HW 1: Sets, Vocabulary, and Measuring

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Contents

Set Theory Problems

Short-answer problems.

Problem 1 *Given two sets X and Y, $X \cup Y$ is the set of elements that are*

Multiple Choice:

(a) *in X or in Y (but not in both).*

(b) *in X or in Y (or both, as the "or" is inclusive).*

(c) *in X and in Y.*

(d) *in X but not in Y.*

(e) *in Y but not in X.*

Problem 2 *Given two sets X and Y, $X \cap Y$ is the set of elements that are*

Multiple Choice:

(a) *in X or in Y (but not in both).*

(b) *in X or in Y (or both, as the "or" is inclusive).*

(c) *in X and in Y.*

(d) *in X but not in Y.*

(e) *in Y but not in X.*

Problem 3 *Given two sets X and Y, $X - Y$ is the set of elements that are*

Multiple Choice:

(a) *in X or in Y (but not in both).*

(b) *in X or in Y (or both, as the "or" is inclusive).*

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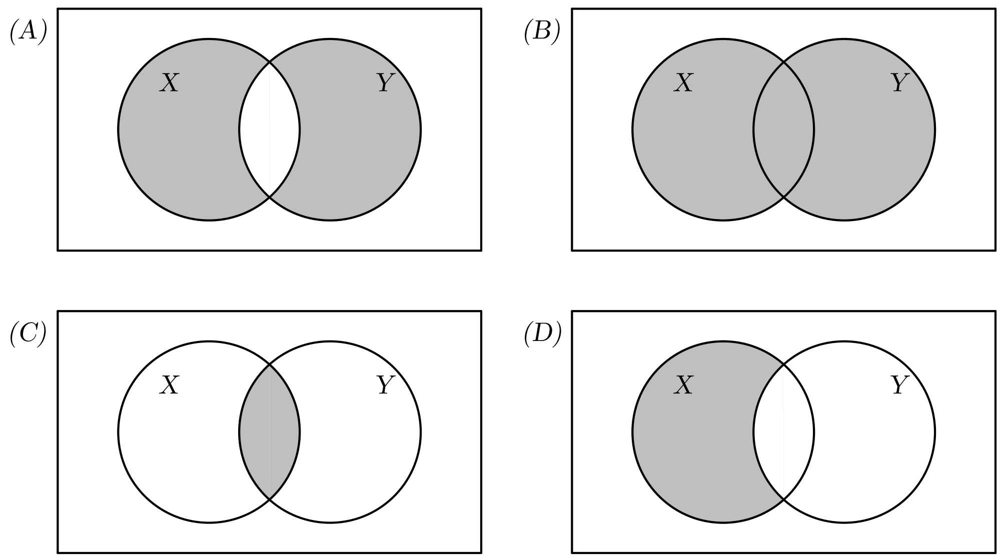
(c) *in X and in Y.*

(d) *in X but not in Y.*

(e) *in Y but not in X.*

Problem 4 *Consider the following Venn diagrams:*



For each set expression below, identify the Venn diagram above in which the corresponding region is shaded:

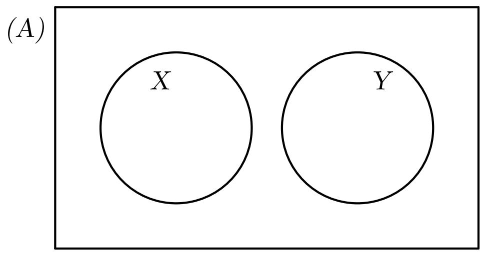
(a) $X \cap Y$ *is figure* $\boxed{?}$

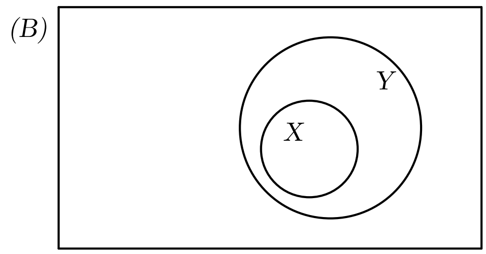
(b) $X \cup Y$ *is figure* $\boxed{?}$.

