Reconstruction of B-Spline Surfaces From Scattered Data Points

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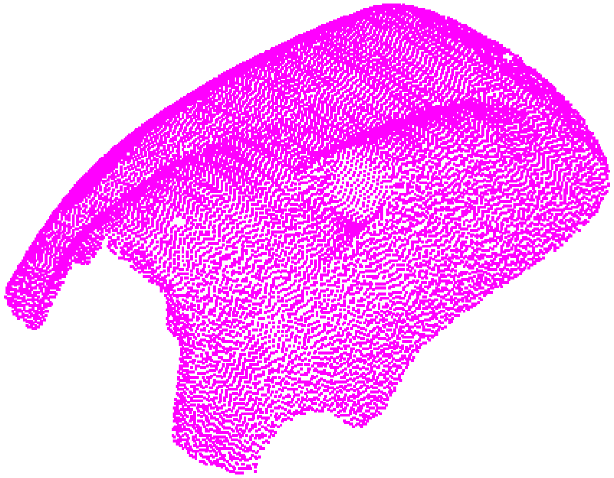
Overview

- Problem Description
- Previous Work
- Algorithm
- Error Metrics
- Conclusions
- Future Work

The Problem

- Scattered Point Sets from Digitization Devices, Range Scanners
- Surfaces (NURBS), Triangle Meshes, Conics for CAD Systems
- Applications
  - Preservation
  - Modeling
  - Reverse Engineering



Previous Work

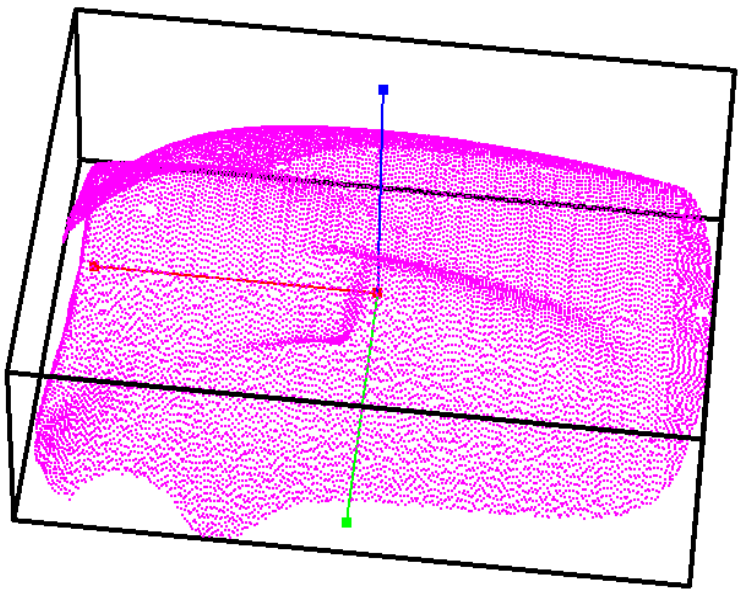
- Types of Scattered data sets
  - Surfaces represented by a single sheet
  - Arbitrary surfaces of any genus
- Reconstructing B-Splines
  - Automatic methods (Hoppe et. al. SIGGRAPH 1996)
  - Interactive methods (Levoy et. al. SIGGRAPH 1996)
- Triangulation algorithms
  - Clustering approaches (Heckel et. al. IEEE VIS 1998)
  - Voronoi Diagrams / Delaunay Triangulations (Amenta et. al. SIGGRAPH 1998)
- Subdivision surfaces
  - Arbitrary topologies (DeRose et. al. SIGGRAPH 1994)

Algorithm Overview

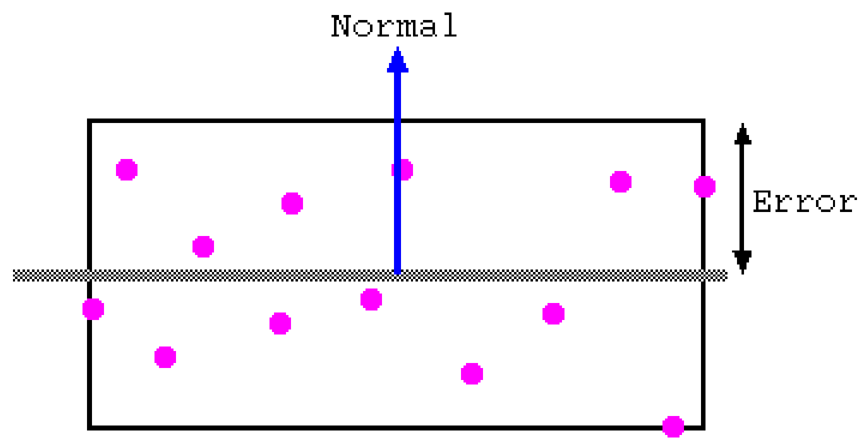
- Decompose Scattered Point Set in a top down fashion using a 3-D strip tree .
- Compute bounding box of data points.
- Re-orient the bounding box along the *principal directions.*
- Recursively subdivide the root box until the resulting set of boxes adequately approximates the data points.
- The resulting tree, called a *strip-tree*, is used to fit surfaces.

