PARALLELS IN GEOMETRY (SUPPLEMENTS)

MATH 1166: SPRING 2019
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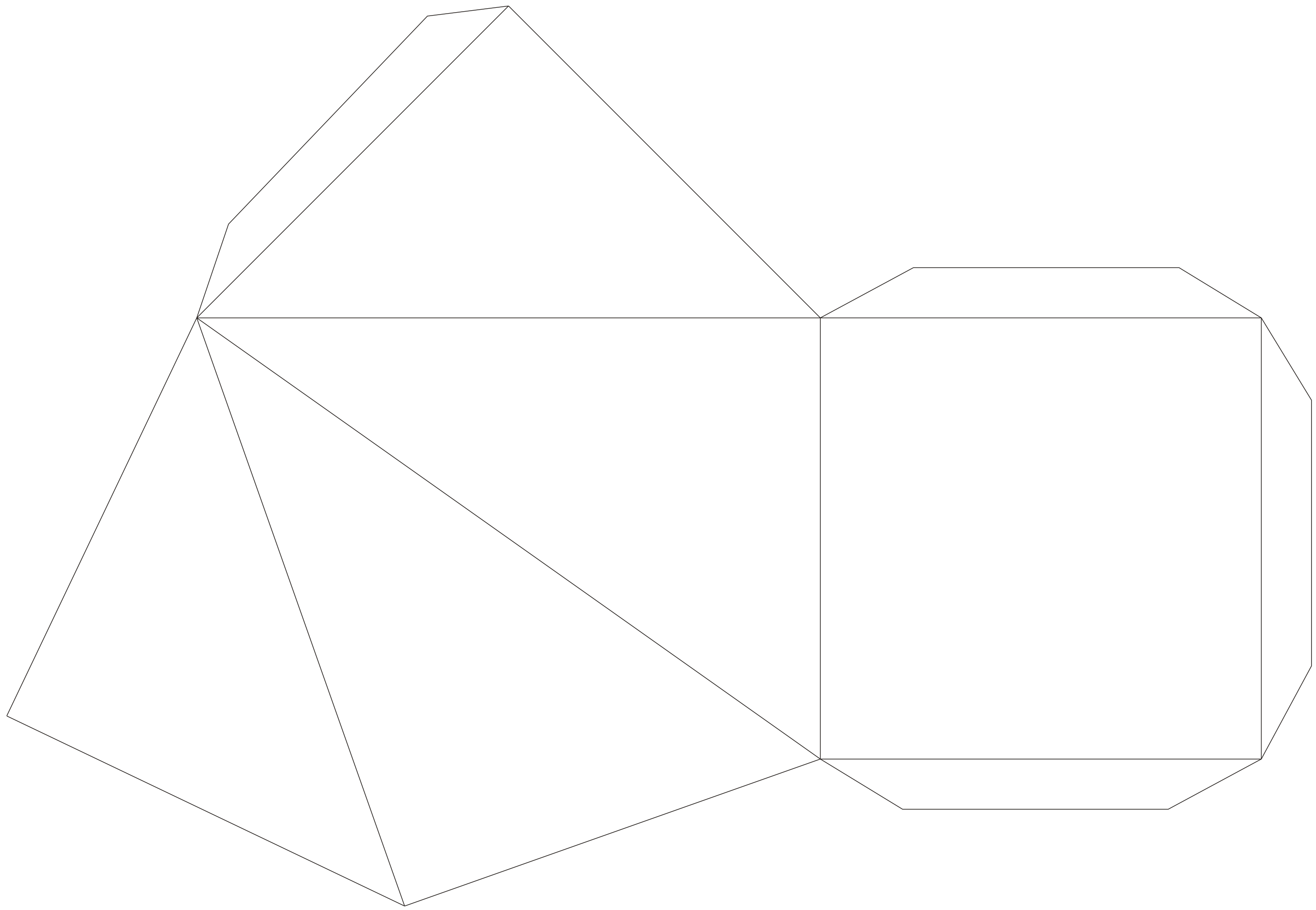
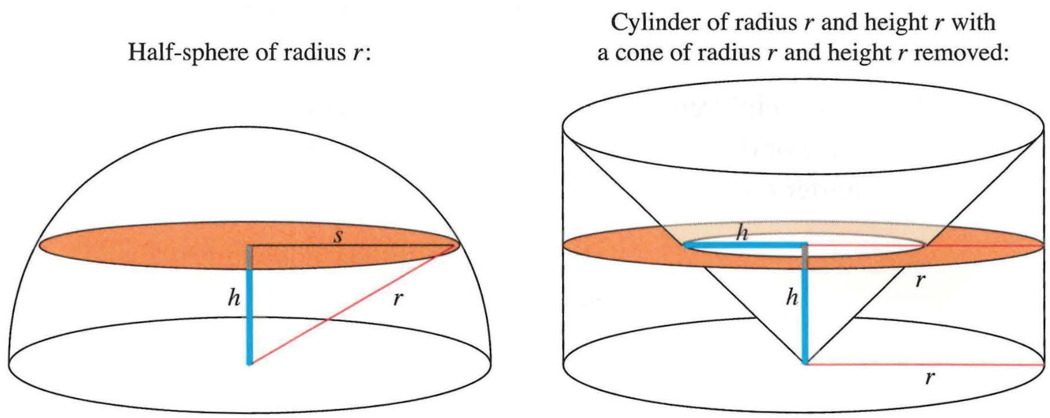
Contents

A Supplemental Activities





From Beckmann, 2014, *Mathematics for Elementary Teachers*

B Supplemental Problems

B.1 Midterm 1 Review, draft 02/08/19

Midterm Exam 1 will cover Chapters 1 – 3 (except section 1.2 and the angle trisection part of 3.1) and activities A.1 – A.18. We did not complete all of these activities in class so you will need to fill in some gaps.

