Ultrafast spectroscopy and control of correlated quantum materials

By

Bryan T. Fichera

B.S., University of Pennsylvania (2017)

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTUTITE OF TECHNOLOGY

May 2024

Authored by: Bryan T. Fichera
Department of Physics
May?, 2024

Certified by: Nuh Gedik
Donner Professor of Physics, Thesis supervisor

Accepted by: ?
? Professor of ?
? Chair of ?

Abstract

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Acknowledgements

Contents

Contents **v**

List of Figures **vii**

List of Tables **viii**

1 Amplitude-mode electromagnon in the spin-spiral multiferroic $CuBr_2$ **9**

1.1 Preface 9
1.2 Abstract 10
1.3 Introduction 10
1.4 Results 12
1.4.1 Equilibrium 12
1.4.2 Nonequilibrium 13
1.5 Discussion 19
1.6 Conclusion 19
1.7 Methods 20
1.8 Supplementary material 20
1.8.1 Electromagnons in $CuBr_2$ 20
1.8.2 Energy of the pseudo-Goldstone mode 21
1.8.3 Fits of time domain signals 22
1.8.4 Error bars in Fig. 1.4 23
1.8.5 Fits to static RASHG data 24
1.8.6 Excluded possibilities for observed results 25
1.8.6.1 $\delta\vec{P}||\hat{x}$ oscillation 25
1.8.6.2 Zone-folded acoustic phonons 26
1.8.6.3 Magnetic SHG 26
1.8.6.4 Multi-phason excitation 28

2 Concluding remarks **29**

Bibliography **33**

List of Figures

1.1 11
1.2 14
1.3 15
1.4 17
1.5 23
1.6 24
1.7 25
1.8 26
1.9 27

List of Tables

Chapter One

Amplitude-mode electromagnon in the spin-spiral multiferroic $CuBr_2$

1.1 Preface

This chapter is based on a manuscript intended for standalone publication and modified to fit the format of this thesis. It was coauthored by myself and Baiqing Lv (as co-first authors), along with Karna Morey, Zongqi Shen, Changmin Lee, Elizabeth Donoway, Alex Liebman-Peláez, Anshul Kogar, Takashi Kurumaji, Martin Rodriguez-Vega, Rodrigo Humberto Aguilera del Toro, Mikel Arruabarrena, Batyr Ilyas, Tianchuang Luo, Peter Müller, Aritz Leonardo, Andres Ayuela, Gregory A. Fiete, Joseph G. Checkelsky, Joseph Orenstein, and Nuh Gedik. It was coauthored by myself, Ajesh Kumar, and Baiqing Lv (as co-first authors), as well as Zongqi Shen, Karna Morey, Qian Song, Riccardo Comin, Todadri Senthil, and Nuh Gedik. Myself, Baiqing Lv, Zonqi Shen, and Karna Morey took the time-resolved second harmonic generation (tr-SHG) measurements, under the supervision of Nuh Gedik. Myself and Ajesh Kumar did the theory and analyzed the data, under the supervision of Nuh Gedik and Todadri Senthil. Qian Song grew the samples, under the supervision of Riccardo Comin. Myself and Ajesh Kumar wrote the paper, and Nuh Gedik supervised the project.

1.2 Abstract

Below a spontaneous symmetry breaking phase transition, the relevant collective excitations may be described as fluctuations in the amplitude and phase of the order parameter, referred to as amplitude and Goldstone modes, respectively. In solids, these modes may take on a different character than the equivalent excitations in particle physics due to the diverse vacuum states accessible in condensed matter. However, the amplitude mode in particular is quite difficult to observe experimentally as it decays quickly into the lower-energy Goldstone bosons and thus has a negligible lifetime in most systems. In this work, we report evidence for a novel amplitude mode in the multiferroic material $CuBr_2$, which shows up as a coherent oscillation in the time-resolved second harmonic generation signal upon excitation with a femtosecond light pulse. Since the spiral spin order in $CuBr_2$ induces a nonzero electric dipole moment in equilibrium, the amplitude mode—which is due to fluctuations in the amplitude of the on-site spin expectation value—is an electromagnon, and thus acquires an inversion quantum number of -1. This is in stark contrast to the amplitude boson of particle physics, which has even parity. Moreover, the excitation described here represents an entirely new type of electromagnon, distinct from the traditional electromagnon in linear spin wave theory which is due to the Goldstone mode. We argue that the amplitude mode in $CuBr_2$ acquires a nontrivial lifetime due to the combination of two features: (i) the quasi-1D nature of the material, and (ii) proximity at zero temperature to a quantum critical point separating the multiferroic ground state from a topological Haldane dimer phase.

1.3 Introduction

When the ground state of a given theory fails to respect one of its symmetries, that symmetry is said to have been be broken spontaneously[7, 22]. The low-energy excitations of this ground state may then be described as excitations of the order parameter either within the subspace of degenerate ground states, or perpendicular to it; these excitations are referred to as Goldstone and amplitude modes, respectively[23] (see Fig. 1.1(a)). This paradigm describes many fundamental phenomena in both particle physics and condensed matter, and the study of these modes has thus

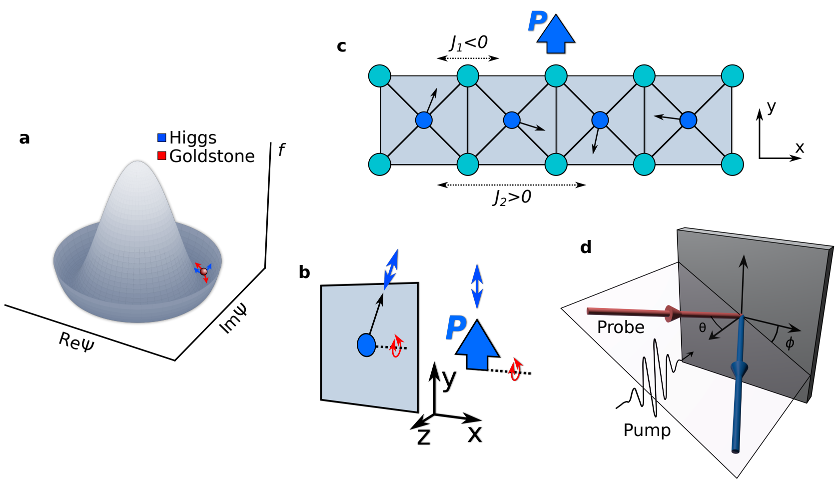

Figure 1.1: (a) Mexican hat potential with amplitude and Goldstone modes indicated. (b) $q = 0$ electromagnons in the quasi-1D spin-spiral in the spin (left) and charge (right) sectors. The amplitude and Goldstone modes are shown in blue and red, respectively. A second Goldstone mode, corresponding to uniform rotations of the spins about the z axis (which does not affect the polarization $\vec{P}$), is not shown. (c) Magnetic ground state of $CuBr_2$. The macroscopic polarization due to the spin order is depicted with a blue arrow. The axis labelled x is parallel to the nominal b axis of the crystal structure. (d) Schematic of the tr-SHG experimental geometry.

emerged as an essential pursuit in both contexts.

A rich interplay exists between these two fields due to the fact that in particle physics we are limited to a single theory (the standard model), but in condensed matter, the theory is determined by the particular system of interest and may differ dramatically from one material to another. Thus, various exotic species of amplitude modes may be studied simply by exploring different material systems with spontaneous symmetry breaking. An important example is in multiferroics, where it has been predicted[19, 20] that the amplitude mode of the magnetic order (corresponding to modulations in the amplitude of the on-site spin excitation value, see Fig. 1.1(b)) should couple to the macroscopic polarization as an electromagnon, and thus acquire a negative parity eigenvalue. This is not the case for the Higgs boson of the standard model, which is of even parity[2]. In addition to its connection to particle physics, the excitation described here is is also fundamentally different from the traditional electromagnon in multiferroics (which is due to the (pseudo-)Goldstone mode[16]), and is thus of great interest for magnetoelectric device applications. Unfortunately, like in particle physics, the amplitude mode in condensed matter is difficult to observe since it may quickly decay into Goldstone bosons upon excitation[14], and the existence of this mode in real multiferroic systems has thus remained an important open question.

In this work, we report evidence for this mode in $CuBr_2$ (a quasi-1D, spin-spiral multiferroic, see Fig. 1.1(c)), observed by launching a coherent oscillation of this mode with a near-infrared light pulse and measuring the induced modulations in the electric polarization using a delayed second harmonic generation (SHG) probe pulse (Fig. 1.1(d)). We find, as expected, that the mode modulates the macroscopic polarization only along the static ordering direction, and that the frequency of the mode decreases on approaching the critical temperature of the multiferroic order. These results provide conclusive evidence for the existence of this electromagnon in $CuBr_2$, solving a decade-old puzzle and paving the way for future study of the amplitude mode in novel condensed-matter contexts.

1.4 Results

1.4.1 Equilibrium

The low-energy spin Hamiltonian of $CuBr_2$ is well approximated by the so-called frustrated 1D XXZ spin chain, where localized spin-$1/2$ electrons interact ferromagnetically ($J_1 < 0$) with nearest neighbors but antiferromagnetically ($J_2 > 0$) with next-nearest neighbors (see Fig. 1.1(c)). When these interaction strengths are of comparable magnitude, the ground state is an incommensurate magnetic spiral, where the ordering wavevector is directed along the chain direction and has the appropriate magnitude so as to balance the two competing interaction terms. According to theory developed by Katsura, Nagaosa, and Balatsky[15], when spin-orbit coupling is strong this ground state induces an electric polarization at each site n given by

$$\vec{P}^n \propto \hat{x} \times (\vec{S}^n \times \vec{S}^{n+1}), \tag{1.1}$$

where we have set the chain direction to lie along $\hat{x}$. If the spins lie in the xy plane, then Eq. 1.1 induces a macroscopic electric polarization which is equal for each bond and directed purely along the $\hat{y}$ direction (Fig. 1.1(c)).

According to powder neutron diffraction, this spiral magnetic phase is realized in $CuBr_2$ below $T_c = 75\,\mathrm{K}$ [31**?** –33], with the propogation vector (in reciprocal lattice units) given by $\vec{k} = (0, k_y, 0.5)$, with $k_y \sim 0.235$[18, 33]. A pyroelectric current turns on at this temperature as well, indicating a macroscopic electric polarization density $|\vec{P}_0|$ of about $8\,\mu\mathrm{C/m}^2$ at $10\,\mathrm{K}$[33].

In a generalized Ginzburg-Landau theory, the SHG susceptibility tensor χ_{ijk} is linearly proportional to this polarization:

$$\chi_{ijk}(T < T_c) = \chi_{ijkl}(T > T_c)P_{0l} = \chi_{ijky}(T > T_c)P_0, \tag{1.2}$$

where χ_{ijkl} is some unknown tensor with the symmetry of the high temperature phase[25], and we have used that $\vec{P}_0||\hat{y}$. Fig. 1.2 shows the temperature dependence of the SHG intensity in $CuBr_2$, indicating a pronounced, order parameter-like enhancement of the SHG intensity at T_c due to Eq. 1.2. Note that other contributions to the SHG intensity due to e.g. magnetic dipole, surface electric dipole, and electric quadrupole terms are allowed above and below T_c and thus cannot explain the intensity increase below T_c. In addition, the c-type electric dipole term purely due to the magnetic order[3] is also not allowed by the magnetic point group of the incommensurate spin spiral (see Supplementary material, section 1.8.6.3).

1.4.2 Nonequilibrium

Having thus demonstrated that the SHG intensity is a direct probe of the electric polarization in $CuBr_2$, we proceed to investigate the low-energy collective excitations in this phase. To do so, we excite the sample with a $150\,\mathrm{fs}$ near-infrared pump pulse, and then probe the SHG intensity with a second pulse delayed in time by an amount Δt. We carry out this procedure in each of four independent polarization channels ($P_{\mathrm{in}}P_{\mathrm{out}}$, $P_{\mathrm{in}}S_{\mathrm{out}}$, $S_{\mathrm{in}}P_{\mathrm{out}}$, and $S_{\mathrm{in}}S_{\mathrm{out}}$, see section 1.7), where each channel probes a different linear combination of the tensor elements χ_{ijk}. The results are shown in Fig. 1.3. Two oscillations, with different dependencies on the

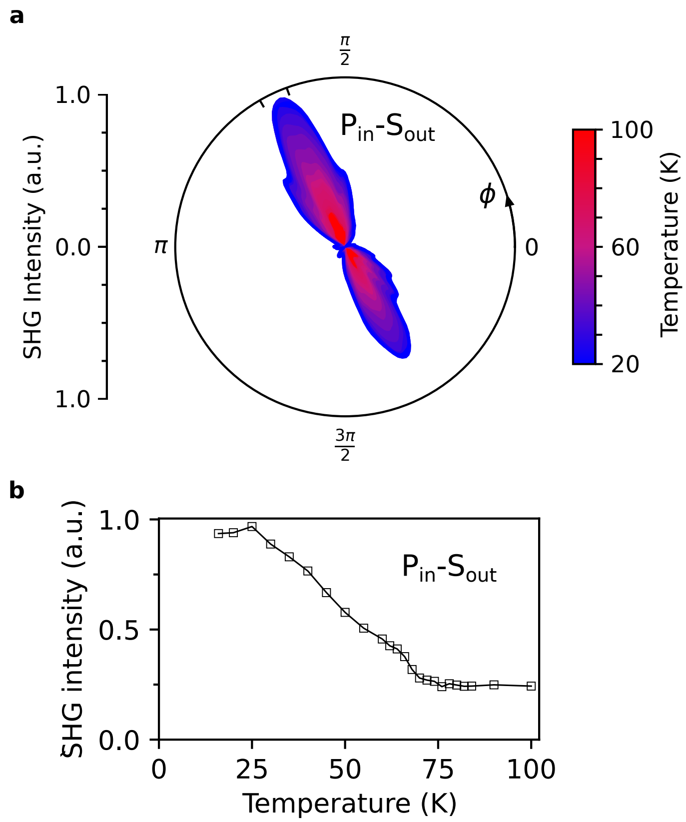

Figure 1.2: (a) SHG intensity as a function of temperature in the $P_{\rm in}S_{\rm out}$ polarization channel. (b) Integrated SHG intensity in the region near $\pi/2$ of **??** marked by the tick marks.

SHG polarization channel, may be observed: one high-frequency mode ($\nu \sim 0.23\,\mathrm{THz}$, $\hbar\omega \sim 1.0\,\mathrm{meV}$), which is only observed in the crossed polarization channels $P_{\rm in}S_{\rm out}$ and $S_{\rm in}P_{\rm out}$, and one low-frequency mode ($\nu \sim 0.05\,\mathrm{THz}$, $\hbar\omega \sim 0.20\,\mathrm{meV}$), which occurs in all polarization channels equally. Both of these are too low to be observed with typical THz or neutron spectroscopies, yet they are readily apparent in the tr-SHG data due to the pump-probe nature of the experiment.