Oriented Bounding Boxes

- Root bounding box for scattered point set
- Oriented using PCA



- Error calculation for an oriented bounding box

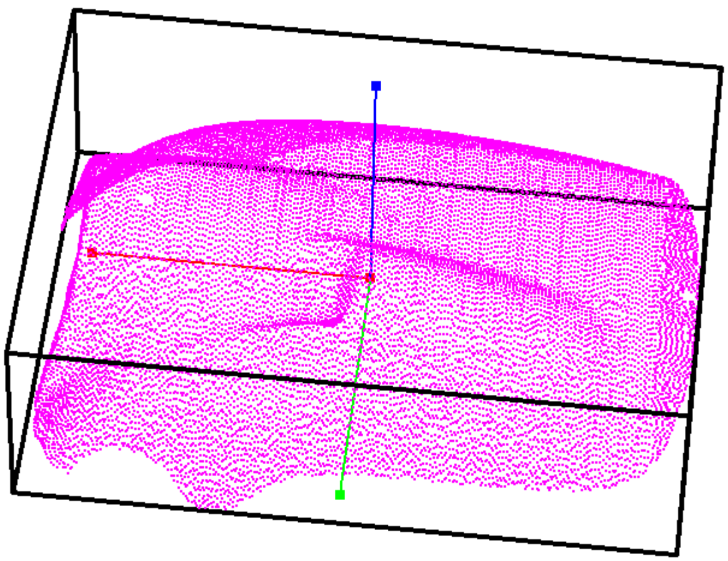


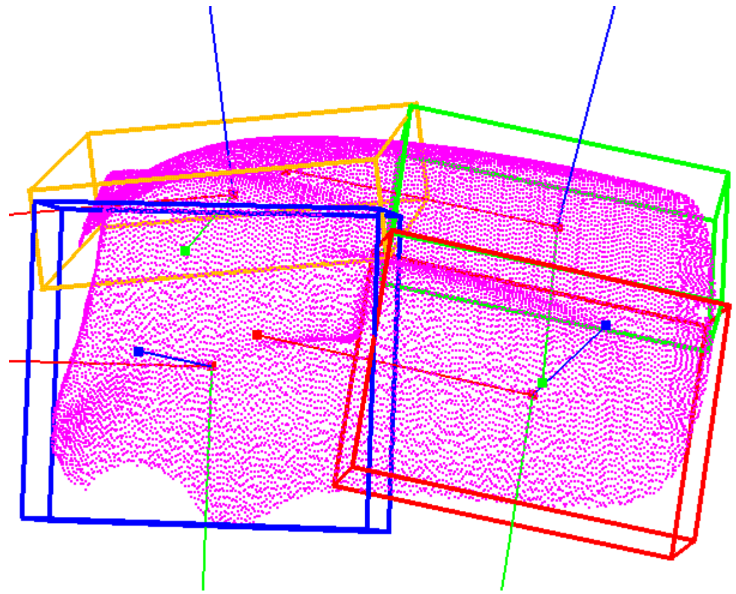
Generalized Strip Tree

- Quad-tree in 3-D space whose nodes are oriented bounding boxes.
- Each level approximates the data points.
- The boxes at one level are decomposed or subdivided to form the next finer level.
- Currently the subdivision is uniform.
  - All nodes must be subdivided.

Bounding Box Subdivision

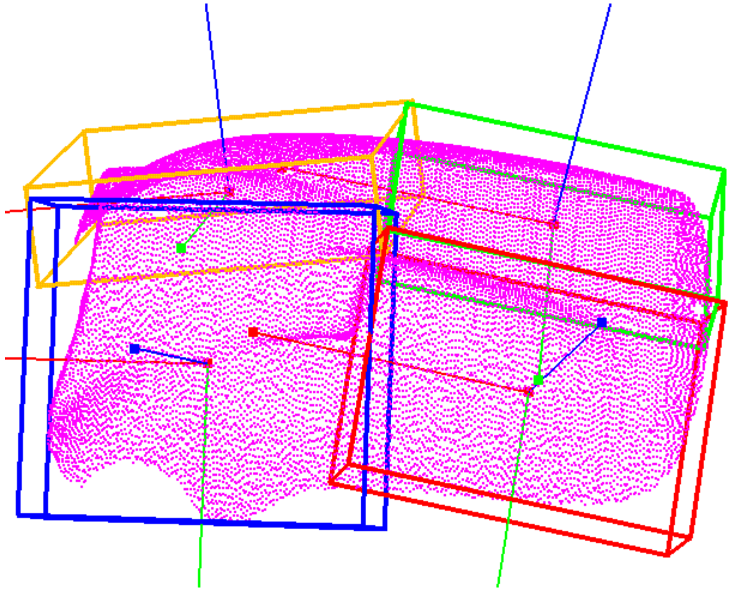
- Find a subdivision point.
- Distribute data points.
- Re-orient the children.



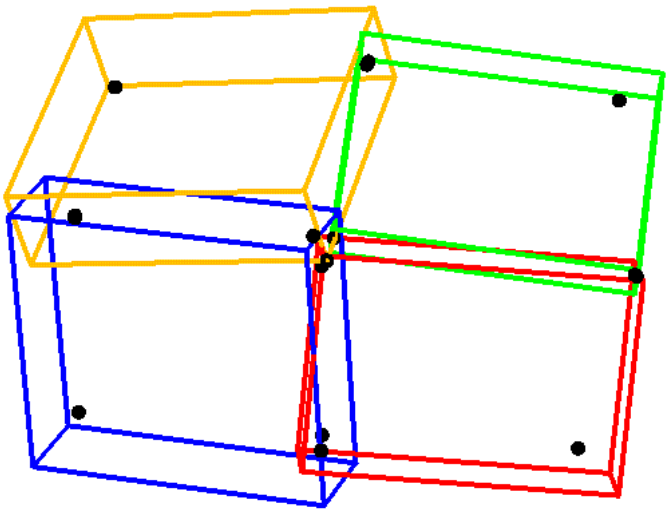


Box Points

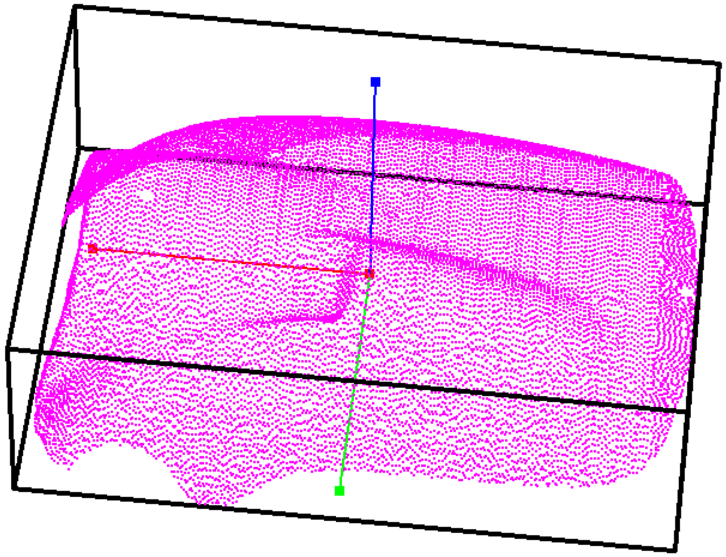
- Strip Tree with four nodes.



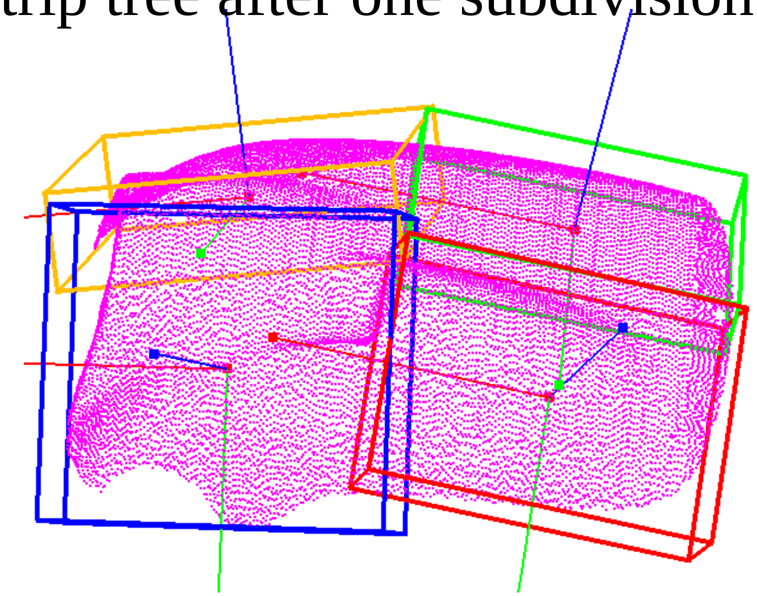
- Box Points for the four nodes.



