

## Set Theory Problems

*Short-answer problems about sets.*

**Problem 1** Given two sets  $X$  and  $Y$ ,  $X \cup Y$  is the set of elements that are

**Multiple Choice:**

- (a) in  $X$  or in  $Y$  (but not in both).
  - (b) in  $X$  or in  $Y$  (or both, as the “or” is inclusive). ✓
  - (c) in  $X$  and in  $Y$ .
  - (d) in  $X$  but not in  $Y$ .
  - (e) in  $Y$  but not in  $X$ .
- 

**Problem 2** Given two sets  $X$  and  $Y$ ,  $X \cap Y$  is the set of elements that are

**Multiple Choice:**

- (a) in  $X$  or in  $Y$  (but not in both).
  - (b) in  $X$  or in  $Y$  (or both, as the “or” is inclusive).
  - (c) in  $X$  and in  $Y$ . ✓
  - (d) in  $X$  but not in  $Y$ .
  - (e) in  $Y$  but not in  $X$ .
- 

**Problem 3** Given two sets  $X$  and  $Y$ ,  $X - Y$  is the set of elements that are

**Multiple Choice:**

- (a) in  $X$  or in  $Y$  (but not in both).
- (b) in  $X$  or in  $Y$  (or both, as the “or” is inclusive).

---

Author(s): Bart Snapp and Brad Findell

- (c) in  $X$  and in  $Y$ .
- (d) in  $X$  but not in  $Y$ . ✓
- (e) in  $Y$  but not in  $X$ .

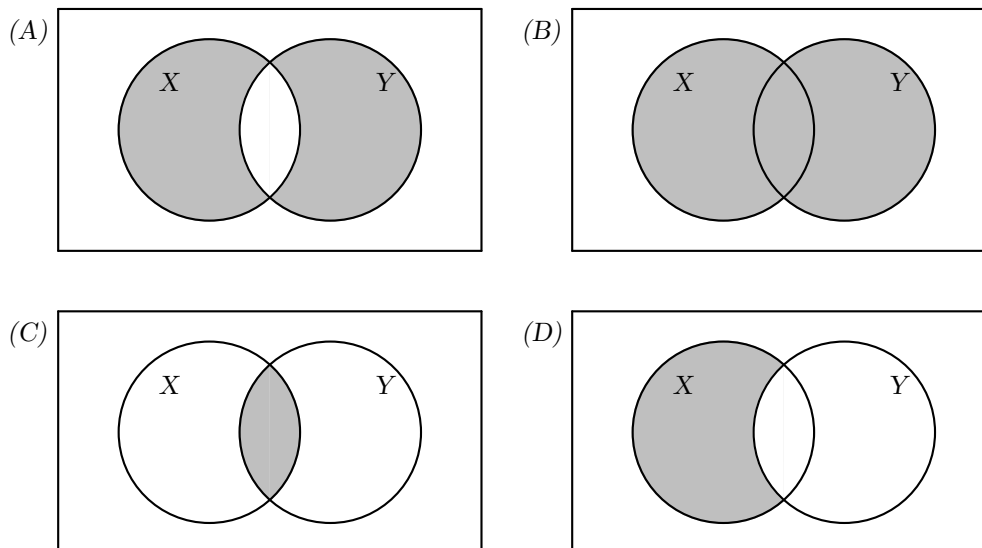
**Problem 4** Explain the difference between the symbols  $\in$  and  $\subseteq$ .

**Free Response:** **Hint:** The notation  $X \in Y$  means that  $X$  is a single element in the set  $Y$ . In this case,  $X$  is probably not a set. The notation  $X \subseteq Y$  requires that both  $X$  and  $Y$  are sets and, furthermore, that every element of  $X$  is also in  $Y$ .

**Problem 5** How is  $\{\emptyset\}$  different from  $\emptyset$ ?

**Free Response:** **Hint:** The empty set,  $\emptyset$ , is a set that contains no elements. That is,  $\emptyset = \{\}$ . The set  $\{\emptyset\}$  contains one element that is itself a set—and that element happens to be the empty set. We could instead write  $\{\{\}\}$ , but that looks ugly.

