Integers

Problems about integers.

Problem 1 *Describe the set of integers. Give some relevant and revealing examples/nonexamples.*

Free Response: ***Hint:*** *The integers are the counting numbers, 0, and the opposites of the counting numbers.*

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Problem 2 *Use the definition of divides to decide whether the following statements are true or false. In each case, an explanation must be given justifying your claim.*

(a) $5|30$ *(True ✓/ False)*

Hint: $30 = 5 \cdot 6$.

(b) $7|41$ *(True/ False ✓)*

Hint: *There is no integer solution to* $41 = 7k$. *But* $41 = 7 \cdot 5 + 6$.

(c) $0|3$ *(True/ False ✓)*

Hint: *There is no integer solution to* $3 = 0k$.

(d) $3|0$ *(True ✓/ False)*

Hint: *The solution to* $0 = 3k$ *is* $k = 0$.

(e) $6|(2^2 \cdot 3^4 \cdot 5 \cdot 7)$. *(True ✓/ False)*

Hint: $6 = 2 \cdot 3$, *and* $2 \cdot 3$ *appears in factorization of the second number.*

(f) $1000|(2^7 \cdot 3^9 \cdot 5^{11} \cdot 17^8)$ *(True ✓/ False)*

Hint: $1000 = 2^3 \cdot 5^3$, *and these primes appear enough times in factorization of the second number.*

Learning outcomes:
Author(s): Bart Snapp and Brad Findell

(g) $6000|(2^{21} \cdot 3^{17} \cdot 5^{89} \cdot 29^{20})$. *(True ✓/ False)*

Hint: $6000 = 2^4 \cdot 3 \cdot 5^3$, *and these primes appear enough times in factorization of the second number.*

JavaScript

```
function isPrime(num) {
  for(var i = 2; i < num; i++)
    if(num % i === 0) return false;
  return num > 1;
}

function isPrimeFactorization(x,y) {
  var terms = x.split('*').map( function(t) { return parseInt(t) } );
  return terms.every( isPrime ) &&
    (terms.reduce( function(a,c) { return a*c; }, 1 )) == parseInt(y);
}
```

Problem 3 *Factor the following integers. Enter the primes in increases order, use * for multiplication, and do not use exponents. If the number is prime, enter the number itself.*

(a) 15 $3 * 5$

(b) 12 $2 * 2 * 3$

(c) 111 $\boxed{3 * 37}$

(d) 1234 $\boxed{2 * 617}$

(e) 2345 $\boxed{5 * 7 * 67}$

(f) 4567 $\boxed{4567}$

(g) 111111 $\boxed{3 * 7 * 11 * 13 * 37}$

Problem 4 *Find the greatest common divisors below:*

(a) $\gcd(462, 1463) = \boxed{77}$

Hint: $462 = 2 \cdot 3 \cdot 7 \cdot 11$, *and* $1463 = 7 \cdot 11 \cdot 19$.

(b) $\gcd(541, 4669) = \boxed{1}$.

Hint: 541 *is prime. And* $4669 = 7 \cdot 23 \cdot 29$.

(c) $\gcd(10000, 2^5 \cdot 3^{19} \cdot 5^7 \cdot 11^{13}) = \boxed{10000}$

Hint: $10000 = 2^5 \cdot 5^5$.

(d) $\gcd(11111, 2^{14} \cdot 7^{21} \cdot 41^5 \cdot 101) = \boxed{41}$

Hint: $11111 = 41 \cdot 271$.

(e) $\gcd(437^5, 8993^3) = \boxed{23^5}$

Hint: $437 = 19 \cdot 23$, *and* $8993 = 17 \cdot 23^2$.

Problem 5 *Consider the following:*

$$20 \div 8 = 2 \text{ remainder } 4,$$
$$28 \div 12 = 2 \text{ remainder } 4.$$

Is it correct to say that $20 \div 8 = 28 \div 12$*? (Yes/ No ✓)*

Explain your reasoning.

Free Response: ***Hint:*** *The answer "2 remainder 4" is not a single number but rather a pair of numbers (a quotient and a remainder) that have different meanings. In particular, the 2 is about different things: groups of 8 versus groups of 12. Calling this pair of numbers "equal" is questionable.*

Problem 6 *Give a formula for the* n*th even number:* $\boxed{2n}$

Problem 7 *Give a formula for the* n*th odd number:* $\boxed{2n-1}$.

Problem 8 *Give a formula for the* n*th multiple of 3:* $\boxed{3n}$

Problem 9 *Give a formula for the* n*th multiple of* -7. $\boxed{-7n}$

Problem 10 *Give a formula for the nth number whose remainder when divided by 5 is 1.*

If the first such number is 1, the formula is $\boxed{5n-4}$.

If the first such number is 6, the formula is $\boxed{5n+1}$.