Math 4480 Online HW

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Contents

Measuring by Sight

Short-answer questions involving measuring.

Careful Measurement by Sight

Adjust the figures to fit the given conditions within **eyeball accuracy**. Enter the requested measurements.

Problem 1 *Geogebra link:* `https://tube.geogebra.org/m/gjf28er6`

In figure above, when point C is adjusted so that $\overline{BC}$ is perpendicular to $\overline{AC}$, $AC = \boxed{2.09}$.

Hint: *When two lines are* perpendicular*, they cross to create four congruent angles.*

Hint: *Use the corner of a piece of paper.*

Problem 2 *Geogebra link:* `https://tube.geogebra.org/m/q32gyaud`

In $\triangle ABC$ above, move point D to make the following measurements. ***Enter -1 if it is not possible.***

(a) *When $\overline{BD}$ is a median, $AD = \boxed{2.25}$.*

Hint: *A median is drawn from a vertex to the midpoint of the opposite side.*

(b) *When $\overline{BD}$ is a angle bisector, $AD = \boxed{2.78}$.*

Hint: *An angle bisector cuts an angle in half. Focus near the vertex of the angle rather than near D.*

(c) *When $\overline{BD}$ is a perpendicular bisector, $AD = \boxed{-1}$.*

Hint: *An perpendicular bisector cuts an segment in half and is perpendicular to it.* ***Enter -1 if it is not possible.***

(d) *When $\overline{BD}$ is a altitude, $AD = \boxed{6.46}$.*

Author(s): Brad Findell

Hint: *An altitude contains a vertex and is perpendicular to the line containing the opposite side.* ***Enter -1 if it is not possible.***

Problem 3 *Geogebra link:* https://tube.geogebra.org/m/a888zyw2

In $\triangle ABC$ *above, the height to base* $\overline{AC}$ *is* $\boxed{3.585}$.

Hint: *You may move point* D*. A height is the length of an altitude, which must be perpendicular to the line containing the chosen base.*

Problem 4 *Geogebra link:* https://tube.geogebra.org/m/kta9hbuf

In $\triangle ABC$ *above, the height to base* $\overline{AC}$ *is* $\boxed{3.511}$.

Hint: *You may move point* D*. A height is the length of an altitude, which must be perpendicular to the line containing the chosen base.*

Measuring Interior Angles

Short-answer questions involving angles in triangles.

Geogebra link: `https://tube.geogebra.org/m/zrapvzpz`

Problem 5 *Measure the interior angles of quadrilateral $ABCD$ above.*

(a) $m\angle A = \boxed{31}$ *degrees.*

(b) $m\angle B = \boxed{26.74}$ *degrees.*

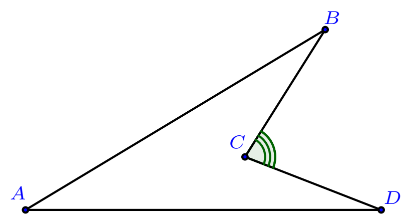
(c) $m\angle C = \boxed{281}$ *degrees.*

(d) $m\angle D = \boxed{21.25}$ *degrees.*

(e) $m\angle A + m\angle B + m\angle C + m\angle D = \boxed{360}$ *degrees.*

Hint: *Be sure to measure interior angle as an amount of turning between the two sides of the angle.*

Problem 6 *Use the measurements from the previous problem to answer the following questions:*



(a) *The marked angle should measure* $\boxed{79}$ *degrees.*

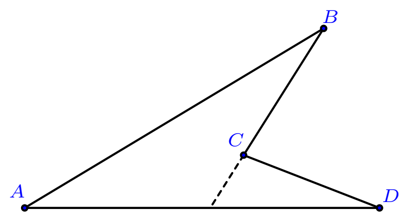
(b) $m\angle A + m\angle B + m\angle D = \boxed{79}$ *degrees.*

(c) *What do you notice?*

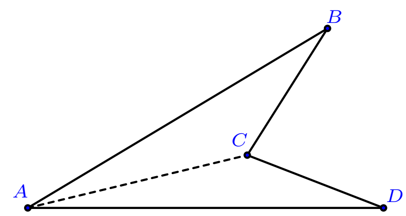
Free Response: ***Hint:*** *They should be the same because, in both cases, adding the interior angle at C should give 360°.*