(c) $X - Y$ *is figure* $\boxed{?}$

Problem 5 *Draw a Venn diagram for the set of elements that are in X or Y* but not both. *How does it differ from the Venn diagram for $X \cup Y$?*

Problem 6 *Consider the following Venn diagrams:*





(a) *If Venn diagram (A) above shows the relationship between sets X and Y, then $X \cap Y =$ (0/ $\emptyset$/ $X \cup Y$) and the sets are said to be (disjoint/ empty / subsets).*

(b) *If Venn diagram (B) above shows the relationship between sets X and Y, then we say that (X and Y are disjoint/ $X \subseteq Y$ / $Y \subseteq X$).*

(c) *If we let X be the set of "right triangles" and we let Y be the set of "equilateral triangles" which diagram above shows the relationship between these two sets?*

Multiple Choice:

(i) *Diagram (A).*

(ii) *Diagram (B).*

(iii) *Neither of these.*

(iv) *Not enough information.*

Explain your reasoning.

Problem 7 *If $X = \{1, 2, 3, 4, 5\}$ and $Y = \{3, 4, 5, 6\}$ find the following: (List elements in ascending order, separated by commas, with no spaces.)*

(a) $X \cup Y = \{\boxed{?}\}$

(b) $X \cap Y = \{\boxed{?}\}$

(c) $X - Y = \{\boxed{?}\}$

(d) $Y - X = \{\boxed{?}\}$

Problem 8 *Let $n\mathbb{Z}$ represent the integer multiples of n. So for example:*

$$3\mathbb{Z} = \{\ldots, -12, -9, -6, -3, 0, 3, 6, 9, 12, \ldots\}$$

Compute the following (use capital Z for $\mathbb{Z}$):

(a) $3\mathbb{Z} \cap 4\mathbb{Z} = \boxed{?}$

(b) $2\mathbb{Z} \cap 5\mathbb{Z} = \boxed{?}$

(c) $3\mathbb{Z} \cap 6\mathbb{Z} = \boxed{?}$

(d) $4\mathbb{Z} \cap 6\mathbb{Z} = \boxed{?}$

(e) $4\mathbb{Z} \cap 10\mathbb{Z} = \boxed{?}$

Problem 9 *Make a general rule for intersecting sets of the form $n\mathbb{Z}$ and $m\mathbb{Z}$. Explain why your rule works.*

Problem 10 *If $X \cup Y = X$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

($X \subseteq Y$ / $X = Y$ / $Y \subseteq X$ / $X = \emptyset$) because every element of (X / Y) must be in (X / Y).

Problem 11 *If $X \cap Y = X$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

($X \subseteq Y$ / $X = Y$ / $Y \subseteq X$ / $X = \emptyset$) because every element of (X / Y) must be in (X / Y).

Problem 12 *If $X - Y = \emptyset$, what can we say about the relationship between the sets X and Y? Explain your reasoning.*

($X \subseteq Y$ / $X = Y$ / $Y \subseteq X$ / $X = \emptyset$) because every element of (X / Y) must be in (X / Y).

Extra Set Theory Problems

More short-answer problems.

Reminders

- Sets are collections of objects such as numbers or points. The objects are called *elements* of the set, and the order elements are listed is not important.
- The notation $\{7, 3\}$ means "The set containing 7 and 3."
- Note that $\{8\}$ is not the same as the number 8 but rather is a set that contains one element that happens to be a number.
- The set containing zero elements, sometimes call the *empty set* is denoted $\{\}$ or $\emptyset$.
- The elements of a set can themselves be sets.

