Key Proofs

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Introduction: Online proofs

Online proof project description.

Student performance is generally quite poor on Ohio's end-of-course exams for Algebra 1, Geometry, Math 1, and Math 2, especially on items involving proof. In response to concerns that most of the proof items are too difficult, the following pages provide more accessible examples of computer-scorable proof items, mostly in Geometry.

The proofs are written to focus on the most important steps and reasons in the argument. Students complete the proofs by filling in blanks, pulling down menus, and selecting correct answers. In the Ximera environment, some answers are checked automatically when they are chosen. Others answers require pressing Enter, clicking the blue question mark, or clicking the blue "Check Work" button.

Example 1. *Some problems are multiple-choice:*

Multiple Choice:

(a) *Don't pick me.*

(b) *Not me either.*

(c) *Pick me!*

(d) *Also an incorrect choice*

Example 2. *Some problems are select-all that are correct:*

Select All Correct Answers:

(a) *Don't pick me.*

(b) *Pick me!*

(c) *Pick me too!*

(d) *I'm a correct choice too.*

Example 3. *Some problems use (purple haze/ purple rain/ pull-down menus).*

Example 4. *Some problems are fill in the blank:* $3 \times 2 = \boxed{?}$

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For the convenience of teachers using an integrated curriculum, items are separated into two groups: those appropriate for Math 1, and those appropriate for Math 2, according to Ohio's assessments.

This is work in progress. Please send comments to Brad Findell, `findell.2@osu.edu`, Department of Mathematics, Ohio State University.

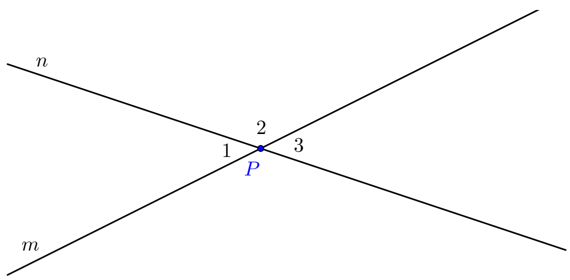
Part I

Math 1

Vertical Angles

Proofs updated.

Problem 1 *Point P is the intersection of lines m and n. Prove that $\angle 1 \cong \angle 3$.*



Fixnote: When students write equations about linear pairs, they often write two equations for non-overlapping linear pairs—which doesn't help. The figure above is intended to help avoid that dead end, but it would be worthwhile to discuss that dead end anyway.

Fixnote: Below are three different brief proofs that separate out details. Which of the details should be included, and which should be omitted?

(a) $\angle 1 \cong \angle 3$ because they are both (*complementary/ supplementary/ opposite / congruent*) to $\angle 2$.

Detail: First write down equations about linear pairs of angles:

$$m\angle 1 + m\angle 2 = 180°$$

$$m\angle 3 + m\angle 2 = 180°$$

By comparing the two equations, it might be clear that $m\angle 1 = m\angle 3$. Other students may need to do some algebra.

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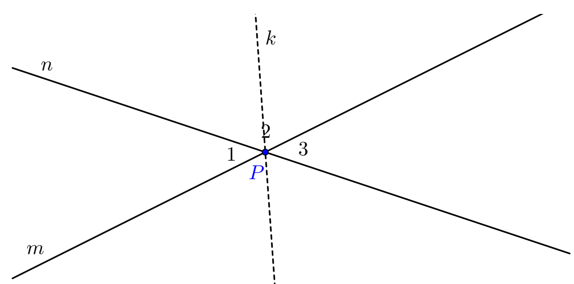
(b) *A rotation of* $180°$ *about* P *maps* m *onto itself, maps* n *onto itself, and swaps* $\angle 1$ *and* $\angle 3$*. Because rotations preserve angle measures, it must be that* $\angle 1 \cong \angle 3$*.*

Detail: Line m *is the union of two opposite rays with endpoint* P*. Check that the* $180°$ *rotation about* P *swaps these opposite rays. The same idea holds for line* n *so that together the sides of* $\angle 1$ *become the sides of* $\angle 3$ *and vice versa.*

(c) *Reflecting about the bisector of* $\angle 2$ *swaps* $\angle 1$ *and* $\angle 3$*. Because reflections preserve angle measures, it follows that* $\angle 1 \cong \angle 3$*.*

