Online Exam 2 Review

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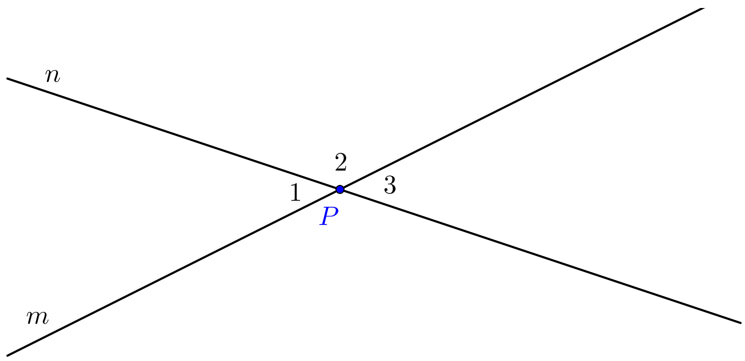
Contents

Vertical Angles

Proofs updated.

Below are three different proofs that vertical angles are congruent. Please consider them separately.

Problem 1 *Point P is the intersection of lines m and n. Prove that $\angle 1 \cong \angle 3$.*



Fix note: When students write equations about linear pairs, they often write two equations for non-overlapping linear pairs—which doesn't help. The figure above is intended to help avoid that dead end, but it would be worthwhile to discuss that dead end anyway.

Proof *Using adjacent angles, $\angle 1 \cong \angle 3$ because they are both (complementary / supplementary / opposite / congruent) to $\angle 2$.* ■

Problem 2 *Point P is the intersection of lines m and n. Prove that $\angle 1 \cong \angle 3$.*

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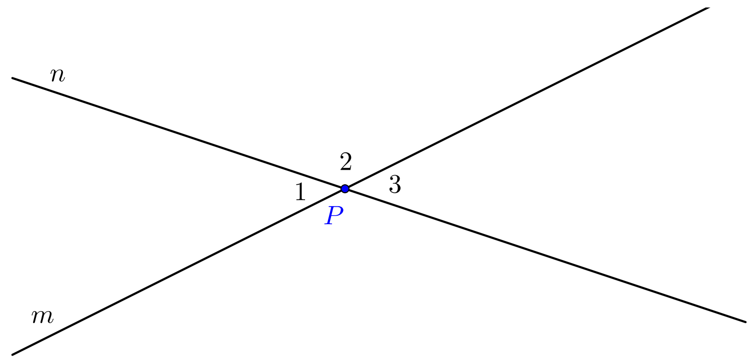
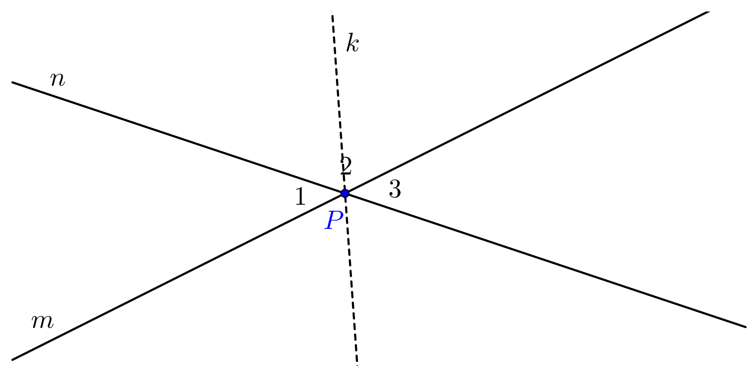


Proof *A rotation of (90° / 180° / 360°) about P maps m onto itself, maps n onto itself, and swaps $\angle 1$ and ($\angle 1$ / $\angle 2$ / $\angle 3$). Because rotations preserve angle measures, it must be that $\angle 1 \cong \angle 3$.* ■

Problem 3 *Point P is the intersection of lines m and n. Prove that $\angle 1 \cong \angle 3$.*



Proof *Reflecting about the (bisector / supplement / opposite) of $\angle 2$ swaps the sides of $\angle 2$ and therefore lines m and n. Thus, that reflection swaps $\angle 1$ and ($\angle 1$ / $\angle 2$ / $\angle 3$). Because reflections preserve angle measures, it follows that $\angle 1 \cong \angle 3$.* ■

Parallel Lines

Proofs updated.

