Similarity

Short-answer problems about similarity.

Question 1 ***Definition.*** *Under a* ***dilation*** *about center O and scale factor $r > 0$, the image of P is a point Q so that Q lies on* (segment/ray/line) $\boxed{OP}$ *and* $OQ = \boxed{rOP}$. *The image of O is* $\boxed{O}$.

Question 2 *Describe, both informally and formally, what it means to say two figures are congruent.*

Free Response: ***Hint:*** *Informally: Two figures are congruent if one can be "placed on another" so that the figures match exactly. This is called the principle of "superposition."*

Formally: Two figures are congruent if there is a sequence of basic rigid motions that maps on onto the other.

Question 3 *Describe, both informally and formally, what it means to say two figures are similar.*

Free Response: ***Hint:*** *Informally: Two figures are similar if they have the same shape. With moving and scaling (zooming in or out), one can be placed on another so that the figures match exactly.*

Formally: Two figures are similar if there is a sequence of basic rigid motions and dilations that maps on onto the other.

Question 4 *Compare and contrast the ideas of equal triangles, congruent triangles, and similar triangles.*

Free Response: ***Hint:*** *When two triangles are coon Two triangles are equal of they are the same sets of points. For example, you might say that $\triangle ABC = \triangle BCA$.*

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Question 5 *Suppose* $\triangle ABC \sim \triangle XYZ$. *Write down some ratios.* $\frac{AB}{XY} = \frac{BC}{\boxed{YZ}}$.

Question 6 *Explain why all equilateral triangles are similar to each other.*

Free Response: ***Hint:*** *Equilateral triangles have interior angles that all measure* 60°. *So all equilateral triangles are similar to one another by the AAA similarity criterion.*

Question 7 *Explain why all isosceles right triangles are similar to each other.*

Free Response: ***Hint:*** *Isosceles right triangles have interior angles that measure* 45°, 45°, *and* 90°. *So all equilateral triangles are similar to one another by the AAA similarity criterion.*

Question 8 *Explain why when given a right triangle, the altitude of the right angle divides the triangle into two smaller triangles each similar to the original right triangle.*

Free Response: ***Hint:*** *Each of the two smaller triangles has a has a right angle and shares an acute angle with the original triangle. So the triangles are congruent by AA similarity.*

Question 9 *The following sets contain lengths of sides of similar triangles. Solve for all unknowns—give all solutions. In each case explain your reasoning.*

(a) $\{3, 4, 5\}$, $\{6, 8, \boxed{10}\}$

(b) $\{3, 3, 5\}$, $\{9, 9, \boxed{15}\}$

(c) $\{5, 5, x\}$, $\{10, 4, y\}$

(d) $\{5, 5, x\}$, $\{10, 8, y\}$

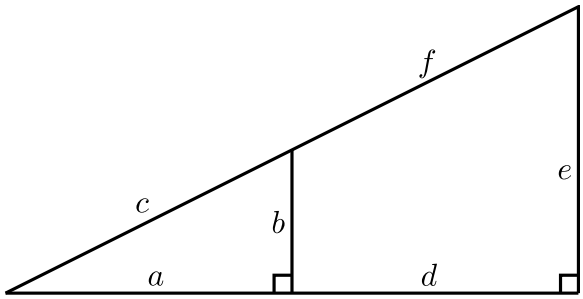
(e) $\{3, 4, x\}$, $\{4, 5, y\}$

Free Response: ***Hint:***

Question 10 *A Pythagorean Triple is a set of three positive integers $\{a, b, c\}$ such that $a^2 + b^2 = c^2$. Write down an infinite list of Pythagorean Triples. Explain your reasoning and justify all claims.*

Free Response: ***Hint:*** *Scale any Pythagorean triple by an integer scale factor greater than 1, and you have another Pythagorean triple. For example, 3, 4, 5 is a Pythagorean triple, and so is $3r$, $4r$, $5r$ if $r > 1$ is an integer.*

Question 11 *Here is a right triangle,* ***not*** *drawn to scale:*



Solve for all unknowns in the following cases. Note: To enter, say, $\sqrt{3}$, type `sqrt(3)` *or use the Math Editor.*

(a) $a = 3$, $b = \boxed{1}$, $c = \boxed{\sqrt{10}}$, $d = 12$, $e = 5$, $f = \boxed{4\sqrt{10}}$

(b) $a = \boxed{12/5}$, $b = 3$, $c = \boxed{3\sqrt{41}/5}$, $d = 8$, $e = 13$, $f = \boxed{2\sqrt{41}}$

(c) $a = 7$, $b = 4$, $c = \boxed{\sqrt{65}}$, $d = \boxed{49/4}$, $e = 11$, $f = \boxed{7\sqrt{65}/4}$

(d) $a = 5$, $b = 2$, $c = \boxed{\sqrt{29}}$, $d = 6$, $e = \boxed{22/5}$, $f = \boxed{6\sqrt{29}/5}$

In each case explain your reasoning.