B.1.1 Review Ideas

- Be able to state definitions of commonly used terms, such as
  - Perpendicular line, parallel line, line segment, ray, angle, circle
  - Concurrent, collinear
  - Equilateral, equiangular, and regular polygons
  - Acute, obtuse, and right angles and triangles; isosceles and scalene triangles
  - Straight, complementary, and supplementary angles
  - Trapezoid (inclusive and exclusive), parallelogram, rhombus, rectangle, square, kite
  - Median, perpendicular bisector, angle bisector, altitude
  - Centroid, circumcenter, incenter, orthocenter
  - Chord, arc, arc measure, central angle, inscribed angle, tangent
- Be able to perform standard constructions and explain why they work.
- Be able to draw (careful) figures satisfying particular conditions.
- Be able to explain key concepts such as area and angle.
- Be able to state precisely the triangle congruence criteria.
- Know the properties of various special quadrilaterals and be able to prove them.
- Know the various centers of a triangle, how to construct them, and whether they can lie outside the triangle.
- Be able to state key theorems and prove them in at least two (2) ways, especially:
  - Isosceles triangle theorem and its converse
  - Pythagorean theorem and its converse
  - The angle sum of a triangle

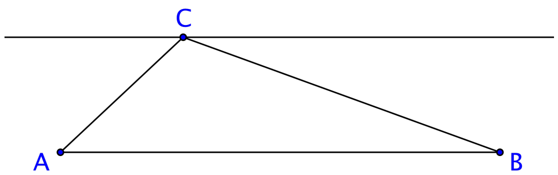
Review Problems

B.1.1) Demonstrate and describe Euclid's (compass and straightedge) construction for an equilateral triangle. Explain why it works.

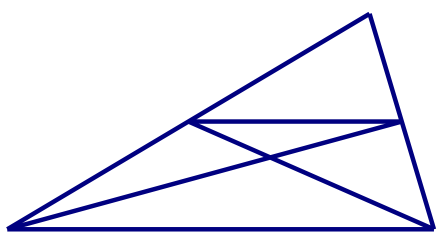
What about a square or a regular hexagon?

B.1.2) Maddie had an idea for proving that the sum of the interior angles in a triangle is 180°: Given $\triangle ABC$, draw a line through C parallel to $\overline{AB}$. Finish Maddie's proof.

What were the approaches we used in class?



B.1.3) Use the picture below to show that a pair of medians intersects at a point 2/3 of the way from the vertex to the opposite side. Then use that fact to argue that the three medians must be concurrent.



B.1.4) Prove that the points on an angle bisector are *exactly those* that are equidistant from the sides of the angle.

What about the perpendicular bisectors?

B.1.5) Prove that the perpendicular bisectors of a triangle are concurrent.

What about the angle bisectors?

B.1.6) Construct a 30-60-90 right triangle. Explain the steps in your construction and how you know it works.

What about a 45-45-90 triangle?

B.1.7) Where is the orthocenter of a right triangle? Explain your reasoning. What about the circumcenter? Again, explain your reasoning.

What about the other centers of a right triangle?

B.1.8) Show that, given any three non-collinear points in the Euclidean plane, there is a unique circle passing through the three points.

B.1.9) Construct a tangent line from a point outside a given circle to the circle.

B.1.10) Give an informal derivation of the relationship between the circumference and area of a circle.

B.1.11) Complete the following statement: When a quadrilateral is inscribed in a circle, opposite angles are . . . . Now prove the statement.

B.1.12) Claim: A radius that is perpendicular to a chord bisects the chord.

Use this problem structure for other problems.

(a) Prove the claim.

(b) State the converse of the claim.

(c) Is the converse true? If not, give a counterexample.

(d) If the converse is true, prove it. If the converse is false, "salvage it" to make a true statement, and prove it.

B.1.13) State and prove the isosceles triangle theorem.

What about the converse?

B.1.14) State and prove the Pythagorean theorem.

What about the converse?

B.1.15) Prove: Opposite sides of a parallelogram are congruent.

What about the converse?

B.1.16) Prove: The diagonals of a rhombus are perpendicular.

What about the converse?

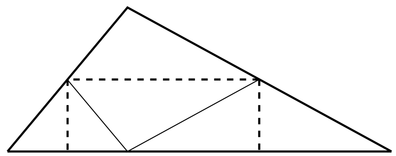
B.1.17) Prove: The diagonals of a rectangle are congruent.

What about the converse?

B.1.18) Given a circle, give a construction that finds its center. Explain why it works.

B.1.19) Draw an arbitrary convex quadrilateral. Form a second quadrilateral by connecting the midpoints of the sides of the first quadrilateral. You will find that the second quadrilateral is a special quadrilateral. Make a conjecture about the second quadrilateral and prove it.

