Rational Numbers

More Problems about Rational Numbers.

Problem 1 *Find yourself a sheet of paper. Now, suppose that this sheet of paper is actually* $4/5$ *of some imaginary larger sheet of paper.*

- *Shade your sheet of paper so that* $3/5$ *of the larger (imaginary) sheet of paper is shaded in. Explain why your shading is correct.*
- *Explain how this shows that*

$$\frac{3/5}{4/5} = \frac{3}{4}.$$

Problem 2 *Try to find the largest rational number smaller than* $3/7$*. Explain your solution or explain why this cannot be done.*

Problem 3 *How many rational numbers are there between* $3/4$ *and* $4/7$*? Find* 3 *of them. Explain your reasoning.*

Problem 4 *A youthful Bart loved to eat hamburgers. He ate* $5/8$ *pounds of hamburger meat a day. After testing revealed that his blood consisted mostly of cholesterol, Bart decided to alter his eating habits by cutting his hamburger consumption by* $3/4$*. How many pounds of hamburger a day did Bart eat on his new "low-cholesterol" diet? Explain your reasoning.*

Problem 5 *Courtney and Paolo are eating popcorn. Unfortunately,* $1/3$*rd of the popcorn kernels are poisoned. If Courtney eats exactly* $5/16$*th of the kernels and Paolo eats exactly* $5/13$*ths of the kernels, did at least one of them eat a poisoned kernel? Explain your reasoning. Also, at least how many kernels of popcorn are in the bowl? Again, explain your reasoning.*

Learning outcomes:
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Problem 6 *Best of clocks, how much of the day is past if there remains twice two-thirds of what is gone? Explain what this strange question is asking and answer the question being sure to explain your reasoning—note this is an old problem from the Greek Anthology compiled by Metrodorus around the year 500.*

Problem 7 *John spent a fifth of his life as a boy growing up, another one-sixth of his life in college, one-half of his life as a bookie, and has spent the last six years in prison. How old is John now? Explain your reasoning*

Problem 8 *Diophantus was a boy for* $1/6$*th of his life, his beard grew after* $1/12$ *more, he married after* $1/7$*th more, and a son was born five years after his marriage. Alas! After attaining the measure of half his father's full life, chill fate took the child. Diophantus spent the last four years of his life consoling his grief through mathematics. How old was Diophantus when he died? Explain your reasoning—note this is an old problem from the Greek Anthology compiled by Metrodorus around the year 500.*

Problem 9 *Wandering around my home town (perhaps trying to find my former self!), I suddenly realized that I had been in my job for one-quarter of my life. Perhaps the melancholia was getting the best of me, but I wondered: How long would it be until I had been in my job for one-third of my life? Explain your reasoning.*

Problem 10 *In a certain adult condominium complex,* $2/3$ *of the men are married to* $3/5$ *of the women. Assuming that men are only married to women (and vice versa), and that married residents' spouses are also residents, what portion of the residents are married?*

(a) *Before any computations are done, use common sense to guess the solution to this problem.*

(b) *Try to get a feel for this problem by choosing numbers for the unknowns and doing some calculations. What do these calculations say about your guess?*

(c) *Use algebra to solve the problem.*

Explain your reasoning in each step above.

Problem 11 *Let a, b, c, and d be positive integers such that*

$$a < b < c < d$$

Is it true that

$$\frac{a}{b} < \frac{c}{d}?$$

Explain your reasoning.

Problem 12 *Let a, b, c, and d be positive consecutive integers such that*

$$a < b < c < d.$$

Is it true that

$$\frac{a}{b} < \frac{c}{d}?$$

Explain your reasoning.

Problem 13 *Let a, b, c, and d be positive consecutive integers such that*

$$a < b < c < d.$$

Is it true that

$$\frac{a}{b} < \frac{b}{c} < \frac{c}{d}?$$

Explain your reasoning.

Problem 14 *Can you generalize Problem and Problem above? Explain your reasoning.*

Let a, b, c, and d be positive integers such that

$$\frac{a}{b} < \frac{c}{d}.$$

Is it true that

$$\frac{a}{a+b} < \frac{c}{c+d}?$$

Explain your reasoning.
