Math 1166: Parallels in Geometry!

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Set Theory Problems

Short-answer problems about sets.

Question 1 *What is your name?*

Problem 2 *Given two sets X and Y, explain what is meant by $X \cup Y$.*

Problem 3 *Given two sets X and Y, explain what is meant by $X \cap Y$.*

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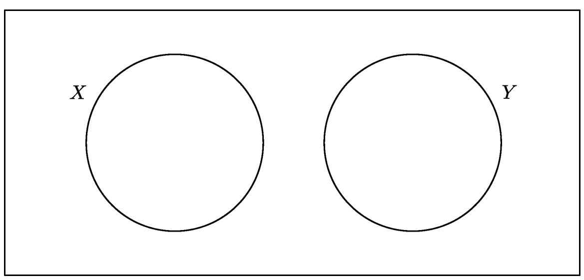
Problem 4 *Given two sets X and Y, explain what is meant by $X - Y$.*

Problem 5 *Explain the difference between the symbols $\in$ and $\subset$.*

Problem 6 *How is $\{\emptyset\}$ different from $\emptyset$?*

Problem 7 *Draw a Venn diagram for the set of elements that are in X or Y* but not both. *How does it differ from the Venn diagram for $X \cup Y$?*

Problem 8 *If we let X be the set of "right triangles" and we let Y be the set of "equilateral triangles" does the picture below show the relationship between these two sets?*



Explain your reasoning.

Problem 9 *If* $X = \{1, 2, 3, 4, 5\}$ *and* $Y = \{3, 4, 5, 6\}$ *find:*

(a) $X \cup Y$

(b) $X \cap Y$

(c) $X - Y$

(d) $Y - X$

Problem 10 *Let* $n\mathbb{Z}$ *represent the integer multiples of* n*. So for example:*

$$3\mathbb{Z} = \{\dots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \dots\}$$

Compute the following:

(a) $3\mathbb{Z} \cap 4\mathbb{Z}$

(b) $2\mathbb{Z} \cap 5\mathbb{Z}$

(c) $3\mathbb{Z} \cap 6\mathbb{Z}$

(d) $4\mathbb{Z} \cap 6\mathbb{Z}$

(e) $4\mathbb{Z} \cap 10\mathbb{Z}$

In each case explain your reasoning.

Problem 11 *Make a general rule for intersecting sets of the form $n\mathbb{Z}$ and $m\mathbb{Z}$. Explain why your rule works.*

Problem 12 *If $X \cup Y = X$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

Problem 13 *If $X \cap Y = X$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

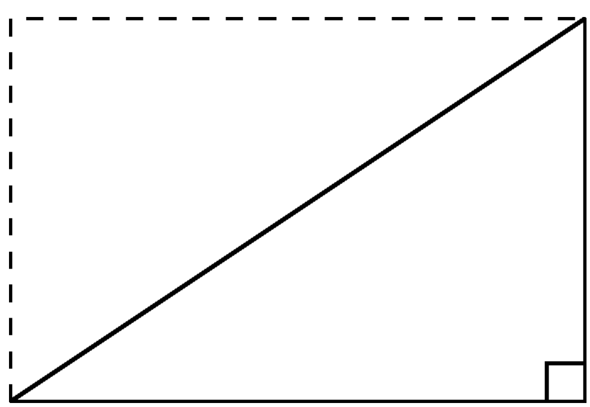
Problem 14 *If $X - Y = \emptyset$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

Proof by Picture

Short-answer proofs by pictures.

Question 1 *What is your name?*

Problem 2 *Explain how the following picture "proves" that the area of a right triangle is half the base times the height.*



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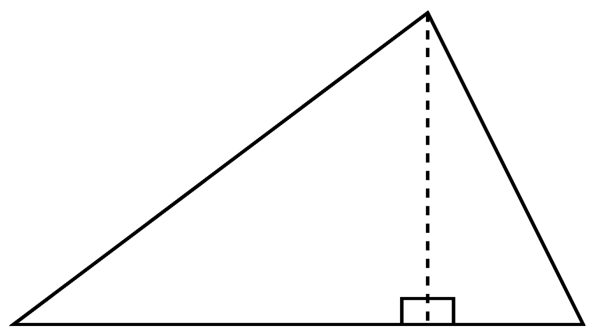
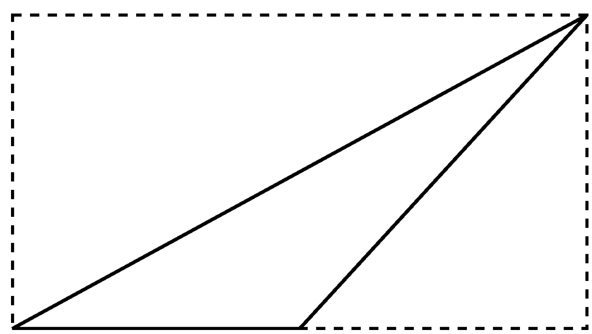
Problem 3 *Suppose you know that the area of a* ***right*** *triangle is half the base times the height. Explain how the following picture "proves" that the area of* ***every*** *triangle is half the base times the height.*

