Counting

Problems about counting and probability.

Problem 1 *Which of the following situations are the same type of counting situation?*

Select All Correct Answers:

(a) *The number of ways to see two threes on four rolls of a 6-sided die.*

(b) *The number of ways to flip a coin four times and see two heads.* ✓

(c) *The number of ways to elect a President and Vice President from a group of four people.*

(d) *The number of ways to choose two students from a class of four.* ✓

(e) *The number of ways to choose two different scoops of ice cream from a shop offering four flavors.* ✓

(f) *The number of ways to choose two scoops of ice cream which are the same flavor from a shop offering four flavors.*

Problem 2 *Use the Binomial Theorem to expand* $(a+b)^4$. $\boxed{a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4}$

Problem 3 *Use the Binomial Theorem to expand* $(3-x)^6$. $\boxed{3^6 - 6 * 3^5x + 15 * 3^4x^2 - 20 * 3^3 - x^3 + 15 * 9x^}$

Problem 4 *Explain why* $\binom{n}{k} = \binom{n}{n-k}$.

Free Response: ***Hint:*** *Using the context of the stop lights,* $\binom{n}{k}$ *represents* k *green lights out of a total of* n *lights. Remembering that* k *green lights also means*

Learning outcomes:
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$n-k$ red lights, we could simply exchange the role of red lights and green lights, we see the result. Remember that you should be able to use two contexts to explain this pattern!

Problem 5 *Explain why the sum of the entries in the n-th row of Pascal's Triangle is 2^n.*

Free Response: ***Hint:*** *Using the context of the pizza shop, the n-th row of Pascal's Triangle represents the total number of pizzas we could make if there are n toppings available. We could also count the number of pizzas as $2 \times 2 \times ... \times 2 = 2^n$, where we have two options for each pizza topping: on or off the pizza. Remember that you should be able to use two contexts to explain this pattern!*

Problem 6 *You flip a coin 5 times. How many different ways are there to flip two heads?* $\boxed{10}$
given

Problem 7 *You flip a coin 5 times. How many different ways are there to flip at least two heads?* $\boxed{26}$
given

Problem 8 *You flip a coin 5 times. How many different outcomes are there?*
$\boxed{2^5}$
given

Problem 9 *You flip a coin 5 times. What is the probability that you flip exactly two heads?* $\boxed{\frac{10}{32}}$
given

Problem 10 *You flip a coin 5 times. What is the probability that you flip at least two heads?* $\boxed{\frac{26}{32}}$
given

Problem 11 *You flip a coin 5 times. What is the probability that your result was HHTTT?* $\boxed{\frac{1}{32}}$
given

Problem 12 *You flip a coin 5 times, and you get two heads. What is the probability that your result was HHTTT?* $\boxed{\frac{1}{10}}$
given

Problem 13 *In your own words, summarize the similarities and differences between the previous problems.*

Free Response:

Problem 14 *A certain passcode is made by choosing two digits in 0 to 9 followed by three shapes (square, triangle, circle, or star). How many such passcodes can be made?* $\boxed{6400}$
given

Problem 15 *A certain passcode is made by choosing two digits in 0 to 9 followed by three shapes (square, triangle, circle, or star). How many such passcodes can be made if you cannot choose the same number or same shape more than once?* $\boxed{2160}$
given

Problem 16 *A certain passcode is made by choosing symbols from the digits in 0 to 9 and three shapes (square, triangle, circle, or star). How many such passcodes can be made if you cannot choose the same number or same shape more than once, but you can choose the numbers and shapes in any order?* $\boxed{240240}$
given

Problem 17 *In your own words, summarize the similarities and differences between the previous problems.*

Free Response:

Problem 18 *In a magical dome, the probability that it will snow on any weekday is* 10%*, while the probability that it will snow on any weekend day is* 60%.

(a) *What is the probability that it will snow on both Monday and Tuesday?* $\boxed{0.01}$

(b) *What is the probability that it will snow on Monday and not snow on Tuesday?* $\boxed{0.09}$

(c) *What is the probability that it will snow on either Monday or Tuesday?* $\boxed{0.18}$

(d) *What is the probability that it will not snow for a whole week?* $\boxed{0.0944784}$

(e) *What is the probability that it will snow on one weekday and not snow on the weekend?* $\boxed{0.052488}$

(f) *What is the probability that it will snow one day during a week?* $\boxed{0.3359232}$