Author(s): Brad Findell

Problem 7 *In order to reason about the sum of the interior angles, Bart and Brad each* triangulated *the figure as shown below.*



Brad's triangulation



Bart's triangulation

Both Bart and Brad claim that because in a triangle the sum of the interior angles is $\boxed{180}$ *degrees, and this quadrilateral is cut into* $\boxed{2}$ *triangles, the angle sum in this quadrilateral should be* $\boxed{360}$ *degrees. What is your judgment?*

Multiple Choice:

(a) *They are both correct.*

(b) *Only Brad is correct.*

(c) *Only Bart is correct.* ✓

(d) *Neither of them are correct.*

Explain your reasoning.

Free Response: ***Hint:*** *In Bart's triangulation, the interior angles of the quadrilateral are composed only of interior angles of the triangles. But in Brad's triangulation, a new angle has been created with a vertex between* A *and* D*, and part of interior angle* C *has been lost.*

Inscribed Angles

Inscribed angles exploration.

Definition 1. *In a circle, a* ***central angle*** *has the center of the circle as its vertex. An* ***inscribed angle*** *has a point on the circle as its vertex. An* ***arc*** *of a circle has both a measure and a length.* ***Arc measure*** *indicates an amount of turning (in degrees). An* ***arc length*** *is a distance.*

Geogebra link: `https://tube.geogebra.org/m/kcq9bpbd`

Problem 8 (a) *Keeping points A and C fixed, when point B moves, $m\angle ABC$ (increases/ stays the same ✓/ decreases/ varies widely).*

(b) *The arc measure is (equal to ✓/ half/ double/ unrelated to) the measure of the corresponding central angle.*

(c) *The measure of an inscribed angle is (equal to/ half ✓/ double/ unrelated to) the measure of the corresponding central angle.*

(d) *The measure of an inscribed angle is (equal to/ half ✓/ double/ unrelated to) the measure of the corresponding arc.*

Maybe some questions about visually estimating angle measures or arc measures.

Author(s): Brad Findell

Vocabulary Review

Short-answer, multiple-choice, and select-all questions about key vocabulary.

Question 9 *An **equilateral quadrilateral** is called a* $\boxed{\textit{rhombus}}$.

Question 10 *An **equiangular quadrilateral** is called a* $\boxed{\textit{rectangle}}$.

Question 11 *An **regular quadrilateral** is called a* $\boxed{\textit{square}}$.

Question 12 *A* $\boxed{\textit{straightangle}}$ *measures* $180°$. *(Hint: Answer with two words.)*

Question 13 *Two angles whose measures sum to* $180°$ *are said to be* $\boxed{\textit{supplementary}}$.

Question 14 *Two angles whose measures sum to* $90°$ *are said to be* $\boxed{\textit{complementary}}$.

Question 15 *Three (or more) points that lie on the same line are said to be* $\boxed{\textit{collinear}}$.

Question 16 *Three (or more) lines that lie on the same point are said to be* $\boxed{\textit{concurrent}}$.

Question 17 *An **altitude** in a triangle ...*

Author(s): Bart Snapp and Brad Findell

Multiple Choice:

(a) *contains the midpoint of the side of a triangle and is perpendicular to that side.*

(b) *contains a vertex of a triangle and is perpendicular to the line containing the other side.* ✓

(c) *contains a vertex of a triangle and the midpoint of the opposite side.*

(d) *contains a vertex and bisects that angle.*

(e) *none of these.*

Question 18 *A **median** in a triangle . . .*

Multiple Choice:

(a) *contains the midpoint of the side of a triangle and is perpendicular to that side.*

(b) *contains a vertex of a triangle and is perpendicular to the line containing the other side.*

(c) *contains a vertex of a triangle and the midpoint of the opposite side.* ✓

(d) *contains a vertex and bisects that angle.*

(e) *none of these.*

Question 19 *The **circumcenter** of a triangle is . . . [select all]*

Select All Correct Answers:

