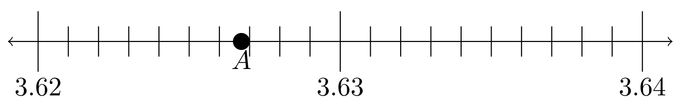
Decimals

Problems about decimal numbers.

Problem 1 *On the number line below, a point is marked A. Select all options which could be candidates for the value of A.*



Select All Correct Answers:

(a) 3.6278 ✓

(b) 3.627783 ✓

(c) 3.68

(d) 3.62788983 ✓

(e) 3.629

Problem 2 *Select all fractions below which have terminating decimal representation.*

Select All Correct Answers:

(a) $\frac{1}{10}$ ✓

(b) $\frac{1}{30}$

(c) $\frac{1}{80}$ ✓

(d) $\frac{1}{64}$ ✓

(e) $\frac{1}{125}$ ✓

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(f) $\dfrac{1}{250}$ ✓

(g) $\dfrac{1}{385}$

(h) $\dfrac{1}{2048}$ ✓

(i) $\dfrac{1}{4228}$

(j) $\dfrac{1}{2^{19} \times 5^{47}}$ ✓

Problem 3 *A harder version of the previous problem: select all fractions below which have terminating decimal representation.*

Select All Correct Answers:

(a) $\dfrac{14}{10}$ ✓

(b) $\dfrac{6}{30}$ ✓

(c) $\dfrac{4}{60}$

(d) $\dfrac{7}{98}$

(e) $\dfrac{11}{125}$ ✓

(f) $\dfrac{3}{150}$ ✓

(g) $\dfrac{11}{385}$

(h) $\dfrac{2}{2049}$

(i) $\dfrac{1057}{4228}$ ✓

(j) $\dfrac{3^4 \times 7^{11} \times 19}{3^2 \times 5^{22} \times 19}$ ✓

Hint: *Don't forget to reduce the fractions to lowest terms!*

Problem 4 *Give an example of an irrational number. For a challenge, don't pick π, e, or $\sqrt{p}$ where p is prime.*

Free Response: ***Hint:*** *One of my favorites is* 0.01001000100001000001 . . . . *This number's decimal representation is neither terminating nor repeating, though it* does *have a pattern!*

Problem 5 *Without doing the long division, after how many places would you expect $\frac{1}{47}$ to repeat?*

We expect the repetition to occur after at most $\boxed{46}$ *places.*
given

Problem 6 *Without doing the long division, after how many places would you expect $\frac{3}{104}$ to repeat?*

We expect the repetition to occur after at most $\boxed{103}$ *places.*
given

Problem 7 *Write each of the following decimals as a fraction using the patterns we observed in class.*

(a) $0.\overline{4} = \frac{\boxed{4}}{\boxed{9}}$

(b) $0.\overline{42} = \frac{\boxed{42}}{\boxed{99}}$

(c) $0.\overline{215} = \frac{\boxed{215}}{\boxed{999}}$

(d) $0.\overline{234584} = \frac{\boxed{234584}}{\boxed{999999}}$

Problem 8 *It is true that $0.\overline{9} = 1$. What do you expect the following to be equal to?*

(a) $1.\overline{9} = \boxed{2}$

(b) $0.5\overline{9} = \boxed{0.6}$

(c) $2.34\overline{9} = \boxed{2.35}$

Problem 9 *Given a prime number p, we will explore a relationship between the number of decimal places in which $\frac{1}{p}$ repeats, and the smallest value of n where p divides $10^n - 1$.*

Consider the case of $p = 3$. We know that $\frac{1}{3} = 0.\overline{\underbrace{\boxed{3}}_{\text{given}}}$, or $\frac{1}{3}$ repeats after $\underbrace{\boxed{1}}_{\text{given}}$ decimal place. What is the smallest value of n so that $3|10^n - 1$?

Hint: *Choose potential values for n in an organized fashion. What is the prime factorization of $10^n - 1$?*

For $p = 3$, we have $n = \boxed{1}$.

Problem 9.1 *Consider the case of $p = 7$. We know that $\frac{1}{7} = 0.\overline{\underbrace{\boxed{142857}}_{\text{given}}}$, or $\frac{1}{7}$ repeats after $\underbrace{\boxed{6}}_{\text{given}}$ places. What is the smallest value of n so that $7|10^n - 1$?*

For $p = 7$, we have $n = \boxed{6}$.

Problem 9.1.1 *Consider the case of $p = 11$. We know that $\frac{1}{11} = 0.\overline{\underbrace{\boxed{09}}_{\text{given}}}$, or $\frac{1}{11}$ repeats after $\underbrace{\boxed{2}}_{\text{given}}$ places. What is the smallest value of n so that $11|10^n - 1$?*

For $p = 11$, we have $n = \boxed{2}$.

Consider the case of $p = 13$. We know that $\frac{1}{13} = 0.\overline{\underbrace{\boxed{076923}}_{\text{given}}}$, or $\frac{1}{13}$ repeats after $\underbrace{\boxed{6}}_{\text{given}}$ places. What is the smallest value of n so that $13|10^n - 1$?

For $p = 13$, we have $n = \boxed{6}$.

Consider the case of $p = 37$. We know that $\frac{1}{37} = 0.\overline{\underbrace{\boxed{027}}_{\text{given}}}$, or $\frac{1}{37}$ repeats after $\underbrace{\boxed{3}}_{\text{given}}$ places. What is the smallest value of n so that $37|10^n - 1$?

For $p = 37$, we have $n = \boxed{3}$.

What pattern are you observing?

Free Response: ***Hint:*** *The number of places in the decimal's repeat is the same as the value of n!*