Scaling in 2D

Short-answer problems about scaling in two dimensions.

Length and Area Under Scaling

In this section, we explore what happens to length, area, and other measures under scaling.

In a previous section, we defined similarity in terms of basic rigid motions and dilations, and we showed that this definition leads to well-known results about similarity, such as the AA criterion for triangle similarity and consistent ratios of lengths between and within similar figures. A key feature of this discussion was the notion of "scale factor," which describes what happens to lengths under a dilation. From the definition of a dilation, it is clear that segments on lines through the center of dilation scale by the scale factor. We used the side-splitter theorems to show that other segments are scaled by the same scale factor.

Question 1 Two students claims that a 3×5 rectangle and a 4×6 rectangle are similar.

Fred says that that they are similar because the angles are the same. How do you respond?

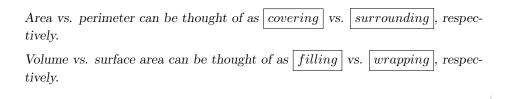
Free Response: Hint: Angles are enough to determine similarity of triangles. But similarity requires a consistent scale factor. For these rectangles the height is scaled by 4/3 whereas the base is scaled by 6/5.

Ned says that they are similar because you can do the same thing (i.e., add 1) to "both sides" of the 3×5 rectangle to get the 4×6 rectangle. How do you respond?

Free Response: Hint: Similarity requires consistent scaling, which is a multiplicative (not additive) relationship.

Question 2 Complete the following sentences using words such as filling, falling, covering, wrapping, hiding, surrounding, or traveling:

Learning outcomes: Author(s): Brad Findell



To explore how measures of figures change under scaling and non-scaling transformations, here are some useful strategies:

- Cutting the figures and rearranging the pieces.
- Using "rep-tiles."
- Using known formulas for perimeters, areas, volumes, or surface areas.
- Approximating with segments, squares, or cubes.
- Shearing.

Question 3 To estimate the length of a curve, imagine approximating it with many small segments. Now apply a similarity transformation with a scale factor of k. Each segment will scale by k, so the length of the curve will be k times the original length.

Question 4 To estimate the area of a non-polygonal region, imagine covering it approximately with a grid of squares of side length s. Piecing together partial squares, suppose you count n squares. Your area estimate is then ns^2 . Now apply a similarity transformation of scale factor k to both the region and the grid. Each square in the scaled grid will have area $(sk)^2$, and piecing together partial squares there will be n squares. Thus, we estimate the area of the scaled region to be $n(sk)^2$, which is k^2 times the area of the original region.

Question 5 When n copies of a plane figure can form a figure similar to the original, the figure is called a rep-n-tile. Explain briefly why any parallelogram is a rep-4-tile and also a rep-9-tile.

Free Response: Hint: If a parallelogram is scaled by a factor of 2, then 4 original parallelograms can make the larger parallelogram. If a parallelogram is scaled by a factor of 3, then 9 original parallelograms can make the larger parallelogram. (Draw pictures.)

Question 6 Use formulas to determine what happens to the perimeter and area of a rectangle when it is scaled by k. Begin with a rectangle of base b and height h. After scaling the rectangle by k, the base will be |kb| and the height will be |kh|. The original perimeter is 2b+2h. After scaling, the perimeter will be 2bk+2hkwhich is precisely |k| times the original perimeter. The original area is bh. After scaling, the area will be (kb)(kh), which is precisely $|k^2|$ times the original area. Question 7 Use formulas to determine what happens to the circumference and area of a circle when it is scaled by k. Begin with a circle of radius r. After scaling the circle by k, its radius will be kr. The original circumference is $|2\pi r|$. After scaling, the circumference will be $2\pi kr$, which is precisely |k| times the original circumference. The original area is πr^2 . After scaling, the area will be $\pi (kr)^2$, which is precisely $|k^2|$ times the original area.

In general, if a parallelogram is scaled by a factor of k, then $|k^2|$ copies of the

original parallelogram can make the scaled version.