

Parallel Lines

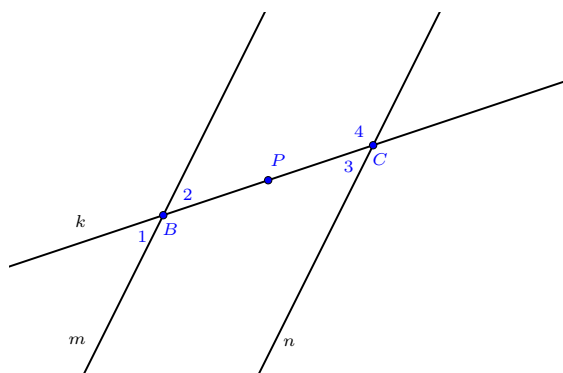
Proofs updated.

Parallel postulate (uniqueness of parallels): Given a line and a point not on the line, there is exactly one line through the given point parallel to the given line.

Theorems to prove:

- (a) A 180° rotation about a point on a line takes the line to itself.
- (b) A 180° rotation about a point not on a line takes the line to a parallel line.
- (c) If two parallel lines are cut by a transversal alternate interior (and corresponding) angles are congruent.
- (d) If two lines are cut by a transversal so that alternate interior (or corresponding) angles are congruent, then the lines are parallel.

Problem 1 Given that parallel lines m and n are cut by transversal k , prove that alternate interior angles are congruent.

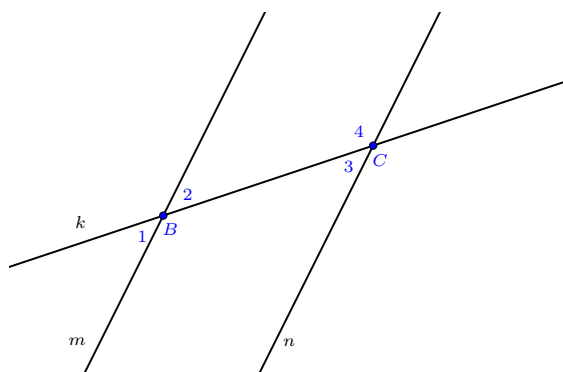


- (a) Let B and C be the intersections of transversal k with lines m and n , respectively. Let P be the midpoint of \overline{BC}
- (b) Rotate 180° about P , which takes k to itself.
- (c) The rotation maps B to C and C to B because distances are preserved.

Learning outcomes:
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- (d) The rotation maps m to a parallel line through C , which must be $(k/m/n)$ by the uniqueness of parallels.
- (e) The rotation maps n to $(k/m/n)$ by the same reasoning.
- (f) The rotation swaps $\angle 2$ and $(\angle 1/\angle 2/\angle 3/\angle 4)$. These alternate interior angles must be congruent because the rotation preserves angle measures.

Problem 2 Given that parallel lines m and n are cut by transversal k , prove that corresponding angles are congruent.



- (a) Let B and C be the intersections of transversal k with lines m and n , respectively.
- (b) Translate to the right along line k by distance BC , which takes k to itself.
- (c) The translation maps B to C , and it maps m to $(k/m/n)$ because the translation maintains parallels, and there is a unique parallel to m through C .
- (d) The translation maps $\angle 1$ to $(\angle 1/\angle 2/\angle 3/\angle 4)$. These corresponding angles must be congruent because the translation preserves angle measures.