

Integers

Problems about integers.

Problem 1 Describe the set of integers. Give some relevant and revealing examples/nonexamples.

Free Response: **Hint:** The integers are the counting numbers, 0, and the opposites of the counting numbers.

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Problem 2 Give some examples of contexts we might use to describe integers. What relevant properties should your context have?

Free Response: **Hint:** Common contexts include temperature, bank balance, and elevation. A good context should have a sense of positive and negative, or a quantity and its opposite.

For the next few problems, identify which operation should be used to solve the problem.

Problem 3 Alison has 34 head bands. April has 38 headbands. How many more headbands does April have? The problem above is an example of:

Multiple Choice:

- (a) Addition.
- (b) Take-away subtraction.
- (c) Comparison subtraction. ✓
- (d) Missing-addend subtraction.

Learning outcomes:

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Problem 4 Gabe has a bank balance of \$17. After he spends \$22, what will Gabe's balance be? The problem above is an example of:

Multiple Choice:

- (a) Addition.
 - (b) Take-away subtraction. ✓
 - (c) Comparison subtraction.
 - (d) Missing-addend subtraction.
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Problem 5 Yesterday, Ollie was 9 miles west of home. Today, Ollie is 6 miles east of home. How far did Ollie travel overnight? The problem above is an example of:

Multiple Choice:

- (a) Addition.
- (b) Take-away subtraction.
- (c) Comparison subtraction.
- (d) Missing-addend subtraction. ✓

Hint: The problem is asking what was added to Ollie's distance from home yesterday in order to get to today's location.

Problem 6 Last week, the average temperature was -2 degrees Celsius. This week, the average temperature is 4 degrees warmer. What is the average temperature this week? The problem above is an example of:

Multiple Choice:

- (a) Addition. ✓
- (b) Take-away subtraction.
- (c) Comparison subtraction.
- (d) Missing-addend subtraction.

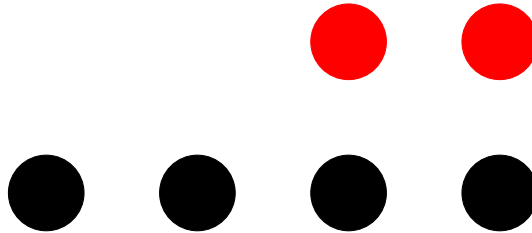
The next few problems use the red and black chips we discussed for modeling integers.

Problem 7 What is the total value of the chips in the following picture?



The total value is $\boxed{1}$.
given

Question 8 What is the total value of all of the chips in the picture below?



The total value of the chips is $\boxed{2}$.
given

Question 9 Suppose you would like to use chips to represent a total value of 8. Which of the following combinations of chips would give you this value?

Select All Correct Answers:

- (a) 8 black chips ✓
- (b) 8 red chips
- (c) 10 black chips and 2 red chips ✓
- (d) 2 black chips and 10 red chips
- (e) 24 black chips and 16 red chips ✓
- (f) 3 black chips and 11 red chips
- (g) 5 black chips and 3 red chips

Question 10 Suppose you would like to use chips to represent a total value of -6 . Which of the following combinations of chips would give you this value?

Select All Correct Answers:

- (a) -6 black chips
- (b) 6 red chips ✓
- (c) 6 black chips and 6 red chips
- (d) 2 black chips and 4 red chips
- (e) 10 black chips and 6 red chips
- (f) 24 black chips and 30 red chips ✓
- (g) 18 black chips and 12 red chips

Question 11 You are working on the subtraction problem $10 - (-2)$, and you would like to solve this problem using red and black chips from a take-away perspective. First, you put your chips down on the table. What chips will you need?

You need to model a total value of $\boxed{10}_{\text{given}}$ using the chips. You will need at least

$\boxed{2}_{\text{given}}$ red chips in your model so that you have enough to take away.

Question 11.1 You decide to use a model with 12 black chips and 2 red chips. How many chips do you take away, and what is left?

You take away $\boxed{2}_{\text{given}}$ (black/ red ✓) chips, leaving $\boxed{12}_{\text{given}}$ black chips and $\boxed{0}_{\text{given}}$ red chips.

Question 11.1.1 What is $10 - (-2)$, from your result with the chips?

$$10 - (-2) = \boxed{12}_{\text{given}}$$

Question 12 You are working on the subtraction problem $(-4) - (-7)$, and you would like to solve this problem using red and black chips from a take-away perspective. First, you put your chips down on the table. What chips will you need?

