Symmetry

Short-answer questions about symmetry.

Question 1 *Categorize the capital letters of the alphabet by their symmetries. Use the following font:*

ABCDEFGHIJKLMNOPQRSTUVWXYZ

- *Vertical line symmetry:* $\boxed{AHIMOTUVWXY}$
- *Horizontal line symmetry:* $\boxed{BCDEHIKOX}$
- 180° *rotational symmetry:* $\boxed{HINOSXZ}$
- *None:* $\boxed{FGJLPQR}$

Feedback(`correct`): *Notes: (1) In many fonts that look much the same, B and K have no symmetry. (2) In this font, the O is slightly taller than it is wide. If it were a circle, there would be more symmetry. (See later problem.)*

Question 2 *Write the words COKE and PEPSI in capital letters so that they read vertically. Use a mirror to look at a reflection of the words. What is different about the reflections of the two words? Explain.*

Free Response:

Hint: *If the K has horizontal line symmetry in the font, then all the letters in COKE have horizontal line symmetry, which becomes vertical line symmetry when the word is written vertically. PEPSI, on the other hand, has several letters without that symmetry.*

Question 3 *We often say a figure is "symmetric" when we notice that it has symmetry, but now we want to be more precise:*

A symmetry *of a figure is a (reflection/ rotation/ transformation ✓/ translation) that maps a figure (to its opposite/ onto itself ✓/ to another figure).*

Learning outcomes:
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Question 4 *Explain why a sequence of two symmetries of a figure must also be a symmetry of that figure.*

Free Response:

Hint: *The first transformation maps the figure onto itself, and the second transformation maps the figure onto itself, so the sequence of two transformations maps the figure* $\boxed{ontoitself}$.

Question 5 *Explain why the identity transformation should be considered a symmetry of* any *figure.*

Some possible explanations:

- *The identity transformation satisfies the definition of a symmetry: It maps the figure* $\boxed{ontoitself}$ *(two words).*
- *If a figure has reflection symmetry R_k about a line k, then R_k followed by R_k is the* $\boxed{identitytransformation}$ *(two words). And by the previous result, this sequence of symmetries must also be a symmetry.*
- *If a figure has rotational symmetry R_α by some angle α about some center, then it must also have a rotational symmetry $R_{-\alpha}$ by the angle $-\alpha$ about the same center. R_α followed by $R_{-\alpha}$ is the* $\boxed{identitytransformation}$ *(two words). And by the previous result, this sequence of symmetries must also be a symmetry.*

Feedback(`correct`): *Note: Sometimes the identity transformation is called the* trivial *symmetry. If the identity transformation is the* only *symmetry of a figure, we usually say the figure is asymmetric or has no (nontrivial) symmetry.*

Question 6 *It is reasonable to call the identity transformation a translation because it is a translation of magnitude* $\boxed{0}$ *in any direction.*

It is reasonable to call the identity transformation a rotation because it is a rotation of $\boxed{0}$ *degrees about any center.*

Question 7 *Indicate the number of rotation and reflection symmetries of the following figures (including the identity rotation):*

(a) *An equilateral triangle:* $\boxed{3}$ *rotation(s) and* $\boxed{3}$ *reflection(s).*

(b) *An isosceles triangle that is not equilateral:* $\boxed{0}$ *rotation(s) and* $\boxed{1}$ *reflection(s).*

(c) *A square:* $\boxed{4}$ *rotation(s) and* $\boxed{4}$ *reflection(s).*

(d) *A rectangle that is not a square:* $\boxed{2}$ *rotation(s) and* $\boxed{2}$ *reflection(s).*

(e) *A rhombus that is not a square:* $\boxed{2}$ *rotation(s) and* $\boxed{2}$ *reflection(s).*

(f) *A (non-special) parallelogram:* $\boxed{2}$ *rotation(s) and* $\boxed{0}$ *reflection(s).*

(g) *A regular* $n-$*gon:* $\boxed{n}$ *rotation(s) and* $\boxed{n}$ *reflection(s).*

Question 8 *Suppose that quadrilateral $ABCD$ has exactly one rotation symmetry (other than the identity transformation) and no reflection symmetry. What kind(s) of quadrilateral could it be? Explain your reasoning.*

Free Response:

Hint: *If a rotation of α is a symmetry of a figure, then a rotation of 2α must also be a symmetry. Thus, for the quadrilateral to have only one (non-identity) rotation symmetry, it must be that* $\alpha = \boxed{180}$ *degrees. That rotation will swap opposite vertices, which implies that the center of rotation is the midpoint of each diagonal, so that the diagonals must* $\boxed{\textit{bisect}}$ *each other. Thus, quadrilateral $ABCD$ is a* $\boxed{\textit{parallelogram}}$ *that is not any more special (or it would have additional symmetry).*

Question 9 *Suppose that quadrilateral $ABCD$ has exactly one reflection symmetry and no rotation symmetry (other than the identity transformation). What kind(s) of quadrilateral could it be? Explain your reasoning.*

Free Response:

Hint: *A parallelogram has 180° rotational symmetry, so quadrilateral $ABCD$ cannot be a parallelogram, which also excludes the special cases: rhombus, rectangle, and square. If the line of symmetry goes through two vertices, it must be a(n)* $\boxed{\textit{kite}}$ *that is not a rhombus. If the line of symmetry goes through two sides, it must be a(n)* $\boxed{\textit{isoscelestrapezoid}}$ *(two words).*

Question 10 *What are the symmetries of a circle?*

Free Response:

Hint: *A circle has rotational symmetry by any* $\boxed{\textit{angle}}$ *about its center. A circle has reflection symmetry about any* $\boxed{\textit{line}}$ *through its center. A circle does not have translation symmetry.*

Question 11 *How can you use the symmetries of a circle to determine whether a figure is indeed a circle?*

Free Response:

Hint: *Perform any of the symmetry transformations to be sure that the circle is actually mapped onto itself.*

Question 12 *What are the symmetries of a line?*

Free Response:

Hint:

(a) *A line has translation symmetry by a vector of any length (parallel ✓/ perpendicular/ opposite) to the line.*

(b) *A line has* $\boxed{180}$ *degree rotational symmetry about any point on the line.*

(c) *A line has reflection symmetry about any (parallel/ perpendicular ✓/ opposite) to the line. A line also has reflection symmetry about itself.*

Question 13 *How can you use the symmetries of a line to determine whether a figure is indeed a line?*

Free Response:

Hint: *Perform any of the symmetry transformations to be sure that the line is actually mapped onto itself.*