# Software-Defined GPS Receiver

July 21, 2019

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# 1 GPS Primer

The Global Positioning System is the Global Navigation Satellite System (GNSS) run by the US Air Force. The basic principle is a simple and powerful concept - a constellation of satellites in Medium Earth Orbit that continuosly broadcast a signal that can be precisely timed by a receiver.

With knowledge of the satellite position and time of transmission of multiple signals, using time-of-flight the user's position can be precisely calculated. With precise knowledge of time (order of nanoseconds), this would require three satellites to solve the three unknowns of three-dimensional position. In general, the user does not have this knowledge, and instead solves for time along with position, which requires a fourth satellite.

The legacy GPS system uses two frequencies - L1 (1575.42 MHz) and L2 (1227.6 MHz). All satellites broadcast on the same frequency using code-division multiple access (CDMA). There are two code sequences in use: the Coarse/Acquisition (C/A) and Precision (P) code. The C/A code is freely avialable to the public, while the P code is encrypted for military use. The C/A code is broadcast on L1, while the P code is broadcast on both L1 and L2. In short, the only legacy signal available to the public is the C/A code on L1. The GPS system is undergoing modernization, and new signals are being added for both the public and the military.

The C/A code is a repeating 1023-bit sequence that is broadcast at 1.023 MHz, or once per millisecond. There are 32 unique C/A code sequences in use by the GPS Constellation, and they are designed to be orthogonal with each other, as well as with themselves at offsets other than 0. The autocorrelation function of the C/A code is triangular between +/- 1 chip, and near zero everywhere else. See Figure 1.

The user requires data in order to determine their position from this signal - mainly the position of the satellite and the time. The Legacy Navigation Message (LNAV) contains this information. It is a framed data stream, with 1500 bits per frame, divided into five 300-bit subframes. Subframes 1-3 contain the time and ephemeris data, while subframes 4 & 5 rotate through 25 pages and contain the Almanac and Constellation status. The LNAV message is modulo-2 added to the C/A bitstream at 50 bps, so each subframe takes 6 seconds, each full frame takes 30 seconds, and all 25 pages take 12.5 minutes to broadcast. This composite bitstream is then modulated onto the carrier using binary phase-shift keying (BPSK). See Figure 2.

The GPS signal is very weak - in fact the spread signal is below the thermal noise floor. The signal is therefore not directly observable — it is indirectly observed using its correlation properties.

