Algorithms

More problems about algorithms.

Problem 1 *Here is an example of a standard addition algorithm:*

$$\begin{array}{r} 11 \\ 892 \\ +398 \\ \hline 1290 \end{array}$$

Learning outcomes:
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(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 2 *Here is an example of the column addition algorithm:*

$$
\begin{array}{r}
892 \\
+398 \\
\hline
10 \\
18 \\
11 \\
\hline
1290
\end{array}
$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 3 *If you check out Problems and , you will learn about "partial" algorithms.*

(a) *Develop a "partial" algorithm for addition, give it a name, and describe how to perform this algorithm.*

(b) *Provide a relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 4 *Here is an example of the banker's addition algorithm:*

$$
\begin{array}{r}
892 \\
+398 \\
\hline
\boldsymbol{1}1\boldsymbol{0} \\
\boldsymbol{1}1\boldsymbol{9} \\
\boldsymbol{12} \\
\hline
1290
\end{array}
$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 5 *Here is an example of a standard subtraction algorithm:*

$$\begin{array}{r} 8 \\ 8\not{9}^{1}2 \\ -\,3\,7\;\;8 \\ \hline 5\,1\;\;4 \end{array}$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 6 *Here is an example of the subtraction by addition algorithm:*

$$\begin{array}{r} 892 \\ -378 \\ \hline 514 \end{array} \quad \leftrightsquigarrow \quad \begin{array}{ll} 8 + \boldsymbol{4} = 12 & \textit{add } 1 \textit{ to } 7 \textit{ to get } 8 \\ 8 + \boldsymbol{1} = 9 & \\ 3 + \boldsymbol{5} = 8 & \end{array}$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 7 *Here is an example of the Austrian subtraction algorithm:*

$$\begin{array}{r} 8\ 9^{1}2 \\ -3^{8}\not{7}\ 8 \\ \hline 5\ 1\ 4 \end{array}$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 8 *If you check out Problems and , you will learn about "partial" algorithms.*

(a) *Develop a "partial" algorithm for subtraction, give it a name, and describe how to perform this algorithm.*

(b) *Provide a relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 9 *Here is an example of a standard multiplication algorithm:*

$$\begin{array}{r} 23 \\ 634 \\ \times \quad 8 \\ \hline 5072 \end{array}$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 10 *Here is an example of the partial-products algorithm:*

$$
\begin{array}{r}
634 \\
\times \quad 8 \\
\hline
4800 \\
240 \\
32 \\
\hline
5072
\end{array}
$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 11 *Here is an example of a standard division algorithm:*

$$
\begin{array}{r}
97\ R\ 1 \\
8\overline{)\,777} \\
72 \\
\hline
57 \\
56 \\
\hline
1
\end{array}
$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 12 *Here is an example of the partial quotients algorithm:*

$$
\begin{array}{r}
7 \\
90 \\
\hline
8\,\big)\,777 \\
720 \\
\hline
57 \\
56 \\
\hline
1
\end{array}
$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 13 *Here is another example of the partial-quotients division algorithm:*

$$
\begin{array}{r}
4 \\
10 \\
10 \\
10 \\
\hline
8 \overline{)277} \\
80 \\
\hline
197 \\
80 \\
\hline
\end{array}
$$

(a) *Describe how to perform this algorithm—be sure to explain how this is different from the scaffolding division algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 14 *Here is an example of a standard multiplication algorithm:*

$$
\begin{array}{r}
634 \\
\times 216 \\
\hline
3804 \\
6340 \\
126800 \\
\hline
136944
\end{array}
$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here—you may*

assume that you already know the algebra behind the standard multiplication algorithm.

Problem 15 *Here is an example of the addition algorithm with decimals:*

$$\begin{array}{r} {\scriptstyle 1} \\ 37.2 \\ +8.74 \\ \hline 45.94 \end{array}$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 16 *Here is an example of the multiplication algorithm with decimals:*

$$\begin{array}{r} 3.40 \\ \times\ \ .21 \\ \hline 340 \\ 6800 \\ \hline .7140 \end{array}$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 17 *Here is an example of the division algorithm without remainder:*

$$
\begin{array}{r}
0.75 \\
\hline
4\,\big)\,3.00 \\
\underline{2\;8} \\
20 \\
\underline{20} \\
\end{array}
$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 18 *In the following addition problem, every digit has been replaced with a letter.*

$$\begin{array}{r} \texttt{MOON} \\ +\ \texttt{SUN} \\ \hline \texttt{PLUTO} \end{array}$$

Recover the original problem and solution. Explain your reasoning. Hint: $\texttt{S} = 6$ *and* $\texttt{U} = 5$.

Problem 19 *In the following addition problem, every digit has been replaced with a letter.*

$$\begin{array}{r} SEND \\ +MORE \\ \hline MONEY \end{array}$$

Recover the original problem and solution. Explain your reasoning.

Problem 20 *In the following subtraction problem, every digit has been replaced with a letter.*

$$\begin{array}{r} DEFER \\ -DU7Y \\ \hline N2G2 \end{array}$$

Recover the original problem and solution. Explain your reasoning.

Problem 21 *In the following two subtraction problems, every digit has been replaced with a letter.*

$$\begin{array}{r} NINE \\ -TEN \\ \hline TWO \end{array} \qquad \begin{array}{r} NINE \\ -ONE \\ \hline ALL \end{array}$$

Using both problems simultaneously, recover the original problems and solutions. Explain your reasoning.

Problem 22 *In the following multiplication problem, every digit has been replaced with a letter.*

$$\begin{array}{r} LET \\ \times\ NO \\ \hline SOT \\ NOT \\ \hline FRET \end{array}$$

Recover the original problem and solution. Explain your reasoning.

Problem 23 *The following is a long division problem where every digit except*

7 was replaced by X.

$$
\begin{array}{r}
\;X\,7X \\
\hline
XX\,\big)\,XXXXX \\
\;X\,7\,7 \\
\cline{2-2}
\;\;X\,7X \\
\;X\,7X \\
\cline{2-2}
\;\;\;XX \\
\;\;\;XX \\
\hline\hline
\end{array}
$$

Recover the digits from this long division problem. Explain your reasoning.

Problem 24 *The following is a long division problem where the various digits were replaced by X except for a single* 8*. The double bar indicates that the remainder is 0.*

$$
\begin{array}{r}
\,XX8XX \\
XXX\overline{)\,XXXXXXXXX} \\
\,\underline{XXX} \\
\,\overline{XXXX} \\
\,\underline{XXX} \\
\,\overline{XXXX} \\
\,\underline{XXXX} \\
\,=\!=\!=\!=
\end{array}
$$

Recover the digits from this long division problem. Explain your reasoning.