Online HW 3: Integers and the Fundamental Theorem

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Contents

Integers

Problems about integers.

Problem 1 *Describe the set of integers. Give some relevant and revealing examples/nonexamples.*

Problem 2 *Explain how to model integer addition with pictures or items. What relevant properties should your model show?*

Problem 3 *Explain how to model integer multiplication with pictures or items. What relevant properties should your model show?*

Problem 4 *Explain what it means for one integer to* divide *another integer. Give some relevant and revealing examples/nonexamples.*

Problem 5 *Use the definition of* divides *to decide whether the following statements are true or false. In each case, an explanation must be given justifying your claim.*

(a) $5|30$ *(True/ False)*

(b) $7|41$ *(True/ False)*

(c) $0|3$ *(True/ False)*

(d) $3|0$ *(True/ False)*

(e) $6|(2^2 \cdot 3^4 \cdot 5 \cdot 7)$. *(True/ False)*

(f) $1000|(2^7 \cdot 3^9 \cdot 5^{11} \cdot 17^8)$ *(True/ False)*

(g) $6000|(2^{21} \cdot 3^{17} \cdot 5^{89} \cdot 29^{20})$. *(True/ False)*

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Problem 6 *Factor the following integers:*

(a) 111 $\boxed{?}$

(b) 1234 $\boxed{?}$

(c) 2345 $\boxed{?}$

(d) 4567 $\boxed{?}$

(e) 111111 $\boxed{?}$

In each case, how large a prime must you check before you can be sure of your answers? Explain your reasoning.

Problem 7 *Find the greatest common divisors below:*

(a) $\gcd(462, 1463) = \boxed{?}$

(b) $\gcd(541, 4669) = \boxed{?}$.

(c) $\gcd(10000, 2^5 \cdot 3^{19} \cdot 5^7 \cdot 11^{13}) = \boxed{?}$

(d) $\gcd(11111, 2^{14} \cdot 7^{21} \cdot 41^5 \cdot 101) = \boxed{?}$

(e) $\gcd(437^5, 8993^3) = \boxed{?}$

Problem 8 *Consider the following:*

$$20 \div 8 = 2 \textit{ remainder } 4,$$
$$28 \div 12 = 2 \textit{ remainder } 4.$$

Is it correct to say that $20 \div 8 = 28 \div 12$*? (Yes/ No)*

Explain your reasoning.

Problem 9 *Give a formula for the* n*th even number:* $\boxed{?}$

Problem 10 *Give a formula for the* n*th odd number:* $\boxed{?}$.

Problem 11 *Give a formula for the* n*th multiple of* 3*:* $\boxed{?}$

Problem 12 *Give a formula for the* n*th multiple of* -7. $\boxed{?}$

Problem 13 *Give a formula for the* n*th number whose remainder when divided by* 5 *is* 1.

If the first such number is 1, the formula is $\boxed{?}$.

If the first such number is 6, the formula is $\boxed{?}$.

Fundamental Theorem

Problems about Unique Factorization.

Problem 14 *Problem*

Problem 15 *Explain what the GCD of two integers is. Give some relevant and revealing examples/nonexamples.*

Problem 16 *Explain what the LCM of two integers is. Give some relevant and revealing examples/nonexamples.*

Problem 17 *How many zeros are at the end of the following numbers:*

(a) $2^2 \cdot 5^8 \cdot 7^3 \cdot 11^5$. *There are* $\boxed{?}$ *zeros.*

(b) 11!. *There are* $\boxed{?}$ *zeros.*

(c) 27!. *There are* $\boxed{?}$ *zeros.*

(d) 99!. *There are* $\boxed{?}$ *zeros.*

(e) 1001!. *There are* $\boxed{?}$ *zeros*

In each case, explain your reasoning.

Problem 18 *Decide whether the following statements are true or false. In each case, a detailed argument and explanation must be given justifying your claim.*

(a) $7|56$. *(True/ False)*

(b) $55|11$. *(True/ False)*

(c) $3|40$. *(True/ False)*

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(d) $100|(2^4 \cdot 3^{17} \cdot 5^2 \cdot 7)$ *(True/ False)*

(e) $5555|(5^{20} \cdot 7^9 \cdot 11^{11} \cdot 13^{23})$ *(True/ False)*

(f) $3|(3 + 6 + 9 + \cdots + 300 + 303)$ *(True/ False)*

Problem 19 *Suppose that*

$$(3^5 \cdot 7^9 \cdot 11^x \cdot 13^y)|(3^a \cdot 7^b \cdot 11^{19} \cdot 13^7)$$

What values of a, b, x and y, make true statements? Explain your reasoning.

- a *($\geqslant$/ $=$/ $\leqslant$)* $\boxed{?}$.
- b *($\geqslant$/ $=$/ $\leqslant$)* $\boxed{?}$.
- x *($\geqslant$/ $=$/ $\leqslant$)* $\boxed{?}$.
- y *($\geqslant$/ $=$/ $\leqslant$)* $\boxed{?}$.

Problem 20 *Decide whether the following statements are true or false. In each case, a detailed argument and explanation must be given justifying your claim.*

(a) *If $7|13a$, then $7|a$. (True/ False)*

(b) *If $6|49a$, then $6|a$. (True/ False)*

(c) *If $10|65a$, then $10|a$. (True/ False)*

(d) *If $14|22a$, then $14|a$. (True/ False)*

(e) $54|931^{21}$. *(True/ False)*

(f) $54|810^{33}$. *(True/ False)*

Problem 21 *Joanna thinks she can see if a number is divisible by 24 by checking to see if it's divisible by 4 and divisible by 6. She claims that if the number is divisible by 4 and by 6, then it must be divisible by 24.*

Lindsay has a similar divisibility test for 24: She claims that if a number is divisible by 3 and by 8, then it must be divisible by 24.

Are either correct? Explain your reasoning.

Joanna is (correct/ incorrect). Lindsay is (correct/ incorrect).

Problem 22 *Decide whether the following statements are true or false. In each case, a detailed argument and explanation must be given justifying your claim.*

(a) *If* $a^2|b^2$, *then* $a|b$. *(True/ False)*

(b) *If* $a|b^2$, *then* $a|b$. *(True/ False)*

(c) *If* $a|b$ *and* $\gcd(a, b) = 1$, *then* $a = 1$. *(True/ False)*

Problem 23 *Suppose* x *and* y *are integers. If* $x^2 = 11 \cdot y$, *what can you say about* y*? Explain your reasoning.*

Problem 24 *Suppose* x *and* y *are integers. If* $x^2 = 25 \cdot y$, *what can you say about* y*? Explain your reasoning.*