Polynomials

Problems about polynomials.

Problem 1 *Explain what is meant by a polynomial in a variable x.*

Free Response: ***Hint:*** *A polynomial in x is an algebraic expression that can be written as a sum of terms, each of which is a whole-number power of x multiplied by some real number.*

Problem 2 *Which of the following expressions are polynomials (select all):*

Select All Correct Answers:

(a) $x - 2$ ✓

(b) $\sqrt{x}$

(c) $1 - x - x^2$ ✓

(d) e^x

(e) 56 ✓

(f) $\dfrac{2}{x}$

Problem 3 *Indicate the degree of the following polynomials. For expressions that are not polynomials, type NP.*

$$\begin{array}{cc}
x-2 & \boxed{1} \\
\sqrt{x} & \boxed{NP} \\
x^{15} & \boxed{15} \\
1-x-x^2 & \boxed{2} \\
e^x & \boxed{NP} \\
56 & \boxed{0} \\
2/x & \boxed{NP}
\end{array}$$

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Problem 4 *Given:*

$$3x^7 - x^5 + x^4 - 16x^3 + 27 = a_7x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x^1 + a_0$$

Find a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7.

Answer: $a_0 = \boxed{27}$, $a_1 = \boxed{0}$, $a_2 = \boxed{0}$, $a_3 = \boxed{-16}$, $a_4 = \boxed{1}$, $a_5 = \boxed{-1}$, $a_6 = \boxed{0}$, $a_7 = \boxed{3}$.

Problem 5 *Given:*

$$6x^5 + a_4x^4 - x^2 + a_0 = a_5x^5 - 24x^4 + a_3x^3 + a_2x^2 - 5$$

Find a_0, a_1, a_2, a_3, a_4, a_5.

Answers: $a_0 = \boxed{-5}$, $a_1 = \boxed{0}$, $a_2 = \boxed{-1}$, $a_3 = \boxed{0}$, $a_4 = \boxed{-24}$, $a_5 = \boxed{6}$.

Problem 6 *Is it true that polynomials are equal if and only if their coefficients are equal? Explain your reasoning.*

Free Response: ***Hint:*** *Yes. This is the fact we used to complete the previous two problems.*

Problem 7 *Is it true that numbers are equal if and only if their digits are equal? Explain your reasoning.*

Free Response: ***Hint:*** *For whole numbers written in the same base, the answer is yes. For decimals, the answer is no. [Note: You will learn why and how later in the course.]*

For fractions, of course, the answer is no. For example, $\frac{2}{3} = \frac{4}{6}$*, and the "digits" of the fraction on the left are clearly different from those of the fraction on the right.*

Problem 8 *Explain how to add two polynomials. Explain, in particular, how "collecting like terms" is an application of the properties of arithmetic.*

Free Response: ***Hint:*** *Use the associative and commutative properties to rearrange the sum so that like terms are consecutive. Then use the distributive property to "collect" the like terms.*

Problem 9 *Explain how to multiply two polynomials.*

Free Response: ***Hint:*** *Use the distributive property: Multiply each term in the first polynomial by each term in the second polynomial. Then collect like terms.*

Problem 10 *Here is an example of the polynomial division algorithm:*

$$\begin{array}{r|l} & \quad\quad\quad\;\; x - 3 \quad R\ 9x+4 \\ \cline{2-2} x^2 + 3x + 1 & x^3 + 0x^2 + x + 1 \\ & \underline{x^3 + 3x^2 + x} \\ & \quad -3x^2 + 0x + 1 \\ & \quad \underline{-3x^2 - 9x - 3} \\ & \quad\quad\quad\quad 9x + 4 \end{array}$$

(a) *Describe how to perform this algorithm.*

(b) *Provide an additional relevant and revealing example demonstrating that you understand the algorithm.*

(c) *Show the "behind-the-scenes" algebra that is going on here.*

Free Response: ***Hint:*** *This is very much like the long division algorithm for counting numbers:*

(a) *Write part of the quotient;*
(b) *Multiply that part of the quotient by the divisor;*
(c) *Subtract that product from the dividend;*
(d) *Repeat until the remaining part of the dividend has a degree less than the degree of the divisor.*

Problem 11 *State the Division Theorem for polynomials. Give some relevant and revealing examples of this theorem in action.*

Free Response: ***Hint:*** *Informally, when dividing a polynomial by a (non-zero) polynomial, we can always find a sensible quotient and remainder (both polynomials). More formally, given $n(x)$ and a non-zero divisor $d(x)$, we can find $q(x)$ and $r(x)$ such that* $\boxed{n(x) = d(x)q(x) + r(x)}$ *with the degree of $r(x)$ (greater than / equal to / less than ✓) the degree of $d(x)$.*