(a) *the point of concurrency of the medians.*

(b) *the point of concurrency of the angle bisectors.*

(c) *the point of concurrency of the perpendicular bisectors.* ✓

(d) *the point of concurrency of the altitudes.*

(e) *the balance point for the triangle.*

(f) *the center in the triangle.*

(g) *the center of the incircle.*

(h) *the center of the circumcircle.* ✓

(i) *equidistant from the sides of the triangle.*

(j) *equidistant from the vertices of the triangle.* ✓

Question 20 *The* ***incenter*** *of a triangle is ... [select all]*

Select All Correct Answers:

(a) *the point of concurrency of the medians.*

(b) *the point of concurrency of the angle bisectors.* ✓

(c) *the point of concurrency of the perpendicular bisectors.*

(d) *the point of concurrency of the altitudes.*

(e) *the balance point for the triangle.*

(f) *the center in the triangle.*

(g) *the center of the incircle.* ✓

(h) *the center of the circumcircle.*

(i) *equidistant from the sides of the triangle.* ✓

(j) *equidistant from the vertices of the triangle.*

Question 21 *The* ***centroid*** *of a triangle is ... [select all]*

Select All Correct Answers:

(a) *the point of concurrency of the medians.* ✓

(b) *the point of concurrency of the angle bisectors.*

(c) *the point of concurrency of the perpendicular bisectors.*

(d) *the point of concurrency of the altitudes.*

(e) *the balance point for the triangle.* ✓

(f) *the center in the triangle.*

(g) *the center of the incircle.*

(h) *the center of the circumcircle.*

(i) *equidistant from the sides of the triangle.*

(j) *equidistant from the vertices of the triangle.*

Question 22 *The* ***orthocenter*** *of a triangle is . . . [select all]*

Select All Correct Answers:

(a) *the point of concurrency of the medians.*

(b) *the point of concurrency of the angle bisectors.*

(c) *the point of concurrency of the perpendicular bisectors.*

(d) *the point of concurrency of the altitudes.* ✓

(e) *the balance point for the triangle.*

(f) *the center in the triangle.*

(g) *the center of the incircle.*

(h) *the center of the circumcircle.*

(i) *equidistant from the sides of the triangle.*

(j) *equidistant from the vertices of the triangle.*

Question 23 *A* ***midsegment*** *in a triangle is . . . [select all]*

Select All Correct Answers:

(a) *a segment in the middle.*

(b) *a segment connecting the midpoints of two sides.* ✓

(c) *parallel to a side of the triangle.* ✓

(d) *perpendicular to a side of the triangle.*

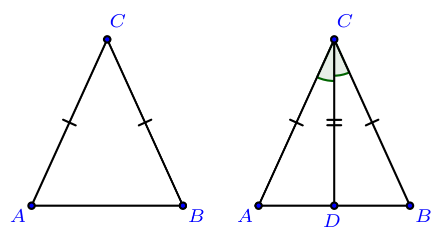
(e) *also called a median.*

Isosceles Triangle Theorem

Proofs updated.

Fix note: Below are several different proofs, along with one that is not a proof. Please consider them separately.
Any (or all) of the proofs might be extended to conclude that, in the case of an isosceles triangle, the perpendicular bisector, angle bisector, median, and altitude all lie on the same line.

Problem 24 *Prove that the base angles of an isosceles triangle are congruent.*

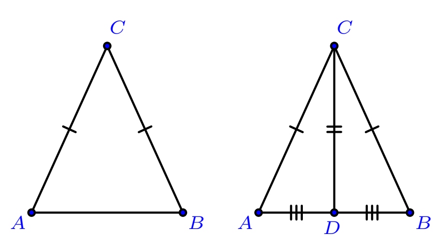


(a) *Beginning with the given figure on the left, Morgan draws $\overline{CD}$ and marks the figure intending that this new segment is a(n) (median/ angle bisector $\checkmark$/ perpendicular bisector/ altitude).*

(b) *Based on the marked figure, Morgan claims that the $\triangle ACD \cong \triangle\boxed{BCD}$ by (SAS $\checkmark$/ SSS/ SSA/ ASA/ HL).*

(c) *Finally, Morgan concludes that $\angle A \cong \angle\boxed{B}$, as they are corresponding parts of congruent triangles.*

