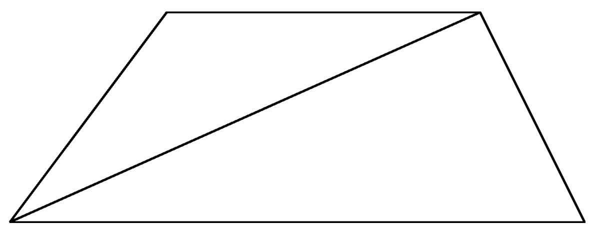
Proof by Picture 2

Short-answer proofs by pictures.

Problem 1 *Recall that a trapezoid is a quadrilateral with two parallel sides. Consider the following picture:*



How does the above picture prove that the area of a trapezoid is

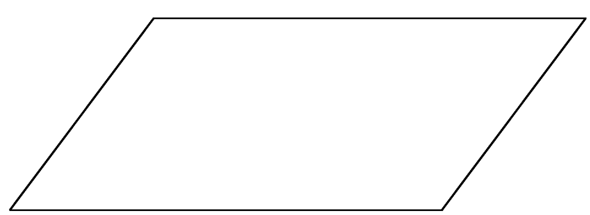
$$\text{area} = \frac{h(b_1 + b_2)}{2}$$

where h is the height of the trapezoid and b_1, b_2, are the lengths of the parallel sides?

Free Response: ***Hint:*** *Using the parallel sides as bases, the two triangles have the same height as the trapezoid. The area of the trapezoid is the sum of the areas of the triangles:*

$$\text{area} = \frac{hb_1}{2} + \frac{hb_2}{2} = \frac{h(b_1 + b_2)}{2}$$

Problem 2 *Look at the previous problem. Can you use a similar idea to prove that the area of a parallelogram*



is the length of the base times the height?

Learning outcomes:

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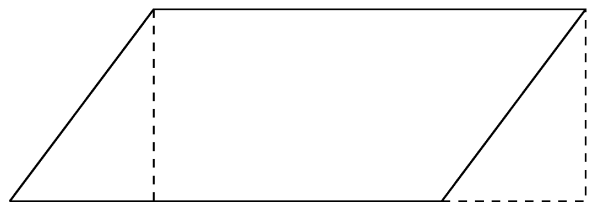
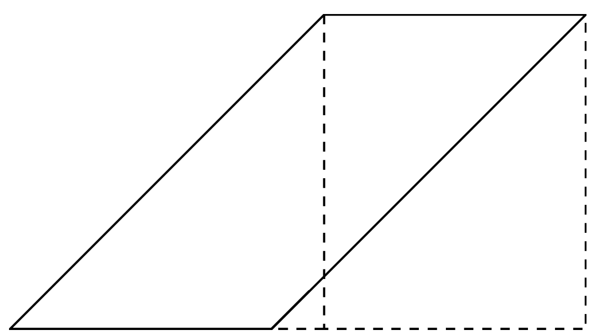
Free Response: ***Hint:*** *Hint: Draw a diagonal. Then there are two triangles with the same base and the same height.*

Problem 3 *Explain how the following picture "proves" that the area of a parallelogram is base times height.*



Free Response: ***Hint:*** *Imagine cutting off the triangle on the left and placing it in the (congruent) dotted region on the right. Then we have a rectangle with the same base and height as the parallelogram. The cutting and rearranging doesn't change the area of the figure, so the area of the parallelogram is the same as the area of the rectangle, which is bh.*

Problem 4 *Now suppose that a student, say Geometry Giorgio attempts to solve a similar problem. In an attempt to prove the formula for the area of a parallelogram, Geometry Giorgio draws the following picture:*



At this point Geometry Giorgio says that he has proved the formula for area of a parallelogram. What do you think of his picture? Give a complete argument based on his picture.

Free Response: ***Hint:*** *Hint: This parallelogram problem is much like Giorgio's triangle problem above. Use the same idea.*