Trigonometry Checkup

This activity is intended to remind you of key ideas from high school trigonometry.

Problem 1 *What are the ratios of side lengths in a* $45°$*-*$45°$*-*$90°$ *triangle from shortest to longest, using 1 for the shortest?* $1 : \boxed{1} : \boxed{\sqrt{2}}$. *(Hint: Type* `sqrt(2)` *for* $\sqrt{2}$*.)*

Explain where the ratios come from, including why they work for any such triangle, no matter what size.

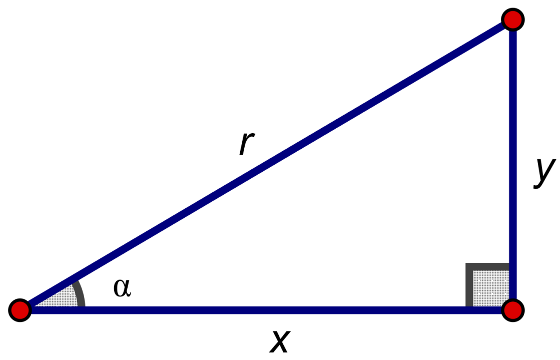
Hint: *Think of half of a square. Then use the Pythagorean Theorem.*

Problem 2 *What are the ratios of side lengths in a* $30°$*-*$60°$*-*$90°$ *triangle, from shortest to longest, using 1 for the shortest?* $1 : \boxed{\sqrt{3}} : \boxed{2}$.

Explain where the ratios come from.

Hint: *Think of half of an equilateral triangle. Then use the Pythagorean Theorem.*

Problem 3 *Consider the right triangle below with an angle of* α*, sides of length* x *and* y*, and hypotenuse of length* r*, as labeled.*



Learning outcomes:
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(a) *Using the triangle above (and your memory of Precalculus), write down the side-length ratios for sine, cosine, and tangent:*

$$\sin\alpha = \boxed{y/r}, \quad \cos\alpha = \boxed{x/r}, \quad \tan\alpha = \boxed{y/x}$$

(b) *What values of α, measured in degrees, make sense in* right triangle trigonometry*? (We overcome these bounds for circular trigonometry.)*

$$\boxed{0} \leqslant \alpha < \boxed{90}$$

Problem 4 *For the problem and triangle above, ...*

(a) *If we imagine angle α is fixed, why are ratios of pairs of side lengths the same, no matter the size of the triangle?*

Hint: *Because the triangles are are similar by AA.*

(b) *What does it mean to say that these ratios depend upon the angle α?*

Hint: *The value of α determines each ratio. (And a different α gives different ratios.)*

(c) *Why is only one of the triangle's three angles necessary in defining these ratios?*

Hint: *The triangle is right, so two angles are directly known. Then the third angle can be determined because the sum of the angles is 180°.*

Free Response:

Problem 5 *Use your work so far to find the following trigonometric ratios:*

(a) $\sin 30^\circ = \boxed{1/2}, \quad \cos 30^\circ = \boxed{\sqrt{3}/2}, \quad \tan 30^\circ = \boxed{1/\sqrt{3}}.$

(b) $\sin 45^\circ = \boxed{1/\sqrt{2}}, \quad \cos 45^\circ = \boxed{1/\sqrt{2}}, \quad \tan 45^\circ = \boxed{1}.$

(c) $\sin 60^\circ = \boxed{\sqrt{3}/2}, \quad \cos 60^\circ = \boxed{1/2}, \quad \tan 60^\circ = \boxed{\sqrt{3}}.$

(d) $\sin 0^\circ = \boxed{0}, \quad \cos 0^\circ = \boxed{1}, \quad \tan 0^\circ = \boxed{0}.$

Problem 6 *You may recall the identity* $\sin^2\vartheta + \cos^2\vartheta = 1$.

(a) *Explain why the equation is true.*

Hint: *For the triangle above,* $x^2 + y^2 = r^2$. *Divide both sides by* r^2*, and the equation becomes* $\frac{x^2}{r^2} + \frac{x^2}{r^2} = 1$. *From the definitions of sine and cosine and the pictured triangle, it follows that* $\sin^2\vartheta + \cos^2\vartheta = 1$.

(b) *Why is it called an identity?*

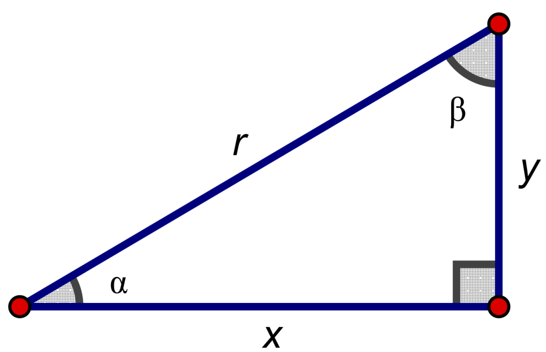
Hint: *Because the equation is true for all values of the variables.*

(c) *Why is it called a Pythagorean identity?*

Hint: *Because the proof is essentially a restatement of the Pythagorean Theorem for the pictured triangle.*

Free Response:

Problem 7 *In right triangle trigonometry, there are two acute angles, as shown in the figure below.*



(a) *How are the angles* α *and* β *related? Explain why.*

Hint: *They are complementary because* $\alpha + \beta + 90 = 180$.

(b) *Using lengths in the above triangle, find the following ratios:*

$$\sin\alpha = \boxed{y/r} \qquad \cos\alpha = \boxed{x/r}$$

$$\sin\beta = \boxed{x/r} \qquad \cos\beta = \boxed{y/r}$$

(c) *We have shown that when angles α and β are complementary,*

$\sin\alpha = \boxed{\cos\beta}$. *Enter* $\cos\alpha$, $\sin\beta$, *or* $\cos\beta$. *Type alpha for* α, *beta for* β.

$\cos\alpha = \boxed{\sin\beta}$. *Enter* $\sin\alpha$, $\sin\beta$, *or* $\cos\beta$.

(d) *Explain why the result makes sense.*

Hint: *We are writing the ratios from the perspective of the other angle, which reverses the roles of x and y.*

Given an angle and a side length of a right triangle, you can find the missing side lengths. This is called "solving the right triangle." And given the sine, cosine, or tangent of an angle, you can find the other two ratios.

Problem 8 *Suppose* $\sin\alpha = \frac{3}{5}$. *Then* $\cos\alpha = \boxed{4/5}$, $\tan\alpha = \boxed{3/4}$.

Hint: *Draw a right triangle with* $\sin\alpha = \frac{3}{5}$, *use the Pythagorean Theorem to find the missing side, and write down the other trigonometric ratios.*

Problem 9 *Ethan stands 120 feet from the trunk of a tree (along flat ground). He measures that his line of sight to the top of the tree is at an angle of 53° from horizontal. How tall is the tree?* $\boxed{120\tan(53\pi/180)}$

Explain your reasoning.

Hint: *Draw a picture with y representing the height of the tree. From the picture,* $\tan 53^\circ = \frac{y}{120}$. *Solve for y.*
