

Set Theory Problems

Short-answer problems about sets.

Problem 1 Given two sets X and Y , $X \cup Y$ is the set of elements that are

Multiple Choice:

- (a) in X or in Y (but not in both).
 - (b) in X or in Y (or both, as the “or” is inclusive). ✓
 - (c) in X and in Y .
 - (d) in X but not in Y .
 - (e) in Y but not in X .
-

Problem 2 Given two sets X and Y , $X \cap Y$ is the set of elements that are

Multiple Choice:

- (a) in X or in Y (but not in both).
 - (b) in X or in Y (or both, as the “or” is inclusive).
 - (c) in X and in Y . ✓
 - (d) in X but not in Y .
 - (e) in Y but not in X .
-

Problem 3 Given two sets X and Y , $X - Y$ is the set of elements that are

Multiple Choice:

- (a) in X or in Y (but not in both).
- (b) in X or in Y (or both, as the “or” is inclusive).

Author(s): Bart Snapp and Brad Findell

- (c) in X and in Y .
- (d) in X but not in Y . ✓
- (e) in Y but not in X .

Problem 4 Explain the difference between the symbols \in and \subseteq .

Free Response: **Hint:** The notation $X \in Y$ means that X is a single element in the set Y . In this case, X is probably not a set. The notation $X \subseteq Y$ requires that both X and Y are sets and, furthermore, that every element of X is also in Y .

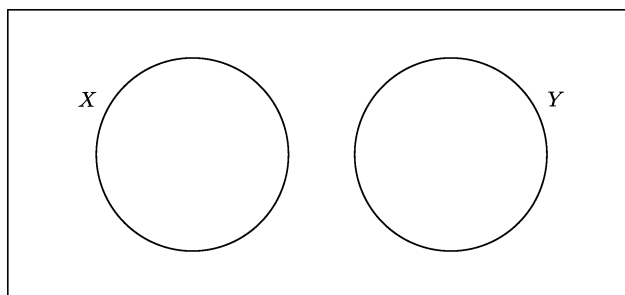
Problem 5 How is $\{\emptyset\}$ different from \emptyset ?

Free Response: **Hint:** The empty set, \emptyset , is a set that contains no elements. That is, $\emptyset = \{\}$. The set $\{\emptyset\}$ contains one element that is itself a set—and that element happens to be the empty set. We could instead write $\{\{\}\}$, but that looks ugly.

Problem 6 Draw a Venn diagram for the set of elements that are in X or Y but not both. How does it differ from the Venn diagram for $X \cup Y$?

Free Response: **Hint:** Same as the Venn diagram for $X \cup Y$, except that the $X \cap Y$ part is not shaded.

Problem 7 If we let X be the set of “right triangles” and we let Y be the set of “equilateral triangles” does the picture below show the relationship between these two sets?



Multiple Choice:

- (a) Yes. ✓
- (b) No.
- (c) Not enough information.

Explain your reasoning.

Free Response: *Hint:* Yes. The picture is accurate because no right triangles are also equilateral triangles.

Problem 8 If $X = \{1, 2, 3, 4, 5\}$ and $Y = \{3, 4, 5, 6\}$ find the following: (List elements in ascending order, separated by commas, with no spaces.)

- (a) $X \cup Y = \{\boxed{1, 2, 3, 4, 5, 6}\}$
- (b) $X \cap Y = \{\boxed{3, 4, 5}\}$
- (c) $X - Y = \{\boxed{1, 2}\}$
- (d) $Y - X = \{\boxed{6}\}$

Problem 9 Let $n\mathbb{Z}$ represent the integer multiples of n . So for example:

$$3\mathbb{Z} = \{\dots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \dots\}$$

Compute the following (use capital Z for \mathbb{Z}):

- (a) $3\mathbb{Z} \cap 4\mathbb{Z} = \boxed{12Z}$
 - (b) $2\mathbb{Z} \cap 5\mathbb{Z} = \boxed{10Z}$
 - (c) $3\mathbb{Z} \cap 6\mathbb{Z} = \boxed{6Z}$
 - (d) $4\mathbb{Z} \cap 6\mathbb{Z} = \boxed{12Z}$
 - (e) $4\mathbb{Z} \cap 10\mathbb{Z} = \boxed{20Z}$
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Problem 10 Make a general rule for intersecting sets of the form $n\mathbb{Z}$ and $m\mathbb{Z}$. Explain why your rule works.

Free Response: **Hint:** The intersection of two sets is what they have in common. The intersection of the set of multiples of n and the set of multiples of m are called common multiples (surprise!), and they are all multiples of the least common multiple of n and m .

Problem 11 If $X \cup Y = X$, what can we say about the relationship between the sets X and Y ? Explain your reasoning.

$(X \subseteq Y / X = Y / Y \subseteq X \checkmark / X = \emptyset)$ because every element of $(X / Y \checkmark)$ must be in $(X \checkmark / Y)$.

Problem 12 If $X \cap Y = X$, what can we say about the relationship between the sets X and Y ? Explain your reasoning.

$(X \subseteq Y \checkmark / X = Y / Y \subseteq X / X = \emptyset)$ because every element of $(X \checkmark / Y)$ must be in $(X / Y \checkmark)$.

Problem 13 If $X - Y = \emptyset$, what can we say about the relationship between the sets X and Y ? Explain your reasoning.

$(X \subseteq Y \checkmark / X = Y / Y \subseteq X / X = \emptyset)$ because every element of $(X \checkmark / Y)$ must be in $(X / Y \checkmark)$.