

Polynomials

Problems about polynomials

Problem 1 Explain what is meant by a *polynomial* in a variable x .

Problem 2 Given:

$$3x^7 - x^5 + x^4 - 16x^3 + 27 = a_7x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x^1 + a_0$$

Find $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7$.

Problem 3 Given:

$$6x^5 + a_4x^4 - x^2 + a_0 = a_5x^5 - 24x^4 + a_3x^3 + a_2x^2 - 5$$

Find $a_0, a_1, a_2, a_3, a_4, a_5$.

Problem 4 Is it true that polynomials are equal if and only if their coefficients are equal? Explain your reasoning.

Problem 5 Is it true that numbers are equal if and only if their digits are equal? Explain your reasoning.

Problem 6 Explain how to add two polynomials. Explain, in particular, how “collecting like terms” is an application of the properties of arithmetic.

Problem 7 Explain how to multiply two polynomials.

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Problem 8 Here is an example of the polynomial division algorithm:

$$\begin{array}{r}
 x^2 + 3x + 1 \overline{) \begin{array}{l} x^3 + 0x^2 + x + 1 \\ x^3 + 3x^2 + x \\ \hline 011 \end{array} } \begin{array}{l} x - 3 \\ R 9x + 4 \end{array} \\
 \begin{array}{l} 011 \end{array} \begin{array}{l} [t] \\ -3x^2 + 0x + 1 \\ \hline -3x^2 - 9x - 3 \\ \hline 412 \end{array} \begin{array}{l} 9x + 4 \end{array}
 \end{array}$$

- Describe how to perform this algorithm.
- Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- Show the “behind-the-scenes” algebra that is going on here.

Problem 9 State the Division Theorem for polynomials. Give some relevant and revealing examples of this theorem in action.

Problem 10 Given a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

can you find two numbers L and U such that $L \leq p(x) \leq U$ for all x ? If so, explain why. If not, explain why not.

Problem 11 Consider all polynomials of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where the a_i 's are integers. If you substitute an integer for x will you always get an integer out? Explain your reasoning.

Problem 12 Consider the following polynomial:

$$p(x) = \frac{x^2}{2} + \frac{x}{2}$$

Will $p(x)$ always returns an integer when an integer is substituted for x ? Explain your reasoning.

Problem 13 Fix some integer value for x and consider all polynomials of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Where the a_i 's are integers greater than or equal to 0. Which numbers can be represented by such polynomials? Explain your reasoning.

Problem 14 Find a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

such that a_i 's are integers greater than or equal to 0 and less than 2 such that $p(2) = 35$. Discuss how your answer compares to the representation of 35 in base two. Explain your reasoning.

Problem 15 Find a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

such that a_i 's are integers greater than or equal to 0 and less than 7 such that $p(7) = 234$. Discuss how your answer compares to the representation of 234 in base seven. Explain your reasoning.

Problem 16 Find a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

such that a_i 's are integers greater than or equal to 0 and less than 10 such that $p(10) = 18$. Discuss how your answer compares to the representation of 18 in base ten. Explain your reasoning.

Problem 17 Find a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

such that a_i 's are integers greater than or equal to 0 and less than 15 such that $p(15) = 201$. Discuss how your answer compares to the representation of 201 in base fifteen. Explain your reasoning.

Problem 18 Fix some integer value for x and consider all polynomials of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Where the a_i 's are integers greater than or equal to 0 and less than x . Which numbers can be represented by such polynomials? Explain your reasoning. Big hint: Base x .

Problem 19 Fix some integer value for x and consider all polynomials of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Where the a_i 's are integers greater than or equal to 0 and less than 10. Which numbers can be represented by such polynomials? Explain your reasoning.

Problem 20 Consider $x^2 + x + 1$. This can be thought of as a “number” in base x . Express this number in base $(x + 1)$, that is, find b_0, b_1, b_2 such that

$$b_2(x + 1)^2 + b_1(x + 1) + b_0 = x^2 + x + 1.$$

Explain your reasoning.

Problem 21 Consider $x^2 + 2x + 3$. This can be thought of as a “number” in base x . Express this number in base $(x - 1)$, that is, find b_0, b_1, b_2 such that

$$b_2(x - 1)^2 + b_1(x - 1) + b_0 = x^2 + 2x + 3.$$

Explain your reasoning.

Problem 22 Consider $x^3 + 2x + 1$. This can be thought of as a “number” in base x . Express this number in base $(x - 1)$, that is, find b_0, b_1, b_2, b_3 such that

$$b_3(x - 1)^3 + b_2(x - 1)^2 + b_1(x - 1) + b_0 = x^3 + 2x + 1.$$

Explain your reasoning.

Problem 23 If the polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

is thought of as a “number” in base x , describe two different ways to find the base $(x - 1)$ coefficients of $p(x)$.