## Scaling in 2D

Short-answer problems about scaling in two dimensions.

## Length and Area Under Scaling

In this section, we explore what happens to length, area, and other measures under scaling.

In a previous section, we defined similarity in terms of basic rigid motions and dilations, and we showed that this definition leads to well-known results about similarity, such as the AA criterion for triangle similarity and consistent ratios of lengths between and within similar figures. A key feature of this discussion was the notion of "scale factor," which describes what happens to lengths under a dilation. From the definition of a dilation, it is clear that segments on lines through the center of dilation scale by the scale factor. We used the side-splitter theorems to show that other segments are scaled by the same scale factor.

**Question 1** Two students claims that a  $3 \times 5$  rectangle and a  $4 \times 6$  rectangle are similar.

Fred says that that they are similar because the angles are the same. How do you respond?

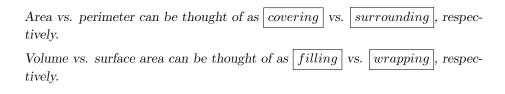
**Free Response:** Hint: Angles are enough to determine similarity of triangles. But similarity requires a consistent scale factor. For these rectangles the height is scaled by 4/3 whereas the base is scaled by 6/5.

Ned says that they are similar because you can do the same thing (i.e., add 1) to "both sides" of the  $3\times 5$  rectangle to get the  $4\times 6$  rectangle. How do you respond?

Free Response: Hint: Similarity requires consistent scaling, which is a multiplicative (not additive) relationship.

**Question 2** Complete the following sentences using words such as filling, falling, covering, wrapping, hiding, surrounding, or traveling:

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To explore how measures of figures change under scaling and non-scaling transformations, here are some useful strategies:

- Cutting the figures and rearranging the pieces.
- Using "rep-tiles."
- Using known formulas for perimeters, areas, volumes, or surface areas.
- Approximating with segments, squares, or cubes.

**Question 3** To estimate the length of a curve, imagine approximating it with many small segments. Now apply a similarity transformation with a scale factor of k. Each segment will scale by k, so the length of the curve will be k times the original length.

**Question 4** To estimate the area of a non-polygonal region, imagine covering it approximately with a grid of squares of side length s. Piecing together partial squares, suppose you count n squares. Your area estimate is then  $ns^2$ . Now apply a similarity transformation of scale factor k to both the region and the grid. Each square in the scaled grid will have area  $(sk)^2$ , and piecing together partial squares there will be n squares. Thus, we estimate the area of the scaled region to be  $n(sk)^2$ , which is  $k^2$  times the area of the original region.

**Question 5** When n copies of a plane figure can form a figure similar to the original, the figure is called a rep-n-tile. Explain briefly why any parallelogram is a rep-4-tile and also a rep-9-tile.

Free Response: Hint: If a parallelogram is scaled by a factor of 2, then 4 original parallelograms can make the larger parallelogram. If a parallelogram is scaled by a factor of 3, then 9 original parallelograms can make the larger parallelogram. (Draw pictures.)

Question 6 Use formulas to determine what happens to the perimeter and area of a rectangle when it is scaled by k. Begin with a rectangle of base b and height h. After scaling the rectangle by k, the base will be |kb| and the height will be |kh|. The original perimeter is 2b+2h. After scaling, the perimeter will be 2bk+2hkwhich is precisely |k| times the original perimeter. The original area is bh. After scaling, the area will be (kb)(kh), which is precisely  $|k^2|$  times the original area. Question 7 Use formulas to determine what happens to the circumference and area of a circle when it is scaled by k. Begin with a circle of radius r. After scaling the circle by k, its radius will be kr. The original circumference is  $|2\pi r|$ . After scaling, the circumference will be  $2\pi kr$ , which is precisely |k| times the original circumference. The original area is  $\pi r^2$ . After scaling, the area will be  $\pi (kr)^2$ , which is precisely  $|k^2|$  times the original area.

In general, if a parallelogram is scaled by a factor of k, then  $|k^2|$  copies of the

original parallelogram can make the scaled version.