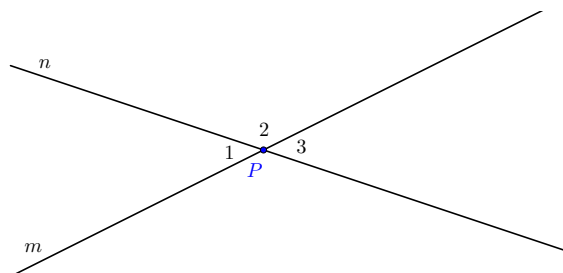


# Vertical Angles

*Proofs updated.*

**Problem 1** Point  $P$  is the intersection of lines  $m$  and  $n$ . Prove that  $\angle 1 \cong \angle 3$ .



*Fixnote: When students write equations about linear pairs, they often write two equations for non-overlapping linear pairs—which doesn't help. The figure above is intended to help avoid that dead end, but it would be worthwhile to discuss that dead end anyway.*

*Fixnote: Below are three different brief proofs that omit details. Which of the details should be included?*

- (a) It must be that  $\angle 1 \cong \angle 3$  because they are both (complementary/ supplementary ✓/ opposite) to  $\angle 2$ .

*Detail: First write down equations about linear pairs of angles:*

$$m\angle 1 + m\angle 2 = 180^\circ$$

$$m\angle 3 + m\angle 2 = 180^\circ$$

*By comparing the two equations, it might be clear that  $m\angle 1 = m\angle 3$ . Other students may need to do some algebra.*

- (b) A rotation of  $180^\circ$  about  $P$  maps  $m$  onto itself, maps  $n$  onto itself, and swaps  $\angle 1$  and  $\angle 3$ . Because rotations preserve angle measures, it must be that  $\angle 1 \cong \angle 3$ .

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Learning outcomes:  
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## Vertical Angles

*Detail: Line  $m$  is the union of two opposite rays with endpoint  $P$ . Check that the  $180^\circ$  rotation about  $P$  swaps these opposite rays. The same idea holds for line  $n$  so that together the sides of  $\angle 1$  become the sides of  $\angle 3$  and vice versa.*

- (c) *Reflecting about the bisector of  $\angle 2$  swaps  $\angle 1$  and  $\angle 3$ . Because reflections preserve angle measures, it follows that  $\angle 1 \cong \angle 3$ .*

*Detail: The reflection swaps the two rays that are the sides of  $\angle 2$ . Because reflections take lines to lines, that reflection must swap not just the rays but lines  $m$  and  $n$ .*

