#### Algorithms

Problems about algorithms.

**Problem 1** Explain what it means for an operation  $\star$  to be associative. Give some relevant and revealing examples and non-examples.

**Problem 2** Consider the following pictures:



Jesse claims that these pictures represent  $(2 \cdot 3) \cdot 4$  and  $2 \cdot (3 \cdot 4)$ .

- (a) Is Jesse's claim correct? Explain your reasoning.
- (b) Do Jesse's pictures show the associativity of multiplication? If so, explain why. If not, draw new pictures representing  $(2\cdot 3)\cdot 4$  and  $2\cdot (3\cdot 4)$  that do show the associativity of multiplication.

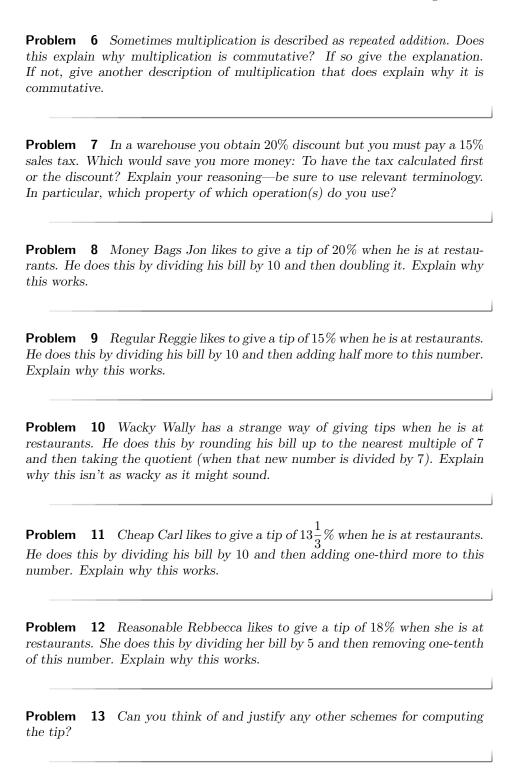
**Problem 3** Explain what it means for an operation  $\star$  to be commutative. Give some relevant and revealing examples and non-examples.

**Problem 4** Explain what it means for an operation  $\star$  to distribute over another operation  $\dagger$ . Give some relevant and revealing examples and non-examples.

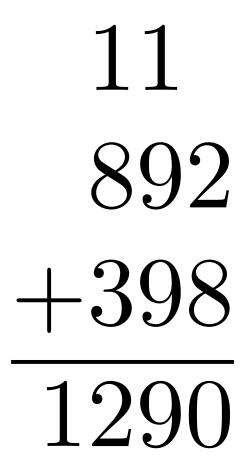
**Problem 5** Explain what it means for an operation  $\star$  to be closed on a set of numbers. Give some relevant and revealing examples and non-examples.

Learning outcomes:

Author(s): Bart Snapp and Brad Findell



**Problem 14** Here is an example of a standard addition algorithm:



- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 15** Here is an example of the column addition algorithm:

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

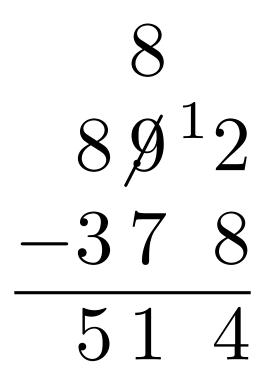
**Problem 16** If you check out Problems and , you will learn about "partial" algorithms.

- (a) Develop a "partial" algorithm for addition, give it a name, and describe how to perform this algorithm.
- (b) Provide a relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 17** Here is an example of the banker's addition algorithm:

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 18** Here is an example of a standard subtraction algorithm:



- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 19** Here is an example of the subtraction by addition algorithm:

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 20** Here is an example of the Austrian subtraction algorithm:

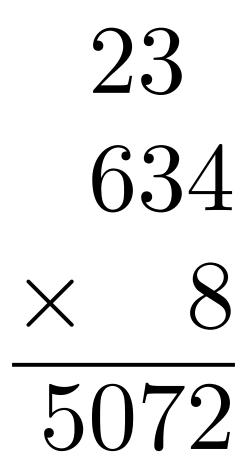
$$89^{1}2$$
 $-3^{8}78$ 
 $\overline{5}14$ 

