

Set Theory Problems

Short-answer problems about sets.

Problem 1 Given two sets X and Y , $X \cup Y$ is the set of elements that are

Multiple Choice:

- (a) in X or in Y (but not in both).
 - (b) in X or in Y (or both, as the “or” is inclusive). ✓
 - (c) in X and in Y .
 - (d) in X but not in Y .
 - (e) in Y but not in X .
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Problem 2 Given two sets X and Y , $X \cap Y$ is the set of elements that are

Multiple Choice:

- (a) in X or in Y (but not in both).
 - (b) in X or in Y (or both, as the “or” is inclusive).
 - (c) in X and in Y . ✓
 - (d) in X but not in Y .
 - (e) in Y but not in X .
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Problem 3 Given two sets X and Y , $X - Y$ is the set of elements that are

Multiple Choice:

- (a) in X or in Y (but not in both).
- (b) in X or in Y (or both, as the “or” is inclusive).

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- (c) in X and in Y .
- (d) in X but not in Y . ✓
- (e) in Y but not in X .

Problem 4 Explain the difference between the symbols \in and \subseteq .

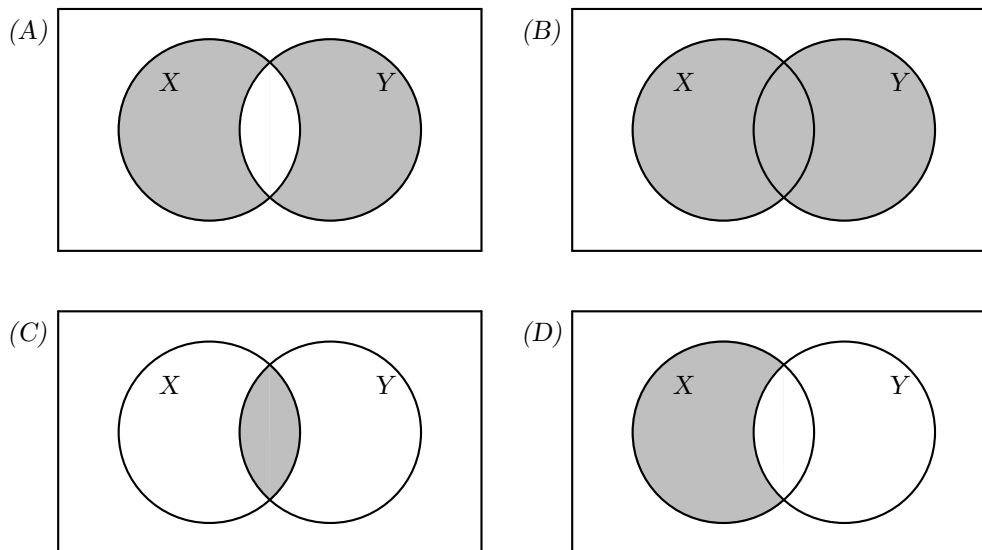
Free Response: **Hint:** The symbol \in means “is an element of,” whereas \subseteq means “is a subset of.”

The notation $X \in Y$ means that X is a single element in the set Y . In this case, X is typically not a set. The notation $X \subseteq Y$, in contrast, requires that both X and Y are sets and, furthermore, that every element of X is also in Y .

Problem 5 How is $\{\emptyset\}$ different from \emptyset ?

Free Response: **Hint:** The empty set, \emptyset , is a set that contains no elements. That is, $\emptyset = \{\}$. The set $\{\emptyset\}$ contains one element that is itself a set—and that element happens to be the empty set. We could instead write $\{\{\}\}$, but that looks ugly.

