## Geometric Sequences

Problems about geometric (and other) sequences.

**Problem** 1 A sequence has first two terms  $1, 2, \ldots$  What type of sequence is this?

## Multiple Choice:

- (a) An arithmetic sequence.
- (b) A geometric sequence.
- (c) A quadratic sequence.
- (d) It is impossible to tell.  $\checkmark$

**Hint:** If we do not know the rule for generating the terms of this sequence, can we be sure we know the next term?

**Problem 2** Sylvie has a bank account which contains \$23 currently. She decides to set up a savings plan for depositing money in her account. The first week, she will deposit \$4 into this account, and then each week afterwards, she will deposit 10% more than she did the previous week.

Would we use a geometric sequence to describe the amount she deposits each week?

## Multiple Choice:

- (a) Yes, this is a geometric sequence.  $\checkmark$
- (b) No, this is not a geometric sequence.

**Problem 3** Sylvie has a bank account which contains \$23 currently. She decides to set up a savings plan for depositing money in her account. The first week, she will deposit \$4 into this account, and then each week afterwards, she will deposit 10% more than she did the previous week.

Learning outcomes:

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Fill out the following table corresponding to the amount Sylvie deposits in her bank account n weeks from now. (Enter the exact amount; do not round to the nearest cent.)

n	Deposit Amount
0	4
1	4.4
2	$\boxed{4.84}$
3	5.324
4	5.8564
5	6.44204
6	7.086244
7	7.7948684
8	8.57435524

**Problem 4** Sylvie has a bank account which contains \$23 currently. She decides to set up a savings plan for depositing money in her account. The first week, she will deposit \$4 into this account, and then each week afterwards, she will deposit 10% more than she did the previous week.

Write a recursive formula for the amount Sylvie deposits in her account in week n, using "Next" and "Now" to describe the situation.

Sylvie will deposit (Next  $\sqrt{\ Now/\ n/\ 4/\ 1.1/\ 0.1}$ ) = (Next/Now  $\sqrt{\ n/\ 4/\ 1.1/\ 0.1}$ ) × (Next/Now/ $n/\ 4/\ 1.1\ \sqrt{\ 0.1}$ ).

**Problem 5** Sylvie has a bank account which contains \$23 currently. She decides to set up a savings plan for depositing money in her account. The first week, she will deposit \$4 into this account, and then each week afterwards, she will deposit 10% more than she did the previous week.

Write an explicit formula for f(n), the amount she deposits in week n.

Sylvie will deposit 
$$f(n) = \underbrace{\boxed{4(1.1)^n}}_{\text{given}}$$
 in week  $n$ .

**Problem 6** Seth is in a lab, measuring the amount of a decaying substance. He knows that each day, he expects to have  $\frac{1}{12}$  of the amount he had the previous day. If he began with 90 grams of the substance, predict how many grams he will have in the future.

How much of the substance will Seth expect to have on day 5? Seth expects to have  $90(11/12)^5$  grams of the substance.

**Problem 7** Seth is in a lab, measuring the amount of a decaying substance. He knows that each day, he expects to have  $\frac{11}{12}$  of the amount he had the previous day. If he began with 90 grams of the substance, predict how many grams he will have in the future.

When will Seth expect to have 75 grams of the substance? Seth expects to have 75 grams of the substance sometime between day 2 and day 3.

**Problem 8** Seth is in a lab, measuring the amount of a decaying substance. He knows that each day, he expects to have  $\frac{11}{12}$  of the amount he had the previous day. If he began with 90 grams of the substance, predict how many grams he will have in the future.

Write a recursive function for the amount of the substance Seth expects to have on day n. Use f(n-1) in your equation. Seth expects to have f(n) = f(n-1)(11/12) grams of the substance.

**Problem 9** Seth is in a lab, measuring the amount of a decaying substance. He knows that each day, he expects to have  $\frac{11}{12}$  of the amount he had the previous day. If he began with 90 grams of the substance, predict how many grams he will have in the future.

Write an explicit function for the amount of the substance Seth expects to have on day n. Seth expects to have  $f(n) = 90(11/12)^n$  grams of the substance.

**Problem 10** Consider a sequence whose first term is -2, whose second term is 1, and which has a constant second difference  $\Delta\Delta(n) = 3$ .

What type of sequence is this?

## Multiple Choice:

- (a) Arithmetic.
- (b) Geometric.
- (c) Quadratic. ✓
- (d) Something else.

**Problem 11** Consider a sequence whose first term is -2, whose second term is 1, and which has a constant second difference  $\Delta\Delta(n) = 3$ .

Fill out the following table of values for this sequence. Then, find explicit formulas for both  $\Delta(n)$  and f(n).

$\overline{n}$	f(n)	$\Delta(n)$	$\Delta\Delta(n)$
1	-2	1	3
2	-1	4	3
3	3	7	3
4	10	10	3
5	20	13	3
6	33	16	3

We find that 
$$\Delta(n) = \boxed{1 + 3(n-1)}$$
 and that  $f(n) = \boxed{-2 + \frac{3}{2}(n)(n-1) - 2(n-1)}$ 

**Problem 12** Assume the sequence below is a geometric sequence. Fill in the blanks.

$$[3]$$
, 6, 12,  $[24]$ ,  $[48]$ ,  $[96]$ ,  $[192]$ , . . .

**Problem** 13 Assume the sequence below is a geometric sequence. Fill in the blanks.

$$\ldots$$
,  $\boxed{-4}$ ,  $6$ ,  $\boxed{-9}$ ,  $\boxed{13.5}$ ,  $-20.25$ ,  $\boxed{30.375}$ ,  $\boxed{-45.5625}$ ,  $\ldots$ 

**Problem** 14 Assume the sequence below is a geometric sequence. Fill in the blanks.

$$\dots, \boxed{16(0.75)^{(-0.5)}}, \boxed{16(0.75)^{(-0.25)}}, 16, \boxed{16(0.75)^{(0.25)}}, \boxed{16(0.75)^{(0.5)}}, \boxed{16(0.75)^{(0.5)}}, \boxed{16(0.75)^{(0.75)}}, 12, \dots$$