Problem 25 *Prove that the base angles of an isosceles triangle are congruent.*



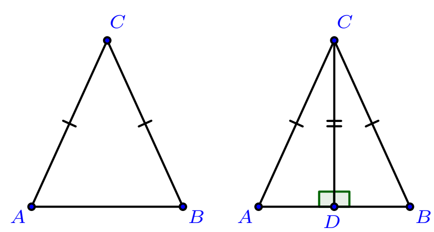
Author(s): Brad Findell

(a) *Beginning with the given figure on the left, Deja draws $\overline{CD}$ and marks the figure intending that this new segment is a(n) (median ✓/ angle bisector / perpendicular bisector/ altitude).*

(b) *Based on the marked figure, Deja claims that the $\triangle ACD \cong \triangle\boxed{BCD}$ by (SAS/ SSS ✓/ SSA/ ASA/ HL).*

(c) *Finally, Deja concludes that $\angle A \cong \angle\boxed{B}$, as they are corresponding parts of congruent triangles.*

Problem 26 *Prove that the base angles of an isosceles triangle are congruent.*

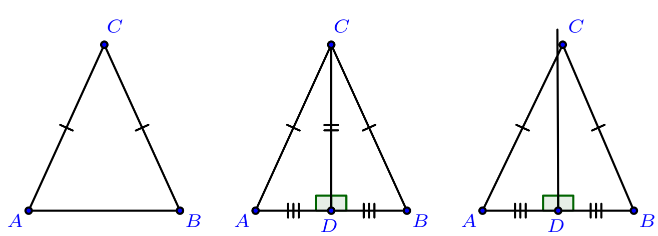


(a) *Beginning with the given figure on the left, Elle draws $\overline{CD}$ and marks the figure intending that this new segment is a(n) (median/ angle bisector/ perpendicular bisector/ altitude ✓).*

(b) *Based on the marked figure, Elle claims that the $\triangle ACD \cong \triangle\boxed{BCD}$ by (SAS/ SSS/ SSA/ ASA/ HL ✓).*

(c) *Finally, Elle concludes that $\angle A \cong \angle\boxed{B}$, as they are corresponding parts of congruent triangles.*

Problem 27 *Simon and Taylor are trying to prove that the base angles of an isosceles triangle are congruent.*



Given figure *Simon's figure* *Taylor's figure*

Beginning with the given figure on the left, Simon draws $\overline{CD}$ and marks the second figure intending that this new segment is a perpendicular bisector of $\overline{AB}$.

Taylor claims that a perpendicular bisector of a side of a triangle usually misses the opposite vertex, so the figure should allow for that possibility.

Fix note: Taylor's claim, the prompt, and the choices below need attention. Simon's figure suggests congruence by "SSAS," which might indicate that too much is being assumed. Would that make sense as a distractor?

Without using other facts about isosceles triangles or perpendicular bisectors, choose the best assessment of their disagreement:

Multiple Choice:

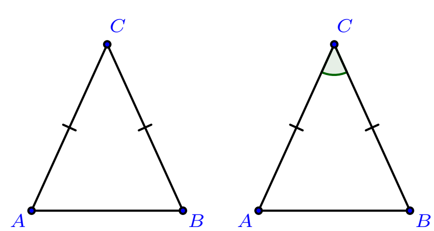
(a) *Simon is correct, and $\triangle ACD \cong \triangle BCD$ by SAS.*

(b) *Simon is correct, and $\triangle ACD \cong \triangle BCD$ by SSS*

(c) *Taylor is correct, and the perpendicular bisector should not be used to complete this proof.* ✓

(d) *Neither of them are correct.*

Problem 28 *Prove that the base angles of an isosceles triangle are congruent.*



Given figure

(a) *Examining the given figure on the left, Lissy notices symmetry in the triangle and claims that the triangle is congruent to itself by a (translation / reflection ✓/ rotation).*

(b) *Based on the marked figure, Lissy claims that the $\triangle ACB \cong \triangle\boxed{BCA}$ by (SAS ✓/ SSS/ SSA/ ASA/ HL).*

