# Integers

*Problems about integers.*

**Problem 1** *Describe the set of integers. Give some relevant and revealing examples/nonexamples.*

***Free Response:*** ***Hint:*** *The integers are the counting numbers, 0, and the opposites of the counting numbers.*

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

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**Problem 2** *Give some examples of contexts we might use to describe integers. What relevant properties should your context have?*

***Free Response:*** ***Hint:*** *Common contexts include temperature, bank balance, and elevation. A good context should have a sense of positive and negative, or a quantity and its opposite.*

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For the next few problems, identify which operation should be used to solve the problem.

**Problem 3** *Alison has 34 head bands. April has 38 headbands. How many more headbands does April have? The problem above is an example of:*

***Multiple Choice:***

(a) *Addition.*

(b) *Take-away subtraction.*

(c) *Comparison subtraction.* ✓

(d) *Missing-addend subtraction.*

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**Problem 4** *Gabe has a bank balance of* \$17. *After he spends* \$22, *what will Gabe's balance be? The problem above is an example of:*

***Multiple Choice:***

(a) *Addition.*

(b) *Take-away subtraction.* ✓

(c) *Comparison subtraction.*

(d) *Missing-addend subtraction.*

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**Problem 5** *Yesterday, Ollie was* 9 *miles west of home. Today, Ollie is* 6 *miles east of home. How far did Ollie travel overnight? The problem above is an example of:*

***Multiple Choice:***

(a) *Addition.*

(b) *Take-away subtraction.*

(c) *Comparison subtraction.*

(d) *Missing-addend subtraction.* ✓

***Hint:*** *The problem is asking what was added to Ollie's distance from home yesterday in order to get to today's location.*

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**Problem 6** *Last week, the average temperature was* $-2$ *degrees Celsius. This week, the average temperature is* 4 *degrees warmer. What is the average temperature this week? The problem above is an example of:*

***Multiple Choice:***

(a) *Addition.* ✓

(b) *Take-away subtraction.*

(c) *Comparison subtraction.*

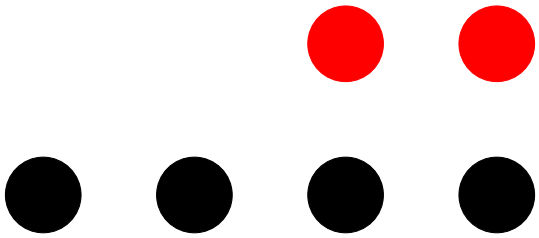
(d) *Missing-addend subtraction.*

The next few problems use the red and black chips we discussed for modeling integers.

**Problem 7** *What is the total value of the chips in the following picture?*

*The total value is* $\underset{\text{given}}{\boxed{1}}$ .

**Question 8** *What is the total value of all of the chips in the picture below?*



*The total value of the chips is* $\underset{\text{given}}{\boxed{2}}$ .

**Question 9** *Suppose you would like to use chips to represent a total value of 8. Which of the following combinations of chips would give you this value?*

***Select All Correct Answers:***

(a) 8 *black chips* ✓

(b) 8 *red chips*

(c) 10 *black chips and* 2 *red chips* ✓

(d) 2 *black chips and* 10 *red chips*

(e) 24 *black chips and* 16 *red chips* ✓

(f) 3 *black chips and* 11 *red chips*

(g) 5 *black chips and* 3 *red chips*

**Question 10** *Suppose you would like to use chips to represent a total value of* $-6$*. Which of the following combinations of chips would give you this value?*

***Select All Correct Answers:***

(a) $-6$ *black chips*

(b) 6 *red chips* ✓

(c) 6 *black chips and* 6 *red chips*

(d) 2 *black chips and* 4 *red chips*

(e) 10 *black chips and* 6 *red chips*

(f) 24 *black chips and* 30 *red chips* ✓

(g) 18 *black chips and* 12 *red chips*

**Question 11** *You are working on the subtraction problem* $10 - (-2)$*, and you would like to solve this problem using red and black chips from a take-away perspective. First, you put your chips down on the table. What chips will you need?*

*You need to model a total value of* $\underset{\text{given}}{\boxed{10}}$ *using the chips. You will need at least* $\underset{\text{given}}{\boxed{2}}$ *red chips in your model so that you have enough to take away.*

**Question 11.1** *You decide to use a model with* 12 *black chips and* 2 *red chips. How many chips do you take away, and what is left?*

*You take away* $\underset{\text{given}}{\boxed{2}}$ *(black/ red ✓) chips, leaving* $\underset{\text{given}}{\boxed{12}}$ *black chips and* $\underset{\text{given}}{\boxed{0}}$ *red chips.*

**Question 11.1.1** *What is* $10 - (-2)$*, from your result with the chips?*

$10 - (-2) = \underset{\text{given}}{\boxed{12}}$

**Question 12** *You are working on the subtraction problem* $(-4)-(-7)$*, and you would like to solve this problem using red and black chips from a take-away perspective. First, you put your chips down on the table. What chips will you need?*

