Math 4407: Online HW 2

Bart Snapp and Brad Findell

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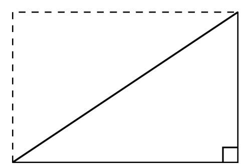
Contents

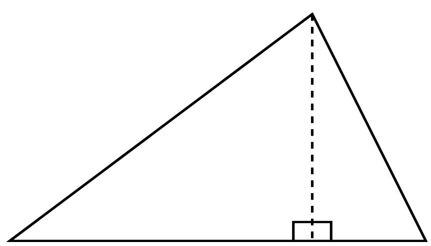
Proof by Picture

Short-answer proofs by pictures.

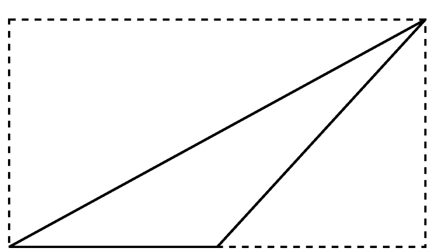
Problem 1 *Explain how the following picture "proves" that the area of a right triangle is half the base times the height.*



Problem 2 *Suppose you know that the area of a **right** triangle is half the base times the height. Explain how the following picture "proves" that the area of **every** triangle is half the base times the height.*



Problem 3 *Now suppose that a student, say Geometry Giorgio attempts to solve a similar problem. Again knowing that the area of a right triangle is half the base times the height, he draws the following picture:*



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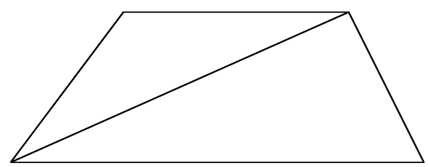
Geometry Giorgio states that the diagonal line cuts the rectangle in half, and thus the area of the triangle is half the base times the height. Is this correct reasoning? If so, give a complete explanation. If not, give correct reasoning based on Geometry Giorgio's picture.

Problem 4 *Recall that a trapezoid is a quadrilateral with two parallel sides. Consider the following picture:*

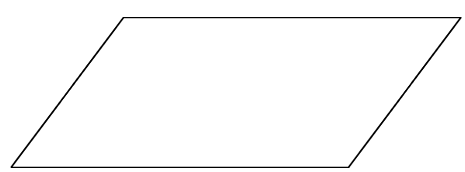


How does the above picture prove that the area of a trapezoid is

$$\text{area} = \frac{h(b_1 + b_2)}{2}$$

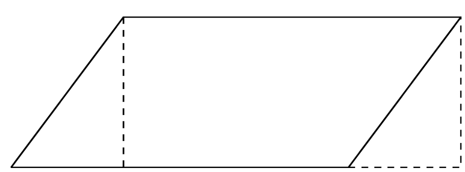
where h is the height of the trapezoid and b_1, b_2, are the lengths of the parallel sides?

Problem 5 *Look at the previous problem. Can you use a similar idea to prove that the area of a parallelogram*

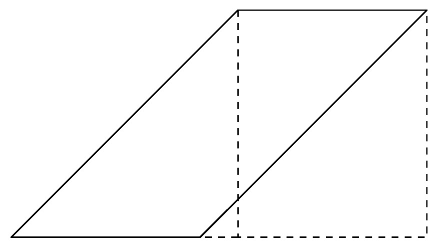


is the length of the base times the height?

Problem 6 *Explain how the following picture "proves" that the area of a parallelogram is base times height.*



Problem 7 *Now suppose that a student, say Geometry Giorgio attempts to solve a similar problem. In an attempt to prove the formula for the area of a parallelogram, Geometry Giorgio draws the following picture:*



At this point Geometry Giorgio says that he has proved the formula for area of a parallelogram. What do you think of his picture? Give a complete argument based on his picture.

Transformations

Short-answer problems about transformations.

Question 8 *To specify a translation, we need a* $\boxed{?}$*. Equivalently, we need a magnitude (or length or* $\boxed{?}$*) and a* $\boxed{?}$*.*

Question 9 *To specify a rotation, we need a* $\boxed{?}$ *and an* $\boxed{?}$ *(assuming an agreement about the direction of rotation).*

Question 10 *To specify a reflection, we need a* $\boxed{?}$*.*

Question 11 *A transformation that does nothing is call the* $\boxed{?}$*. (Hint: Two words.)*

Sometimes a sequence of transformations can be described as a single translation, rotation, or reflection.

Question 12 *What kind of transformation is a translation followed by a translation? Explain. Be sure to consider any special cases.*

Answer: *Usually a* $\boxed{?}$*. If the vectors are opposites of each other, the result is the* $\boxed{?}$ *(two words).*

Question 13 *What kind of transformation is a rotation followed by a rotation? Explain. Be sure to consider any special cases.*

Answer: *Usually a* $\boxed{?}$*. If the angles sum to a multiple of* 360° *and the centers are different, then the result is a* $\boxed{?}$*. If the centers of rotation are the same* ***and*** *the angles sum to a multiple of* 360°*, the result is the* $\boxed{?}$ *(two words).*

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Question 14 *What kind of transformation is a reflection followed by another reflection? Explain. Be sure to consider any special cases.*

Answer: *If the reflection lines intersect, the result is a* $\boxed{?}$. *If the reflection lines are parallel, the result is a* $\boxed{?}$. *If the reflection lines are the same line, the result is the* $\boxed{?}$.

Question 15 *Will the letter F look like an F after a reflection? What about after a sequence of two reflections? What about after a sequence of 73 or 124 reflections? Explain your reasoning.*

Question 16 *How will your answer to the previous problem change if you use a capital D? Explain.*

Question 17 *Given a figure and its image after a translation, how do find the direction and distance of the translation? How many points and images do you need?*

Question 18 *Given a figure and its image after a reflection, how do you find the line of reflection? How many points and images do you need?*

Question 19 *Given a figure and its image after a rotation, how do you find the center and the angle of the rotation? How many points and images do you need?*

Symmetry

Short-answer questions about symmetry.

