

# Algorithms

*More problems about algorithms.*

**Problem 1** Here is an example of a standard addition algorithm:

$$\begin{array}{r} 11 \\ 892 \\ +398 \\ \hline 1290 \end{array}$$

- (a) Describe how to perform this algorithm.
  - (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
  - (c) Show the “behind-the-scenes” algebra that is going on here.
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**Problem 2** Here is an example of the column addition algorithm:

$$\begin{array}{r} 892 \\ +398 \\ \hline 10 \\ 18 \\ \hline 11 \\ \hline 1290 \end{array}$$

- (a) Describe how to perform this algorithm.
  - (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
  - (c) Show the “behind-the-scenes” algebra that is going on here.
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**Problem 3** If you check out Problems ?? and ??, you will learn about “partial” algorithms.

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- (a) Develop a “partial” algorithm for addition, give it a name, and describe how to perform this algorithm.
  - (b) Provide a relevant and revealing example demonstrating that you understand the algorithm.
  - (c) Show the “behind-the-scenes” algebra that is going on here.
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**Problem 4** Here is an example of the banker’s addition algorithm:

$$\begin{array}{r}
 892 \\
 +398 \\
 \hline
 10 \\
 19 \\
 \mathbf{12} \\
 \hline
 1290
 \end{array}$$

- (a) Describe how to perform this algorithm.
  - (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
  - (c) Show the “behind-the-scenes” algebra that is going on here.
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**Problem 5** Here is an example of a standard subtraction algorithm:

$$\begin{array}{r}
 8 \\
 89^{12} \\
 -378 \\
 \hline
 514
 \end{array}$$

- (a) Describe how to perform this algorithm.
  - (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
  - (c) Show the “behind-the-scenes” algebra that is going on here.
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**Problem 6** Here is an example of the subtraction by addition algorithm:

$$\begin{array}{r}
 892 \\
 -378 \\
 \hline
 514
 \end{array}
 \quad \longleftrightarrow \quad
 \begin{array}{l}
 8 + \mathbf{4} = 12 \quad \text{add 1 to 7 to get 8} \\
 8 + \mathbf{1} = 9 \\
 3 + \mathbf{5} = 8
 \end{array}$$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the “behind-the-scenes” algebra that is going on here.

**Problem 7** Here is an example of the Austrian subtraction algorithm:

$$\begin{array}{r}
 8 \ 9^1 2 \\
 -3 \ 8^7 8 \\
 \hline
 5 \ 1 \ 4
 \end{array}$$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the “behind-the-scenes” algebra that is going on here.

**Problem 8** If you check out Problems ?? and ??, you will learn about “partial” algorithms.

- (a) Develop a “partial” algorithm for subtraction, give it a name, and describe how to perform this algorithm.
- (b) Provide a relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the “behind-the-scenes” algebra that is going on here.

**Problem 9** Here is an example of a standard multiplication algorithm:

$$\begin{array}{r} 23 \\ 634 \\ \times 8 \\ \hline 5072 \end{array}$$

- (a) Describe how to perform this algorithm.
  - (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
  - (c) Show the “behind-the-scenes” algebra that is going on here.
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**Problem 10** Here is an example of the partial-products algorithm:

$$\begin{array}{r} 634 \\ \times 8 \\ \hline 4800 \\ 240 \\ 32 \\ \hline 5072 \end{array}$$

- (a) Describe how to perform this algorithm.
  - (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
  - (c) Show the “behind-the-scenes” algebra that is going on here.
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**Problem 11** Here is an example of a standard division algorithm:

$$\begin{array}{r} 97 \text{ R } 1 \\ 8 \overline{)777} \\ \underline{72} \phantom{00} \\ 57 \phantom{00} \\ \underline{56} \phantom{00} \\ 1 \phantom{00} \end{array}$$

- (a) Describe how to perform this algorithm.

- (b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*
  - (c) *Show the “behind-the-scenes” algebra that is going on here.*
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**Problem 12** *Here is an example of the partial quotients algorithm:*

$$\begin{array}{r}
 7 \\
 90 \\
 8 \overline{) 777} \\
 \underline{720} \\
 57 \\
 \underline{56} \\
 1
 \end{array}$$

- (a) *Describe how to perform this algorithm.*
  - (b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*
  - (c) *Show the “behind-the-scenes” algebra that is going on here.*
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**Problem 13** *Here is another example of the partial-quotients division algorithm:*

$$\begin{array}{r}
 4 \\
 10 \\
 10 \\
 10 \\
 10 \\
 8 \overline{) 277} \\
 \underline{80} \\
 197 \\
 \underline{80} \\
 117 \\
 \underline{80} \\
 37 \\
 \underline{32} \\
 5
 \end{array}$$

