Set Theory Problems

Short-answer problems about sets.

Problem 1 *Given two sets X and Y, explain what is meant by $X \cup Y$.*

Free Response: $X \cup Y$ *is the set of elements that are in X or in Y (or both, as the "or" is inclusive).*

Problem 2 *Given two sets X and Y, explain what is meant by $X \cap Y$.*

Free Response: $X \cap Y$ *is the set of elements that are in X and in Y.*

Problem 3 *Given two sets X and Y, explain what is meant by $X - Y$.*

Free Response: $X - Y$ *is the set of elements that are in X but not in Y.*

Problem 4 *Explain the difference between the symbols $\in$ and $\subset$.*

Free Response: *The notation $X \in Y$ would mean that X is a single element in the set Y. In this case, X might not be a set. The notation $X \subset Y$ would require that both X and Y are sets and also that every element of X is also in Y.*

Problem 5 *How is $\{\emptyset\}$ different from $\emptyset$?*

Free Response: *The empty set, $\emptyset$, is a set that contains no elements. The set $\{\emptyset\}$ contains 1 element that is itself a set.*

Learning outcomes:

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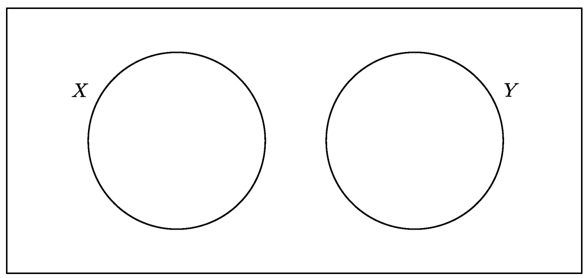
Problem 6 *Draw a Venn diagram for the set of elements that are in X or Y* but not both. *How does it differ from the Venn diagram for $X \cup Y$?*

Free Response: *Same as the Venn diagram for $X \cup Y$, except that the $X \cap Y$ part is not shaded.*

Problem 7 *If we let X be the set of "right triangles" and we let Y be the set of "equilateral triangles" does the picture below show the relationship between these two sets?*



Explain your reasoning.

Free Response: *Yes. The picture is accurate because no right triangles are also equilateral triangles.*

Problem 8 *If $X = \{1, 2, 3, 4, 5\}$ and $Y = \{3, 4, 5, 6\}$ find:*

(a) $X \cup Y$

(b) $X \cap Y$

(c) $X - Y$

(d) $Y - X$

Free Response: (a) $X \cup Y = \{1, 2, 3, 4, 5, 6\}$

(b) $X \cap Y = \{3, 4, 5\}$

(c) $X - Y = \{1, 2\}$

(d) $Y - X = \{6\}$

Problem 9 *Let $n\mathbb{Z}$ represent the integer multiples of n. So for example:*

$$3\mathbb{Z} = \{\dots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \dots\}$$

Compute the following:

(a) $3\mathbb{Z} \cap 4\mathbb{Z}$

(b) $2\mathbb{Z} \cap 5\mathbb{Z}$

(c) $3\mathbb{Z} \cap 6\mathbb{Z}$

(d) $4\mathbb{Z} \cap 6\mathbb{Z}$

(e) $4\mathbb{Z} \cap 10\mathbb{Z}$

In each case explain your reasoning.

Free Response: (a) $3\mathbb{Z} \cap 4\mathbb{Z} = 12\mathbb{Z}$

(b) $2\mathbb{Z} \cap 5\mathbb{Z} = 10\mathbb{Z}$

(c) $3\mathbb{Z} \cap 6\mathbb{Z} = 6\mathbb{Z}$

(d) $4\mathbb{Z} \cap 6\mathbb{Z} = 12\mathbb{Z}$

(e) $4\mathbb{Z} \cap 10\mathbb{Z} = 20\mathbb{Z}$

Problem 10 *Make a general rule for intersecting sets of the form $n\mathbb{Z}$ and $m\mathbb{Z}$. Explain why your rule works.*

Free Response: *The intersection of two sets is what they have in* common. *The intersection of set of multiples of n and the set of multiples of m are, in fact, called* common multiples, *and they are all multiples of the least common multiple of n and m.*

Problem 11 *If $X \cup Y = X$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

Free Response: *$Y \subset X$ because every element of Y must already be in X.*

Problem 12 *If $X \cap Y = X$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

Free Response: *$X \subset Y$ because every element of X must already be in Y.*

Problem 13 *If $X - Y = \emptyset$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

Free Response: *$X \subset Y$ because that would mean X contains no elements that are not also in Y.*
