Special Number Two

The Parity Tensor and the Equilibrium Role of 2

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Abstract

This paper explains why integers are *even* or *odd* through a simple, lawful tensor built from three coordinates: multiplication $M(n) = n$, prime complexity $A(n) = \mathrm{PC}(n) = \sum a_i p_i$, and the pull $P(n) = n - \mathrm{PC}(n)$. Reducing each coordinate *modulo two* yields a parity tensor $T(n) = (M \bmod 2,\ A \bmod 2,\ P \bmod 2)$. We prove that its structure is governed by one principle—$M \equiv A + P \pmod 2$—and one asymmetry: the prime 2. The result is a four-class classification that captures the equilibrium role of two and clarifies why the odd primes all line up with the signature $(1, 1, 0)$ while powers of two sit at $(0, 0, 0)$. Our presentation is educational by design, but every claim is mathematically precise and certified by a small, reusable Python proof DAG.

Contents

1 Reader's Orientation

Parity—"even or odd"—is the first property most of us learn about integers. Here we show that parity is not just a single bit: it is the front door to a *tensor* that records how multiplication, addition of primes, and their difference interact modulo two.

Two ideas carry the load:

1. The **three coordinates**: $M(n) = n$, $A(n) = \mathrm{PC}(n) = \sum a_i p_i$, and $P(n) = n - \mathrm{PC}(n)$. They satisfy the identity $M = A + P$.
2. The **mod-two view**: reduce each coordinate modulo two, obtaining $T(n) = (M \bmod 2,\ A \bmod 2,\ P \bmod 2)$.

With these in hand, patterns that look folkloric become theorems, and the uniqueness of the prime 2 becomes an equilibrium principle rather than a curiosity.

2 The Parity Tensor

Definition 2.1 (Parity Tensor)**.** For $n \geq 2$, define

$$T(n) := (\, M(n) \bmod 2,\ A(n) \bmod 2,\ P(n) \bmod 2\,) \in (\mathbb{Z}/2\mathbb{Z})^3.$$

Small examples illustrate the idea:

n	Factorization	Coordinates (M, A, P)	Tensor $T(n)$
6	$2 \cdot 3$	(6, 5, 1)	$(0, 1, 1)$
9	3^2	(9, 6, 3)	$(1, 0, 1)$
10	$2 \cdot 5$	(10, 7, 3)	$(0, 1, 1)$
12	$2^2 \cdot 3$	(12, 7, 5)	$(0, 1, 1)$
27	3^3	(27, 9, 18)	$(1, 1, 0)$
32	2^5	(32, 10, 22)	$(0, 0, 0)$

3 The Fundamental Parity Theorem

Write the factorization of n as $n = 2^a \prod_{i=1}^{k} p_i^{a_i}$ where the p_i are distinct odd primes. Define the odd-exponent count

$$\omega_{\text{odd}}(n) := \#\{\, i : a_i \text{ is odd }\}.$$

Theorem 3.1 (Fundamental Parity)**.** *For every $n \geq 2$,*

$$A(n) \equiv \omega_{\text{odd}}(n) \pmod 2.$$

Proof. Since $A(n) = \sum a_i p_i$ and $2 \equiv 0 \pmod 2$, the 2-power contributes $a \cdot 2 \equiv 0$. Each odd prime contributes $a_i p_i \equiv a_i \cdot 1 \equiv a_i \pmod 2$. Summing over odd primes counts exactly those with odd exponent. $\square$

Corollary 3.2 (Parity of P)**.** *For all n,*

$$P(n) \equiv n + A(n) \equiv n + \omega_{\text{odd}}(n) \pmod 2.$$

4 Worked Examples

We include short, concrete tables that you can read line-by-line. Each row shows the factorization, the three coordinates (M, A, P), and the tensor $T(n) = (M \bmod 2, A \bmod 2, P \bmod 2)$.

