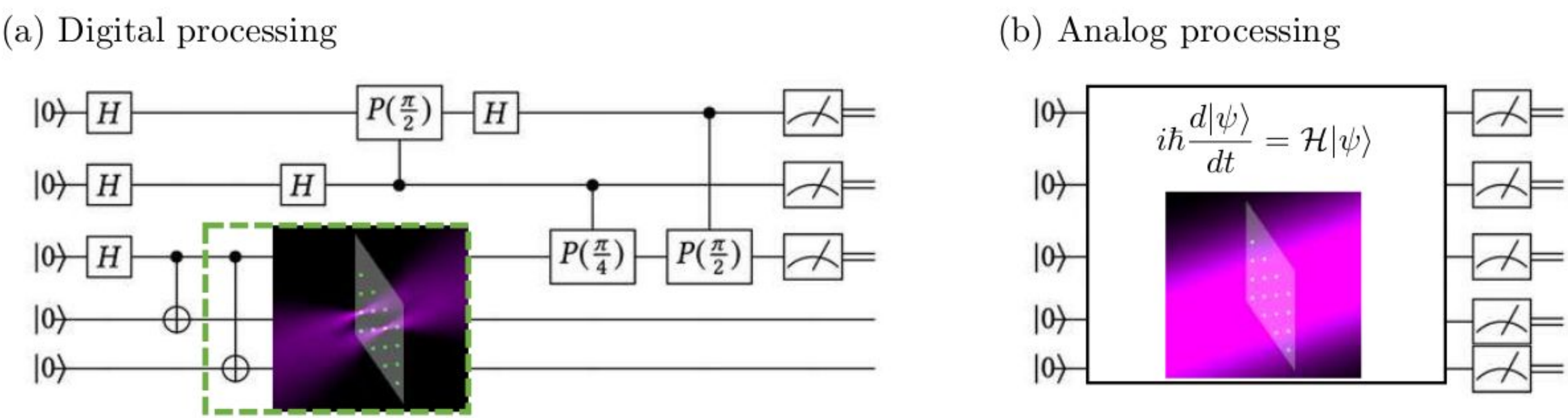


Figure 6: Digital- vs analog processing. (a) In digital processing, a succession of gates is applied to the qubits to implement a quantum algorithm. Each gate is performed by addressing the qubits individually with laser beams. (b) In analog processing the qubits evolve under a tailored Hamiltonian $\mathcal{H}$, for instance by illuminating the whole register with a laser beam. The wavefunction $|\psi\rangle$ of the system follows the Schrödinger equation.

Single-qubit operations

General single-qubit unitary

The most general form of a single-qubit unitary writes:

$$U(\gamma,\theta,\phi) = \begin{bmatrix} \cos\frac{\theta}{2} & -ie^{i\phi}\sin\frac{\theta}{2} \\ -ie^{i\gamma}\sin\frac{\theta}{2} & e^{i(\gamma+\phi)}\cos\frac{\theta}{2} \end{bmatrix}$$

- $\theta \in [0, \pi]$: amplitude parameter
- $\phi \in [0, 2\pi]$: phase parameter
- $\gamma \in [0, 2\pi]$: phase parameter

Single-qubit operations

Pauli operators

The simplest single-qubit gates are the Pauli operators: X, Y and Z. Their action is to perform a half rotation of the Bloch sphere around the x, y and z axes:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = U\left(\frac{\pi}{2}, \pi, \frac{\pi}{2}\right)$$

$$|\pm\rangle \equiv \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} \qquad \begin{cases} X|+\rangle = |+\rangle \\ X|-\rangle = -|-\rangle \end{cases}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = U\left(\pi, \pi, 0\right)$$

$$|\pm i\rangle \equiv \frac{|0\rangle \pm i|1\rangle}{\sqrt{2}} \qquad \begin{cases} Y|+i\rangle = |+i\rangle \\ Y|-i\rangle = -|-i\rangle \end{cases}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = U\left(0, 0, \pi\right)$$

