

Set Theory Problems

Extra problems about sets.

Reminders

- Sets are collections of objects such as numbers or points. The objects are called *elements* of the set, and the order elements are listed is not important.
- The notation $\{7, 3\}$ means “The set containing 7 and 3.”
- Note that $\{8\}$ is not the same as the number 8 but rather is a set that contains one element that happens to be a number.
- The set containing zero elements, sometimes call the *empty set* is denoted $\{\}$ or \emptyset .
- The elements of a set can themselves be sets.

Problem 1 *Indicate the number of elements in each set:*

- (a) The set $\{3, 5, 6, 9, 10\}$ has element(s).
- (b) The set $\{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$ has element(s).
- (c) The set $\{\{\}\}$ has element(s).
- (d) The set $\{\}$ has element(s).
- (e) The set \emptyset has element(s).
- (f) The set $\{\emptyset\}$ has element(s).

Problem 2 *Indicate whether each statement is true or false:*

- (a) $2 \in \{3, 2, 5\}$. (True ✓/ False)
- (b) $2 \subseteq \{3, 2, 5\}$. (True/ False ✓)

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- (c) $\{2\} \in \{3, 2, 5\}$. (True/ False ✓)
- (d) $\{2\} \subseteq \{3, 2, 5\}$. (True ✓/ False)
- (e) $\emptyset = \{\}$. (True ✓/ False)
- (f) $\emptyset = \{\emptyset\}$. (True/ False ✓)
- (g) $\{\emptyset\} = \{\{\}\}$. (True ✓/ False)
- (h) $\emptyset \in \{\emptyset\}$. (True ✓/ False)
- (i) $\emptyset \subseteq \{\emptyset\}$. (True ✓/ False)
- (j) $2 \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. (True/ False ✓)
- (k) $2 \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. (True/ False ✓)
- (l) $\{2\} \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. (True ✓/ False)
- (m) $\{2\} \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. (True/ False ✓)
- (n) $\{\{2\}\} \in \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. (True/ False ✓)
- (o) $\{\{2\}\} \subseteq \{\{3, 2, 7\}, \{4, 5\}, \{2\}, \emptyset\}$. (True ✓/ False)

Problem 3 Explain the difference between the symbols \in and \subseteq .

Free Response: **Hint:** The symbol \in means “is an element of,” whereas \subseteq means “is a subset of.” The notation $X \in Y$ means that X is a single element in the set Y . In this case, X is typically not a set. The notation $X \subseteq Y$, in contrast, requires that both X and Y are sets and, furthermore, that every element of X is also in Y .

Problem 4 How is $\{\emptyset\}$ different from \emptyset ?

Free Response: **Hint:** The empty set, \emptyset , is a set that contains no elements. That is, $\emptyset = \{\}$. The set $\{\emptyset\}$ contains one element that is itself a set—and that element happens to be the empty set. We could instead write $\{\{\}\}$, but that looks ugly.