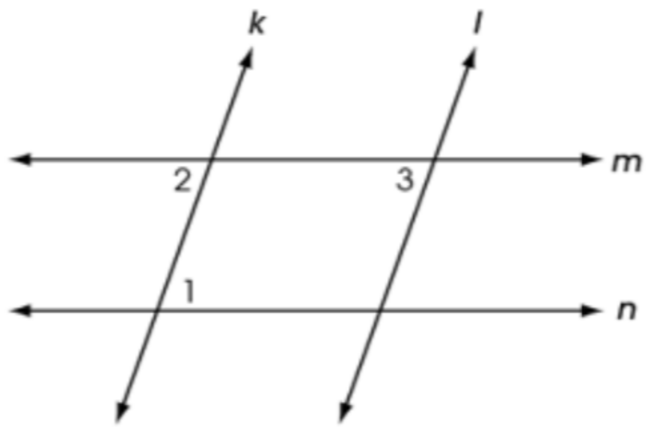
(c) *Finally, Lissy concludes that $\angle A \cong \angle\boxed{B}$, as they are corresponding parts of congruent triangles.*

Quadrilaterals

Proof.

Problem 29 *Adapted from Ohio's 2017 Geometry released item 13.*

Two pairs of parallel lines intersect to form a parallelogram as shown.



Complete the following proof that opposite angles of a parallelogram are congruent:

(a) $\angle 1 \cong \angle 2$ *as (opposite angles/ alternate interior angles ✓/ corresponding angles) for parallel lines (*m *and* n ✓/ k *and* l*).*

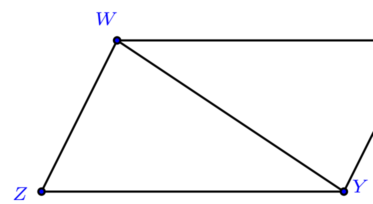
(b) $\angle 3 \cong \angle 2$ *as (opposite angles/ alternate interior angles/ corresponding angles ✓)for parallel lines (*m *and* n/ k *and* l ✓*).*

(c) *Then* $\angle 1 \cong \angle 3$ *because they are both congruent to* $\angle 2$.

Problem 30 *Adapted from Ohio's 2018 Geometry released item 21.*

Given the parallelogram $WXYZ$*, prove that* $\overline{WX} \cong \overline{YZ}$.

Author(s): Brad Findell



Fix note: It really would help to have an online environment that allows students to mark diagrams.

Complete the proof below:

(a) $\angle ZWY \cong \angle XYW$ as (alternate interior angles ✓/ corresponding angles/ opposite angles) for parallel segments ($\overline{WZ}$ and $\overline{XY}$ ✓/ $\overline{WX}$ and $\overline{YZ}$).

(b) $\angle ZYW \cong \angle XWY$ for the same reason, this time for parallel segments ($\overline{WZ}$ and $\overline{XY}$ / $\overline{WX}$ and $\overline{YZ}$ ✓).

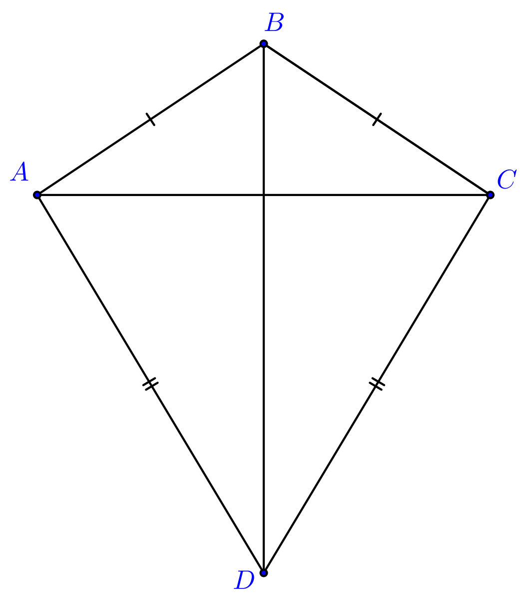
(c) $\overline{WY} \cong \overline{YW}$ because a segment is congruent to itself.

(d) $\triangle WYZ \cong \triangle YWX$ by (SAS/ ASA ✓/ SSS).

(e) Then $\overline{YZ} \cong \overline{WX}$ as corresponding parts of congruent triangles.

Fix note: Maybe number the angles.