$$\begin{cases} Z|0\rangle = |0\rangle \\ Z|1\rangle = -|1\rangle \end{cases}$$

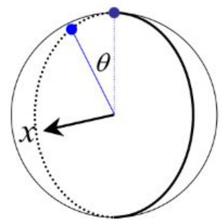
Single-qubit operations

Rotation operators

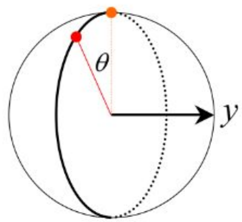
The Pauli operators give rise to three useful classes of unitary operators when they are exponentiated, the rotations operators about the x, y and z axes of the Bloch sphere:

$$R_P(\theta) = e^{-i\theta P} = \cos\frac{\theta}{2} I - i \sin\frac{\theta}{2} P$$

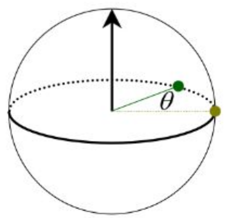
$$R_X(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} = U\left(0, \theta, 0\right)$$



$$R_Y(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} = U\left(\frac{\pi}{2}, \theta, -\frac{\pi}{2}\right)$$



$$R_Z(\theta) = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} = U\left(0, 0, \theta\right)$$



Single-qubit operations

Rotation operators

Any single-qubit unitary can be decomposed as a serie of rotation gates:

$$U(\gamma, \theta, \phi) = R_z(\gamma) R_x(\theta) R_z(\phi)$$

Single-qubit operations

Clifford operators

The Clifford gates are the elements of the Clifford group, a set of mathematical transformations which effect permutations of the Pauli operators:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = U\left(\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = U\left(0, 0, \frac{\pi}{2}\right)$$

$$HZH = X$$
$$HYH = -Y$$
$$HXH = Z$$

$$SXS^\dagger = Y$$
$$SYS^\dagger = -X$$
$$SZS^\dagger = Z$$

Single-qubit operations

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$$S = \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix} = U\left(0, 0, \frac{\pi}{2}\right)$$

$$T = \begin{bmatrix}1 & 0\\ 0 & e^{i\frac{\pi}{4}}\end{bmatrix} = U\left(0, 0, \frac{\pi}{4}\right)$$

$$HZH = X$$
$$HYH = -Y$$
$$HXH = Z$$

$$SXS^\dagger = Y$$
$$SYS^\dagger = -X$$
$$SZS^\dagger = Z$$

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Single-qubit operations

Physical implementation

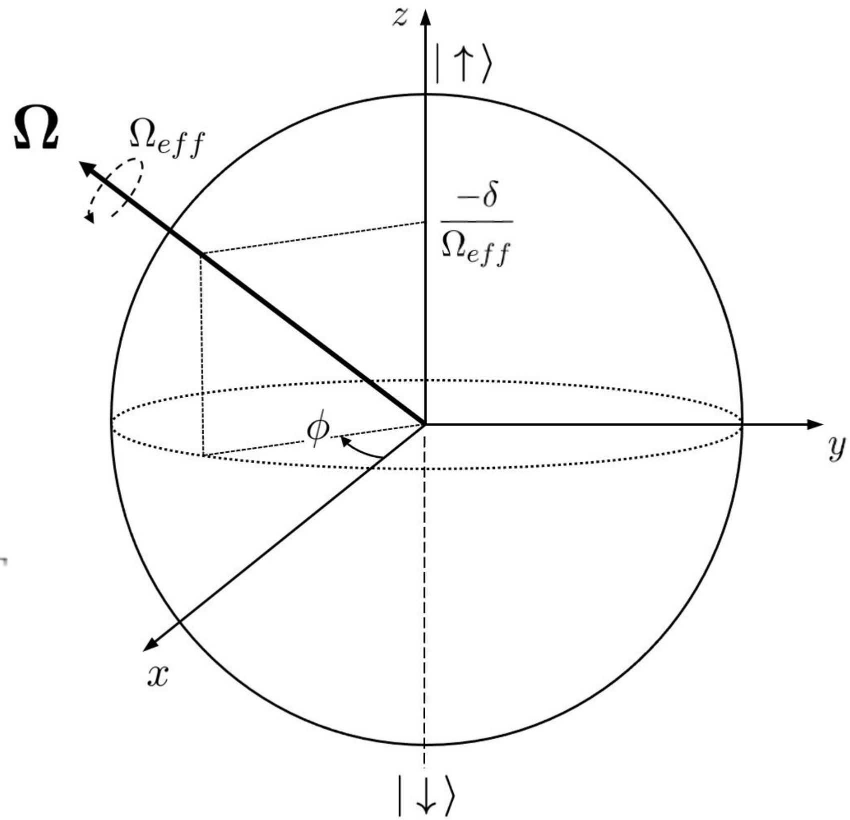
At a single-qubit level, an atomic transition is described by the drive Hamiltonian:

$$H^D(t) = \frac{\hbar}{2}\boldsymbol{\Omega}\cdot\sigma$$

- Pauli vector: $\sigma = (X, Y, Z)^T$
- Rotation vector:

$$\boldsymbol{\Omega}(t) = (\Omega(t)\cos(\phi), -\Omega(t)\sin(\phi), -\delta(t))^T$$

- Angular velocity: $\Omega_{eff} = |\boldsymbol{\Omega}| = \sqrt{\Omega^2 + \delta^2}$



Single-qubit operations

Physical implementation

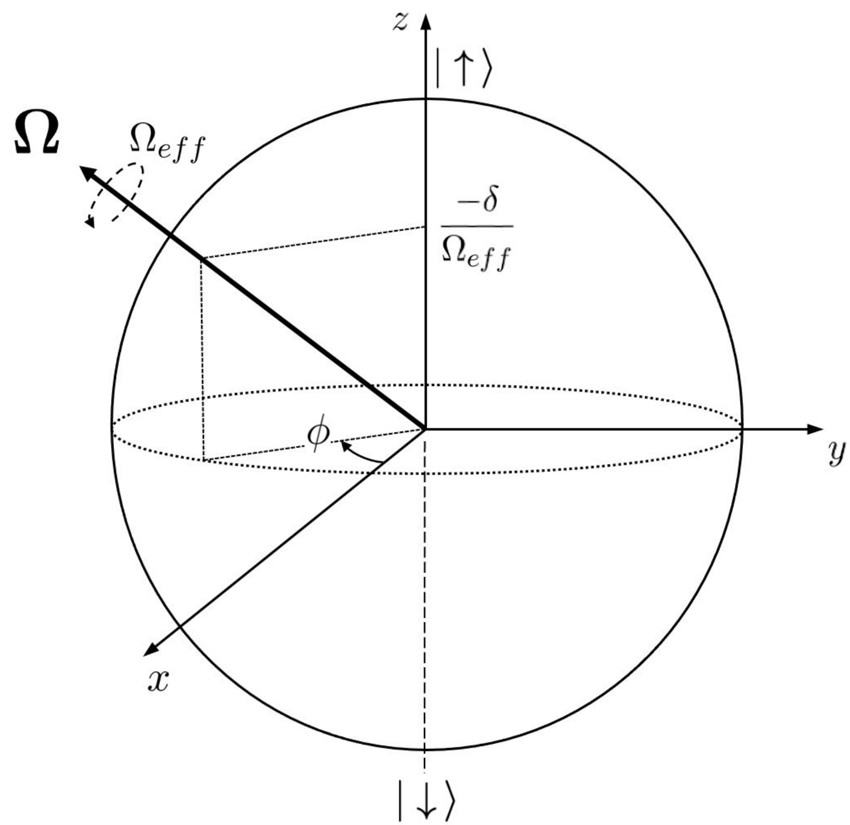
At a single-qubit level, an atomic transition is described by the drive Hamiltonian:

$$H^D(t) = \frac{\hbar}{2}\boldsymbol{\Omega}\cdot\sigma$$

The unitary operator associated with this evolution is:

$$U(\boldsymbol{\Omega},\tau) = \exp\left[-\frac{i}{2}\int_0^\tau \boldsymbol{\Omega}\cdot\sigma\,dt\right]$$

It describes a rotation around the time-dependent axis $\boldsymbol{\Omega}(t)$.