Question 20 *Categorize the capital letters of the alphabet by their symmetries. Use the following font:*

ABCDEFGHIJKLMNOPQRSTUVWXYZ

- *Vertical line symmetry:* $\boxed{?}$
- *Horizontal line symmetry:* $\boxed{?}$
- 180° *rotational symmetry:* $\boxed{?}$
- *None:* $\boxed{?}$

Question 21 *Write the words COKE and PEPSI in capital letters so that they read vertically. Use a mirror to look at a reflection of the words. What is different about the reflections of the two words? Explain.*

Question 22 *We often say a figure is "symmetric" when we notice that it has symmetry, but now we want to be more precise:*

A symmetry *of a figure is a (reflection/ rotation/ transformation/ translation) that maps a figure (to its opposite/ onto itself/ to another figure).*

Question 23 *Explain why a sequence of two symmetries of a figure must also be a symmetry of that figure.*

Question 24 *Explain why the identity transformation should be considered a symmetry of* any *figure.*

Some possible explanations:

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- *The identity transformation satisfies the definition of a symmetry: It maps the figure* $\boxed{?}$ *(two words).*
- *If a figure has reflection symmetry* R_k *about a line* k*, then* R_k *followed by* R_k *is the* $\boxed{?}$ *(two words). And by the previous result, this sequence of symmetries must also be a symmetry.*
- *If a figure has rotational symmetry* R_α *by some angle* α *about some center, then it must also have a rotational symmetry* $R_{-\alpha}$ *by the angle* $-\alpha$ *about the same center.* R_α *followed by* $R_{-\alpha}$ *is the* $\boxed{?}$ *(two words). And by the previous result, this sequence of symmetries must also be a symmetry.*

Question 25 *It is reasonable to call the identity transformation a translation because it is a translation of magnitude* $\boxed{?}$ *in any direction.*

It is reasonable to call the identity transformation a rotation because it is a rotation of $\boxed{?}$ *degrees about any center.*

Question 26 *Indicate the number of rotation and reflection symmetries of the following figures (including the identity rotation):*

(a) *An equilateral triangle:* $\boxed{?}$ *rotation(s) and* $\boxed{?}$ *reflection(s).*

(b) *An isosceles triangle that is not equilateral:* $\boxed{?}$ *rotation(s) and* $\boxed{?}$ *reflection(s).*

(c) *A square:* $\boxed{?}$ *rotation(s) and* $\boxed{?}$ *reflection(s).*

(d) *A rectangle that is not a square:* $\boxed{?}$ *rotation(s) and* $\boxed{?}$ *reflection(s).*

(e) *A rhombus that is not a square:* $\boxed{?}$ *rotation(s) and* $\boxed{?}$ *reflection(s).*

(f) *A (non-special) parallelogram:* $\boxed{?}$ *rotation(s) and* $\boxed{?}$ *reflection(s).*

(g) *A regular* $n-$*gon:* $\boxed{?}$ *rotation(s) and* $\boxed{?}$ *reflection(s).*

Question 27 *Suppose that quadrilateral* $ABCD$ *has exactly one rotation symmetry (other than the identity transformation) and no reflection symmetry. What kind(s) of quadrilateral could it be? Explain your reasoning.*

Question 28 *Suppose that quadrilateral $ABCD$ has exactly one reflection symmetry and no rotation symmetry (other than the identity transformation). What kind(s) of quadrilateral could it be? Explain your reasoning.*

Question 29 *What are the symmetries of a circle?*

Question 30 *How can you use the symmetries of a circle to determine whether a figure is indeed a circle?*

Question 31 *What are the symmetries of a line?*

Question 32 *How can you use the symmetries of a line to determine whether a figure is indeed a line?*

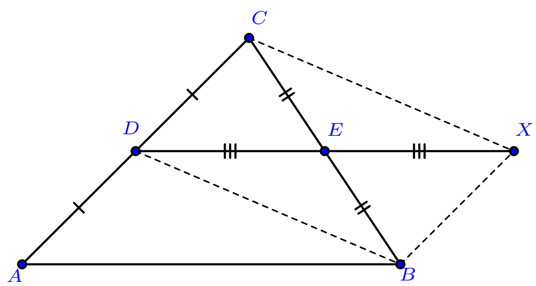
Midsegment Theorem

Proofs updated.

Theorem 1. *Midsegment Theorem: The segment joining the midpoints of two sides of a triangle is parallel to and half the length of the third side.*

In preparation for the midsegment theorem, the class proved several useful theorems about parallelograms.

Problem 33 *To prove the midsegment theorem for $\triangle ABC$ with midpoints D and E of sides AC and BC, respectively, Mitch extended $\overline{DE}$ to a point X such that $EX = DE$, as shown in the marked figure. Then he added dotted lines to the figure to show parallelograms.*



Mitch organized his reasoning in the following flow chart:

Fix note: The flowchart omits reasons to reduce clutter. The most significant steps are green whereas the details are blue.

