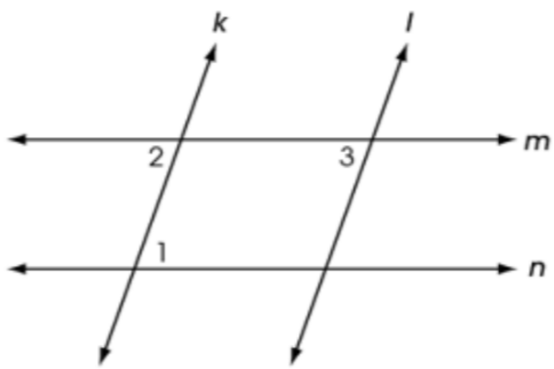
Quadrilaterals

Proof.

Problem 1 *Adapted from Ohio's 2017 Geometry released item 13.*

Two pairs of parallel lines intersect to form a parallelogram as shown.



Complete the following proof that opposite angles of a parallelogram are congruent:

(a) $\angle 1 \cong \angle 2$ *as (opposite angles/ alternate interior angles ✓/ corresponding angles) for parallel lines (m and n ✓/ k and l).*

(b) $\angle 3 \cong \angle 2$ *as (opposite angles/ alternate interior angles/ corresponding angles ✓)for parallel lines (m and n/ k and l ✓).*

(c) *Then* $\angle 1 \cong \angle 3$ *because they are both congruent to* $\angle 2$.

Problem 2 *Adapted from Ohio's 2018 Geometry released item 21.*

Given the parallelogram $WXYZ$, *prove that* $\overline{WX} \cong \overline{YZ}$.

Learning outcomes:
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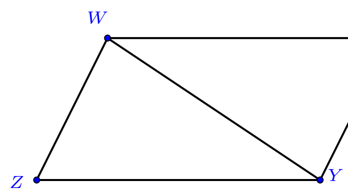


Fix note: It really would help to have an online environment that allows students to mark diagrams.

Complete the proof below:

(a) $\angle ZWY \cong \angle XYW$ *as (alternate interior angles* ✓/ *corresponding angles/ opposite angles) for parallel segments (*$\overline{WZ}$ and $\overline{XY}$ ✓/ $\overline{WX}$ and $\overline{YZ}$*).*

(b) $\angle ZYW \cong \angle XWY$ *for the same reason, this time for parallel segments (* $\overline{WZ}$ and $\overline{XY}$ / $\overline{WX}$ and $\overline{YZ}$ ✓*).*

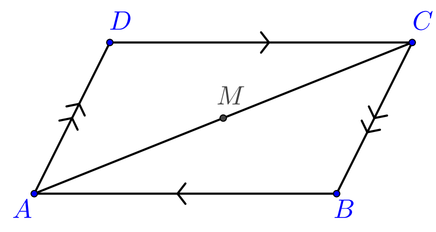
(c) $\overline{WY} \cong \overline{YW}$ *because a segment is congruent to itself.*

(d) $\triangle WYZ \cong \triangle YWX$ *by (SAS/ ASA* ✓*/ SSS).*

(e) *Then* $\overline{YZ} \cong \overline{WX}$ *as corresponding parts of congruent triangles.*

Fix note: Maybe number the angles.

Problem 3 *Use symmetry to prove properties of parallelograms.*



(a) *Let* M *be the midpoint of* $\overline{AC}$, *and let* R *be a* $180°$ *rotation about* M.

(b) *Then* $R(A) = C$ *and* $R(C) = A$ *because a rotation about a point on a line takes the line to itself and preserves lengths.*

(c) *Now a* 180° *rotation about* M *takes lines not containing* M *to parallel lines. Thus, the uniqueness of parallels implies that the parallel sides of the parallelogram must swap.*

(d) *Furthermore, the image of their intersections must be the intersection of their images, which means that* $R(B) = D$ *and* $R(D) = B$*.*

(e) *Therefore,* R *maps the parallelogram onto itself, which implies that*

- *opposite sides are congruent, and*
- *opposite angles are congruent.*

(f) *Because* R *is a* 180° *rotation,* $\overrightarrow{MB}$ *and* $\overrightarrow{MD}$ *are opposite rays, so that* B*,* M*, and* D *are collinear.*

(g) *Because* R *preserves lengths,* $MB = MD$*, so that* M *is also the midpoint of* $\overline{BD}$*, which means that the diagonals bisect each other.*