**Problem 6** Consider the following Venn diagrams:



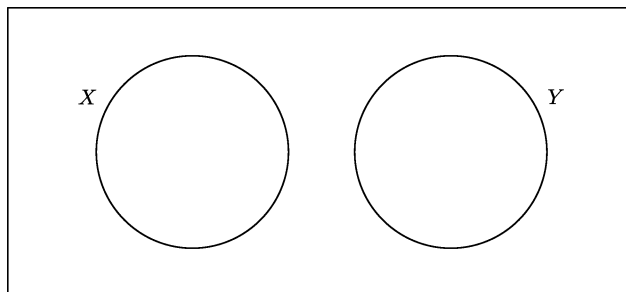
For each set expression below, identify the Venn diagram above in which the region is shaded:

- (a)  $X \cap Y$  is figure
- (b)  $X \cup Y$  is figure .
- (c)  $X - Y$  is figure

**Problem 7** Draw a Venn diagram for the set of elements that are in  $X$  or  $Y$  but not both. How does it differ from the Venn diagram for  $X \cup Y$ ?

**Free Response:** **Hint:** The elements in  $X$  or  $Y$  but not both is shown in figure (A) from the previous problem.

**Problem 8** Consider the following image:



If the above Venn diagram shows the relationship between sets  $X$  and  $Y$ , then  $X \cap Y = (\emptyset / \emptyset \checkmark / X \cup Y)$  and the sets are said to be .

If we let  $X$  be the set of “right triangles” and we let  $Y$  be the set of “equilateral triangles” does the picture below show the relationship between these two sets?

**Multiple Choice:**

- (a) Yes. ☒
- (b) No.
- (c) Not enough information.

Explain your reasoning.

**Free Response:** **Hint:** Yes. The picture is accurate because no right triangles are also equilateral triangles.

**Problem 9** If  $X = \{1, 2, 3, 4, 5\}$  and  $Y = \{3, 4, 5, 6\}$  find the following: (List elements in ascending order, separated by commas, with no spaces.)

(a)  $X \cup Y = \{ \boxed{1, 2, 3, 4, 5, 6} \}$

(b)  $X \cap Y = \{ \boxed{3, 4, 5} \}$

(c)  $X - Y = \{ \boxed{1, 2} \}$

(d)  $Y - X = \{ \boxed{6} \}$

**Problem 10** Let  $n\mathbb{Z}$  represent the integer multiples of  $n$ . So for example:

$$3\mathbb{Z} = \{\dots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \dots\}$$

Compute the following (use capital Z for  $\mathbb{Z}$ ):

(a)  $3\mathbb{Z} \cap 4\mathbb{Z} = \boxed{12\mathbb{Z}}$

(b)  $2\mathbb{Z} \cap 5\mathbb{Z} = \boxed{10\mathbb{Z}}$

(c)  $3\mathbb{Z} \cap 6\mathbb{Z} = \boxed{6\mathbb{Z}}$

(d)  $4\mathbb{Z} \cap 6\mathbb{Z} = \boxed{12\mathbb{Z}}$

(e)  $4\mathbb{Z} \cap 10\mathbb{Z} = \boxed{20\mathbb{Z}}$

**Problem 11** Make a general rule for intersecting sets of the form  $n\mathbb{Z}$  and  $m\mathbb{Z}$ . Explain why your rule works.

**Free Response:** **Hint:** The intersection of two sets is what they have in common. The intersection of the set of multiples of  $n$  and the set of multiples of  $m$  are called common multiples (surprise!), and they are all multiples of the least common multiple of  $n$  and  $m$ .

**Problem 12** If  $X \cup Y = X$ , what can we say about the relationship between the sets  $X$  and  $Y$ ? Explain your reasoning.

( $X \subseteq Y / X = Y / Y \subseteq X \checkmark / X = \emptyset$ ) because every element of ( $X / Y \checkmark$ ) must be in ( $X \checkmark / Y$ ).

**Problem 13** If  $X \cap Y = X$ , what can we say about the relationship between the sets  $X$  and  $Y$ ? Explain your reasoning.

$(X \subseteq Y \checkmark / X = Y / Y \subseteq X / X = \emptyset)$  because every element of  $(X \checkmark / Y)$  must be in  $(X / Y \checkmark)$ .

---

**Problem 14** If  $X - Y = \emptyset$ , what can we say about the relationship between the sets  $X$  and  $Y$ ? Explain your reasoning.

$(X \subseteq Y \checkmark / X = Y / Y \subseteq X / X = \emptyset)$  because every element of  $(X \checkmark / Y)$  must be in  $(X / Y \checkmark)$ .

---