Online HW 3: Integers and the Fundamental Theorem

Bart Snapp and Brad Findell

December 14, 2018

Contents

Integers

Problems about integers.

Problem 1 *Describe the set of integers. Give some relevant and revealing examples/nonexamples.*

Problem 2 *Use the definition of divides to decide whether the following statements are true or false. In each case, an explanation must be given justifying your claim.*

(a) $5|30$ *(True/ False)*

(b) $7|41$ *(True/ False)*

(c) $0|3$ *(True/ False)*

(d) $3|0$ *(True/ False)*

(e) $6|(2^2 \cdot 3^4 \cdot 5 \cdot 7)$. *(True/ False)*

(f) $1000|(2^7 \cdot 3^9 \cdot 5^{11} \cdot 17^8)$ *(True/ False)*

(g) $6000|(2^{21} \cdot 3^{17} \cdot 5^{89} \cdot 29^{20})$. *(True/ False)*

JavaScript

```javascript
function isPrime(num) {
  for(var i = 2; i < num; i++)
    if(num % i === 0) return false;
  return num > 1;
}

function isPrimeFactorization(x,y) {
  var terms = x.split('*').map( function(t) { return parseInt(t) } );
  return terms.every( isPrime ) &&
    (terms.reduce( function(a,c) { return a*c; }, 1 )) == parseInt(y);
}
```

Author(s): Bart Snapp and Brad Findell

Problem 3 *Factor the following integers. Enter the primes in increases order, use * for multiplication, and do not use exponents. If the number is prime, enter the number itself.*

(a) 15 $3 * 5$

(b) 12 $2 * 2 * 3$

(c) 111 $\boxed{?}$

(d) 1234 $\boxed{?}$

(e) 2345 $\boxed{?}$

(f) 4567 $\boxed{?}$

(g) 111111 $\boxed{?}$

Problem 4 *Find the greatest common divisors below:*

(a) $\gcd(462, 1463) = \boxed{?}$

(b) $\gcd(541, 4669) = \boxed{?}$.

(c) $\gcd(10000, 2^5 \cdot 3^{19} \cdot 5^7 \cdot 11^{13}) = \boxed{?}$

(d) $\gcd(11111, 2^{14} \cdot 7^{21} \cdot 41^5 \cdot 101) = \boxed{?}$

(e) $\gcd(437^5, 8993^3) = \boxed{?}$

Problem 5 *Consider the following:*

$$20 \div 8 = 2 \textit{ remainder } 4,$$
$$28 \div 12 = 2 \textit{ remainder } 4.$$

Is it correct to say that $20 \div 8 = 28 \div 12$*? (Yes/ No)*

Explain your reasoning.

Problem 6 *Give a formula for the* n*th even number:* $\boxed{?}$

Problem 7 *Give a formula for the* n*th odd number:* $\boxed{?}$.

Problem 8 *Give a formula for the* n*th multiple of* 3*:* $\boxed{?}$

Problem 9 *Give a formula for the* n*th multiple of* -7. $\boxed{?}$

Problem 10 *Give a formula for the* n*th number whose remainder when divided by* 5 *is* 1.

If the first such number is 1, the formula is $\boxed{?}$.

If the first such number is 6, the formula is $\boxed{?}$.

Fundamental Theorem

Problems about unique factorization.

Problem 11 *Explain what the GCD of two integers is. Give some relevant and revealing examples/nonexamples.*

Problem 12 *Explain what the LCM of two integers is. Give some relevant and revealing examples/nonexamples.*

Problem 13 *How many zeros are at the end of the following numbers:*

(a) $2^2 \cdot 5^8 \cdot 7^3 \cdot 11^5$. *There are* $\boxed{?}$ *zeros.*

(b) $11!$. *There are* $\boxed{?}$ *zeros.*

(c) $27!$. *There are* $\boxed{?}$ *zeros.*

(d) $99!$. *There are* $\boxed{?}$ *zeros.*

(e) $1001!$. *There are* $\boxed{?}$ *zeros.*

In each case, explain your reasoning.

Problem 14 *Decide whether the following statements are true or false. In each case, a detailed argument and explanation must be given justifying your claim.*

(a) $7|56$. *(True/ False)*

(b) $55|11$. *(True/ False)*

(c) $3|40$. *(True/ False)*

(d) $100|(2^4 \cdot 3^{17} \cdot 5^2 \cdot 7)$ *(True/ False)*

(e) $5555|(5^{20} \cdot 7^9 \cdot 11^{11} \cdot 13^{23})$ *(True/ False)*

Author(s): Bart Snapp and Brad Findell

(f) $3|(3+6+9+\cdots+300+303)$ *(True/ False)*

Problem 15 *Suppose that*

$$(3^5 \cdot 7^9 \cdot 11^x \cdot 13^y)|(3^a \cdot 7^b \cdot 11^{19} \cdot 13^7)$$

What values of a, b, x and y, make true statements? Explain your reasoning.

- a *($\geqslant$/ $=$/ $\leqslant$)* $\boxed{?}$.
- b *($\geqslant$/ $=$/ $\leqslant$)* $\boxed{?}$.
- x *($\geqslant$/ $=$/ $\leqslant$)* $\boxed{?}$.
- y *($\geqslant$/ $=$/ $\leqslant$)* $\boxed{?}$.

Problem 16 *Decide whether the following statements are true or false. In each case, a detailed argument and explanation must be given justifying your claim.*

(a) *If* $7|13a$*, then* $7|a$*. (True/ False)*

(b) *If* $6|49a$*, then* $6|a$*. (True/ False)*

(c) *If* $10|65a$*, then* $10|a$*. (True/ False)*

(d) *If* $14|22a$*, then* $14|a$*. (True/ False)*

(e) $54|931^{21}$. *(True/ False)*

(f) $54|810^{33}$. *(True/ False)*

Problem 17 *Joanna thinks she can see if a number is divisible by 24 by checking to see if it's divisible by 4 and divisible by 6. She claims that if the number is divisible by 4 and by 6, then it must be divisible by 24.*

Lindsay has a similar divisibility test for 24: She claims that if a number is divisible by 3 and by 8, then it must be divisible by 24.

Are either correct? Explain your reasoning.

Joanna is (correct/ incorrect). Lindsay is (correct/ incorrect).

Problem 18 *Decide whether the following statements are true or false. In each case, a detailed argument and explanation must be given justifying your claim.*

(a) *If* $a^2|b^2$*, then* $a|b$*. (True/ False)*

(b) *If* $a|b^2$*, then* $a|b$*. (True/ False)*

(c) *If* $a|b$ *and* $\gcd(a, b) = 1$*, then* $a = 1$*. (True/ False)*

Problem 19 *Suppose* x *and* y *are integers. If* $x^2 = 11 \cdot y$*, what can you say about* y*? Explain your reasoning.*

Problem 20 *Suppose* x *and* y *are integers. If* $x^2 = 25 \cdot y$*, what can you say about* y*? Explain your reasoning.*