Key Proofs

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Part I

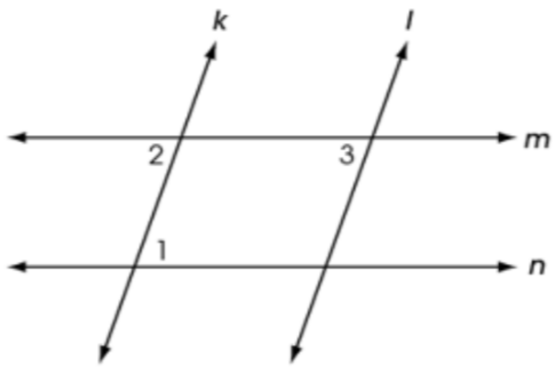
Math 1

Parallelogram

Proof.

Problem 1 *Adapted from Ohio's 2017 Geometry released item 13.*

Two pairs of parallel lines intersect to form a parallelogram as shown.



Complete the following proof that opposite angles of a parallelogram are congruent:

(a) $\angle 1 \cong \angle 2$ *as (opposite angles / alternate interior angles / corresponding angles) for parallel lines (m and n / k and l).*

(b) $\angle 3 \cong \angle 2$ *as (opposite angles / alternate interior angles / corresponding angles)for parallel lines (m and n / k and l).*

(c) *Then* $\angle 1 \cong \angle 3$ *because they are both congruent to* $\angle 2$*.*

Problem 2 *Adapted from Ohio's 2018 Geometry released item 21.*

Given the parallelogram $WXYZ$, prove that $\overline{WX} \cong \overline{YZ}$.

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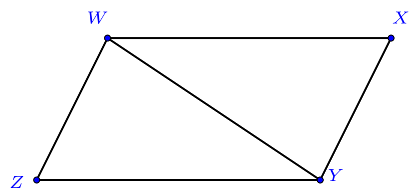
Parallelogram



Complete the proof below:

(a) $\angle ZWY \cong \angle XYW$ *as alternate interior angles for parallel segments* $\overline{WZ}$ *and* $\overline{XY}$

(b) $\angle ZYW \cong \angle XWY$ *as alternate interior angles for parallel segments* $\overline{WX}$ *and* $\overline{YZ}$.

(c) $\overline{WY} \cong \overline{WY}$ *because a segment is congruent to itself.*

(d) $\triangle WYZ \cong \triangle YWX$ *by the ASA criterion.*

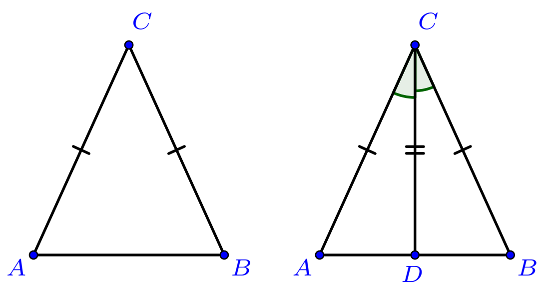
(e) *Then* $\overline{WX} \cong \overline{YZ}$ *as corresponding parts of congruent triangles.*

Fixnote: Use drop-down menus. Maybe number the angles.

The Isosceles Triangle Theorem

Proofs.

Problem 3 *Prove that the base angles of an isosceles triangle are congruent.*

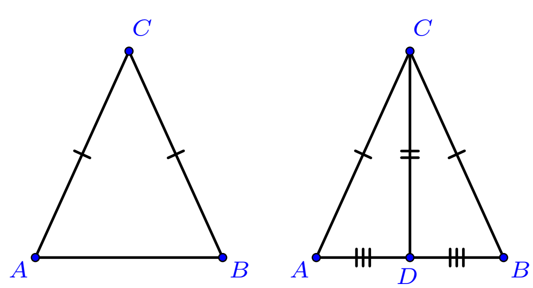


(a) *Beginning with the given figure on the left, Morgan draws $\overline{CD}$ and marks the figure intending that this new segment is a(n) (median/ angle bisector / perpendicular bisector/ altitude).*

(b) *Based on the marked figure, Morgan claims that the $\triangle ACD \cong \triangle\boxed{?}$ by (SAS/ SSS/ SSA/ ASA/ HL).*

(c) *Finally, Morgan concludes that $\angle A \cong \angle\boxed{?}$, as they are corresponding parts of congruent triangles.*

Problem 4 *Prove that the base angles of an isosceles triangle are congruent.*



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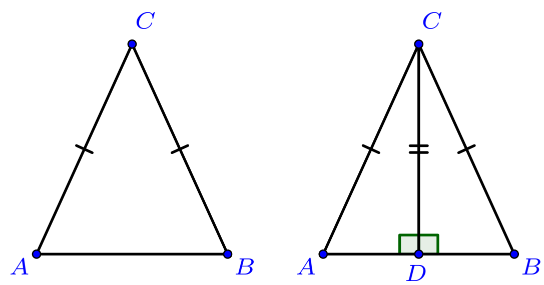
(a) *Beginning with the given figure on the left, Deja draws* $\overline{CD}$ *and marks the figure intending that this new segment is a(n) (median/ angle bisector/ perpendicular bisector/ altitude).*