Odd primes and powers of two

n	Factorization	(M, A, P)	$T(n)$
2	2	(2, 2, 0)	$(0, 0, 0)$
3	3	(3, 3, 0)	$(1, 1, 0)$
5	5	(5, 5, 0)	$(1, 1, 0)$
7	7	(7, 7, 0)	$(1, 1, 0)$
8	2^3	(8, 6, 2)	$(0, 0, 0)$
32	2^5	(32, 10, 22)	$(0, 0, 0)$

Odd squares and odd semiprimes

n	Factorization	(M, A, P)	$T(n)$
9	3^2	(9, 6, 3)	$(1, 0, 1)$
25	5^2	(25, 10, 15)	$(1, 0, 1)$
15	$3 \cdot 5$	(15, 8, 7)	$(1, 0, 1)$
21	$3 \cdot 7$	(21, 10, 11)	$(1, 0, 1)$
35	$5 \cdot 7$	(35, 12, 23)	$(1, 0, 1)$

Even semiprimes $2p$ with p odd

n	Factorization	(M, A, P)	$T(n)$
6	$2 \cdot 3$	(6, 5, 1)	$(0, 1, 1)$
10	$2 \cdot 5$	(10, 7, 3)	$(0, 1, 1)$
22	$2 \cdot 11$	(22, 13, 9)	$(0, 1, 1)$

Checking $A(n) \equiv \omega_{\text{odd}}(n) \pmod 2$

n	Factorization	$\omega_{\text{odd}}(n)$	$A(n) \bmod 2$	Match?
12	$2^2 \cdot 3$	1	1	
18	$2 \cdot 3^2$	0	0	
45	$3^2 \cdot 5$	1	1	
75	$3 \cdot 5^2$	1	1	
90	$2 \cdot 3^2 \cdot 5$	1	1	

A single multiplication, worked end-to-end

Take $m = 12 = 2^2 \cdot 3$ and $n = 15 = 3 \cdot 5$. We compute modulo two:

	M mod 2	A mod 2	P mod 2
$m = 12$	0	1	1
$n = 15$	1	0	1
$mn = 180$	0	1	1

Checks: $M(mn) \equiv (0)(1) = 0$; $A(mn) \equiv 1 + 0 = 1$; $P(mn) \equiv 0 + 1 + 0 = 1$. This illustrates **??** and the relations of § **??** below in a single glance.

5 The Four Parity Classes

The identity $M = A + P$ implies $M \equiv A + P \pmod 2$. This leaves only four possibilities for $T(n)$:

Tensor	Condition	Examples
$(0, 0, 0)$	n even, ω_{odd} even	$2, 4, 8, 18, 30$
$(0, 1, 1)$	n even, ω_{odd} odd	$6, 10, 12, 14, 20$
$(1, 0, 1)$	n odd, ω_{odd} even	$9, 15, 25, 49$
$(1, 1, 0)$	n odd, ω_{odd} odd	all odd primes

Proposition 5.1 (Completeness). *No other parity tensors occur:* $(0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1)$ *are impossible.*

Proof. If $M \equiv 0$ then $A \equiv P$; if $M \equiv 1$ then $A \not\equiv P$. This eliminates the four excluded patterns. ☐

6 Special Signatures and Why 2 is Special

Proposition 6.1 (Odd primes). *Every odd prime p has $T(p) = (1, 1, 0)$.*

Proof. For p odd, $n \equiv 1$, and the only odd prime in the factorization has exponent one, so $A \equiv 1$ and $P \equiv 0$. ☐

Proposition 6.2 (Powers of two). *Every $n = 2^k$ has $T(n) = (0, 0, 0)$.*

Proof. Here $n \equiv 0$, there are no odd primes at all, so $A \equiv 0$ and $P \equiv 0$. ☐

Proposition 6.3 (Odd prime squares and odd semiprimes). *If $n = p^2$ with p odd, or $n = pq$ with distinct odd primes, then $T(n) = (1, 0, 1)$.*

Proof. In both cases $n \equiv 1$ and the number of odd primes with odd exponent is even (0 or 2), so $A \equiv 0$ and $P \equiv 1$. ☐

The equilibrium role of 2. Two plays two roles at once: it flips the multiplicative bit M mod 2, and it never contributes to ω_{odd}. This decoupling makes 2 the pivot of the tensor: it sets the "evenness" of n without changing the odd-prime counter that governs A mod 2. The four-class picture is the equilibrium of those two influences.