Detail: The reflection swaps the two rays that are the sides of $\angle 2$*. Because reflections take lines to lines, that reflection must swap not just the rays but lines* m *and* n*.*

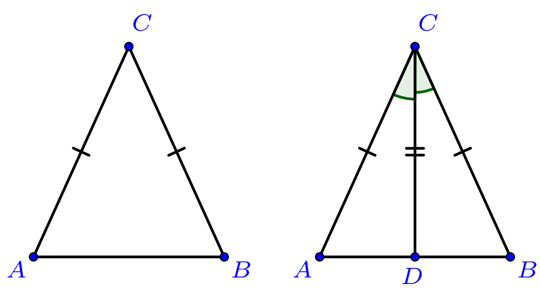


The Isosceles Triangle Theorem

Proofs updated.

Problem 2 *Prove that the base angles of an isosceles triangle are congruent.*

Fixnote: Below are several different proofs, along with one that is not a proof. Please consider them separately.



(a) *Beginning with the given figure on the left, Morgan draws $\overline{CD}$ and marks the figure intending that this new segment is a(n) (median/ angle bisector / perpendicular bisector/ altitude).*

(b) *Based on the marked figure, Morgan claims that the $\triangle ACD \cong \triangle\boxed{?}$ by (SAS/ SSS/ SSA/ ASA/ HL).*

(c) *Finally, Morgan concludes that $\angle A \cong \angle\boxed{?}$, as they are corresponding parts of congruent triangles.*

Problem 3 *Prove that the base angles of an isosceles triangle are congruent.*

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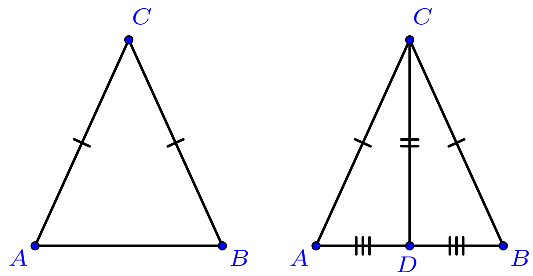
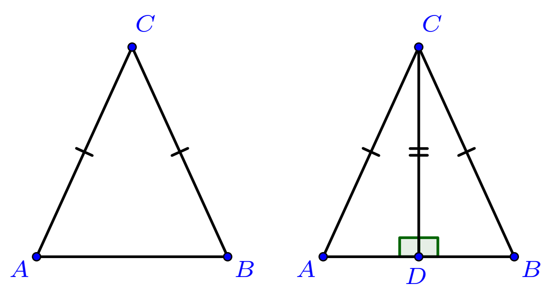


(a) *Beginning with the given figure on the left, Deja draws* $\overline{CD}$ *and marks the figure intending that this new segment is a(n) (median/ angle bisector/ perpendicular bisector/ altitude).*

(b) *Based on the marked figure, Deja claims that the* $\triangle ACD \cong \triangle\boxed{?}$ *by (SAS / SSS/ SSA/ ASA/ HL).*

(c) *Finally, Deja concludes that* $\angle A \cong \angle\boxed{?}$*, as they are corresponding parts of congruent triangles.*

Problem 4 *Prove that the base angles of an isosceles triangle are congruent.*

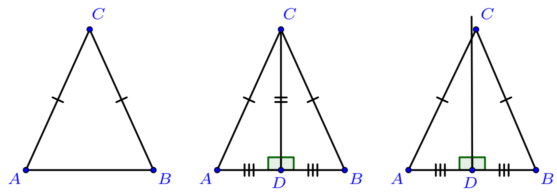


(a) *Beginning with the given figure on the left, Elle draws* $\overline{CD}$ *and marks the figure intending that this new segment is a(n) (median/ angle bisector/ perpendicular bisector/ altitude).*

(b) *Based on the marked figure, Deja claims that the* $\triangle ACD \cong \triangle\boxed{?}$ *by (SAS / SSS/ SSA/ ASA/ HL).*

(c) *Finally, Deja concludes that* $\angle A \cong \angle\boxed{?}$*, as they are corresponding parts of congruent triangles.*

Problem 5 *Simon and Taylor are trying to prove that the base angles of an isosceles triangle are congruent.*



Given figure   Simon's figure   Taylor's figure

Beginning with the given figure on the left, Simon draws $\overline{CD}$ and marks the second figure intending that this new segment is a perpendicular bisector of $\overline{AB}$.