To show that these two modes are directly related to the multiferroic transition at T_c, we measure the pump-induced change in the SHG intensity as a function of Δt for a series of temperatures approaching T_c

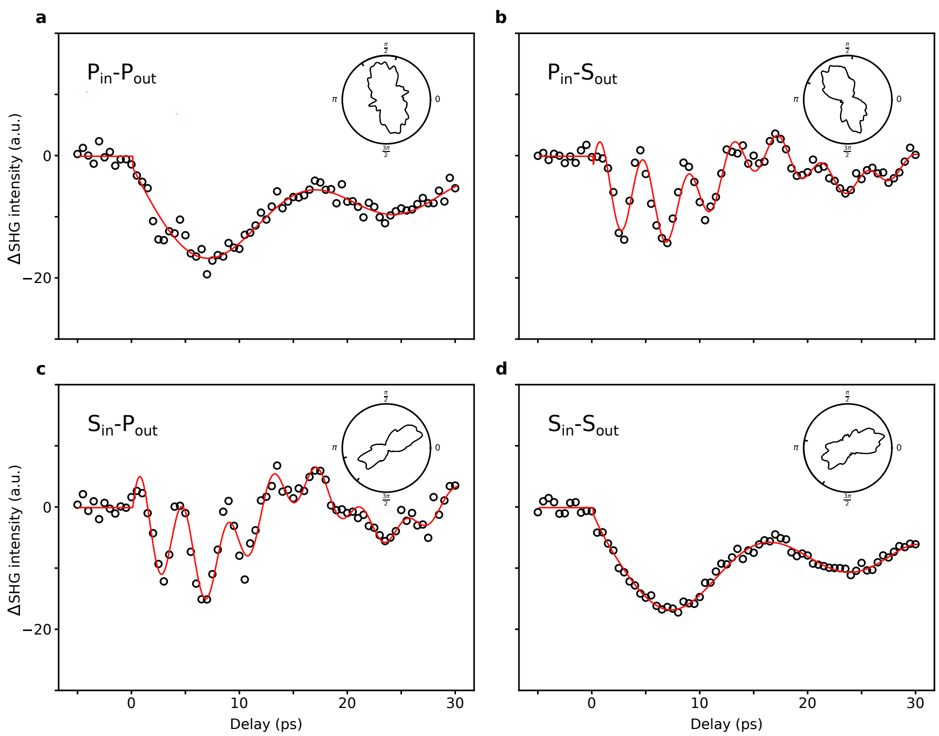

Figure 1.3: Pump-induced change in the SHG intensity at $15\,\mathrm{K}$ in the four polarization channels (a) $P_{\mathrm{in}}P_{\mathrm{out}}$ (b) $P_{\mathrm{in}}S_{\mathrm{out}}$ (c) $S_{\mathrm{in}}P_{\mathrm{out}}$, and (d) $S_{\mathrm{in}}S_{\mathrm{out}}$. Insets depict the static SHG intensity in each polarization channel. The time-domain signals are computed by performing an azimuthal integration at each delay of the full SHG pattern over the angles specified by the additional tick marks in each inset.

Fig. 1.4. By fitting the respective time-domain traces to damped harmonic oscillators (see Supplementary material, section 1.8.3), we can extract the natural frequency of each collective mode as a function of temperature (Fig. 1.4). Both modes exhibit a pronounced softening on approaching T_c, confirming their direct involvement in the multiferroic transition. We also note that both modes disappear above T_c, which is sensible given that the macroscopic polarization $\vec{P}_0$ also disappears above this temperature.

To clarify the microscopic origin of these polarization oscillations, we begin by performing density functional theory (DFT)+U and finite-displacement lattice dynamics calculations[] (see Supplementary material, section 1.8.6.2) to compare their energies with those of the zone-center phonon modes. The lowest zone-center optical phonon in this calculation appears at $7.4\,\mathrm{meV}$, in excellent agreement with Raman spectroscopy[30], and the calculated acoustic phonon branches (which agree with inelastic neutron scattering (INS)[30]) disperse too rapidly to form a zone-folded acoustic phonon mode at the Γ point with an energy low enough to match the frequencies observed in the tr-SHG experiment. Thus, the modes observed in Fig. 1.3 and **??** are not phonons. The only remaining possibility is that these modes are magnons of the incommensurate spin spiral, which imprint themselves on the polarization via Eq. 1.1; i.e., they are electromagnons.

In linear spin wave theory (LSWT), there is only one spin boson which couples to the polarization in the spiral magnetic phase of $CuBr_2$ (see Supplementary material, section 1.8.1); it is the so-called pseudo-goldstone mode of the magnetic order[16], which corresponds to a rotation of the spin plane about the chain direction (Fig. 1.1(b)). This mode has zero energy if the system is isotropic about the chain axis, but in the presence of an anisotropy term it acquires a finite energy. In $CuBr_2$, this energy is expected to lie around $1.25\,\mathrm{meV}$ (see Supplementary material, section 1.8.2), which is close to the value observed for our high-frequency oscillation ($1.0\,\mathrm{meV}$). Additionally, since this mode involves a rotation of the spin plane about the chain direction, the effect of this mode (from Eq. 1.1) on the polarization is to tilt the vector $\vec{P}_0$ into the $\hat{z}$ direction (Fig. 1.1(b)). Since the equilibrium polarization lies along $\hat{y}$, Eq. 1.2 implies that a canting of the polarization $\delta\vec{P}||\hat{z}$ involves new elements of the tensor $\chi_{ijkz}(T > T_c)$ which are not present in equilibrium. The result is that this mode may appear in different polarization channels with different magnitudes; we

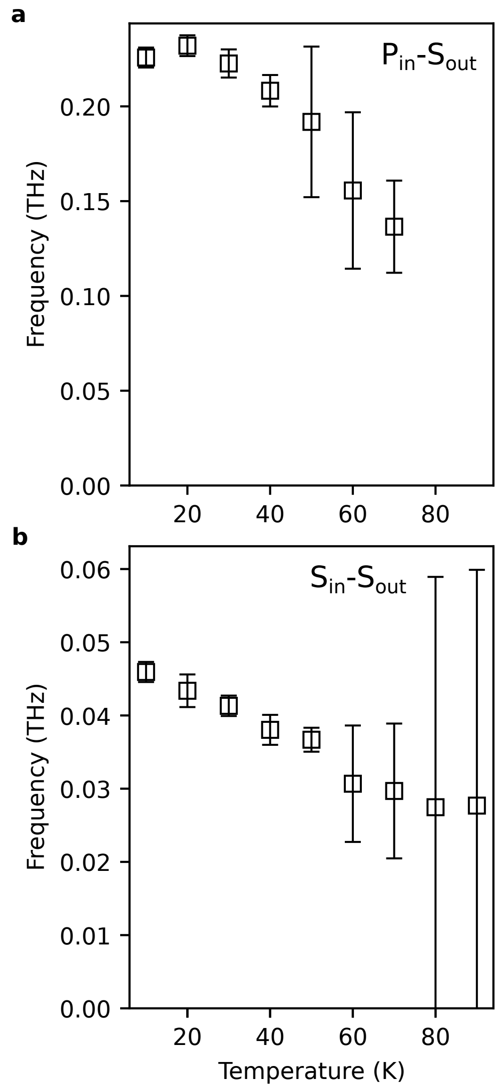

Figure 1.4: Temperature dependece of the frequencies extracted from the (a) $P_{\mathrm{in}}S_{\mathrm{out}}$ and (b) $S_{\mathrm{in}}S_{\mathrm{out}}$ time-domain signals (Supplementary material, section 1.8.3) in a damped harmonic oscillator model. Error bars denote $95\,\%$ confidence intervals estimated within a parametric bootstrap (see Supplementary material, section 1.8.4).

thus identify the fast, $0.23\,\mathrm{Thz}$ oscillation in Fig. 1.3 with this mode.

The observation of a *second* mode in the tr-SHG, however, is impossible to explain in LSWT, and represents the most striking aspect of this work. To understand the origin of this second mode, we note that there are only three normal modes of the polarization which occur at the Γ point in the Brillouin zone, corresponding to polarization oscillations $\delta\vec{P}$ along $\hat{x}$, $\hat{y}$, and $\hat{z}$. Since the $\delta\vec{P}||\hat{z}$ mode is already accounted for by the pseudo-goldstone mode, that leaves only $\delta\vec{P}||\hat{x}$ and $\delta\vec{P}||\hat{y}$ as possibilities. The $\delta\vec{P}||\hat{x}$ mode does not couple to the spin order in this compound[16], and in any case is not observable in the geometry of our experiment (see Supplementary material, section 1.8.6.1). Thus, the only polarization oscillation which is consistent with the observation of a second mode is an oscillation with $\delta\vec{P}||\hat{y}$. Since the equilibrium polarization is also directed along $\hat{y}$, this mode simply corresponds to an oscillation in the total amplitude of the polarization, and is thus expected to modulate the overall SHG intensity irrespective of the polarization channel – in excellent agreement with Fig. 1.3, which shows the low-frequency oscillation appearing in all four polarization channels with equal magnitude.

Naively, electromagnons with $\delta\vec{P}||\hat{y}$ do not exist in LSWT. The key insight, however, is that LSWT neglects dynamics associated with the magnitude of the onsite spin expectation value. By Eq. 1.1, such dynamics change the magnitude of the induced polarization only, not its direction; i.e., they induce oscillations $\delta\vec{P}||\hat{y}$. In fact, it is possible to show (see Supplementary material, section 1.8.1) that oscillations along $\hat{y}$ of the induced polarization *necessarily* involve modulations in the amplitude of the onsite spin excpectation value; that is, the only mode which couples to $\delta\vec{P}||\hat{y}$ is the amplitude mode of the magnetic spiral. Naturally, this mode should soften on approaching T_c, in agreement with Fig. 1.4. The $0.05\,\mathrm{THz}$ oscillation observed in our experiment (which is of similar energy to the amplitude mode in non-ferroelectric quantum magnetsHong et al. [13]) is therefore direct evidence for this mode in $CuBr_2$, with the additional information that it couples to the electric polarization (i.e., it is an electromagnon) via Eq. 1.1.

1.5 Discussion

Let us make two important remarks about this mode in $CuBr_2$. First, we note that this mode is fundamentally distinct from the amplitude mode in non-multiferroic magnets – since the multiferroic order breaks inversion symmetry, the amplitude mode of this phase has a parity quantum number $l = -1$ rather than $+1$. The static polarization $\vec{P}_0$ which appears at T_c can thus be viewed as arising from the odd-parity nature of its amplitude mode. Second, we remark that the amplitude mode in $CuBr_2$ can in principle decay – as in non-multiferroic magnets – quite rapidly into the Goldstone modes of the magnetic order (which in the spin-spiral phase of Fig. 1.1(c) are gapless and correspond to uniform rotations of each spin about the $\hat{z}$ direction), and thus should not exist as a well-defined quasiparticle unless these decay channels are quenched by some mechanism.

In non-multiferroic magnets, two such mechanisms have been identified. First, the amplitude mode may be stabilized by bringing the system close to a quantum critical point (QCP)[13, 14, 24, 27], which suppresses the Goldstone bosons and thus stabilizes the amplitude mode. A second option is to lower the dimensionality[1, 4, 8, 26, 34]; in 1D, enhanced fluctuations weaken the long range magnetic order and also reduce the spectral weight of the Goldstone bosons[34]. In $CuBr_2$, not only is the system fundamentally one-dimensional, but is thought also to lie in close proximity to a zero-temperature QCP[10] separating the spiral phase considered here and a paraelectric Haldane dimer phase. Both of these mechanisms thus likely contribute to stabilizing the amplitude electromagnon in $CuBr_2$.

1.6 Conclusion

To summarize, we have presented evidence of a novel electromagnon arising from the amplitude mode of the spiral magnetic order in $CuBr_2$. This mode appears alongside the pseudo-Goldstone mode in the tr-SHG data as a low-frequency oscillation in the longitudinal component of the electric polarization, which softens on warming close to T_c. Looking forward, we note that the two mechanisms we identified for stabilizing this mode in $CuBr_2$ – low dimensionality and potential proximity to a QCP – are not necessarily unique to this material. Thus, the amplitude electromagnon presented here may in fact be a common feature of 1D multiferroics, and its observation could indicate a wealth of new opportunities to explore the amplitude mode of particle physics in novel condensed-matter contexts.

1.7 Methods

Tr-SHG measurements were carried out using a fast-rotating optical grating setup described previously[9, 12, 29]. $100\,\mathrm{fs}$ ultrashort pulses from a regeneratively amplified $5\,\mathrm{kHz}$ Ti:Sapphire laser were used to pump an optical parametric amplifier (OPA), producing $1300\,\mathrm{nm}$ (Fig. 1.3) or $1650\,\mathrm{nm}$ (Fig. 1.4) pump pulses which were delayed with an optical delay line and focused at normal incidence to a $300\,\mathrm{um}$-diameter spot on the sample. The pump fluence was $\sim 1\,\mathrm{mJ}\cdot\mathrm{cm}^{-2}$ for all measurements. A small portion of the Ti:Sapphire output was used for the SHG probe experiment, the output of which was spectrally filtered with a $400\,\mathrm{nm}$ bandpass filter, collected by a photomultiplier tube, filtered with a lock-in amplifier, and correlated with the plane of incidence angle using an optical rotary encoder. To measure the pump-induced change in the SHG signal, the pump pulses were chopped at a frequency of $2.5\,\mathrm{kHz}$, and the lock-in amplifier was set to that frequency so as to measure $I_{\mathrm{pump+probe}} - I_{\mathrm{probe}}$. For the pump-probe rotational anisotropy SHG (RA-SHG) measurements, the plane of incidence was rotated while the delay stage was moved and the polarizers were controlled automatically using homebuilt polarization rotators described in Morey et al. [21]. For the single-angle tr-SHG measurements, the plane of incidence was parked at the angle which maximized the static SHG intensity in the respective polarization channel.