Question 12 *Suppose you have two similar triangles. What can you say about the area of one in terms of the area of the other? Be specific and explain your reasoning.*

Free Response: ***Hint:*** *If the triangles are similar with a scale factor of r, then the ratio of their areas is r^2.*

Question 13 *During a solar eclipse we see that the apparent diameter of the Sun and Moon are nearly equal. If the Moon is around* $240,000$ *miles from Earth, the Moon's diameter is about* 2000 *miles, and the Sun's diameter is about* $865,000$ *miles how far is the Sun from the Earth?*

Distance to sun $\approx \boxed{(865000/2000)240000}$ *miles.*

Question 14 *When jets fly above* $8,000$ *meters in the air they form a vapor trail. Cruising altitude for a commercial airliner is around* $10,000$ *meters. One day I reached my arm into the sky and measured the length of the vapor trail with my hand—my hand could just span the entire trail. If my hand spans* 9 *inches and my arm extends* 25 *inches from my eye, how long is the vapor trail in* ***kilometers****?*

Length of vapor trail $\approx \boxed{(10/25)9}$ *km.*

Question 15 *David proudly owns a 42 inch (measured diagonally) flat screen TV. Michael proudly owns a 13 inch (measured diagonally) flat screen TV. Dave sits comfortably with his dog Fritz at a distance of 10 feet. How far must Michael stand from his TV to have the "same" viewing experience? Explain your reasoning.*

Standing distance $\approx \boxed{(42/13)10}$ *feet.*

Question 16 *Here is a personal problem: Suppose you are out somewhere and you see that when you stretch out your arm, the width of your thumb is the same apparent size as a distant object. How far away is the object if you know the object is:*

(a) *6' long (as tall as a person).*

(b) *16' long (as long as a car).*

(c) *40' long (as long as a school bus).*

(d) *220' long (as long as a large passenger airplane).*

(e) *340' long (as long as an aircraft carrier).*

Explain your reasoning.

Free Response: ***Hint:***

Question 17 *I was walking down Woody Hayes Drive, standing in front of St. John Arena when a car pulled up and the driver asked, "Where is Ohio Stadium?" At this point I was a bit perplexed, but nevertheless I answered, "Do you see the enormous concrete building on the other side of the street that looks like the Roman Colosseum? That's it."*

The person in the car then asked, "Where are the Twin-Towers then?" Looking up, I realized that the towers were in fact just covered by top of Ohio Stadium. I told the driver to just drive around the stadium until they found two enormous identical towers—that would be them. They thanked me and I suppose they met their destiny.

I am about 2 meters tall, I was standing about 100 meters from the Ohio Stadium and Ohio Stadium is about 40 meters tall. If the Towers are around 500 meters from the rotunda (the front entrance of the stadium), how tall could they be and still be obscured by the stadium? Explain your reasoning—for the record, the towers are about 80 meters tall.

Free Response: ***Hint:***

Question 18 *Explain how to use the notion of similar triangles to multiply numbers with your answer expressed as a segment of the appropriate length.*

Free Response: ***Hint:***

Question 19 *Explain how to use the notion of similar triangles to divide numbers with your answer expressed as a segment of the appropriate length.*

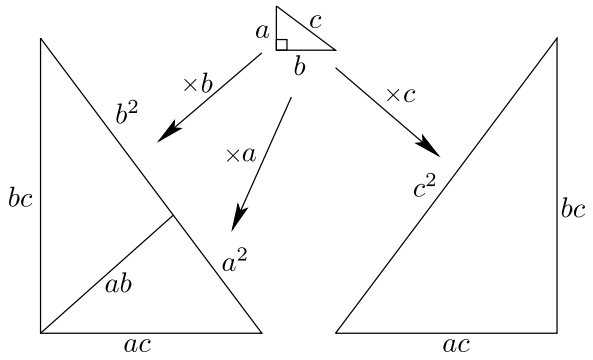
Free Response: ***Hint:***

Question 20 *Consider the following combinations of S's and A's. Which of them produce a Congruence Theorem? Which of them produce a Similarity Theorem? Explain your reasoning.*

SSS, *SSA*, *SAS*, *SAA*, *ASA*, *AAA*

Free Response: ***Hint:***

Question 21 *Explain how the following picture "proves" the Pythagorean Theorem.*



Free Response: ***Hint:***