Confidential

# QUANTUM DISCOVERY

Analog quantum computing

PASQAL
www.pasqal.com
office@pasqal.com
7 rue Léonard de Vinci
91300 Massy
France

# Combinatorial optimization

## Problem statement

Given binary constraints:

$$x \in \{0,1\}^n \qquad C_a(x) = \begin{cases} 1 & \text{if } x \text{ satisfies the constraint } a \\ 0 & \text{if } x \text{ doesn't} \end{cases}$$

maximize

$$C(x) = \sum_a C_a(x)$$

# Combinatorial optimization

## QUBO formulation

Given the quadratic form:

Upper triangular matrix

$$f_Q(x) = x^\top Q x = \sum_{i=1}^{n} \sum_{j=i}^{n} Q_{ij} \boxed{x_i x_j}$$

Binary vector of size n

$= \begin{cases} \text{1 if both x\_i, x\_j are equal to one} \\ \text{0 otherwise} \end{cases}$

find:

$$x^* = \arg\min_{x \in \mathbb{B}^n} f_Q(x)$$

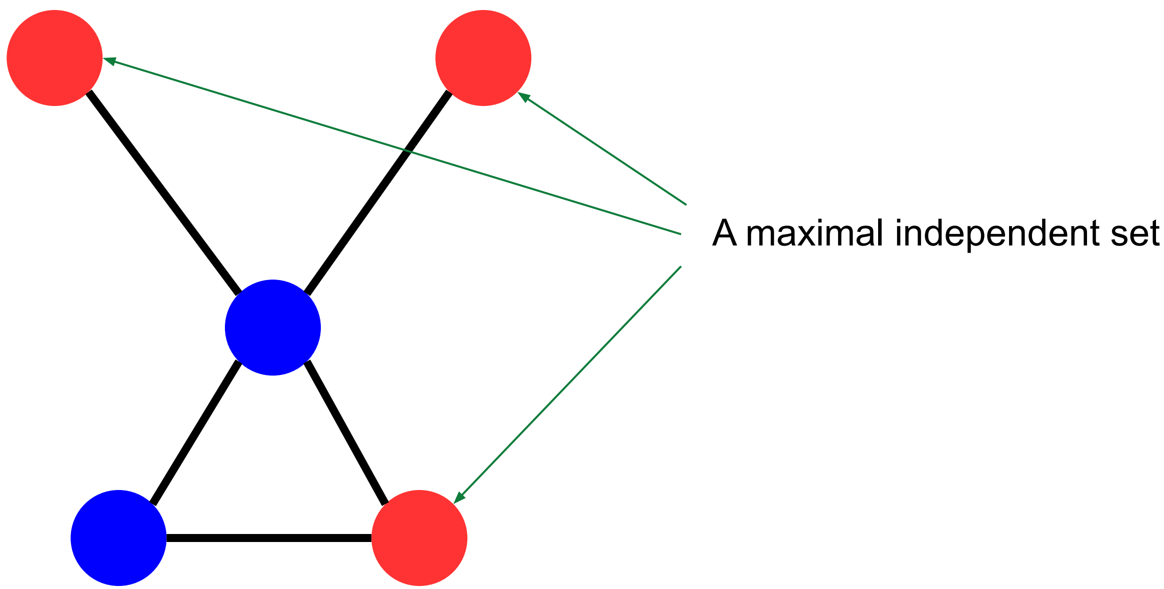
PASQAL

# Combinatorial optimization

## Maximal Independent Set

**Example optimization problem (MIS-problem):**

G(V, E):

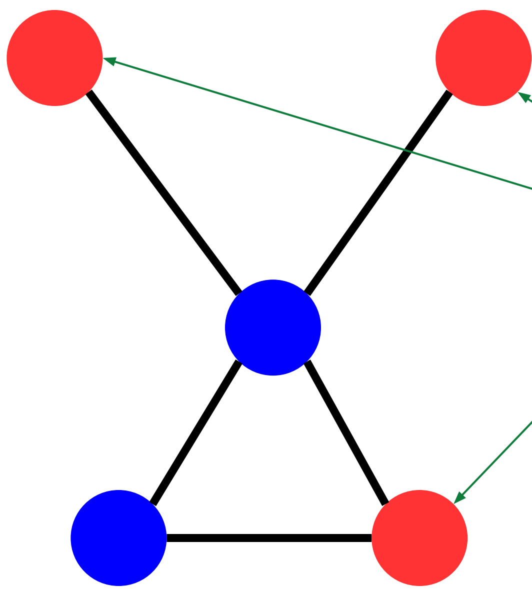


# Combinatorial optimization

## Maximal Independent Set

**<u>Example optimization problem (MIS-problem):</u>**

G(V, E):



$$\begin{cases} z_i = 1 & \text{if the } i\text{-th vertex is attributed to the indep. set} \\ z_i = 0 & \text{otherwise} \end{cases}$$

