# Online HW 7: Parallels

**Bart Snapp and Brad Findell**

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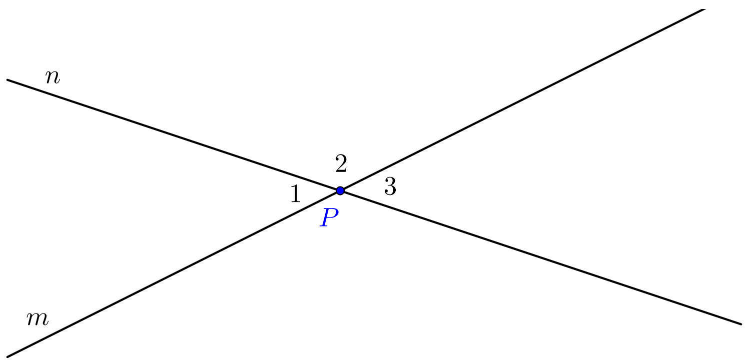
# Contents

# Vertical Angles

*Proofs updated.*

Fix note: Below are three different proofs. Please consider them separately. And in each proof, which of the details should be included, and which should be omitted?

**Problem 1** *Point $P$ is the intersection of lines $m$ and $n$. Prove that $\angle 1 \cong \angle 3$.*




*Fix note: When students write equations about linear pairs, they often write two equations for non-overlapping linear pairs—which doesn't help. The figure above is intended to help avoid that dead end, but it would be worthwhile to discuss that dead end anyway.*

(a) $\angle 1 \cong \angle 3$ *because they are both (complementary/ supplementary/ opposite / congruent) to* $\angle 2$.

*Detail: First write down equations about linear pairs of angles:*

$$m\angle 1 + m\angle 2 = 180^\circ$$

$$m\angle 3 + m\angle 2 = 180^\circ$$

*By comparing the two equations, some students will see clearly that $m\angle 1 = m\angle 3$. A more formal approach would be to do some algebra.*

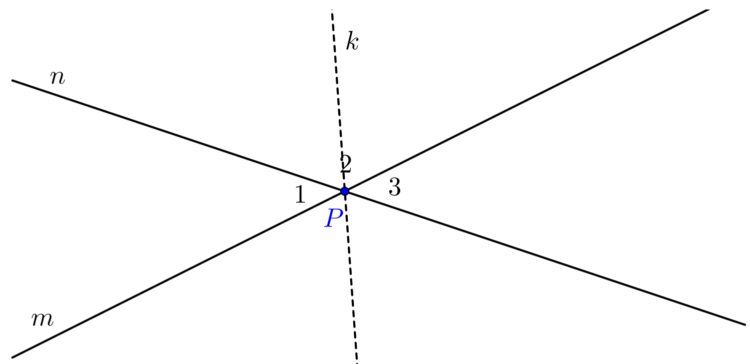
Author(s): Brad Findell

(b) *A rotation of* $(90^\circ\,/\,180^\circ\,/\,360^\circ)$ *about* $P$ *maps* $m$ *onto itself, maps* $n$ *onto itself, and swaps* $\angle 1$ *and* $\angle 3$*. Because rotations preserve angle measures, it must be that* $\angle 1 \cong \angle 3$.

*Detail: Line* $m$ *is the union of two opposite rays with endpoint* $P$*. Check that the* $180^\circ$ *rotation about* $P$ *swaps these opposite rays. The same idea holds for line* $n$ *so that together the sides of* $\angle 1$ *become the sides of* $\angle 3$ *and vice versa.*

(c) *Reflecting about the (bisector/ supplement/ opposite) of* $\angle 2$ *swaps* $\angle 1$ *and* $\angle 3$*. Because reflections preserve angle measures, it follows that* $\angle 1 \cong \angle 3$.

*Detail: The reflection swaps the two rays that are the sides of* $\angle 2$*. Because reflections take lines to lines, that reflection must swap not just the rays but lines* $m$ *and* $n$*.*



# Parallel Lines

*Proofs updated.*

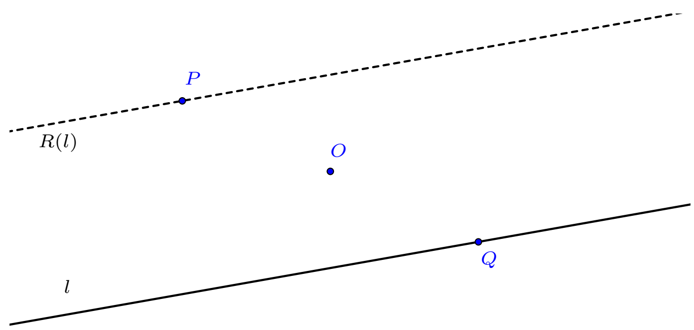
Parallel postulate (uniqueness of parallels): Given a line and a point not on the line, there is exactly one line through the given point parallel to the given line.

