Set Theory Problems

Short-answer problems about sets.

Problem 1 *Given two sets X and Y, $X \cup Y$ is the set of elements that are*

Multiple Choice:

(a) *in X or in Y (but not in both).*
(b) *in X or in Y (or both, as the "or" is inclusive).* ✓
(c) *in X and in Y.*
(d) *in X but not in Y.*
(e) *in Y but not in X.*

Problem 2 *Given two sets X and Y, $X \cap Y$ is the set of elements that are*

Multiple Choice:

(a) *in X or in Y (but not in both).*
(b) *in X or in Y (or both, as the "or" is inclusive).*
(c) *in X and in Y.* ✓
(d) *in X but not in Y.*
(e) *in Y but not in X.*

Problem 3 *Given two sets X and Y, $X - Y$ is the set of elements that are*

Multiple Choice:

(a) *in X or in Y (but not in both).*
(b) *in X or in Y (or both, as the "or" is inclusive).*

Learning outcomes:
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(c) *in X and in Y.*

(d) *in X but not in Y.* ✓

(e) *in Y but not in X.*

Problem 4 *Explain the difference between the symbols $\in$ and $\subseteq$.*

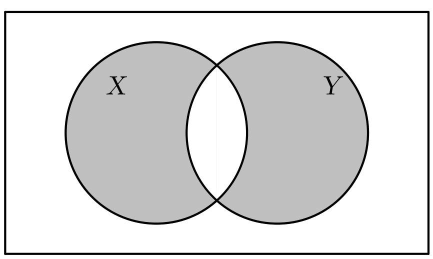
Free Response: ***Hint:*** *The symbol $\in$ means "is an element of," whereas $\subseteq$ means "is a subset of." The notation $X \in Y$ means that X is a single element in the set Y. In this case, X is typically not a set. The notation $X \subseteq Y$, in contrast, requires that both X and Y are sets and, furthermore, that every element of X is also in Y.*

Problem 5 *How is $\{\emptyset\}$ different from $\emptyset$?*

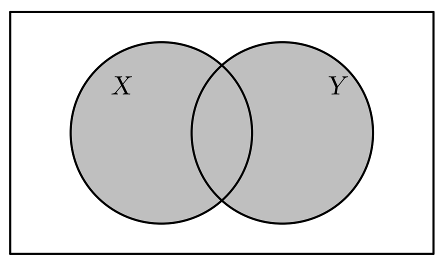
Free Response: ***Hint:*** *The empty set, $\emptyset$, is a set that contains no elements. That is, $\emptyset = \{\}$. The set $\{\emptyset\}$ contains one element that is itself a set—and that element happens to be the empty set. We could instead write $\{\{\}\}$, but that looks ugly.*

Problem 6 *Consider the following Venn diagrams:*

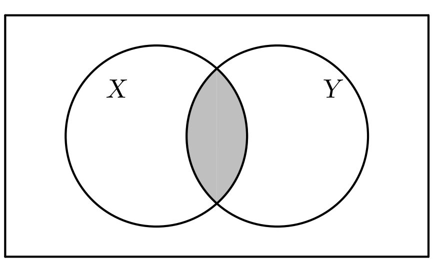
(A)



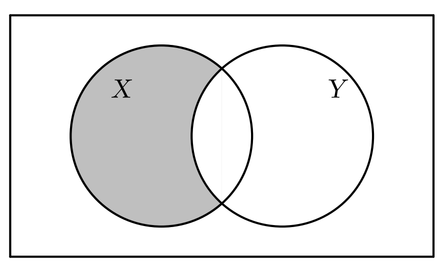
(B)



(C)



(D)



For each set expression below, identify the Venn diagram above in which the corresponding region is shaded:

(a) *$X \cap Y$ is figure* $\boxed{C}$

(b) *$X \cup Y$ is figure* $\boxed{B}$.