B.1.20) The following picture shows a triangle that has been folded along the dotted lines:

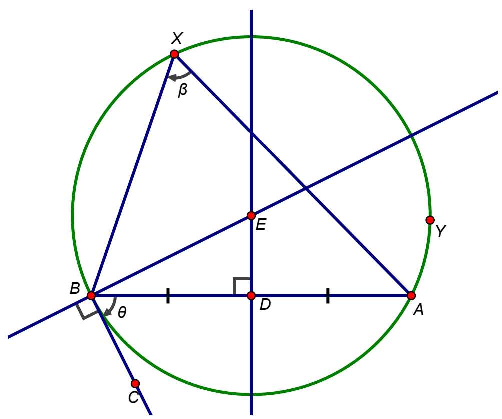


Explain how the picture "proves" the following statements:

(a) The interior angles of a triangle sum to $180°$.

(b) The area of a triangle is given by $bh/2$.

B.1.21) The figure below illustrates a construction given an angle ϑ and a segment $\overline{AB}$. Line $\overleftrightarrow{DE}$ is the perpendicular bisector of $\overline{AB}$, and $\overleftrightarrow{BE}$ is perpendicular to $\overrightarrow{BC}$, as marked.



(a) Prove that $\angle\vartheta = \angle\beta$.

(b) What will happen to $\angle\beta$ if point X is moved over to point Y? Say how you know.

B.2 Midterm 2 Review

Midterm 2 will cover Chapter 4, Sections 5.1 and 5.2, Activities A.19–A.35, and A.38. Here are some specific reminders:

- Know the distinction between synthetic and analytic geometry.
- Know the basic rigid motions, what is required to specify them, and their properties.
- Know what it means to say that transformations of the plane are functions.
- Know how to define congruence in terms of basic rigid motions.
- Know how to define similarity in terms of dilations and basic rigid motions.
- Know and be able to use criteria for congruence and similarity of triangles.
- Know how to use transformations to describe symmetries of figures, including tessellations.
- Know the focus and directrix definition of parabola and be able to use it in both synthetic and analytic geometry.
- Know the definition of circle and be able to use it in both synthetic and analytic geometry.
- Be aware of assumptions underlying Euclidean geometry and how those assumptions can be different in other geometries (such as spherical geometry).
- Use similarity to find missing lengths by reasoning from the scale factor or from within-figure comparisons.
- Understand right-triangle trigonometry as similarity, and use trigonometry to solve problems. (See activity A.27 for a review.)
- Reason about length, area, and volume in similarity situations. How are rep-tiles related to this question?
- Use shearing and Cavalieri's principle to reason about area and volume.
- If you know the area of a rectangle, what can you say about its perimeter? What about more general figures?
- If you know the perimeter of a rectangle, what can you say about its area? What about more general figures?
- In coordinate geometry, what is a point? What is a line?
- Know how to find an equation of the line containing two given points.
- Know how to derive and explain the distance and midpoint formulas.

B.2.1 Midterm 2 Review Problems

B.2.1) Ethan stands 120 feet from the trunk of a tree (along flat ground). He measures that his line of sight to the top of the tree is at an angle of 53° from horizontal. How tall is the tree? Explain your reasoning.

B.2.2) During a solar eclipse we see that the apparent diameter of the Sun and Moon are nearly equal. If the Moon is around 240,000 miles from Earth, the Moon's diameter is about 2000 miles, and the Sun's diameter is about 865,000 miles how far is the Sun from the Earth?

(a) Draw a relevant (and helpful) picture showing the important points of this problem.

(b) Write an expression that gives the solution to this problem—show all work.

B.2.3) David proudly owns a 42 inch (measured diagonally) flat screen TV. Michael proudly owns a 13 inch (measured diagonally) flat screen TV. Dave sits comfortably with his dog Fritz at a distance of 10 feet. How close must Michael sit from his TV to have the "same" viewing experience? Explain your reasoning.

(a) Draw a relevant (and helpful) picture showing the important points of this problem.

(b) Solve this problem, and be sure to explain your reasoning.

B.2.4) Right triangle trigonometry: α, β, and γ are acute angles.

(a) Suppose $\sin\alpha = 3/4$. Find $\cos\alpha$ and $\tan\alpha$.

(b) Suppose $\cos\beta = 2/3$. Find $\sin\beta$ and $\tan\beta$.

(c) Suppose $\tan\gamma = 2/3$. Find $\sin\gamma$ and $\cos\gamma$.

B.2.5) Some drugs work best when dosages are proportional to body surface area. Other drugs work best when dosages are proportional to blood volume. A typical adult male (5 ft 10 in, 175 lbs) has a body surface area of about 2 square meters and about 5 liters of blood. Scale these values up to estimate LeBron's body surface area and blood volume.

B.2.6) Consider a version of LeBron that is d times as tall. How would following quantities compare between the scaled version and the real LeBron: leather in the sole of a shoe, shoe size, inseam, fabric in a T-shirt, lung capacity, neck circumference, and hat size. (Cool fact: The size of a hat is the diameter (in inches) of the hat when it is reshaped into a circle. Most adults have hat sizes between $6\frac{3}{4}$ and 8.) Explain briefly.

