Set Theory Problems

Short-answer problems.

Problem   **1**   *Given two sets X and Y, $X \cup Y$ is the set of elements that are*

Multiple Choice:

(a) *in X or in Y (but not in both).*

(b) *in X or in Y (or both, as the "or" is inclusive).* ✓

(c) *in X and in Y.*

(d) *in X but not in Y.*

(e) *in Y but not in X.*

Problem   **2**   *Given two sets X and Y, $X \cap Y$ is the set of elements that are*

Multiple Choice:

(a) *in X or in Y (but not in both).*

(b) *in X or in Y (or both, as the "or" is inclusive).*

(c) *in X and in Y.* ✓

(d) *in X but not in Y.*

(e) *in Y but not in X.*

Problem   **3**   *Given two sets X and Y, $X - Y$ is the set of elements that are*

Multiple Choice:

(a) *in X or in Y (but not in both).*

(b) *in X or in Y (or both, as the "or" is inclusive).*

(c) *in X and in Y.*

(d) *in X but not in Y.* ✓

(e) *in Y but not in X.*

Problem 4 *Consider the following Venn diagrams:*

For each set expression below, identify the Venn diagram above in which the corresponding region is shaded:

(a) $X \cap Y$ *is figure* $\boxed{C}$

(b) $X \cup Y$ *is figure* $\boxed{B}$.

(c) $X - Y$ *is figure* $\boxed{D}$

Problem 5 *Draw a Venn diagram for the set of elements that are in X or Y but not both. How does it differ from the Venn diagram for $X \cup Y$?*

Free Response: **Hint:** *A Venn diagram for elements in X or Y but not both is shown in figure (A) from the previous problem.*

Problem 6 *Consider the following Venn diagrams:*

(a) *If Venn diagram (A) above shows the relationship between sets X and Y, then $X \cap Y = (0 / \emptyset$ ✓/ $X \cup Y$) and the sets are said to be (disjoint ✓/ empty/ subsets).*

(b) *If Venn diagram (B) above shows the relationship between sets X and Y, then we say that (X and Y are disjoint/ $X \subseteq Y$ ✓/ $Y \subseteq X$).*

(c) *If we let X be the set of "right triangles" and we let Y be the set of "equilateral triangles" which diagram above shows the relationship between these two sets?*

> **Multiple Choice:**
>
> (i) *Diagram (A).* ✓
> (ii) *Diagram (B).*
> (iii) *Neither of these.*

(iv) *Not enough information.*

Explain your reasoning.

Free Response: **Hint:**   *Diagram (A) is accurate because no right triangles are also equilateral triangles.*

Problem   7   *If $X = \{1, 2, 3, 4, 5\}$ and $Y = \{3, 4, 5, 6\}$ find the following: (List elements in ascending order, separated by commas, with no spaces.)*

(a) $X \cup Y = \{\boxed{1, 2, 3, 4, 5, 6}\}$

(b) $X \cap Y = \{\boxed{3, 4, 5}\}$

(c) $X - Y = \{\boxed{1, 2}\}$

(d) $Y - X = \{\boxed{6}\}$

Problem   8   *Let $n\mathbb{Z}$ represent the integer multiples of n. So for example:*

$$3\mathbb{Z} = \{\dots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \dots\}$$

Compute the following (use capital Z for $\mathbb{Z}$):

(a) $3\mathbb{Z} \cap 4\mathbb{Z} = \boxed{12Z}$

(b) $2\mathbb{Z} \cap 5\mathbb{Z} = \boxed{10Z}$

(c) $3\mathbb{Z} \cap 6\mathbb{Z} = \boxed{6Z}$

(d) $4\mathbb{Z} \cap 6\mathbb{Z} = \boxed{12Z}$

(e) $4\mathbb{Z} \cap 10\mathbb{Z} = \boxed{20Z}$

Problem   9   *Make a general rule for intersecting sets of the form $n\mathbb{Z}$ and $m\mathbb{Z}$. Explain why your rule works.*

Free Response: **Hint:**   *The intersection of two sets is what they have in common. The intersection of the set of multiples of n and the set of multiples of m are called common multiples (surprise!), and they are all multiples of the least common multiple of n and m.*

Problem   10   *If $X \cup Y = X$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

$(X \subseteq Y \,/\, X = Y \,/\, Y \subseteq X \,\checkmark/\, X = \emptyset)$ *because every element of* $(X \,/\, Y \,\checkmark)$ *must be in* $(X \,\checkmark/\, Y)$.

Problem   11   *If $X \cap Y = X$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

$(X \subseteq Y \,\checkmark/\, X = Y \,/\, Y \subseteq X \,/\, X = \emptyset)$ *because every element of* $(X \,\checkmark/\, Y)$ *must be in* $(X \,/\, Y \,\checkmark)$.

Problem   12   *If $X - Y = \emptyset$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

$(X \subseteq Y \,\checkmark/\, X = Y \,/\, Y \subseteq X \,/\, X = \emptyset)$ *because every element of* $(X \,\checkmark/\, Y)$ *must be in* $(X \,/\, Y \,\checkmark)$.