7 Multiplicative Relations modulo two

Theorem 7.1 (Relations under multiplication)**.** *For all $m, n \geq 2$,*

$$M(mn) \equiv M(m)\,M(n), \qquad A(mn) \equiv A(m)+A(n), \qquad P(mn) \equiv M(m)M(n)+A(m)+A(n) \pmod 2.$$

Proof. The first is $mn \equiv (m \bmod 2)(n \bmod 2)$. For A, additivity modulo two follows from **??** below, or directly from the odd-exponent count. The relation for P is $P = M - A$. $\square$

8 Certification and Reproducibility

Every claim above is certified by an executable proof DAG:

- Core definitions $M, A = \mathrm{PC}, P$ live in `new_foundations/python/core.py`.
- Parity modules live in `special_number_two/python/`. Run `python -m special_number_two.python.certif` from the repo root.

This mirrors the intended Lean structure: one theorem per file with explicit imports. The same dependency graph can be used to order formal proofs.

9 Appendix: Extended Tables

This appendix is auto-generated from the certification modules. Regenerate with the Makefile target `make tables` in `special_number_two/paper/`.

n	Factorization	$M(n)$	$A(n),\ P(n)$	$T(n)$
2	2	2	2, 0	$(0,0,0)$
3	3	3	3, 0	$(1,1,0)$
4	2^2	4	4, 0	$(0,0,0)$
5	5	5	5, 0	$(1,1,0)$
6	$2 \cdot 3$	6	5, 1	$(0,1,1)$
7	7	7	7, 0	$(1,1,0)$
8	2^3	8	6, 2	$(0,0,0)$
9	3^2	9	6, 3	$(1,0,1)$
10	$2 \cdot 5$	10	7, 3	$(0,1,1)$
11	11	11	11, 0	$(1,1,0)$
12	$2^2 \cdot 3$	12	7, 5	$(0,1,1)$
13	13	13	13, 0	$(1,1,0)$
14	$2 \cdot 7$	14	9, 5	$(0,1,1)$
15	$3 \cdot 5$	15	8, 7	$(1,0,1)$
16	2^4	16	8, 8	$(0,0,0)$
17	17	17	17, 0	$(1,1,0)$
18	$2 \cdot 3^2$	18	8, 10	$(0,0,0)$
19	19	19	19, 0	$(1,1,0)$
20	$2^2 \cdot 5$	20	9, 11	$(0,1,1)$
21	$3 \cdot 7$	21	10, 11	$(1,0,1)$
22	$2 \cdot 11$	22	13, 9	$(0,1,1)$
23	23	23	23, 0	$(1,1,0)$
24	$2^3 \cdot 3$	24	9, 15	$(0,1,1)$
25	5^2	25	10, 15	$(1,0,1)$

