Set Theory Problems

Extra problems about sets.

Problem 1 *Indicate the number of elements in each set:*

(a) *The set* $\{3, 5, 6, 9, 10\}$ *has* $\boxed{5}$ *element(s).*

(b) *The set* $\{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$ *has* $\boxed{4}$ *element(s).*

(c) *The set* $\{\{\}\}$ *has* $\boxed{1}$ *element(s).*

(d) *The set* $\{\}$ *has* $\boxed{0}$ *element(s).*

(e) *The set* $\emptyset$ *has* $\boxed{0}$ *element(s).*

(f) *The set* $\{\emptyset\}$ *has* $\boxed{1}$ *element(s).*

Problem 2 *Indicate whether each statement is true or false:*

(a) $2 \in \{3, 2, 5\}$. *(True ✓/ False)*

(b) $2 \subseteq \{3, 2, 5\}$. *(True/ False ✓)*

(c) $\{2\} \in \{3, 2, 5\}$. *(True/ False ✓)*

(d) $\{2\} \subseteq \{3, 2, 5\}$. *(True ✓/ False)*

(e) $\emptyset = \{\}$. *(True ✓/ False)*

(f) $\emptyset = \{\emptyset\}$. *(True/ False ✓)*

(g) $\{\emptyset\} = \{\{\}\}$. *(True ✓/ False)*

(h) $\emptyset \in \{\emptyset\}$. *(True ✓/ False)*

(i) $\emptyset \subseteq \{\emptyset\}$. *(True ✓/ False)*

(j) $2 \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False ✓)*

(k) $2 \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False ✓)*

Learning outcomes:
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(l) $\{2\} \in \{\{3,2,7\},\{4,5\},\{2\},\emptyset\}$. *(True ✓/ False)*

(m) $\{2\} \subseteq \{\{3,2,7\},\{4,5\},\{2\},\emptyset\}$. *(True/ False ✓)*

(n) $\{\{2\}\} \in \{\{3,2,7\},\{4,5\},\{2\},\emptyset\}$. *(True/ False ✓)*

(o) $\{\{2\}\} \subseteq \{\{3,2,7\},\{4,5\},\{2\},\emptyset\}$. *(True ✓/ False)*

Problem 3 *Explain the difference between the symbols $\in$ and $\subseteq$.*

Free Response: ***Hint:*** *The symbol $\in$ means "is an element of," whereas $\subseteq$ means "is a subset of." The notation $X \in Y$ means that X is a single element in the set Y. In this case, X is typically not a set. The notation $X \subseteq Y$, in contrast, requires that both X and Y are sets and, furthermore, that every element of X is also in Y.*

Problem 4 *How is $\{\emptyset\}$ different from $\emptyset$?*

Free Response: ***Hint:*** *The empty set, $\emptyset$, is a set that contains no elements. That is, $\emptyset = \{\}$. The set $\{\emptyset\}$ contains one element that is itself a set—and that element happens to be the empty set. We could instead write $\{\{\}\}$, but that looks ugly.*