Solving Equations

Problems about solving equations.

Problem 1 *Jess is solving the equation* $6x + 13 = 25$. *Here is their work.*

$$25 - 13 = 12 \div 6 = 2$$

What is the issue with this work?

Multiple Choice:

(a) The algebra is incorrect.

(b) The equals sign does not mean equal here. ✓

(c) The solution is not related to the original equation.

(d) There is no issue with this work.

Problem 2 *Give a polynomial* $p(x)$ *whose leading coefficient is* 1, *and which has* $x = 12$ *and* $x = -1$ *as roots (and no other roots).*

$p(x) = \underbrace{\boxed{(x-12)(x+1)}}_{\text{given}}$

Problem 3 *Give a polynomial* $p(x)$ *whose leading coefficient is* 3, *and which has* $x = \frac{2}{3}$ *and* $x = 2$ *as roots (and no other roots).*

$p(x) = \underbrace{\boxed{(3x-2)(x-2)}}_{\text{given}}$

Problem 4 *Give a polynomial* $p(x)$ *of degree 3 whose leading coefficient is* 1, *and which has* $x = -5$ *as a root (and no other roots).*

$p(x) = \underbrace{\boxed{(x+5)^3}}_{\text{given}}$

Learning outcomes:

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Problem 5 *Give a polynomial $p(x)$ of degree 4 whose leading coefficient is 1, and which has $x = 8$, $x = 1 + \sqrt{3}$ and $x = 1 - \sqrt{3}$ as roots (and no other roots).*

$p(x) = \underset{\text{given}}{\boxed{(x-8)^2(x-(1+\sqrt{3}))(x-(1-\sqrt{3}))}}$

Problem 6 *Solve the problem below by completing the square. Practice drawing a diagram to help! Enter your answers from smallest to largest.*

$$x^2 + 8x = 20$$

$\underset{\text{given}}{\boxed{-10}}$, $\underset{\text{given}}{\boxed{2}}$

Problem 7 *Solve the problem below by completing the square. Practice drawing a diagram to help! Enter your answers from smallest to largest.*

$$4x^2 + 9x = 3$$

$\underset{\text{given}}{\boxed{-\frac{9}{8} - \frac{\sqrt{129}}{8}}}$, $\underset{\text{given}}{\boxed{\frac{\sqrt{129}}{8} - \frac{9}{8}}}$

Problem 8 *According to the Fundamental Theorem of Algebra, how many roots should the polynomial $p(x) = x^4 - 3x^3 + x - 2$ have?* $\underset{\text{given}}{\boxed{4}}$

Hint: *Remember that the Fundamental Theorem of Algebra counts real and complex roots, and also repeated roots.*

Problem 9 *According to the Fundamental Theorem of Algebra, how many roots should the polynomial $p(x) = x^{16} - 1$ have?* $\underset{\text{given}}{\boxed{16}}$

Problem 10 *The Rational Root Theorem says that if $\pm\frac{a}{b}$ is a root of a polynomial (written in lowest terms), then a must be a factor of the constant term, and b must be a factor of the leading term.*

For the polynomial $p(x) = x^3 + 2x^2 - 8x + 4$, which of the following could be rational roots of $p(x)$? (Do not solve this problem by plugging in the answers to the polynomial!)

Select All Correct Answers:

(a) 1 ✓

(b) -1 ✓

(c) $\frac{3}{2}$

(d) $-\frac{2}{3}$

(e) -4 ✓

(f) $\frac{1}{4}$

Problem 11 *The Rational Root Theorem says that if $\pm\frac{a}{b}$ is a root of a polynomial (written in lowest terms), then a must be a factor of the constant term, and b must be a factor of the leading term.*

For the polynomial $p(x) = 12x^8+5x^7+3x^5+14x^3-x+20$, which of the following could be rational roots of $p(x)$? (Do not solve this problem by plugging in the answers to the polynomial!)

Select All Correct Answers:

(a) 3

(b) -1 ✓

(c) $\frac{5}{7}$

(d) $-\frac{5}{3}$ ✓

(e) -4 ✓

(f) $\frac{1}{4}$ ✓

Problem 12 *Find all solutions to the equation $x^5 - 31x^4 + 310x^3 - 1240x^2 + 1984x - 1024 = 0$.*

Hint: *The Rational Root Theorem combined with some division of polynomials might help!*

Enter your answers in order from smallest to largest.

$\boxed{1}$, $\boxed{2}$, $\boxed{4}$, $\boxed{8}$, $\boxed{16}$
given given given given given

Problem 13 *Find all solutions to the equation* $x^5 - 28x^4 + 288x^3 - 1358x^2 + 2927x - 2310 = 0$.

Hint: *The Rational Root Theorem combined with some division of polynomials might help!*

Enter your answers in order from smallest to largest.

$\boxed{2}$, $\boxed{3}$, $\boxed{5}$, $\boxed{7}$, $\boxed{11}$
given given given given given

Problem 14 *Find all solutions to the equation* $x^5 - x^4 - 25x^3 + x^2 + 168x + 144 = 0$.

Hint: *The Rational Root Theorem combined with some division of polynomials might help!*

Enter your answers in order from smallest to largest.

$\boxed{-3}$, $\boxed{-3}$, $\boxed{-1}$, $\boxed{4}$, $\boxed{4}$
given given given given given
