

Integers

Problems about integers.

Problem 1 Describe the set of integers. Give some relevant and revealing examples/nonexamples.

Free Response: **Hint:** The integers are the counting numbers, 0, and the opposites of the counting numbers.

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Problem 2 Use the definition of divides to decide whether the following statements are true or false. In each case, an explanation must be given justifying your claim.

(a) $5|30$ (True ✓/ False)

Hint: $30 = 5 \cdot 6$.

(b) $7|41$ (True/ False ✓)

Hint: There is no integer solution to $41 = 7k$. But $41 = 7 \cdot 5 + 6$.

(c) $0|3$ (True/ False ✓)

Hint: There is no integer solution to $3 = 0k$.

(d) $3|0$ (True ✓/ False)

Hint: The solution to $0 = 3k$ is $k = 0$.

(e) $6|(2^2 \cdot 3^4 \cdot 5 \cdot 7)$. (True ✓/ False)

Hint: $6 = 2 \cdot 3$, and $2 \cdot 3$ appears in factorization of the second number.

(f) $1000|(2^7 \cdot 3^9 \cdot 5^{11} \cdot 17^8)$ (True ✓/ False)

Hint: $1000 = 2^3 \cdot 5^3$, and these primes appear enough times in factorization of the second number.

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(g) $6000 \mid (2^{21} \cdot 3^{17} \cdot 5^{89} \cdot 29^{20})$. (True \checkmark / False)

Hint: $6000 = 2^4 \cdot 3 \cdot 5^3$, and these primes appear enough times in factorization of the second number.

JavaScript

```

1 function isPrime(num) {
2   for(var i = 2; i < num; i++)
3     if(num % i === 0) return false;
4   return num > 1;
5 }
6
7 function isPrimeFactorization(x,y) {
8   var terms = x.split('*').map( function(t) { return parseInt(t) } );
9   return terms.every( isPrime ) &&
10    (terms.reduce( function(a,c) { return a*c; }, 1 )) == parseInt(y);
11 }

```

Problem 3 Factor the following integers. Use * for multiplication and do not use exponents. Enter the number itself if it is prime.

(a) 15 $3 * 5$

(b) 12 $2 * 2 * 3$

(c) 111 $3 * 37$

(d) 1234 $2 * 617$

(e) 2345 $5 * 7 * 67$

(f) 4567 4567

(g) 111111 $3 * 7 * 11 * 13 * 37$

Problem 4 Find the greatest common divisors below:

(a) $\gcd(462, 1463) =$ 77

Hint: $462 = 2 \cdot 3 \cdot 7 \cdot 11$, and $1463 = 7 \cdot 11 \cdot 19$.

(b) $\gcd(541, 4669) =$ 1.

Hint: 541 is prime. And $4669 = 7 \cdot 23 \cdot 29$.

(c) $\gcd(10000, 2^5 \cdot 3^{19} \cdot 5^7 \cdot 11^{13}) = \boxed{10000}$

Hint: $10000 = 2^5 \cdot 5^5$.

(d) $\gcd(11111, 2^{14} \cdot 7^{21} \cdot 41^5 \cdot 101) = \boxed{41}$

Hint: $11111 = 41 \cdot 271$.

(e) $\gcd(437^5, 8993^3) = \boxed{23^5}$

Hint: $437 = 19 \cdot 23$, and $8993 = 17 \cdot 23^2$.

Problem 5 Consider the following:

$$20 \div 8 = 2 \text{ remainder } 4,$$

$$28 \div 12 = 2 \text{ remainder } 4.$$

Is it correct to say that $20 \div 8 = 28 \div 12$? (Yes/ No ✓)

Explain your reasoning.

Free Response: **Hint:** The answer “2 remainder 4” is not a single number but rather a pair of numbers (a quotient and a remainder) that have different meanings. In particular, the 2 is about different things: groups of 8 versus groups of 12. Calling this pair of numbers “equal” is questionable.

Problem 6 Give a formula for the n th even number: $\boxed{2n}$

Problem 7 Give a formula for the n th odd number: $\boxed{2n - 1}$.

Problem 8 Give a formula for the n th multiple of 3: $\boxed{3n}$

Problem 9 Give a formula for the n th multiple of -7 . $\boxed{-7n}$

Problem 10 Give a formula for the n th number whose remainder when divided by 5 is 1.

If the first such number is 1, the formula is $\boxed{5n - 4}$.

If the first such number is 6, the formula is $\boxed{5n + 1}$.