*You need to model a total value of* $\underset{\text{given}}{\boxed{-4}}$ *using the chips. You will need at least* $\underset{\text{given}}{\boxed{7}}$ *red chips in your model so that you have enough to take away.*

**Question 12.1** *You decide to use a model with* 7 *black chips and* 10 *red chips. How many chips do you take away, and what is left?*

*You take away* $\underset{\text{given}}{\boxed{7}}$ *(black/ red ✓) chips, leaving* $\underset{\text{given}}{\boxed{7}}$ *black chips and* $\underset{\text{given}}{\boxed{3}}$ *red chips.*

**Question 12.1.1** *What is* $(-4)-(-7)$*, from your result with the chips?*

$(-4)-(-7)=\underset{\text{given}}{\boxed{3}}$

**Question 13** *You are working on the subtraction problem* $1-5$*, and you would like to solve this problem using red and black chips from a missing-addend perspective. First, you change your problem into the related addition problem. What is the result?*

*The related addition problem is* $\underset{\text{given}}{\boxed{1}}=\underset{\text{given}}{\boxed{5}}+?$.

**Question 13.1** *We now need to begin with a model for* 5 *and decide how to get to* 1*. What chips would we lay down to do this?*

*We lay down* $\underset{\text{given}}{\boxed{4}}$ *(black/ red ✓) chips.*

**Question 13.1.1** *Using our work with the chips, what is* $1-5$*?*

$1-5=\underset{\text{given}}{\boxed{-4}}$

**Question 14** *You are working on the subtraction problem* $(-8)-7$*, and you would like to solve this problem using red and black chips from a missing-addend perspective. First, you change your problem into the related addition problem. What is the result?*

*The related addition problem is* $\underset{\text{given}}{\boxed{-8}} = \underset{\text{given}}{\boxed{7}} + ?$.

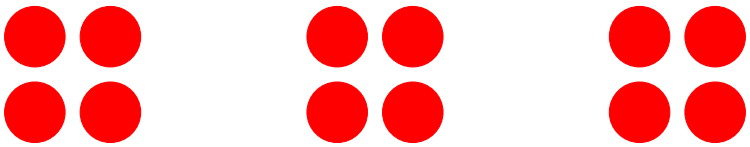
**Question 14.1** *We now need to begin with a model for* 7 *and decide how to get to* $-8$*. What chips would we lay down to do this?*

*We lay down* $\underset{\text{given}}{\boxed{15}}$ *(black/ red ✓) chips.*

**Question 14.1.1** *Using our work with the chips, what is* $(-8)-7$*?*

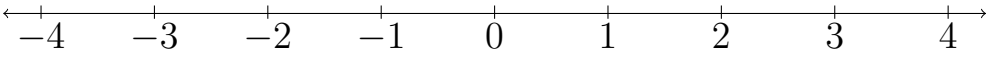
$(-8)-7 = \underset{\text{given}}{\boxed{-15}}$

**Question 15** *In the picture below, use our definition of multiplication to find the total value of the chips.*



*The total value of the chips is* $\underset{\text{given}}{\boxed{3}} \times \underset{\text{given}}{\boxed{-4}}$.

**Example 1.** *Imagine using a number line like the one below to solve the subtraction problem* $-8-4$*.*

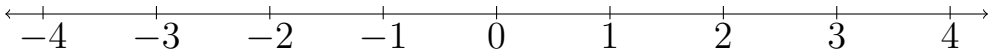



*We begin by standing on the number line at the tick marked with* $\underset{\text{given}}{\boxed{-8}}$*. Since we are subtracting, we face towards the (right/ left ✓). We will move* $\underset{\text{given}}{\boxed{4}}$ *spaces*

*(forward ✓/ backward), since* 4 *is positive. Where on the number line are we now?*

*We are located at the tick labeled* $\underset{\text{given}}{\boxed{-12}}$.

**Example 2.** *Imagine using a number line like the one below to solve the subtraction problem* $(-22) - (-54)$.




*We begin by standing on the number line at the tick marked with* $\underset{\text{given}}{\boxed{-22}}$. *Since we are subtracting, we face towards the (right/ left ✓). We will move* $\underset{\text{given}}{\boxed{54}}$ *spaces (forward/ backward ✓), since* 54 *is negative. Where on the number line are we now?*

*We are located at the tick labeled* $\underset{\text{given}}{\boxed{32}}$.

**Question 16** *In your own words, what is the difference between subtraction and negation?*

***Free Response:*** ***Hint:*** *Subtraction is an operation, requiring two numbers. With subtraction, we have two quantities that we are trying to relate. With our models, subtraction requires some movement: removing chips, adding chips, or walking on the number line. Negation, on the other hand, only requires a single value. With negation, we are essentially just asking, "What is the opposite?" With our models, this is essentially a switching rather than a moving: we switch from black chips to red, or we face backward instead of forward.*