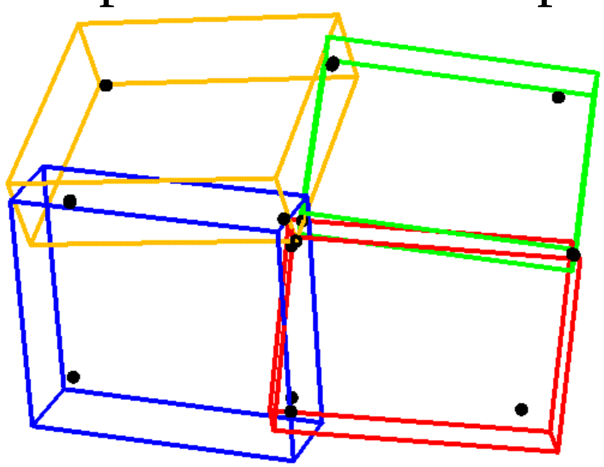
Root Bounding Box



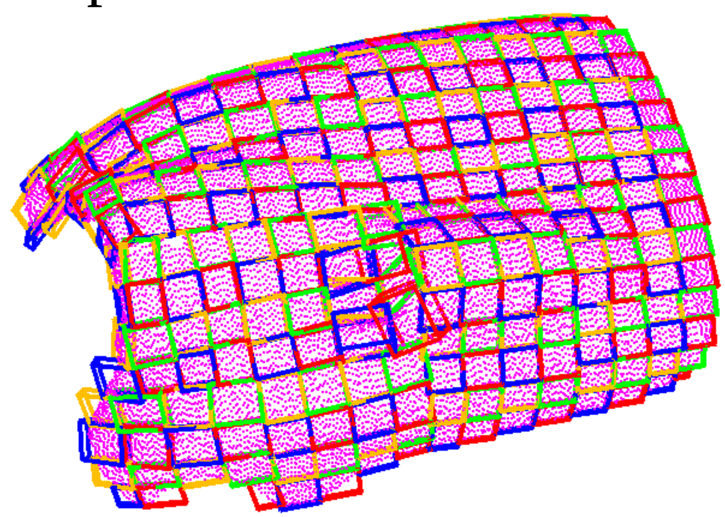
Strip tree after one subdivision



Box points for the strip tree



Strip tree after 4 subdivisions



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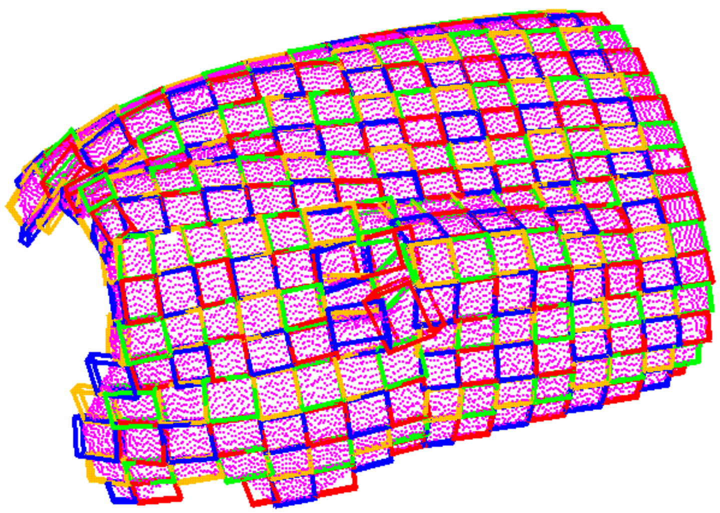
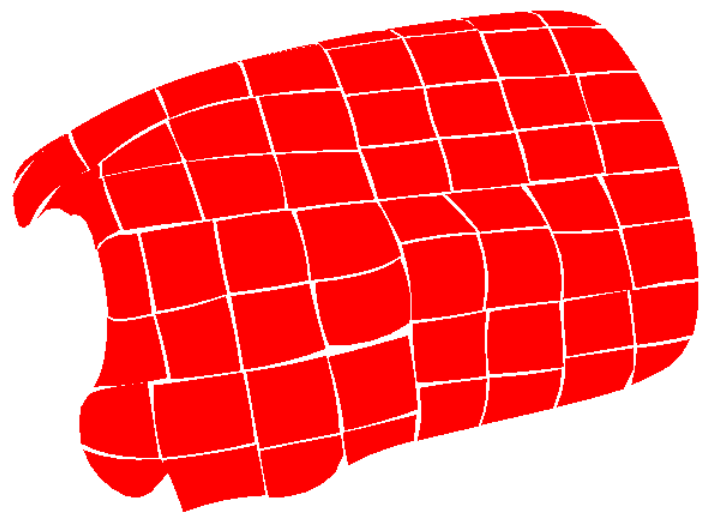
Reconstruction of B-Spline Surfaces

Fitting Surfaces

- Decompose strip tree.
- Use next-to-last level to fit surfaces.
- Fit bi-quadratic surface to box points.
- Elevate to bi-cubic.





Reconstruction of B-Spline Surfaces

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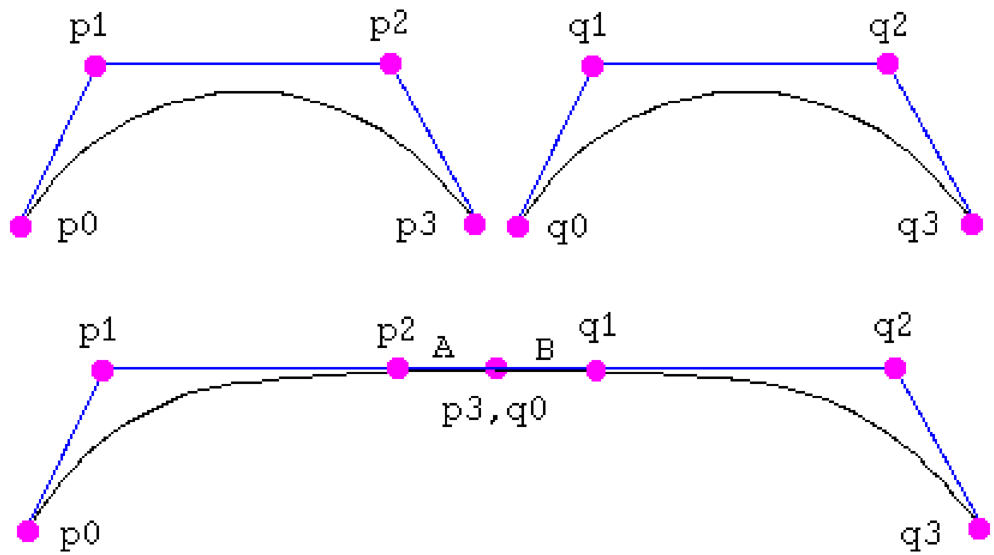
Blending Surfaces

