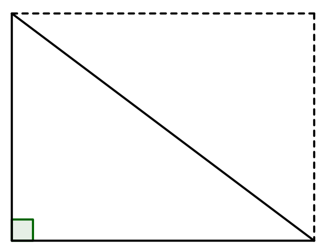
Proof by Picture 1

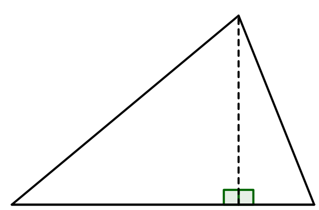
Short-answer proofs about triangle area.

Problem 1 *Explain how the following picture "proves" that the area of a right triangle is half the base times the height.*



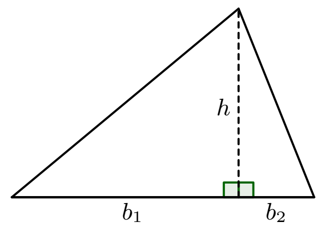
Free Response: ***Hint:*** *The area of the rectangle is base times height. The rectangle is made up of two congruent right triangles. Because congruent triangles have the same area, the area of each right triangle is (equal to/ half ✓/ double) the area of the rectangle.*

Problem 2 *Suppose you know that the area of a* ***right*** *triangle is half the base times the height. Explain how the following picture "proves" that the area of* ***every*** *triangle is half the base times the height.*



Free Response: ***Hint:*** *The whole triangle is made up of two right triangles. Call the bases of the small right triangles b_1 and b_2, and call the base of the large (combined) triangle b.*

Author(s): Bart Snapp and Brad Findell

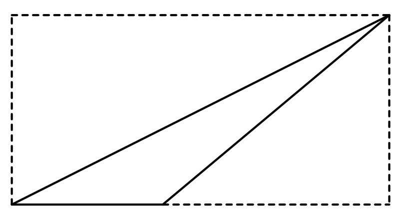


Then $b = b_1 + b_2$, *and the three triangles all have the same height,* h. *The area of the whole triangle is the sum of the areas of the two right triangles:*

$$\text{area} = \frac{hb_1}{2} + \frac{hb_2}{2} = \frac{h(b_1 + b_2)}{2} = \boxed{bh/2}$$

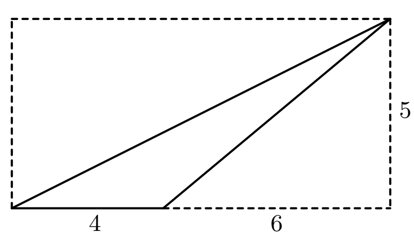
because $b = b_1 + b_2$.

Problem 3 *Now suppose that a student, say Geometry Giorgio attempts to solve a similar problem. Again knowing that the area of a right triangle is half the base times the height, he draws the following picture:*



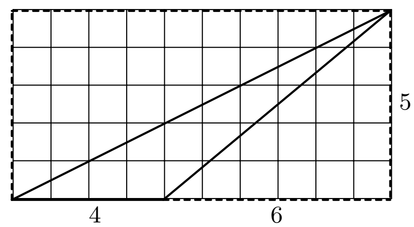
Geometry Giorgio states that the diagonal line cuts the rectangle in half, and thus the area of the triangle is half the base times the height. Is this correct reasoning? If so, give a complete explanation. If not, give correct reasoning based on Geometry Giorgio's picture.

Free Response: ***Hint:*** *First try some numbers.*

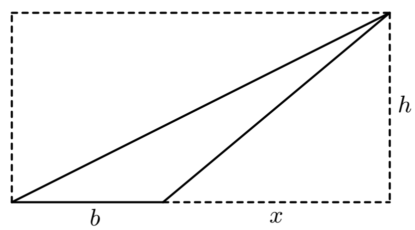


Find the area of this triangle (which is much less than half the rectangle). Then generalize the approach.

Hint: *Maybe a grid will help.*



Hint: *The triangle is not half the rectangle. Furthermore, the rectangle does not have the same base as the triangle, so "base times height" is unclear. The following picture allows us to distinguish these bases:*



One way to compute the area of the solid triangle is to (1) compute the area right triangle that is the lower half of the rectangle (with base $b + x$*) and then (2) subtract the area of the small right triangle (with base* x*):*

$$\text{area} = \boxed{\frac{h(b+x)}{2}} - \boxed{\frac{hx}{2}} = \frac{hb}{2} + \frac{hx}{2} - \frac{hx}{2} = \boxed{\frac{hb}{2}}$$