1.8 Supplementary material

1.8.1 Electromagnons in $CuBr_2$

The observed co-existence of spiral magnetic order and ferroelectricity is due to the spin-orbit coupling enabled interaction term between the spins

$\vec{S}$ and the electronic polarization $\vec{P}$ []:

$$H_{s-P} = \lambda \sum_i \vec{P}_i \cdot (\hat{x} \times \vec{S}_i \times \vec{S}_{i+1}) \tag{1.3}$$

The ordered state for $T < T_N$ is a multiferroic with spontaneous polarization

$$\left\langle \vec{P}_i \right\rangle = P_0 \hat{y} \tag{1.4}$$

and spiral spin ordering

$$\left\langle \vec{S}_i \right\rangle \equiv \vec{S}_{0,i} = S_0 \left(\cos(\vec{Q} \cdot \vec{R}_i)\hat{x} + \sin(\vec{Q} \cdot \vec{R}_i)\hat{y} \right), \tag{1.5}$$

where $\vec{Q}$ is the spin-ordering wavevector and $\vec{R}_i$ are the spatial coordinates of the Cu atoms. Let us consider fluctuations about this ordered state and ask which fluctuations are detectable via SHG. Representing fluctuations in the polarization by $\delta\vec{P}_i$ and spin by $\delta\vec{S}_i$, we get the following fluctuation Hamiltonian:

$$H^f_{s-P} = \lambda \sum_i \delta\vec{P}_i \cdot (\hat{x} \times \delta\vec{S}_i \times \vec{S}_{0,i+1} + \hat{x} \times \vec{S}_{0,i} \times \delta\vec{S}_{i+1}) + \mathcal{O}(\delta\vec{P}^2, \delta\vec{S}^2). \tag{1.6}$$

Expanding the spin fluctuations along all directions, we find that they couple only to polarization fluctuations along $\hat{y}$ and $\hat{z}$. Focusing on zero-momentum polarization fluctuations (since they are sensitive to SHG),

$$\begin{aligned} H^f_{s-P} =& i \sin(\vec{Q} \cdot \vec{a}) \delta P_z(\vec{q}=0) \left(\delta S_z(-\vec{Q}) - \delta S_z(\vec{Q}) \right) \\ &+ \sin(\vec{Q} \cdot \vec{a}) \delta P_y(\vec{q}=0) \left(-\delta S_x(-\vec{Q}) + i\delta S_y(-\vec{Q}) - \delta S_x(\vec{Q}) - i\delta S_y(\vec{Q}) \right) \\ &+ \mathcal{O}(\delta\vec{P}^2, \delta\vec{S}^2) \end{aligned} \tag{1.7}$$

where $\vec{a}$ is the lattice vector along the chain. Transverse polarization fluctuations $\delta P \sim \hat{z}$ couple to a uniform rotation of the spin-plane about the x axis. These are the electromagnons discussed in Katsura et al. [16]. The longitudinal fluctuations, on the other hand, couple to longitudinal fluctuations of the magnetization on each site.

1.8.2 Energy of the pseudo-Goldstone mode

An expression for the frequency of the pseudo-Goldstone mode in the presence of an easy-plane anisotropy is given by Katsura et al. [16] as

$$\omega_- = \sqrt{A(Q)B(Q)}, \tag{1.8}$$

where

$$A(q) = 2S\left[\frac{2J(Q) - J(Q+q) - J(Q-q)}{2}\right], \tag{1.9}$$

$$B(q) = 2S\left[J(Q) - J(q) + D\right], \tag{1.10}$$

$$J(q) = 2\left[J_1\cos(qa) + J_2\cos(2qa)\right], \tag{1.11}$$

D is the anisotropy energy, and $2S$ is the amplitude of the spin in units of μ_B.

Using $Q = 0.235\,\mathrm{rlu}$[33], $J_1 = 8.8\,\mathrm{meV}$[17], $J_2 = -22.2\,\mathrm{meV}$[17], $D = 0.15\,\mathrm{meV/Cu}$[18], and $2S = 0.38$[18]), we have

$$\omega_- = 1.3\,\mathrm{meV} \tag{1.12}$$

in good agreement with the experiment.

1.8.3 Fits of time domain signals

Time-domain plots corresponding to the frequencies in Fig. 1.4 are illustrated in Fig. 1.5. Each plot is a least-squares fit of the data to a damped harmonic oscillator model

$$I_p^{\mathrm{SHG}}(t,\theta) = P_p^0\,\delta P_p(t,\theta) + [\delta P_p(t)]^2, \tag{1.13}$$

where

$$\delta P_p = A_p e^{-\gamma_p t}\cos\left(\sqrt{(2\pi\nu_p)^2 - \gamma_p^2}\,t + \psi_p\right), \tag{1.14}$$

$p \in \{P_{\mathrm{in}}S_{\mathrm{out}}, S_{\mathrm{in}}S_{\mathrm{out}}\}$, and θ denotes the set of free parameters to be estimated.

The main conclusion of these fits is that the frequency of the two modes (most notably, the low-frequency $S_{\mathrm{in}}S_{\mathrm{out}}$ mode) soften on approaching T_c. This may also be seen heuristically from the time-domain signals without doing any fits. Fig. 1.6 shows an enlarged (i.e., scaled to account for the

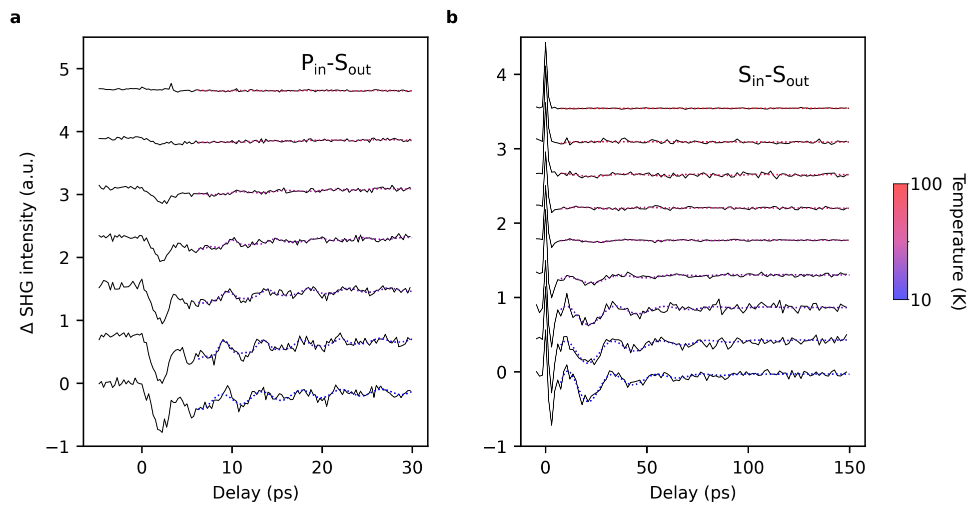

Figure 1.5: Time-domain signals corresponding to (a) Fig. 1.4(a) and (b) Fig. 1.4(b). Dashed lines depict least-squares fits to the data in a damped harmonic oscillator model, see Supplementary material, section 1.8.3.

decrease in signal amplitude) view of the $S_{\text{in}}S_{\text{out}}$ signal for three temperatures below T_c, showing a clear decrease in the oscillation frequency at high temperature. Fig. 1.7 shows an alternative fit where the frequency parameter ν_{SS} is constrained to be constant as a function of temperature, showing that our data is not consistent with a hypothetical model where the frequency shift with temperature in Eq. 1.14 is only attributed to the damping term γ_{SS}.

1.8.4 Error bars in Fig. 1.4

In this section, we describe how the uncertainties in the least square estimates of the frequency parameter ν of Eq. 1.14, which are depicted as a function of temperature in Fig. 1.4, were calculated from the the time-domain signals in Fig. 1.5. For each temperature and polarization channel, a Levenberg-Marquardt (LM) algorithm was used to find the

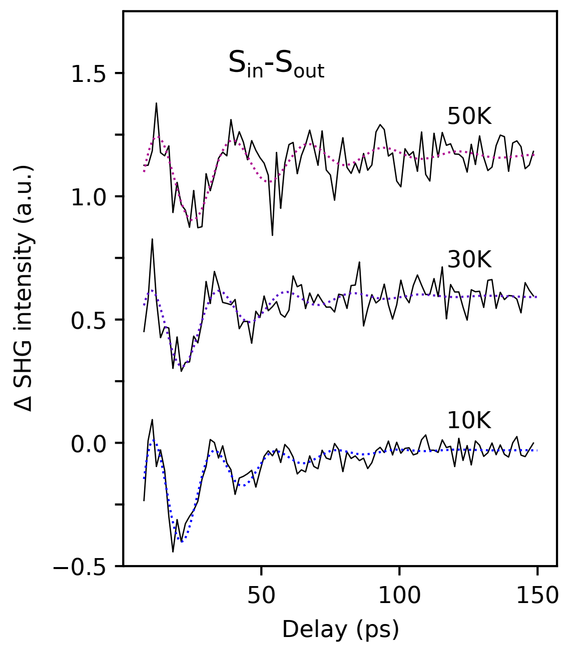

Figure 1.6: Rescaled $S_{\text{in}}S_{\text{out}}$ time-domain signals (see Fig. 1.5(b)) for select temperatures below T_c.

minimum θ_0 of the objective function

$$f_p(\theta) \propto \sum_{n=0}^{N-1} \left(I_p^{\text{SHG}}(t_n, \theta) - I_{p,n}^{\text{SHG}}\right)^2, \tag{1.15}$$

where $\{(t_n, I_{p,n}^{\text{SHG}}), n \in (0, 1, \dots, N-1)\}$ are the data points in Fig. 1.5, and we have assumed the noise level is independent of delay. The uncertainty in each parameter is estimated within a parametric bootstrap[5]: for each temperature, 1000 bootstrap samples are generated by adding noise (normally distributed, with variance given by the variance of data points at long times where the signal is constant) to the LM estimate $I_p^{\text{SHG}}(t_n, \theta_0)$. For each bootstrap sample s, an estimate θ_s is computed by minimizing Eq. 1.15 as above, and the $95\,\%$ confidence interval reported in Fig. 1.4 is taken to be 1.96 times the standard deviation of the distribution $\{\theta_s - \theta_0\}$.

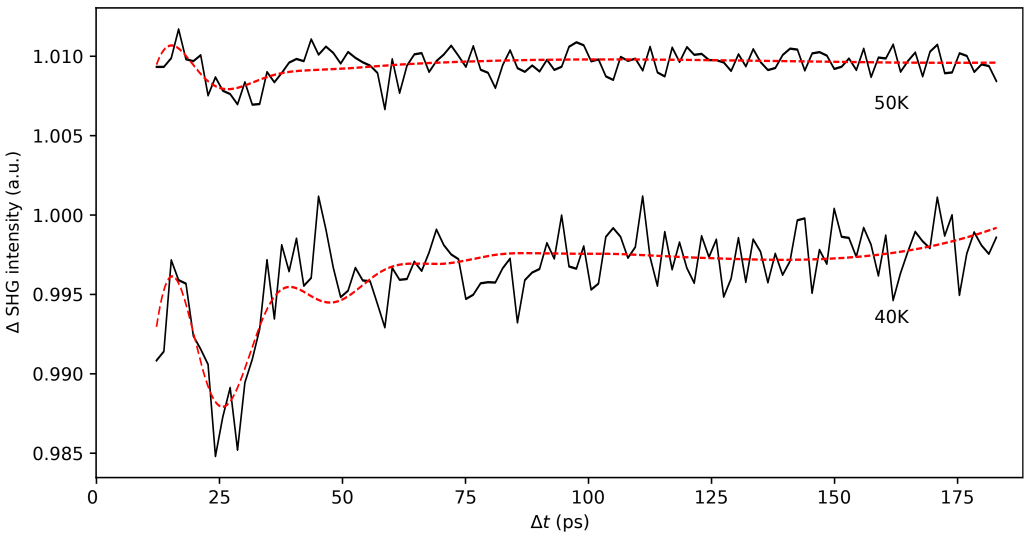

Figure 1.7: $S_{\text{in}}S_{\text{out}}$ time-domain signals (see Fig. 1.5(b)) for select temperatures approaching T_c. Dashed lines depict least-squares fits to the data in a variant of Eq. 1.14 where ν_{SS} is constrained not to vary with temperature.

1.8.5 Fits to static RASHG data

The static SHG intensity was fit by Eq. 1.18. The susceptibility tensor was taken to be the form Eq. 1.19, plus an additional C_1 component (likely due to surface adsorbates). The result is shown in Fig. 1.8.

1.8.6 Excluded possibilities for observed results

1.8.6.1 $\delta\vec{P}||\hat{x}$ oscillation

Without loss of generality, let the maximum of the SHG in $S_{\text{in}}P_{\text{out}}$ occur when the incoming electric field is along $\hat{x}$. Then, we have:

$$\Delta I_{\text{SP}}^{\text{SHG}} \propto |\hat{e}_i^{\text{out}}\chi_{ijkl}\hat{e}_j^{\text{in}}\hat{e}_k^{\text{in}}[P_{0l} + \delta P_l]|^2 - |\hat{e}_i^{\text{out}}\chi_{ijkl}\hat{e}_j^{\text{in}}\hat{e}_k^{\text{in}}P_{0l}|^2 \tag{1.16}$$

with $\hat{e}_i^{\text{in}}||x$ and $P_{0l}||y$, we have

$$\Delta I_{\text{SHG}} \propto 2\hat{e}_i^{\text{out}}\hat{e}_j^{\text{out}}\chi_{ixxy}\chi_{jxxx}P_{0y}\delta P_x + \hat{e}_i^{\text{out}}\hat{e}_j^{\text{out}}\chi_{ixxx}\chi_{jxxx}\delta P_x\delta P_x \tag{1.17}$$

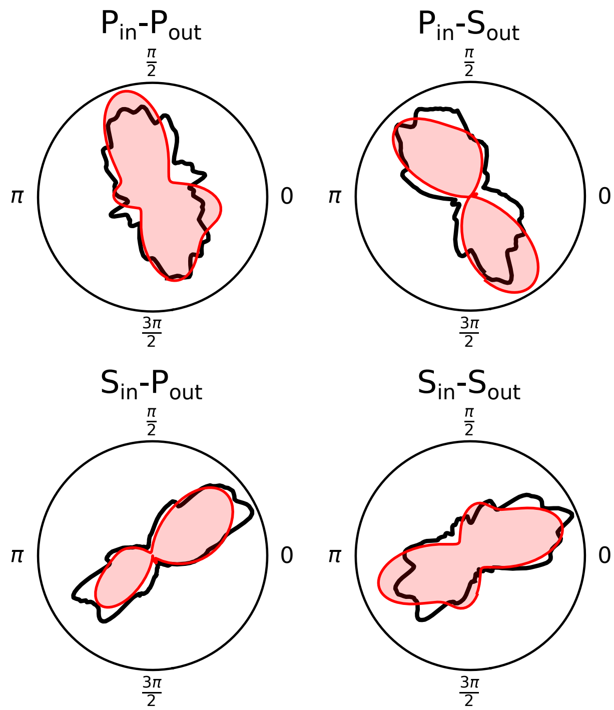

Figure 1.8: Fits (red) to static RA-SHG data (black) depicted in Fig. 1.3.

Since we are in $S_{\rm in}P_{\rm out}$, $\hat{e}_i^{\rm out} \perp x$; thus, Eq. 1.17 involves the tensor elements χ_{yxxx} and χ_{zxxx}. Both of these elements are zero due to the $x \to -x$ mirror symmetry. Thus, the $\delta\vec{P}||\hat{x}$ oscillation is not visible in our experiment.

