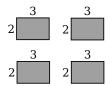
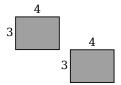
Algorithms

Problems about algorithms.

Problem 1 Explain what it means for an operation \star to be associative. Give some relevant and revealing examples and non-examples.

Problem 2 Consider the following pictures:





Jesse claims that these pictures represent $(2 \cdot 3) \cdot 4$ and $2 \cdot (3 \cdot 4)$.

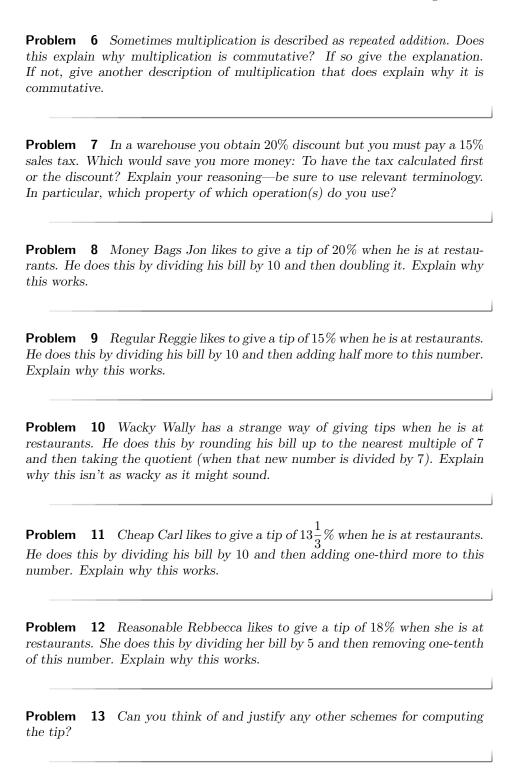
- (a) Is Jesse's claim correct? Explain your reasoning.
- (b) Do Jesse's pictures show the associativity of multiplication? If so, explain why. If not, draw new pictures representing $(2\cdot 3)\cdot 4$ and $2\cdot (3\cdot 4)$ that do show the associativity of multiplication.

Problem 3 Explain what it means for an operation \star to be commutative. Give some relevant and revealing examples and non-examples.

Problem 4 Explain what it means for an operation \star to distribute over another operation \dagger . Give some relevant and revealing examples and non-examples.

Problem 5 Explain what it means for an operation \star to be closed on a set of numbers. Give some relevant and revealing examples and non-examples.

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Problem 14 Here is an example of a standard addition algorithm:

 $\begin{array}{r}
 11 \\
 892 \\
 +398 \\
 \hline
 1290
 \end{array}$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 15 Here is an example of the column addition algorithm:

$$\begin{array}{r}
 892 \\
 +398 \\
 \hline
 10 \\
 18 \\
 \hline
 11 \\
 \hline
 1290
 \end{array}$$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 16 If you check out Problems and , you will learn about "partial" algorithms.

- (a) Develop a "partial" algorithm for addition, give it a name, and describe how to perform this algorithm.
- (b) Provide a relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 17 Here is an example of the banker's addition algorithm:

$$892 \\ +398 \\ \hline 10 \\ 19 \\ \hline 12 \\ \hline 1290$$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 18 Here is an example of a standard subtraction algorithm:

$$8 \\ 8 \cancel{9}^{1} 2 \\ -3 7 8 \\ \hline 5 1 4$$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 19 Here is an example of the subtraction by addition algorithm:

$$892$$
 -378
 514
 $8 + 4 = 12$ add 1 to 7 to get 8
 $8 + 1 = 9$
 $3 + 5 = 8$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 20 Here is an example of the Austrian subtraction algorithm:

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 21 If you check out Problems and , you will learn about "partial" algorithms.

