

Set Theory Problems

Short-answer problems about sets.

Problem 1 Given two sets X and Y , explain what is meant by $X \cup Y$.

Free Response: **Hint:** $X \cup Y$ is the set of elements that are in X or in Y (or both, as the “or” is inclusive).

Problem 2 Given two sets X and Y , explain what is meant by $X \cap Y$.

Free Response: **Hint:** $X \cap Y$ is the set of elements that are in X and in Y .

Problem 3 Given two sets X and Y , explain what is meant by $X - Y$.

Free Response: **Hint:** $X - Y$ is the set of elements that are in X but not in Y .

Problem 4 Explain the difference between the symbols \in and \subset .

Free Response: **Hint:** The notation $X \in Y$ means that X is a single element in the set Y . In this case, X is probably not a set. The notation $X \subset Y$ requires that both X and Y are sets and, furthermore, that every element of X is also in Y .

Problem 5 How is $\{\emptyset\}$ different from \emptyset ?

Free Response: **Hint:** The empty set, \emptyset , is a set that contains no elements. That is, $\emptyset = \{\}$. The set $\{\emptyset\}$ contains one element that is itself a set—and that element happens to be the empty set. We could instead write $\{\{\}\}$, but that looks ugly.

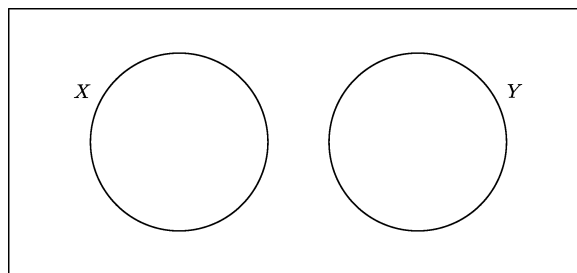
Learning outcomes:

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Problem 6 Draw a Venn diagram for the set of elements that are in X or Y but not both. How does it differ from the Venn diagram for $X \cup Y$?

Free Response: **Hint:** Same as the Venn diagram for $X \cup Y$, except that the $X \cap Y$ part is not shaded.

Problem 7 If we let X be the set of “right triangles” and we let Y be the set of “equilateral triangles” does the picture below show the relationship between these two sets?



Explain your reasoning.

Free Response: **Hint:** Yes. The picture is accurate because no right triangles are also equilateral triangles.

Problem 8 If $X = \{1, 2, 3, 4, 5\}$ and $Y = \{3, 4, 5, 6\}$ find:

- (a) $X \cup Y$
- (b) $X \cap Y$
- (c) $X - Y$
- (d) $Y - X$

Free Response: **Hint:**

- (a) $X \cup Y = \{1, 2, 3, 4, 5, 6\}$
- (b) $X \cap Y = \{3, 4, 5\}$
- (c) $X - Y = \{1, 2\}$
- (d) $Y - X = \{6\}$

Problem 9 Let $n\mathbb{Z}$ represent the integer multiples of n . So for example:

$$3\mathbb{Z} = \{\dots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \dots\}$$

Compute the following:

- (a) $3\mathbb{Z} \cap 4\mathbb{Z}$
- (b) $2\mathbb{Z} \cap 5\mathbb{Z}$
- (c) $3\mathbb{Z} \cap 6\mathbb{Z}$
- (d) $4\mathbb{Z} \cap 6\mathbb{Z}$
- (e) $4\mathbb{Z} \cap 10\mathbb{Z}$

In each case explain your reasoning.

Free Response: **Hint:** (a) $3\mathbb{Z} \cap 4\mathbb{Z} = 12\mathbb{Z}$

- (b) $2\mathbb{Z} \cap 5\mathbb{Z} = 10\mathbb{Z}$
- (c) $3\mathbb{Z} \cap 6\mathbb{Z} = 6\mathbb{Z}$
- (d) $4\mathbb{Z} \cap 6\mathbb{Z} = 12\mathbb{Z}$
- (e) $4\mathbb{Z} \cap 10\mathbb{Z} = 20\mathbb{Z}$

Problem 10 Make a general rule for intersecting sets of the form $n\mathbb{Z}$ and $m\mathbb{Z}$. Explain why your rule works.

Free Response: **Hint:** The intersection of two sets is what they have in common. The intersection of the set of multiples of n and the set of multiples of m are called common multiples (surprise!), and they are all multiples of the least common multiple of n and m .

Problem 11 If $X \cup Y = X$, what can we say about the relationship between the sets X and Y ? Explain your reasoning.

Free Response: **Hint:** $Y \subset X$ because every element of Y must already be in X .

Problem 12 If $X \cap Y = X$, what can we say about the relationship between the sets X and Y ? Explain your reasoning.

Free Response: *Hint:* $X \subset Y$ because every element of X must already be in Y .

Problem 13 If $X - Y = \emptyset$, what can we say about the relationship between the sets X and Y ? Explain your reasoning.

Free Response: *Hint:* $X \subset Y$ because that would mean X contains no elements that are not also in Y .