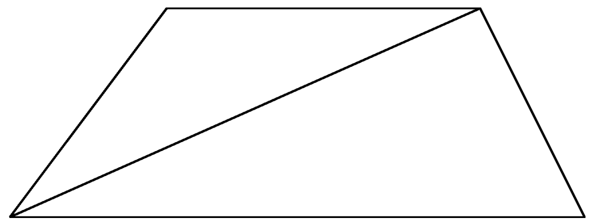


Now suppose that a student, say Geometry Giorgio attempts to solve a similar problem. Again knowing that the area of a right triangle is half the base times the height, he draws the following picture:



Geometry Giorgio states that the diagonal line cuts the rectangle in half, and thus the area of the triangle is half the base times the height. Is this correct reasoning? If so, give a complete explanation. If not, give correct reasoning based on Geometry Giorgio's picture.

Problem 4 *Recall that a trapezoid is a quadrilateral with two parallel sides. Consider the following picture:*

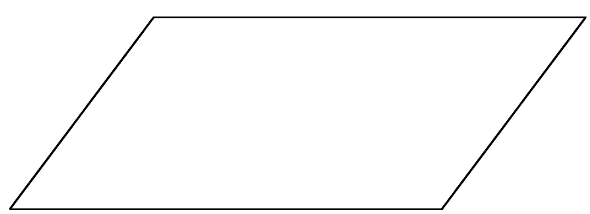


How does the above picture prove that the area of a trapezoid is

$$\text{area} = \frac{h(b_1 + b_2)}{2}$$

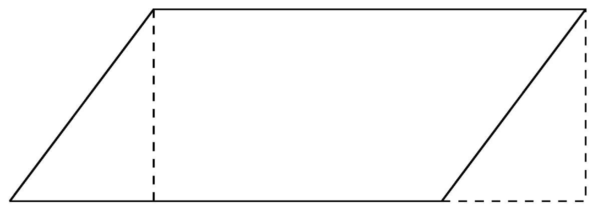
where h is the height of the trapezoid and b_1, b_2, are the lengths of the parallel sides?

Problem 5 *Look at the previous problem. Can you use a similar idea to prove that the area of a parallelogram*

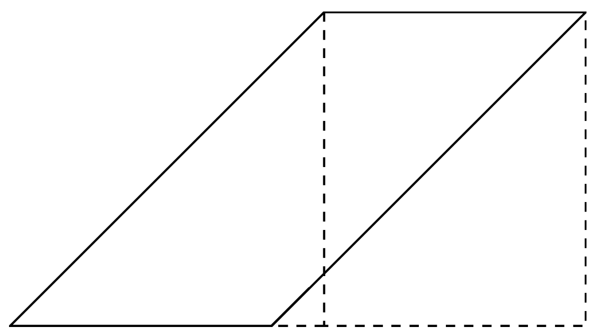


is the length of the base times the height?

Problem 6 *Explain how the following picture "proves" that the area of a parallelogram is base times height.*



Now suppose that a student, say Geometry Giorgio *attempts to solve a similar problem. In an attempt to prove the formula for the area of a parallelogram,* Geometry Giorgio *draws the following picture:*



At this point Geometry Giorgio *says that he has proved the formula for area of a parallelogram. What do you think of his picture? Give a complete argument based on his picture.*

Constructions Problems

Short-answer problems about constructions.

Question 1 *What is your name?*

Problem 2 *Given a line segment, construct an equilateral triangle whose edge has the length of the given segment. Explain the steps in your construction and how you know it works.*

Problem 3 *Use a compass and straightedge to bisect a given line segment. Explain the steps in your construction and how you know it works.*

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Problem 4 *Given a line segment with a point on it, construct a line perpendicular to the segment that passes through the given point. Explain the steps in your construction and how you know it works.*

Problem 5 *Use a compass and straightedge to bisect a given angle. Explain the steps in your construction and how you know it works.*

Problem 6 *Given an angle and some point [or a ray], use a compass and straightedge to copy the angle so that the new angle has as its vertex the given point [or a ray as one side of the angle]. Explain the steps in your construction and how you know it works.*

Problem 7 *Given a point and line, construct a line perpendicular to the given line that passes through the given point. Explain the steps in your construction and how you know it works.*

Problem 8 *Given a point and line, construct a line parallel to the given line that passes through the given point. Explain the steps in your construction and how you know it works.*

Problem 9 *Construct a* 30-60-90 *right triangle. Explain the steps in your construction and how you know it works.*

Problem 10 *Construct an isosceles right triangle. Explain the steps in your construction and how you know it works.*

Anatomy of Figures

Short-answer problems about centers of triangles.