- (a) Develop a "partial" algorithm for subtraction, give it a name, and describe how to perform this algorithm.
- (b) Provide a relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 22 Here is an example of a standard multiplication algorithm:

 $\begin{array}{r}
 23 \\
 634 \\
 \times 8 \\
 \hline
 5072
 \end{array}$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

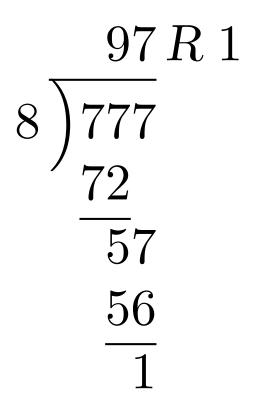
Problem 23 Here is an example of the partial-products algorithm:

 $\begin{array}{r}
 634 \\
 \times 8 \\
 \hline
 4800 \\
 240 \\
 \hline
 5072
\end{array}$

(a) Describe how to perform this algorithm.

- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 24 Here is an example of a standard division algorithm:



- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 25 Here is an example of the partial quotients algorithm:

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 26 Here is another example of the partial-quotients division algorithm:

- (a) Describe how to perform this algorithm—be sure to explain how this is different from the scaffolding division algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 27 Here is an example of a standard multiplication algorithm:

 $\begin{array}{r}
 634 \\
 \times 216 \\
 \hline
 3804 \\
 6340 \\
 \hline
 126800 \\
 \hline
 136944
\end{array}$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here—you may assume that you already know the algebra behind the standard multiplication algorithm.

Problem 28 Here is an example of the addition algorithm with decimals:

 $\begin{array}{r}
 1 \\
 37.2 \\
 +8.74 \\
 \hline
 45.94
 \end{array}$

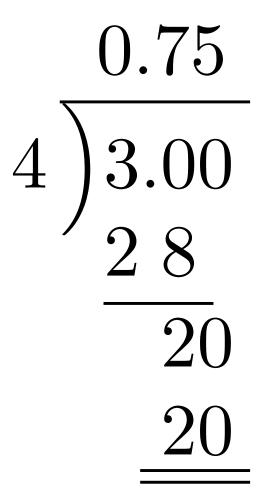
- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 29 Here is an example of the multiplication algorithm with decimals:

$$\begin{array}{r}
3.40 \\
\times .21 \\
\hline
340 \\
6800 \\
\hline
.7140
\end{array}$$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 30 Here is an example of the division algorithm without remainder:



- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

Problem 31 In the following addition problem, every digit has been replaced with a letter.

 $\frac{\textit{MOON}}{+\textit{SUN}}$

Recover the original problem and solution. Explain your reasoning. Hint: S=6 and U=5.

Problem 32 In the following addition problem, every digit has been replaced with a letter.

 $\frac{\textit{SEND}}{\textit{MONEY}}$

Recover the original problem and solution. Explain your reasoning.

Problem 33 In the following subtraction problem, every digit has been replaced with a letter.

$DEFER \\ -DU7Y \\ \hline N2G2$

Recover the original problem and solution. Explain your reasoning.

Problem 34 In the following two subtraction problems, every digit has been

replaced with a letter.

$$\begin{array}{cc} \textit{NINE} & \textit{NINE} \\ -\textit{TEN} & -\textit{ONE} \\ \hline \textit{TWO} & ALL \end{array}$$

Using both problems simultaneously, recover the original problems and solutions. Explain your reasoning.

 $\begin{array}{lll} \textbf{Problem} & \textbf{35} & \text{In the following multiplication problem, every digit has been} \\ & \text{replaced with a letter.} \end{array}$

$$\frac{LET}{\times NO}$$

$$\frac{SOT}{FRET}$$

Recover the original problem and solution. Explain your reasoning.

Problem 36 The following is a long division problem where every digit except 7 was replaced by X.

$$\begin{array}{c} X7X \\ XX \\ XXXXX \\ \underline{X77} \\ X7X \\ \underline{X7X} \\ \underline{X7X} \\ \underline{XX} \\ \underline{XX} \end{array}$$

Recover the digits from this long division problem. Explain your reasoning.

Problem 37 The following is a long division problem where the various digits were replaced by X except for a single 8. The double bar indicates that the remainder is 0.

Recover the digits from this long division problem. Explain your reasoning.

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