Single-qubit operations

Physical implementation

For a resonant pulse, we get a rotation angle:

$$\theta = \int_0^T \Omega(t)\, dt$$

around the fixed axis:

$$\hat{\mathbf{n}}(\phi) = (\cos(\phi), -\sin(\phi), 0)^T$$

situated on the equator of the Bloch sphere.

Single-qubit operations

Physical implementation

If $\hat{\mathbf{n}} = (n_x, n_y, n_z)^T$ is a real unit vector in three dimensions, then rotating around this vector of angle θ actually corresponds to implementing the rotation matrix:

$$R_{\hat{\mathbf{n}}(\phi)}(\theta) \equiv \exp\left(-i\theta\,\hat{\mathbf{n}}\cdot\sigma/2\right) = \cos\frac{\theta}{2}\,I - i\sin\frac{\theta}{2}\,\hat{\mathbf{n}}\cdot\sigma$$

Thus, if $\hat{\mathbf{n}}(\phi) = (\cos(\phi), -\sin(\phi), 0)^T$ - i.e resonant pulse:

$$\begin{aligned} R_{\hat{\mathbf{n}}(\phi)}(\theta) &= e^{-i\frac{\theta}{2}(\cos(\phi)X - \sin(\phi)Y)} \\ &= e^{i\frac{\phi}{2}Z}e^{-i\frac{\theta}{2}X}e^{-i\frac{\phi}{2}Z} \\ R_{\hat{\mathbf{n}}(\phi)}(\theta) &= R_z(-\phi)R_x(\theta)R_z(\phi) \end{aligned}$$

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Single-qubit operations

Physical implementation

Finally, any single qubit unitary can be constructed by implementing an additional phase shift right after a resonant pulse:

$$U(\gamma,\theta,\phi) = \underbrace{\boxed{R_z(\gamma+\phi)}}_{\text{Phase reference frame change}}\underbrace{\boxed{R_{\hat{\mathbf{n}}(\phi)}(\theta)}}_{\text{Resonant pulse}} = R_z(\gamma)R_x(\theta)R_z(\phi)$$

Two-qubit operations

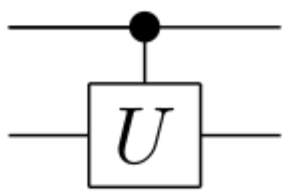
Controlled operations

Suppose U is a single-qubit operation with matrix representation:

$$U = \begin{pmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{pmatrix}$$

A controlled version of U is:

$$C_U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & u_{00} & u_{01} \\ 0 & 0 & u_{10} & u_{11} \end{pmatrix}$$



i.e. we apply U on the target qubit (here the rightmost qubit) only if the control qubit (here the leftmost qubit) is in state one. The state of the control qubit is left unchanged.

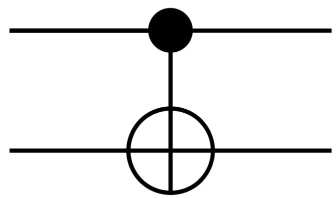
Two-qubit operations

Controlled operations

A critical operator in quantum computing is the C-NOT gate:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$|c, t\rangle \longrightarrow |c, c \oplus t\rangle$$

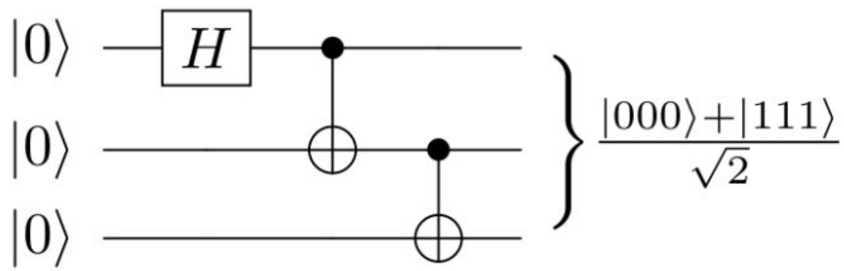


It's used to entangle qubits.