This page develops important results regarding parallel lines and transversals. **Read carefully, and complete the proofs.**

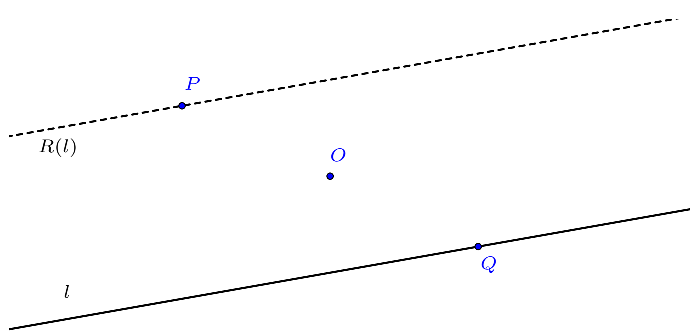
Axiom 1. *Parallel postulate (uniqueness of parallels): Given a line and a point not on the line, there is exactly one line through the given point parallel to the given line.*

Theorem 1. *A 180° rotation about a point on a line takes the line to itself.*

Proof Suppose point P is on line k. The point cuts the line into two opposite $\boxed{?}$. A 180° rotation about P swaps the two opposite rays, thereby mapping the line onto itself. ■

Theorem 2. *A 180° rotation about a point not on a line takes the line to a parallel line.*

Proof Let O be a point not on line l. Let P be an arbitrary point on $R(l)$, the rotated image of l. To show that $R(l)$ is parallel to l, it is sufficient to show that P cannot lie also on l.



Because P is on $R(l)$, there is a point Q on l such that $P = R(Q)$. The rotated image of $\overrightarrow{OQ}$ is ($\overrightarrow{QO}$ / $\overrightarrow{OP}$ / $\overrightarrow{QP}$), and because $\angle QOP$ is 180°, it follows that Q, O, and P are $\boxed{?}$. Call that line k. We know line k is distinct from l because point $\boxed{?}$ is on k but not on l. Now, if P were on l, then points P and Q would be on two distinct lines, k and l, contradicting the assumption that on two points there is a unique line. The theorem is proved. ■

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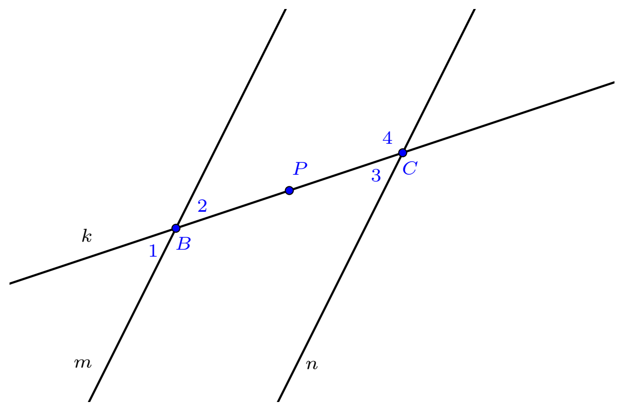
Theorem 3. *If two parallel lines are cut by a transversal, alternate interior angles and corresponding angles are congruent.*

Proof Given that parallel lines m and n are cut by transversal k, prove that alternate interior angles are congruent.



Let B and C be the intersections of transversal k with lines m and n, respectively. Let P be the midpoint of $\overline{BC}$.

(a) Rotate 180° about P, which takes k to (itself/ m/ n).

(b) The rotation maps B to $\boxed{?}$ because $PB = PC$ and the rotation preserves distances.

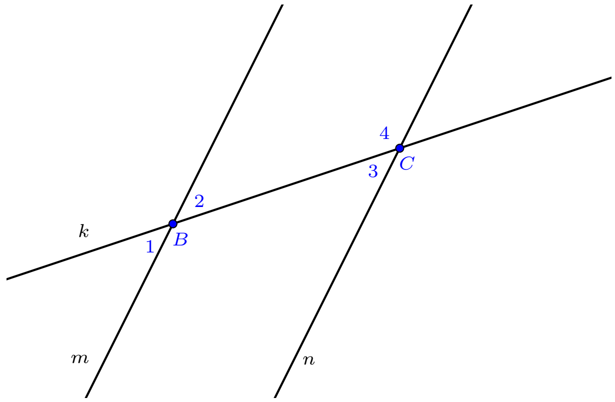
(c) Because P is not on m, the rotation maps m to a parallel line through C, which must be (k/ m/ n) by the uniqueness of parallels.

(d) Thus, the rotation maps $\angle 2$ to ($\angle 1$/ $\angle 2$/ $\angle 3$/ $\angle 4$). These alternate interior angles must be congruent because the rotation preserves angle measures.

■

Note: The congruence of corresponding angles now follows from the congruence of vertical angles. But here is another approach that uses a translation.