Taylor claims that a perpendicular bisector of a side of a triangle usually misses the opposite vertex. So without using other properties of isosceles triangles or perpendicular bisectors, the figure should allow for that possibility.

Fixnote: Taylor's claim needs some work, as is the case for the choices below.

Choose the best response to their argument:

Multiple Choice:

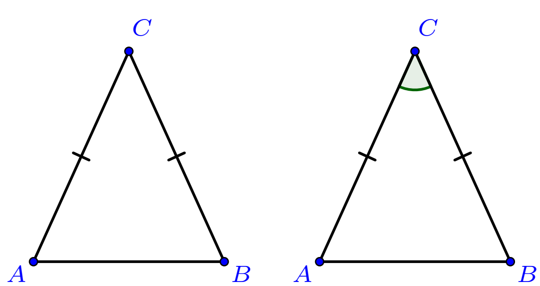
(a) *Simon is correct, and $\triangle ACD \cong \triangle BCD$ by SAS.*

(b) *Simon is correct, and $\triangle ACD \cong \triangle BCD$ by SSS*

(c) *Taylor is correct, and the perpendicular bisector cannot be used to complete this proof.*

(d) *Neither of them are correct.*

Problem 6 *Prove that the base angles of an isosceles triangle are congruent.*



Given figure

(a) *Examining the given figure on the left, Lissy notices symmetry in the triangle and claims that the triangle is congruent to itself by a (translation / reflection/ rotation).*

(b) *Based on the marked figure, Lissy claims that the* $\triangle ACB \cong \triangle\boxed{?}$ *by (SAS/ SSS/ SSA/ ASA/ HL).*

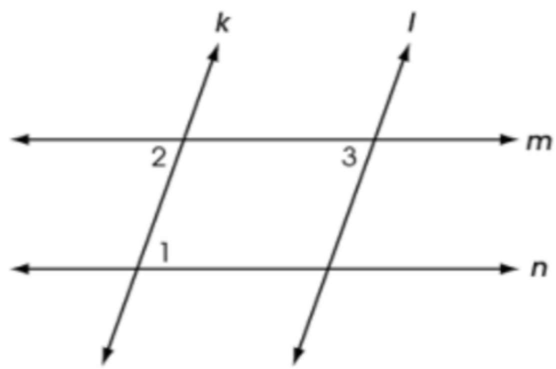
(c) *Finally, Lissy concludes that* $\angle A \cong \angle\boxed{?}$, *as they are corresponding parts of congruent triangles.*

Quadrilaterals

Proof.

Problem 7 *Adapted from Ohio's 2017 Geometry released item 13.*

Two pairs of parallel lines intersect to form a parallelogram as shown.



Complete the following proof that opposite angles of a parallelogram are congruent:

(a) $\angle 1 \cong \angle 2$ *as (opposite angles / alternate interior angles / corresponding angles) for parallel lines* (m *and* n / k *and* l).

(b) $\angle 3 \cong \angle 2$ *as (opposite angles / alternate interior angles / corresponding angles)for parallel lines* (m *and* n / k *and* l).

(c) *Then* $\angle 1 \cong \angle 3$ *because they are both congruent to* $\angle 2$.

Problem 8 *Adapted from Ohio's 2018 Geometry released item 21.*

Given the parallelogram $WXYZ$*, prove that* $\overline{WX} \cong \overline{YZ}$.

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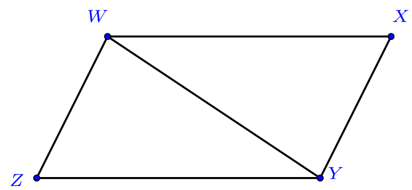


Fixnote: It really would help to have an online environment that allows students to mark diagrams.

Complete the proof below:

(a) $\angle ZWY \cong \angle XYW$ as (alternate interior angles/ corresponding angles/ opposite angles) for parallel segments ($\overline{WZ}$ and $\overline{XY}$ / $\overline{WX}$ and $\overline{YZ}$).

(b) $\angle ZYW \cong \angle XWY$ for the same reason, this time for parallel segments ($\overline{WZ}$ and $\overline{XY}$ / $\overline{WX}$ and $\overline{YZ}$).