- (a) *Describe how to perform this algorithm—be sure to explain how this is different from the scaffolding division algorithm.*

- (b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*
  - (c) *Show the “behind-the-scenes” algebra that is going on here.*
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**Problem 14** *Here is an example of a standard multiplication algorithm:*

$$\begin{array}{r}
 634 \\
 \times 216 \\
 \hline
 3804 \\
 6340 \\
 126800 \\
 \hline
 136944
 \end{array}$$

- (a) *Describe how to perform this algorithm.*
  - (b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*
  - (c) *Show the “behind-the-scenes” algebra that is going on here—you may assume that you already know the algebra behind the standard multiplication algorithm.*
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**Problem 15** *Here is an example of the addition algorithm with decimals:*

$$\begin{array}{r}
 1 \\
 37.2 \\
 +8.74 \\
 \hline
 45.94
 \end{array}$$

- (a) *Describe how to perform this algorithm.*
  - (b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*
  - (c) *Show the “behind-the-scenes” algebra that is going on here.*
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**Problem 16** *Here is an example of the multiplication algorithm with decimals:*

$$\begin{array}{r}
 3.40 \\
 \times .21 \\
 \hline
 340 \\
 6800 \\
 \hline
 .7140
 \end{array}$$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the “behind-the-scenes” algebra that is going on here.

**Problem 17** Here is an example of the division algorithm without remainder:

$$\begin{array}{r}
 0.75 \\
 4 \overline{) 3.00} \\
 \underline{28} \phantom{0} \\
 20 \\
 \underline{20} \\
 \hline
 \hline
 \end{array}$$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the “behind-the-scenes” algebra that is going on here.

**Problem 18** In the following addition problem, every digit has been replaced with a letter.

$$\begin{array}{r}
 MOON \\
 + SUN \\
 \hline
 PLUTO
 \end{array}$$

Recover the original problem and solution. Explain your reasoning. Hint:  $S = 6$  and  $U = 5$ .

**Problem 19** In the following addition problem, every digit has been replaced with a letter.

$$\begin{array}{r} \text{SEND} \\ + \text{MORE} \\ \hline \text{MONEY} \end{array}$$

Recover the original problem and solution. Explain your reasoning.

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**Problem 20** In the following subtraction problem, every digit has been replaced with a letter.

$$\begin{array}{r} \text{DEFER} \\ - \text{DU7Y} \\ \hline \text{N2G2} \end{array}$$

Recover the original problem and solution. Explain your reasoning.

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**Problem 21** In the following two subtraction problems, every digit has been replaced with a letter.

$$\begin{array}{r} \text{NINE} \\ - \text{TEN} \\ \hline \text{TWO} \end{array} \qquad \begin{array}{r} \text{NINE} \\ - \text{ONE} \\ \hline \text{ALL} \end{array}$$

Using both problems simultaneously, recover the original problems and solutions. Explain your reasoning.

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**Problem 22** In the following multiplication problem, every digit has been replaced with a letter.

$$\begin{array}{r} \text{LET} \\ \times \text{NO} \\ \hline \text{SOT} \\ \text{NOT} \\ \hline \text{FRET} \end{array}$$



Recover the original problem and solution. Explain your reasoning.

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**Problem 23** The following is a long division problem where every digit except 7 was replaced by X.

$$\begin{array}{r}
 \phantom{XX}X7X \\
 XX \overline{)XXXXX} \\
 \underline{X77} \phantom{X} \\
 X7X \phantom{X} \\
 \underline{X7X} \phantom{X} \\
 \phantom{X}XX \\
 \underline{\phantom{X}XX} \\
 \phantom{X}\phantom{X}
 \end{array}$$

Recover the digits from this long division problem. Explain your reasoning.

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**Problem 24** The following is a long division problem where the various digits were replaced by X except for a single 8. The double bar indicates that the remainder is 0.

$$\begin{array}{r}
 \phantom{XXX}XX8XX \\
 XXX \overline{)XXXXXXXX} \\
 \underline{XXX} \phantom{X} \\
 XXXX \phantom{X} \\
 \underline{XXX} \phantom{X} \\
 XXXX \phantom{X} \\
 \underline{XXXX} \\
 \phantom{X}\phantom{X}\phantom{X}
 \end{array}$$

Recover the digits from this long division problem. Explain your reasoning.

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