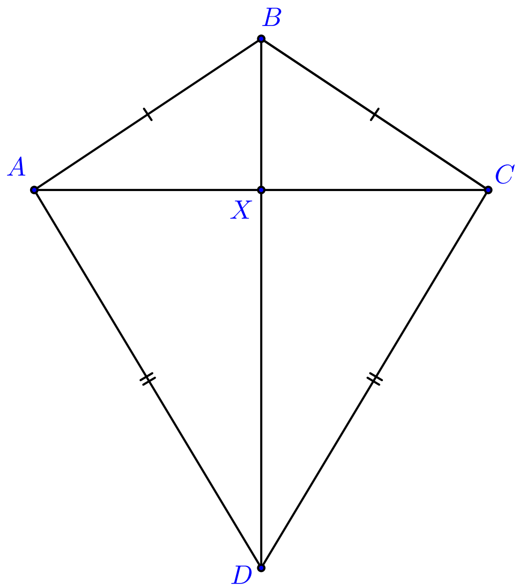
Problem 31 *Quadrilateral $ABCD$ is a kite as marked. Prove that $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.*



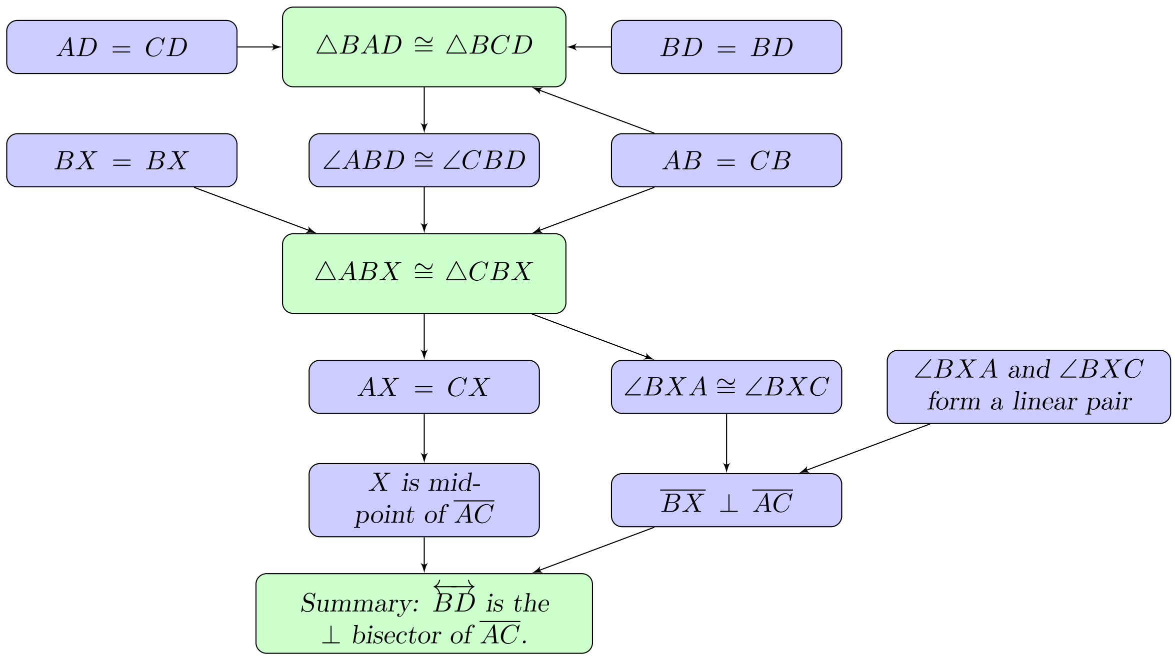
Key theorem: The points on a perpendicular bisector are exactly those that are equidistant from the endpoints of a segment.

Proof: Because B and D are each $\boxed{\textit{equidistant}}$ *from A and C, they each must lie on the perpendicular bisector of segment* $\boxed{AC}$*, which implies that $\overleftrightarrow{BD}$ is its perpendicular bisector.*

Problem 32 *Quadrilateral $ABCD$ is a kite as marked. Prove that $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.*



A proof that makes use of triangle congruence:



Fix note: Do we need a step about $\overleftrightarrow{BX}$ and $\overleftrightarrow{BD}$ being the same line?