n	Factorization	$M(n)$	$A(n),\ P(n)$	$T(n)$
26	$2 \cdot 13$	26	15, 11	$(0,1,1)$
27	3^3	27	9, 18	$(1,1,0)$
28	$2^2 \cdot 7$	28	11, 17	$(0,1,1)$
29	29	29	29, 0	$(1,1,0)$
30	$2 \cdot 3 \cdot 5$	30	10, 20	$(0,0,0)$
31	31	31	31, 0	$(1,1,0)$
32	2^5	32	10, 22	$(0,0,0)$
33	$3 \cdot 11$	33	14, 19	$(1,0,1)$
34	$2 \cdot 17$	34	19, 15	$(0,1,1)$
35	$5 \cdot 7$	35	12, 23	$(1,0,1)$
36	$2^2 \cdot 3^2$	36	10, 26	$(0,0,0)$
37	37	37	37, 0	$(1,1,0)$
38	$2 \cdot 19$	38	21, 17	$(0,1,1)$
39	$3 \cdot 13$	39	16, 23	$(1,0,1)$
40	$2^3 \cdot 5$	40	11, 29	$(0,1,1)$
41	41	41	41, 0	$(1,1,0)$
42	$2 \cdot 3 \cdot 7$	42	12, 30	$(0,0,0)$
43	43	43	43, 0	$(1,1,0)$
44	$2^2 \cdot 11$	44	15, 29	$(0,1,1)$
45	$3^2 \cdot 5$	45	11, 34	$(1,1,0)$
46	$2 \cdot 23$	46	25, 21	$(0,1,1)$
47	47	47	47, 0	$(1,1,0)$
48	$2^4 \cdot 3$	48	11, 37	$(0,1,1)$
49	7^2	49	14, 35	$(1,0,1)$
50	$2 \cdot 5^2$	50	12, 38	$(0,0,0)$
51	$3 \cdot 17$	51	20, 31	$(1,0,1)$
52	$2^2 \cdot 13$	52	17, 35	$(0,1,1)$
53	53	53	53, 0	$(1,1,0)$
54	$2 \cdot 3^3$	54	11, 43	$(0,1,1)$
55	$5 \cdot 11$	55	16, 39	$(1,0,1)$
56	$2^3 \cdot 7$	56	13, 43	$(0,1,1)$
57	$3 \cdot 19$	57	22, 35	$(1,0,1)$
58	$2 \cdot 29$	58	31, 27	$(0,1,1)$
59	59	59	59, 0	$(1,1,0)$
60	$2^2 \cdot 3 \cdot 5$	60	12, 48	$(0,0,0)$
61	61	61	61, 0	$(1,1,0)$
62	$2 \cdot 31$	62	33, 29	$(0,1,1)$
63	$3^2 \cdot 7$	63	13, 50	$(1,1,0)$
64	2^6	64	12, 52	$(0,0,0)$
65	$5 \cdot 13$	65	18, 47	$(1,0,1)$
66	$2 \cdot 3 \cdot 11$	66	16, 50	$(0,0,0)$
67	67	67	67, 0	$(1,1,0)$
68	$2^2 \cdot 17$	68	21, 47	$(0,1,1)$
69	$3 \cdot 23$	69	26, 43	$(1,0,1)$
70	$2 \cdot 5 \cdot 7$	70	14, 56	$(0,0,0)$
71	71	71	71, 0	$(1,1,0)$
72	$2^3 \cdot 3^2$	72	12, 60	$(0,0,0)$
73	73	73	73, 0	$(1,1,0)$
74	$2 \cdot 37$	74	39, 35	$(0,1,1)$
75	$3 \cdot 5^2$	75	13, 62	$(1,1,0)$
76	$2^2 \cdot 19$	76	23, 53	$(0,1,1)$
77	$7 \cdot 11$	77	18, 59	$(1,0,1)$
78	$2 \cdot 3 \cdot 13$	78	18, 60	$(0,0,0)$
79	79	79	79, 0	$(1,1,0)$
80	$2^4 \cdot 5$	80	13, 67	$(0,1,1)$
81	3^4	81	12, 69	$(1,0,1)$
82	$2 \cdot 41$	82	43, 39	$(0,1,1)$

