Geometric Sequences

Problems about geometric (and other) sequences.

Problem 1 *A sequence has first two terms* $1, 2, \ldots$. *What type of sequence is this?*

Multiple Choice:

(a) *An arithmetic sequence.*

(b) *A geometric sequence.*

(c) *A quadratic sequence.*

(d) *It is impossible to tell.* ✓

Hint: *If we do not know the rule for generating the terms of this sequence, can we be sure we know the next term?*

Problem 2 *Sylvie has a bank account which contains* \$23 *currently. She decides to set up a savings plan for depositing money in her account. The first week, she will deposit* \$4 *into this account, and then each week afterwards, she will deposit* 10% *more than she did the previous week.*

Would we use a geometric sequence to describe the amount she deposits each week?

Multiple Choice:

(a) *Yes, this is a geometric sequence.* ✓

(b) *No, this is not a geometric sequence.*

Problem 3 *Sylvie has a bank account which contains* \$23 *currently. She decides to set up a savings plan for depositing money in her account. The first week, she will deposit* \$4 *into this account, and then each week afterwards, she will deposit* 10% *more than she did the previous week.*

Learning outcomes:
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Fill out the following table corresponding to the amount Sylvie deposits in her bank account n weeks from now. (Enter the exact amount; do not round to the nearest cent.)

n	*Deposit Amount*
0	$\boxed{4}$
1	$\boxed{4.4}$
2	$\boxed{4.84}$
3	$\boxed{5.324}$
4	$\boxed{5.8564}$
5	$\boxed{6.44204}$
6	$\boxed{7.086244}$
7	$\boxed{7.7948684}$
8	$\boxed{8.57435524}$

Problem 4 *Sylvie has a bank account which contains $23 currently. She decides to set up a savings plan for depositing money in her account. The first week, she will deposit $4 into this account, and then each week afterwards, she will deposit 10% more than she did the previous week.*

Write a recursive formula for the amount Sylvie deposits in her account in week n, using "Next" and "Now" to describe the situation.

Sylvie will deposit (Next ✓/ Now/ n/ 4/ 1.1/ 0.1) = (Next/ Now ✓/ n/ 4/ 1.1/ 0.1) × (Next/ Now/ n/ 4/ 1.1 ✓/ 0.1).

Problem 5 *Sylvie has a bank account which contains $23 currently. She decides to set up a savings plan for depositing money in her account. The first week, she will deposit $4 into this account, and then each week afterwards, she will deposit 10% more than she did the previous week.*

Write an explicit formula for $f(n)$, the amount she deposits in week n.

Sylvie will deposit $f(n) = \underbrace{\boxed{4(1.1)^n}}_{\text{given}}$ in week n.

Problem 6 *Seth is in a lab, measuring the amount of a decaying substance. He knows that each day, he expects to have $\frac{11}{12}$ of the amount he had the previous day. If he began with 90 grams of the substance, predict how many grams he will have in the future.*

How much of the substance will Seth expect to have on day 5? Seth expects to have $\underbrace{\boxed{90(11/12)^5}}_{\text{given}}$ grams of the substance.

Problem 7 *Seth is in a lab, measuring the amount of a decaying substance. He knows that each day, he expects to have* $\frac{11}{12}$ *of the amount he had the previous day. If he began with* 90 *grams of the substance, predict how many grams he will have in the future.*

When will Seth expect to have 75 *grams of the substance? Seth expects to have* 75 *grams of the substance sometime between day* $\underset{\text{given}}{\boxed{2}}$ *and day* $\underset{\text{given}}{\boxed{3}}$ *.*

Problem 8 *Seth is in a lab, measuring the amount of a decaying substance. He knows that each day, he expects to have* $\frac{11}{12}$ *of the amount he had the previous day. If he began with* 90 *grams of the substance, predict how many grams he will have in the future.*

Write a recursive function for the amount of the substance Seth expects to have on day n*. Use* $f(n-1)$ *in your equation. Seth expects to have* $f(n) = \underset{\text{given}}{\boxed{f(n-1)(11/12)}}$ *grams of the substance.*

Problem 9 *Seth is in a lab, measuring the amount of a decaying substance. He knows that each day, he expects to have* $\frac{11}{12}$ *of the amount he had the previous day. If he began with* 90 *grams of the substance, predict how many grams he will have in the future.*

Write an explicit function for the amount of the substance Seth expects to have on day n*. Seth expects to have* $f(n) = \underset{\text{given}}{\boxed{90(11/12)^n}}$ *grams of the substance.*

Problem 10 *Consider a sequence whose first term is* -2*, whose second term is* 1*, and which has a constant second difference* $\Delta\Delta(n) = 3$*.*

What type of sequence is this?

Multiple Choice:

(a) *Arithmetic.*

(b) *Geometric.*

(c) *Quadratic.* ✓

(d) *Something else.*

Problem 11 *Consider a sequence whose first term is -2, whose second term is 1, and which has a constant second difference $\Delta\Delta(n) = 3$.*

Fill out the following table of values for this sequence. Then, find explicit formulas for both $\Delta(n)$ and $f(n)$.

n	$f(n)$	$\Delta(n)$	$\Delta\Delta(n)$
1	-2	$\boxed{1}$	$\boxed{3}$
2	-1	$\boxed{4}$	$\boxed{3}$
3	$\boxed{3}$	$\boxed{7}$	$\boxed{3}$
4	$\boxed{10}$	$\boxed{10}$	$\boxed{3}$
5	$\boxed{20}$	$\boxed{13}$	$\boxed{3}$
6	$\boxed{33}$	$\boxed{16}$	$\boxed{3}$

We find that $\Delta(n) = \underbrace{\boxed{1 + 3(n-1)}}_{\text{given}}$ *and that* $f(n) = \underbrace{\boxed{-2 + \frac{3}{2}(n)(n-1) - 2(n-1)}}_{\text{given}}$.

Problem 12 *Assume the sequence below is a geometric sequence. Fill in the blanks.*

$$\boxed{3}, 6, 12, \boxed{24}, \boxed{48}, \boxed{96}, \boxed{192}, \ldots$$

Problem 13 *Assume the sequence below is a geometric sequence. Fill in the blanks.*

$$\ldots, \boxed{-4}, 6, \boxed{-9}, \boxed{13.5}, -20.25, \boxed{30.375}, \boxed{-45.5625}, \ldots$$

Problem 14 *Assume the sequence below is a geometric sequence. Fill in the blanks.*

$$\ldots, \boxed{16(0.75)^{(-0.5)}}, \boxed{16(0.75)^{(-0.25)}}, 16, \boxed{16(0.75)^{(0.25)}}, \boxed{16(0.75)^{(0.5)}}, \boxed{16(0.75)^{(0.75)}}, 12, \ldots$$