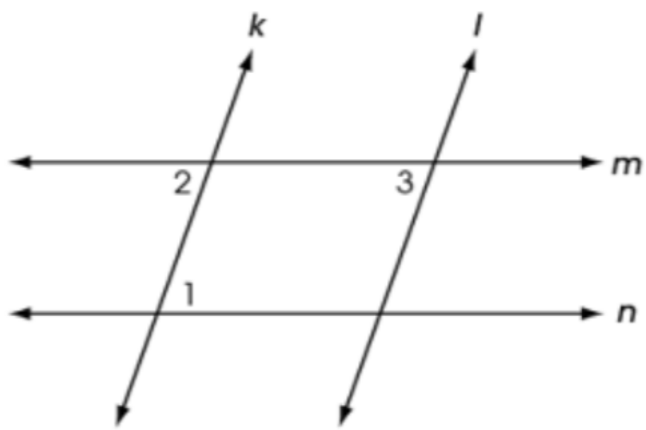
Quadrilaterals

Proof.

Problem 1 *Adapted from Ohio's 2017 Geometry released item 13.*
Two pairs of parallel lines intersect to form a parallelogram as shown.



Complete the following proof that opposite angles of a parallelogram are congruent:

(a) $\angle 1 \cong \angle 2$ as *(opposite angles/ alternate interior angles* ✓*/ corresponding angles) for parallel lines* (m *and* n ✓*/* k *and* l *).*

(b) $\angle 3 \cong \angle 2$ as *(opposite angles/ alternate interior angles/ corresponding angles* ✓*)for parallel lines* (m *and* n */* k *and* l ✓*).*

(c) *Then* $\angle 1 \cong \angle 3$ *because they are both congruent to* $\angle 2$*.*

Problem 2 *Adapted from Ohio's 2018 Geometry released item 21.*
Given the parallelogram $WXYZ$*, prove that* $\overline{WX} \cong \overline{YZ}$*.*

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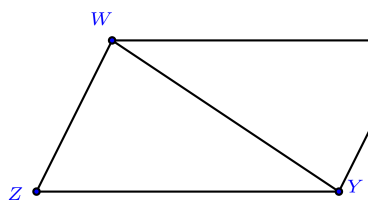


Fix note: It really would help to have an online environment that allows students to mark diagrams.

Complete the proof below:

(a) $\angle ZWY \cong \angle XYW$ as (alternate interior angles ✓/ corresponding angles/ opposite angles) for parallel segments ($\overline{WZ}$ and $\overline{XY}$ ✓/ $\overline{WX}$ and $\overline{YZ}$).

(b) $\angle ZYW \cong \angle XWY$ for the same reason, this time for parallel segments ($\overline{WZ}$ and $\overline{XY}$ / $\overline{WX}$ and $\overline{YZ}$ ✓).

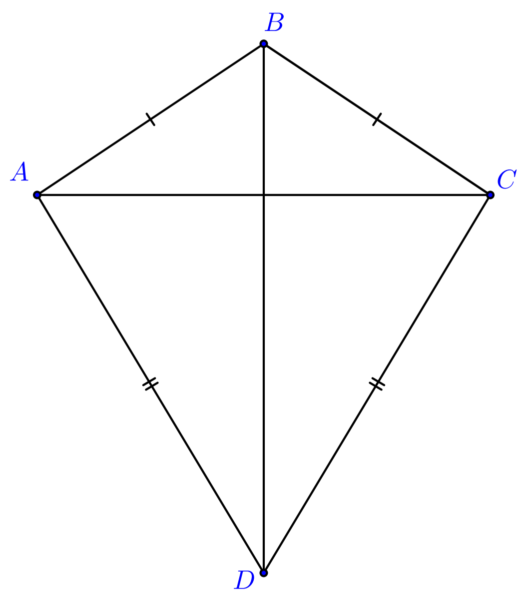
(c) $\overline{WY} \cong \overline{YW}$ because a segment is congruent to itself.

(d) $\triangle WYZ \cong \triangle YWX$ by (SAS/ ASA ✓/ SSS).

(e) Then $\overline{YZ} \cong \overline{WX}$ as corresponding parts of congruent triangles.

Fix note: Maybe number the angles.

