

# Solving Equations

*Problems about solving equations.*

**Problem 1** Jess is solving the equation  $6x + 13 = 25$ . Here is their work.

$$25 - 13 = 12 \div 6 = 2$$

What is the issue with this work?

**Multiple Choice:**

- (a) The algebra is incorrect.
- (b) The equals sign does not mean equal here. ✓
- (c) The solution is not related to the original equation.
- (d) There is no issue with this work.

**Problem 2** Give a polynomial  $p(x)$  whose leading coefficient is 1, and which has  $x = 12$  and  $x = -1$  as roots (and no other roots).

$$p(x) = \boxed{(x - 12)(x + 1)}$$

given

**Problem 3** Give a polynomial  $p(x)$  whose leading coefficient is 3, and which has  $x = \frac{2}{3}$  and  $x = 2$  as roots (and no other roots).

$$p(x) = \boxed{(3x - 2)(x - 2)}$$

given

**Problem 4** Give a polynomial  $p(x)$  of degree 3 whose leading coefficient is 1, and which has  $x = -5$  as a root (and no other roots).

$$p(x) = \boxed{(x + 5)^3}$$

given

**Problem 5** Give a polynomial  $p(x)$  of degree 4 whose leading coefficient is 1, and which has  $x = 8$ ,  $x = 1 + \sqrt{3}$  and  $x = 1 - \sqrt{3}$  as roots (and no other roots).

$$p(x) = \boxed{(x-8)^2(x-(1+\sqrt{3}))(x-(1-\sqrt{3}))}$$

given

**Problem 6** Solve the problem below by completing the square. Practice drawing a diagram to help! Enter your answers from smallest to largest.

$$x^2 + 8x = 20$$

$$\boxed{-10}, \boxed{2}$$

given      given

**Problem 7** Solve the problem below by completing the square. Practice drawing a diagram to help! Enter your answers from smallest to largest.

$$4x^2 + 9x = 3$$

$$\boxed{-\frac{9}{8} - \frac{\sqrt{129}}{8}}, \boxed{\frac{\sqrt{129}}{8} - \frac{9}{8}}$$

given      given

**Problem 8** According to the Fundamental Theorem of Algebra, how many roots should the polynomial  $p(x) = x^4 - 3x^3 + x - 2$  have?  $\boxed{4}$

given

**Hint:** Remember that the Fundamental Theorem of Algebra counts real and complex roots, and also repeated roots.

**Problem 9** According to the Fundamental Theorem of Algebra, how many roots should the polynomial  $p(x) = x^{16} - 1$  have?  $\boxed{16}$

given

**Problem 10** The Rational Root Theorem says that if  $\pm \frac{a}{b}$  is a root of a polynomial (written in lowest terms), then  $a$  must be a factor of the constant term, and  $b$  must be a factor of the leading term.

For the polynomial  $p(x) = x^3 + 2x^2 - 8x + 4$ , which of the following could be rational roots of  $p(x)$ ? (Do not solve this problem by plugging in the answers to the polynomial!)

**Select All Correct Answers:**

- (a) 1 ✓
- (b) -1 ✓
- (c)  $\frac{3}{2}$
- (d)  $-\frac{2}{3}$
- (e) -4 ✓
- (f)  $\frac{1}{4}$

**Problem 11** The Rational Root Theorem says that if  $\pm \frac{a}{b}$  is a root of a polynomial (written in lowest terms), then  $a$  must be a factor of the constant term, and  $b$  must be a factor of the leading term.

For the polynomial  $p(x) = 12x^8 + 5x^7 + 3x^5 + 14x^3 - x + 20$ , which of the following could be rational roots of  $p(x)$ ? (Do not solve this problem by plugging in the answers to the polynomial!)

**Select All Correct Answers:**

- (a) 3
- (b) -1 ✓
- (c)  $\frac{5}{7}$
- (d)  $-\frac{5}{3}$  ✓
- (e) -4 ✓
- (f)  $\frac{1}{4}$  ✓

**Problem 12** Find all solutions to the equation  $x^5 - 31x^4 + 310x^3 - 1240x^2 + 1984x - 1024 = 0$ .

**Hint:** The Rational Root Theorem combined with some division of polynomials might help!

Enter your answers in order from smallest to largest.

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**Problem 13** Find all solutions to the equation  $x^5 - 28x^4 + 288x^3 - 1358x^2 + 2927x - 2310 = 0$ .

**Hint:** The Rational Root Theorem combined with some division of polynomials might help!

Enter your answers in order from smallest to largest.

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**Problem 14** Find all solutions to the equation  $x^5 - x^4 - 25x^3 + x^2 + 168x + 144 = 0$ .

**Hint:** The Rational Root Theorem combined with some division of polynomials might help!

Enter your answers in order from smallest to largest.

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