## Home Base

Problems about numbers in various bases.

**Problem 1** Explain why the following "joke" is "funny:" There are 10 types of people in the world. Those who understand base two and those who don't.

**Problem 2** You meet some Tripod aliens, they tally by threes. Thankfully for everyone involved, they use the symbols 0, 1, and 2.

- (a) Can you explain how a Tripod would count from 11 to 201? Be sure to carefully explain what number comes after 22.
- (b) What number comes immediately before 10? 210? 20110? Explain your reasoning.

**Problem 3** You meet some people who tally by sevens. They use the symbols O, A, B, C, D, E, and F.

- (a) What do the individual symbols O, A, B, C, D, E, and F mean?
- (b) Can you explain how they would count from DD to AOC? Be sure to carefully explain what number comes after FF.
- (c) What number comes immediately before AO? ABO? EOFFA? Explain your reasoning.

**Problem 4** Now, suppose that you meet a hermit who tallies by thirteens. Explain how he might count. Give some relevant and revealing examples.

**Problem 5** While visiting Mos Eisley spaceport, you stop by Chalmun's Cantina. After you sit down, you notice that one of the other aliens is holding a discussion on fractions. Much to your surprise, they explain that 1/6 of 36

Learning outcomes:

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is 7. You are unhappy with this, knowing that 1/6 of 36 is in fact 6, yet their audience seems to agree with it, not you. Next the alien challenges its audience by asking, "What is 1/4 of 10?" What is the correct answer to this question, and how many fingers do the aliens have? Explain your reasoning.

**Problem 6** When the first Venusian to visit Earth attended a sixth grade class, it watched the teacher show that

$$\frac{3}{12} = \frac{1}{4}.$$

"How strange," thought the Venusian. "On Venus,  $\frac{4}{12} = \frac{1}{4}$ ." What base do Venusians use? Explain your reasoning.

**Problem 7** When the first Martian to visit Earth attended a high school algebra class, it watched the teacher show that the only solution of the equation

$$5x^2 - 50x + 125 = 0$$

is x = 5.

"How strange," thought the Martian. "On Mars, x=5 is a solution of this equation, but there also is another solution." If Martians have more fingers than humans, how many fingers do Martians have on both hands? Explain your reasoning.

Problem 8 In one of your many space-time adventures, you see the equation

$$\frac{3}{10} + \frac{4}{13} = \frac{21}{20}$$

written on a napkin. How many fingers did the beast who wrote this have? Explain your reasoning.

**Problem 9** What is the smallest number of weights needed to produce every integer-valued mass from 0 grams to say n grams? Explain your reasoning.

**Problem 10** Starting at zero, how high can you count using just your fingers?

- (a) Explain how to count to 10.
- (b) Explain how to count to 35.
- (c) Explain how to count to 1023.
- (d) Explain how to count to 59048.
- (e) Can you count even higher?

Explain your reasoning.