**Theorem 1.** *A $180^\circ$ rotation about a point on a line takes the line to itself.*

***Proof*** Suppose point $P$ is on line $k$. The point cuts the line into two opposite rays. A $180^\circ$ rotation about $P$ swaps the two opposite rays, thereby mapping the line onto itself. ■

**Theorem 2.** *A $180^\circ$ rotation about a point not on a line takes the line to a parallel line.*

***Proof*** Let $O$ be a point not on line $l$. Let $P$ be an arbitrary point on $R(l)$, the rotated image of $l$. To show that $R(l)$ is parallel to $l$, it is sufficient to show that $P$ cannot lie also on $l$.




Because $P$ is on $R(l)$, there is a point $Q$ on $l$ such that $P = R(Q)$. The rotated image of $\overrightarrow{OQ}$ is $\overrightarrow{OP}$, and because $\angle QOP$ is $180^\circ$, it follows that $Q$, $O$, and $P$ are collinear. Call that line $k$. We know line $k$ is distinct from $l$ because $O$ is on $k$ but not on $l$. Now, if $P$ were on $l$, then points $P$ and $Q$ would be on two distinct lines, $k$ and $l$, contradicting the assumption that on two points there is a unique line. The theorem is proved. ■

**Theorem 3.** *If two parallel lines are cut by a transversal alternate interior angles (and corresponding angles) are congruent.*

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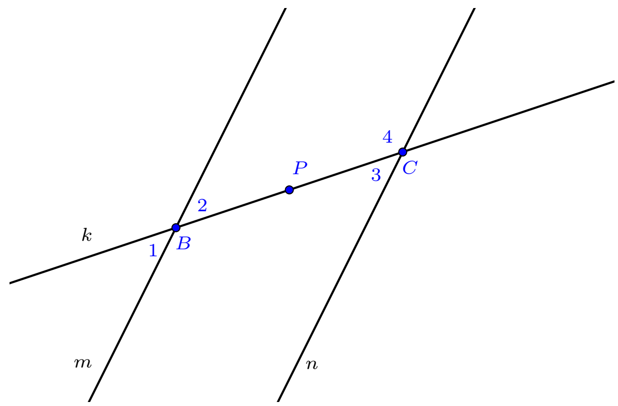
***Proof*** Given that parallel lines $m$ and $n$ are cut by transversal $k$, prove that alternate interior angles are congruent.




(a) Let $B$ and $C$ be the intersections of transversal $k$ with lines $m$ and $n$, respectively. Let $P$ be the midpoint of $\overline{BC}$

(b) Rotate $180^\circ$ about $P$, which takes $k$ to itself.

(c) The rotation maps $B$ to $C$ and $C$ to $B$ because distances are preserved.

(d) The rotation maps $m$ to a parallel line through $C$, which must be $(k/\ m\ /\ n)$ by the uniqueness of parallels.

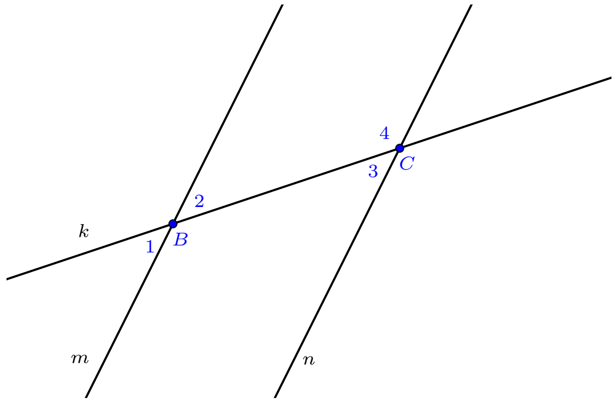
(e) The rotation maps $n$ to $(k/\ m/\ n)$ by the same reasoning.

(f) The rotation swaps $\angle 2$ and $(\angle 1/\ \angle 2/\ \angle 3/\ \angle 4)$. These alternate interior angles must be congruent because the rotation preserves angle measures.

■

**Theorem 4.** *If two parallel lines are cut by a transversal corresponding angles (and alternate interior angles) are congruent.*

***Proof*** Given that parallel lines $m$ and $n$ are cut by transversal $k$, prove that corresponding angles are congruent.




(a) Let $B$ and $C$ be the intersections of transversal $k$ with lines $m$ and $n$, respectively.

(b) Translate to the right along line $k$ by distance $BC$, which takes $k$ to itself.

(c) The translation maps $B$ to $C$, and it maps $m$ to ( $k$ / $m$ / $n$ ) because the translation maintains parallels, and there is a unique parallel to $m$ through $C$.

(d) The translation maps $\angle 1$ to ($\angle 1$/ $\angle 2$/ $\angle 3$/ $\angle 4$). These corresponding angles must be congruent because the translation preserves angle measures.

■

**Theorem 5.** *If two lines are cut by a transversal so that alternate interior (and corresponding) angles are congruent, then the lines are parallel.*