

Circle Intersections

These problems provide practice solving systems of equations involving circles.

To solve the systems of equations below, we use two common solution methods: $\boxed{?}$ and $\boxed{?}$. In interpreting the solutions below, you are encouraged to use Desmos (see <https://www.desmos.com/>) or GeoGebra Classic (see <https://www.geogebra.org/>).

A Line and a Circle

When the two equations are of a circle and of a line, a common approach uses substitution. Solve the line for one of the variables (say, y) and then substitute into the equation of the circle. This approach yields a (linear, / quadratic, / exponential, / radical) equation in (0, / 1, / 2) variable(s), which can be solved with the $\boxed{?}$ formula.

Problem 1 Solve the following equations simultaneously. Interpret the solutions.

$$(x - 3)^2 + (y - 2)^2 = 14, \quad x - y = -4$$

$$x = \boxed{?} \pm \boxed{?}, y = \boxed{?} \pm \boxed{?}$$

Problem 2 Solve the following equations simultaneously. Interpret the solutions.

$$(x - 3)^2 + (y - 3)^2 = 8, \quad x - y = -4$$

$$x = \boxed{?} \pm \boxed{?}, y = \boxed{?} \pm \boxed{?}$$

Problem 3 Solve the following equations simultaneously. Interpret the solutions.

$$(x - 3)^2 + (y - 2)^2 = 12, \quad x - y = -4$$

$$x = \boxed{?} \pm \boxed{?}, y = \boxed{?} \pm \boxed{?}$$

Two Circles

Given equations of two circles, to find their intersections, first use the method of elimination. Subtract one equation from the other to eliminate the squared terms. The result will (usually) be an equation of a (line, / parabola, / circle). Then proceed as before, using the equation of the line and the equation of either circle.

Problem 4 Solve the following equations simultaneously. Interpret the solutions.

$$(x - 2)^2 + (y - 1)^2 = 3, \quad x^2 + y^2 = 2$$

The elimination step yields the line $y = \boxed{?}$. Then substitution yields the following solutions:

$$x = \boxed{?} \pm \boxed{?}, y = \boxed{?} \mp \boxed{?}$$

Summary

Finding the intersection of a line and a circle or the intersection of two circles (usually) leads to quadratic equations in one variable, which can be solved using the quadratic formula, which will reveal 0, 1, or 2 real solutions.

Note: When the solutions to the quadratic are (nonreal) complex numbers, we say that there are 0 real solutions.

Question 5 If there are 2 real solutions to the quadratic, then

Multiple Choice:

- (a) the figures do not intersect in the xy -plane.
- (b) the figures are tangent in the xy -plane.
- (c) the figures intersect twice in the xy -plane.
- (d) none of these.

Question 6 If there is 1 real solution to the quadratic, then

Multiple Choice:

- (a) *the figures do not intersect in the xy -plane.*
 - (b) *the figures are tangent in the xy -plane.*
 - (c) *the figures intersect twice in the xy -plane.*
 - (d) *none of these.*
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Question 7 *If there are 0 real solutions to the quadratic, then*

Multiple Choice:

- (a) *the figures do not intersect in the xy -plane.*
 - (b) *the figures are tangent in the xy -plane.*
 - (c) *the figures intersect twice in the xy -plane.*
 - (d) *none of these.*
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