Scaling in 3D

Short-answer problems about scaling in 3 dimensions.

Length, Area, and Volume Under Scaling

In this section, we explore what happens to length, area, volume, and other measures under scaling.

To explore how measures of figures change under scaling and non-scaling transformations, here are some useful strategies:

- Cutting the figures and rearranging the pieces.
- Using "rep-tiles."
- Using known formulas for perimeters, areas, volumes, or surface areas.
- Approximating with segments, squares, or cubes.
- Shearing.

Question 1 *Fiona's fudge is shipped in boxes that are right rectangular prisms, all similar to 3 cm by 4 cm by 5 cm. Compute the following measures:*

(a) *The volume of the box =* $\boxed{60}$ *cubic centimeters.*

(b) *The volume of a box scaled by* $k =$ $\boxed{60k^3}$ *cubic centimeters.*

(c) *The area of the box =* $\boxed{12+12+15+15+20+20}$ *square centimeters.*

(d) *The area of the box a box scaled by* $k =$ $\boxed{(12+12+15+15+20+20)k^2}$ *square centimeters.*

(e) *The length + girth of the box =* $\boxed{5+3+3+4+4}$ *cm.*

(f) *The length + girth of a box scaled by* $k =$ $\boxed{(5+3+3+4+4)k}$ *cm.*

Hint: *The U.S. postal services measures packages by this formula. For a right rectangular prism, the length is the longest measure; the girth is the distance around the other two measures.*

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Question 2 *Given a cylinder of radius* 5 *and height* 8*, how many times greater will be the volume of . . .*

(a) *. . . a cylinder with k times the radius and the same height?* $\boxed{k^2}$

(b) *. . . a cylinder with the same radius and k times the height?* $\boxed{k}$

(c) *. . . cylinder with k times the radius and k times the height?* $\boxed{k^3}$

Question 3 *Which of the following cylinders is similar to the given a cylinder of radius 5 and height 8?*

Select All Correct Answers:

(a) *A cylinder with k times the radius and the same height?*

(b) *A cylinder with the same radius and k times the height?*

(c) *A cylinder with k times the radius and k times the height?* ✓

(d) *None of the above*

The similar cylinder will have surface area $\boxed{k^2}$ *times the surface area of the original cylinder.*

Problem 4 *Some drugs work best when dosages are proportional to body surface area. Other drugs work best when dosages are proportional to blood volume. A typical adult male (5 ft 10 in, 175 lbs) has a body surface area of about 2 square meters and about 5 liters of blood. Scale these values up to estimate LeBron's body surface area and blood volume. Recall: LeBron is 6 ft 8 in, 250 lbs.)*

LeBron's body surface area = $\boxed{2(8/7)^2}$ *square meters.*

LeBron's blood volume = $\boxed{5(8/7)^3}$ *liters.*

Problem 5 *Consider a version of LeBron that is d times as tall. How would following quantities compare between the scaled version and the real LeBron:*

leather in the sole of a shoe, shoe size, inseam, fabric in a T-shirt, lung capacity, neck circumference, and hat size? Explain briefly.

Cool fact: The size of a hat is the diameter (in inches) of the hat when it is reshaped into a circle. Most adults have hat sizes between $6\frac{3}{4}$ *and* 8.

Hint:
- *Inseam and neck circumference are lengths, so they will be d times as big.*
- *Hat size is proportional to head circumference, which is a length, so it will be d times as big.*
- *Shoe sizes are not really lengths, and men's, women's, and children's shoe sizes are all different. All three vary linearly with foot length but none are proportional to actual length measurements.*
- *Leather in the sole of the shoe and fabric in the T-shirt are both about area because their thicknesses stay constant, so they will be d^2 times as big.*
- *Lung capacity is about volume, so it will be d^3 times as big.*

Problem 6 *A typical adult male gorilla is about 5.5 feet tall and weighs about 400 pounds. Suppose King Kong was about 22 feet tall and proportioned like a typical adult male gorilla.*

(a) *What is the scale factor between from the typical gorilla to King Kong?* $\boxed{22/5.5}$

(b) *Approximates King Kong's weight.* $\boxed{(400)4^3}$ *pounds.*

Briefly explain your reasoning.

Hint: *Weight is proportional to volume.*

(c) *The circumference of the neck of a typical adult male gorilla is 36 inches. Approximately what would be the circumference of King Kong's neck?* $\boxed{(36)4}$ *inches.*

Briefly explain your reasoning.

Hint: *Circumference is a length.*

(d) *Suppose an Ohio State sweatshirt for a typical adult male gorilla requires 3 square yards of fabric. Approximately how much fabric would be required for an Ohio State sweatshirt for King Kong?* $\boxed{(3)4^2}$ *square yards of fabric.*

Briefly explain your reasoning.

Hint: *Fabric is about area.*