Proof Given that parallel lines m and n are cut by transversal k, prove that corresponding angles are congruent.



Let B and C be the intersections of transversal k with lines m and n, respectively.

(a) Translate to the right along line k by distance BC, which takes k to (itself / m/ n).

(b) The translation maps B to $\boxed{?}$, and it maps m to (k / m / n) because the translation maintains parallels, and there is a unique parallel to m through C.

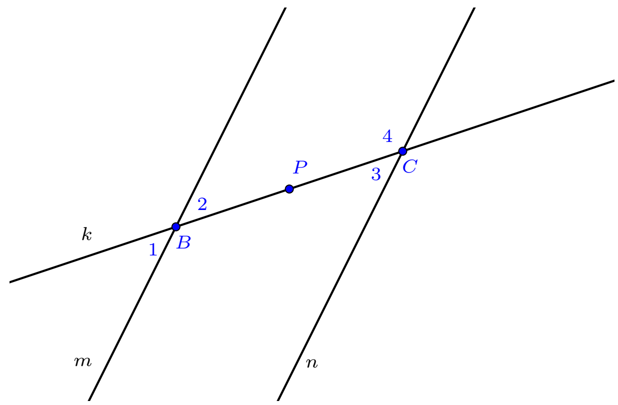
(c) The translation maps $\angle 1$ to ($\angle 1$/ $\angle 2$/ $\angle 3$/ $\angle 4$). These corresponding angles must be congruent because the translation preserves angle measures.

■

Theorem 4. *If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.*

Note: *This theorem is the* $\boxed{?}$ *of the previous theorem about alternate interior angles.*

Proof Given that m and n are cut by transversal k with alternate interior angles congruent, prove that lines m and n are parallel.



Let B and C be the intersections of transversal k with lines m and n, respectively. Let P be the midpoint of $\overline{BC}$.

(a) Rotate 180° about P, which takes k to (itself/ m/ n), and which swaps B and $\boxed{?}$ because distances are preserved.

(b) Because $\angle 2 \cong \angle 3$ and because a side of $\angle 2$ (i.e., $\overrightarrow{BP}$) is mapped to a side of $\angle 3$ (i.e., ($\overrightarrow{CP}$/ $\overrightarrow{PC}$/ $\overrightarrow{BP}$)), it must be that the other side of $\angle 2$ (which lies on m) is mapped to the other side of $\angle 3$ (which lies on line $\boxed{?}$). Thus, n is the image of m.

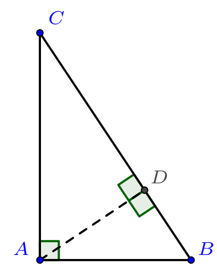
(c) Because P is not on m, the 180° rotation maps m to a parallel line through C. Thus, n must be parallel to m.

■

Similar Right Triangles

Proofs.

Problem 4 *Adapted from Ohio's 2017 Geometry released item 17.*



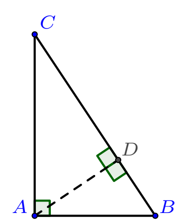
Complete the following proof that $\triangle DAC$ *is similar to* $\triangle DBA$*:*

(a) $\triangle DBA \sim \triangle\boxed{?}$ *by (AA similarity / CPCTC / right triangle similarity) because they share* $\angle B$ *and they each have a right angle.*

(b) $\triangle DAC \sim \triangle\boxed{?}$ *for the same reason because they share (*$\angle A$ */* $\angle B$ */* $\angle C$*) and they each have a right angle.*

(c) $\triangle DAC \sim \triangle DBA$ *because (CPCTC / right triangle similarity / they are both similar to* $\triangle ABC$*).*

Problem 5 *A different proof, also adapted from Ohio's 2017 Geometry released item 17.*



Complete the following proof that $\triangle DAC$ *is similar to* $\triangle DBA$*:*

Author(s): Brad Findell

(a) $\angle B$ and $\angle BAD$ are $\boxed{?}$ *because they are acute angles in a right triangle.*

(b) $\angle DAC$ and $\angle BAD$ *are complementary because they are adjacent angles that form* $\angle BAC$, *which is (right/ acute/ obtuse).*

(c) $\angle B \cong \angle DAC$ *because they are both complementary to* $\angle BAD$.

(d) $\triangle DAC \sim \triangle DBA$ *by (AA similarity/ CPCTC/ right triangle similarity) because* $\angle B \cong \angle DAC$ *and they each have a right angle.*

Trigonometry Checkup

This activity is intended to remind you of key ideas from high school trigonometry.