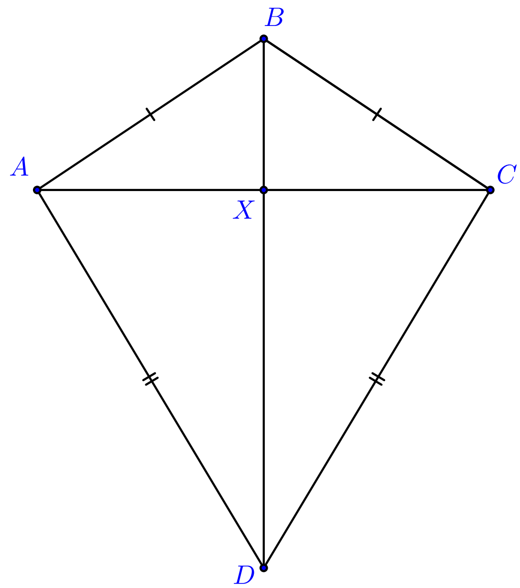
Problem 3 *Quadrilateral $ABCD$ is a kite as marked. Prove that $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.*



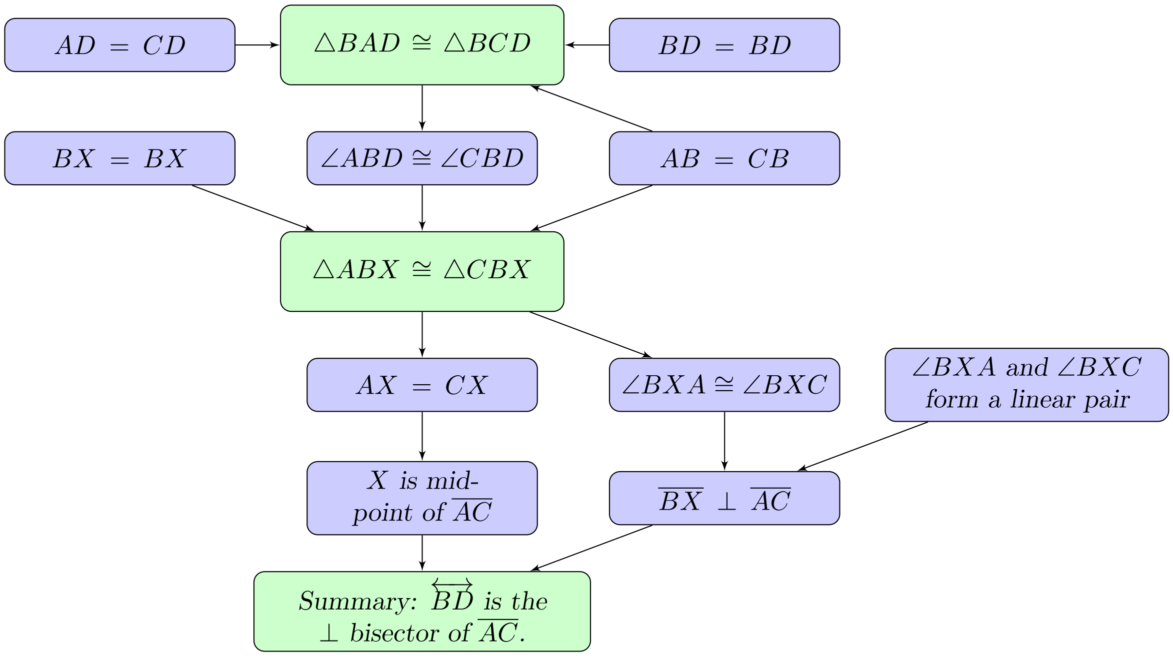
Key theorem: The points on a perpendicular bisector are exactly those that are equidistant from the endpoints of a segment.

Proof: Because B and D are each $\boxed{\textit{equidistant}}$ *from A and C, they each must lie on the perpendicular bisector of segment* $\boxed{AC}$*, which implies that $\overleftrightarrow{BD}$ is its perpendicular bisector.*

Problem 4 *Quadrilateral $ABCD$ is a kite as marked. Prove that $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.*



A proof that makes use of triangle congruence:



Fix note: Do we need a step about $\overleftrightarrow{BX}$ and $\overleftrightarrow{BD}$ being the same line?

In the proof above, $\triangle BAD \cong \triangle BCD$ by $\boxed{SSS}$, and $\triangle ABX \cong \triangle CBX$ by $\boxed{SAS}$.

Feedback(*`correct`*): *Paragraph proof:*
$\overline{BD} \cong \overline{BD}$, *so that* $\triangle BAD \cong \triangle BCD$ *by SSS.*
$\angle ABD \cong \angle CBD$ *by CPCTC.*
$\overline{BX} \cong \overline{BX}$, *so that* $\triangle ABX \cong \triangle CBX$ *by SAS.*
$\angle BXA \cong \angle BXC$ *by CPCTC, and they are a linear pair, so* $\overline{BX} \perp \overline{AC}$.
$\overline{AX} \cong \overline{CX}$ *by CPCTC, so* X *is the midpoint of* $\overline{AC}$.
Thus, $\overleftrightarrow{BD}$ *is the perpendicular bisector of* $\overline{AC}$.