- Initial fitting process yields a collection of B-Spline surfaces.
- Blending Process consists of three steps:
  - C^0 continuity is achieved by averaging control points of adjacent surfaces.
  - C^1 continuity is achieved by using the strip tree to approximate derivatives.
  - Average twist-vectors are computed and the interior control points are adjusted.

Curve Blending



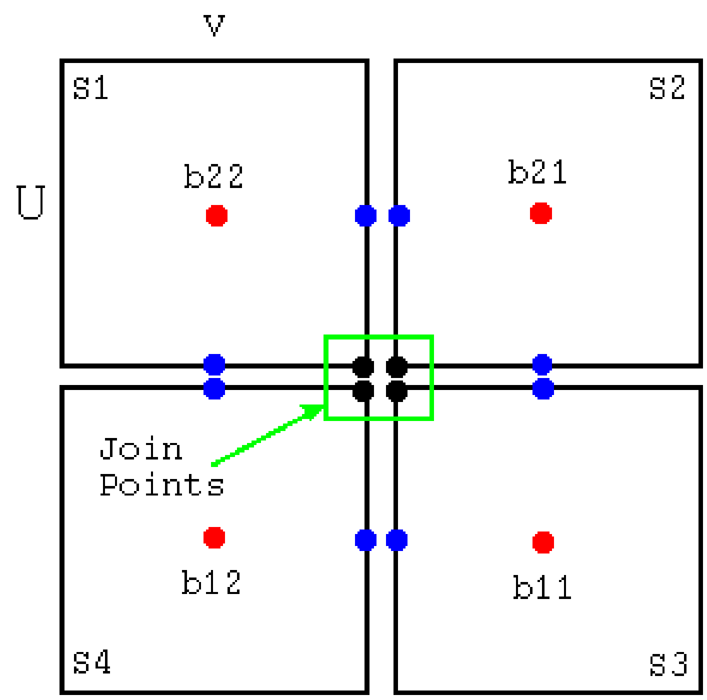
C^0 : p3 = q0

C^1 : (p2 p3) : (q0 q1) = A : B

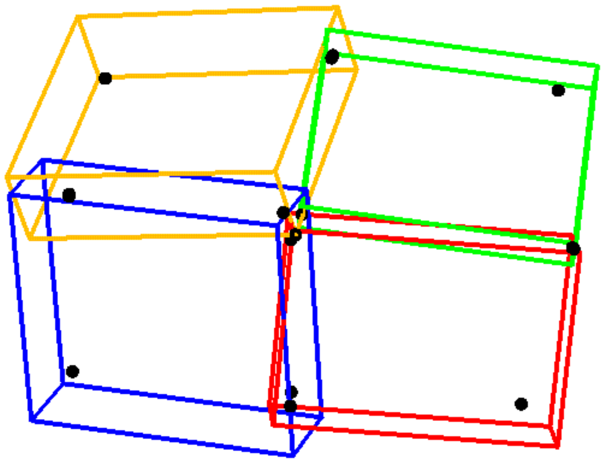
A and B are the parameter ranges of the curves

Surface Blending

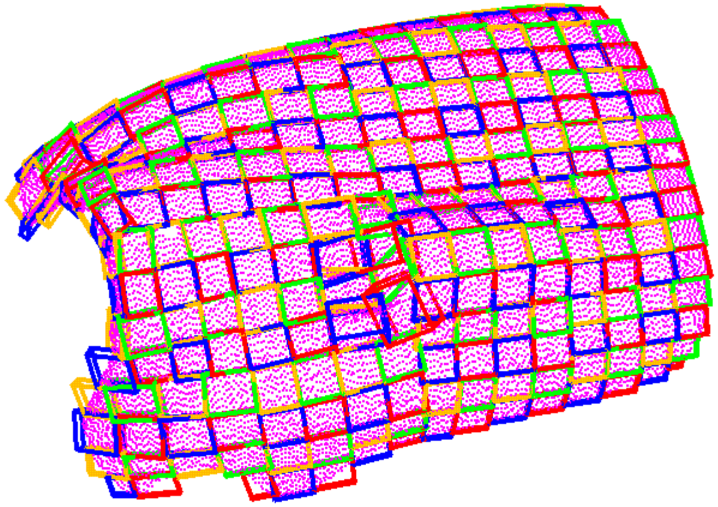
- C^0 continuity
  - Equate four black points each blue pair
- C^1 continuity
  - Derivatives boundary curves in u,v directions (adjust blue and black points)
  - Twist vectors (adjust red points)



