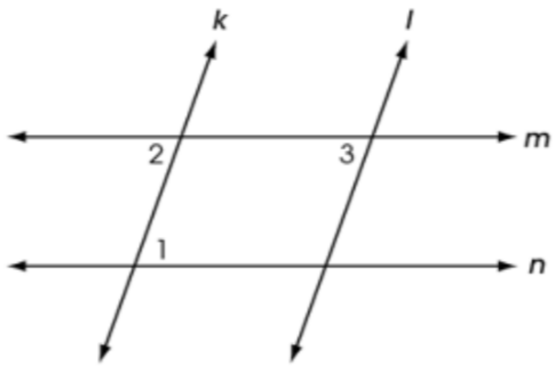
Quadrilaterals

Proof.

Problem 1 *Adapted from Ohio's 2017 Geometry released item 13.*

Two pairs of parallel lines intersect to form a parallelogram as shown.



Complete the following proof that opposite angles of a parallelogram are congruent:

(a) $\angle 1 \cong \angle 2$ *as (opposite angles/ alternate interior angles ✓/ corresponding angles) for parallel lines (m and n ✓/ k and l).*

(b) $\angle 3 \cong \angle 2$ *as (opposite angles/ alternate interior angles/ corresponding angles ✓)for parallel lines (m and n/ k and l ✓).*

(c) *Then* $\angle 1 \cong \angle 3$ *because they are both congruent to* $\angle 2$.

Problem 2 *Adapted from Ohio's 2018 Geometry released item 21.*

Given the parallelogram $WXYZ$, prove that $\overline{WX} \cong \overline{YZ}$.

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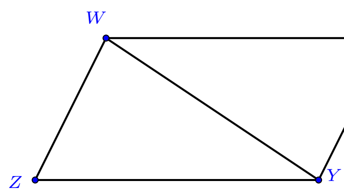


Fix note: It really would help to have an online environment that allows students to mark diagrams.

Complete the proof below:

(a) $\angle ZWY \cong \angle XYW$ as (alternate interior angles ✓/ corresponding angles/ opposite angles) for parallel segments ($\overline{WZ}$ and $\overline{XY}$ ✓/ $\overline{WX}$ and $\overline{YZ}$).

(b) $\angle ZYW \cong \angle XWY$ for the same reason, this time for parallel segments ($\overline{WZ}$ and $\overline{XY}$ / $\overline{WX}$ and $\overline{YZ}$ ✓).

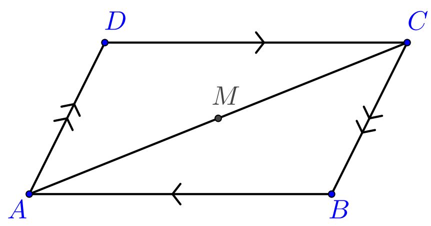
(c) $\overline{WY} \cong \overline{YW}$ because a segment is congruent to itself.

(d) $\triangle WYZ \cong \triangle YWX$ by (SAS/ ASA ✓/ SSS).

(e) Then $\overline{YZ} \cong \overline{WX}$ as corresponding parts of congruent triangles.

Fix note: Maybe number the angles.

Problem 3 *Use symmetry to prove properties of parallelograms.*



(a) Let M be the midpoint of $\overline{AC}$, and let R be a 180° rotation about M.

(b) Then $R(A) = C$ and $R(C) = A$ because a rotation about a point on a line takes the line to itself and preserves lengths.

(c) *Now a 180° rotation about M takes lines not containing M to parallel lines. Thus, the uniqueness of parallels implies that the parallel sides of the parallelogram must swap.*

(d) *Furthermore, the image of their intersections must be the intersection of their images, which means that $R(B) = D$ and $R(D) = B$.*

(e) *Therefore, R maps the parallelogram onto itself, which implies that*

- *opposite sides are congruent, and*
- *opposite angles are congruent.*

(f) *Because R is a 180° rotation, $\overrightarrow{MB}$ and $\overrightarrow{MD}$ are opposite rays, so that B, M, and D are collinear.*

(g) *Because R preserves lengths, $MB = MD$, so that M is also the midpoint of $\overline{BD}$, which means that the diagonals bisect each other.*