Fractions

Problems about rational numbers.

Problem 1 *Select all numbers below which are fractions.*

Select All Correct Answers:

(a) $\frac{4}{9}$ ✓

(b) 0

(c) e

(d) $\frac{\sqrt{3}}{12}$ ✓

(e) $\frac{-\pi}{3}$ ✓

(f) $\frac{-23}{387}$ ✓

(g) -2.9734

Problem 2 *Select all numbers below which are rational numbers.*

Select All Correct Answers:

(a) $\frac{4}{9}$ ✓

(b) 0 ✓

(c) e

(d) $\frac{\sqrt{3}}{12}$

(e) $\frac{-\pi}{3}$

(f) $\frac{-23}{387}$ ✓

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(g) -2.9734 ✓

Problem 3 *Here's a tricky one! True or false:* $\dfrac{\sqrt{2}}{\sqrt{8}}$ *is a rational number.*

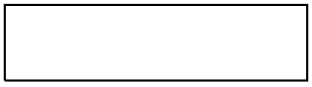
Hint: *Can you simplify this number at all?*

Multiple Choice:

(a) *True* ✓

(b) *False*

Problem 4 *Ashleigh has a brownie recipe that calls for* $\dfrac{2}{9}$ *of a tablespoon of baking powder. (She has some crazy measuring cups at home!). To represent her tablespoon of baking powder, she draws the following rectangle.*



To represent her $\dfrac{2}{9}$ *of a tablespoon, into how many equal-sized pieces should she cut the entire rectangle? Give the most basic answer you can.*

She should cut the rectangle into $\underset{\text{given}}{\boxed{9}}$ *equal-sized pieces.*

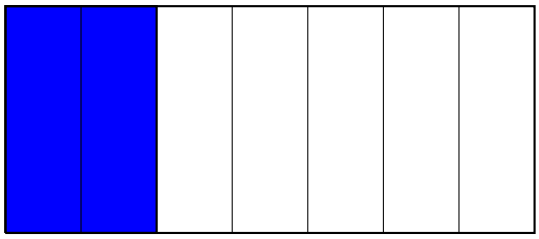
To represent her $\dfrac{2}{9}$ *of a tablespoon, how many of those pieces should she shade? Give the most basic answer you can.*

She should shade $\underset{\text{given}}{\boxed{2}}$ *pieces.*

Problem 5 *In the previous problem, the question asked for "the most basic answer" that you could give. Why was the question phrased in that way? What other kinds of answers might there be?*

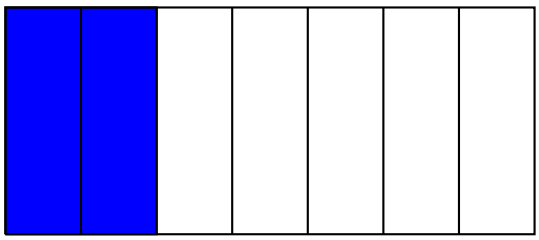
Free Response: ***Hint:*** *There are infinitely many fractions which are equivalent to* $\dfrac{2}{9}$*. If we were not looking for the most basic answer, we might cut our whole into* 27 *pieces, and shade* 6 *of them. Or, we might cut our whole into* 90 *pieces, and shade* 20 *of those pieces.*

Problem 6 *The image below depicts a fraction whose whole is the entire rectangle. What fraction of the entire rectangle is shaded?*



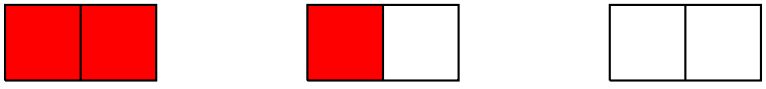
We see that $\frac{\boxed{2}}{\boxed{7}}$ (given / given) *of the rectangle is shaded.*

Problem 7 *The image below depicts a fraction whose whole is the blue shaded region. What fraction is the entire drawing of the blue shaded region?*



We see that the rectangle is $\frac{\boxed{7}}{\boxed{2}}$ (given / given) *of the shaded region.*

Problem 8 *Consider the following picture.*



What fraction of the entire picture is shaded?