Additionally, since the $\delta\vec{P}||\hat{x}$ mode does not couple to the spin order in this compound[16], its frequency should be far above the frequencies observed in our experiment (which are determined by the energy scales of the spin Hamiltonian).

1.8.6.2 Zone-folded acoustic phonons

Phonon band structure calculations were carried out using the finite displacement method[28] with a distance of $0.01\,\text{Å}$ within a $3 \times 3 \times 3$ supercell. Forces were calculated via the DFT-D2 method[11] and LDA+U method[6] ($U_{\rm Cu} = 3\,\rm eV$) using a $7 \times 7 \times 5$ k-mesh with 122 irreducible k-points and a plane-wave cutoff energy of $100\,\rm eV$. The result is shown in Fig. 1.9. The acoustic phonons in Fig. 1.9 all disperse too rapidly for the

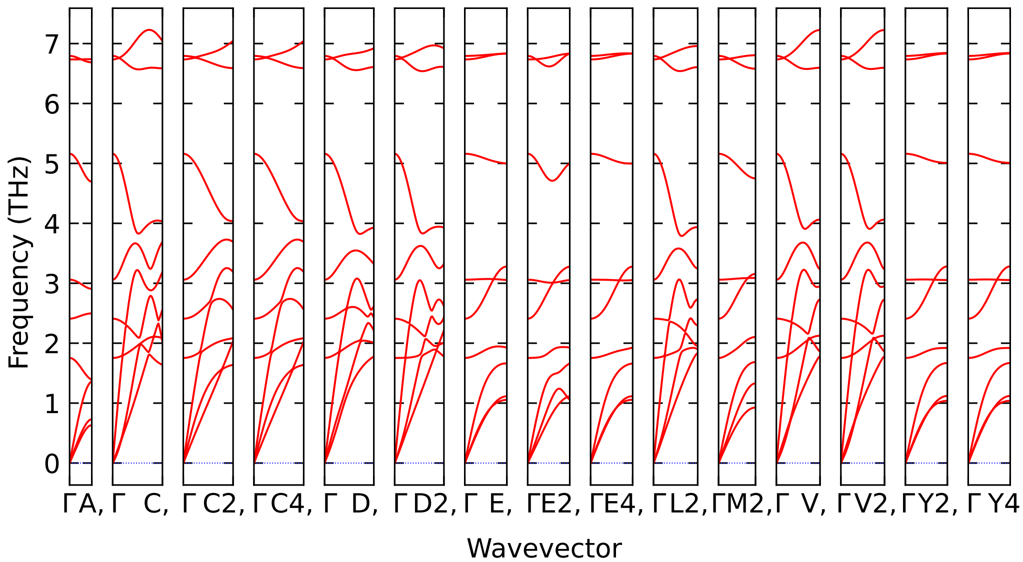

Figure 1.9: Phonon band structure of $CuBr_2$ within a finite displacement calculation.

$0.05\,\mathrm{THz}$ oscillation in the tr-SHG to be consistent with a zone-folded (at $k = (0, 0.235, 0.5)$) acoustic phonon.

1.8.6.3 Magnetic SHG

In principle, magnetic systems with broken inversion symmetry may generate electric-dipole SHG with or without a static electric dipole moment. In this section, we wish to show that this is not the case in $CuBr_2$; i.e., in $CuBr_2$, the SHG intensity is a direct measure of the macroscopic electric dipole moment.

Indeed, in the presence of such a static electric dipole moment $\vec{P}_0$, we typically expect the SHG response to be directly proportional to it; i.e.

$$I(2\omega) \propto |\hat{e}_i^{\mathrm{out}} \chi_{ijk} \hat{e}_j^{\mathrm{in}} \hat{e}_k^{\mathrm{in}}|^2, \tag{1.18}$$

where

$$\chi_{ijk} = \chi_{ijkl} P_{0l}, \tag{1.19}$$

and $\hat{e}^{\mathrm{in}}$, $\hat{e}^{\mathrm{out}}$ are unit vectors in the direction of the incoming and outgoing electric fields, respectively. In $CuBr_2$, we have

$$\vec{P}_0 = \sum_{\langle i,j \rangle} \hat{x} \times \vec{S}_i \times \vec{S}_j, \tag{1.20}$$

i.e., the static polarization is quadratic in the spin degree of freedom.

The question, then, is whether there exists some additional term

$$\chi'_{ijk} = \chi_{ijkl} G_{0l}, \tag{1.21}$$

where $\vec{G}_0$ is either (a) linear in spin, or (b) quadratic in the spins but not of the form $\sum_{\langle ij \rangle} \vec{S}_i \times \vec{S}_j$. For case (b), note that the term $\sum_{\langle ij \rangle} \vec{S}_i \times \vec{S}_j$ is the only quadratic form which is simultaneously antisymmetric in the bond direction and $\vec{q} = 0$ (i.e. each bond has the same coefficient).

For case (a), we argue here that any such term is weak due to the approximate time-reversal symmetry of the spiral magnetic order. Consider first a four-site commensurate approximant of the incommensurate spin spiral. This phase has a symmetry element consisting of the time-reversal operation followed by a translation by half of the magnetic supercell. Thus, the point group contains time-reversal symmetry. Since $\vec{G}_0$ is linear in spin, time-reversal takes $\vec{G}_0 \to -\vec{G}_0$; but since time-reversal is a symmetry, it must also take $\chi'_{ijk} \to \chi'_{ijk}$ and $\chi_{ijkl} \to \chi_{ijkl}$. Thus, $\chi'_{ijk} = 0$ in the commensurate approximation.

In the incommensurate case, note that the magnetic point group of an incommensurate magnetic phase is defined as the set of point-group operations present in the operations belonging to the superspace group. Thus, for a single-k incommensurate magnetic structure, time-reversal is always an element of the magnetic point group. This is due to the fact that the lattice constant in the chain direction is 3.51 Å, so lengthscales associated with translations in the space group are much smaller than the probe wavelength ($\sim 800\,\mathrm{nm}$). The symmetry group "seen" by the probe thus contains time reversal to a very good approximation.

1.8.6.4 Multi-phason excitation

While the amplitude mode of the spin spiral in $CuBr_2$ is the only single-particle excitation which couples to δP_y (see Eq. 1.7), in principle multiparticle excitations consisting of, e.g., two phasons with opposite momenta are also allowed. However, note that the relevant energy scale which defines the phason sound velocity depends on the intra-chain coupling terms, which are $\mathcal{O}(10\,\mathrm{meV})$; the peak in the phason joint density of states thus occurs at this high energy scale, which is much larger than the $0.2\,\mathrm{meV}$ energy of our low-frequency oscillation. Multi-phason excitations are thus not consistent with the long-lived oscillation observed in our experiment.

Chapter Two

Concluding remarks

*

Bibliography

[1] Ian Affleck and Greg F. Wellman.
Longitudinal modes in quasi-one-dimensional antiferromagnets.
Phys. Rev. B, 46(14):8934–8953, October 1992.
doi: 10.1103/PhysRevB.46.8934.

[2] ATLAS collaboration, G. Aad, B. Abbott, J. Abdallah, O. Abdinov, R. Aben, M. Abolins, O. S. AbouZeid, H. Abramowicz, H. Abreu, R. Abreu, Y. Abulaiti, B. S. Acharya, L. Adamczyk, D. L. Adams, J. Adelman, S. Adomeit, T. Adye, A. A. Affolder, T. Agatonovic-Jovin, J. A. Aguilar-Saavedra, M. Agustoni, S. P. Ahlen, F. Ahmadov, G. Aielli, H. Akerstedt, T. P. A. Åkesson, G. Akimoto, A. V. Akimov, G. L. Alberghi, J. Albert, S. Albrand, M. J. Alconada Verzini, M. Aleksa, I. N. Aleksandrov, C. Alexa, G. Alexander, T. Alexopoulos, M. Alhroob, G. Alimonti, L. Alio, J. Alison, S. P. Alkire, B. M. M. Allbrooke, P. P. Allport, A. Aloisio, A. Alonso, F. Alonso, C. Alpigiani, A. Altheimer, B. Alvarez Gonzalez, D. Álvarez Piqueras, M. G. Alviggi, K. Amako, Y. Amaral Coutinho, C. Amelung, D. Amidei, S. P. Amor Dos Santos, A. Amorim, S. Amoroso, N. Amram, G. Amundsen, C. Anastopoulos, L. S. Ancu, N. Andari, T. Andeen, C. F. Anders, G. Anders, K. J. Anderson, A. Andreazza, V. Andrei, S. Angelidakis, I. Angelozzi, P. Anger, A. Angerami, F. Anghinolfi, A. V. Anisenkov, N. Anjos, A. Annovi, M. Antonelli, A. Antonov, J. Antos, F. Anulli, M. Aoki, L. Aperio Bella, G. Arabidze, Y. Arai, J. P. Araque, A. T. H. Arce, F. A Arduh, J.-F. Arguin, S. Argyropoulos, M. Arik, A. J. Armbruster, O. Arnaez, V. Arnal, H. Arnold, M. Arratia, O. Arslan, A. Artamonov, G. Artoni, S. Asai, N. Asbah, A. Ashkenazi, B. Åsman, L. Asquith, K. Assamagan, R. Astalos, M. Atkinson, N. B. Atlay, B. Auer-

bach, K. Augsten, M. Aurousseau, G. Avolio, B. Axen, M. K. Ayoub, G. Azuelos, M. A. Baak, A. E. Baas, C. Bacci, H. Bachacou, K. Bachas, M. Backes, M. Backhaus, E. Badescu, P. Bagiacchi, P. Bagnaia, Y. Bai, T. Bain, J. T. Baines, O. K. Baker, P. Balek, T. Balestri, F. Balli, E. Banas, Sw. Banerjee, A. A. E. Bannoura, H. S. Bansil, L. Barak, S. P. Baranov, E. L. Barberio, D. Barberis, M. Barbero, T. Barillari, M. Barisonzi, T. Barklow, N. Barlow, S. L. Barnes, B. M. Barnett, R. M. Barnett, Z. Barnovska, A. Baroncelli, G. Barone, A. J. Barr, F. Barreiro, J. Barreiro Guimarães da Costa, R. Bartoldus, A. E. Barton, P. Bartos, A. Bassalat, A. Basye, R. L. Bates, S. J. Batista, J. R. Batley, M. Battaglia, M. Bauce, F. Bauer, H. S. Bawa, J. B. Beacham, M. D. Beattie, T. Beau, P. H. Beauchemin, R. Beccherle, P. Bechtle, H. P. Beck, K. Becker, M. Becker, S. Becker, M. Beckingham, C. Becot, A. J. Beddall, A. Beddall, V. A. Bednyakov, C. P. Bee, L. J. Beemster, T. A. Beermann, M. Begel, J. K. Behr, C. Belanger-Champagne, P. J. Bell, W. H. Bell, G. Bella, L. Bellagamba, A. Bellerive, M. Bellomo, K. Belotskiy, O. Beltramello, O. Benary, D. Benchekroun, M. Bender, K. Bendtz, N. Benekos, Y. Benhammou, E. Benhar Noccioli, J. A. Benitez Garcia, D. P. Benjamin, J. R. Bensinger, S. Bentvelsen, L. Beresford, M. Beretta, D. Berge, E. Bergeaas Kuutmann, N. Berger, F. Berghaus, J. Beringer, C. Bernard, N. R. Bernard, C. Bernius, F. U. Bernlochner, T. Berry, P. Berta, C. Bertella, G. Bertoli, F. Bertolucci, C. Bertsche, D. Bertsche, M. I. Besana, G. J. Besjes, O. Bessidskaia Bylund, M. Bessner, N. Besson, C. Betancourt, S. Bethke, A. J. Beven, W. Bhimji, R. M. Bianchi, L. Bianchini, M. Bianco, O. Biebel, S. P. Bieniek, M. Biglietti, J. Bilbao De Mendizabal, H. Bilokon, M. Bindi, S. Binet, A. Bingul, C. Bini, C. W. Black, J. E. Black, K. M. Black, D. Blackburn, R. E. Blair, J.-B. Blanchard, J.E. Blanco, T. Blazek, I. Bloch, C. Blocker, W. Blum, U. Blumenschein, G. J. Bobbink, V. S. Bobrovnikov, S. S. Bocchetta, A. Bocci, C. Bock, M. Boehler, J. A. Bogaerts, A. G. Bogdanchikov, C. Bohm, V. Boisvert, T. Bold, V. Boldea, A. S. Boldyrev, M. Bomben, M. Bona, M. Boonekamp, A. Borisov, G. Borissov, S. Borroni, J. Bortfeldt, V. Bortolotto, K. Bos, D. Boscherini, M. Bosman, J. Boudreau, J. Bouffard, E. V. Bouhova-Thacker, D. Boumediene, C. Bourdarios, N. Bousson, A. Boveia, J. Boyd, I. R. Boyko, I. Bozic, J. Bracinik, A. Brandt, G. Brandt, O. Brandt, U. Bratzler,

