Transformations

Short-answer problems about transformations.

Question 1 *To specify a translation, we need a* $\boxed{vector}$. *Equivalently, we need a magnitude (or length or* $\boxed{distance}$*) and a* $\boxed{direction}$.

Question 2 *To specify a rotation, we need a* $\boxed{center}$ *and an* $\boxed{angle}$ *(assuming an agreement about the direction of rotation).*

Question 3 *To specify a reflection, we need a* $\boxed{line}$.

Question 4 *A transformation that does nothing is call the* $\boxed{identitytransformation}$. *(Hint: Two words.)*

Sometimes a sequence of transformations can be described as a single translation, rotation, or reflection.

Question 5 *What kind of transformation is a translation followed by a translation? Explain. Be sure to consider any special cases.*

Answer: *Usually a* $\boxed{translation}$. *If the vectors are opposites of each other, the result is the* $\boxed{identitytransformation}$ *(two words).*

Feedback(*`correct`*): *The identity transformation can be thought of as a translation by a vector of magnitude* 0.

Question 6 *What kind of transformation is a rotation followed by a rotation? Explain. Be sure to consider any special cases.*

Answer: *Usually a* $\boxed{rotation}$. *If the angles sum to a multiple of* 360° *and the centers are different, then the result is a* $\boxed{translation}$. *If the centers of*

Author(s): Bart Snapp and Brad Findell

rotation are the same ***and*** *the angles sum to a multiple of* 360°*, the result is the* $\boxed{identitytransformation}$ *(two words).*

Feedback(`correct` ***):*** *The identity transformation can be thought of as a rotation by an angle of* 0°*.*

Question 7 *What kind of transformation is a reflection followed by another reflection? Explain. Be sure to consider any special cases.*

Answer: *If the reflection lines intersect, the result is a* $\boxed{rotation}$*. If the reflection lines are parallel, the result is a* $\boxed{translation}$*. If the reflection lines are the same line, the result is the* $\boxed{identitytransformation}$*.*

Feedback(`correct` ***):*** *If the lines intersect, the center of the resulting rotation is the intersection of the lines. (What is the angle?) If the lines are parallel, the resulting translation vector is perpendicular to the two lines. (What is the distance?)*

Question 8 *Will the letter F look like an F after a reflection? What about after a sequence of two reflections? What about after a sequence of 73 or 124 reflections? Explain your reasoning.*

Free Response: ***Hint:*** *Ignoring which side is up, after a reflection the F will look like a "backwards F". More generally, after an odd number of reflections, the F will look like a backwards F. After an even number of reflections, the F will look like a typical F.*

Question 9 *How will your answer to the previous problem change if you use a capital D? Explain.*

Free Response: ***Hint:*** *Ignoring which side is up, the D will always look like a D. Because of its line symmetry, a reflection doesn't appear to reverse its "orientation."*

Question 10 *Given a figure and its image after a translation, how do find the direction and distance of the translation? How many points and images do you need?*

Free Response: ***Hint:*** *Draw a vector from any point to its image. The vector provides both the direction and the magnitude. Any point and its image will do.*

Question 11 *Given a figure and its image after a reflection, how do you find the line of reflection? How many points and images do you need?*

Free Response: ***Hint:*** *Draw a segment from a point to its image. The perpendicular bisector of that segment is the line of reflection. Any point and its image will do, (as long the point moves under the reflection).*

Question 12 *Given a figure and its image after a rotation, how do you find the center and the angle of the rotation? How many points and images do you need?*

Free Response: ***Hint:*** *Draw a segment from a point P to its image P'. The center of rotation is somewhere on the perpendicular bisector of that segment. Draw a segment from a second point Q to its image Q'. The center of rotation is also somewhere on the the perpendicular bisector of that segment. As long as the segments $\overline{PP'}$ and $\overline{QQ'}$ are not parallel, the two perpendicular bisectors will intersect at a point C, which is the unique center of the rotation.*

To find the angle of rotation, measure $\angle PCP'$ or $\angle QCQ'$.

Two points and their images are enough, (as long as the segments $\overline{PP'}$ and $\overline{QQ'}$ are not parallel).