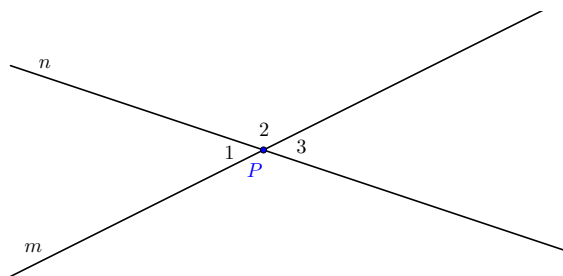


Vertical Angles

Proofs updated.

Problem 1 Point P is the intersection of lines m and n . Prove that $\angle 1 \cong \angle 3$.



Fixnote: When students write equations about linear pairs, they often write two equations for non-overlapping linear pairs—which doesn't help. The figure above is intended to help avoid that dead end, but it would be worthwhile to discuss that dead end anyway.

Fixnote: Below are three different brief proofs that omit details. Which of the details should be included?

- (a) It must be that $\angle 1 \cong \angle 3$ because they are both (complementary/ supplementary ✓/ opposite) to $\angle 2$.

Detail: First write down equations about linear pairs of angles:

$$m\angle 1 + m\angle 2 = 180^\circ$$

$$m\angle 3 + m\angle 2 = 180^\circ$$

By comparing the two equations, it might be clear that $m\angle 1 = m\angle 3$. Other students may need to do some algebra.

- (b) A rotation of 180° about P maps m onto itself, maps n onto itself, and swaps $\angle 1$ and $\angle 3$. Because rotations preserve angle measures, it must be that $\angle 1 \cong \angle 3$.

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Vertical Angles

Detail: Line m is the union of two opposite rays with endpoint P . Check that the 180° rotation about P swaps these opposite rays. The same idea holds for line n so that together the sides of $\angle 1$ become the sides of $\angle 3$ and vice versa.

- (c) *Reflecting about the bisector of $\angle 2$ swaps $\angle 1$ and $\angle 3$. Because reflections preserve angle measures, it follows that $\angle 1 \cong \angle 3$.*

Detail: The reflection swaps the two rays that are the sides of $\angle 2$. Because reflections take lines to lines, that reflection must swap not just the rays but lines m and n .