B.2.7) A typical adult male gorilla is about 5.5 feet tall and weighs about 400 pounds. Suppose King Kong was about 22 feet tall and proportioned like a typical adult male gorilla.

(a) Approximates King Kong's weight. Briefly explain your reasoning.

(b) The circumference of the neck of a typical adult male gorilla is 36 inches. Approximately what would be the circumference of King Kong's neck? Briefly explain your reasoning.

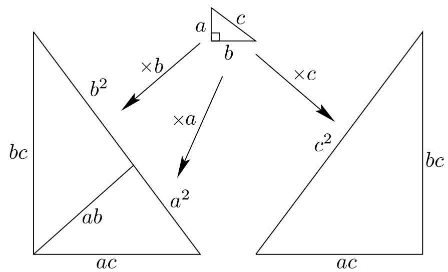
(c) Suppose an Ohio State sweatshirt for a typical adult male gorilla requires 3 square yards of fabric. Approximately how much fabric would be required for an Ohio State sweatshirt for King Kong? Briefly explain your reasoning.

B.2.8) Brenah is drinking fruit punch from a glass shaped like an inverted cone. Suppose the glass has a height 5 in. and a base of radius 2 in. What is the volume of the glass? What is the height of the fruit punch when the glass is half full? Generalize your result for any glass shaped like an inverted cone.

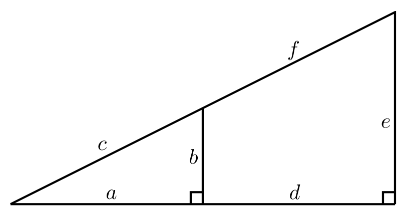
B.2.9) A cup has a circular opening, a circular base, and circular cross sections at every height parallel to the base. The opening has a diameter of 9 cm, the base has a diameter of 6 cm, and the cup is 12 cm high. What is the volume of the cup? Explain your reasoning.

B.2.10) Suppose you use a photocopier to enlarge a figure to 125% of its original size. What is the scale factor of the enlargement? What happens to areas under the enlargement? If you lost the original figure, what reduction percentage would you use on the enlargment to create a figure congruent to the original? What is the scale factor for the reduction?

B.2.11) Explain how the following picture “proves” the Pythagorean Theorem.



B.2.12) Here is a right triangle, note it is **not** drawn to scale:



Solve for all unknowns in the following cases.

(a) $a = 3$, $b = ?$, $c = ?$, $d = 12$, $e = 5$, $f = ?$

(b) $a = ?$, $b = 3$, $c = ?$, $d = 8$, $e = 13$, $f = ?$

(c) $a = 7$, $b = 4$, $c = ?$, $d = ?$, $e = 11$, $f = ?$

(d) $a = 5$, $b = 2$, $c = ?$, $d = 6$, $e = ?$, $f = ?$

In each case explain your reasoning.

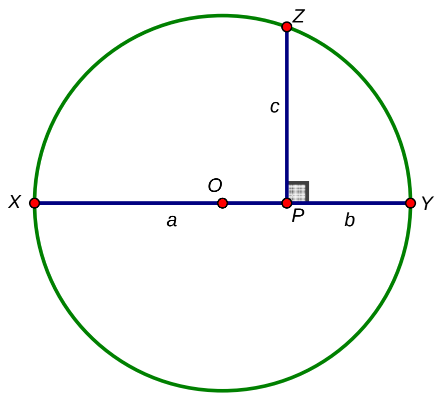
B.2.13) Describe a general (and foolproof) way of demonstrating that any two parabolas are similar.

B.2.14) Prove that the angle sum of a triangle is $180°$.

B.2.15) Give an informal derivation of the relationship between the circumference and area of a circle.

B.2.16) Given a figure and a rotation of that figure, find the center and angle of rotation.

B.2.17) In the figure below O is the center of the circle, $\overline{XY}$ is a diameter, $a = PX$, $b = PY$, and $c = PZ$.



(a) Show that $c = \sqrt{ab}$.

(b) Use the figure to explain the Arithmetic-Geometric Mean Inequality: $\dfrac{a+b}{2} \geqslant \sqrt{ab}$.

B.2.18) If the perimeter of a rectangle is 20 feet, what is the most one can say about the rectangle's area? If the perimeter of any simple closed 2-dimensional shape is 20 feet, what is the most anyone can say about its area?

B.2.19) If the surface area of a rectangular prism is 20 square feet, what is the most one can say about the prism's volume? If the surface area of any simple closed 3-dimensional shape is 20 square feet, what is the most one can say about its volume?

B.2.20) Why do cute furry animals curl up to stay warm in the winter? Why are most ugly desert reptiles long and skinny?

B.2.21) Simple closed curve A is contained entirely inside simple closed curve B.

(a) True or False: The area enclosed by A is less than the area enclosed by B. Explain

(b) True or False: The perimeter of A is less than the perimeter of B. Explain.

B.2.22) Is it correct to say that “area is length times width”? Think about what these three quantities mean. When would it be correct in the numerical sense and why? (Make sure you use the meaning of multiplication.)

