Integers

Problems about integers.

Problem 1 *Describe the set of integers. Give some relevant and revealing examples/nonexamples.*

Free Response: ***Hint:*** *Hint*

Problem 2 *Explain how to model integer addition with pictures or items. What relevant properties should your model show?*

Free Response: ***Hint:*** *Hint*

Problem 3 *Explain how to model integer multiplication with pictures or items. What relevant properties should your model show?*

Problem 4 *Explain what it means for one integer to* divide *another integer. Give some relevant and revealing examples/nonexamples.*

Free Response: ***Hint:*** *Hint*

Problem 5 *Use the definition of* divides *to decide whether the following statements are true or false. In each case, an explanation must be given justifying your claim.*

(a) $5|30$ *(True ✓/ False)*

Hint: $30 = 5 \cdot 6$.

(b) $7|41$ *(True ✓/ False)*

Hint: *There is no integer solution to* $41 = 7k$. *But* $41 = 7 \cdot 5 + 6$.

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(c) $0|3$ *(True/ False ✓)*

Hint: *There is no integer solution to* $3 = 0k$*.*

(d) $3|0$ *(True ✓/ False)*

Hint: *The solution to* $0 = 3k$ *is* $k = 0$*.*

(e) $6|(2^2 \cdot 3^4 \cdot 5 \cdot 7)$. *(True ✓/ False)*

(f) $1000|(2^7 \cdot 3^9 \cdot 5^{11} \cdot 17^8)$ *(True ✓/ False)*

(g) $6000|(2^{21} \cdot 3^{17} \cdot 5^{89} \cdot 29^{20})$. *(True ✓/ False)*

Hint: $6 = 2 \cdot 3$, $1000 = 2^3 \cdot 5^3$, *and* $6000 = 2^4 \cdot 3 \cdot 5^3$*. In each case, the primes appear enough times in the second number.*

Problem 6 *Factor the following integers:*

(a) 111 $\boxed{3 \cdot 37}$

(b) 1234 $\boxed{2 \cdot 617}$

(c) 2345 $\boxed{5 \cdot 7 \cdot 67}$

(d) 4567 $\boxed{\textit{prime}}$

(e) 111111 $\boxed{3 \cdot 7 \cdot 11 \cdot 13 \cdot 37}$

In each case, how large a prime must you check before you can be sure of your answers? Explain your reasoning.

Problem 7 *Find the greatest common divisors below:*

(a) $\gcd(462, 1463) = \boxed{77}$

Hint: $462 = 2 \cdot 3 \cdot 7 \cdot 11$*;* $1463 = 7 \cdot 11 \cdot 19$*.*

(b) $\gcd(541, 4669) = \boxed{1}$.

Hint: 541 *is prime.* $4669 = 7 \cdot 23 \cdot 29$

(c) $\gcd(10000, 2^5 \cdot 3^{19} \cdot 5^7 \cdot 11^{13}) = \boxed{10000}$

Hint: $10000 = 2^5 \cdot 5^5$.

(d) $\gcd(11111, 2^{14} \cdot 7^{21} \cdot 41^5 \cdot 101) = \boxed{41}$

Hint: $11111 = 41 \cdot 271$

(e) $\gcd(437^5, 8993^3) = \boxed{23^5}$

Hint: $437 = 19 \cdot 23$, *and* $8993 = 17 \cdot 23^2$.

Problem 8 *Consider the following:*

$$20 \div 8 = 2 \text{ remainder } 4,$$
$$28 \div 12 = 2 \text{ remainder } 4.$$

Is it correct to say that $20 \div 8 = 28 \div 12$*? (Yes/ No ✓)*

Explain your reasoning.

Free Response: ***Hint:*** *The answer "2 remainder 4" is not a single number but rather a pair of numbers that play different roles. Calling this pair of numbers "equal" is questionable. Furthermore, in the division problems, the 2 is about different things: groups of 8 versus groups of 12.*

Problem 9 *Give a formula for the* n*th even number:* $\boxed{2n}$

Problem 10 *Give a formula for the* n*th odd number:* $\boxed{2n-1}$.

Problem 11 *Give a formula for the* n*th multiple of* 3*:* $\boxed{3n}$

Problem 12 *Give a formula for the* n*th multiple of* -7. $\boxed{-7n}$

Problem 13 *Give a formula for the* n*th number whose remainder when divided by* 5 *is* 1.

If the first such number is 1, the formula is $\boxed{5n-4}$.

If the first such number is 6, the formula is $\boxed{5n+1}$.