Two-qubit operations

Controlled operations

Example application (CNOT gate):



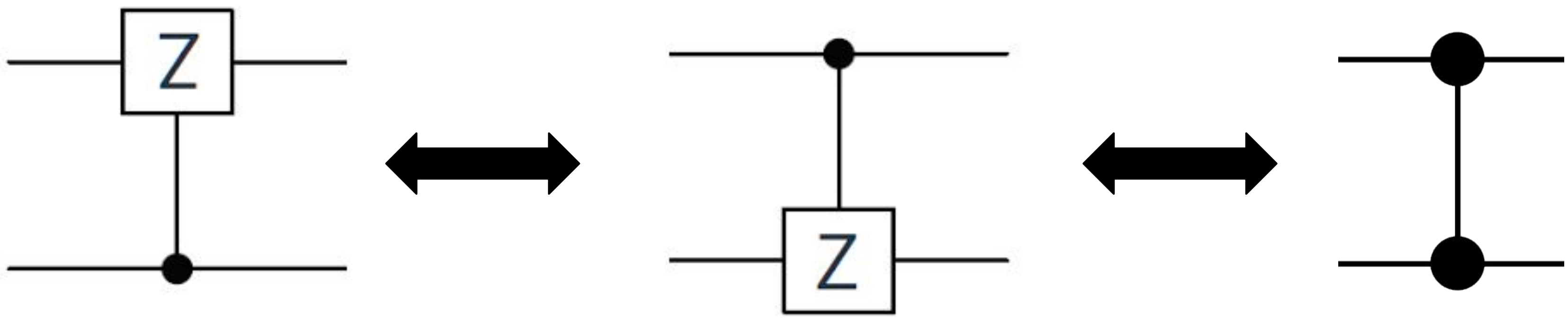
- Step 1 (H-gate on q1): $|000\rangle \longrightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}|00\rangle = \frac{|000\rangle + |100\rangle}{\sqrt{2}}$
- Step 2 (CNOT-gate q1 → q2): $\frac{|000\rangle + |100\rangle}{\sqrt{2}} \longrightarrow \frac{|000\rangle + |110\rangle}{\sqrt{2}}$
- Step 3 (CNOT-gate q2 → q3): $\frac{|000\rangle + |110\rangle}{\sqrt{2}} \longrightarrow \frac{|000\rangle + |111\rangle}{\sqrt{2}}$

Two-qubit operations

Controlled operations

The CZ gate flips the phase of a target qubit conditionally to the state of a control qubit:

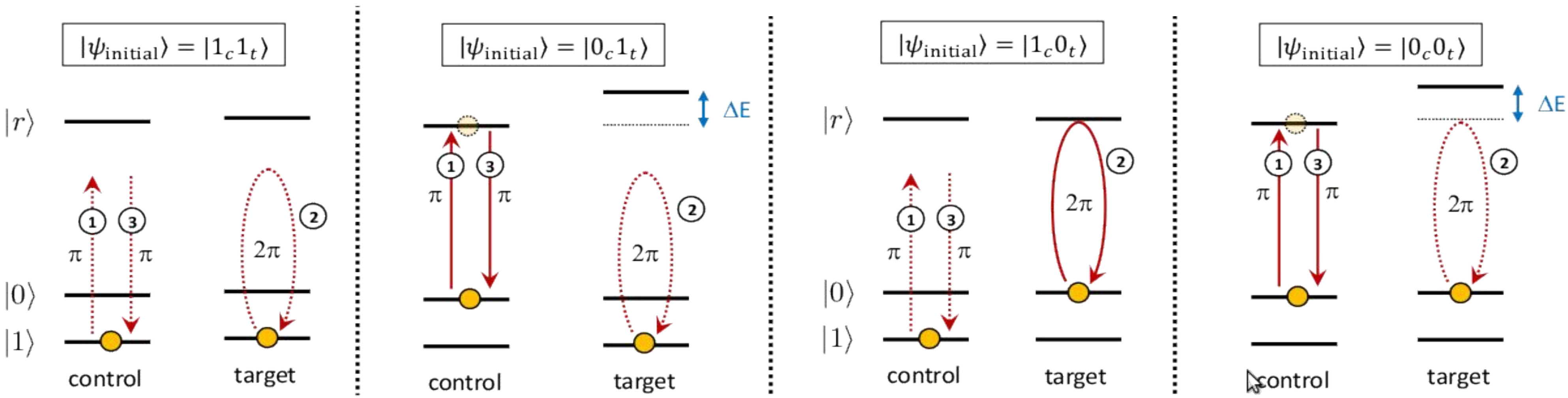
$$\mathrm{CZ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$



Two-qubit operations

Controlled operations with neutral atoms

CZ implementation protocol, where three consecutive pulses address the ground-rydberg transition:



Full (dashed) red lines illustrates if a given pulse is effective (not effective).