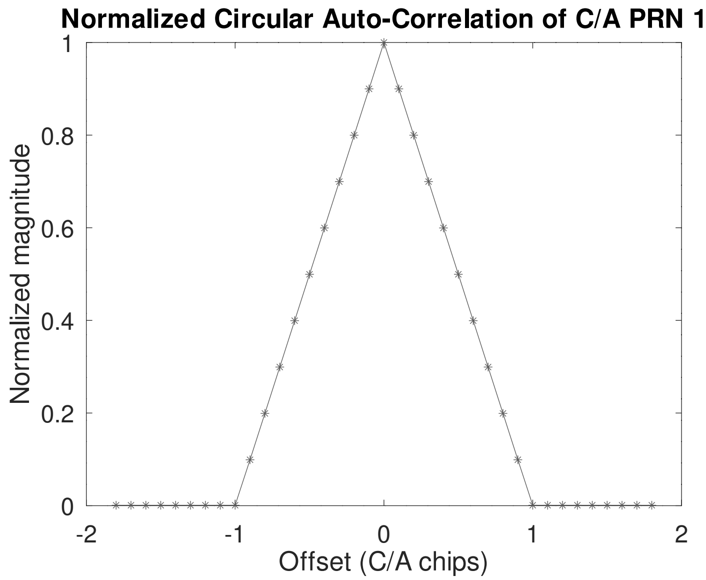


Figure 1: C/A Code Auto Correlation

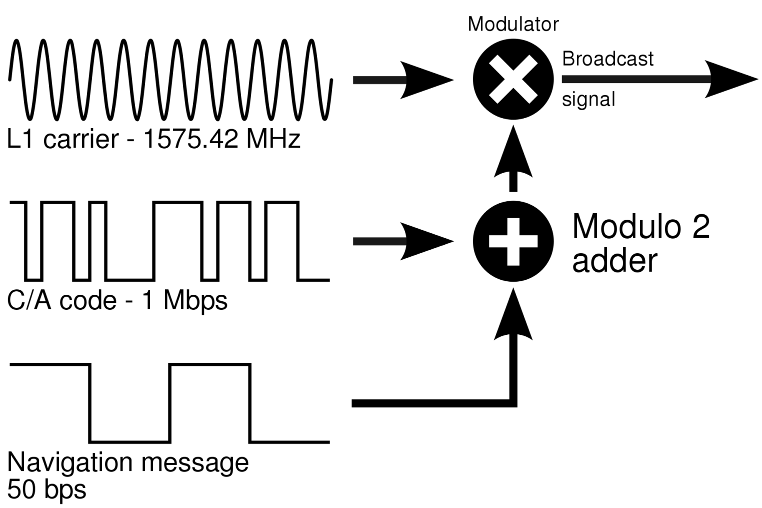


Figure 2: C/A Code Generation[1]

# 2 Receiver Operation

The GPS Receiver moves through several states to acquire position. In order, they are:

1. Search for satellites
2. Begin tracking satellites
3. Find nav-bit edges
4. Find preamble (and fix inversion if found)
5. Demodulate LNAV data
6. Calculate position and transmit time for all satellites
7. Calculate user position and time

This section will walk through each step and the way they are implemented.

## 2.1 Search

In order to find a satellite, we have to conduct a two-dimensional search for the signal over chip-offset and frequency-offset. The frequency offset is caused primarily by Doppler and user clock error. SDGPSR searches over a 10 KHz range of frequency offsets in 500 Hz intervals, and across all 1023 chips in 0.5 chip intervals.

The naive way of conducting the search is to do a time-domain cross-correlation between the known chipping sequence and the received signal. For a sample length N, each chip-offset bucket would cost N multiplications and N additions. With roughly N buckets per frequency, the cost is $O(N^2)$. However, by converting both signals into the frequency domain, conducting element-wise multiplication, then converting back to the time domain, the order of the cost is reduced to that of the FFT, which is $O(NlogN)$.

A search is conducted as the sum of 128 non-coherent integration periods over the 2-dimensional search space for all satellites, at which point each search window is evaluated to determine if a satellite has been found. When a strong signal is present, it appears as shown in Figure 3.

