

# Transformations

*Short-answer problems about the definitions of basic rigid motions and dilations.*

To define a translation, rotation, or reflection, we specify the image of a generic point  $P$  under the transformation as specified.

**Question 1** Under a **rotation** about center  $O$  counterclockwise by an angle  $\vartheta$ , the image of  $P$  is a point  $Q$  so that  $OQ = OP$  and  $m\angle POQ = \vartheta$ , measured counterclockwise from  $\overrightarrow{OP}$  to  $\overrightarrow{OQ}$  if  $\vartheta > 0$  and measured clockwise if  $\vartheta < 0$ . If  $P = O$  then  $Q = O = P$ .

**Question 2** Under a **translation** by the distance and direction from  $A$  to  $B$ , the image of  $P$  is a point  $Q$  so that  $\overrightarrow{PQ} \parallel \overrightarrow{AB}$ ,  $PQ = AB$ , and  $\overrightarrow{PQ}$  is in the same direction as  $\overrightarrow{AB}$ .

**Question 3** Under a **reflection** about a line  $\ell$ , if  $P$  is on  $\ell$ , then  $P$  is mapped to itself. If  $P$  is not on  $\ell$ , then the image of  $P$  is a point  $Q$  so that  $\ell$  is the perpendicular bisector of  $\overline{PQ}$ .

**Question 4** Under a **dilation** about center  $O$  and scale factor  $r > 0$ , the image of  $P$  is a point  $Q$  so that  $Q$  lies on  $\overrightarrow{OP}$  and  $OQ = rOP$ . The image of  $O$  is  $O$ .