You need to model a total value of $\boxed{-4}$ _{given} using the chips. You will need at least

$\boxed{7}$ _{given} red chips in your model so that you have enough to take away.

Question 12.1 You decide to use a model with 7 black chips and 10 red chips. How many chips do you take away, and what is left?

You take away $\boxed{7}$ _{given} (black/ red ✓) chips, leaving $\boxed{7}$ _{given} black chips and $\boxed{3}$ _{given} red chips.

Question 12.1.1 What is $(-4) - (-7)$, from your result with the chips?

$$(-4) - (-7) = \boxed{3}$$
_{given}

Question 13 You are working on the subtraction problem $1 - 5$, and you would like to solve this problem using red and black chips from a missing-addend perspective. First, you change your problem into the related addition problem. What is the result?

The related addition problem is $\boxed{1}$ _{given} = $\boxed{5}$ _{given} + ?.

Question 13.1 We now need to begin with a model for 5 and decide how to get to 1. What chips would we lay down to do this?

We lay down $\boxed{4}$ _{given} (black/ red ✓) chips.

Question 13.1.1 Using our work with the chips, what is $1 - 5$?

$$1 - 5 = \boxed{-4}$$
_{given}

Question 14 You are working on the subtraction problem $(-8) - 7$, and you would like to solve this problem using red and black chips from a missing-addend perspective. First, you change your problem into the related addition problem. What is the result?

The related addition problem is $\boxed{-8}_{\text{given}} = \boxed{7}_{\text{given}} + ?$.

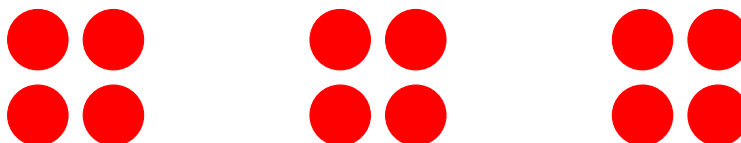
Question 14.1 We now need to begin with a model for 7 and decide how to get to -8 . What chips would we lay down to do this?

We lay down $\boxed{15}_{\text{given}}$ (black/ red ✓) chips.

Question 14.1.1 Using our work with the chips, what is $(-8) - 7$?

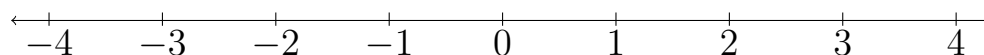
$(-8) - 7 = \boxed{-15}_{\text{given}}$

Question 15 In the picture below, use our definition of multiplication to find the total value of the chips.



The total value of the chips is $\boxed{3}_{\text{given}} \times \boxed{-4}_{\text{given}}$.

Example 1. Imagine using a number line like the one below to solve the subtraction problem $-8 - 4$.

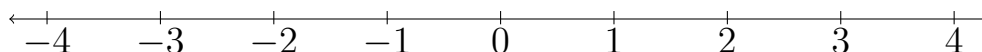


We begin by standing on the number line at the tick marked with $\boxed{-8}_{\text{given}}$. Since we are subtracting, we face towards the (right/ left ✓). We will move $\boxed{4}_{\text{given}}$ spaces

(forward ✓/ backward), since 4 is positive. Where on the number line are we now?

We are located at the tick labeled $\boxed{-12}$.
given

Example 2. Imagine using a number line like the one below to solve the subtraction problem $(-22) - (-54)$.



We begin by standing on the number line at the tick marked with $\boxed{-22}$. Since
given
we are subtracting, we face towards the (right/ left ✓). We will move $\boxed{54}$ spaces
given
(forward/ backward ✓), since 54 is negative. Where on the number line are we now?

We are located at the tick labeled $\boxed{32}$.
given

Question 16 In your own words, what is the difference between subtraction and negation?

Free Response: **Hint:** Subtraction is an operation, requiring two numbers. With subtraction, we have two quantities that we are trying to relate. With our models, subtraction requires some movement: removing chips, adding chips, or walking on the number line. Negation, on the other hand, only requires a single value. With negation, we are essentially just asking, “What is the opposite?” With our models, this is essentially a switching rather than a moving: we switch from black chips to red, or we face backward instead of forward.