Similarity

Short-answer problems about similarity.

Question 1 ***Definition.*** *Under a* ***dilation*** *about center O and scale factor $r > 0$, the image of P is a point Q so that Q lies on* (segment/ray/line) $\boxed{OP}$ *and* $OQ = \boxed{rOP}$. *The image of O is* $\boxed{O}$.

Question 2 *Describe, both informally and formally, what it means to say two figures are congruent.*

Free Response: ***Hint:*** *Informally: Two figures are congruent if one can be "placed upon" the other so that the figures match exactly. This is called the principle of "superposition."*

Formally: Two figures are congruent if there is a sequence of basic rigid motions that maps on onto the other.

Question 3 *Describe, both informally and formally, what it means to say two figures are similar.*

Free Response: ***Hint:*** *Informally: Two figures are similar if they have the same shape. With moving and scaling (zooming in or out), one can be placed on another so that the figures match exactly.*

Formally: Two figures are similar if there is a sequence of basic rigid motions and dilations that maps on onto the other.

Question 4 *Compare and contrast the ideas of equal triangles, congruent triangles, and similar triangles.*

Free Response: ***Hint:*** *When two triangles are coon Two triangles are equal of they are the same sets of points. For example, you might say that $\triangle ABC = \triangle BCA$.*

Author(s): Bart Snapp and Brad Findell

Question 5 *Suppose $\triangle ABC \sim \triangle XYZ$. Complete the following equation relating ratios **within** the figures:*

$$\frac{AB}{\boxed{BC}} = \frac{\boxed{XY}}{YZ}.$$

*Complete the following equation relating ratios **between** or **across** the figures:*

$$\frac{AB}{\boxed{XY}} = \frac{\boxed{BC}}{YZ}.$$

A ratio between (or across) corresponding lengths in two similar figures is called a $\boxed{\textit{scale}}$ factor.

Question 6 *Explain why all equilateral triangles are similar to each other.*

Free Response: ***Hint:*** *Equilateral triangles have interior angles that all measure 60°. So all equilateral triangles are similar to one another by the AAA similarity criterion.*

Question 7 *Explain why all isosceles right triangles are similar to each other.*

Free Response: ***Hint:*** *Isosceles right triangles have interior angles that measure 45°, 45°, and 90°. So all equilateral triangles are similar to one another by the AAA similarity criterion.*

Question 8 *Explain why when given a right triangle, the altitude of the right angle divides the triangle into two smaller triangles each similar to the original right triangle.*

Free Response: ***Hint:*** *Each of the two smaller triangles has a has a right angle and shares an acute angle with the original triangle. By AA similarity, each of the smaller triangles is similar to the large triangle, and thus the three triangles must be similar to one another.*

Question 9 *The following sets contain lengths of sides of similar triangles. Solve for all unknowns—give all solutions. In each case explain your reasoning.*

(a) $\{3, 4, 5\}$, $\{6, 8, \boxed{10}\}$

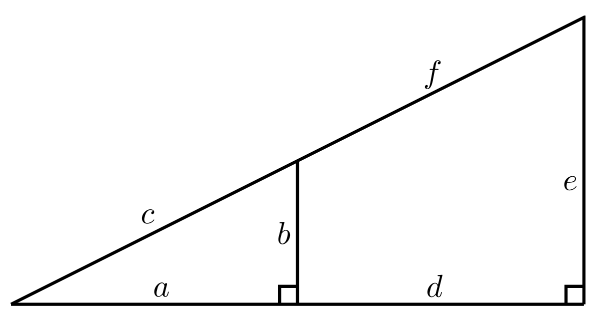
(b) $\{3, 3, 5\}$, $\{9, 9, \boxed{15}\}$

Free Response: ***Hint:***

Question 10 *A Pythagorean Triple is a set of three positive integers* $\{a, b, c\}$ *such that* $a^2 + b^2 = c^2$. *Write down an infinite list of Pythagorean Triples. Explain your reasoning and justify all claims.*

Free Response: ***Hint:*** *Scale any Pythagorean triple by an integer scale factor greater than 1, and you have another Pythagorean triple. For example, 3, 4, 5 is a Pythagorean triple, and so is* $3n$, $4n$, $5n$ *if* $n > 1$ *is an integer.*