B. Brau, J. E. Brau, H. M. Braun, S. F. Brazzale, K. Brendlinger, A. J. Brennan, L. Brenner, R. Brenner, S. Bressler, K. Bristow, T. M. Bristow, D. Britton, D. Britzger, F. M. Brochu, I. Brock, R. Brock, J. Bronner, G. Brooijmans, T. Brooks, W. K. Brooks, J. Brosamer, E. Brost, J. Brown, P. A. Bruckman de Renstrom, D. Bruncko, R. Bruneliere, A. Bruni, G. Bruni, M. Bruschi, L. Bryngemark, T. Buanes, Q. Buat, P. Buchholz, A. G. Buckley, S. I. Buda, I. A. Budagov, F. Buehrer, L. Bugge, M. K. Bugge, O. Bulekov, H. Burckhart, S. Burdin, B. Burghgrave, S. Burke, I. Burmeister, E. Busato, D. Büscher, V. Büscher, P. Bussey, C. P. Buszello, J. M. Butler, A. I. Butt, C. M. Buttar, J. M. Butterworth, P. Butti, W. Buttinger, A. Buzatu, R. Buzykaev, S. Cabrera Urbán, D. Caforio, O. Cakir, P. Calafiura, A. Calandri, G. Calderini, P. Calfayan, L. P. Caloba, D. Calvet, S. Calvet, R. Camacho Toro, S. Camarda, D. Cameron, L. M. Caminada, R. Caminal Armadans, S. Campana, M. Campanelli, A. Campoverde, V. Canale, A. Canepa, M. Cano Bret, J. Cantero, R. Cantrill, T. Cao, M. D. M. Capeans Garrido, I. Caprini, M. Caprini, M. Capua, R. Caputo, R. Cardarelli, T. Carli, G. Carlino, L. Carminati, S. Caron, E. Carquin, G. D. Carrillo-Montoya, J. R. Carter, J. Carvalho, D. Casadei, M. P. Casado, M. Casolino, E. Castaneda-Miranda, A. Castelli, V. Castillo Gimenez, N. F. Castro, P. Catastini, A. Catinaccio, J. R. Catmore, A. Cattai, J. Caudron, V. Cavaliere, D. Cavalli, M. Cavalli-Sforza, V. Cavasinni, F. Ceradini, B. Cerio, K. Cerny, A. S. Cerqueira, A. Cerri, L. Cerrito, F. Cerutti, M. Cerv, A. Cervelli, S. A. Cetin, A. Chafaq, D. Chakraborty, I. Chalupkova, P. Chang, B. Chapleau, J. D. Chapman, D. G. Charlton, C. C. Chau, C. A. Chavez Barajas, S. Cheatham, A. Chegwidden, S. Chekanov, S. V. Chekulaev, G. A. Chelkov, M. A. Chelstowska, C. Chen, H. Chen, K. Chen, L. Chen, S. Chen, X. Chen, Y. Chen, H. C. Cheng, Y. Cheng, A. Cheplakov, E. Cheremushkina, R. Cherkaoui El Moursli, V. Chernyatin, E. Cheu, L. Chevalier, V. Chiarella, J. T. Childers, G. Chiodini, A. S. Chisholm, R. T. Chislett, A. Chitan, M. V. Chizhov, K. Choi, S. Chouridou, B. K. B. Chow, V. Christodoulou, D. Chromek-Burckhart, M. L. Chu, J. Chudoba, A. J. Chuinard, J. J. Chwastowski, L. Chytka, G. Ciapetti, A. K. Ciftci, D. Cinca, V. Cindro, I. A. Cioara, A. Ciocio, Z. H. Citron, M. Ciubancan, A. Clark, B. L. Clark, P. J. Clark, R. N. Clarke, W. Cleland, C. Clement, Y. Coadou, M. Cobal, A. Coccaro, J. Cochran, L. Coffey, J. G. Cogan, B. Cole, S. Cole, A. P. Colijn, J. Collot, T. Colombo, G. Compostella, P. Conde Muiño, E. Coniavitis, S. H. Connell, I. A. Connelly, S. M. Consonni, V. Consorti, S. Constantinescu, C. Conta, G. Conti, F. Conventi, M. Cooke, B. D. Cooper, A. M. Cooper-Sarkar, K. Copic, T. Cornelissen, M. Corradi, F. Corriveau, A. Corso-Radu, A. Cortes-Gonzalez, G. Cortiana, G. Costa, M. J. Costa, D. Costanzo, D. Côté, G. Cottin, G. Cowan, B. E. Cox, K. Cranmer, G. Cree, S. Crépé-Renaudin, F. Crescioli, W. A. Cribbs, M. Crispin Ortuzar, M. Cristinziani, V. Croft, G. Crosetti, T. Cuhadar Donszelmann, J. Cummings, M. Curatolo, C. Cuthbert, H. Czirr, P. Czodrowski, S. D'Auria, M. D'Onofrio, M. J. Da Cunha Sargedas De Sousa, C. Da Via, W. Dabrowski, A. Dafinca, T. Dai, O. Dale, F. Dallaire, C. Dallapiccola, M. Dam, J. R. Dandoy, A. C. Daniells, M. Danninger, M. Dano Hoffmann, V. Dao, G. Darbo, S. Darmora, J. Dassoulas, A. Dattagupta, W. Davey, C. David, T. Davidek, E. Davies, M. Davies, P. Davison, Y. Davygora, E. Dawe, I. Dawson, R. K. Daya-Ishmukhametova, K. De, R. de Asmundis, S. De Castro, S. De Cecco, N. De Groot, P. de Jong, H. De la Torre, F. De Lorenzi, L. De Nooij, D. De Pedis, A. De Salvo, U. De Sanctis, A. De Santo, J. B. De Vivie De Regie, W. J. Dearnaley, R. Debbe, C. Debenedetti, D. V. Dedovich, I. Deigaard, J. Del Peso, T. Del Prete, D. Delgove, F. Deliot, C. M. Delitzsch, M. Deliyergiyev, A. Dell'Acqua, L. Dell'Asta, M. Dell'Orso, M. Della Pietra, D. della Volpe, M. Delmastro, P. A. Delsart, C. Deluca, D. A. DeMarco, S. Demers, M. Demichev, A. Demilly, S. P. Denisov, D. Derendarz, J. E. Derkaoui, F. Derue, P. Dervan, K. Desch, C. Deterre, P. O. Deviveiros, A. Dewhurst, S. Dhaliwal, A. Di Ciaccio, L. Di Ciaccio, A. Di Domenico, C. Di Donato, A. Di Girolamo, B. Di Girolamo, A. Di Mattia, B. Di Micco, R. Di Nardo, A. Di Simone, R. Di Sipio, D. Di Valentino, C. Diaconu, M. Diamond, F. A. Dias, M. A. Diaz, E. B. Diehl, J. Dietrich, S. Diglio, A. Dimitrievska, J. Dingfelder, F. Dittus, F. Djama, T. Djobava, J. I. Djuvsland, M. A. B. do Vale, D. Dobos, M. Dobre, C. Doglioni, T. Dohmae, J. Dolejsi, Z. Dolezal, B. A. Dolgoshein, M. Donadelli, S. Donati, P. Dondero, J. Donini, J. Dopke, A. Doria, M. T. Dova, A. T. Doyle, E. Drechsler, M. Dris, E. Dubreuil, E. Duchovni, G. Duckeck, O. A. Ducu, D. Duda, A. Dudarev, L. Duflot, L. Duguid,

M. Dührssen, M. Dunford, H. Duran Yildiz, M. Düren, A. Durglishvili, D. Duschinger, M. Dwuznik, M. Dyndal, C. Eckardt, K. M. Ecker, W. Edson, N. C. Edwards, W. Ehrenfeld, T. Eifert, G. Eigen, K. Einsweiler, T. Ekelof, M. El Kacimi, M. Ellert, S. Elles, F. Ellinghaus, A. A. Elliot, N. Ellis, J. Elmsheuser, M. Elsing, D. Emeliyanov, Y. Enari, O. C. Endner, M. Endo, R. Engelmann, J. Erdmann, A. Ereditato, G. Ernis, J. Ernst, M. Ernst, S. Errede, E. Ertel, M. Escalier, H. Esch, C. Escobar, B. Esposito, A. I. Etienvre, E. Etzion, H. Evans, A. Ezhilov, L. Fabbri, G. Facini, R. M. Fakhrutdinov, S. Falciano, R. J. Falla, J. Faltova, Y. Fang, M. Fanti, A. Farbin, A. Farilla, T. Farooque, S. Farrell, S. M. Farrington, P. Farthouat, F. Fassi, P. Fassnacht, D. Fassouliotis, A. Favareto, L. Fayard, P. Federic, O. L. Fedin, W. Fedorko, S. Feigl, L. Feligioni, C. Feng, E. J. Feng, H. Feng, A. B. Fenyuk, P. Fernandez Martinez, S. Fernandez Perez, S. Ferrag, J. Ferrando, A. Ferrari, P. Ferrari, R. Ferrari, D. E. Ferreira de Lima, A. Ferrer, D. Ferrere, C. Ferretti, A. Ferretto Parodi, M. Fiascaris, F. Fiedler, A. Filipčič, M. Filipuzzi, F. Filthaut, M. Fincke-Keeler, K. D. Finelli, M. C. N. Fiolhais, L. Fiorini, A. Firan, A. Fischer, C. Fischer, J. Fischer, W. C. Fisher, E. A. Fitzgerald, M. Flechl, I. Fleck, P. Fleischmann, S. Fleischmann, G. T. Fletcher, G. Fletcher, T. Flick, A. Floderus, L. R. Flores Castillo, M. J. Flowerdew, A. Formica, A. Forti, D. Fournier, H. Fox, S. Fracchia, P. Francavilla, M. Franchini, D. Francis, L. Franconi, M. Franklin, M. Fraternali, D. Freeborn, S. T. French, F. Friedrich, D. Froidevaux, J. A. Frost, C. Fukunaga, E. Fullana Torregrosa, B. G. Fulsom, J. Fuster, C. Gabaldon, O. Gabizon, A. Gabrielli, A. Gabrielli, S. Gadatsch, S. Gadomski, G. Gagliardi, P. Gagnon, C. Galea, B. Galhardo, E. J. Gallas, B. J. Gallop, P. Gallus, G. Galster, K. K. Gan, J. Gao, Y. Gao, Y. S. Gao, F. M. Garay Walls, F. Garberson, C. García, J. E. García Navarro, M. Garcia-Sciveres, R. W. Gardner, N. Garelli, V. Garonne, C. Gatti, A. Gaudiello, G. Gaudio, B. Gaur, L. Gauthier, P. Gauzzi, I. L. Gavrilenko, C. Gay, G. Gaycken, E. N. Gazis, P. Ge, Z. Gecse, C. N. P. Gee, D. A. A. Geerts, Ch. Geich-Gimbel, M. P. Geisler, C. Gemme, M. H. Genest, S. Gentile, M. George, S. George, D. Gerbaudo, A. Gershon, H. Ghazlane, N. Ghodbane, B. Giacobbe, S. Giagu, V. Giangiobbe, P. Giannetti, B. Gibbard, S. M. Gibson, M. Gilchriese, T. P. S. Gillam, D. Gillberg, G. Gilles, D. M. Gingrich, N. Giokaris, M. P. Giordani, F. M. Giorgi, F. M. Giorgi, P. F. Giraud, P. Giromini, D. Giugni, C. Giuliani, M. Giulini, B. K. Gjelsten, S. Gkaitatzis, I. Gkialas, E. L. Gkougkousis, L. K. Gladilin, C. Glasman, J. Glatzer, P. C. F. Glaysher, A. Glazov, G. L. Glonti, M. Goblirsch-Kolb, J. R. Goddard, J. Godlewski, S. Goldfarb, T. Golling, D. Golubkov, A. Gomes, R. Gonçalo, J. Goncalves Pinto Firmino Da Costa, L. Gonella, S. González de la Hoz, G. Gonzalez Parra, S. Gonzalez-Sevilla, L. Goossens, P. A. Gorbounov, H. A. Gordon, I. Gorelov, B. Gorini, E. Gorini, A. Gorišek, E. Gornicki, A. T. Goshaw, C. Gössling, M. I. Gostkin, D. Goujdami, A. G. Goussiou, N. Govender, H. M. X. Grabas, L. Graber, I. Grabowska-Bold, P. Grafström, K-J. Grahn, J. Gramling, E. Gramstad, S. Grancagnolo, V. Grassi, V. Gratchev, H. M. Gray, E. Graziani, Z. D. Greenwood, K. Gregersen, I. M. Gregor, P. Grenier, J. Griffiths, A. A. Grillo, K. Grimm, S. Grinstein, Ph. Gris, Y. V. Grishkevich, J.-F. Grivaz, J. P. Grohs, A. Grohsjean, E. Gross, J. Grosse-Knetter, G. C. Grossi, Z. J. Grout, L. Guan, J. Guenther, F. Guescini, D. Guest, O. Gueta, E. Guido, T. Guillemin, S. Guindon, U. Gul, C. Gumpert, J. Guo, S. Gupta, P. Gutierrez, N. G. Gutierrez Ortiz, C. Gutschow, C. Guyot, C. Gwenlan, C. B. Gwilliam, A. Haas, C. Haber, H. K. Hadavand, N. Haddad, P. Haefner, S. Hageböck, Z. Hajduk, H. Hakobyan, M. Haleem, J. Haley, D. Hall, G. Halladjian, G. D. Hallewell, K. Hamacher, P. Hamal, K. Hamano, M. Hamer, A. Hamilton, S. Hamilton, G. N. Hamity, P. G. Hamnett, L. Han, K. Hanagaki, K. Hanawa, M. Hance, P. Hanke, R. Hanna, J. B. Hansen, J. D. Hansen, M. C. Hansen, P. H. Hansen, K. Hara, A. S. Hard, T. Harenberg, F. Hariri, S. Harkusha, R. D. Harrington, P. F. Harrison, F. Hartjes, M. Hasegawa, S. Hasegawa, Y. Hasegawa, A. Hasib, S. Hassani, S. Haug, R. Hauser, L. Hauswald, M. Havranek, C. M. Hawkes, R. J. Hawkings, A. D. Hawkins, T. Hayashi, D. Hayden, C. P. Hays, J. M. Hays, H. S. Hayward, S. J. Haywood, S. J. Head, T. Heck, V. Hedberg, L. Heelan, S. Heim, T. Heim, B. Heinemann, L. Heinrich, J. Hejbal, L. Helary, S. Hellman, D. Hellmich, C. Helsens, J. Henderson, R. C. W. Henderson, Y. Heng, C. Hengler, A. Henrichs, A. M. Henriques Correia, S. Henrot-Versille, G. H. Herbert, Y. Hernández Jiménez, R. Herrberg-Schubert, G. Herten, R. Hertenberger, L. Hervas, G. G. Hesketh, N. P. Hessey, J. W. Hetherly, R. Hickling, E. Higón-Rodriguez, E. Hill, J. C. Hill, K. H. Hiller, S. J. Hillier, I. Hinchliffe, E. Hines, R. R. Hinman, M. Hirose, D. Hirschbuehl, J. Hobbs, N. Hod, M. C. Hodgkinson, P. Hodgson, A. Hoecker, M. R. Hoeferkamp, F. Hoenig, M. Hohlfeld, D. Hohn, T. R. Holmes, T. M. Hong, L. Hooft van Huysduynen, W. H. Hopkins, Y. Horii, A. J. Horton, J-Y. Hostachy, S. Hou, A. Hoummada, J. Howard, J. Howarth, M. Hrabovsky, I. Hristova, J. Hrivnac, T. Hryn'ova, A. Hrynevich, C. Hsu, P. J. Hsu, S.-C. Hsu, D. Hu, Q. Hu, X. Hu, Y. Huang, Z. Hubacek, F. Hubaut, F. Huegging, T. B. Huffman, E. W. Hughes, G. Hughes, M. Huhtinen, T. A. Hülsing, N. Huseynov, J. Huston, J. Huth, G. Iacobucci, G. Iakovidis, I. Ibragimov, L. Iconomidou-Fayard, E. Ideal, Z. Idrissi, P. Iengo, O. Igonkina, T. Iizawa, Y. Ikegami, K. Ikematsu, M. Ikeno, Y. Ilchenko, D. Iliadis, N. Ilic, Y. Inamaru, T. Ince, P. Ioannou, M. Iodice, K. Iordanidou, V. Ippolito, A. Irles Quiles, C. Isaksson, M. Ishino, M. Ishitsuka, R. Ishmukhametov, C. Issever, S. Istin, J. M. Iturbe Ponce, R. Iuppa, J. Ivarsson, W. Iwanski, H. Iwasaki, J. M. Izen, V. Izzo, S. Jabbar, B. Jackson, M. Jackson, P. Jackson, M. R. Jaekel, V. Jain, K. Jakobs, S. Jakobsen, T. Jakoubek, J. Jakubek, D. O. Jamin, D. K. Jana, E. Jansen, R. W. Jansky, J. Janssen, M. Janus, G. Jarlskog, N. Javadov, T. Javůrek, L. Jeanty, J. Jejelava, G.-Y. Jeng, D. Jennens, P. Jenni, J. Jentzsch, C. Jeske, S. Jézéquel, H. Ji, J. Jia, Y. Jiang, S. Jiggins, J. Jimenez Pena, S. Jin, A. Jinaru, O. Jinnouchi, M. D. Joergensen, P. Johansson, K. A. Johns, K. Jon-And, G. Jones, R. W. L. Jones, T. J. Jones, J. Jongmanns, P. M. Jorge, K. D. Joshi, J. Jovicevic, X. Ju, C. A. Jung, P. Jussel, A. Juste Rozas, M. Kaci, A. Kaczmarska, M. Kado, H. Kagan, M. Kagan, S. J. Kahn, E. Kajomovitz, C. W. Kalderon, S. Kama, A. Kamenshchikov, N. Kanaya, M. Kaneda, S. Kaneti, V. A. Kantserov, J. Kanzaki, B. Kaplan, A. Kapliy, D. Kar, K. Karakostas, A. Karamaoun, N. Karastathis, M. J. Kareem, M. Karnevskiy, S. N. Karpov, Z. M. Karpova, K. Karthik, V. Kartvelishvili, A. N. Karyukhin, L. Kashif, R. D. Kass, A. Kastanas, Y. Kataoka, A. Katre, J. Katzy, K. Kawagoe, T. Kawamoto, G. Kawamura, S. Kazama, V. F. Kazanin, M. Y. Kazarinov, R. Keeler, R. Kehoe, M. Keil, J. S. Keller, J. J. Kempster, H. Keoshkerian, O. Kepka, B. P. Kerševan, S. Kersten, R. A. Keyes, F. Khalil-zada, H. Khandanyan, A. Khanov, A.G. Kharlamov, T. J. Khoo, G. Khoriauli, V. Khovanskiy, E. Khramov, J. Khubua, H. Y. Kim, H. Kim, S. H. Kim, Y. Kim, N. Kimura, O. M. Kind, B. T. King, M. King, R. S. B. King, S. B. King, J. Kirk, A. E. Kiryunin, T. Kishimoto, D. Kisielewska, F. Kiss, K. Kiuchi, O. Kivernyk, E. Kladiva, M. H. Klein, M. Klein, U. Klein, K. Kleinknecht, P. Klimek, A. Klimentov, R. Klingenberg, J. A. Klinger, T. Klioutchnikova, P. F. Klok, E.-E. Kluge, P. Kluit, S. Kluth, E. Kneringer, E. B. F. G. Knoops, A. Knue, D. Kobayashi, T. Kobayashi, M. Kobel, M. Kocian, P. Kodys, T. Koffas, E. Koffeman, L. A. Kogan, S. Kohlmann, Z. Kohout, T. Kohriki, T. Koi, H. Kolanoski, I. Koletsou, A. A. Komar, Y. Komori, T. Kondo, N. Kondrashova, K. Köneke, A. C. König, S. König, T. Kono, R. Konoplich, N. Konstantinidis, R. Kopeliansky, S. Koperny, L. Köpke, A. K. Kopp, K. Korcyl, K. Kordas, A. Korn, A. A. Korol, I. Korolkov, E. V. Korolkova, O. Kortner, S. Kortner, T. Kosek, V. V. Kostyukhin, V. M. Kotov, A. Kotwal, A. Kourkoumeli-Charalampidi, C. Kourkoumelis, V. Kouskoura, A. Koutsman, R. Kowalewski, T. Z. Kowalski, W. Kozanecki, A. S. Kozhin, V. A. Kramarenko, G. Kramberger, D. Krasnopevtsev, M. W. Krasny, A. Krasznahorkay, J. K. Kraus, A. Kravchenko, S. Kreiss, M. Kretz, J. Kretzschmar, K. Kreutzfeldt, P. Krieger, K. Krizka, K. Kroeninger, H. Kroha, J. Kroll, J. Kroseberg, J. Krstic, U. Kruchonak, H. Krüger, N. Krumnack, Z. V. Krumshteyn, A. Kruse, M. C. Kruse, M. Kruskal, T. Kubota, H. Kucuk, S. Kuday, S. Kuehn, A. Kugel, F. Kuger, A. Kuhl, T. Kuhl, V. Kukhtin, Y. Kulchitsky, S. Kuleshov, M. Kuna, T. Kunigo, A. Kupco, H. Kurashige, Y. A. Kurochkin, R. Kurumida, V. Kus, E. S. Kuwertz, M. Kuze, J. Kvita, T. Kwan, D. Kyriazopoulos, A. La Rosa, J. L. La Rosa Navarro, L. La Rotonda, C. Lacasta, F. Lacava, J. Lacey, H. Lacker, D. Lacour, V. R. Lacuesta, E. Ladygin, R. Lafaye, B. Laforge, T. Lagouri, S. Lai, L. Lambourne, S. Lammers, C. L. Lampen, W. Lampl, E. Lançon, U. Landgraf, M. P. J. Landon, V. S. Lang, J. C. Lange, A. J. Lankford, F. Lanni, K. Lantzsch, S. Laplace, C. Lapoire, J. F. Laporte, T. Lari, F. Lasagni Manghi, M. Lassnig, P. Laurelli, W. Lavrijsen, A. T. Law, P. Laycock, O. Le Dortz, E. Le Guirriec, E. Le Menedeu, M. LeBlanc, T. LeCompte, F. Ledroit-Guillon, C. A. Lee, S. C. Lee, L. Lee, G. Lefebvre, M. Lefebvre, F. Legger, C. Leggett, A. Lehan, G. Lehmann Miotto, X. Lei, W. A. Leight, A. Leisos, A. G. Leister, M. A. L. Leite, R. Leitner, D. Lellouch, B. Lemmer, K. J. C. Leney,