Two-qubit operations

Controlled operations with neutral atoms

Effect of the pulse sequence in the computational basis:

Initial state	$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
Final state	$-\|00\rangle$	$-\|01\rangle$	$-\|10\rangle$	$\|11\rangle$

More precisely, the pulse sequence implements a CZ gate up to a global phase factor:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathrm{e}^{i\pi} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \mathrm{e}^{i\pi}\mathrm{CZ}$$

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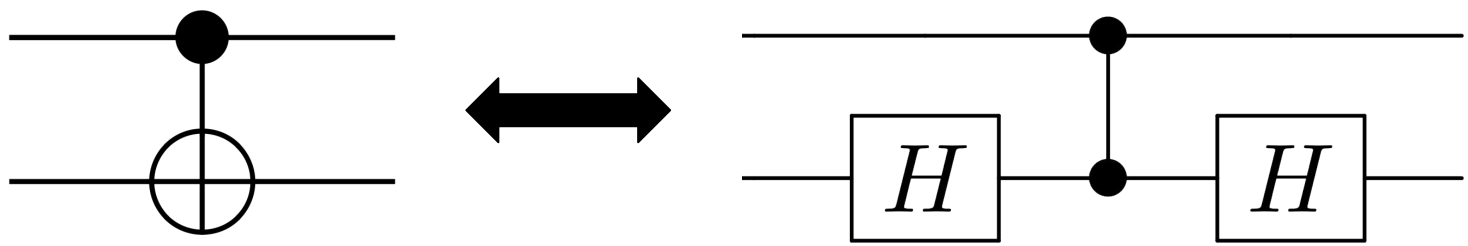
Two-qubit operations

Controlled operations with neutral atoms

Combining a CZ gate with two Hadamard gates recreates a C-NOT gate:



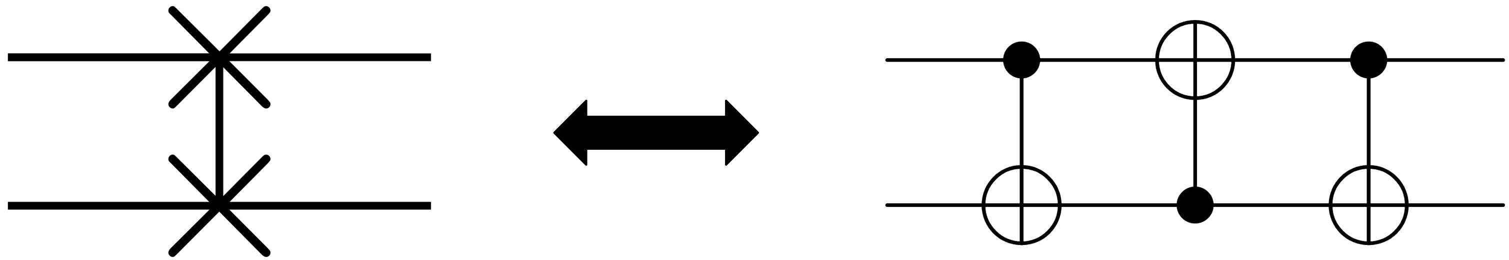
Two-qubit operations

SWAP operation

The SWAP gate exchanges the states of two qubits:

$$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$|a, b\rangle \longrightarrow |b, a\rangle$$



Conclusion

- ➔ Digital quantum computer are universal quantum computers
- ➔ In digital processing, a succession of gates is applied to implement an algorithm
- ➔ In neutral atom devices, single-qubit gates are implemented through Raman transition with resonant pulses (and virtual phase shift)
- ➔ While two-qubit gates are implemented through Rydberg transitions so to make qubits interact
- ➔ In neutral atom devices, the native two-qubit gate is the CZ gate