
Key Proofs

Bart Snapp and Brad Findell

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Contents

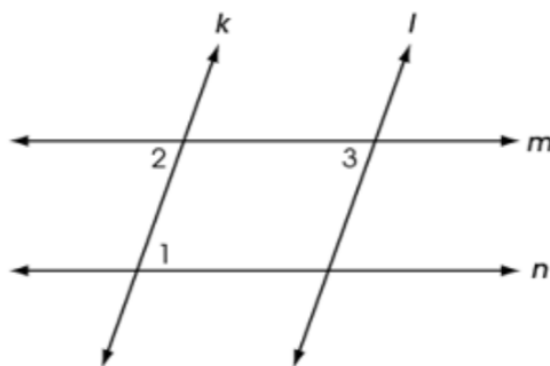
Parallelogram	3
The Isosceles Triangle Theorem	4
Inscribed Angles	7
Similar Right Triangles	9

Parallelogram

Proof.

Problem 1 Adapted from Ohio's 2017 Geometry released item 13.

Two pairs of parallel lines intersect to form a parallelogram as shown.



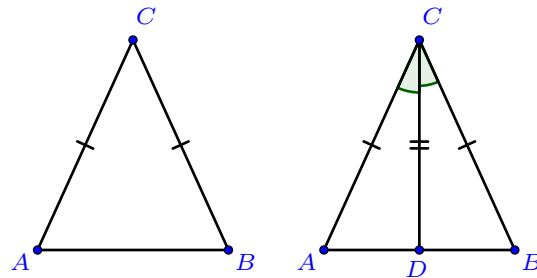
Complete the following proof that opposite angles of a parallelogram are congruent:

- (a) $\angle 1 \cong \angle 2$ as (opposite angles / alternate interior angles / corresponding angles) for parallel lines (m and n / k and l).
- (b) $\angle 3 \cong \angle 2$ as (opposite angles / alternate interior angles / corresponding angles) for parallel lines (m and n / k and l).
- (c) Then $\angle 1 \cong \angle 3$ because they are both congruent to $\angle 2$.

The Isosceles Triangle Theorem

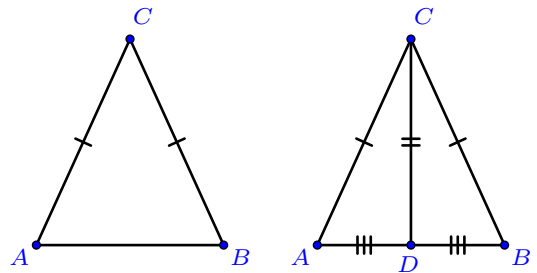
Proofs.

Problem 2 Prove that the base angles of an isosceles triangle are congruent.



- (a) Beginning with the given figure on the left, Morgan draws \overline{CD} and marks the figure intending that this new segment is a(n) (median/ angle bisector / perpendicular bisector/ altitude).
- (b) Based on the marked figure, Morgan claims that the $\triangle ACD \cong \triangle \boxed{?}$ by (SAS/ SSS/ SSA/ ASA/ HL).
- (c) Finally, Morgan concludes that $\angle A \cong \angle \boxed{?}$, as they are corresponding parts of congruent triangles.

Problem 3 Prove that the base angles of an isosceles triangle are congruent.

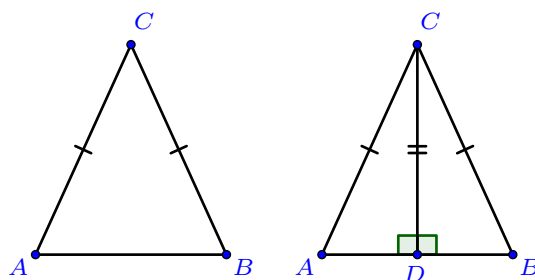


Author(s): Brad Findell

The Isosceles Triangle Theorem

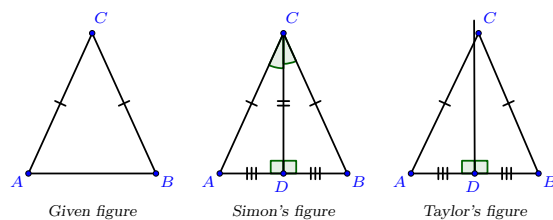
- (a) Beginning with the given figure on the left, Deja draws \overline{CD} and marks the figure intending that this new segment is a(n) (median / angle bisector / perpendicular bisector / altitude).
- (b) Based on the marked figure, Deja claims that the $\triangle ACD \cong \triangle \square$ by (SAS / SSS / SSA / ASA / HL).
- (c) Finally, Deja concludes that $\angle A \cong \angle \square$, as they are corresponding parts of congruent triangles.

Problem 4 Prove that the base angles of an isosceles triangle are congruent.



- (a) Beginning with the given figure on the left, Elle draws \overline{CD} and marks the figure intending that this new segment is a(n) (median / angle bisector / perpendicular bisector / altitude).
- (b) Based on the marked figure, Deja claims that the $\triangle ACD \cong \triangle \square$ by (SAS / SSS / SSA / ASA / HL).
- (c) Finally, Deja concludes that $\angle A \cong \angle \square$, as they are corresponding parts of congruent triangles.

Problem 5 Simon and Taylor are trying to prove that the base angles of an isosceles triangle are congruent.