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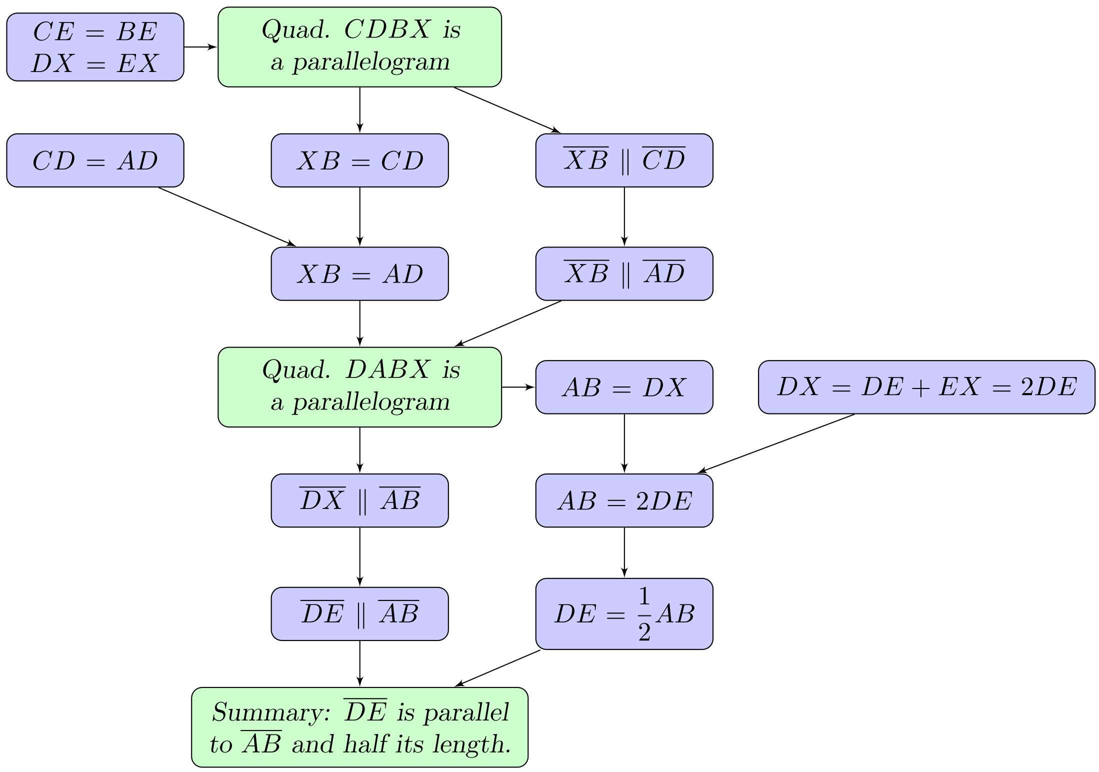


In the proof above, which theorem may Mitch use to conclude that quadrilateral $CDBX$ a parallelogram?

Multiple Choice:

(a) *If a pair of sides of a quadrilateral are congruent and parallel, then it is a parallelogram.*

(b) *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.*

(c) *If opposite sides of a quadrilateral are congruent, then it is a parallelogram.*

(d) *If opposite angles of a quadrilateral are congruent, then it is a parallelogram.*

(e) *The Pythagorean Theorem.*

(f) *None of these.*

In the proof above, which theorem may Mitch use to conclude that quadrilateral $DABX$ a parallelogram?

Multiple Choice:

(a) *If one pair of sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.*

(b) *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.*

(c) *If opposite sides of a quadrilateral are congruent, then it is a parallelogram.*

(d) *If opposite angles of a quadrilateral are congruent, then it is a parallelogram.*

(e) *The Pythagorean Theorem.*

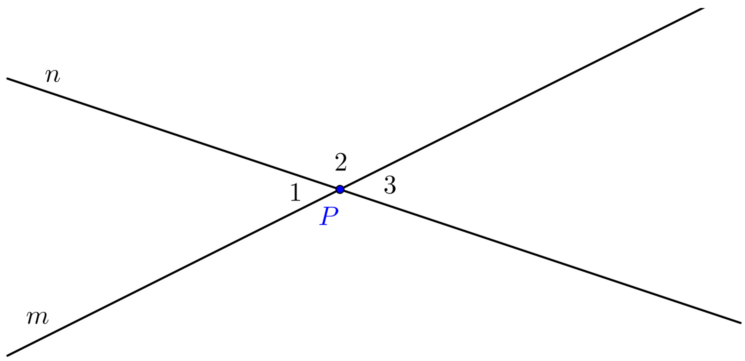
(f) *None of these.*

Vertical Angles

Proofs updated.

Below are three different proofs that vertical angles are congruent. Please consider them separately.

Problem 34 *Point P is the intersection of lines m and n. Prove that $\angle 1 \cong \angle 3$.*



Fix note: When students write equations about linear pairs, they often write two equations for non-overlapping linear pairs—which doesn't help. The figure above is intended to help avoid that dead end, but it would be worthwhile to discuss that dead end anyway.

Proof *Using adjacent angles, $\angle 1 \cong \angle 3$ because they are both (complementary / supplementary / opposite / congruent) to $\angle 2$.* ■

Problem 35 *Point P is the intersection of lines m and n. Prove that $\angle 1 \cong \angle 3$.*

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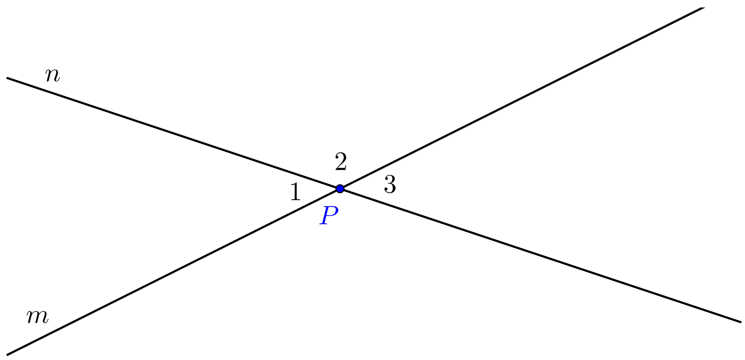
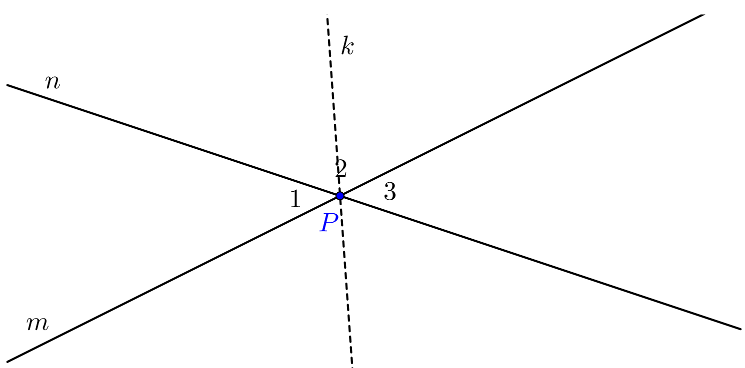


Proof *A rotation of* $(90^\circ\,/\,180^\circ\,/\,360^\circ\,)$ *about* P *maps* m *onto itself, maps* n *onto itself, and swaps* $\angle 1$ *and* $(\angle 1/\,\angle 2/\,\angle 3)$*. Because rotations preserve angle measures, it must be that* $\angle 1 \cong \angle 3$*.* ■

Problem 36 *Point* P *is the intersection of lines* m *and* n*. Prove that* $\angle 1 \cong \angle 3$*.*



Proof *Reflecting about the (bisector/ supplement/ opposite) of* $\angle 2$ *swaps the sides of* $\angle 2$ *and therefore lines* m *and* n*. Thus, that reflection swaps* $\angle 1$ *and* $(\angle 1/\,\angle 2/\,\angle 3)$*. Because reflections preserve angle measures, it follows that* $\angle 1 \cong \angle 3$*.* ■

Parallel Lines

Proofs updated.