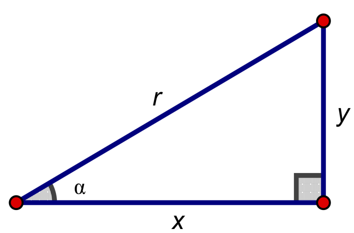
Problem 6 *What are the ratios of side lengths in a 45°-45°-90° triangle from shortest to longest, using 1 for the shortest?* $1 : \boxed{?} : \boxed{?}$. *(Hint: Type* `sqrt(2)` *for* $\sqrt{2}$*.)*

Explain where the ratios come from, including why they work for any such triangle, no matter what size.

Problem 7 *What are the ratios of side lengths in a 30°-60°-90° triangle, from shortest to longest, using 1 for the shortest?* $1 : \boxed{?} : \boxed{?}$.

Explain where the ratios come from.

Problem 8 *Consider the right triangle below with an angle of α, sides of length x and y, and hypotenuse of length r, as labeled.*



(a) *Using the triangle above (and your memory of Precalculus), write down the side-length ratios for sine, cosine, and tangent:*

$$\sin\alpha = \boxed{?}, \quad \cos\alpha = \boxed{?}, \quad \tan\alpha = \boxed{?}$$

(b) *What values of α, measured in degrees, make sense in* right triangle trigonometry*? (We overcome these bounds for circular trigonometry.)*

$$\boxed{?} \leqslant \alpha < \boxed{?}$$

Author(s): Bart Snapp and Brad Findell

Problem 9 *For the problem and triangle above, ...*

(a) *If we imagine angle α is fixed, why are ratios of pairs of side lengths the same, no matter the size of the triangle?*

(b) *What does it mean to say that these ratios depend upon the angle α?*

(c) *Why is only one of the triangle's three angles necessary in defining these ratios?*

Problem 10 *Use your work so far to find the following trigonometric ratios:*

(a) $\sin 30^\circ = \boxed{?}$, $\cos 30^\circ = \boxed{?}$, $\tan 30^\circ = \boxed{?}$.

(b) $\sin 45^\circ = \boxed{?}$, $\cos 45^\circ = \boxed{?}$, $\tan 45^\circ = \boxed{?}$.

(c) $\sin 60^\circ = \boxed{?}$, $\cos 60^\circ = \boxed{?}$, $\tan 60^\circ = \boxed{?}$.

(d) $\sin 0^\circ = \boxed{?}$, $\cos 0^\circ = \boxed{?}$, $\tan 0^\circ = \boxed{?}$.

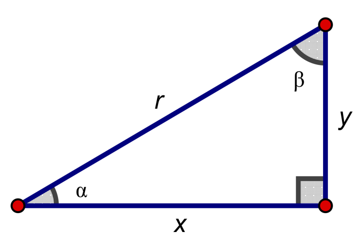
Problem 11 *You may recall the identity* $\sin^2\vartheta + \cos^2\vartheta = 1$.

(a) *Explain why the equation is true.*

(b) *Why is it called an identity?*

(c) *Why is it called a Pythagorean identity?*

Problem 12 *In right triangle trigonometry, there are two acute angles, as shown in the figure below.*



(a) *How are the angles α and β related? Explain why.*

(b) *Using lengths in the above triangle, find the following ratios:*

$$\sin\alpha = \boxed{?} \qquad \cos\alpha = \boxed{?}$$

$$\sin\beta = \boxed{?} \qquad \cos\beta = \boxed{?}$$

(c) *We have shown that when angles α and β are complementary,*

$\sin\alpha = \boxed{?}$. *Enter* $\cos\alpha$, $\sin\beta$, *or* $\cos\beta$. *Type alpha for* α, *beta for* β.

$\cos\alpha = \boxed{?}$. *Enter* $\sin\alpha$, $\sin\beta$, *or* $\cos\beta$.

(d) *Explain why the result makes sense.*

Given an angle and a side length of a right triangle, you can find the missing side lengths. This is called "solving the right triangle." And given the sine, cosine, or tangent of an angle, you can find the other two ratios.

Problem 13 *Suppose* $\sin\alpha = \dfrac{3}{5}$. *Then* $\cos\alpha = \boxed{?}$, $\tan\alpha = \boxed{?}$.

Problem 14 *Ethan stands 120 feet from the trunk of a tree (along flat ground). He measures that his line of sight to the top of the tree is at an angle of* 53° *from horizontal. How tall is the tree?* $\boxed{?}$

Explain your reasoning.