The Isosceles Triangle Theorem

Beginning with the given figure on the left, Simon draws \overline{CD} and marks the second figure intending that this new segment is a perpendicular bisector of \overline{AB} .

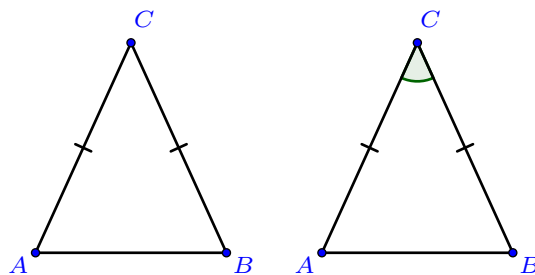
Taylor claims that a perpendicular bisector of a side of a triangle usually misses the opposite vertex. So without using properties of isosceles triangles, the figure must allow for that possibility.

Choose the best response to their argument:

Multiple Choice:

- (a) Simon is correct, and $\triangle ACD \cong \triangle BCD$ by SAS.
- (b) Simon is correct, and $\triangle ACD \cong \triangle BCD$ by SSS
- (c) Taylor is correct, and the perpendicular bisector cannot be used to complete this proof.
- (d) Neither of them are correct.

Problem 6 Prove that the base angles of an isosceles triangle are congruent.



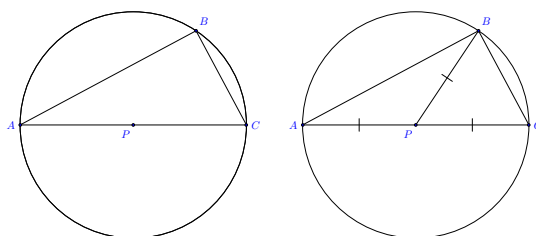
Given figure

- (a) Examining the given figure on the left, Lissy notices symmetry in the triangle and claims that the triangle is congruent to itself by a (translation / reflection / rotation).
- (b) Based on the marked figure, Lissy claims that the $\triangle ACB \cong \triangle \boxed{?}$ by (SAS / SSS / SSA / ASA / HL).
- (c) Finally, Lissy concludes that $\angle A \cong \angle \boxed{?}$, as they are corresponding parts of congruent triangles.

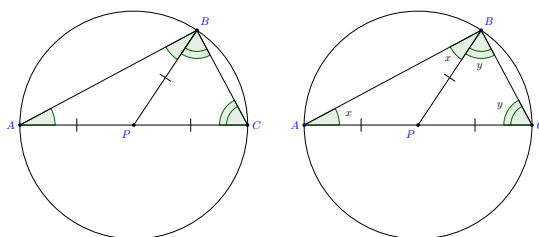
Inscribed Angles

Proofs.

Problem 7 In the figure below, \overline{AB} is a diameter of a circle with center P . Prove that $\angle B$ is a right angle.



- (a) Beginning with the diagram on the left, Natalia draws \overline{PB} and marks the diagram to show segments that she knows to be congruent because each one is a ? of the circle.



- (b) Natalia sees that $\triangle APB$ and $\triangle BPC$ are ? triangles, so she marks the figure to show angles that must be congruent. (Note: Do we need a statement or citation of the theorem?)
- (c) In order to do some algebra with these congruent angles, Natalia labels their measures x and y , as shown in the picture on the right.
- (d) She writes an equation for the sum of the angles of $\triangle ABC$:

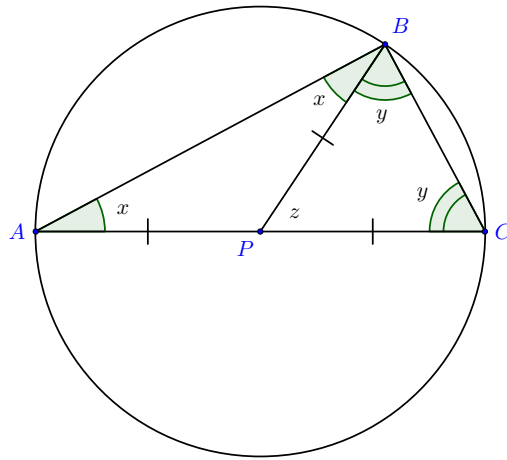
$$\boxed{?} = 180^\circ$$

(Note: Need a question about dividing the equation by 2.)

Author(s): Brad Findell

- (e) Since $m\angle B = \boxed{?}$, she concludes that $m\angle B = 90^\circ$. (Note: Should call it $\angle ABC$ because of the new segment, or maybe note this earlier.)

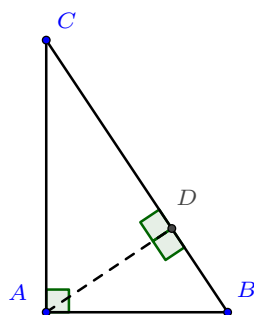
Problem 8 New problem about relationship between inscribed angle and central angle.



Similar Right Triangles

Proofs.

Problem 9 Adapted from Ohio's 2017 Geometry released item 17.



Complete the following proof that $\triangle DAC$ is similar to $\triangle DBA$:

- (a) $\triangle ABC \sim \triangle \square$ by AA because they share $\angle B$ and they each have a right angle.
- (b) $\triangle ABC \sim \triangle \square$ by AA because they share $\angle C$ and they each have a right angle.
- (c) $\triangle DAC \sim \triangle \square$ because they are both similar to $\triangle ABC$.
