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# Key Proofs

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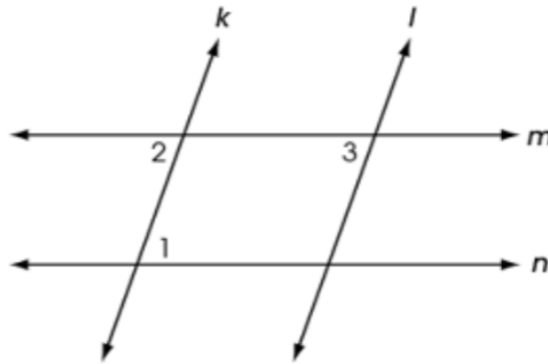
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## End-of-Course Exam Proofs for Math 1

*Proofs.*

**Problem 1** *Adapted from Ohio's 2017 Geometry released item 13.*

Two pairs of parallel lines intersect to form a parallelogram as shown.



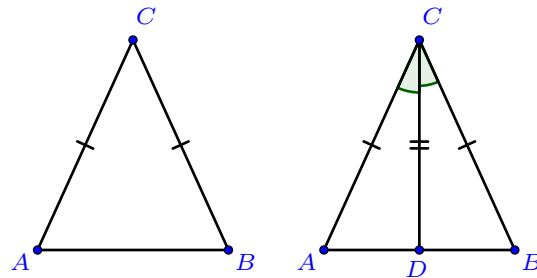
Complete the following proof that opposite angles of a parallelogram are congruent:

- (a)  $\angle 1 \cong \angle 2$  as (opposite angles / alternate interior angles / corresponding angles) for parallel lines ( $m$  and  $n$  /  $k$  and  $l$ ).
- (b)  $\angle 3 \cong \angle 2$  as (opposite angles / alternate interior angles / corresponding angles) for parallel lines ( $m$  and  $n$  /  $k$  and  $l$ ).
- (c) Then  $\angle 1 \cong \angle 3$  because they are both congruent to  $\angle 2$ .

# The Isosceles Triangle Theorem

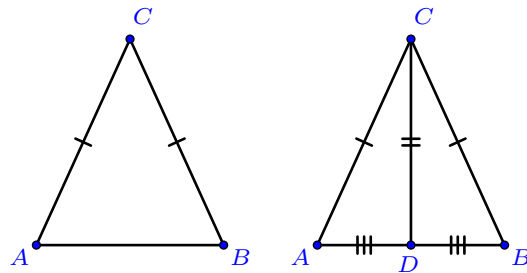
*Proofs.*

**Problem 2** Prove that the base angles of an isosceles triangle are congruent.



- Beginning with the given figure on the left, Morgan draws  $\overline{CD}$  and marks the figure intending that this new segment is a(n) (median/ angle bisector / perpendicular bisector/ altitude).
- Based on the marked figure, Morgan claims that the  $\triangle ACD \cong \triangle \boxed{?}$  by ( SAS/ SSS/ SSA/ ASA/ HL ).
- Finally, Morgan concludes that  $\angle A \cong \angle \boxed{?}$ , as they are corresponding parts of congruent triangles.

**Problem 3** Prove that the base angles of an isosceles triangle are congruent.

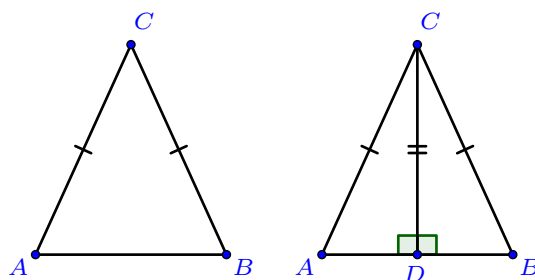


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## The Isosceles Triangle Theorem

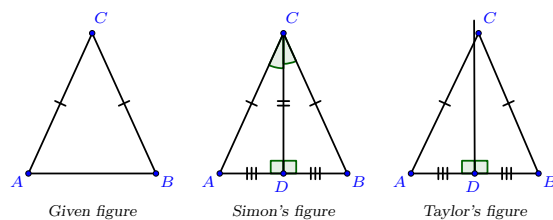
- (a) Beginning with the given figure on the left, Deja draws  $\overline{CD}$  and marks the figure intending that this new segment is a(n) (median / angle bisector / perpendicular bisector / altitude).
- (b) Based on the marked figure, Deja claims that the  $\triangle ACD \cong \triangle \square$  by (SAS / SSS / SSA / ASA / HL).
- (c) Finally, Deja concludes that  $\angle A \cong \angle \square$ , as they are corresponding parts of congruent triangles.

**Problem 4** Prove that the base angles of an isosceles triangle are congruent.



- (a) Beginning with the given figure on the left, Elle draws  $\overline{CD}$  and marks the figure intending that this new segment is a(n) (median / angle bisector / perpendicular bisector / altitude).
- (b) Based on the marked figure, Deja claims that the  $\triangle ACD \cong \triangle \square$  by (SAS / SSS / SSA / ASA / HL).
- (c) Finally, Deja concludes that  $\angle A \cong \angle \square$ , as they are corresponding parts of congruent triangles.

