Online HW 2: Proof by Pictures

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Contents

Set Theory Problems

Extra problems about sets.

Reminders

- Sets are collections of objects such as numbers or points. The objects are called *elements* of the set, and the order elements are listed is not important.
- The notation $\{7, 3\}$ means "The set containing 7 and 3."
- Note that $\{8\}$ is not the same as the number 8 but rather is a set that contains one element that happens to be a number.
- The set containing zero elements, sometimes call the *empty set* is denoted $\{\}$ or $\emptyset$.
- The elements of a set can themselves be sets.

Problem 1 *Indicate the number of elements in each set:*

(a) *The set* $\{3, 5, 6, 9, 10\}$ *has* $\boxed{?}$ *element(s).*

(b) *The set* $\{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$ *has* $\boxed{?}$ *element(s).*

(c) *The set* $\{\{\}\}$ *has* $\boxed{?}$ *element(s).*

(d) *The set* $\{\}$ *has* $\boxed{?}$ *element(s).*

(e) *The set* $\emptyset$ *has* $\boxed{?}$ *element(s).*

(f) *The set* $\{\emptyset\}$ *has* $\boxed{?}$ *element(s).*

Problem 2 *Indicate whether each statement is true or false:*

(a) $2 \in \{3, 2, 5\}$. *(True/ False)*

(b) $2 \subseteq \{3, 2, 5\}$. *(True/ False)*

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(c) $\{2\} \in \{3, 2, 5\}$. *(True/ False)*

(d) $\{2\} \subseteq \{3, 2, 5\}$. *(True/ False)*

(e) $\emptyset = \{\}$. *(True/ False)*

(f) $\emptyset = \{\emptyset\}$. *(True/ False)*

(g) $\{\emptyset\} = \{\{\}\}$. *(True/ False)*

(h) $\emptyset \in \{\emptyset\}$. *(True/ False)*

(i) $\emptyset \subseteq \{\emptyset\}$. *(True/ False)*

(j) $2 \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

(k) $2 \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

(l) $\{2\} \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

(m) $\{2\} \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

(n) $\{\{2\}\} \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

(o) $\{\{2\}\} \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

Problem 3 *Explain the difference between the symbols $\in$ and $\subseteq$.*

Problem 4 *How is $\{\emptyset\}$ different from $\emptyset$?*

Measuring by Sight

Short-answer questions involving measuring.

Careful Measurement by Sight

Adjust the figures to fit the given conditions within **eyeball accuracy**. Enter the requested measurements.

Problem 5 *Geogebra link:* `https://tube.geogebra.org/m/gjf28er6`

In figure above, when point C is adjusted so that $\overline{BC}$ is perpendicular to $\overline{AC}$, $AC = \boxed{?}$.

Problem 6 *Geogebra link:* `https://tube.geogebra.org/m/q32gyaud`

In $\triangle ABC$ above, move point D to make the following measurements. ***Enter -1 if it is not possible.***

(a) *When $\overline{BD}$ is a median, $AD = \boxed{?}$.*

(b) *When $\overline{BD}$ is a angle bisector, $AD = \boxed{?}$.*

(c) *When $\overline{BD}$ is a perpendicular bisector, $AD = \boxed{?}$.*

(d) *When $\overline{BD}$ is a altitude, $AD = \boxed{?}$.*

Problem 7 *Geogebra link:* `https://tube.geogebra.org/m/a888zyw2`

In $\triangle ABC$ above, the height to base $\overline{AC}$ is $\boxed{?}$.

Problem 8 *Geogebra link:* `https://tube.geogebra.org/m/kta9hbuf`

In $\triangle ABC$ above, the height to base $\overline{AC}$ is $\boxed{?}$.

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Measuring Interior Angles

Short-answer questions involving length, angle, and area.

Geogebra link: `https://tube.geogebra.org/m/zrapvzpz`

Problem 9 *Measure the interior angles of quadrilateral $ABCD$ above.*

(a) $m\angle A = \boxed{?}$ *degrees.*

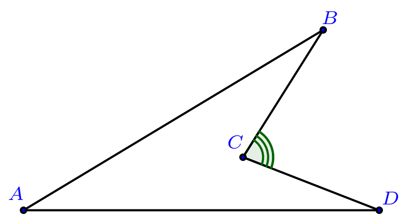
(b) $m\angle B = \boxed{?}$ *degrees.*

(c) $m\angle C = \boxed{?}$ *degrees.*

(d) $m\angle D = \boxed{?}$ *degrees.*

(e) $m\angle A + m\angle B + m\angle C + m\angle D = \boxed{?}$.

Problem 10 *Use the measurements from the previous problem to answer the following questions:*



(a) *The marked angle should measure* $\boxed{?}$ *degrees.*

(b) $m\angle A + m\angle B + m\angle D = \boxed{?}$ degrees.

