Polynomials

Problems about polynomials

Problem 1 *Explain what is meant by a polynomial in a variable x.*

Problem 2 *Given:*

$$3x^7 - x^5 + x^4 - 16x^3 + 27 = a_7x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x^1 + a_0$$

Find a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7.

Problem 3 *Given:*

$$6x^5 + a_4x^4 - x^2 + a_0 = a_5x^5 - 24x^4 + a_3x^3 + a_2x^2 - 5$$

Find a_0, a_1, a_2, a_3, a_4, a_5.

Problem 4 *Is it true that polynomials are equal if and only if their coefficients are equal? Explain your reasoning.*

Problem 5 *Is it true that numbers are equal if and only if their digits are equal? Explain your reasoning.*

Problem 6 *Explain how to add two polynomials. Explain, in particular, how "collecting like terms" is an application of the properties of arithmetic.*

Problem 7 *Explain how to multiply two polynomials.*

Problem 8 *Here is an example of the polynomial division algorithm:*

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$$
\begin{array}{r|l}
\multicolumn{2}{r}{x-3 \quad R\;9x+4} \\
x^2+3x+1 & x^3+0x^2+x+1 \\
 & \underline{x^3+3x^2+x} \\
 & \quad -3x^2+0x+1 \\
 & \quad \underline{-3x^2-9x-3} \\
 & \qquad\qquad 9x+4
\end{array}
$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Problem 9 *State the Division Theorem for polynomials. Give some relevant and revealing examples of this theorem in action.*

Problem 10 *Given a polynomial*

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

can you find two numbers L and U such that $L \leq p(x) \leq U$ for all x? If so, explain why. If not, explain why not.

Problem 11 *Consider all polynomials of the form*

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where the a_i's are integers. If you substitute an integer for x will you always get an integer out? Explain your reasoning.

Problem 12 *Consider the following polynomial:*

$$p(x) = \frac{x^2}{2} + \frac{x}{2}$$

Will $p(x)$ always returns an integer when an integer is substituted for x? Explain your reasoning.

Problem 13 *Fix some integer value for x and consider all polynomials of the form*

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Where the a_i's are integers greater than or equal to 0. Which numbers can be represented by such polynomials? Explain your reasoning.

Problem 14 *Find a polynomial*

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

such that a_i's are integers greater than or equal to 0 and less than 2 such that $p(2) = 35$. Discuss how your answer compares to the representation of 35 in base two. Explain your reasoning.

Problem 15 *Find a polynomial*

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

such that a_i's are integers greater than or equal to 0 and less than 7 such that $p(7) = 234$. Discuss how your answer compares to the representation of 234 in base seven. Explain your reasoning.

Problem 16 *Find a polynomial*

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

such that a_i's are integers greater than or equal to 0 and less than 10 such that $p(10) = 18$. Discuss how your answer compares to the representation of 18 in base ten. Explain your reasoning.

Problem 17 *Find a polynomial*

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

such that a_i's are integers greater than or equal to 0 and less than 15 such that $p(15) = 201$. Discuss how your answer compares to the representation of 201 in base fifteen. Explain your reasoning.

Problem 18 *Fix some integer value for x and consider all polynomials of the form*

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

Where the a_i's are integers greater than or equal to 0 and less than x. Which numbers can be represented by such polynomials? Explain your reasoning. Big hint: Base x.

Problem 19 *Fix some integer value for x and consider all polynomials of the form*

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

Where the a_i's are integers greater than or equal to 0 and less than 10. Which numbers can be represented by such polynomials? Explain your reasoning.

Problem 20 *Consider $x^2 + x + 1$. This can be thought of as a "number" in base x. Express this number in base $(x+1)$, that is, find b_0, b_1, b_2 such that*

$$b_2(x+1)^2 + b_1(x+1) + b_0 = x^2 + x + 1.$$

Explain your reasoning.

Problem 21 *Consider $x^2 + 2x + 3$. This can be thought of as a "number" in base x. Express this number in base $(x-1)$, that is, find b_0, b_1, b_2 such that*

$$b_2(x-1)^2 + b_1(x-1) + b_0 = x^2 + 2x + 3.$$

Explain your reasoning.

Problem 22 *Consider $x^3 + 2x + 1$. This can be thought of as a "number" in base x. Express this number in base $(x-1)$, that is, find b_0, b_1, b_2, b_3 such that*

$$b_3(x-1)^3 + b_2(x-1)^2 + b_1(x-1) + b_0 = x^3 + 2x + 1.$$

Explain your reasoning.

Problem 23 *If the polynomial*

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

is thought of as a "number" in base x, describe two different ways to find the base $(x-1)$ coefficients of $p(x)$.