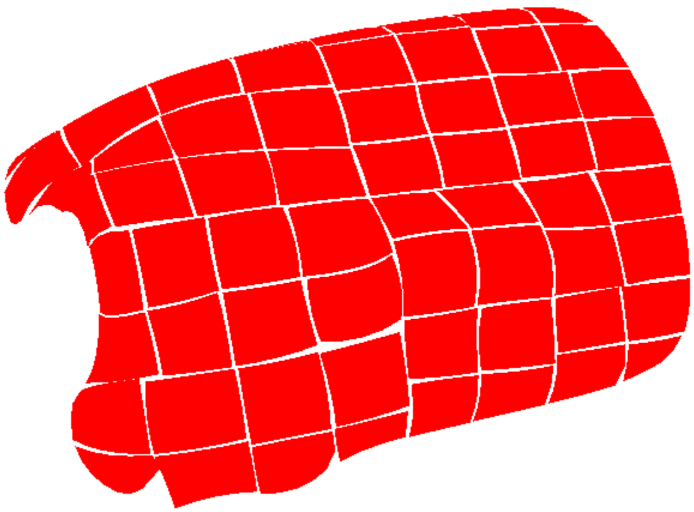
Box points for strip tree



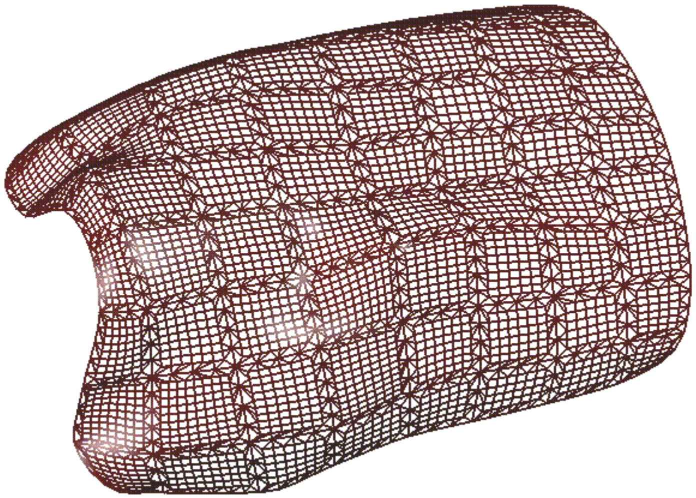
Strip tree after 4 subdivisions



Initial surfaces for the strip tree



Wireframe of final surfaces



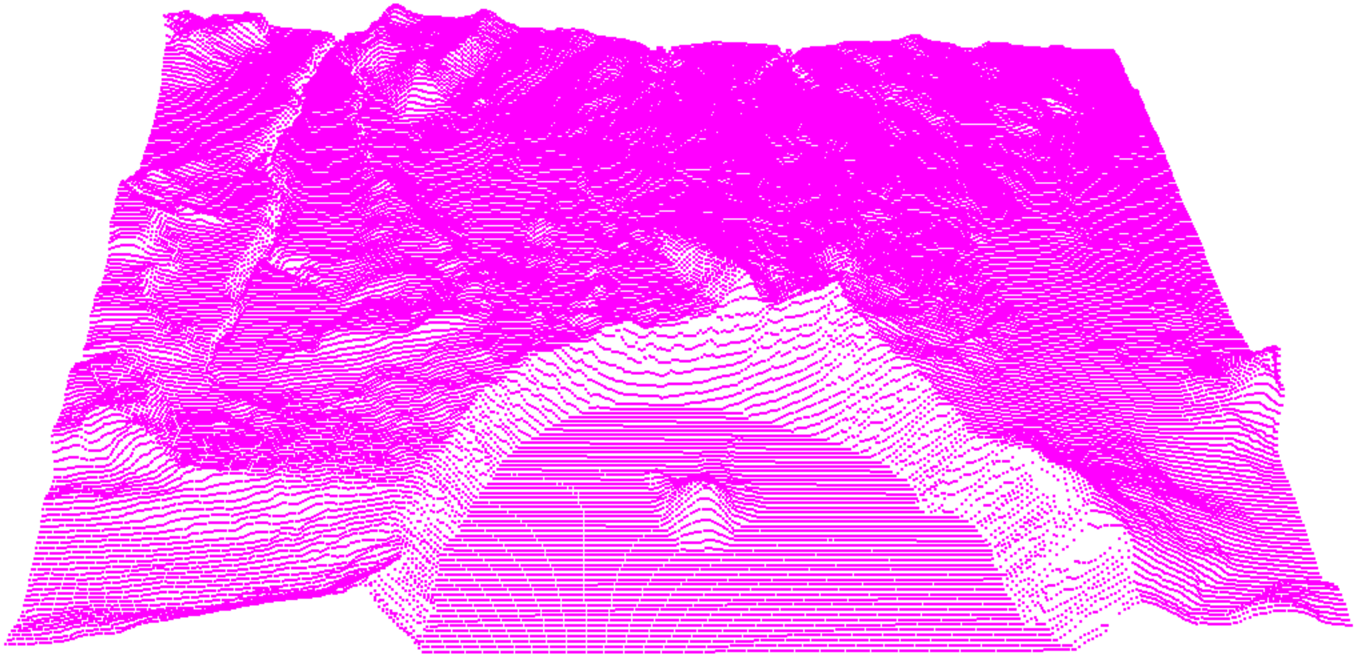
Error Calculations

- Error due to bounding box approximation
  - Size of the bounding box in the direction of the best-fit plane's normal vector.
- Error due to blending process.
  - Difference between the control points of the surfaces before and after blending.
- Total Error is the sum of these two errors.
- Upper bound of the actual deviation.

Performance Analysis

- For n data points, PCA runs in O(n).
- Expected O(n log n) time is necessary to completely subdivide the tree.
  - Assuming points are divided uniformly
- Worst case $O(n^2)$
  - A good subdivision point is necessary.
- Memory Usage
  - Storage of data points. (small n)
  - Size of strip tree. (large n)