T. Lenz, G. Lenzen, B. Lenzi, R. Leone, S. Leone, C. Leonidopoulos, S. Leontsinis, C. Leroy, C. G. Lester, M. Levchenko, J. Levêque, D. Levin, L. J. Levinson, M. Levy, A. Lewis, A. M. Leyko, M. Leyton, B. Li, H. Li, H. L. Li, L. Li, L. Li, S. Li, Y. Li, Z. Liang, H. Liao, B. Liberti, A. Liblong, P. Lichard, K. Lie, J. Liebal, W. Liebig, C. Limbach, A. Limosani, S. C. Lin, T. H. Lin, F. Linde, B. E. Lindquist, J. T. Linnemann, E. Lipeles, A. Lipniacka, M. Lisovyi, T. M. Liss, D. Lissauer, A. Lister, A. M. Litke, B. Liu, D. Liu, J. Liu, J. B. Liu, K. Liu, L. Liu, M. Liu, M. Liu, Y. Liu, M. Livan, A. Lleres, J. Llorente Merino, S. L. Lloyd, F. Lo Sterzo, E. Lobodzinska, P. Loch, W. S. Lockman, F. K. Loebinger, A. E. Loevschall-Jensen, A. Loginov, T. Lohse, K. Lohwasser, M. Lokajicek, B. A. Long, J. D. Long, R. E. Long, K. A. Looper, L. Lopes, D. Lopez Mateos, B. Lopez Paredes, I. Lopez Paz, J. Lorenz, N. Lorenzo Martinez, M. Losada, P. Loscutoff, P. J. Lösel, X. Lou, A. Lounis, J. Love, P. A. Love, N. Lu, H. J. Lubatti, C. Luci, A. Lucotte, F. Luehring, W. Lukas, L. Luminari, O. Lundberg, B. Lund-Jensen, M. Lungwitz, D. Lynn, R. Lysak, E. Lytken, H. Ma, L. L. Ma, G. Maccarrone, A. Macchiolo, C. M. Macdonald, J. Machado Miguens, D. Macina, D. Madaffari, R. Madar, H. J. Maddocks, W. F. Mader, A. Madsen, S. Maeland, T. Maeno, A. Maevskiy, E. Magradze, K. Mahboubi, J. Mahlstedt, C. Maiani, C. Maidantchik, A. A. Maier, T. Maier, A. Maio, S. Majewski, Y. Makida, N. Makovec, B. Malaescu, Pa. Malecki, V. P. Maleev, F. Malek, U. Mallik, D. Malon, C. Malone, S. Maltezos, V. M. Malyshev, S. Malyukov, J. Mamuzic, G. Mancini, B. Mandelli, L. Mandelli, I. Mandić, R. Mandrysch, J. Maneira, A. Manfredini, L. Manhaes de Andrade Filho, J. Manjarres Ramos, A. Mann, P. M. Manning, A. Manousakis-Katsikakis, B. Mansoulie, R. Mantifel, M. Mantoani, L. Mapelli, L. March, G. Marchiori, M. Marcisovsky, C. P. Marino, M. Marjanovic, F. Marroquim, S. P. Marsden, Z. Marshall, L. F. Marti, S. Marti-Garcia, B. Martin, T. A. Martin, V. J. Martin, B. Martin dit Latour, M. Martinez, S. Martin-Haugh, V. S. Martoiu, A. C. Martyniuk, M. Marx, F. Marzano, A. Marzin, L. Masetti, T. Mashimo, R. Mashinistov, J. Masik, A. L. Maslennikov, I. Massa, L. Massa, N. Massol, P. Mastrandrea, A. Mastroberardino, T. Masubuchi, P. Mättig, J. Mattmann, J. Maurer, S. J. Maxfield, D. A. Maximov, R. Mazini, S. M. Mazza, L. Mazzaferro, G. Mc Goldrick, S. P. Mc Kee,

A. McCarn, R. L. McCarthy, T. G. McCarthy, N. A. McCubbin, K. W. McFarlane, J. A. Mcfayden, G. Mchedlidze, S. J. McMahon, R. A. McPherson, M. Medinnis, S. Meehan, S. Mehlhase, A. Mehta, K. Meier, C. Meineck, B. Meirose, B. R. Mellado Garcia, F. Meloni, A. Mengarelli, S. Menke, E. Meoni, K. M. Mercurio, S. Mergelmeyer, P. Mermod, L. Merola, C. Meroni, F. S. Merritt, A. Messina, J. Metcalfe, A. S. Mete, C. Meyer, C. Meyer, J-P. Meyer, J. Meyer, R. P. Middleton, S. Miglioranzi, L. Mijović, G. Mikenberg, M. Mikestikova, M. Mikuž, M. Milesi, A. Milic, D. W. Miller, C. Mills, A. Milov, D. A. Milstead, A. A. Minaenko, Y. Minami, I. A. Minashvili, A. I. Mincer, B. Mindur, M. Mineev, Y. Ming, L. M. Mir, T. Mitani, J. Mitrevski, V. A. Mitsou, A. Miucci, P. S. Miyagawa, J. U. Mjörnmark, T. Moa, K. Mochizuki, S. Mohapatra, W. Mohr, S. Molander, R. Moles-Valls, K. Mönig, C. Monini, J. Monk, E. Monnier, J. Montejo Berlingen, F. Monticelli, S. Monzani, R. W. Moore, N. Morange, D. Moreno, M. Moreno Llácer, P. Morettini, M. Morgenstern, M. Morii, V. Morisbak, S. Moritz, A. K. Morley, G. Mornacchi, J. D. Morris, S. S. Mortensen, A. Morton, L. Morvaj, H. G. Moser, M. Mosidze, J. Moss, K. Motohashi, R. Mount, E. Mountricha, S. V. Mouraviev, E. J. W. Moyse, S. Muanza, R. D. Mudd, F. Mueller, J. Mueller, K. Mueller, R. S. P. Mueller, T. Mueller, D. Muenstermann, P. Mullen, Y. Munwes, J. A. Murillo Quijada, W. J. Murray, H. Musheghyan, E. Musto, A. G. Myagkov, M. Myska, O. Nackenhorst, J. Nadal, K. Nagai, R. Nagai, Y. Nagai, K. Nagano, A. Nagarkar, Y. Nagasaka, K. Nagata, M. Nagel, E. Nagy, A. M. Nairz, Y. Nakahama, K. Nakamura, T. Nakamura, I. Nakano, H. Namasivayam, G. Nanava, R. F. Naranjo Garcia, R. Narayan, T. Naumann, G. Navarro, R. Nayyar, H. A. Neal, P. Yu. Nechaeva, T. J. Neep, P. D. Nef, A. Negri, M. Negrini, S. Nektarijevic, C. Nellist, A. Nelson, S. Nemecek, P. Nemethy, A. A. Nepomuceno, M. Nessi, M. S. Neubauer, M. Neumann, R. M. Neves, P. Nevski, P. R. Newman, D. H. Nguyen, R. B. Nickerson, R. Nicolaidou, B. Nicquevert, J. Nielsen, N. Nikiforou, A. Nikiforov, V. Nikolaenko, I. Nikolic-Audit, K. Nikolopoulos, J. K. Nilsen, P. Nilsson, Y. Ninomiya, A. Nisati, R. Nisius, T. Nobe, M. Nomachi, I. Nomidis, T. Nooney, S. Norberg, M. Nordberg, O. Novgorodova, S. Nowak, M. Nozaki, L. Nozka, K. Ntekas, G. Nunes Hanninger, T. Nunnemann,