Question 11 *Here is a right triangle,* ***not*** *drawn to scale:*



Solve for all unknowns in the following cases. Note: To enter, say, $\sqrt{3}$, *type* `sqrt(3)` *or use the Math Editor.*

(a) $a = 3$, $b = \boxed{1}$, $c = \boxed{\sqrt{10}}$, $d = 12$, $e = 5$, $f = \boxed{4\sqrt{10}}$

(b) $a = \boxed{12/5}$, $b = 3$, $c = \boxed{3\sqrt{41}/5}$, $d = 8$, $e = 13$, $f = \boxed{2\sqrt{41}}$

(c) $a = 7$, $b = 4$, $c = \boxed{\sqrt{65}}$, $d = \boxed{49/4}$, $e = 11$, $f = \boxed{7\sqrt{65}/4}$

(d) $a = 5$, $b = 2$, $c = \boxed{\sqrt{29}}$, $d = 6$, $e = \boxed{22/5}$, $f = \boxed{6\sqrt{29}/5}$

Question 12 *Suppose you have two similar triangles. What can you say about the area of one in terms of the area of the other? Be specific and explain your reasoning.*

Free Response: ***Hint:*** *If the triangles are similar with a scale factor of r, then the ratio of their areas is r^2.*

Question 13 *During a solar eclipse we see that the apparent diameter of the Sun and Moon are nearly equal. If the Moon is around $240,000$ miles from Earth, the Moon's diameter is about 2000 miles, and the Sun's diameter is about $865,000$ miles how far is the Sun from the Earth?*

Distance to sun $\approx \boxed{(865000/2000)240000}$ *miles.*

Question 14 *When jets fly above $8,000$ meters in the air they form a vapor trail. Cruising altitude for a commercial airliner is around $10,000$ meters. One day I reached my arm into the sky and measured the length of the vapor trail with my hand—my hand could just span the entire trail. If my hand spans 9 inches and my arm extends 25 inches from my eye, how long is the vapor trail in **kilometers**?*

Length of vapor trail $\approx \boxed{(10/25)9}$ *km.*

Question 15 *David proudly owns a 42 inch (measured diagonally) flat screen TV. Michael proudly owns a 13 inch (measured diagonally) flat screen TV. Dave sits comfortably with his dog Fritz at a distance of 10 feet. How far must Michael stand from his TV to have the "same" viewing experience? Explain your reasoning.*

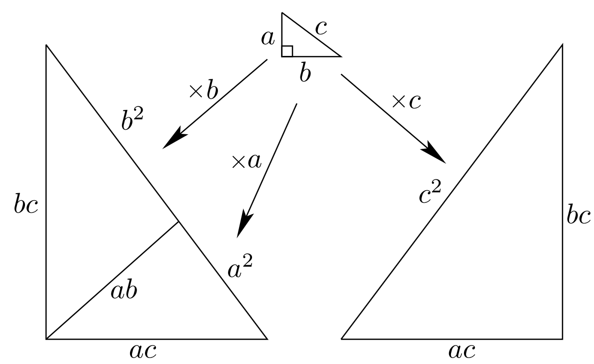
Standing distance $\approx \boxed{(42/13)10}$ *feet.*

Question 16 *Use the definition of similarity (in terms of transformations) to prove that all circles are similar.*

Free Response: ***Hint:*** *Comment: We need to find a sequence of basic rigid motions and dilations that maps one circle onto the other. There are many ways to go about this, but here is one straightforward way that always works:*

Sketch of proof: Translate one circle so that centers coincide. Dilate about the (now common) center by the ratio of the radii. Then the circles coincide.

Question 17 *Explain how the following picture "proves" the Pythagorean Theorem.*



Free Response: ***Hint:*** *We are given a small right triangle with legs a and b and hypotenuse c. We want to show that $a^2 + b^2 = c^2$.*

Sketch of proof: The arrows represent scaling of that triangle by scale factors b, a, and c, resulting in three right triangles with side lengths as labeled. The segments of length a^2 and b^2 are collinear (why?), which implies the two medium-sized triangles make a single large triangle on the left. The large triangle on the left is a right triangle. (Why?) The two large triangles are congruent. (Why?) Then $a^2 + b^2 = c^2$. (Why?)