MATH 140 Quiz 5 (Home)

2.5

17) $f(x) = (2x-3)^4(x^2+x+1)^5$

$f'(x) = 4(2x-3)^3(2)(x^2+x+1)^5 + (2x-3)^4 \cdot 5(x^2+x+1)^4(2x+1)$

$= 8(2x-3)^3(x^2+x+1)^5 + (10x+5)(2x-3)^4(x^2+x+1)^4$

25) $h(\theta) = \tan(\theta^2 \sin\theta)$

$h'(\theta) = \sec^2(\theta^2\sin\theta) \cdot \frac{d}{d\theta}(\theta^2\sin\theta)$

$= \sec^2(\theta^2\sin\theta) \cdot [2\theta\sin\theta + \theta^2\cos\theta]$

27) $y = \dfrac{\cos x}{\sqrt{1+\sin x}}$

$y' = \dfrac{\sqrt{1+\sin x}\,(-\sin x) - \cos x\left(\frac{1}{2\sqrt{1+\sin x}} \cdot \cos x\right)}{\left(\sqrt{1+\sin x}\right)^2}$

$= \dfrac{-\sin x\sqrt{1+\sin x} - \cos^2 x\left(\frac{1}{2\sqrt{1+\sin x}}\right)}{1+\sin x}$

$= \dfrac{-2\sin x(1+\sin x) - \cos^2 x}{2(1+\sin x)^{3/2}}$

$= \dfrac{-2\sin x - 2\sin^2 x - \cos^2 x}{2(1+\sin x)^{3/2}} = -\left(\dfrac{1+2\sin x+\sin^2 x}{2(1+\sin x)^{3/2}}\right)$

37) $y = \cot^2(\sin\theta) = \left[\cot(\sin\theta)\right]^2$

$$\frac{dy}{d\theta} = 2\cot(\sin\theta)\cdot\frac{d}{d\theta}\left[\cot(\sin\theta)\right]$$

$$= 2\cot(\sin\theta)\cdot\left(-\csc^2(\sin\theta)\cdot\frac{d}{d\theta}(\sin\theta)\right)$$

$$= -2\cot(\sin\theta)\cdot\csc^2(\sin\theta)\cdot\cos\theta$$

2.6

7) $x^4 + x^2y^2 + y^3 = 5$

$$\Rightarrow 4x^3 + \left[2xy^2 + x^2\left(2y\frac{dy}{dx}\right)\right] + 3y^2\frac{dy}{dx} = 0$$

$$2x^2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -4x^3 - 2xy^2$$

$$\frac{dy}{dx} = \frac{-4x^3 - 2xy^2}{2x^2y + 3y^2}$$

13) $\sqrt{x+y} = x^4 + y^4$

$$\Rightarrow \frac{1}{2\sqrt{x+y}}\cdot\left(1 + \frac{dy}{dx}\right) = 4x^3 + 4y^3\frac{dy}{dx}$$

$$1 + \frac{dy}{dx} = 8x^3\sqrt{x+y} + 8y^3\sqrt{x+y}\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx} - 8y^3\sqrt{x+y}\cdot\frac{dy}{dx} = 8x^3\sqrt{x+y} - 1$$

$$\frac{dy}{dx} = \frac{8x^3\sqrt{x+y} - 1}{1 - 8y^3\sqrt{x+y}}$$

19) $\sin(xy) = \cos(x+y)$

$$\Rightarrow \cos(xy)\left[y + x\frac{dy}{dx}\right] = -\sin(x+y)\cdot\left[1 + \frac{dy}{dx}\right]$$

$$y\cos(xy) + x\cos(xy)\frac{dy}{dx} = -\sin(x+y) - \sin(x+y)\frac{dy}{dx}$$

$$x\cos(xy)\frac{dy}{dx} + \sin(x+y)\frac{dy}{dx} = -y\cos(xy) - \sin(x+y)$$

$$\frac{dy}{dx} = \frac{-y\cos(xy) - \sin(x+y)}{x\cos(xy) + \sin(x+y)}$$

31) $2(x^2+y^2)^2 = 25(x^2-y^2), \quad (3, 1)$

$$4(x^2+y^2)\left[2x + 2y\frac{dy}{dx}\right] = 50x - 50y\frac{dy}{dx}$$

$x=3$, $y=1$ $\Rightarrow$

$$4(9+1)\left[6 + 2\frac{dy}{dx}\right] = 150 - 50\frac{dy}{dx}$$

$$240 + 80\frac{dy}{dx} = 150 - 50\frac{dy}{dx}$$

$$130\frac{dy}{dx} = -90$$

$$\Rightarrow \text{slope} = -\frac{9}{13}$$

Equation: $\boxed{y - 1 = -\frac{9}{13}(x-3)}$