E. Nurse, F. Nuti, B. J. O'Brien, F. O'grady, D. C. O'Neil, V. O'Shea, F. G. Oakham, H. Oberlack, T. Obermann, J. Ocariz, A. Ochi, I. Ochoa, S. Oda, S. Odaka, H. Ogren, A. Oh, S. H. Oh, C. C. Ohm, H. Ohman, H. Oide, W. Okamura, H. Okawa, Y. Okumura, T. Okuyama, A. Olariu, S. A. Olivares Pino, D. Oliveira Damazio, E. Oliver Garcia, A. Olszewski, J. Olszowska, A. Onofre, P. U. E. Onyisi, C. J. Oram, M. J. Oreglia, Y. Oren, D. Orestano, N. Orlando, C. Oropeza Barrera, R. S. Orr, B. Osculati, R. Ospanov, G. Otero y Garzon, H. Otono, M. Ouchrif, E. A. Ouellette, F. Ould-Saada, A. Ouraou, K. P. Oussoren, Q. Ouyang, A. Ovcharova, M. Owen, R. E. Owen, V. E. Ozcan, N. Ozturk, K. Pachal, A. Pacheco Pages, C. Padilla Aranda, M. Pagáčová, S. Pagan Griso, E. Paganis, C. Pahl, F. Paige, P. Pais, K. Pajchel, G. Palacino, S. Palestini, M. Palka, D. Pallin, A. Palma, Y. B. Pan, E. Panagiotopoulou, C. E. Pandini, J. G. Panduro Vazquez, P. Pani, S. Panitkin, L. Paolozzi, Th. D. Papadopoulou, K. Papageorgiou, A. Paramonov, D. Paredes Hernandez, M. A. Parker, K. A. Parker, F. Parodi, J. A. Parsons, U. Parzefall, E. Pasqualucci, S. Passaggio, F. Pastore, Fr. Pastore, G. Pásztor, S. Pataraia, N. D. Patel, J. R. Pater, T. Pauly, J. Pearce, B. Pearson, L. E. Pedersen, M. Pedersen, S. Pedraza Lopez, R. Pedro, S. V. Peleganchuk, D. Pelikan, H. Peng, B. Penning, J. Penwell, D. V. Perepelitsa, E. Perez Codina, M. T. Pérez García-Estañ, L. Perini, H. Pernegger, S. Perrella, R. Peschke, V. D. Peshekhonov, K. Peters, R. F. Y. Peters, B. A. Petersen, T. C. Petersen, E. Petit, A. Petridis, C. Petridou, E. Petrolo, F. Petrucci, N. E. Pettersson, R. Pezoa, P. W. Phillips, G. Piacquadio, E. Pianori, A. Picazio, E. Piccaro, M. Piccinini, M. A. Pickering, R. Piegaia, D. T. Pignotti, J. E. Pilcher, A. D. Pilkington, J. Pina, M. Pinamonti, J. L. Pinfold, A. Pingel, B. Pinto, S. Pires, M. Pitt, C. Pizio, L. Plazak, M.-A. Pleier, V. Pleskot, E. Plotnikova, P. Plucinski, D. Pluth, R. Poettgen, L. Poggioli, D. Pohl, G. Polesello, A. Policicchio, R. Polifka, A. Polini, C. S. Pollard, V. Polychronakos, K. Pommès, L. Pontecorvo, B. G. Pope, G. A. Popeneciu, D. S. Popovic, A. Poppleton, S. Pospisil, K. Potamianos, I. N. Potrap, C. J. Potter, C. T. Potter, G. Poulard, J. Poveda, V. Pozdnyakov, P. Pralavorio, A. Pranko, S. Prasad, S. Prell, D. Price, J. Price, L. E. Price, M. Primavera, S. Prince, M. Proissl, K. Prokofiev, F. Prokoshin, E. Protopapadaki, S. Protopopescu, J. Proudfoot, M. Przybycien,

E. Ptacek, D. Puddu, E. Pueschel, D. Puldon, M. Purohit, P. Puzo, J. Qian, G. Qin, Y. Qin, A. Quadt, D. R. Quarrie, W. B. Quayle, M. Queitsch-Maitland, D. Quilty, S. Raddum, V. Radeka, V. Radescu, S. K. Radhakrishnan, P. Radloff, P. Rados, F. Ragusa, G. Rahal, S. Rajagopalan, M. Rammensee, C. Rangel-Smith, F. Rauscher, S. Rave, T. Ravenscroft, M. Raymond, A. L. Read, N. P. Readioff, D. M. Rebuzzi, A. Redelbach, G. Redlinger, R. Reece, K. Reeves, L. Rehnisch, H. Reisin, M. Relich, C. Rembser, H. Ren, A. Renaud, M. Rescigno, S. Resconi, O. L. Rezanova, P. Reznicek, R. Rezvani, R. Richter, S. Richter, E. Richter-Was, O. Ricken, M. Ridel, P. Rieck, C. J. Riegel, J. Rieger, M. Rijssenbeek, A. Rimoldi, L. Rinaldi, B. Ristić, E. Ritsch, I. Riu, F. Rizatdinova, E. Rizvi, S. H. Robertson, A. Robichaud-Veronneau, D. Robinson, J. E. M. Robinson, A. Robson, C. Roda, S. Roe, O. Røhne, S. Rolli, A. Romaniouk, M. Romano, S. M. Romano Saez, E. Romero Adam, N. Rompotis, M. Ronzani, L. Roos, E. Ros, S. Rosati, K. Rosbach, P. Rose, P. L. Rosendahl, O. Rosenthal, V. Rossetti, E. Rossi, L. P. Rossi, R. Rosten, M. Rotaru, I. Roth, J. Rothberg, D. Rousseau, C. R. Royon, A. Rozanov, Y. Rozen, X. Ruan, F. Rubbo, I. Rubinskiy, V. I. Rud, C. Rudolph, M. S. Rudolph, F. Rühr, A. Ruiz-Martinez, Z. Rurikova, N. A. Rusakovich, A. Ruschke, H. L. Russell, J. P. Rutherfoord, N. Ruthmann, Y. F. Ryabov, M. Rybar, G. Rybkin, N. C. Ryder, A. F. Saavedra, G. Sabato, S. Sacerdoti, A. Saddique, H. F-W. Sadrozinski, R. Sadykov, F. Safai Tehrani, M. Saimpert, H. Sakamoto, Y. Sakurai, G. Salamanna, A. Salamon, M. Saleem, D. Salek, P. H. Sales De Bruin, D. Salihagic, A. Salnikov, J. Salt, D. Salvatore, F. Salvatore, A. Salvucci, A. Salzburger, D. Sampsonidis, A. Sanchez, J. Sánchez, V. Sanchez Martinez, H. Sandaker, R. L. Sandbach, H. G. Sander, M. P. Sanders, M. Sandhoff, C. Sandoval, R. Sandstroem, D. P. C. Sankey, M. Sannino, A. Sansoni, C. Santoni, R. Santonico, H. Santos, I. Santoyo Castillo, K. Sapp, A. Sapronov, J. G. Saraiva, B. Sarrazin, O. Sasaki, Y. Sasaki, K. Sato, G. Sauvage, E. Sauvan, G. Savage, P. Savard, C. Sawyer, L. Sawyer, J. Saxon, C. Sbarra, A. Sbrizzi, T. Scanlon, D. A. Scannicchio, M. Scarcella, V. Scarfone, J. Schaarschmidt, P. Schacht, D. Schaefer, R. Schaefer, J. Schaeffer, S. Schaepe, S. Schaetzel, U. Schäfer, A. C. Schaffer, D. Schaile, R. D. Schamberger, V. Scharf, V. A. Schegelsky, D. Scheirich, M. Schernau, C. Schiavi, C. Schillo, M. Schioppa,

S. Schlenker, E. Schmidt, K. Schmieden, C. Schmitt, S. Schmitt, S. Schmitt, B. Schneider, Y. J. Schnellbach, U. Schnoor, L. Schoeffel, A. Schoening, B. D. Schoenrock, E. Schopf, A. L. S. Schorlemmer, M. Schott, D. Schouten, J. Schovancova, S. Schramm, M. Schreyer, C. Schroeder, N. Schuh, M. J. Schultens, H.-C. Schultz-Coulon, H. Schulz, M. Schumacher, B. A. Schumm, Ph. Schune, C. Schwanenberger, A. Schwartzman, T. A. Schwarz, Ph. Schwegler, Ph. Schwemling, R. Schwienhorst, J. Schwindling, T. Schwindt, M. Schwoerer, F. G. Sciacca, E. Scifo, G. Sciolla, F. Scuri, F. Scutti, J. Searcy, G. Sedov, E. Sedykh, P. Seema, S. C. Seidel, A. Seiden, F. Seifert, J. M. Seixas, G. Sekhniaidze, S. J. Sekula, K. E. Selbach, D. M. Seliverstov, N. Semprini-Cesari, C. Serfon, L. Serin, L. Serkin, T. Serre, R. Seuster, H. Severini, T. Sfiligoj, F. Sforza, A. Sfyrla, E. Shabalina, M. Shamim, L. Y. Shan, R. Shang, J. T. Shank, M. Shapiro, P. B. Shatalov, K. Shaw, A. Shcherbakova, C. Y. Shehu, P. Sherwood, L. Shi, S. Shimizu, C. O. Shimmin, M. Shimojima, M. Shiyakova, A. Shmeleva, D. Shoaleh Saadi, M. J. Shochet, S. Shojaii, S. Shrestha, E. Shulga, M. A. Shupe, S. Shushkevich, P. Sicho, O. Sidiropoulou, D. Sidorov, A. Sidoti, F. Siegert, Dj. Sijacki, J. Silva, Y. Silver, S. B. Silverstein, V. Simak, O. Simard, Lj. Simic, S. Simion, E. Simioni, B. Simmons, D. Simon, R. Simoniello, P. Sinervo, N. B. Sinev, G. Siragusa, A. N. Sisakyan, S. Yu. Sivoklokov, J. Sjölin, T. B. Sjursen, M. B. Skinner, H. P. Skottowe, P. Skubic, M. Slater, T. Slavicek, M. Slawinska, K. Sliwa, V. Smakhtin, B. H. Smart, L. Smestad, S. Yu. Smirnov, Y. Smirnov, L. N. Smirnova, O. Smirnova, M. N. K. Smith, M. Smizanska, K. Smolek, A. A. Snesarev, G. Snidero, S. Snyder, R. Sobie, F. Socher, A. Soffer, D. A. Soh, C. A. Solans, M. Solar, J. Solc, E. Yu. Soldatov, U. Soldevila, A. A. Solodkov, A. Soloshenko, O. V. Solovyanov, V. Solovyev, P. Sommer, H. Y. Song, N. Soni, A. Sood, A. Sopczak, B. Sopko, V. Sopko, V. Sorin, D. Sosa, M. Sosebee, C. L. Sotiropoulou, R. Soualah, P. Soueid, A. M. Soukharev, D. South, S. Spagnolo, M. Spalla, F. Spanò, W. R. Spearman, F. Spettel, R. Spighi, G. Spigo, L. A. Spiller, M. Spousta, T. Spreitzer, R. D. St. Denis, S. Staerz, J. Stahlman, R. Stamen, S. Stamm, E. Stanecka, C. Stanescu, M. Stanescu-Bellu, M. M. Stanitzki, S. Stapnes, E. A. Starchenko, J. Stark, P. Staroba, P. Starovoitov, R. Staszewski, P. Stavina, P. Steinberg, B. Stelzer, H. J. Stelzer,

O. Stelzer-Chilton, H. Stenzel, S. Stern, G. A. Stewart, J. A. Stillings, M. C. Stockton, M. Stoebe, G. Stoicea, P. Stolte, S. Stonjek, A. R. Stradling, A. Straessner, M. E. Stramaglia, J. Strandberg, S. Strandberg, A. Strandlie, E. Strauss, M. Strauss, P. Strizenec, R. Ströhmer, D. M. Strom, R. Stroynowski, A. Strubig, S. A. Stucci, B. Stugu, N. A. Styles, D. Su, J. Su, R. Subramaniam, A. Succurro, Y. Sugaya, C. Suhr, M. Suk, V. V. Sulin, S. Sultansoy, T. Sumida, S. Sun, X. Sun, J. E. Sundermann, K. Suruliz, G. Susinno, M. R. Sutton, S. Suzuki, Y. Suzuki, M. Svatos, S. Swedish, M. Swiatlowski, I. Sykora, T. Sykora, D. Ta, C. Taccini, K. Tackmann, J. Taenzer, A. Taffard, R. Tafirout, N. Taiblum, H. Takai, R. Takashima, H. Takeda, T. Takeshita, Y. Takubo, M. Talby, A. A. Talyshev, J. Y. C. Tam, K. G. Tan, J. Tanaka, R. Tanaka, S. Tanaka, S. Tanaka, B. B. Tannenwald, N. Tannoury, S. Tapprogge, S. Tarem, F. Tarrade, G. F. Tartarelli, P. Tas, M. Tasevsky, T. Tashiro, E. Tassi, A. Tavares Delgado, Y. Tayalati, F. E. Taylor, G. N. Taylor, W. Taylor, F. A. Teischinger, M. Teixeira Dias Castanheira, P. Teixeira-Dias, K. K. Temming, H. Ten Kate, P. K. Teng, J. J. Teoh, F. Tepel, S. Terada, K. Terashi, J. Terron, S. Terzo, M. Testa, R. J. Teuscher, J. Therhaag, T. Theveneaux-Pelzer, J. P. Thomas, J. Thomas-Wilsker, E. N. Thompson, P. D. Thompson, R. J. Thompson, A. S. Thompson, L. A. Thomsen, E. Thomson, M. Thomson, R. P. Thun, M. J. Tibbetts, R. E. Ticse Torres, V. O. Tikhomirov, Yu. A. Tikhonov, S. Timoshenko, E. Tiouchichine, P. Tipton, S. Tisserant, T. Todorov, S. Todorova-Nova, J. Tojo, S. Tokár, K. Tokushuku, K. Tollefson, E. Tolley, L. Tomlinson, M. Tomoto, L. Tompkins, K. Toms, E. Torrence, H. Torres, E. Torró Pastor, J. Toth, F. Touchard, D. R. Tovey, T. Trefzger, L. Tremblet, A. Tricoli, I. M. Trigger, S. Trincaz-Duvoid, M. F. Tripiana, W. Trischuk, B. Trocmé, C. Troncon, M. Trottier-McDonald, M. Trovatelli, P. True, M. Trzebinski, A. Trzupek, C. Tsarouchas, J. C-L. Tseng, P. V. Tsiareshka, D. Tsionou, G. Tsipolitis, N. Tsirintanis, S. Tsiskaridze, V. Tsiskaridze, E. G. Tskhadadze, I. I. Tsukerman, V. Tsulaia, S. Tsuno, D. Tsybychev, A. Tudorache, V. Tudorache, A. N. Tuna, S. A. Tupputi, S. Turchikhin, D. Turecek, R. Turra, A. J. Turvey, P. M. Tuts, A. Tykhonov, M. Tylmad, M. Tyndel, I. Ueda, R. Ueno, M. Ughetto, M. Ugland, M. Uhlenbrock, F. Ukegawa, G. Unal, A. Undrus, G. Unel, F. C. Ungaro, Y. Unno, C. Unverdorben, J. Urban, P. Urquijo, P. Urrejola, G. Usai, A. Usanova, L. Vacavant, V. Vacek, B. Vachon, C. Valderanis, N. Valencic, S. Valentinetti, A. Valero, L. Valery, S. Valkar, E. Valladolid Gallego, S. Vallecorsa, J. A. Valls Ferrer, W. Van Den Wollenberg, P. C. Van Der Deijl, R. van der Geer, H. van der Graaf, R. Van Der Leeuw, N. van Eldik, P. van Gemmeren, J. Van Nieuwkoop, I. van Vulpen, M. C. van Woerden, M. Vanadia, W. Vandelli, R. Vanguri, A. Vaniachine, F. Vannucci, G. Vardanyan, R. Vari, E. W. Varnes, T. Varol, D. Varouchas, A. Vartapetian, K. E. Varvell, F. Vazeille, T. Vazquez Schroeder, J. Veatch, F. Veloso, T. Velz, S. Veneziano, A. Ventura, D. Ventura, M. Venturi, N. Venturi, A. Venturini, V. Vercesi, M. Verducci, W. Verkerke, J. C. Vermeulen, A. Vest, M. C. Vetterli, O. Viazlo, I. Vichou, T. Vickey, O. E. Vickey Boeriu, G. H. A. Viehhauser, S. Viel, R. Vigne, M. Villa, M. Villaplana Perez, E. Vilucchi, M. G. Vincter, V. B. Vinogradov, I. Vivarelli, F. Vives Vaque, S. Vlachos, D. Vladoiu, M. Vlasak, M. Vogel, P. Vokac, G. Volpi, M. Volpi, H. von der Schmitt, H. von Radziewski, E. von Toerne, V. Vorobel, K. Vorobev, M. Vos, R. Voss, J. H. Vossebeld, N. Vranjes, M. Vranjes Milosavljevic, V. Vrba, M. Vreeswijk, R. Vuillermet, I. Vukotic, Z. Vykydal, P. Wagner, W. Wagner, H. Wahlberg, S. Wahrmund, J. Wakabayashi, J. Walder, R. Walker, W. Walkowiak, C. Wang, F. Wang, H. Wang, H. Wang, J. Wang, J. Wang, K. Wang, R. Wang, S. M. Wang, T. Wang, X. Wang, C. Wanotayaroj, A. Warburton, C. P. Ward, D. R. Wardrope, M. Warsinsky, A. Washbrook, C. Wasicki, P. M. Watkins, A. T. Watson, I. J. Watson, M. F. Watson, G. Watts, S. Watts, B. M. Waugh, S. Webb, M. S. Weber, S. W. Weber, J. S. Webster, A. R. Weidberg, B. Weinert, J. Weingarten, C. Weiser, H. Weits, P. S. Wells, T. Wenaus, T. Wengler, S. Wenig, N. Wermes, M. Werner, P. Werner, M. Wessels, J. Wetter, K. Whalen, A. M. Wharton, A. White, M. J. White, R. White, S. White, D. Whiteson, F. J. Wickens, W. Wiedenmann, M. Wielers, P. Wienemann, C. Wiglesworth, L. A. M. Wiik-Fuchs, A. Wildauer, H. G. Wilkens, H. H. Williams, S. Williams, C. Willis, S. Willocq, A. Wilson, J. A. Wilson, I. Wingerter-Seez, F. Winklmeier, B. T. Winter, M. Wittgen, J. Wittkowski, S. J. Wollstadt, M. W. Wolter, H. Wolters, B. K. Wosiek, J. Wotschack, M. J. Woudstra, K. W. Wozniak, M. Wu, M. Wu, S. L. Wu, X. Wu, Y. Wu, T. R. Wyatt, B. M. Wynne, S. Xella,

