

# Set Theory Problems

*Extra problems about sets.*

## Reminders

- Sets are collections of objects such as numbers or points. The objects are called *elements* of the set, and the order elements are listed is not important.
- The notation  $\{7, 3\}$  means “The set containing 7 and 3.”
- Note that  $\{8\}$  is not the same as the number 8 but rather is a set that contains one element that happens to be a number.
- The set containing zero elements, sometimes call the *empty set* is denoted  $\{\}$  or  $\emptyset$ .
- The elements of a set can themselves be sets.

**Problem 1** *Indicate the number of elements in each set:*

- (a) The set  $\{3, 5, 6, 9, 10\}$  has  element(s).
- (b) The set  $\{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$  has  element(s).
- (c) The set  $\{\{\}\}$  has  element(s).
- (d) The set  $\{\}$  has  element(s).
- (e) The set  $\emptyset$  has  element(s).
- (f) The set  $\{\emptyset\}$  has  element(s).

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**Problem 2** *Indicate whether each statement is true or false:*

- (a)  $2 \in \{3, 2, 5\}$ . (True ✓/ False)
- (b)  $2 \subseteq \{3, 2, 5\}$ . (True/ False ✓)

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Learning outcomes:  
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- (c)  $\{2\} \in \{3, 2, 5\}$ . (True/ False ✓)
- (d)  $\{2\} \subseteq \{3, 2, 5\}$ . (True ✓/ False)
- (e)  $\emptyset = \{\}$ . (True ✓/ False)
- (f)  $\emptyset = \{\emptyset\}$ . (True/ False ✓)
- (g)  $\{\emptyset\} = \{\{\}\}$ . (True ✓/ False)
- (h)  $\emptyset \in \{\emptyset\}$ . (True ✓/ False)
- (i)  $\emptyset \subseteq \{\emptyset\}$ . (True ✓/ False)
- (j)  $2 \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$ . (True/ False ✓)
- (k)  $2 \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$ . (True/ False ✓)
- (l)  $\{2\} \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$ . (True ✓/ False)
- (m)  $\{2\} \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$ . (True/ False ✓)
- (n)  $\{\{2\}\} \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$ . (True/ False ✓)
- (o)  $\{\{2\}\} \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$ . (True ✓/ False)

**Problem 3** Explain the difference between the symbols  $\in$  and  $\subseteq$ .

**Free Response:** **Hint:** The symbol  $\in$  means “is an element of,” whereas  $\subseteq$  means “is a subset of.” The notation  $X \in Y$  means that  $X$  is a single element in the set  $Y$ . In this case,  $X$  is typically not a set. The notation  $X \subseteq Y$ , in contrast, requires that both  $X$  and  $Y$  are sets and, furthermore, that every element of  $X$  is also in  $Y$ .

**Problem 4** How is  $\{\emptyset\}$  different from  $\emptyset$ ?

**Free Response:** **Hint:** The empty set,  $\emptyset$ , is a set that contains no elements. That is,  $\emptyset = \{\}$ . The set  $\{\emptyset\}$  contains one element that is itself a set—and that element happens to be the empty set. We could instead write  $\{\{\}\}$ , but that looks ugly.