**Problem 5** Simon and Taylor are trying to prove that the base angles of an isosceles triangle are congruent.



## The Isosceles Triangle Theorem

Beginning with the given figure on the left, Simon draws  $\overline{CD}$  and marks the second figure intending that this new segment is a perpendicular bisector of  $\overline{AB}$ .

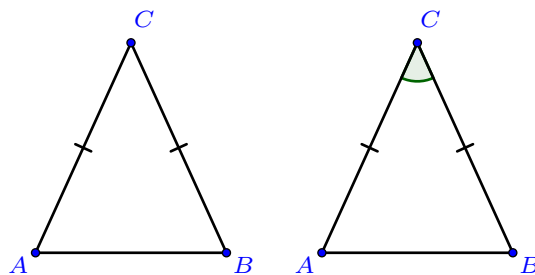
Taylor claims that a perpendicular bisector of a side of a triangle usually misses the opposite vertex. So without using properties of isosceles triangles, the figure must allow for that possibility.

Choose the best response to their argument:

### Multiple Choice:

- (a) Simon is correct, and  $\triangle ACD \cong \triangle BCD$  by SAS.
- (b) Simon is correct, and  $\triangle ACD \cong \triangle BCD$  by SSS
- (c) Taylor is correct, and the perpendicular bisector cannot be used to complete this proof.
- (d) Neither of them are correct.

**Problem 6** Prove that the base angles of an isosceles triangle are congruent.



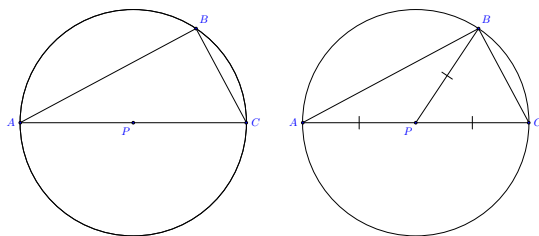
Given figure

- (a) Examining the given figure on the left, Lissy notices symmetry in the triangle and claims that the triangle is congruent to itself by a (translation / reflection / rotation).
- (b) Based on the marked figure, Lissy claims that the  $\triangle ACB \cong \triangle \boxed{?}$  by ( SAS / SSS / SSA / ASA / HL ).
- (c) Finally, Lissy concludes that  $\angle A \cong \angle \boxed{?}$ , as they are corresponding parts of congruent triangles.

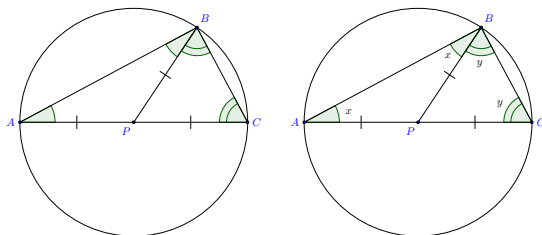
# Inscribed Angles

*Proofs.*

**Problem 7** In the figure below,  $\overline{AB}$  is a diameter of a circle with center  $P$ . Prove that  $\angle B$  is a right angle.



- (a) Beginning with the diagram on the left, Natalia draws  $\overline{PB}$  and marks the diagram to show segments that she knows to be congruent because each one is a ? of the circle.



- (b) Natalia sees that  $\triangle APB$  and  $\triangle BPC$  are ? triangles, so she marks the figure to show congruent angles.
- (c) In order to do some algebra with these congruent angles, Natalia labels their measures  $x$  and  $y$ , as shown in the picture on the right.
- (d) She writes an equation for the sum of the angles of  $\triangle ABC$ :

$$\boxed{?} = 180^\circ$$

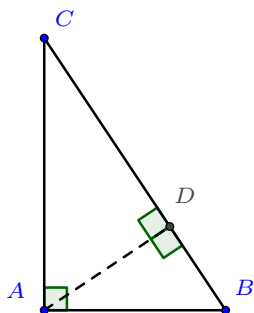
Note: Need a question about dividing the equation by 2.

- (e) Since  $m\angle B = \boxed{?}$ , she concludes that  $m\angle B = 90^\circ$ .

# Proofs for Math 2

*Proofs.*

**Problem 8** *Adapted from Ohio's 2017 Geometry released item 17.*



Complete the following proof that  $\triangle DAC$  is similar to  $\triangle DBA$ :

- (a)  $\triangle ABC \sim \triangle \boxed{?}$  by AA because they share  $\angle B$  and they each have a right angle.
- (b)  $\triangle ABC \sim \triangle \boxed{?}$  by AA because they share  $\angle C$  and they each have a right angle.
- (c)  $\triangle DAC \sim \triangle \boxed{?}$  because they are both similar to  $\triangle ABC$ .

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