Question 1 *What is your name?*

Problem 2 *Compare and contrast the idea of "intersecting sets" with the idea of "intersecting lines."*

Problem 3 *Place three points in the plane. Give a detailed discussion explaining how they may or may not be on a line.*

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Problem 4 *Place three lines in the plane. Give a detailed discussion explaining how they may or may not intersect.*

Problem 5 *Explain how a perpendicular bisector is different from an altitude. Draw an example to illustrate the difference.*

Problem 6 *Explain how a median is different from an angle bisector. Draw an example to illustrate the difference.*

Problem 7 *What is the name of the point that is the same distance from all three sides of a triangle? Explain your reasoning.*

Problem 8 *What is the name of the point that is the same distance from all three vertexes of a triangle? Explain your reasoning.*

Problem 9 *Could the circumcenter be outside the triangle? If so, draw a picture and explain. If not, explain why not using pictures as necessary.*

Problem 10 *Could the orthocenter be outside the triangle? If so, draw a picture and explain. If not, explain why not using pictures as necessary.*

Problem 11 *Could the incenter be outside the triangle? If so, draw a picture and explain. If not, explain why not using pictures as necessary.*

Problem 12 *Could the centroid be outside the triangle? If so, draw a picture and explain. If not, explain why not using pictures as necessary.*

Problem 13 *Are there shapes that do not contain their centroid? If so, draw a picture and explain. If not, explain why not using pictures as necessary.*

Problem 14 *Draw an equilateral triangle. Now draw the lines containing the altitudes of this triangle. How many orthocenters do you have as intersections of lines in your drawing? Hints:*

(a) *More than one.*

(b) *How many triangles are in the picture you drew?*

Problem 15 *Given a triangle, construct the circumcenter. Explain the steps in your construction.*

Problem 16 *Given a triangle, construct the orthocenter. Explain the steps in your construction.*

Problem 17 *Given a triangle, construct the incenter. Explain the steps in your construction.*

Problem 18 *Given a triangle, construct the centroid. Explain the steps in your construction.*

Problem 19 *Given a triangle, construct the incircle. Explain the steps in your construction.*

Problem 20 *Given a triangle, construct the circumcircle. Explain the steps in your construction.*

Problem 21 *Given a circle, give a construction that finds its center.*

Problem 22 *Where is the circumcenter of a right triangle? Explain your reasoning.*

Problem 23 *Where is the orthocenter of a right triangle? Explain your reasoning.*

Problem 24 *Can you draw a triangle where the circumcenter, orthocenter, incenter, and centroid are all the same point? If so, draw a picture and explain. If not, explain why not using pictures as necessary.*

Problem 25 *True or False: Explain your conclusions.*

(a) *An altitude of a triangle is always perpendicular to a line containing some side of the triangle.*

(b) *An altitude of a triangle always bisects some side of the triangle.*

(c) *The incenter is always inside the triangle.*

(d) *The circumcenter, the centroid, and the orthocenter always lie in a line.*

(e) *The circumcenter can be outside the triangle.*

(f) *The orthocenter is always inside the triangle.*

(g) *The centroid is always inside the incircle.*

Problem 26 *Given 3 distinct points not all in a line, construct a circle that passes through all three points. Explain the steps in your construction.*

Important Definitions

Multiple-choice questions about definitions.

Question 1 *An equilateral quadrilateral is called ...*

Multiple Choice:

(a) *a square*

(b) *a rectangle*

(c) *a rhombus*

(d) *a trapezoid*

Question 2 *The incenter of a triangle is . . .*

Select All Correct Answers:

(a) *the point of concurrency of the medians.*

(b) *the point of concurrency of the angle bisectors.*

(c) *the point of concurrency of the perpendicular bisectors.*

(d) *the point of concurrency of the altitudes.*

(e) *the balance point for the triangle.*

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(f) *the center in the triangle.*

(g) *the center of the incircle.*

(h) *the center of the circumcircle.*

(i) *equidistant from the sides of the triangle.*

(j) *equidistant from the vertices of the triangle.*