This page develops important results regarding parallel lines and transversals. **Read carefully, and complete the proofs.**

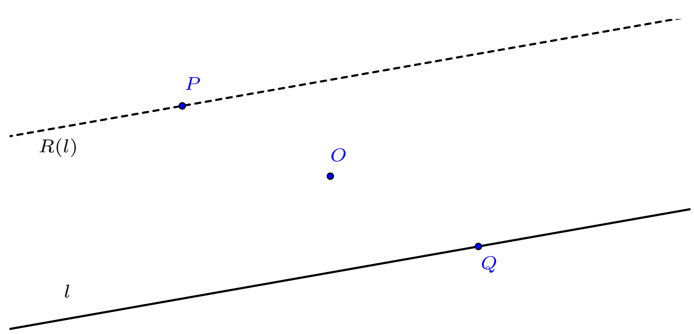
Axiom 1. *Parallel postulate (uniqueness of parallels): Given a line and a point not on the line, there is exactly one line through the given point parallel to the given line.*

Theorem 2. *A* $180°$ *rotation about a point on a line takes the line to itself.*

Proof Suppose point P is on line k. The point cuts the line into two opposite $\boxed{?}$. A $180°$ rotation about P swaps the two opposite rays, thereby mapping the line onto itself. ■

Theorem 3. *A* $180°$ *rotation about a point not on a line takes the line to a parallel line.*

Proof Let O be a point not on line l. Let P be an arbitrary point on $R(l)$, the rotated image of l. To show that $R(l)$ is parallel to l, it is sufficient to show that P cannot lie also on l.



Because P is on $R(l)$, there is a point Q on l such that $P = R(Q)$. The rotated image of $\overrightarrow{OQ}$ is ($\overrightarrow{QO}$ / $\overrightarrow{OP}$ / $\overrightarrow{QP}$), and because $\angle QOP$ is $180°$, it follows that Q, O, and P are $\boxed{?}$. Call that line k. We know line k is distinct from l because point $\boxed{?}$ is on k but not on l. Now, if P were on l, then points P and Q would be on two distinct lines, k and l, contradicting the assumption that on two points there is a unique line. The theorem is proved. ■

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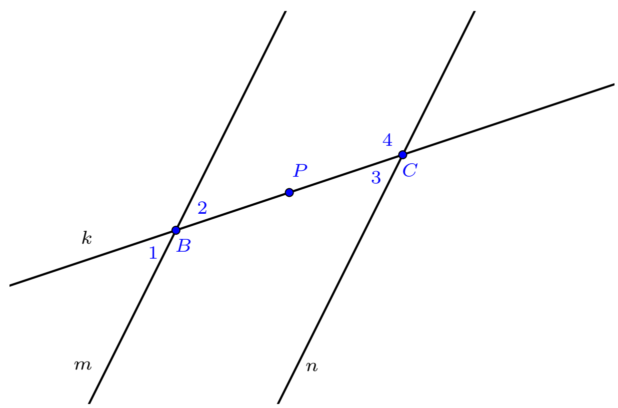
Theorem 4. *If two parallel lines are cut by a transversal, alternate interior angles and corresponding angles are congruent.*

Proof Given that parallel lines m and n are cut by transversal k, prove that alternate interior angles are congruent.



Let B and C be the intersections of transversal k with lines m and n, respectively. Let P be the midpoint of $\overline{BC}$.

(a) Rotate 180° about P, which takes k to (itself/ m/ n).

(b) The rotation maps B to $\boxed{?}$ because $PB = PC$ and the rotation preserves distances.

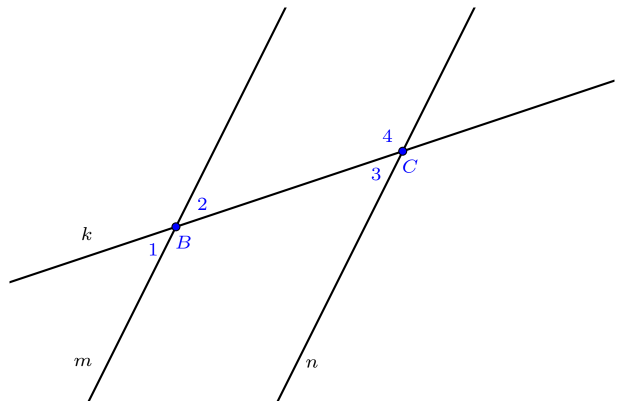
(c) Because P is not on m, the rotation maps m to a parallel line through C, which must be (k/ m/ n) by the uniqueness of parallels.

(d) Thus, the rotation maps $\angle 2$ to ($\angle 1$/ $\angle 2$/ $\angle 3$/ $\angle 4$). These alternate interior angles must be congruent because the rotation preserves angle measures.

■

Note: The congruence of corresponding angles now follows from the congruence of vertical angles. But here is another approach that uses a translation.

Proof Given that parallel lines m and n are cut by transversal k, prove that corresponding angles are congruent.



Let B and C be the intersections of transversal k with lines m and n, respectively.

(a) Translate to the right along line k by distance BC, which takes k to (itself / m / n).

(b) The translation maps B to $\boxed{?}$, and it maps m to (k / m / n) because the translation maintains parallels, and there is a unique parallel to m through C.