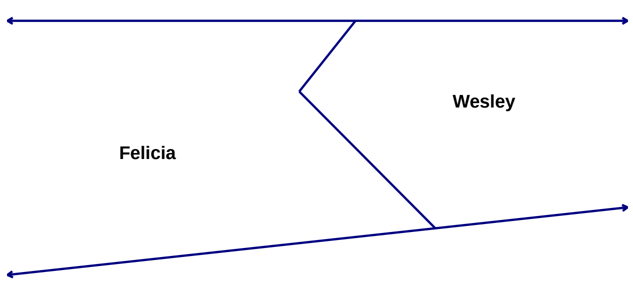
B.2.23) Is it correct to say that “volume is length times width times height”? What must be true about a figure so that the numerical volume can be more easily measured by “area times height”?

B.2.24) Are there figures for which there is no formula for measuring length, area, and volume? Explain. What does your answer to this question imply about the teaching of geometric measurement?

B.2.25) Convert 25 yards to meters (and 25 meters to yards) using “2.54 cm in each inch” as the only Metric-English unit conversion. Now convert 25 square yards to square meters and 25 square meters to square yards. Do the same with cubic yards and cubic meters.

B.2.26) In track and field, 1600 meters is often called the “one mile,” but this is not exactly correct. Is 1600 meters longer or shorter than one mile? By how much?

B.2.27) Felicia and Wesley are neighbors. The common boundary between their properties consists of two line segments, as shown below.



They would prefer their common boundary to be a single straight segment. How might they change their boundary so that they each have the same area as they have now?

B.2.28) Write an equation of the line through $(2, 4)$ parallel to $5x - 3y = 1$. Now write an equation of the line through (x_1, y_1) parallel to $ax + by = c$.

B.2.29) Write an equation of the line through $(2, 4)$ perpendicular to $5x - 3y = 1$. Now write an equation of the line through (x_1, y_1) perpendicular to $ax + by = c$.

B.2.30) Intersections of lines.

(a) Find the intersection of the lines $2x - 3y = 4$ and $3x - 5y = 3$.

(b) Find the intersection of the lines $2x - 3y = 4$ and $-4x + 6y = -8$.

(c) Find the intersection of the lines $2x - 3y = 4$ and $-4x + 6y = 5$.

(d) How might you have predicted in advance how many solutions to expect for each previous system of equations?

(e) Use algebra to help explain why lines intersect in zero, one, or infinitely many points. (You know this geometrically, of course. Here you demonstrate how algebra gives the same result.) Indicate clearly the algebraic conditions for which you get zero, one, or infinitely many points.

B.2.31) Suppose you have a rectangle with vertices at $(0, 0)$, $(a, 0)$, (a, b) and $(0, b)$. Use algebra to prove that the diagonals have the same length.

B.2.32) Use a general (non-special) triangle to explain why every triangle is a rep-4-tile. Use the same triangle to explain why every triangle tessellates the plane. Then use your tessellation to explain why every triangle is a rep-n^2-tile for any positive integer n.

B.2.33) Concurrency of angle bisectors.

(a) For an arbitrary triangle, draw carefully to demonstrate that the angle bisectors of a triangle are concurrent at the incenter.

(b) Prove that the angle bisectors of a triangle are concurrent. (Hint: You may use the result, proved in lecture, that the points on an angle bisector are exactly those that are equidistant from the sides of the angles.)

B.3 Math 1166: Final Exam Review, Draft 2018

Note: The final exam is cumulative, so be sure to include the **previous midterm review documents** as part of your review. Since the second midterm, we will have covered Sections 5.3, 5.4 (except 5.4.4), 6.1 and 6.2 from your notes as well as Activities A.36 – A.48, except A.46. Below is a summary of that content.

Some of the following problems (as well as a few others) will be part of an online supplementary review, available by Reading Day.

Coordinate Geometry

- What are the benefits of the standard form, the point-slope form, and the slope-intercept form of a line?
- Find an equation of a line through a given point parallel to a given line. (Try this in standard form.)
- Find an equation of a line through a given point perpendicular to a given line. (Try this in standard form.)
- Given two lines, two circles, or a line and a circle, find their intersection(s) if any. Describe what is happening geometrically when the algebra yields no solutions, exactly one solution, exactly two solution(s) or infinitely many solutions.
- Given an equation of a circle or a parabola, complete the square to find the center of the circle or the vertex of the parabola.
- What are constructible numbers? What are some numbers that are not constructible?

City Geometry

- Know the distance formula in city geometry, be able to use it, and explain its meaning.
- Given a center and a radius, graph a city-geometry circle and write its equation.
- Given two points in city geometry, graph their midset and write an equation of the midset.
- Given a focus and a directrix, graph the city-geometry parabola and write its equation.

- Explain the absolute value function in several ways.
- Graph and analyze absolute value equations (such as equations of city-geometry circles, midsets, or parabolas) by checking cases.

Functions

- What is a function? What do domain and range mean?
- Why is “Is this a function?” a poor question. What is a better question?
- Graph and analyze parametric equations describing a path in the plane
- In what sense are transformations of the plane functions? What are the input and output values? What are the domain and range of an isometry or dilation of the plane?
- Describe and analyze functions involving several related variables, such as length, width, area, and perimeter of rectangles. When fixing one of these quantities, what kinds of functions can you find among the other quantities?

B.3.1 Supplemental Review Problems

The problems below target content since the second midterm exam.

B.3.1) Consider a nonzero vector defined by the ordered pair (a, b). Let m be the magnitude (length) of this vector, **use algebra** to explain why

$$\frac{(a, b)}{m}$$

is a new vector whose magnitude is 1 and whose direction is the same as (a, b).