Problem 13 *Indicate the number of elements in each set:*

(a) *The set* $\{3, 5, 6, 9, 10\}$ *has* $\boxed{?}$ *element(s).*

(b) *The set* $\{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$ *has* $\boxed{?}$ *element(s).*

(c) *The set* $\{\{\}\}$ *has* $\boxed{?}$ *element(s).*

(d) *The set* $\{\}$ *has* $\boxed{?}$ *element(s).*

(e) *The set* $\emptyset$ *has* $\boxed{?}$ *element(s).*

(f) *The set* $\{\emptyset\}$ *has* $\boxed{?}$ *element(s).*

Problem 14 *Indicate whether each statement is true or false:*

(a) $2 \in \{3, 2, 5\}$. *(True/ False)*

(b) $2 \subseteq \{3, 2, 5\}$. *(True/ False)*

(c) $\{2\} \in \{3, 2, 5\}$. *(True/ False)*

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(d) $\{2\} \subseteq \{3, 2, 5\}$. *(True/ False)*

(e) $\emptyset = \{\}$. *(True/ False)*

(f) $\emptyset = \{\emptyset\}$. *(True/ False)*

(g) $\{\emptyset\} = \{\{\}\}$. *(True/ False)*

(h) $\emptyset \in \{\emptyset\}$. *(True/ False)*

(i) $\emptyset \subseteq \{\emptyset\}$. *(True/ False)*

(j) $2 \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

(k) $2 \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

(l) $\{2\} \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

(m) $\{2\} \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

(n) $\{\{2\}\} \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

(o) $\{\{2\}\} \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. *(True/ False)*

Problem 15 *Explain the difference between the symbols $\in$ and $\subseteq$.*

Problem 16 *How is $\{\emptyset\}$ different from $\emptyset$?*

Vocabulary and Notation

Short-answer questions.

Question 17 *An **equilateral quadrilateral** is called a* $\boxed{?}$.

Question 18 *An **equiangular quadrilateral** is called a* $\boxed{?}$.

Question 19 *An **regular quadrilateral** is called a* $\boxed{?}$. *(Note: A **regular polygon** is both equilateral and equiangular.)*

Question 20 *If A and B are points, then*

(a) $\overleftrightarrow{AB}$ *denotes a* $\boxed{?}$,

(b) $\overrightarrow{AB}$ *denotes a* $\boxed{?}$ *with endpoint* $\boxed{?}$,

(c) $\overline{AB}$ *denotes a* $\boxed{?}$, *and*

(d) AB *denotes the* $\boxed{?}$ *of* $\overline{AB}$ *or (equivalently) the* $\boxed{?}$ *from A to B.*

Question 21 *As a set of points, an **angle** is the* $\boxed{?}$ *of two* $\boxed{?}$ *with a common* $\boxed{?}$, *which is called the* $\boxed{?}$ *of the angle.*

Question 22 *When two lines intersect so that all four angles are congruent, the angles are said to be* $\boxed{?}$ *and the lines are said to be* $\boxed{?}$.

Question 23 *A* $\boxed{?}$ *measures* 180°. *(Hint: Answer with two words.)*

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Question 24 *Two angles whose measures sum to* 180° *are said to be* $\boxed{?}$.

Question 25 *Two angles whose measures sum to* 90° *are said to be* $\boxed{?}$.

Triangle Area

Short-answer measuring problems.

Problem 26 *Carefully measure heights to compute the area of the triangle below in three ways.*

Geogebra link: https://tube.geogebra.org/m/mjscvuw3

(a) *When AB is the base, the height is* $\boxed{?}$, *so the area is* $\boxed{?}$.

(b) *When BC is the base, the height is* $\boxed{?}$, *so the area is* $\boxed{?}$.

(c) *When CA is the base, the height is* $\boxed{?}$, *so the area is* $\boxed{?}$.

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Measuring by Sight

Short-answer measuring problems.

Instructions

Adjust the figures to fit the given conditions within **eyeball accuracy**. Enter the requested measurements.

Problem 27 *Geogebra link:* `https://tube.geogebra.org/m/gjf28er6`

In figure above, when point C is adjusted so that $\overline{BC}$ is perpendicular to $\overline{AC}$, $AC = \boxed{?}$.

Problem 28 *Geogebra link:* `https://tube.geogebra.org/m/a888zyw2`

In $\triangle ABC$ above, the height to base $\overline{AC}$ is $\boxed{?}$.

Problem 29 *Geogebra link:* `https://tube.geogebra.org/m/kta9hbuf`

In $\triangle ABC$ above, the height to base $\overline{AC}$ is $\boxed{?}$.

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