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

 $\begin{array}{ll} \textbf{Problem} & \textbf{21} & \textit{If you check out Problems} & \textit{and , you will learn about "partial"} \\ & \textit{algorithms}. \end{array}$ 

- (a) Develop a "partial" algorithm for subtraction, give it a name, and describe how to perform this algorithm.
- (b) Provide a relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 22** Here is an example of a standard multiplication algorithm:



- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 23** Here is an example of the partial-products algorithm:

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 24** Here is an example of a standard division algorithm:

$$\begin{array}{r}
 97 R 1 \\
 \hline
 8 ) 777 \\
 \hline
 72 \\
 \hline
 57 \\
 \hline
 56 \\
 \hline
 1
 \end{array}$$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 25** Here is an example of the partial quotients algorithm:

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 26** Here is another example of the partial-quotients division algorithm:

- (a) Describe how to perform this algorithm—be sure to explain how this is different from the scaffolding division algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 27** Here is an example of a standard multiplication algorithm:

### 634 $\times 216$ 3804 6340 126800136944

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here—you may

assume that you already know the algebra behind the standard multiplication algorithm.

**Problem 28** Here is an example of the addition algorithm with decimals:

$$1 \\ 37.2 \\ +8.74 \\ \hline 45.94$$

- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

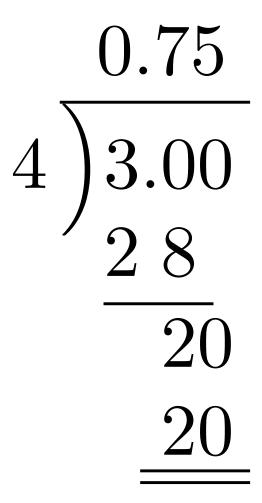
**Problem 29** Here is an example of the multiplication algorithm with decimals:

$$3.40$$
 $\times .21$ 
 $340$ 
 $6800$ 
 $7140$ 

(a) Describe how to perform this algorithm.

- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.
- (c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 30** Here is an example of the division algorithm without remainder:



- (a) Describe how to perform this algorithm.
- (b) Provide an additional relevant and revealing example demonstrating that you understand the algorithm.

(c) Show the "behind-the-scenes" algebra that is going on here.

**Problem 31** In the following addition problem, every digit has been replaced with a letter.

## $\frac{\textit{MOON}}{+ \textit{SUN}}$

Recover the original problem and solution. Explain your reasoning. Hint: S=6 and U=5.

**Problem 32** In the following addition problem, every digit has been replaced with a letter.

### SEND +MORE MONEY

Recover the original problem and solution. Explain your reasoning.

**Problem 33** In the following subtraction problem, every digit has been replaced with a letter.

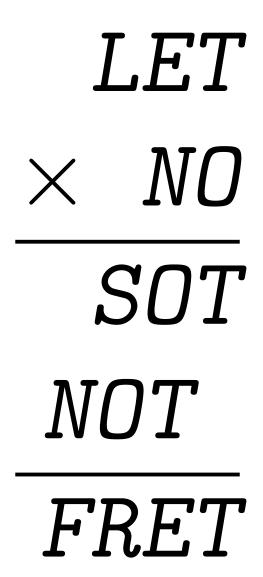
# DEFER - DU7Y N2G2

Recover the original problem and solution. Explain your reasoning.

**Problem 34** In the following two subtraction problems, every digit has been replaced with a letter.

Using both problems simultaneously, recover the original problems and solutions. Explain your reasoning.

**Problem 35** In the following multiplication problem, every digit has been replaced with a letter.



Recover the original problem and solution. Explain your reasoning.

**Problem 36** The following is a long division problem where every digit except

7 was replaced by X.

$$\begin{array}{c} X7X \\ XXXXXX \\ \underline{X77} \\ \overline{X7X} \\ \underline{X7X} \\ \underline{X7X} \\ \underline{XX} \end{array}$$

Recover the digits from this long division problem. Explain your reasoning.

**Problem 37** The following is a long division problem where the various digits were replaced by X except for a single 8. The double bar indicates that the remainder is 0.

Recover the digits from this long division problem. Explain your reasoning.

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