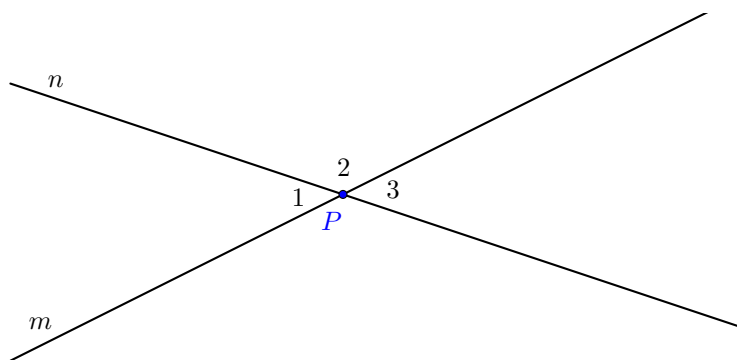


Vertical Angles

Proofs updated.

Below are three different proofs that vertical angles are congruent. Please consider them separately.

Problem 1 Point P is the intersection of lines m and n . Prove that $\angle 1 \cong \angle 3$.



Fix note: When students write equations about linear pairs, they often write two equations for non-overlapping linear pairs—which doesn't help. The figure above is intended to help avoid that dead end, but it would be worthwhile to discuss that dead end anyway.

Proof Using adjacent angles, $\angle 1 \cong \angle 3$ because they are both (complementary / supplementary ✓/ opposite/ congruent) to $\angle 2$. ■

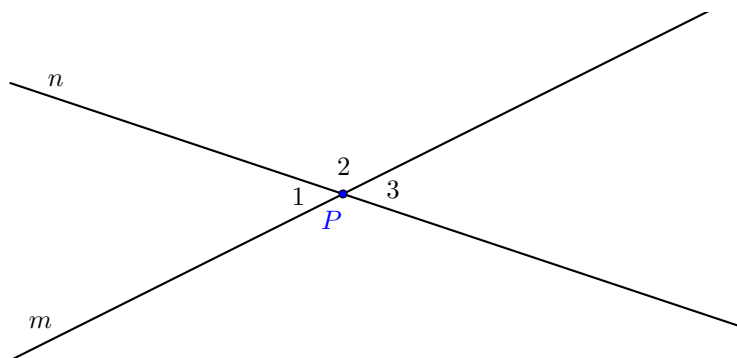
Feedback(correct): Additional detail: First write down equations about linear pairs of angles:

$$m\angle 1 + m\angle 2 = \boxed{180} \text{ degrees}$$

$$m\angle 3 + m\angle 2 = \boxed{180} \text{ degrees}$$

By comparing the two equations, one might see that $m\angle 1 = m\angle 3$. Alternatively, one may do some algebra to conclude that $m\angle 1 = 180^\circ - m\angle 2 = m\angle 3$, which is essentially what the one-sentence proof says.

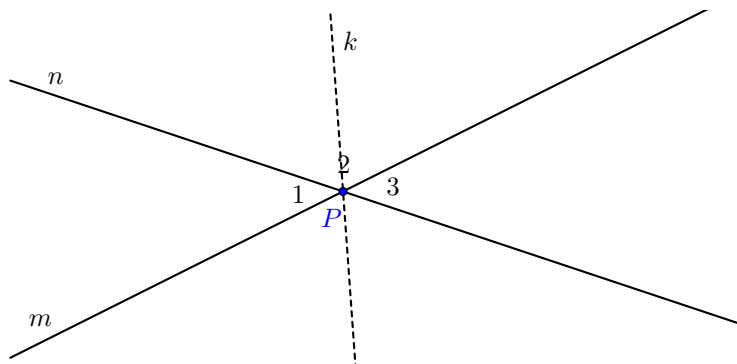
Problem 2 Point P is the intersection of lines m and n . Prove that $\angle 1 \cong \angle 3$.



Proof A rotation of $(90^\circ / 180^\circ \checkmark / 360^\circ)$ about P maps m onto itself, maps n onto itself, and swaps $\angle 1$ and $(\angle 1 / \angle 2 / \angle 3 \checkmark)$. Because rotations preserve angle measures, it must be that $\angle 1 \cong \angle 3$. ■

Feedback(correct): Additional detail: Line m is the union of two opposite rays with endpoint P . The rotation about P swaps these opposite rays, and the same idea holds for line n . That rotation maps the sides of $\angle 1$ onto the sides of $\angle 3$ and vice versa.

Problem 3 Point P is the intersection of lines m and n . Prove that $\angle 1 \cong \angle 3$.



Proof Reflecting about the (bisector \checkmark / supplement / opposite) of $\angle 2$ swaps the sides of $\angle 2$ and therefore lines m and n . Thus, that reflection swaps $\angle 1$ and $(\angle 1 / \angle 2 / \angle 3 \checkmark)$. Because reflections preserve angle measures, it follows that $\angle 1 \cong \angle 3$. ■

Vertical Angles

Feedback(correct): Additional detail: Because reflections take lines to *lines*, the reflection that swaps the sides of $\angle 2$ must swap not just the rays but lines m and n , which contain the rays.