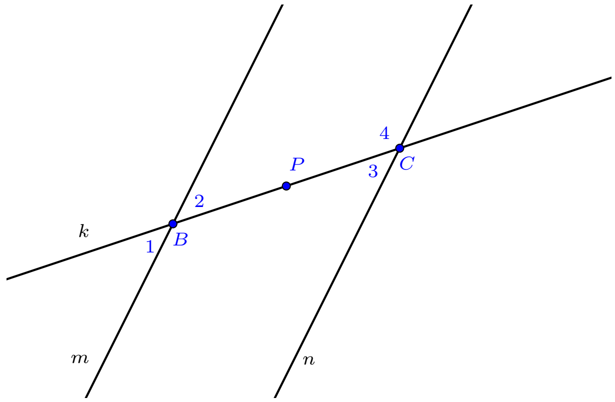
(c) The translation maps $\angle 1$ to ($\angle 1$/ $\angle 2$/ $\angle 3$/ $\angle 4$). These corresponding angles must be congruent because the translation preserves angle measures.

■

Theorem 5. *If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.*

Note: *This theorem is the* $\boxed{?}$ *of the previous theorem about alternate interior angles.*

Proof Given that m and n are cut by transversal k with alternate interior angles congruent, prove that lines m and n are parallel.



Let B and C be the intersections of transversal k with lines m and n, respectively. Let P be the midpoint of $\overline{BC}$.

(a) Rotate 180° about P, which takes k to (itself/ m/ n), and which swaps B and $\boxed{?}$ because distances are preserved.

(b) Because $\angle 2 \cong \angle 3$ and because a side of $\angle 2$ (i.e., $\overrightarrow{BP}$) is mapped to a side of $\angle 3$ (i.e., ($\overrightarrow{CP}$/ $\overrightarrow{PC}$/ $\overrightarrow{BP}$)), it must be that the other side of $\angle 2$ (which lies on m) is mapped to the other side of $\angle 3$ (which lies on line $\boxed{?}$). Thus, n is the image of m.

(c) Because P is not on m, the 180° rotation maps m to a parallel line through C. Thus, n must be parallel to m.

■

Similarity

Short-answer problems about similarity.

Question 37 ***Definition.*** *Under a* ***dilation*** *about center O and scale factor $r > 0$, the image of P is a point Q so that Q lies on (segment/ ray/ line)* $\boxed{?}$ *and $OQ =$* $\boxed{?}$*. The image of O is* $\boxed{?}$.

Question 38 *Describe, both informally and formally, what it means to say two figures are congruent.*

Question 39 *Describe, both informally and formally, what it means to say two figures are similar.*

Question 40 *Compare and contrast the ideas of equal triangles, congruent triangles, and similar triangles.*

Question 41 *A student says that any two rectangles are similar because all the angles are the same. Is the student correct? (Yes. / No. / Not enough information.) Explain.*

Question 42 *Suppose $\triangle ABC \sim \triangle XYZ$. Complete the following equation relating ratios* ***within*** *the figures:*

$$\frac{AB}{\boxed{?}} = \frac{\boxed{?}}{YZ}.$$

Complete the following equation relating ratios ***between*** *(or across) the figures:*

$$\frac{AB}{\boxed{?}} = \frac{\boxed{?}}{YZ}.$$

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A ratio between (or across) corresponding lengths from two similar figures is called a $\boxed{?}$ *factor.*

Question 43 *Are all equilateral triangles are similar to each other? (Yes. / No. / It depends.) Explain.*

Question 44 *Are all isosceles right triangles similar to each other? (Yes. / No. / It depends.) Explain.*

Question 45 *Explain why when given a right triangle, the altitude of the right angle divides the triangle into two smaller triangles each similar to the original right triangle.*

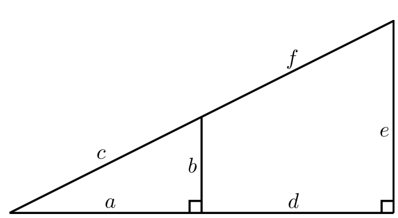
Question 46 *The following sets contain lengths of sides of similar triangles. Solve for all unknowns—give all solutions. In each case explain your reasoning.*

(a) $\{3, 4, 5\}$, $\{6, 8, \boxed{?}\}$

(b) $\{3, 3, 5\}$, $\{9, 9, \boxed{?}\}$

Question 47 *A Pythagorean Triple is a set of three positive integers* $\{a, b, c\}$ *such that* $a^2 + b^2 = c^2$. *Write down an infinite list of Pythagorean Triples. Explain your reasoning and justify all claims.*

Question 48 *Here is a right triangle,* ***not*** *drawn to scale:*



Solve for all unknowns in the following cases. Note: To enter, say, $\sqrt{3}$, *type* `sqrt(3)` *or use the Math Editor.*

(a) $a = 3$, $b = \boxed{?}$, $c = \boxed{?}$, $d = 12$, $e = 5$, $f = \boxed{?}$

(b) $a = \boxed{?}$, $b = 3$, $c = \boxed{?}$, $d = 8$, $e = 13$, $f = \boxed{?}$

(c) $a = 7$, $b = 4$, $c = \boxed{?}$, $d = \boxed{?}$, $e = 11$, $f = \boxed{?}$

(d) $a = 5$, $b = 2$, $c = \boxed{?}$, $d = 6$, $e = \boxed{?}$, $f = \boxed{?}$

Question 49 *Suppose you have two similar triangles. What can you say about the area of one in terms of the area of the other? Be specific and explain your reasoning.*

Question 50 *During a solar eclipse we see that the apparent diameter of the Sun and Moon are nearly equal. If the Moon is around* $240,000$ *miles from Earth, the Moon's diameter is about* 2000 *miles, and the Sun's diameter is about* $865,000$ *miles how far is the Sun from the Earth?*

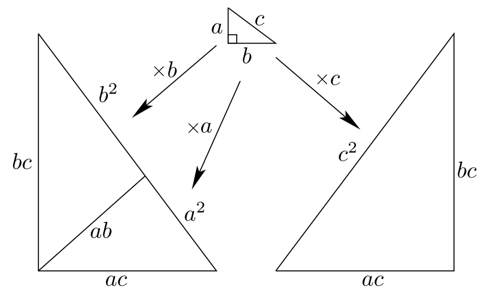
Distance to sun $\approx \boxed{?}$ *miles.*

Question 51 *When jets fly above* $8,000$ *meters in the air they form a vapor trail. Cruising altitude for a commercial airliner is around* $10,000$ *meters. One day I reached my arm into the sky and measured the length of the vapor trail with my hand—my hand could just span the entire trail. If my hand spans* 9 *inches and my arm extends* 25 *inches from my eye, how long is the vapor trail in **kilometers**?*

Length of vapor trail $\approx \boxed{?}$ *km.*

Question 52 *Use the definition of similarity (in terms of transformations) to prove that all circles are similar.*

Question 53 *Explain how the following picture "proves" the Pythagorean Theorem.*


a
c
b
×b
×a
×c
b^2
bc
ab
a^2
ac
c^2
bc
ac

Measurement

Short-answer problems about measurement.