Crater Lake



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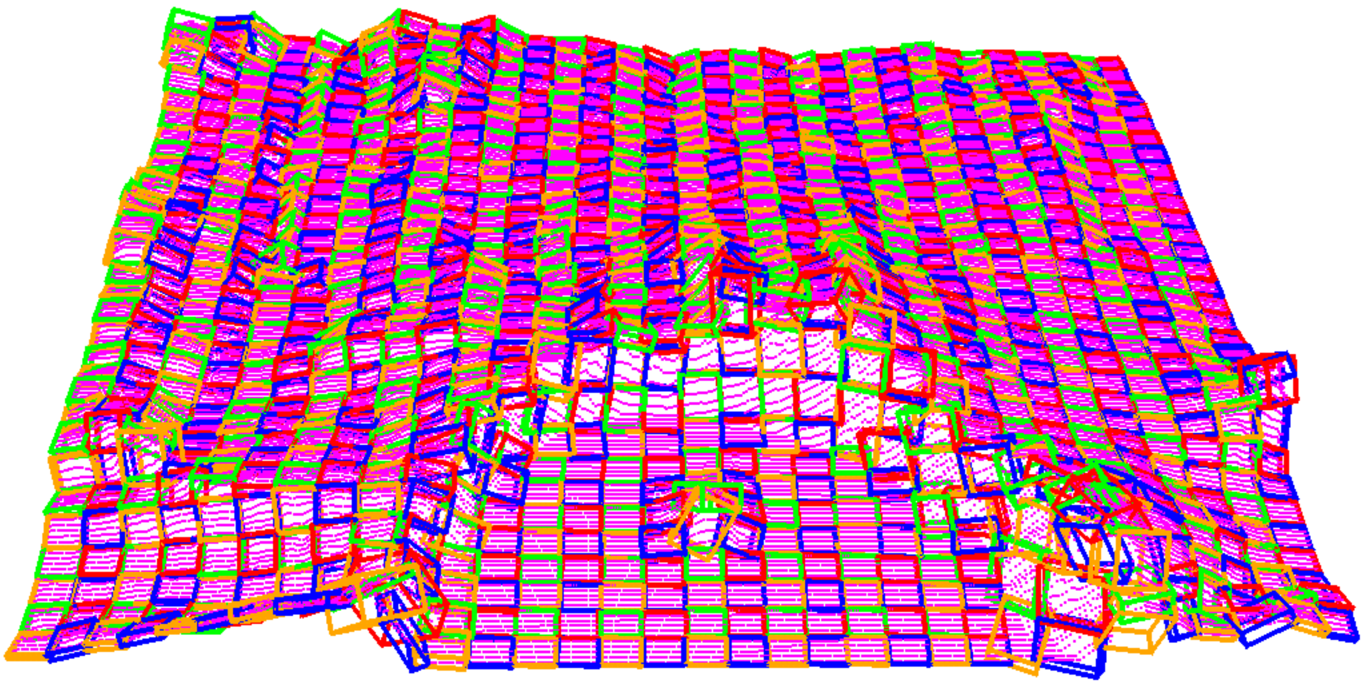
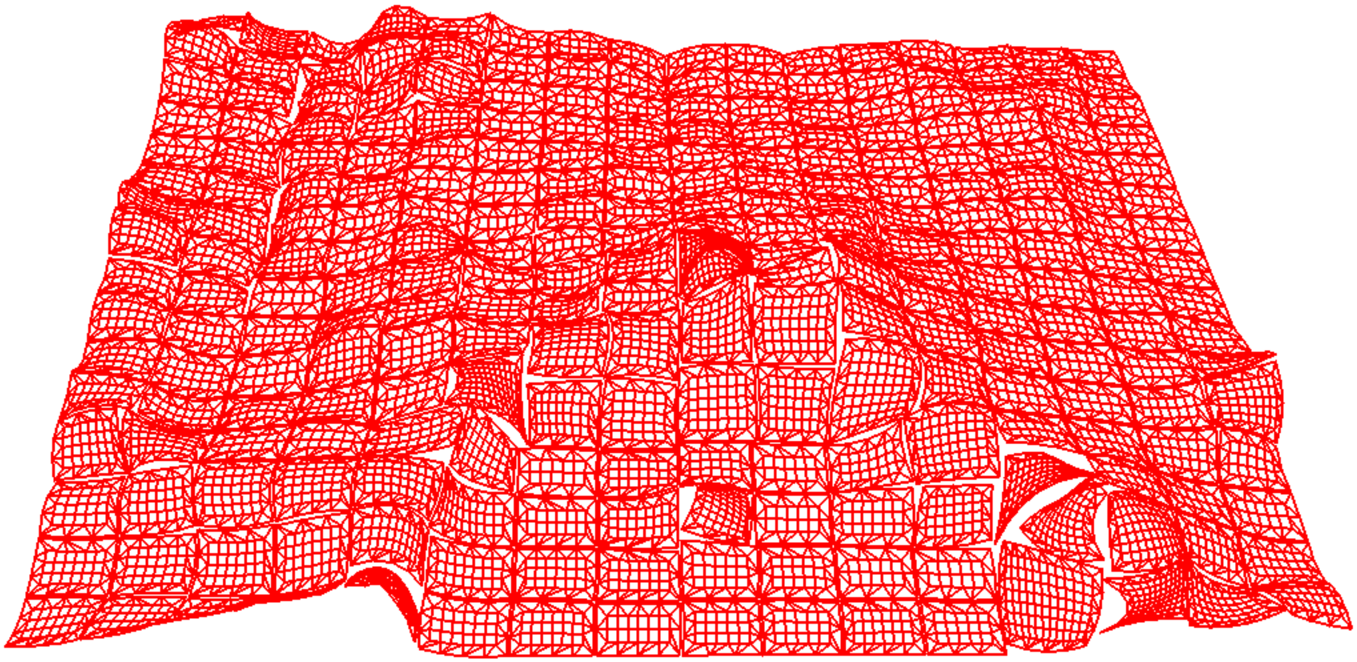
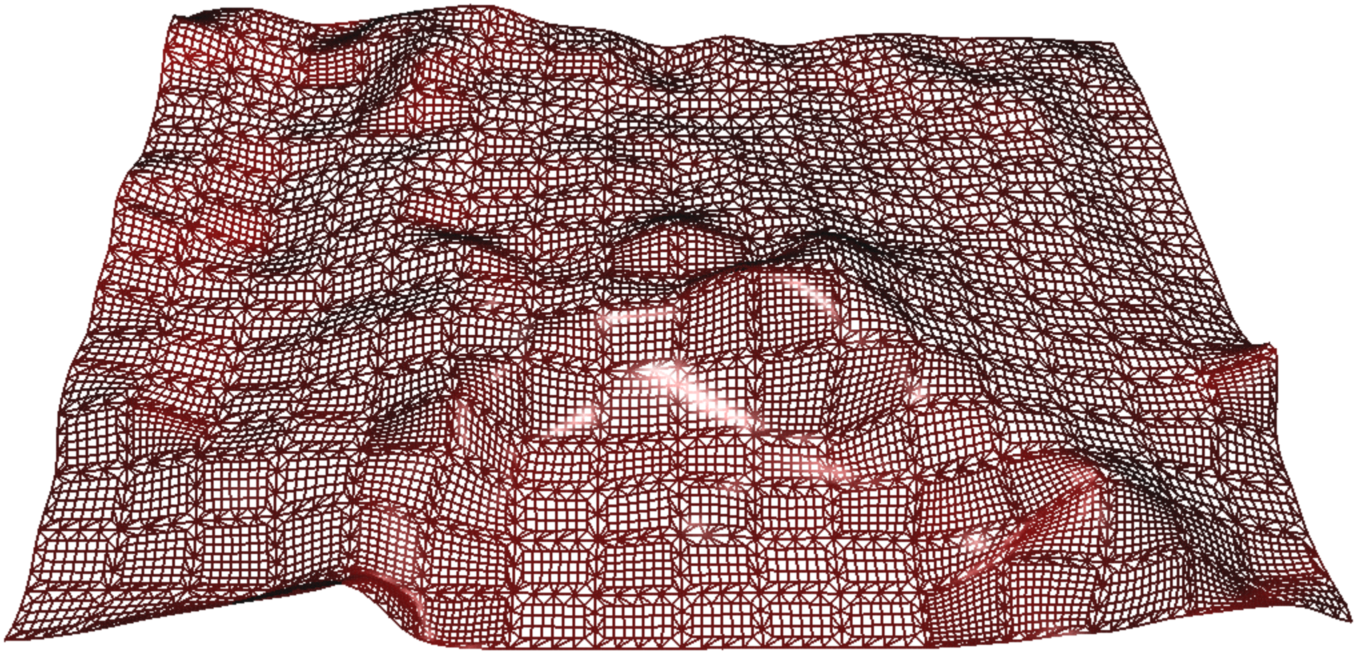
Strip Tree



