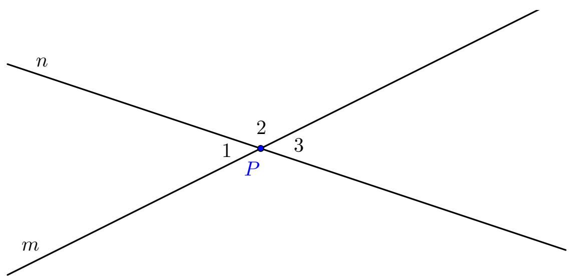
Vertical Angles

Proofs updated.

Problem 1 *Point P is the intersection of lines m and n. Prove that $\angle 1 \cong \angle 3$.*



Fixnote: When students write equations about linear pairs, they often write two equations for non-overlapping linear pairs—which doesn't help. The figure above is intended to help avoid that dead end, but it would be worthwhile to discuss that dead end anyway.

Fixnote: Below are three different brief proofs that separate out details. Which of the details should be included, and which should be omitted?

(a) $\angle 1 \cong \angle 3$ *because they are both (complementary / supplementary ✓/ opposite/ congruent) to* $\angle 2$.

Detail: First write down equations about linear pairs of angles:

$$m\angle 1 + m\angle 2 = 180^\circ$$

$$m\angle 3 + m\angle 2 = 180^\circ$$

By comparing the two equations, it might be clear that $m\angle 1 = m\angle 3$. Other students may need to do some algebra.

(b) *A rotation of 180° about P maps m onto itself, maps n onto itself, and swaps $\angle 1$ and $\angle 3$. Because rotations preserve angle measures, it must be that $\angle 1 \cong \angle 3$.*

Learning outcomes:

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Detail: Line m is the union of two opposite rays with endpoint P. Check that the 180° rotation about P swaps these opposite rays. The same idea holds for line n so that together the sides of $\angle 1$ become the sides of $\angle 3$ and vice versa.

(c) *Reflecting about the bisector of $\angle 2$ swaps $\angle 1$ and $\angle 3$. Because reflections preserve angle measures, it follows that $\angle 1 \cong \angle 3$.*

Detail: The reflection swaps the two rays that are the sides of $\angle 2$. Because reflections take lines to lines, that reflection must swap not just the rays but lines m and n.