Scaling in 2D

Short-answer problems about scaling in two dimensions.

Length and Area Under Scaling

In this section, we explore what happens to length, area, and other measures under scaling.

In a previous section, we defined similarity in terms of basic rigid motions and dilations, and we showed that this definition leads to well-known results about similarity, such as the AA criterion for triangle similarity and consistent ratios of lengths between and within similar figures. A key feature of this discussion was the notion of "scale factor," which describes what happens to lengths under a dilation. From the definition of a dilation, it is clear that segments on lines through the center of dilation scale by the scale factor. We used the side-splitter theorems to show that other segments are scaled by the same scale factor.

Question 1 *Two students claims that a 3×5 rectangle and a 4×6 rectangle are similar.*

Fred says that that they are similar because the angles are the same. How do you respond?

Free Response: ***Hint:*** *Angles are enough to determine similarity of triangles. But similarity requires a consistent scale factor. For these rectangles the height is scaled by $4/3$ whereas the base is scaled by $6/5$.*

Ned says that they are similar because you can do the same thing (i.e., add 1) to "both sides" of the 3×5 rectangle to get the 4×6 rectangle. How do you respond?

Free Response: ***Hint:*** *Similarity requires consistent scaling, which is a multiplicative (not additive) relationship.*

Question 2 *Complete the following sentences using words such as filling, falling, covering, wrapping, hiding, surrounding, or traveling:*

Learning outcomes:
Author(s): Brad Findell

Area vs. perimeter can be thought of as $\boxed{\textit{covering}}$ *vs.* $\boxed{\textit{surrounding}}$*, respectively.*

Volume vs. surface area can be thought of as $\boxed{\textit{filling}}$ *vs.* $\boxed{\textit{wrapping}}$*, respectively.*

To explore how measures of figures change under scaling and non-scaling transformations, here are some useful strategies:

- Cutting the figures and rearranging the pieces.
- Using "rep-tiles."
- Using known formulas for perimeters, areas, volumes, or surface areas.
- Approximating with segments, squares, or cubes.
- Shearing.

Question 3 *To estimate the length of a curve, imagine approximating it with many small segments. Now apply a similarity transformation with a scale factor of* k*. Each segment will scale by* $\boxed{k}$*, so the length of the curve will be* $\boxed{k}$ *times the original length.*

Question 4 *To estimate the area of a non-polygonal region, imagine covering it approximately with a grid of squares of side length* s*. Piecing together partial squares, suppose you count* n *squares. Your area estimate is then* $\boxed{ns^2}$*.*

Now apply a similarity transformation of scale factor k *to both the region and the grid. Each square in the scaled grid will have area* $\boxed{(sk)^2}$*, and piecing together partial squares there will be* $\boxed{n}$ *squares. Thus, we estimate the area of the scaled region to be* $\boxed{n(sk)^2}$*, which is* $\boxed{k^2}$ *times the area of the original region.*

Question 5 *When* n *copies of a plane figure can form a figure similar to the original, the figure is called a rep-*n*-tile. Explain briefly why any parallelogram is a rep-4-tile and also a rep-9-tile.*

Free Response: ***Hint:*** *If a parallelogram is scaled by a factor of 2, then 4 original parallelograms can make the larger parallelogram. If a parallelogram is scaled by a factor of 3, then 9 original parallelograms can make the larger parallelogram. (Draw pictures.)*

In general, if a parallelogram is scaled by a factor of k, then $\boxed{k^2}$ *copies of the original parallelogram can make the scaled version.*

Question 6 *Use formulas to determine what happens to the perimeter and area of a rectangle when it is scaled by k.*

Begin with a rectangle of base b and height h. After scaling the rectangle by k, the base will be $\boxed{kb}$ *and the height will be* $\boxed{kh}$.

The original perimeter is $\boxed{2b + 2h}$. *After scaling, the perimeter will be* $\boxed{2bk + 2hk}$, *which is precisely* $\boxed{k}$ *times the original perimeter.*

The original area is $\boxed{bh}$. *After scaling, the area will be* $\boxed{(kb)(kh)}$, *which is precisely* $\boxed{k^2}$ *times the original area.*

Question 7 *Use formulas to determine what happens to the circumference and area of a circle when it is scaled by k.*

Begin with a circle of radius r. After scaling the circle by k, its radius will be $\boxed{kr}$.

The original circumference is $\boxed{2\pi r}$. *After scaling, the circumference will be* $\boxed{2\pi kr}$, *which is precisely* $\boxed{k}$ *times the original circumference.*

The original area is $\boxed{\pi r^2}$. *After scaling, the area will be* $\boxed{\pi(kr)^2}$, *which is precisely* $\boxed{k^2}$ *times the original area.*