B.3.2) Suppose you have a parametric plot defined by $x(t)$ and $y(t)$.

(a) Compare and contrast the plots of

$$\Big(x(t), y(t)\Big) \qquad \text{and} \qquad \Big(x(t-6), y(t-6)\Big).$$

(b) Suppose that there are two bugs whose positions are given by:

$$\text{bug}_1(t) = \Big(x(t), y(t)\Big) \qquad \text{and} \qquad \text{bug}_2 = \Big(x(t-6), y(t-6)\Big).$$

where t represents time in seconds. Describe what happens as t runs from 0 seconds to 36 seconds.

(c) Now suppose that there are two bugs whose positions are given by:

$$\text{bug}_1(t) = \Big(x(t), y(t)\Big) \qquad \text{and} \qquad \text{bug}_2 = \Big(x(t) - 6, y(t) - 6\Big).$$

where t represents time in seconds. Describe what happens as t runs from 0 seconds to 36 seconds.

B.3.3) Find the intersection of the lines

$$x_1(t) = -6 + 9t \qquad\qquad x_2(t) = 3 + t$$
$$y_1(t) = 3 - 2t \qquad\qquad y_2(t) = -4 - 2t$$

If $(x_1(t), y_1(t))$ gives the position of jogger$_1$ and $(x_2(t), y_2(t))$ gives the position of jogger$_2$, what is the significance of the point of intersection of these lines, from the perspective of the joggers?

B.3.4) A bug moves according to the following parametric equations, where t is measured in seconds and x and y are measured in centimeters: $x = 2t^2$, $y = t - 2$. (Suppose t can be any real number.)

(a) Describe the path of the bug.

(b) Is the bug's position a function of time?

(c) On the path, is y a function of x?

(d) Is x a function of y?

(e) If you know one of x, y, or t, can you determine the other two? How does this question relate to the previous two questions?

(f) In school mathematics, students are often given a graph and asked, "Is it a function." Explain why this is a poor question. What better questions could you ask?

B.3.5) Consider the following equations:

$$x^2 - y^2 = 0 \qquad x^2 = y^2 \qquad |y| = |x| \qquad y = \pm x$$
$$(x - y)(x + y) = 0 \qquad x = \pm y \qquad y = \pm|x| \qquad x = \pm|y|$$

(a) Which equations are equivalent to which other equations? Say how you know. (Be sure to state what it means for the equations to be equivalent.)

(b) For each set of equivalent equations, graph the solution set, and describe how each of the equations provides a different way about thinking about that solution set.

B.3.6) Distance formulas and circle equations across dimensions.

(a) What is the (Euclidean) distance formula in 2 dimensions, on the xy-plane?

(b) What is the distance formula in 3 dimensions?

(c) What is the distance formula in 1 dimension?

(d) Write an equation of the circle of radius r and center (a, b).

(e) Explain how a circle is a one-dimensional figure living in a two-dimensional "space."

(f) In three-dimensional space, write an equation of the two-dimensional "circle" of radius r and center (a, b, c).

(g) In one-dimensional space, write an equation of the zero-dimensional "circle" of radius r and center a.

B.3.7) Recall the method in Euclidean geometry of constructing an equilateral triangle on a given segment. Suppose a "city geometry compass" draws a city geometry circle. Imagine using such a "city geometry compass" below.

(a) Construct a "city geometry equilateral triangle" on the segment defined by the points $(0, 0)$ and $(4, 0)$. Explain your steps.

(b) Now construct a "city geometry equilateral triangle" on the segment defined by the points $(0, 0)$ and $(2, 2)$. Explain your steps.

(c) Will the construction always give a (unique!) equilateral triangle? What does "unique" mean in this context? Give a detailed discussion.

B.3.8) A fundamental feature of the basic rigid motions in Euclidean geometry is that they preserve distance and angle. In city geometry, some basic rigid motions preserve both distance and angle and others fail for various reasons. Explain. (Hint: Some but not all rotations preserve both distance and angle.)

B.3.9) Clearly identify which of the following numbers are constructible and which numbers are not constructible.

$$4 \qquad \sqrt[3]{2} \qquad 3.1415926 \qquad \sqrt[3]{125} \qquad \sqrt[6]{7} \qquad \frac{6}{1+\sqrt{5}}$$

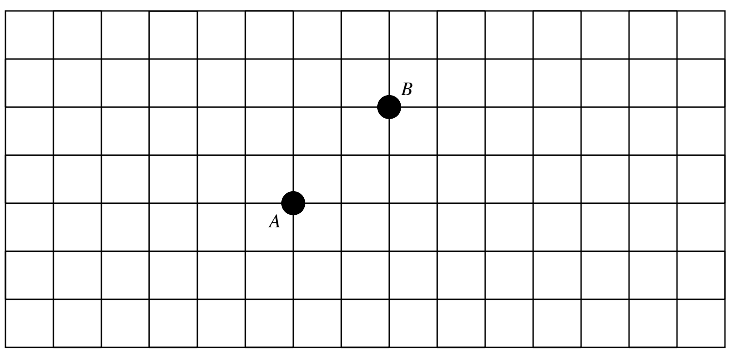
B.3.10) Consider $x = (y-2)^2 + 3$.