Numbers, Units, and Quantities

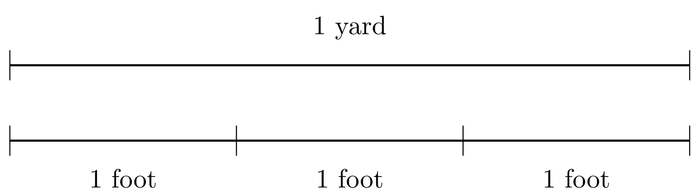
In this section, we develop and explain the algebra of measurement calculations and unit conversions.

Question 54 *Brad measured his driveway to be 21 feet long. How long is it in yards?* $\boxed{?}$ *yards.*

We all know that 3 feet and 1 yard express the same length. Let's write an equation to express this equality of lengths:

$$3 \text{ ft} = 1 \text{ yd}$$

Here is a picture illustrating this idea:



The equation 3 ft = 1 yd seems to suggest that you (multiply/ divide) feet by 3 to get yards. But when converting feet to yards in the question above you (multiplied/ divided) the number of feet by 3.

Why do these answers appear to be opposites of each other?

The resolution of this conundrum requires that we distinguish numbers (e.g., 21) from units (e.g., feet). Quantities measuring, say, length, weight, or speed involve both numbers and units, but the numbers behave differently from the units, as we see in the example above.

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With a little algebraic manipulation, the equation 3 ft = 1 yd is equivalent to the following equations:

$$\frac{3 \text{ feet}}{1 \text{ yard}} = 1 \qquad \text{and} \qquad \frac{1 \text{ yard}}{3 \text{ feet}} = 1.$$

In both equations, the 1 on the right is "dimensionless" in the sense that it is without units. Think of it as a scale factor of 1, which leaves lengths unchanged. These equations are useful for demonstrating the conversion from feet to yards and vice versa:

$$21 \text{ feet} = 21 \cancel{\text{feet}} \cdot \frac{1 \text{ yard}}{3 \cancel{\text{feet}}} = 7 \text{ yards}$$

$$7 \text{ yards} = 7 \cancel{\text{yards}} \cdot \frac{3 \text{ feet}}{1 \cancel{\text{yard}}} = 21 \text{ feet}$$

These examples illustrate an *algebra of units*, in which units behave much like algebraic variables. In both conversions, we multiply a quantity by a dimensionless 1, so that the calculation doesn't change the amount that the quantity represents. The cancellation of units helps to confirm that we are doing the calculation correctly.

At a conceptual level, this algebra of units helps illuminate how the numbers and units behave in apparently opposite ways:

- Yards are three times as big as feet, so there are one-third as many in a given length.
- Feet are one-third the size of yards, so there are three times as many in a given length.

Thus, we must be careful to consider whether the letters represent numbers or units.

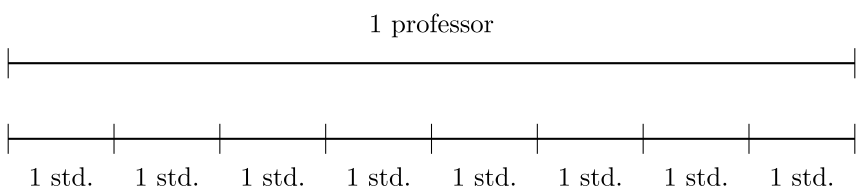
Let's try another problem.

Question 55 *At Big State University, the student-professor ratio is 8 to 1. Write an equation relating the number of students, s, to the number of professors, p.*

Answer: $s = \boxed{?}$.

In response to this question, it is tempting to write $8s = p$ or its equivalent, $s = p/8$, and these turn out to be very common incorrect answers among students. But let's try some specific numbers: If there are 160 students, then there should be 20 professors, but the equation says that $p = 8s = 8 \cdot 160 = 1280$, which would mean 1280 professors. Clearly the equation is backwards: It caused us to multiply the number of students by 8 when we should have divided.

But here is another way of thinking about this incorrect answer. If "students" and "professors" were units (like yards and feet), then the equation would be correct. Here is a picture to help:



If professors and students are units of measurement with this consistent relationship between them, then to "convert" 20 professors to students, we proceed just as we converted yards to feet:

$$20 \text{ professors} = 20 \cancel{\text{ professors}} \cdot \frac{8 \text{ students}}{1 \cancel{\text{ professor}}} = 160 \text{ students}$$

The above question states, however, that the letters are to be the *numbers* of students and professors, not units of measurement. So let's repeat the previous calculation generally, with p representing the number of professors and s representing the number of students:

$$p \text{ professors} = p \cancel{\text{ professors}} \cdot \frac{8 \text{ students}}{1 \cancel{\text{ professor}}} = 8p \text{ students} = s \text{ students}$$

So the correct equation is indeed $s = 8p$. But again, notice how the numbers and the units work in apparently opposite ways.

Because of the common confusion between numbers and units, some mathematics educators recommend against using the first letter of a unit to indicate the number of that unit in a measurement. Indeed, the expressions "p professors" and "s students" are hard to read and interpret—and "$8p$ students" might be worse.

Whether you accept this recommendation or not, here is some good advice:

- When making calculations with quantities that include units, be very clear and careful about whether the letters represent units or numbers.
- When writing equations, it is very easy to get the relationships backwards, so check the equations with easy numbers.