$\boxed{\frac{3}{6}}$ given

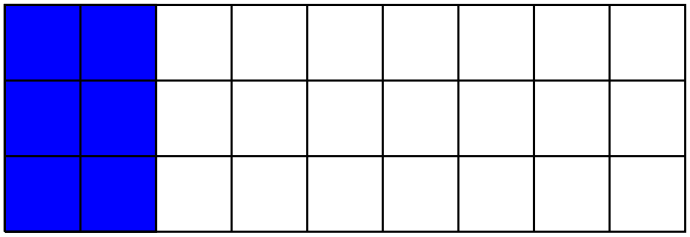
Problem 8.1 *Consider the picture as being composed of three rectangles. What fraction of one rectangle is shaded?*

$\underset{\text{given}}{\boxed{\frac{3}{2}}}$

Problem 8.1.1 *Consider the picture as being composed of six squares. What fraction of one square is shaded?*

$\underset{\text{given}}{\boxed{\frac{3}{1}}}$

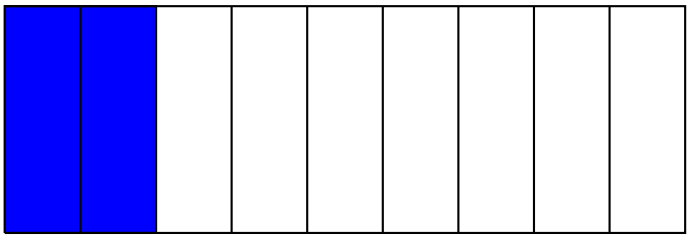
Problem 9 *The image below depicts a fraction whose whole is the entire rectangle. What fraction of the entire rectangle is shaded?*



Rectangle A

We see that $\frac{\underset{\text{given}}{\boxed{6}}}{\underset{\text{given}}{\boxed{27}}}$ *of the rectangle is shaded.*

Problem 9.1 *Compare the rectangle above (Rectangle A) with the one below (Rectangle B).*



Rectangle B

How could we have obtained Rectangle A's drawing from the drawing of Rectangle B? Choose the best answer below.

Multiple Choice:

(a) *The pictures are unrelated.*

(b) *We multiplied the picture by* 3.

(c) *We split each of the pieces in the whole for Rectangle B into three equal pieces.* ✓

(d) *We drew two more horizontal lines on the picture.*

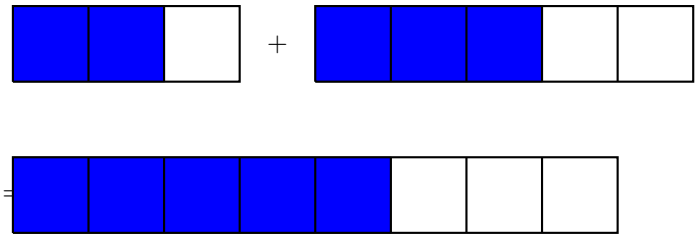
Problem 9.1.1 *Comparing Rectangle A and Rectangle B, we can see the equivalence of which two fractions?*

We see that $\frac{\boxed{2}}{9}$ *given* *is equivalent to* $\frac{\boxed{6}}{27}$ *given*.

Problem 9.1.1.1 *Explain exactly how we can see from the two diagrams that the fractions are equivalent.*

Free Response: ***Hint:*** *First, notice that we start with the same whole in each picture. When we cut each of the nine pieces of Rectangle B into three pieces, we end up with* 27 *pieces making up our whole. At the same time (without doing any more work!) we have also managed to cut each of the original two shaded pieces into three pieces each, leaving us with* 6 *shaded pieces. The shading didn't change at all, and the total amount didn't change at all, so the quantities represented by the two fractions have to be the same.*

Problem 10 *Zeke is adding* $\frac{2}{3} + \frac{3}{5}$. *He uses the following picture.*