(a) Plot this curve.

(b) Explain how this might not be the plot of a function.

(c) Explain how this could be the plot of a function.

B.3.11) Below are two points in City Geometry.



(a) Sketch the City Geometry midset of these two points.

(b) Suppose that $A = (0, 0)$ and $B = (2, 2)$. Using taxicab distance in City Geometry, write an equation that must be satisfied by any point (x, y) that is in the midset of A and B.

(c) Explain the connection between the geometry in part (a) and the algebra in part (b) for the case $x > 2$ and $y < 0$.

B.3.12) Use coordinate constructions to derive the formula for the parabola whose focus is the point $(-3, -4)$ and whose directrix is the line $y = 2$. Show your work. Note, you should use the Euclidean distance formula, and your final answer should start with "$y =$".

B.3.13) Comparing Celsius and Fahrenheit.

(a) Water freezes at 0° C, which is 32° F. Water boils at 100° C, which is 212° F. Use this information to derive a Celsius to Fahrenheit conversion formula.

(b) Suppose the temperature of a model house increases from 24° C to 30° C, which seems to be a 25% increase. Convert these temperatures to Fahrenheit and compute the percent change in degrees Fahrenheit.

(c) Use your conversion formula to explain why the percent changes are not the same in Fahrenheit as in Celsius.

B.3.14) Surface area of a cylinder.

(a) Derive and explain a formula for the surface area of a right cylinder of radius of radius r and height h.

(b) Assuming the radius is fixed, how does the surface area vary with the height? In other words, what kind of function is it? Explain briefly.

(c) Assuming the height is fixed, how does the surface area vary with the radius? Explain briefly.

B.3.15) Volume of a cylinder.

(a) Derive and explain a formula for the volume of a right cylinder of radius of radius r and height h.

(b) Assuming the radius is fixed, how does the volume vary with the height? In other words, what kind of function is it? Explain briefly.

(c) Assuming the height is fixed, how does the volume vary with the radius? Explain briefly.

B.3.16) Standard televisions usually have an aspect ratio (width:length) of 4:3. Wide-screen televisions have an aspect ratio of 16:9. Brad's first wide-screen television was a 36 inch (diagonal) model. Although the new television was clearly wider than the 27 inch (diagonal) standard television it replaced, he was surprised that it did not seem taller than the old television. Which television was actually taller or shorter? By how much? Explain your reasoning.

B.3.17) Given an equation of a line in standard form, $ax + by = c$:

(a) Find an efficient way to write down an equation of a perpendicular line through the point (p, q).

(b) Explain why efficient the method works.

B.4 Math 4407 Exam Review

The exam will sample from Chapters 1 – 4 and Activities 1–32 and 38 of your course notes. Exam problems will focus on Euclidean geometry. In a few of the problems, you will be asked *to compare the Euclidean result to spherical or hyperbolic geometry and to explain your reasoning*.

B.4.1 Review Ideas

- Be able to state definitions of commonly used terms, such as
  - Perpendicular line, parallel line, line segment, ray, angle, circle
  - Concurrent, collinear
  - Equilateral, equiangular, and regular polygons
  - Acute, obtuse, and right angles and triangles; isosceles and scalene triangles
  - Straight, complementary, and supplementary angles
  - Trapezoid (inclusive and exclusive), parallelogram, rhombus, rectangle, square, kite
  - Median, perpendicular bisector, angle bisector, altitude
  - Centroid, circumcenter, incenter, orthocenter
  - Chord, arc, arc measure, central angle, inscribed angle, tangent
- Be able to perform standard constructions and explain why they work.
- Be able to draw (careful) figures satisfying particular conditions.
- Be able to explain key concepts such as area and angle.
- Be able to state precisely the triangle congruence criteria.
- Know the properties of various special quadrilaterals and be able to prove them.
- Know the various centers of a triangle, how to construct them, and whether they can lie outside the triangle.
- Be able to state key theorems and prove them in at least two (2) ways, especially:
  - Isosceles triangle theorem and its converse

  - Pythagorean theorem and its converse
  - The angle sum of a triangle

- Know the distinction between synthetic and analytic geometry.
- Know the basic rigid motions, what is required to specify them, and their properties.
- Know what it means to say that transformations of the plane are functions.
- Know how to define congruence in terms of basic rigid motions.
- Know how to define similarity in terms of dilations and basic rigid motions.
- Know and be able to use criteria for congruence and similarity of triangles.
- Know how to use transformations to describe symmetries of figures, including tessellations.
- Know the definition of circle and be able to use it in both synthetic and analytic geometry.
- Be aware of assumptions underlying Euclidean geometry and how those assumptions can be different in other geometries (such as spherical geometry).
- Use similarity to find missing lengths by reasoning from the scale factor or from within-figure comparisons.
- Understand right-triangle trigonometry as similarity, and use trigonometry to solve problems. (See activity A.27 for a review.)

Review Problems

B.4.1) Describe Euclid's (compass and straightedge) construction for an equilateral triangle, and explain why it works.

B.4.2) Prove that the points on an angle bisector are *exactly those* that are equidistant from the sides of the angle.

B.4.3) Construct a 30-60-90 right triangle. Explain the steps in your construction and how you know it works.

B.4.4) Construct a 45-45-90 right triangle. Explain the steps in your construction and how you know it works.