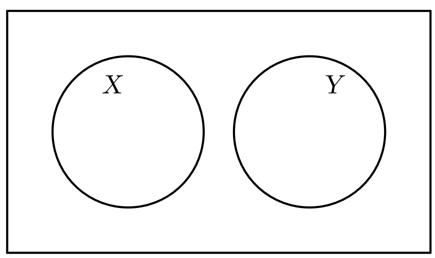
(c) *$X - Y$ is figure* $\boxed{D}$

Problem 7 *Draw a Venn diagram for the set of elements that are in X or Y* but not both. *How does it differ from the Venn diagram for $X \cup Y$?*

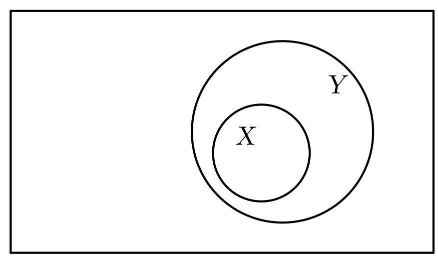
Free Response: ***Hint:*** *A Venn diagram for elements in X or Y* but not both *is shown in figure (A) from the previous problem.*

Problem 8 *Consider the following Venn diagrams:*

(A)



(B)



(a) *If Venn diagram (A) above shows the relationship between sets X and Y, then $X \cap Y = (0 / \emptyset \checkmark / X \cup Y)$ and the sets are said to be (disjoint $\checkmark$/ empty/ subsets).*

(b) *If Venn diagram (B) above shows the relationship between sets X and Y, then we say that (X and Y are disjoint/ $X \subseteq Y$ $\checkmark$/ $Y \subseteq X$).*

(c) *If we let X be the set of "right triangles" and we let Y be the set of "equilateral triangles" which diagram above shows the relationship between these two sets?*

Multiple Choice:

(i) *Diagram (A).* $\checkmark$

(ii) *Diagram (B).*

(iii) *Neither of these.*

(iv) *Not enough information.*

Explain your reasoning.

Free Response: ***Hint:*** *Diagram (A) is accurate because no right triangles are also equilateral triangles.*

Problem 9 *If $X = \{1, 2, 3, 4, 5\}$ and $Y = \{3, 4, 5, 6\}$ find the following: (List elements in ascending order, separated by commas, with no spaces.)*

(a) $X \cup Y = \{\boxed{1, 2, 3, 4, 5, 6}\}$

(b) $X \cap Y = \{\boxed{3, 4, 5}\}$

(c) $X - Y = \{\boxed{1, 2}\}$

(d) $Y - X = \{\boxed{6}\}$

Problem 10 *Let $n\mathbb{Z}$ represent the integer multiples of n. So for example:*

$$3\mathbb{Z} = \{\dots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \dots\}$$

Compute the following (use capital Z for $\mathbb{Z}$):

(a) $3\mathbb{Z} \cap 4\mathbb{Z} = \boxed{12Z}$

(b) $2\mathbb{Z} \cap 5\mathbb{Z} = \boxed{10Z}$

(c) $3\mathbb{Z} \cap 6\mathbb{Z} = \boxed{6Z}$

(d) $4\mathbb{Z} \cap 6\mathbb{Z} = \boxed{12Z}$

(e) $4\mathbb{Z} \cap 10\mathbb{Z} = \boxed{20Z}$

Problem 11 *Make a general rule for intersecting sets of the form $n\mathbb{Z}$ and $m\mathbb{Z}$. Explain why your rule works.*

Free Response: ***Hint:*** *The intersection of two sets is what they have in common. The intersection of the set of multiples of n and the set of multiples of m are called* common multiples *(surprise!), and they are all multiples of the least common multiple of n and m.*

Problem 12 *If $X \cup Y = X$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

($X \subseteq Y$ / $X = Y$ / $Y \subseteq X$ ✓/ $X = \emptyset$) because every element of (X / Y ✓) must be in (X ✓/ Y).

Problem 13 *If $X \cap Y = X$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

($X \subseteq Y$ ✓/ $X = Y$ / $Y \subseteq X$ / $X = \emptyset$) because every element of (X ✓/ Y) must be in (X / Y ✓).

Problem 14 *If $X - Y = \emptyset$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

($X \subseteq Y$ ✓/ $X = Y$ / $Y \subseteq X$ / $X = \emptyset$) because every element of (X ✓/ Y) must be in (X / Y ✓).