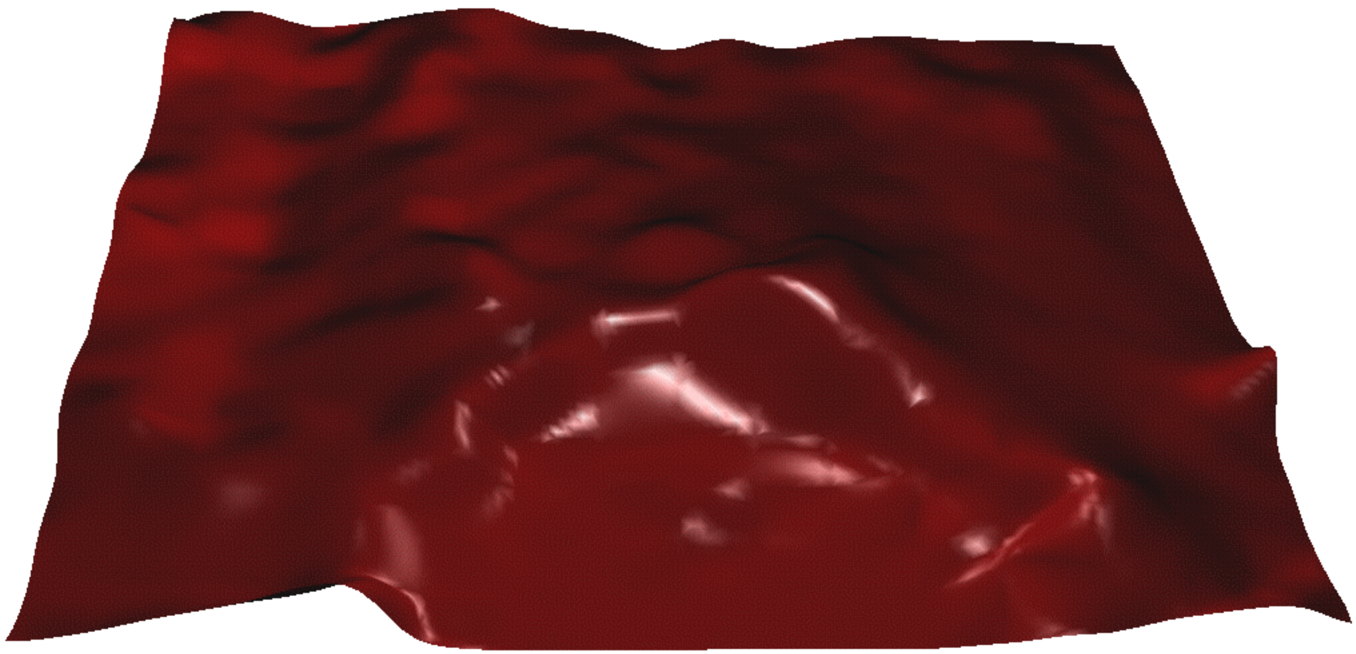
Initial Surfaces



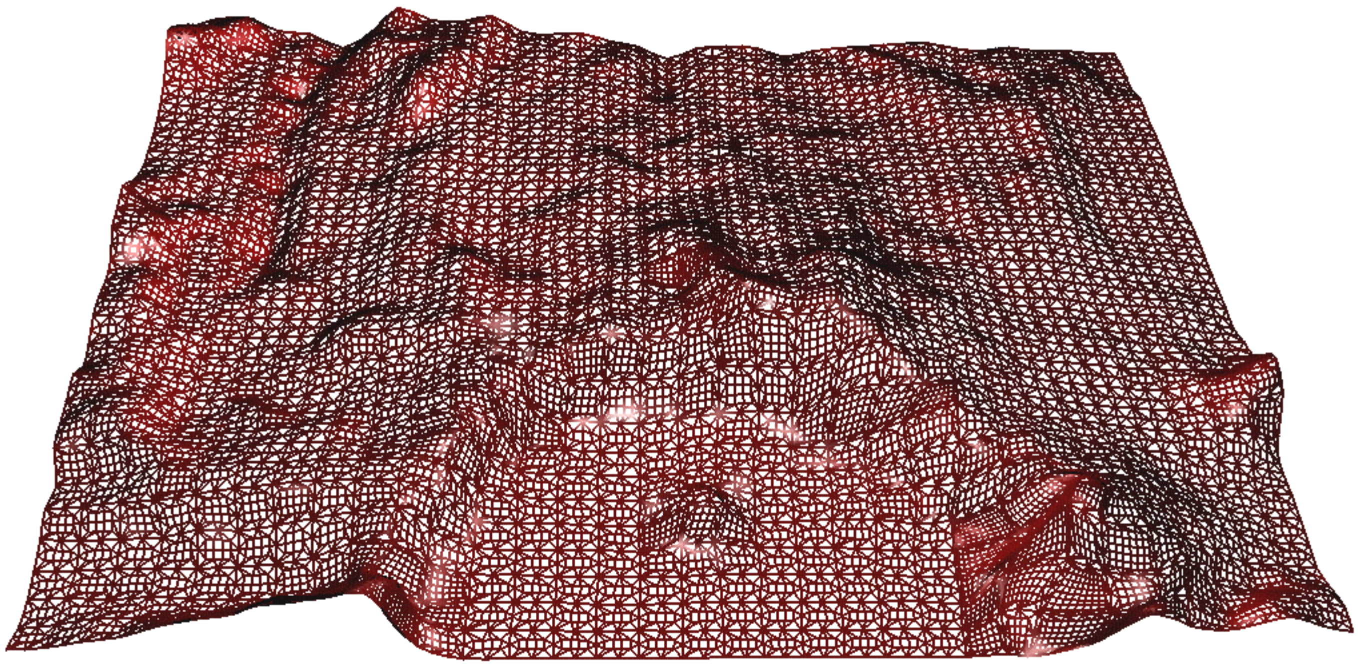
Wireframe Surfaces



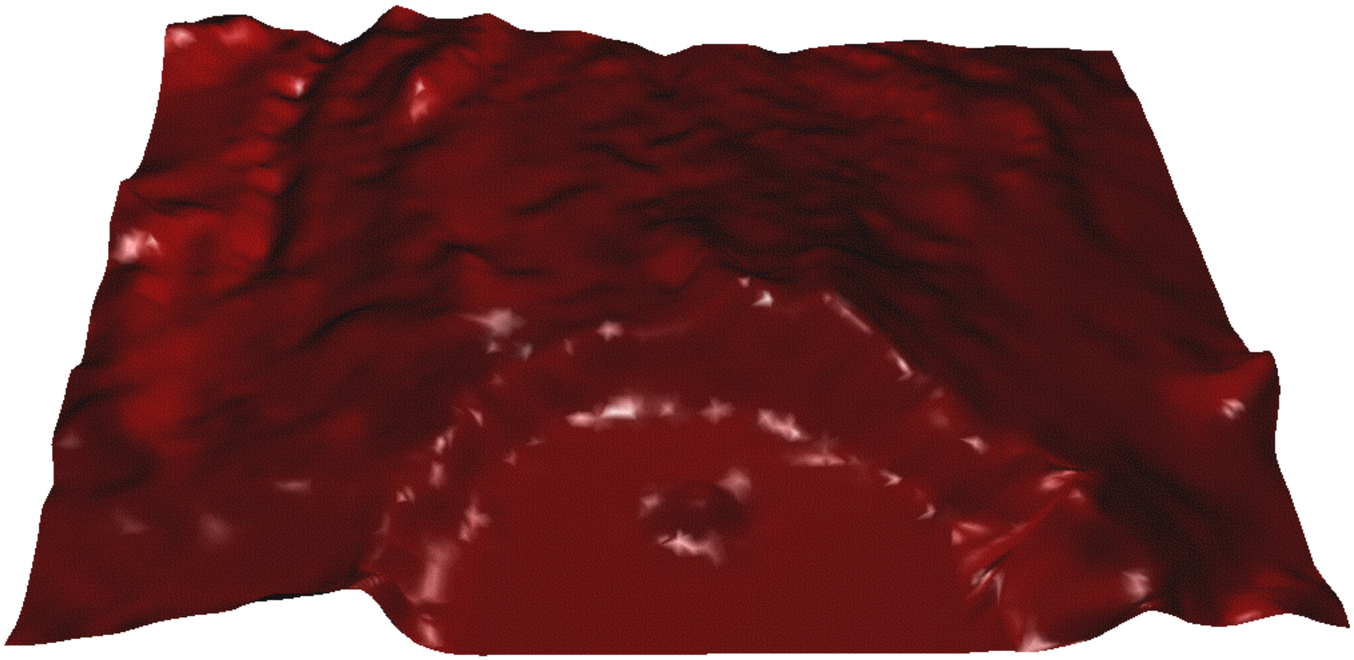
Final Surfaces

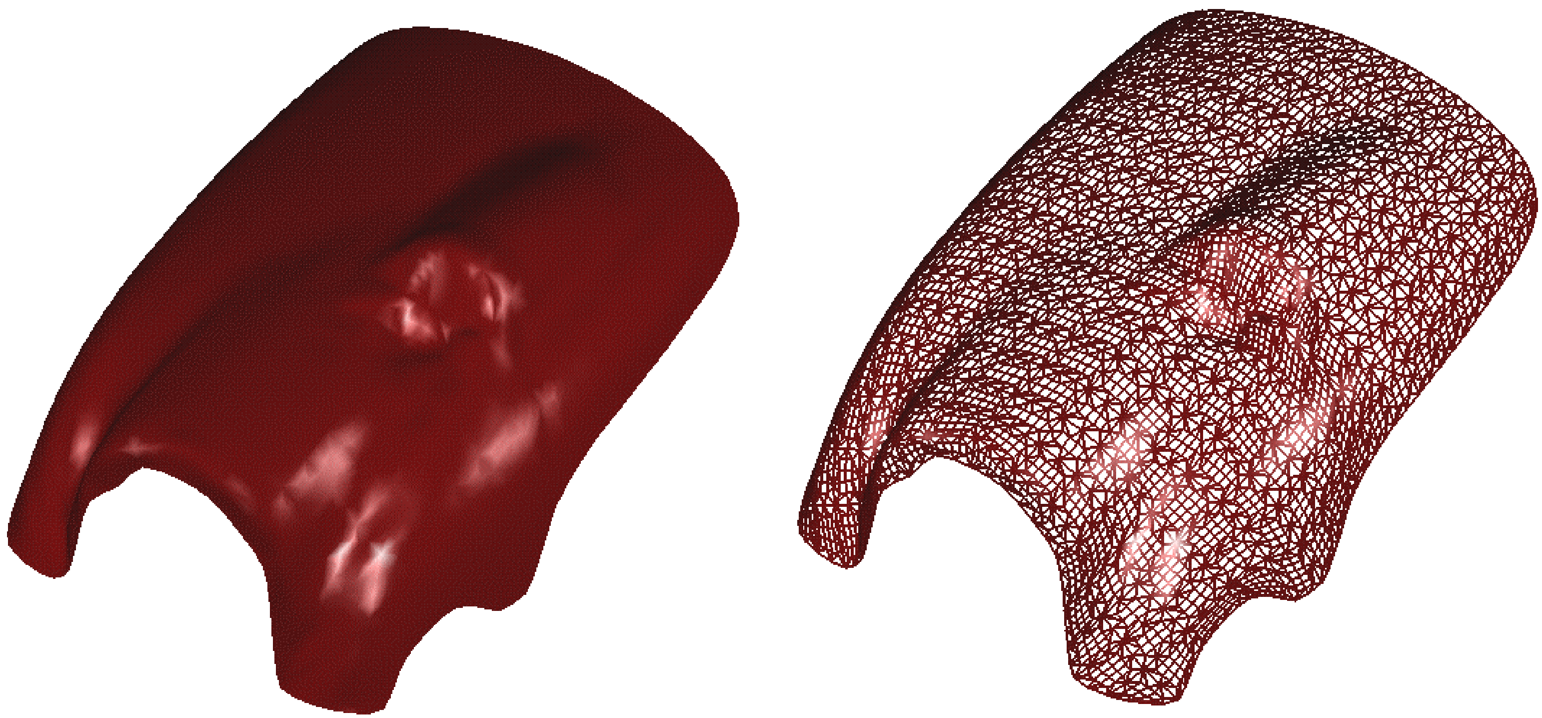


Wireframe Surfaces



Final Surfaces





Reconstruction of B-Spline Surfaces

Conclusions

- Introduced a strip tree structure for approximating scattered data.
- Construct surfaces that approximate the data.
- Works well on scattered data that represents a smooth surface.
- Does not work well for twisting or self intersecting surfaces.
- Limitations arise from the Quad-Tree structure in the strip tree.

Future Work

- Incorporate non-uniform subdivision
  – Adaptive refinement in regions with more data, and more complicated behavior.
- Use an adaptive fitting process for regions of higher curvature.
- Approximate sharper features, darts, cliffs, etc…
- Extend algorithm to operate on more topologically complex data sets.
  – Represent the strip tree nodes as Voronoi tiles.
  – Use strip tree to develop a “curve on surface” scheme.
  – Construct subdivision surface from strip tree nodes.

Acknowledgements

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