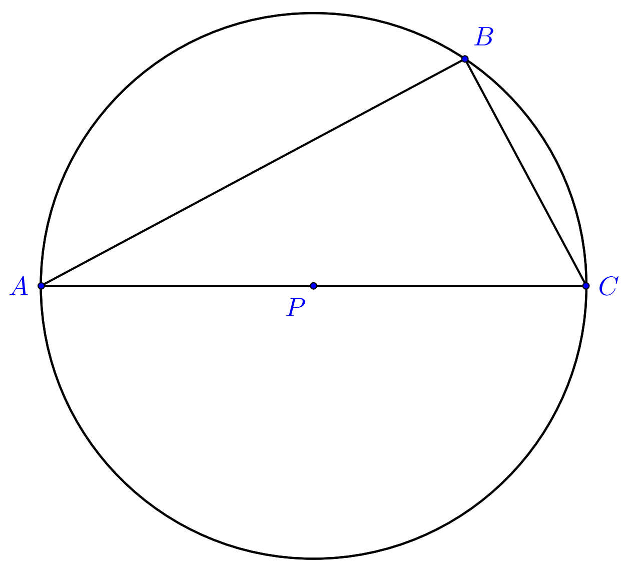
In the proof above, $\triangle BAD \cong \triangle BCD$ by $\boxed{SSS}$, and $\triangle ABX \cong \triangle CBX$ by $\boxed{SAS}$.

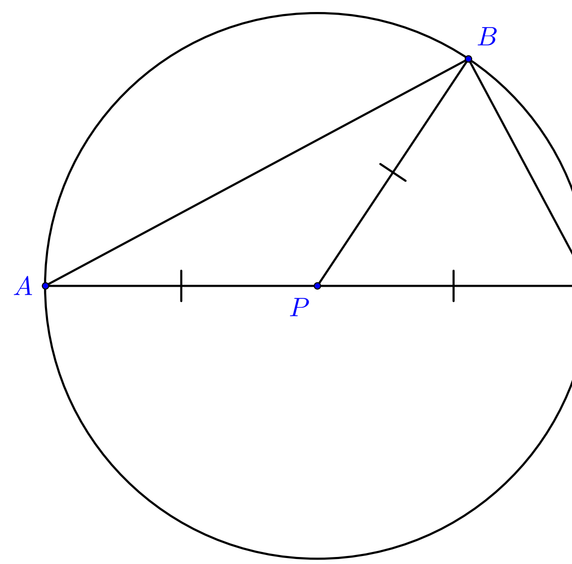
Detail: Paragraph proof:
$\overline{BD} \cong \overline{BD}$, *so that* $\triangle BAD \cong \triangle BCD$ *by SSS.*
$\angle ABD \cong \angle CBD$ *by CPCTC.*
$\overline{BX} \cong \overline{BX}$, *so that* $\triangle ABX \cong \triangle CBX$ *by SAS.*
$\angle BXA \cong \angle BXC$ *by CPCTC, and they are a linear pair, so* $\overline{BX} \perp \overline{AC}$.
$\overline{AX} \cong \overline{CX}$ *by CPCTC, so* X *is the midpoint of* $\overline{AC}$.
Thus, $\overleftrightarrow{BD}$ *is the perpendicular bisector of* $\overline{AC}$.

Inscribed Angles

Proofs updated.

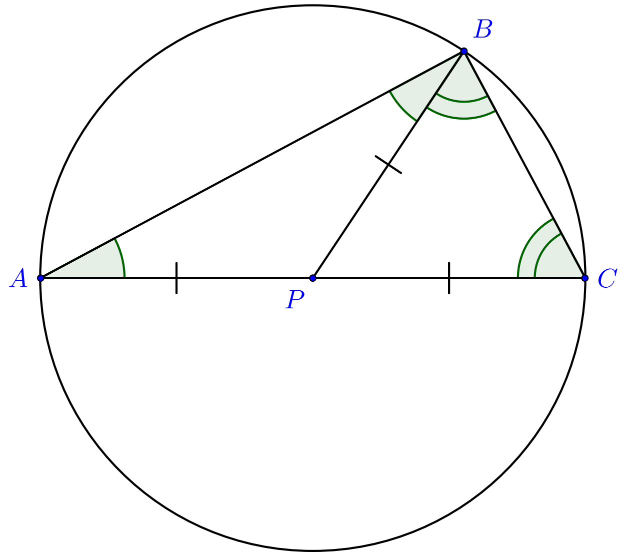
Problem 33 *In the figure below, $\overline{AC}$ is a diameter of a circle with center P. Prove that $\angle ABC$ is a right angle.*