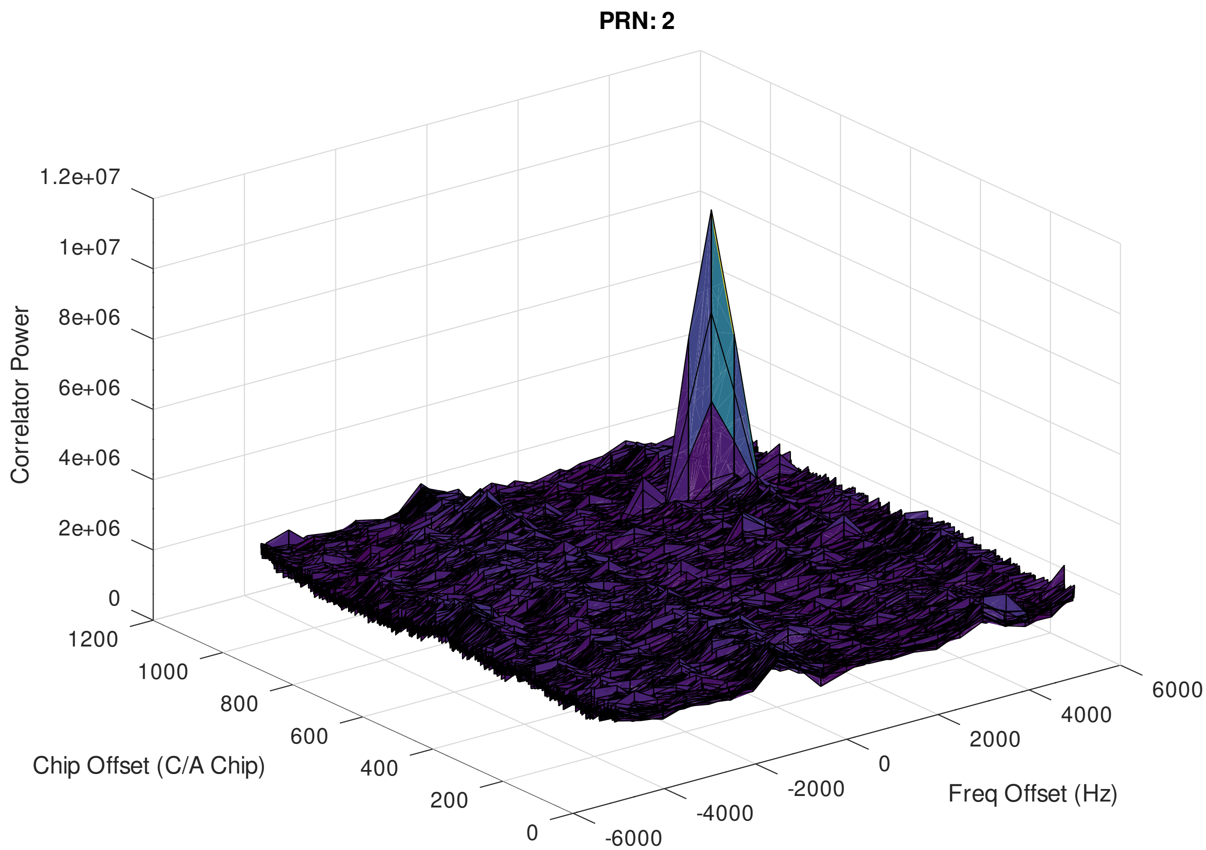


Figure 3: GPS Search Window

## 2.2 Track

Each satellites that is found has a tracking channel created for it. The tracking channel is initialized with several signal trackers, all with different frequency offsets centered around the estimated frequency offset. Each of these attempts to track the satellite, and once one of them gets a lock, the others are deleted.

Each signal tracker begins with an estimated chip and frequency offset, and proceeds through several states in order to fully lock onto the signal. Integration starts off in 1-ms intervals, since it is unknown where the nav-bit edge is. The signal tracker starts off in state 1, which is closing the carrier Frequency-Locked Loop (FLL). If at any point the code locked loop has a loss of lock, the entire signal tracker goes into loss of lock mode and will be deleted.

Once the carrier FLL is locked, the tracker proceeds to state 2 — closing the carrier Phase-Locked Loop (PLL), which uses a Costas loop. As soon as the PLL is locked, the phase error of each integration period is monitored. A phase error greater than 90 degrees is considered a candidate nav-bit edge, and a series of 5 candidates at intervals that are multiples of 20 ms (the nav-bit period) from each other are used to confirm the nav-bit edge was found. The integration interval then increases to 20 ms, spanning each nav-bit. This is considered state 4 — full track.

Figures 4 & 5 show the output of the three correlators and the tracking state of a strong signal during acquisition and subsequent tracking. The channel rapidly acquires full lock on the signal, at which point the correlation interval increases from 1 to 20 ms - hence the increase in correlator power after about 0.6 second.

## 2.3 Data Demodulation

Once the channel is in full track, the result of each integration period is evaluated as a nav-bit. This is passed to the LNAV_Data class, which begins attempting to correlate the nav-bits with the 8-bit preamble on word 1 of each subframe. If a correlation is found and it is negative, all of the candidate bits are flipped. The checksum of the candidate word is evaluated, and if found to be good, the LNAV_Data class begins filling up the data structures. Only subframes 1-3 are processed at this time, the data from subframes 4 & 5 is ignored.

All subframes have time of week (TOW). Subframe 1 is required for time corrections and week number. Subframes 2 & 3 are required for ephemeris data. The IODE variable (Issue of Data, Ephemeris) is incremented every time the satellite changes the ephemeris data. In order to ensure the receiver never uses split ephemeris data, as the data is read in it is stored as the “nextEphemeris.” Once a complete set of matching data is collected, the “currentEphemeris” can