Problem 6 Consider the following Venn diagrams:



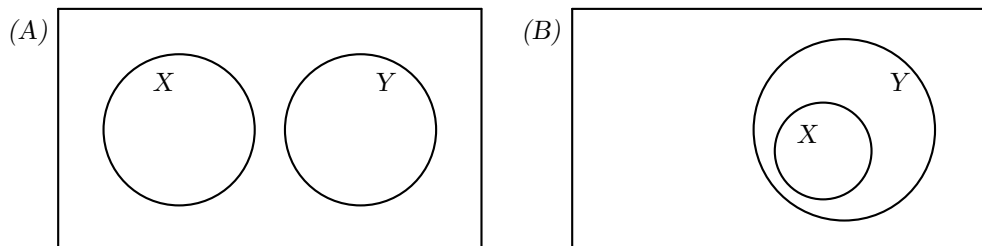
For each set expression below, identify the Venn diagram above in which the corresponding region is shaded:

- (a) $X \cap Y$ is figure
- (b) $X \cup Y$ is figure .
- (c) $X - Y$ is figure

Problem 7 Draw a Venn diagram for the set of elements that are in X or Y but not both. How does it differ from the Venn diagram for $X \cup Y$?

Free Response: **Hint:** A Venn diagram for elements in X or Y but not both is shown in figure (A) from the previous problem.

Problem 8 Consider the following Venn diagrams:



- (a) If Venn diagram (A) above shows the relationship between sets X and Y , then $X \cap Y = (\emptyset / \emptyset \checkmark / X \cup Y)$ and the sets are said to be .
- (b) If Venn diagram (A) above shows the relationship between sets X and Y , then we say that (X and Y are disjoint / $X \subseteq Y \checkmark / Y \subseteq X$).
- (c) If we let X be the set of “right triangles” and we let Y be the set of “equilateral triangles” which diagram above show the relationship between these two sets?

Multiple Choice:

- (i) Diagram (A). ☒
- (ii) Diagram (B). ☐
- (iii) Neither of these. ☐
- (iv) Not enough information. ☐

Explain your reasoning.

Free Response: **Hint:** Diagram (A) is accurate because no right triangles are also equilateral triangles.

Problem 9 If $X = \{1, 2, 3, 4, 5\}$ and $Y = \{3, 4, 5, 6\}$ find the following: (List elements in ascending order, separated by commas, with no spaces.)

(a) $X \cup Y = \{ \boxed{1, 2, 3, 4, 5, 6} \}$

(b) $X \cap Y = \{ \boxed{3, 4, 5} \}$

(c) $X - Y = \{ \boxed{1, 2} \}$

(d) $Y - X = \{ \boxed{6} \}$

Problem 10 Let $n\mathbb{Z}$ represent the integer multiples of n . So for example:

$$3\mathbb{Z} = \{\dots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \dots\}$$

Compute the following (use capital Z for \mathbb{Z}):

(a) $3\mathbb{Z} \cap 4\mathbb{Z} = \boxed{12Z}$

(b) $2\mathbb{Z} \cap 5\mathbb{Z} = \boxed{10Z}$

(c) $3\mathbb{Z} \cap 6\mathbb{Z} = \boxed{6Z}$

(d) $4\mathbb{Z} \cap 6\mathbb{Z} = \boxed{12Z}$

(e) $4\mathbb{Z} \cap 10\mathbb{Z} = \boxed{20Z}$

Problem 11 Make a general rule for intersecting sets of the form $n\mathbb{Z}$ and $m\mathbb{Z}$. Explain why your rule works.

Free Response: **Hint:** The intersection of two sets is what they have in common. The intersection of the set of multiples of n and the set of multiples of m are called common multiples (surprise!), and they are all multiples of the least common multiple of n and m .

Problem 12 If $X \cup Y = X$, what can we say about the relationship between the sets X and Y ? Explain your reasoning.

$(X \subseteq Y / X = Y / Y \subseteq X \checkmark / X = \emptyset)$ because every element of $(X / Y \checkmark)$ must be in $(X \checkmark / Y)$.

Problem 13 If $X \cap Y = X$, what can we say about the relationship between the sets X and Y ? Explain your reasoning.

$(X \subseteq Y \checkmark / X = Y / Y \subseteq X / X = \emptyset)$ because every element of $(X \checkmark / Y)$ must be in $(X / Y \checkmark)$.

Problem 14 If $X - Y = \emptyset$, what can we say about the relationship between the sets X and Y ? Explain your reasoning.

$(X \subseteq Y \checkmark / X = Y / Y \subseteq X / X = \emptyset)$ because every element of $(X \checkmark / Y)$ must be in $(X / Y \checkmark)$.