n	Factorization	$M(n)$	$A(n),\ P(n)$	$T(n)$
83	83	83	83, 0	$(1,1,0)$
84	$2^2\cdot 3\cdot 7$	84	14, 70	$(0,0,0)$
85	$5\cdot 17$	85	22, 63	$(1,0,1)$
86	$2\cdot 43$	86	45, 41	$(0,1,1)$
87	$3\cdot 29$	87	32, 55	$(1,0,1)$
88	$2^3\cdot 11$	88	17, 71	$(0,1,1)$
89	89	89	89, 0	$(1,1,0)$
90	$2\cdot 3^2\cdot 5$	90	13, 77	$(0,1,1)$
91	$7\cdot 13$	91	20, 71	$(1,0,1)$
92	$2^2\cdot 23$	92	27, 65	$(0,1,1)$
93	$3\cdot 31$	93	34, 59	$(1,0,1)$
94	$2\cdot 47$	94	49, 45	$(0,1,1)$
95	$5\cdot 19$	95	24, 71	$(1,0,1)$
96	$2^5\cdot 3$	96	13, 83	$(0,1,1)$
97	97	97	97, 0	$(1,1,0)$
98	$2\cdot 7^2$	98	16, 82	$(0,0,0)$
99	$3^2\cdot 11$	99	17, 82	$(1,1,0)$
100	$2^2\cdot 5^2$	100	14, 86	$(0,0,0)$
101	101	101	101, 0	$(1,1,0)$
102	$2\cdot 3\cdot 17$	102	22, 80	$(0,0,0)$
103	103	103	103, 0	$(1,1,0)$
104	$2^3\cdot 13$	104	19, 85	$(0,1,1)$
105	$3\cdot 5\cdot 7$	105	15, 90	$(1,1,0)$
106	$2\cdot 53$	106	55, 51	$(0,1,1)$
107	107	107	107, 0	$(1,1,0)$
108	$2^2\cdot 3^3$	108	13, 95	$(0,1,1)$
109	109	109	109, 0	$(1,1,0)$
110	$2\cdot 5\cdot 11$	110	18, 92	$(0,0,0)$
111	$3\cdot 37$	111	40, 71	$(1,0,1)$
112	$2^4\cdot 7$	112	15, 97	$(0,1,1)$
113	113	113	113, 0	$(1,1,0)$
114	$2\cdot 3\cdot 19$	114	24, 90	$(0,0,0)$
115	$5\cdot 23$	115	28, 87	$(1,0,1)$
116	$2^2\cdot 29$	116	33, 83	$(0,1,1)$
117	$3^2\cdot 13$	117	19, 98	$(1,1,0)$
118	$2\cdot 59$	118	61, 57	$(0,1,1)$
119	$7\cdot 17$	119	24, 95	$(1,0,1)$
120	$2^3\cdot 3\cdot 5$	120	14, 106	$(0,0,0)$
121	11^2	121	22, 99	$(1,0,1)$
122	$2\cdot 61$	122	63, 59	$(0,1,1)$
123	$3\cdot 41$	123	44, 79	$(1,0,1)$
124	$2^2\cdot 31$	124	35, 89	$(0,1,1)$
125	5^3	125	15, 110	$(1,1,0)$
126	$2\cdot 3^2\cdot 7$	126	15, 111	$(0,1,1)$
127	127	127	127, 0	$(1,1,0)$
128	2^7	128	14, 114	$(0,0,0)$
129	$3\cdot 43$	129	46, 83	$(1,0,1)$
130	$2\cdot 5\cdot 13$	130	20, 110	$(0,0,0)$
131	131	131	131, 0	$(1,1,0)$
132	$2^2\cdot 3\cdot 11$	132	18, 114	$(0,0,0)$
133	$7\cdot 19$	133	26, 107	$(1,0,1)$
134	$2\cdot 67$	134	69, 65	$(0,1,1)$
135	$3^3\cdot 5$	135	14, 121	$(1,0,1)$
136	$2^3\cdot 17$	136	23, 113	$(0,1,1)$
137	137	137	137, 0	$(1,1,0)$
138	$2\cdot 3\cdot 23$	138	28, 110	$(0,0,0)$
139	139	139	139, 0	$(1,1,0)$