D. Xu, L. Xu, B. Yabsley, S. Yacoob, R. Yakabe, M. Yamada, Y. Yamaguchi, A. Yamamoto, S. Yamamoto, T. Yamanaka, K. Yamauchi, Y. Yamazaki, Z. Yan, H. Yang, H. Yang, Y. Yang, L. Yao, W-M. Yao, Y. Yasu, E. Yatsenko, K. H. Yau Wong, J. Ye, S. Ye, I. Yeletskikh, A. L. Yen, E. Yildirim, K. Yorita, R. Yoshida, K. Yoshihara, C. Young, C. J. S. Young, S. Youssef, D. R. Yu, J. Yu, J. M. Yu, J. Yu, L. Yuan, A. Yurkewicz, I. Yusuff, B. Zabinski, R. Zaidan, A. M. Zaitsev, J. Zalieckas, A. Zaman, S. Zambito, L. Zanello, D. Zanzi, C. Zeitnitz, M. Zeman, A. Zemla, K. Zengel, O. Zenin, T. Ženiš, D. Zerwas, D. Zhang, F. Zhang, J. Zhang, L. Zhang, R. Zhang, X. Zhang, Z. Zhang, X. Zhao, Y. Zhao, Z. Zhao, A. Zhemchugov, J. Zhong, B. Zhou, C. Zhou, L. Zhou, L. Zhou, N. Zhou, C. G. Zhu, H. Zhu, J. Zhu, Y. Zhu, X. Zhuang, K. Zhukov, A. Zibell, D. Zieminska, N. I. Zimine, C. Zimmermann, R. Zimmermann, S. Zimmermann, Z. Zinonos, M. Zinser, M. Ziolkowski, L. Živković, G. Zobernig, A. Zoccoli, M. zur Nedden, G. Zurzolo, and L. Zwalinski.
Determination of spin and parity of the Higgs boson in the $$WW^*\rightarrow e \nu \mu \nu $$ W W * → e ν μ ν decay channel with the ATLAS detector.
Eur. Phys. J. C, 75(5):231, May 2015.
doi: 10.1140/epjc/s10052-015-3436-3.

[3] Robert R. Birss.
Symmetry and Magnetism.
North-Holland Pub. Co., 1964.

[4] C. M. Canali and S. M. Girvin.
Theory of Raman scattering in layered cuprate materials.
Phys. Rev. B, 45(13):7127–7160, April 1992.
doi: 10.1103/PhysRevB.45.7127.

[5] F.M. Dekking, C. Kraaikamp, H.P. Lopuhaä, and L.E. Meester.
A Modern Introduction to Probability and Statistics: Understanding Why and How.
Springer Texts in Statistics. Springer London, 2006.

[6] S. L. Dudarev, G. A. Botton, S. Y. Savrasov, C. J. Humphreys, and A. P. Sutton.

Electron-energy-loss spectra and the structural stability of nickel oxide: An LSDA+U study.
Phys. Rev. B, 57:1505–1509, 1998.
doi: 10.1103/PhysRevB.57.1505.

[7] Von H.-P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder, and K. Yamazaki.
Zur Theorie der Elementarteilchen.
Zeitschrift für Kristallographie – Crystalline Materials, 14(5-6):441, 1959.
doi: https://doi.org/10.1515/zna-1959-5-601.

[8] Fabian H. L. Essler, Alexei M. Tsvelik, and Gesualdo Delfino.
Quasi-one-dimensional spin- 1 2 Heisenberg magnets in their ordered phase: Correlation functions.
Phys. Rev. B, 56(17):11001–11013, November 1997.
doi: 10.1103/PhysRevB.56.11001.

[9] Bryan T. Fichera, Anshul Kogar, Linda Ye, Bilal Gökce, Alfred Zong, Joseph G. Checkelsky, and Nuh Gedik.
Second harmonic generation as a probe of broken mirror symmetry.
Phys. Rev. B, 101(24):241106, June 2020.
doi: 10.1103/PhysRevB.101.241106.

[10] Shunsuke Furukawa, Masahiro Sato, Shigeki Onoda, and Akira Furusaki.
Ground-state phase diagram of a spin- 1 2 frustrated ferromagnetic XXZ chain: Haldane dimer phase and gapped/gapless chiral phases.
Phys. Rev. B, 86(9):094417, September 2012.
doi: 10.1103/PhysRevB.86.094417.

[11] Stefan Grimme.
Semiempirical GGA-type density functional constructed with a long-range dispersion correction.
J Comput Chem, 27(15):1787–1799, November 2006.
doi: 10.1002/jcc.20495.

[12] J. W. Harter, L. Niu, A. J. Woss, and D. Hsieh.
High-speed measurement of rotational anisotropy nonlinear optical harmonic generation using position-sensitive detection.

Opt. Lett., 40(20):4671, October 2015.
doi: 10.1364/OL.40.004671.

[13] Tao Hong, Masashige Matsumoto, Yiming Qiu, Wangchun Chen, Thomas R. Gentile, Shannon Watson, Firas F. Awwadi, Mark M. Turnbull, Sachith E. Dissanayake, Harish Agrawal, Rasmus Toft-Petersen, Bastian Klemke, Kris Coester, Kai P. Schmidt, and David A. Tennant.
Higgs amplitude mode in a two-dimensional quantum antiferromagnet near the quantum critical point.
Nature Phys, 13(7):638–642, July 2017.
doi: 10.1038/nphys4182.

[14] A. Jain, M. Krautloher, J. Porras, G. H. Ryu, D. P. Chen, D. L. Abernathy, J. T. Park, A. Ivanov, J. Chaloupka, G. Khaliullin, B. Keimer, and B. J. Kim.
Higgs mode and its decay in a two-dimensional antiferromagnet.
Nature Phys, 13(7):633–637, July 2017.
doi: 10.1038/nphys4077.

[15] Hosho Katsura, Naoto Nagaosa, and Alexander V. Balatsky.
Spin Current and Magnetoelectric Effect in Noncollinear Magnets.
Phys. Rev. Lett., 95(5):057205, July 2005.
doi: 10.1103/PhysRevLett.95.057205.

[16] Hosho Katsura, Alexander V. Balatsky, and Naoto Nagaosa.
Dynamical Magnetoelectric Coupling in Helical Magnets.
Phys. Rev. Lett., 98(2):027203, January 2007.
doi: 10.1103/PhysRevLett.98.027203.

[17] S. Lebernegg, M. Schmitt, A. A. Tsirlin, O. Janson, and H. Rosner.
Magnetism of CuX2 frustrated chains (X = F, Cl, Br): the role of covalency.
Phys. Rev. B, 87(15):155111, April 2013.
doi: 10.1103/PhysRevB.87.155111.
arXiv:1303.4063 [cond-mat].

[18] C. Lee, Jia Liu, M.-H. Whangbo, H.-J. Koo, R. K. Kremer, and A. Simon.

Investigation of the spin exchange interactions and the magnetic structure of the high-temperature multiferroic CuBr 2.
Phys. Rev. B, 86(6):060407, August 2012.
doi: 10.1103/PhysRevB.86.060407.

[19] Masashige Matsumoto.
Electromagnon as a Probe of Higgs (Longitudinal) Mode in Collinear and Noncollinear Magnetically Ordered States.
J. Phys. Soc. Jpn., 83(8):084704, August 2014.
doi: 10.7566/JPSJ.83.084704.

[20] Masashige Matsumoto.
Electromagnon excitation and longitudinal mode studied on the basis of symmetric spin-dependent electric polarization.
J. Phys.: Conf. Ser., 592:012123, March 2015.
doi: 10.1088/1742-6596/592/1/012123.

[21] K Morey, Bryan T. Fichera, Baiqing Lv, Zongqi Shen, and Nuh Gedik.
Automated polarization rotation for multi-axis rotational-anisotropy second harmonic generation experiments.
In preparation.

[22] Yoichiro Nambu.
Axial Vector Current Conservation in Weak Interactions.
Phys. Rev. Lett., 4(7):380–382, April 1960.
doi: 10.1103/PhysRevLett.4.380.

[23] David Pekker and C.M. Varma.
Amplitude/Higgs Modes in Condensed Matter Physics.
Annu. Rev. Condens. Matter Phys., 6(1):269–297, March 2015.
doi: 10.1146/annurev-conmatphys-031214-014350.

[24] Ch. Rüegg, B. Normand, M. Matsumoto, A. Furrer, D. F. McMorrow, K. W. Krämer, H. U. Güdel, S. N. Gvasaliya, H. Mutka, and M. Boehm.
Quantum Magnets under Pressure: Controlling Elementary Excitations in TlCuCl 3.
Phys. Rev. Lett., 100(20):205701, May 2008.
doi: 10.1103/PhysRevLett.100.205701.

[25] D. Sa, R. Valentí, and C. Gros.
A generalized Ginzburg-Landau approach to second harmonic generation.
Eur. Phys. J. B, 14(2):301–305, March 2000.
doi: 10.1007/s100510050133.

[26] H. J. Schulz.
Dynamics of Coupled Quantum Spin Chains.
Phys. Rev. Lett., 77(13):2790–2793, September 1996.
doi: 10.1103/PhysRevLett.77.2790.

[27] Ying Su, A. Masaki-Kato, Wei Zhu, Jian-Xin Zhu, Yoshitomo Kamiya, and Shi-Zeng Lin.
Stable Higgs mode in anisotropic quantum magnets.
Phys. Rev. B, 102(12):125102, September 2020.
doi: 10.1103/PhysRevB.102.125102.

[28] Atsushi Togo and Isao Tanaka.
First principles phonon calculations in materials science.
Scripta Materialia, 108:1–5, November 2015.
doi: 10.1016/j.scriptamat.2015.07.021.

[29] Darius H. Torchinsky, Hao Chu, Tongfei Qi, Gang Cao, and David Hsieh.
A low temperature nonlinear optical rotational anisotropy spectrometer for the determination of crystallographic and electronic symmetries.
Review of Scientific Instruments, 85(8):083102, August 2014.
doi: 10.1063/1.4891417.

[30] Chong Wang, Daiwei Yu, Xiaoqiang Liu, Rongyan Chen, Xinyu Du, Biaoyan Hu, Lichen Wang, Kazuki Iida, Kazuya Kamazawa, Shuichi Wakimoto, Ji Feng, Nanlin Wang, and Yuan Li.
Observation of magnetoelastic effects in a quasi-one-dimensional spiral magnet.
Phys. Rev. B, 96(8):085111, August 2017.
doi: 10.1103/PhysRevB.96.085111.

[31] Rui-Qi Wang, Jia-Cheng Zheng, Tao Chen, Peng-Shuai Wang, Jin-Shan Zhang, Yi Cui, Chao Wang, Yuan Li, Sheng Xu, Feng Yuan, and Wei-Qiang Yu.
NMR evidence of charge fluctuations in multiferroic $CuBr_2$.
Chinese Phys. B, 27(3):037502, March 2018.
doi: 10.1088/1674-1056/27/3/037502.

[32] J. S. Zhang, Yiqi Xie, X. Q. Liu, A. Razpopov, V. Borisov, C. Wang, J. P. Sun, Y. Cui, J. C. Wang, X. Ren, Hongshan Deng, Xia Yin, Yang Ding, Yuan Li, J. G. Cheng, Ji Feng, R. Valentí, B. Normand, and Weiqiang Yu.
Giant pressure-enhancement of multiferroicity in CuBr 2.
Phys. Rev. Research, 2(1):013144, February 2020.
doi: 10.1103/PhysRevResearch.2.013144.

[33] Li. Zhao, Tsu-Lien Hung, Ching-Chien Li, Yang-Yuan Chen, Maw-Kuen Wu, Reinhard K. Kremer, Michael G. Banks, Arndt Simon, Myung-Hwan Whangbo, Changhoon Lee, Jun Sung Kim, Ingyu Kim, and Kee Hoon Kim.
CuBr2 - A New Multiferroic Material with High Critical Temperature.
Adv. Mater., 24(18):2469–2473, May 2012.
doi: 10.1002/adma.201200734.

[34] Chengkang Zhou, Zheng Yan, Han-Qing Wu, Kai Sun, Oleg A. Starykh, and Zi Yang Meng.
Amplitude Mode in Quantum Magnets via Dimensional Crossover.
Phys. Rev. Lett., 126(22):227201, June 2021.
doi: 10.1103/PhysRevLett.126.227201.