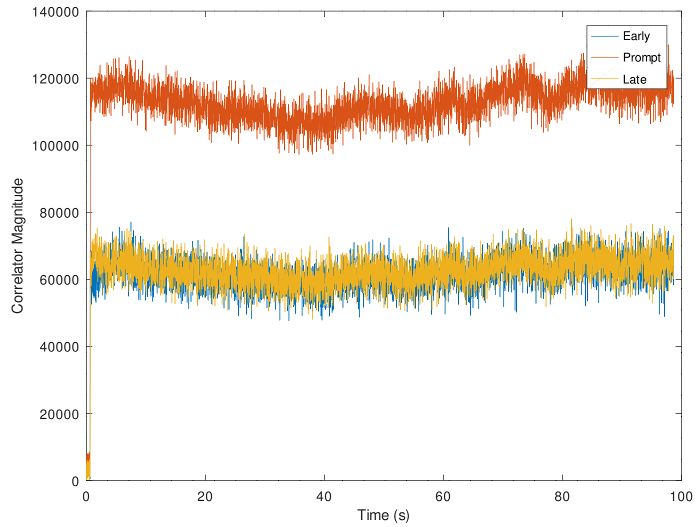


Figure 4: Correlators during track

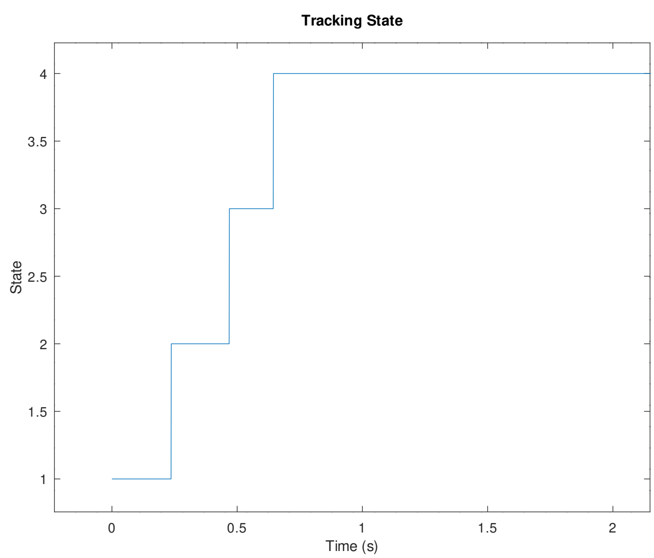


Figure 5: Signal Tracker state during acquisition

be updated with the “nextEphemeris.”

### 2.4 Satellite position and transmission time

Satellite transmission time is determined based on the TOW count in each subframe, the week number, clock correction parameters, relativistic errors, and the code tracking loop. Satellite transmission time along with the ephemeris is used to calculate the position of the satellite at signal transmission according to the orbital equations of IS-GPS-200.

### 2.5 User position and time

Once subframes 1-3 have been downloaded from at least 4 satellites, the user time and position can be calculated.

The performance of the receiver was evaluated based on the mean navigation solution, as well as the error between solutions. Figure 6 shows the time error that accumulates between successive nav solutions - this is due to bias on the SDR clock, which is not currently being tracked. Figure 7 shows the error in ECEF position between successive navigation solutions, which appears to be unbiased. The receiver was stationary during this test — since velocity is not currently tracked, any motion of the receiver would cause a bias in these measurements. Both Figures 6 & 7 have a jump in error around 4386 sec — this is due to an additional satellite being included in the measurement.

Then, the difference between range and pseudorange (the residuals) was plotted on Figure 8. This clearly shows a bias starting at about -200 m and ending at about -180 m, and a distribution over 20-30 m. The bias is due to the clock error - this can be confirmed by multiplying the bias in Figure 6 by the speed of light. The spread of the errors is presumably due at least in part to atmospheric delays (Troposphere & Ionosphere), which are not currently corrected for.

Lastly, the mean position was compared with the position of the antenna, which was found to have a horizontal error of less than 1 m, and a vertical error of 12 m. This squares with expectations for a receiver that is not correcting for atmospheric errors.

