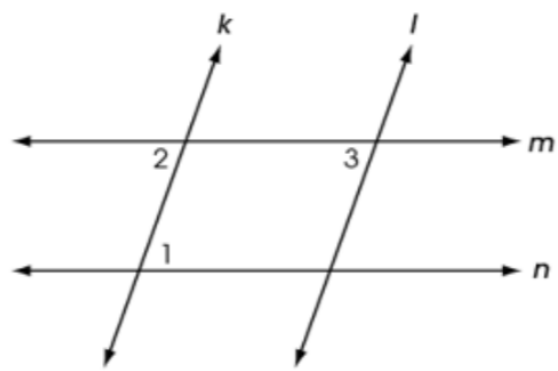
Quadrilaterals

Proof.

Problem 1 *Adapted from Ohio's 2017 Geometry released item 13.*

Two pairs of parallel lines intersect to form a parallelogram as shown.



Complete the following proof that opposite angles of a parallelogram are congruent:

(a) $\angle 1 \cong \angle 2$ *as (opposite angles/ alternate interior angles ✓/ corresponding angles) for parallel lines (*m *and* n ✓/ k *and* l*).*

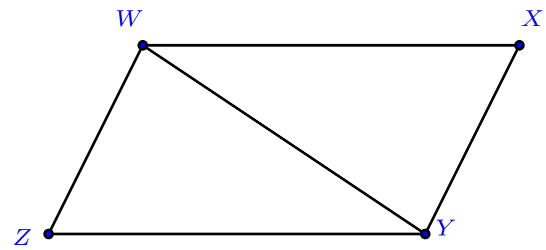
(b) $\angle 3 \cong \angle 2$ *as (opposite angles/ alternate interior angles/ corresponding angles ✓)for parallel lines (*m *and* n/ k *and* l ✓*).*

(c) *Then* $\angle 1 \cong \angle 3$ *because they are both congruent to* $\angle 2$*.*

Problem 2 *Adapted from Ohio's 2018 Geometry released item 21.*

Given the parallelogram $WXYZ$*, prove that* $\overline{WX} \cong \overline{YZ}$*.*

Learning outcomes:
Author(s): Brad Findell



Fix note: It really would help to have an online environment that allows students to mark diagrams.

Complete the proof below:

(a) $\angle ZWY \cong \angle XYW$ *as (alternate interior angles ✓/ corresponding angles/ opposite angles) for parallel segments (*$\overline{WZ}$ and $\overline{XY}$ ✓/ $\overline{WX}$ and $\overline{YZ}$*).*

(b) $\angle ZYW \cong \angle XWY$ *for the same reason, this time for parallel segments (* $\overline{WZ}$ and $\overline{XY}$ / $\overline{WX}$ and $\overline{YZ}$ ✓*).*

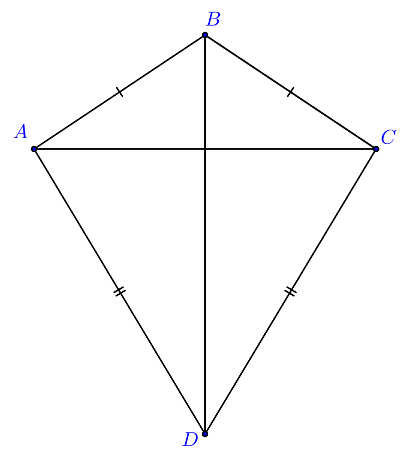
(c) $\overline{WY} \cong \overline{YW}$ *because a segment is congruent to itself.*

(d) $\triangle WYZ \cong \triangle YWX$ *by (SAS/ ASA ✓/ SSS).*

(e) *Then* $\overline{YZ} \cong \overline{WX}$ *as corresponding parts of congruent triangles.*

Fix note: Maybe number the angles.

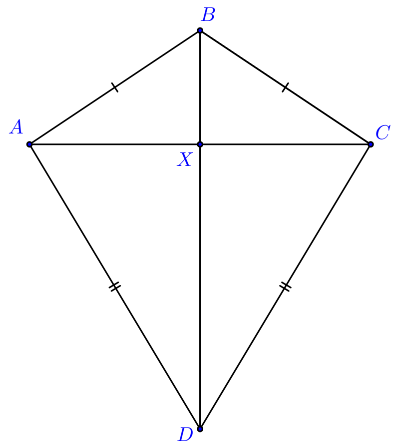
Problem 3 *Quadrilateral* $ABCD$ *is a kite as marked. Prove that* $\overleftrightarrow{BD}$ *is the perpendicular bisector of* $\overline{AC}$*.*



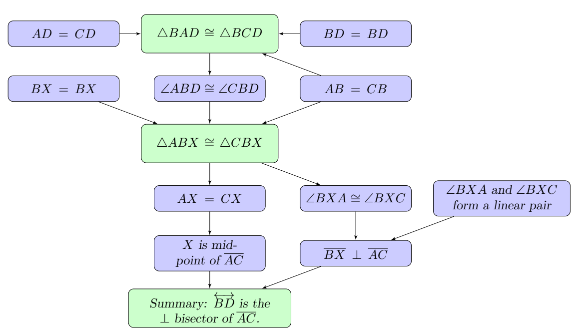
Key theorem: The points on a perpendicular bisector are exactly those that are equidistant from the endpoints of a segment.

Proof: Because B and D are each $\boxed{\textit{equidistant}}$ *from A and C, they each must lie on the perpendicular bisector of segment* $\boxed{AC}$*, which implies that $\overleftrightarrow{BD}$ is its perpendicular bisector.*

Problem 4 *Quadrilateral $ABCD$ is a kite as marked. Prove that $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.*



A proof that makes use of triangle congruence:



Fix note: Do we need a step about $\overleftrightarrow{BX}$ and $\overleftrightarrow{BD}$ being the same line?

In the proof above, $\triangle BAD \cong \triangle BCD$ by $\boxed{SSS}$, and $\triangle ABX \cong \triangle CBX$ by $\boxed{SAS}$.

Detail: Paragraph proof:
$\overline{BD} \cong \overline{BD}$, *so that* $\triangle BAD \cong \triangle BCD$ *by SSS.*
$\angle ABD \cong \angle CBD$ *by CPCTC.*
$\overline{BX} \cong \overline{BX}$, *so that* $\triangle ABX \cong \triangle CBX$ *by SAS.*
$\angle BXA \cong \angle BXC$ *by CPCTC, and they are a linear pair, so* $\overline{BX} \perp \overline{AC}$.
$\overline{AX} \cong \overline{CX}$ *by CPCTC, so* X *is the midpoint of* $\overline{AC}$.
Thus, $\overleftrightarrow{BD}$ *is the perpendicular bisector of* $\overline{AC}$.