Math teachers benefit from knowing conversions from memory. Here are some that you might not know:

$$\begin{aligned} 1 \text{ mile} &= 5280 \text{ feet} \\ 1 \text{ inch} &= 2.54 \text{ cm (exactly)} \\ 1 \text{ kg} &\approx 2.205 \text{ lbs} \end{aligned}$$

And here is a useful uncommon conversion that makes use of common ones:

$$60 \text{ mph} = \frac{60 \text{ miles}}{1 \text{ hour}} = \frac{60 \cancel{\text{miles}}}{1 \cancel{\text{hour}}} \cdot \frac{1 \cancel{\text{hour}}}{3600 \text{ sec}} \cdot \frac{5280 \text{ feet}}{1 \cancel{\text{mile}}} = \frac{88 \text{ feet}}{1 \text{ sec}} = 88 \text{ fps}$$

Question 56 *Use the conversions above to convert 1 meter to inches and to yards.*

$$1 \textit{ meter} = \boxed{?} \textit{ inches} = \boxed{?} \textit{ yards}$$

Question 57 *College and school tracks used to be 1/4 mile around, or 440 yards, so that a half-mile race was two laps, and a one-mile race was four laps. Today, most college and school tracks have been converted to metric, with one lap measuring 400 meters, which is close to 440 yards. In high-school track, the "mile" is usually run as 1600 meters.*

The 1600 meters race is $\boxed{?}$ *yards (shorter/ longer) than 1 mile.*

A four-minute miler should run 1600 meters about $\boxed{?}$ *seconds (to the closest 0.05 seconds) (faster/ slower) than four minutes.*

Square Units and Cubic Units

Because 3 ft = 1 yd, it is tempting to conclude that 3 square feet = 1 square yards. But . . .

Question 58 *What is a square foot? What is a square yard?*

Question 59 *How many square feet are in a square yard?* $\boxed{?}$.

Provide a geometric explanation. Provide an explanation based on the algebra of units.

Problem 60 *Convert 25 yards to meters (and 25 meters to yards) using "2.54 cm in each inch" as the only Metric-English unit conversion. Now convert 25 square yards to square meters and 25 square meters to square yards. Do the same with cubic yards and cubic meters.*

$25\ \textit{yards} = \boxed{?}\ \textit{meters}$

$25\ \textit{meters} = \boxed{?}\ \textit{yards}$

$25\ \textit{square yards} = \boxed{?}\ \textit{square meters}$

$25\ \textit{square meters} = \boxed{?}\ \textit{square yards}$

$25\ \textit{cubic yards} = \boxed{?}\ \textit{cubic meters}$

$25\ \textit{cubic meters} = \boxed{?}\ \textit{cubic yards}$

Scaling in 2D

Short-answer problems about scaling in two dimensions.

Length and Area Under Scaling

In this section, we explore what happens to length, area, and other measures under scaling.

In a previous section, we defined similarity in terms of basic rigid motions and dilations, and we showed that this definition leads to well-known results about similarity, such as the AA criterion for triangle similarity and consistent ratios of lengths between and within similar figures. A key feature of this discussion was the notion of "scale factor," which describes what happens to lengths under a dilation. From the definition of a dilation, it is clear that segments on lines through the center of dilation scale by the scale factor. We used the side-splitter theorems to show that other segments are scaled by the same scale factor.

Question 61 *Two students claims that a 3×5 rectangle and a 4×6 rectangle are similar.*

Fred says that that they are similar because the angles are the same. How do you respond?

Ned says that they are similar because you can do the same thing (i.e., add 1) to "both sides" of the 3×5 rectangle to get the 4×6 rectangle. How do you respond?

Question 62 *Complete the following sentences using words such as filling, falling, covering, wrapping, hiding, surrounding, or traveling:*

Area vs. perimeter can be thought of as $\boxed{?}$ *vs.* $\boxed{?}$*, respectively.*

Volume vs. surface area can be thought of as $\boxed{?}$ *vs.* $\boxed{?}$*, respectively.*

To explore how measures of figures change under scaling and non-scaling transformations, here are some useful strategies:

- Cutting the figures and rearranging the pieces.

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- Using “rep-tiles.”
- Using known formulas for perimeters, areas, volumes, or surface areas.
- Approximating with segments, squares, or cubes.
- Shearing.

Question 63 *To estimate the length of a curve, imagine approximating it with many small segments. Now apply a similarity transformation with a scale factor of k. Each segment will scale by* $\boxed{?}$*, so the length of the curve will be* $\boxed{?}$ *times the original length.*

Question 64 *To estimate the area of a non-polygonal region, imagine covering it approximately with a grid of squares of side length s. Piecing together partial squares, suppose you count n squares. Your area estimate is then* $\boxed{?}$*.*

Now apply a similarity transformation of scale factor k to both the region and the grid. Each square in the scaled grid will have area $\boxed{?}$*, and piecing together partial squares there will be* $\boxed{?}$ *squares. Thus, we estimate the area of the scaled region to be* $\boxed{?}$*, which is* $\boxed{?}$ *times the area of the original region.*

Question 65 *When n copies of a plane figure can form a figure similar to the original, the figure is called a rep-n-tile. Explain briefly why any parallelogram is a rep-4-tile and also a rep-9-tile.*

In general, if a parallelogram is scaled by a factor of k, then $\boxed{?}$ *copies of the original parallelogram can make the scaled version.*

Question 66 *Use formulas to determine what happens to the perimeter and area of a rectangle when it is scaled by k.*

Begin with a rectangle of base b and height h. After scaling the rectangle by k, the base will be $\boxed{?}$ *and the height will be* $\boxed{?}$*.*

The original perimeter is $\boxed{?}$*. After scaling, the perimeter will be* $\boxed{?}$*, which is precisely* $\boxed{?}$ *times the original perimeter.*

The original area is $\boxed{?}$*. After scaling, the area will be* $\boxed{?}$*, which is precisely* $\boxed{?}$ *times the original area.*

Question 67 *Use formulas to determine what happens to the circumference and area of a circle when it is scaled by k.*

Begin with a circle of radius r. After scaling the circle by k, its radius will be $\boxed{?}$.