n	Factorization	$M(n)$	$A(n),\ P(n)$	$T(n)$
140	$2^2 \cdot 5 \cdot 7$	140	16, 124	$(0,0,0)$
141	$3 \cdot 47$	141	50, 91	$(1,0,1)$
142	$2 \cdot 71$	142	73, 69	$(0,1,1)$
143	$11 \cdot 13$	143	24, 119	$(1,0,1)$
144	$2^4 \cdot 3^2$	144	14, 130	$(0,0,0)$
145	$5 \cdot 29$	145	34, 111	$(1,0,1)$
146	$2 \cdot 73$	146	75, 71	$(0,1,1)$
147	$3 \cdot 7^2$	147	17, 130	$(1,1,0)$
148	$2^2 \cdot 37$	148	41, 107	$(0,1,1)$
149	149	149	149, 0	$(1,1,0)$
150	$2 \cdot 3 \cdot 5^2$	150	15, 135	$(0,1,1)$
151	151	151	151, 0	$(1,1,0)$
152	$2^3 \cdot 19$	152	25, 127	$(0,1,1)$
153	$3^2 \cdot 17$	153	23, 130	$(1,1,0)$
154	$2 \cdot 7 \cdot 11$	154	20, 134	$(0,0,0)$
155	$5 \cdot 31$	155	36, 119	$(1,0,1)$
156	$2^2 \cdot 3 \cdot 13$	156	20, 136	$(0,0,0)$
157	157	157	157, 0	$(1,1,0)$
158	$2 \cdot 79$	158	81, 77	$(0,1,1)$
159	$3 \cdot 53$	159	56, 103	$(1,0,1)$
160	$2^5 \cdot 5$	160	15, 145	$(0,1,1)$
161	$7 \cdot 23$	161	30, 131	$(1,0,1)$
162	$2 \cdot 3^4$	162	14, 148	$(0,0,0)$
163	163	163	163, 0	$(1,1,0)$
164	$2^2 \cdot 41$	164	45, 119	$(0,1,1)$
165	$3 \cdot 5 \cdot 11$	165	19, 146	$(1,1,0)$
166	$2 \cdot 83$	166	85, 81	$(0,1,1)$
167	167	167	167, 0	$(1,1,0)$
168	$2^3 \cdot 3 \cdot 7$	168	16, 152	$(0,0,0)$
169	13^2	169	26, 143	$(1,0,1)$
170	$2 \cdot 5 \cdot 17$	170	24, 146	$(0,0,0)$
171	$3^2 \cdot 19$	171	25, 146	$(1,1,0)$
172	$2^2 \cdot 43$	172	47, 125	$(0,1,1)$
173	173	173	173, 0	$(1,1,0)$
174	$2 \cdot 3 \cdot 29$	174	34, 140	$(0,0,0)$
175	$5^2 \cdot 7$	175	17, 158	$(1,1,0)$
176	$2^4 \cdot 11$	176	19, 157	$(0,1,1)$
177	$3 \cdot 59$	177	62, 115	$(1,0,1)$
178	$2 \cdot 89$	178	91, 87	$(0,1,1)$
179	179	179	179, 0	$(1,1,0)$
180	$2^2 \cdot 3^2 \cdot 5$	180	15, 165	$(0,1,1)$
181	181	181	181, 0	$(1,1,0)$
182	$2 \cdot 7 \cdot 13$	182	22, 160	$(0,0,0)$
183	$3 \cdot 61$	183	64, 119	$(1,0,1)$
184	$2^3 \cdot 23$	184	29, 155	$(0,1,1)$
185	$5 \cdot 37$	185	42, 143	$(1,0,1)$
186	$2 \cdot 3 \cdot 31$	186	36, 150	$(0,0,0)$
187	$11 \cdot 17$	187	28, 159	$(1,0,1)$
188	$2^2 \cdot 47$	188	51, 137	$(0,1,1)$
189	$3^3 \cdot 7$	189	16, 173	$(1,0,1)$
190	$2 \cdot 5 \cdot 19$	190	26, 164	$(0,0,0)$
191	191	191	191, 0	$(1,1,0)$
192	$2^6 \cdot 3$	192	15, 177	$(0,1,1)$
193	193	193	193, 0	$(1,1,0)$
194	$2 \cdot 97$	194	99, 95	$(0,1,1)$
195	$3 \cdot 5 \cdot 13$	195	21, 174	$(1,1,0)$
196	$2^2 \cdot 7^2$	196	18, 178	$(0,0,0)$

n	Factorization	$M(n)$	$A(n),\ P(n)$	$T(n)$
197	197	197	197, 0	$(1, 1, 0)$
198	$2 \cdot 3^2 \cdot 11$	198	19, 179	$(0, 1, 1)$
199	199	199	199, 0	$(1, 1, 0)$
200	$2^3 \cdot 5^2$	200	16, 184	$(0, 0, 0)$

Acknowledgments

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