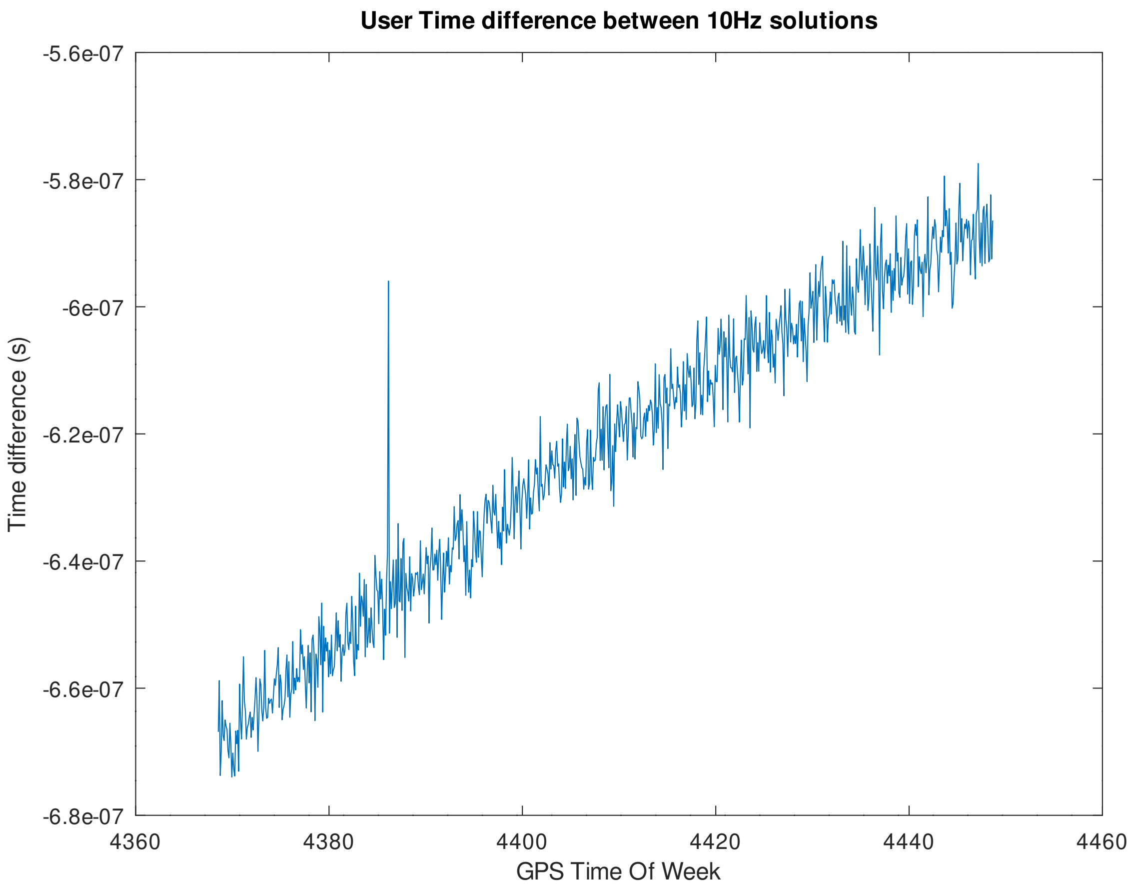


Figure 6: Time error between successive Nav solutions

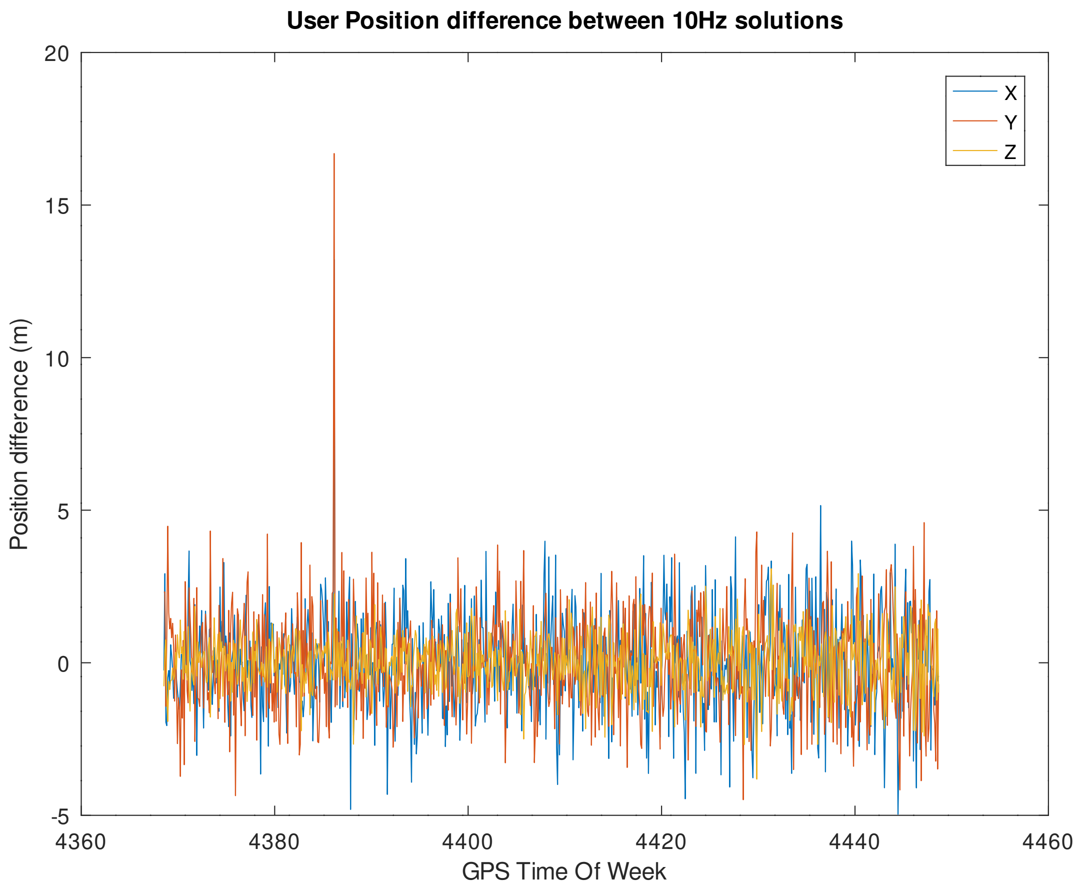


Figure 7: ECEF Position error between successive Nav solutions

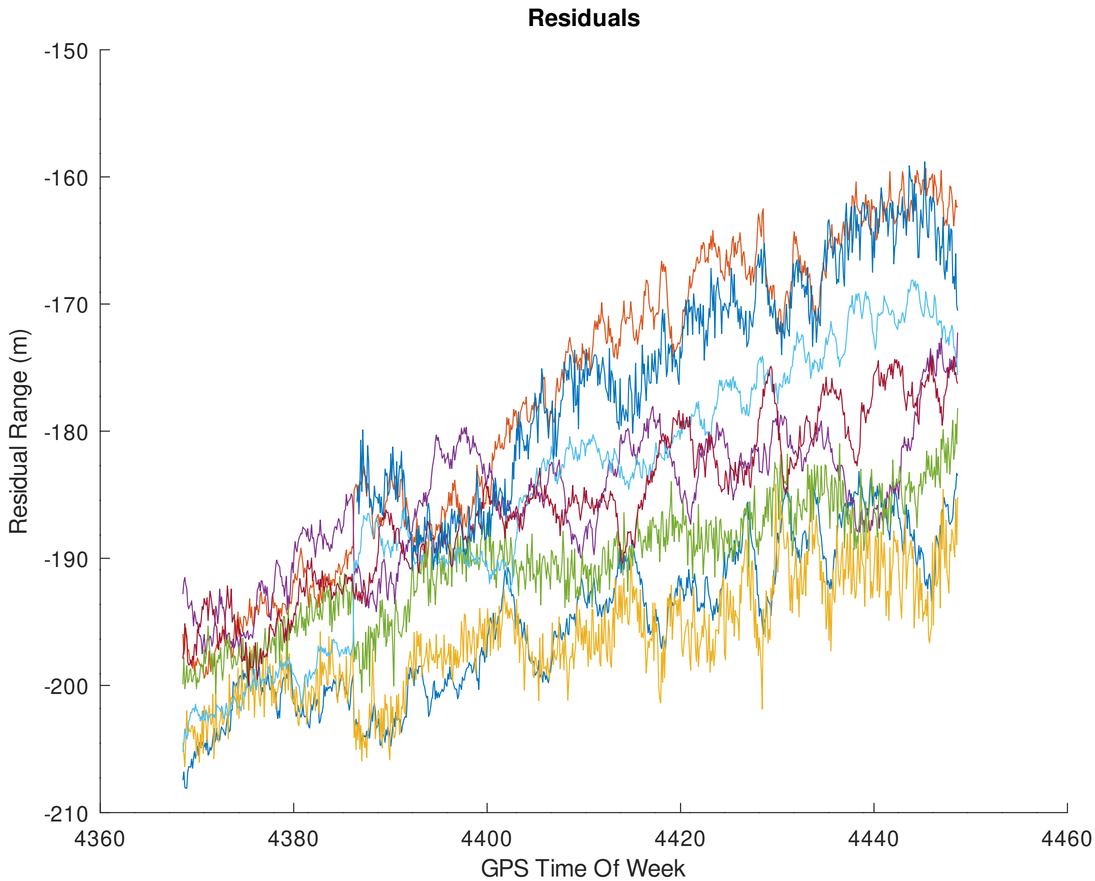


Figure 8: Difference between calculated range and pseudorange for each satellite between successive Nav solutions

## References


[1] Lammertsma, P.F. *Satellite Navigation*
`paul.luminos.nl/download/document/satellite_navigation.pdf`

[2] Elliott D. Kaplan, Christopher J. Hegarty *Understanding GPS: Principles and Applications, 2nd edition* Artech House, Inc, Norwood, MA, 2006