He then reports that the answer is $\frac{5}{8}$. *What is right with Zeke's reasoning? What is wrong with Zeke's reasoning?*

Free Response: ***Hint:*** *Zeke is right that we want to combine these two fractions. However, if we were to combine them, we need to have the same size whole for the addition problem to make sense. First, he should shade his fractions out of the same whole. Then, he should cut all of his pieces so that they are the same size, and then combine those same-size pieces.*

Problem 11 *What is the reasoning behind "making a common denominator"?*

Multiple Choice:

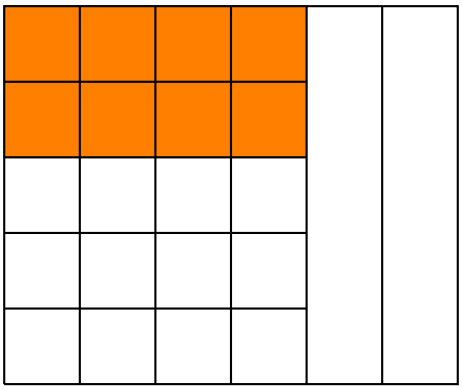
(a) *We always make common denominators when working with fractions.*

(b) *We make common denominators when we need the pieces in our fractions to be the same size.* ✓

(c) *We make common denominators because you cannot add fractions if you don't make common denominators.*

(d) *We make common denominators so that the fractions are out of the same whole.*

Problem 12 *What fraction of the entire rectangle is shaded?*



We see that $\underbrace{\boxed{\frac{8}{30}}}_{\text{given}}$ *of the rectangle is shaded.*

Problem 12.1 *Let's think of the orange shaded area as the result of multiplying two fractions. What are these fractions?*

If we imagine extending the horizontal lines all the way across our rectangle, we can view one whole group as containing $\underbrace{\boxed{\frac{2}{5}}}_{\text{given}}$ *of the entire rectangle. (This would be the continuation of the orange region horizontally across the rectangle.) Then, we can see that we have shaded* $\underbrace{\boxed{\frac{4}{6}}}_{\text{given}}$ *of that group. Thus, our multiplication problem would be* $\underbrace{\boxed{\frac{4}{6}}}_{\text{given}} \times \underbrace{\boxed{\frac{2}{5}}}_{\text{given}}$.

Hint: *Remember: for our meaning of multiplication, the order of the factors matters!*

Problem 13 *A brownie recipe calls for* $\frac{3}{2}$ *of a cup of flour. If you make* $\frac{1}{12}$ *of the brownie recipe, how much flour would you need?*

You would need $\underbrace{\boxed{\frac{3}{24}}}_{\text{given}}$ *of a cup of flour.*

Problem 14 *Yesterday,* $\frac{8}{3}$ *of an inch of rain fell in Doug's garden. Today,* $\frac{2}{5}$ *of an inch of rain fell in Doug's garden. Over the two-day span, how much rain fell in Doug's garden?*

$\underbrace{\boxed{\frac{8}{3}+\frac{2}{5}}}_{\text{given}}$ *of an inch of rain*

Problem 15 *Sue bought* $19\frac{3}{4}$ *pounds of rice to host a very large party. Sam bought* $22\frac{5}{8}$ *pounds of rice to host an even larger party. How much more rice did Sam buy than Sue?*

Sam bought $\underbrace{\boxed{\frac{23}{8}}}_{\text{given}}$ *more pounds of rice.*

Problem 16 *For several years, Mya has been depositing* $\frac{2}{5}$ *of her monthly paycheck into a savings account. This month, Mya has to pay for car repairs, so she can only deposit* $\frac{7}{8}$ *of her usual deposit amount. How much of her paycheck will Mya deposit this month?*

Mya will deposit $\underbrace{\boxed{\frac{14}{40}}}_{\text{given}}$ *of (her paycheck ✓/ her usual deposit amount / a dollar) in her savings account.*
