Polynomials

Problems about polynomials

Problem 1 *Given a polynomial*

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

can you find two numbers L and U such that $L \leqslant p(x) \leqslant U$ for all x? If so, explain why. If not, explain why not.

Problem 2 *Consider all polynomials of the form*

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where the a_i's are integers. If you substitute an integer for x will you always get an integer out? Explain your reasoning.

Problem 3 *Consider the following polynomial:*

$$p(x) = \frac{x^2}{2} + \frac{x}{2}$$

Will $p(x)$ always returns an integer when an integer is substituted for x? Explain your reasoning.

Problem 4 *Fix some integer value for x and consider all polynomials of the form*

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

Where the a_i's are integers greater than or equal to 0. Which numbers can be represented by such polynomials? Explain your reasoning.

Problem 5 *Find a polynomial*

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

Learning outcomes:
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such that a_i's are integers greater than or equal to 0 and less than 2 such that $p(2) = 35$. Discuss how your answer compares to the representation of 35 in base two. Explain your reasoning.

Problem 6 *Find a polynomial*

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

such that a_i's are integers greater than or equal to 0 and less than 7 such that $p(7) = 234$. Discuss how your answer compares to the representation of 234 in base seven. Explain your reasoning.

Problem 7 *Find a polynomial*

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

such that a_i's are integers greater than or equal to 0 and less than 10 such that $p(10) = 18$. Discuss how your answer compares to the representation of 18 in base ten. Explain your reasoning.

Problem 8 *Find a polynomial*

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

such that a_i's are integers greater than or equal to 0 and less than 15 such that $p(15) = 201$. Discuss how your answer compares to the representation of 201 in base fifteen. Explain your reasoning.

Problem 9 *Fix some integer value for x and consider all polynomials of the form*

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

Where the a_i's are integers greater than or equal to 0 and less than x. Which numbers can be represented by such polynomials? Explain your reasoning. Big hint: Base x.

Problem 10 *Fix some integer value for x and consider all polynomials of the form*

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

Where the a_i's are integers greater than or equal to 0 and less than 10. Which numbers can be represented by such polynomials? Explain your reasoning.

Problem 11 *Consider $x^2 + x + 1$. This can be thought of as a "number" in base x. Express this number in base $(x + 1)$, that is, find b_0, b_1, b_2 such that*

$$b_2(x+1)^2 + b_1(x+1) + b_0 = x^2 + x + 1.$$

Explain your reasoning.

Problem 12 *Consider $x^2 + 2x + 3$. This can be thought of as a "number" in base x. Express this number in base $(x - 1)$, that is, find b_0, b_1, b_2 such that*

$$b_2(x-1)^2 + b_1(x-1) + b_0 = x^2 + 2x + 3.$$

Explain your reasoning.

Problem 13 *Consider $x^3 + 2x + 1$. This can be thought of as a "number" in base x. Express this number in base $(x - 1)$, that is, find b_0, b_1, b_2, b_3 such that*

$$b_3(x-1)^3 + b_2(x-1)^2 + b_1(x-1) + b_0 = x^3 + 2x + 1.$$

Explain your reasoning.

Problem 14 *If the polynomial*

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

is thought of as a "number" in base x, describe two different ways to find the base $(x - 1)$ coefficients of $p(x)$.