(c) *What do you notice?*

Problem 11 *In order to reason about the sum of the interior angles, Bart and Brad each* triangulated *the figure as shown below.*

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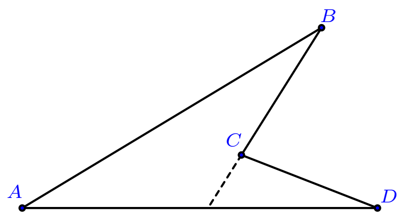
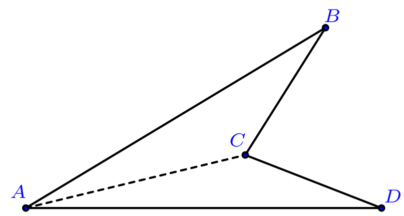


Brad's triangulation



Bart's triangulation

Both Bart and Brad claim that because in a triangle the sum of the interior angles is $\boxed{?}$ *degrees, and this quadrilateral is cut into* $\boxed{?}$ *triangles, the angle sum in this quadrilateral should be* $\boxed{?}$ *degrees. What is your judgment?*

Multiple Choice:

(a) *They are both correct.*

(b) *Only Brad is correct.*

(c) *Only Bart is correct.*

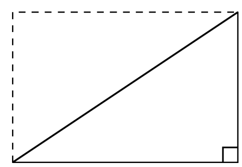
(d) *Neither of them are correct.*

Explain your reasoning.

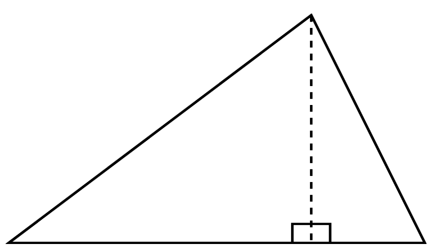
Proof by Picture

Short-answer proofs by pictures.

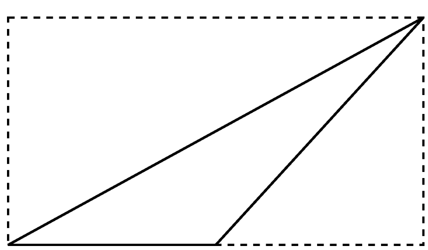
Problem 12 *Explain how the following picture "proves" that the area of a right triangle is half the base times the height.*



Problem 13 *Suppose you know that the area of a **right** triangle is half the base times the height. Explain how the following picture "proves" that the area of **every** triangle is half the base times the height.*



Now suppose that a student, say Geometry Giorgio attempts to solve a similar problem. Again knowing that the area of a right triangle is half the base times the height, he draws the following picture:



Geometry Giorgio states that the diagonal line cuts the rectangle in half, and thus the area of the triangle is half the base times the height. Is this correct

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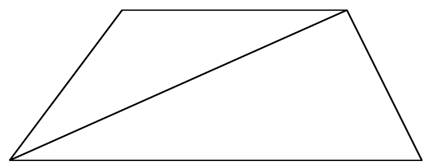
reasoning? If so, give a complete explanation. If not, give correct reasoning based on Geometry Giorgio's picture.

Problem 14 *Recall that a trapezoid is a quadrilateral with two parallel sides. Consider the following picture:*

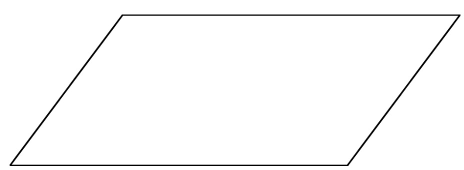


How does the above picture prove that the area of a trapezoid is

$$\text{area} = \frac{h(b_1 + b_2)}{2}$$

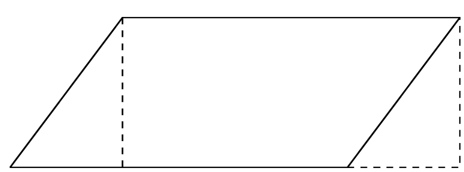
where h is the height of the trapezoid and b_1, b_2, are the lengths of the parallel sides?

Problem 15 *Look at the previous problem. Can you use a similar idea to prove that the area of a parallelogram*

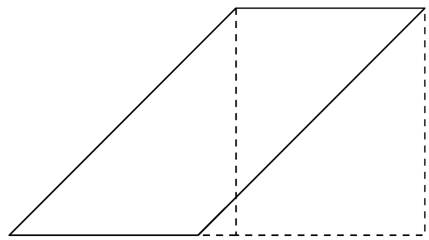


is the length of the base times the height?

Problem 16 *Explain how the following picture "proves" that the area of a parallelogram is base times height.*



Now suppose that a student, say Geometry Giorgio attempts to solve a similar problem. In an attempt to prove the formula for the area of a parallelogram, Geometry Giorgio draws the following picture:



At this point Geometry Giorgio says that he has proved the formula for area of a parallelogram. What do you think of his picture? Give a complete argument based on his picture.