B.4.5) Where is the orthocenter of a right triangle? Explain your reasoning. What about the circumcenter? Again, explain your reasoning.

B.4.6) Show that, given any three non-collinear points in the Euclidean plane, there is a unique circle passing through the three points.

B.4.7) Prove: A radius that is perpendicular to a chord bisects the chord.

B.4.8) Prove: A radius that bisects a chord is perpendicular to the chord.

B.4.9) Given a circle, give a construction that finds its center.

B.4.10) State and prove a condition about the opposite angles of any quadrilateral that is inscribed in a circle.

B.4.11) Construct a tangent line from a point outside a given circle to the circle.

B.4.12) Give an informal derivation of the relationship between the circumference and area of a circle.

B.4.13) Prove: If a quadrilateral is a parallelogram, then opposite sides are congruent.

B.4.14) Prove: If opposite sides of a quadrilateral are congruent, then it is a parallelogram.

B.4.15) Claim: The diagonals of a rhombus are perpendicular.

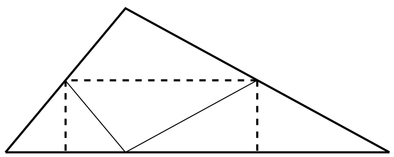
(a) Prove the claim.

(b) State the converse of the claim.

(c) Is the converse true? If so, prove it. If not, "salvage it" to make a true statement, and prove it.

B.4.16) Draw an arbitrary convex quadrilateral. Form a second quadrilateral by connecting the midpoints of the sides of the first quadrilateral. You will notice that the second quadrilateral is a special quadrilateral. Make a conjecture about the second quadrilateral and prove it.

B.4.17) The following picture shows a triangle that has been folded along the dotted lines:



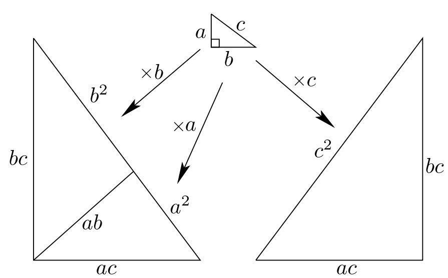
Explain how the picture "proves" the following statements:

(a) The interior angles of a triangle sum to 180°.

(b) The area of a triangle is given by $bh/2$.

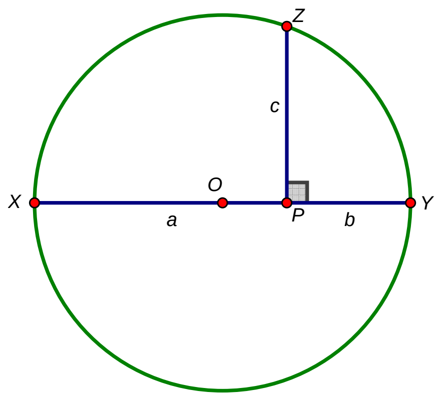
B.4.18) Suppose you use a photocopier to enlarge a figure to 125% of its original size. What is the scale factor of the enlargement? What happens to areas under the enlargement? If you lost the original figure, what reduction percentage would you use on the enlargement to create a figure congruent to the original? What is the scale factor for the reduction?

B.4.19) Explain how the following picture "proves" the Pythagorean Theorem.



B.4.20) Given a figure and a rotation of that figure, find the center and angle of rotation.

B.4.21) In the figure below O is the center of the circle, $\overline{XY}$ is a diameter, $a = PX$, $b = PY$, and $c = PZ$.



(a) Show that $c = \sqrt{ab}$.

(b) Use the figure to explain the Arithmetic-Geometric Mean Inequality: $\dfrac{a+b}{2} \geqslant \sqrt{ab}$.

B.4.22) Concurrency of angle bisectors.

(a) For an arbitrary triangle, draw carefully to demonstrated that the angle bisectors of a triangle are concurrent at the incenter.

(b) Prove that the angle bisectors of a triangle are concurrent. (Hint: You may use the result, proved in lecture, that the points on an angle bisector are exactly those that are equidistant from the sides of the angles.)

B.4.23) Explain how the ASA congruence criterion follows from the definition of congruence in terms of rigid motions. Be sure to indicate, using the two given angles and the included side, why the sequence of rigid motions guarantees triangle congruence.

B.4.24) Find the intersection of the lines

$$x_1(t) = -6 + 9t \qquad\qquad x_2(t) = 3 + t$$
$$y_1(t) = 3 - 2t \qquad\qquad y_2(t) = -4 - 2t$$

If $(x_1(t), y_1(t))$ gives the position of jogger$_1$ and $(x_2(t), y_2(t))$ gives the position of jogger$_2$, what is the significance of the point of intersection of these lines, from the perspective of the joggers?

B.4.25) A bug moves according to the following parametric equations, where t is measured in seconds and x and y are measured in centimeters: $x = 2t^2$, $y = t - 2$. (Suppose t can be any real number.)

(a) Describe the path of the bug.

(b) Is the bug's position a function of time?

(c) On the path, is y a function of x?

(d) Is x a function of y?

(e) If you know one of x, y, or t, can you determine the other two? How does this question relate to the previous two questions?

(f) In school mathematics, students are often given a graph and asked, "Is it a function." Explain why this is a poor question. What better questions could you ask?