The original circumference is $\boxed{?}$. *After scaling, the circumference will be* $\boxed{?}$, *which is precisely* $\boxed{?}$ *times the original circumference.*

The original area is $\boxed{?}$. *After scaling, the area will be* $\boxed{?}$, *which is precisely* $\boxed{?}$ *times the original area.*

Scaling in 3D

Short-answer problems about scaling in 3 dimensions.

Length, Area, and Volume Under Scaling

In this section, we explore what happens to length, area, volume, and other measures under scaling.

To explore how measures of figures change under scaling and non-scaling transformations, here are some useful strategies:

- Cutting the figures and rearranging the pieces.
- Using "rep-tiles."
- Using known formulas for perimeters, areas, volumes, or surface areas.
- Approximating with segments, squares, or cubes.
- Shearing.

Question 68 *Fiona's fudge is shipped in boxes that are right rectangular prisms, all similar to 3 cm by 4 cm by 5 cm. Compute the following measures:*

(a) *The volume of the box =* $\boxed{?}$ *cubic centimeters.*

(b) *The volume of a box scaled by* k = $\boxed{?}$ *cubic centimeters.*

(c) *The surface area of the box =* $\boxed{?}$ *square centimeters.*

(d) *The surface area of the box a box scaled by* k = $\boxed{?}$ *square centimeters.*

(e) *The length + girth of the box =* $\boxed{?}$ *cm.*

(f) *The length + girth of a box scaled by* k = $\boxed{?}$ *cm.*

Question 69 *Given a cylinder of radius* 5 *and height* 8*, how will the following volumes compare?*

Author(s): Brad Findell

(a) *A cylinder with k times the radius and the same height will have* $\boxed{?}$ *times the volume.*

(b) *A cylinder with the same radius and k times the height will have* $\boxed{?}$ *times the volume.*

(c) *A cylinder with k times the radius and k times the height will have* $\boxed{?}$ *times the volume.*

Question 70 *Which of the following cylinders are similar to the given cylinder of radius 5 and height 8?*

Select All Correct Answers:

(a) *A cylinder with k times the radius and the same height?*

(b) *A cylinder with the same radius and k times the height?*

(c) *A cylinder with k times the radius and k times the height?*

(d) *None of the above*

The similar cylinder will have surface area $\boxed{?}$ *times the surface area of the original cylinder.*

Problem 71 *Some drugs work best when dosages are proportional to body surface area. Other drugs work best when dosages are proportional to blood volume. A typical adult male (5 ft 10 in, 175 lbs) has a body surface area of about 2 square meters and about 5 liters of blood. Scale these values up to estimate LeBron's body surface area and blood volume. (Reminder: LeBron is 6 ft 8 in, 250 lbs.)*

LeBron's body surface area = $\boxed{?}$ *square meters.*

LeBron's blood volume = $\boxed{?}$ *liters.*

Problem 72 *Consider a version of LeBron that is d times as tall. How would following quantities compare between the scaled version and the real LeBron: leather in the sole of a shoe, shoe size, inseam, fabric in a T-shirt, lung capacity, neck circumference, and hat size? Explain briefly.*

Cool fact: The size of a hat is the diameter (in inches) of the hat when it is reshaped into a circle. Most adults have hat sizes between $6\frac{3}{4}$ *and 8.*

Problem 73 *A typical adult male gorilla is about 5.5 feet tall and weighs about 400 pounds. Suppose King Kong was about 22 feet tall and proportioned like a typical adult male gorilla.*

(a) *What is the scale factor between from the typical gorilla to King Kong?* $\boxed{?}$

(b) *King Kong's weight* $\approx \boxed{?}$ *pounds.*

Briefly explain your reasoning.

(c) *The circumference of the neck of a typical adult male gorilla is 36 inches. King Kong's neck circumference* $\approx \boxed{?}$ *inches.*

Briefly explain your reasoning.

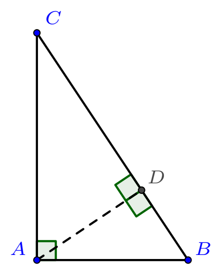
(d) *Suppose an Ohio State sweatshirt for a typical adult male gorilla requires 3 square yards of fabric. An Ohio State sweatshirt for King Kong would need* $\approx \boxed{?}$ *square yards of fabric.*

Briefly explain your reasoning.

Similar Right Triangles

Proofs.

Problem 74 *Adapted from Ohio's 2017 Geometry released item 17.*



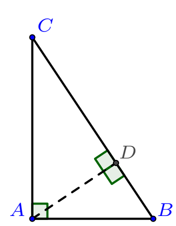
Complete the following proof that $\triangle DAC$ *is similar to* $\triangle DBA$*:*

(a) $\triangle DBA \sim \triangle\boxed{?}$ *by* (*AA similarity*/ *CPCTC*/ *right triangle similarity*) *because they share* $\angle B$ *and they each have a right angle.*

(b) $\triangle DAC \sim \triangle\boxed{?}$ *for the same reason because they share* ($\angle A$/ $\angle B$/ $\angle C$) *and they each have a right angle.*

(c) $\triangle DAC \sim \triangle DBA$ *because* (*CPCTC*/ *right triangle similarity*/ *they are both similar to*$\triangle ABC$).

Problem 75 *A different proof, also adapted from Ohio's 2017 Geometry released item 17.*



Complete the following proof that $\triangle DAC$ *is similar to* $\triangle DBA$*:*

Author(s): Brad Findell

(a) $\angle B$ and $\angle BAD$ are $\boxed{?}$ *because they are acute angles in a right triangle.*

(b) $\angle DAC$ and $\angle BAD$ *are complementary because they are adjacent angles that form* $\angle BAC$, *which is (right/ acute/ obtuse).*

(c) $\angle B \cong \angle DAC$ *because they are both complementary to* $\angle BAD$.

(d) $\triangle DAC \sim \triangle DBA$ *by (AA similarity/ CPCTC/ right triangle similarity) because* $\angle B \cong \angle DAC$ *and they each have a right angle.*