A maximal independent set

Penalty coefficient

Minimize:

$$C(z_1, \ldots, z_N) = -\sum_{i=1}^{N} z_i + U \sum_{(i,j)\in E} z_i z_j$$

# Analog quantum computing

## Analog problems

**Example optimization problem (MIS-problem):** The last two terms of the neutral atoms Ising Hamiltonian are of the same form as the cost function of the MIS problem:

$$H = \sum_{i=1}^{N} \frac{\hbar\Omega}{2}\sigma_i^x \boxed{- \sum_{i=1}^{N} \frac{\hbar\delta}{2}\sigma_i^z + \sum_{j<i} \frac{C_6}{|\mathbf{r}_i - \mathbf{r}_j|^6} n_i n_j}$$

$$\Updownarrow$$

$$C(z_1, \ldots, z_N) = -\sum_{i=1}^{N} z_i + U \sum_{(i,j)\in E} z_i z_j$$

PASQAL

# Analog quantum computing

## Analog quantum algorithms framework

In analog quantum computing, we consider a Hamiltonian of the following form:

$$H(t) = u(t)H_M + (1 - u(t))H_C$$

where:

- $H_C$ : the “problem” / “cost” Hamiltonian, it encodes the optimization task that is trying to be solved
- $H_M$ : the “mixer” Hamiltonian, it encodes quantum mixing (e.g. a uniform transverse field on qubits)
- $u(t) \in [0, 1]$ : the control function

# Analog quantum computing

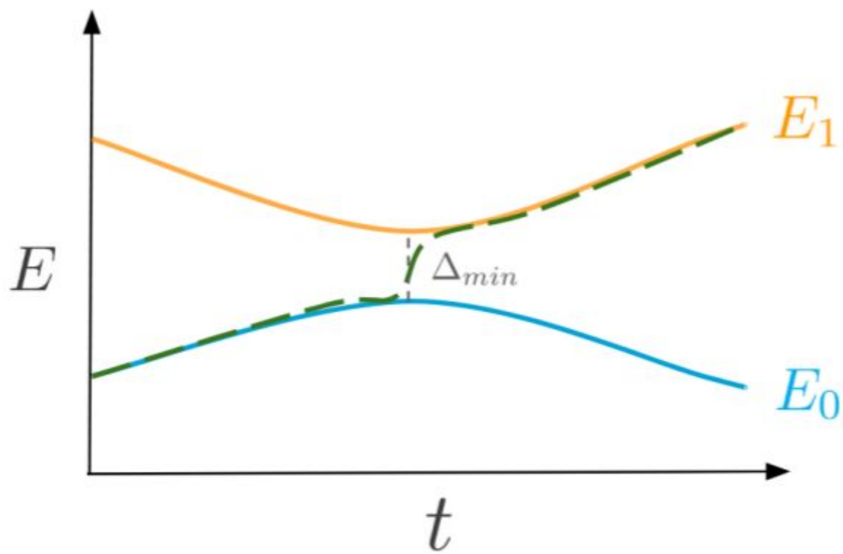
## Analog quantum algorithms framework

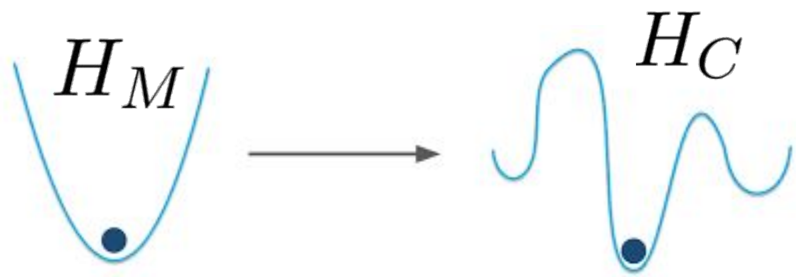
**Example optimization problem (MIS-problem):** The Ising Hamiltonian can be splitted into a cost and a mixer Hamiltonian:

$$H = \underbrace{\sum_{i=1}^{N} \frac{\hbar\Omega}{2}\sigma_i^x}_{H_M} \underbrace{- \sum_{i=1}^{N} \frac{\hbar\delta}{2}\sigma_i^z + \sum_{j<i} \frac{C_6}{|\mathbf{r}_i - \mathbf{r}_j|^6} n_i n_j}_{H_C}$$

# Analog quantum computing

## Adiabatic theorem




$$\begin{cases} H(0) = H_M \\ H(t_f) = H_C \end{cases} \qquad t_f \gg \frac{1}{\Delta_{min}^2}$$

**Figure 3.** Ground-state and excited state energies of $H(t)$ along a trajectory between $H_M$ and $H_C$. The minimum gap $\Delta_{\min}$ occurs in the middle of the path and a jump to the excited state (as illustrated by the dashed green line) is likely to occur at this point if $\Delta_{\min}$ is too small and the speed too fast.

# Conclusion

- ➔ In analog quantum computing, we implement a Hamiltonian simulation to compute
- ➔ Starting from the ground state of a mixer Hamiltonian, the evolution leads the system to the ground state of a cost Hamiltonian analogous to the problem cost function
- ➔ On PASQAL's machines, all the atoms of the register are excited simultaneously with a single laser when implementing an analog quantum computation
- ➔ An adiabatic evolution ensures that the analog quantum computation is successful
- ➔ Analog quantum computers require less operations than digital quantum computers
- ➔ Analog quantum computers are non universal machines