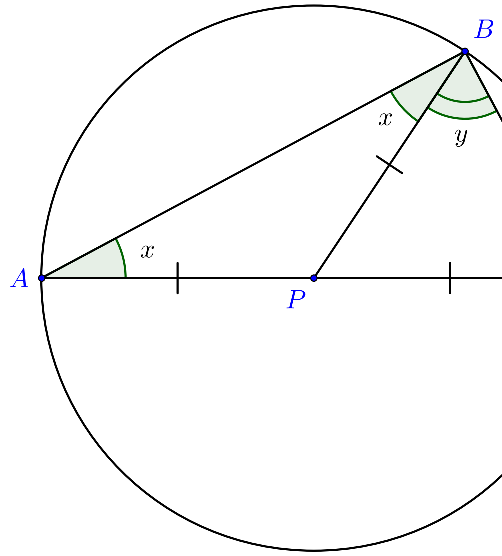




(a) *Beginning with the diagram on the left, Natalia draws $\overline{PB}$ and marks the diagram to show segments that she knows to be congruent because each one is a* $\boxed{\textit{radius}}$ *of the circle.*

Author(s): Brad Findell





(b) *Natalia sees that* $\triangle APB$ *and* $\triangle BPC$ *are* $\boxed{\textit{isosceles}}$ *triangles, so she marks the figure to show angles that must be congruent.*

(c) *In order to do some algebra with these congruent angles, Natalia labels their measures* x *and* y*, as shown in the picture on the right.*

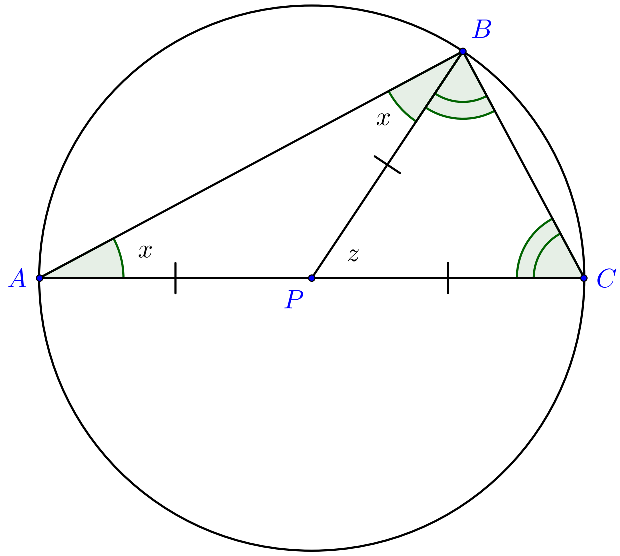
(d) *She writes an equation for the sum of the angles of* $\triangle ABC$*:*

$$\boxed{x + (x + y) + y} = 180^\circ$$

(e) *She divides that equation by* $\boxed{2}$ *to conclude that* $m\angle ABC = x + y = \boxed{90}$ *degrees.*

Problem 34 *A special case of the relationship between an inscribed angle and the corresponding central angle.*

In the figure below, $\overline{AC}$ *is a diameter of a circle with center* P*. Prove that* $z = 2x$*.*



Because z *is the measure of an angle exterior to* $\triangle\boxed{APB}$, *it is equal to the sum of the measures of the (opposite/ adjacent/ remote interior ✓/ alternate interior) angles. In other words* $z = 2x$.

Alternatively, without using the exterior angle theorem, one might proceed as follows:

(a) $\angle APB + x + x = 180°$ *because of the angle sum in* $\triangle\boxed{ABP}$.

(b) $\angle APB + z = 180°$ *because they form a linear pair.*

(c) *Then* $z = 2x$ *by comparing the two equations.*

Fix note: This handles the special case in which one side of the inscribed angle is a diameter. For the general result, consider two cases: (1) When the center of the circle is in the interior of the inscribed angle; and (2) When the center of the circle is not in the interior of the inscribed angle.