(c) $\overline{WY} \cong \overline{YW}$ because a segment is congruent to itself.

(d) $\triangle WYZ \cong \triangle YWX$ by (SAS/ ASA/ SSS).

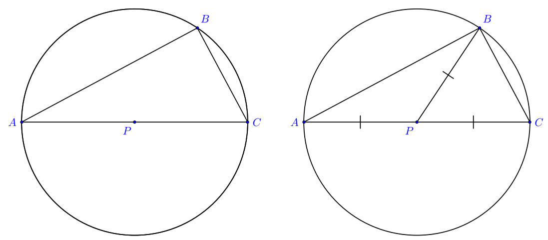
(e) Then $\overline{YZ} \cong \overline{WX}$ as corresponding parts of congruent triangles.

Fixnote: Maybe number the angles.

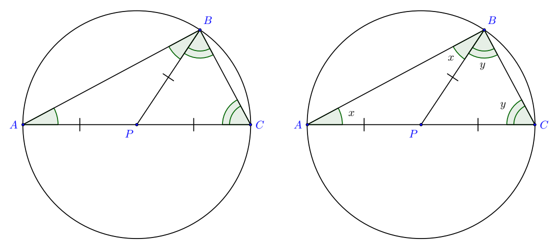
Inscribed Angles

Proofs updated.

Problem 9 *In the figure below, $\overline{AC}$ is a diameter of a circle with center P. Prove that $\angle ABC$ is a right angle.*



(a) *Beginning with the diagram on the left, Natalia draws $\overline{PB}$ and marks the diagram to show segments that she knows to be congruent because each one is a* $\boxed{?}$ *of the circle.*



(b) *Natalia sees that $\triangle APB$ and $\triangle BPC$ are* $\boxed{?}$ *triangles, so she marks the figure to show angles that must congruent.*

Fixnote: Do we need a statement or citation of the theorem?

(c) *In order to do some algebra with these congruent angles, Natalia labels their measures x and y, as shown in the picture on the right.*

(d) *She writes an equation for the sum of the angles of $\triangle ABC$:*

$$\boxed{?} = 180°$$

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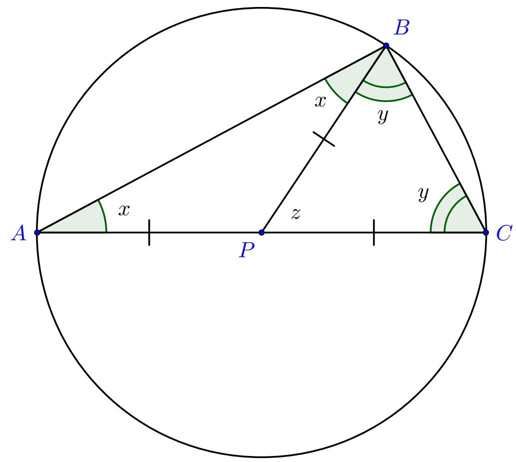
Fixnote: Need a prompt about dividing the equation by 2.

(e) *Since* $m\angle ABC = \boxed{?}$, *she concludes that* $m\angle ABC = 90°$.

Problem 10 *Fixnote: New problem about relationship between inscribed angle and central angle.*



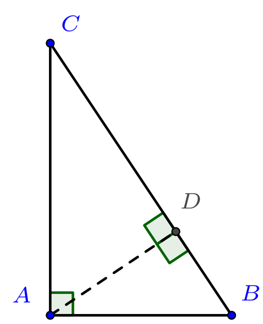
Part II

Math 2

Similar Right Triangles

Proofs updated.

Problem 11 *Adapted from Ohio's 2017 Geometry released item 17.*



Complete the following proof that $\triangle DAC$ *is similar to* $\triangle DBA$:

(a) $\triangle ABC \sim \triangle\boxed{?}$ *by (AA similarity / CPCTC / right triangle similarity) because they share* $\angle B$ *and they each have a right angle.*

(b) $\triangle ABC \sim \triangle DAC$ *for the same reason because they share* ($\angle A$ / $\angle B$ / $\angle C$) *and they each have a right angle.*

(c) $\triangle DAC \sim \triangle\boxed{?}$ *because they are both similar to* $\triangle ABC$.

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