(b) *Based on the marked figure, Deja claims that the* $\triangle ACD \cong \triangle\boxed{?}$ *by (SAS / SSS/ SSA/ ASA/ HL).*

(c) *Finally, Deja concludes that* $\angle A \cong \angle\boxed{?}$*, as they are corresponding parts of congruent triangles.*

Problem 5 *Prove that the base angles of an isosceles triangle are congruent.*

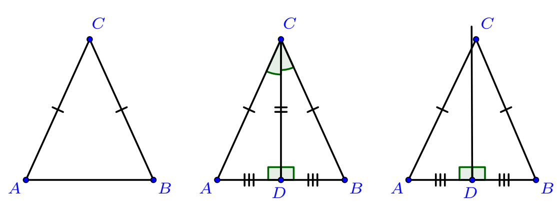


(a) *Beginning with the given figure on the left, Elle draws* $\overline{CD}$ *and marks the figure intending that this new segment is a(n) (median/ angle bisector/ perpendicular bisector/ altitude).*

(b) *Based on the marked figure, Deja claims that the* $\triangle ACD \cong \triangle\boxed{?}$ *by (SAS / SSS/ SSA/ ASA/ HL).*

(c) *Finally, Deja concludes that* $\angle A \cong \angle\boxed{?}$*, as they are corresponding parts of congruent triangles.*

Problem 6 *Simon and Taylor are trying to prove that the base angles of an isosceles triangle are congruent.*



Given figure *Simon's figure* *Taylor's figure*

Beginning with the given figure on the left, Simon draws $\overline{CD}$ and marks the second figure intending that this new segment is a perpendicular bisector of $\overline{AB}$.

Taylor claims that a perpendicular bisector of a side of a triangle usually misses the opposite vertex. So without using properties of isosceles triangles, the figure must allow for that possibility.

Choose the best response to their argument:

Multiple Choice:

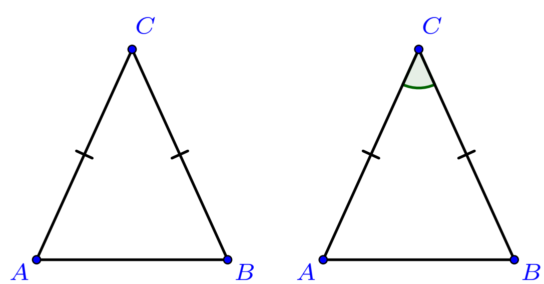
(a) *Simon is correct, and $\triangle ACD \cong \triangle BCD$ by SAS.*

(b) *Simon is correct, and $\triangle ACD \cong \triangle BCD$ by SSS*

(c) *Taylor is correct, and the perpendicular bisector cannot be used to complete this proof.*

(d) *Neither of them are correct.*

Problem 7 *Prove that the base angles of an isosceles triangle are congruent.*



Given figure

(a) *Examining the given figure on the left, Lissy notices symmetry in the triangle and claims that the triangle is congruent to itself by a (translation / reflection/ rotation).*

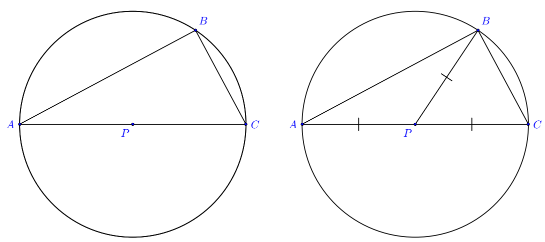
(b) *Based on the marked figure, Lissy claims that the $\triangle ACB \cong \triangle\boxed{?}$ by (SAS/ SSS/ SSA/ ASA/ HL).*

(c) *Finally, Lissy concludes that $\angle A \cong \angle\boxed{?}$, as they are corresponding parts of congruent triangles.*

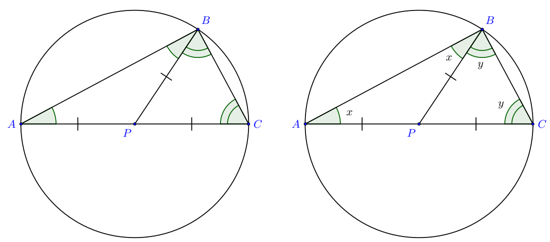
Inscribed Angles

Proofs.

Problem 8 *In the figure below, $\overline{AB}$ is a diameter of a circle with center P. Prove that $\angle B$ is a right angle.*



(a) *Beginning with the diagram on the left, Natalia draws $\overline{PB}$ and marks the diagram to show segments that she knows to be congruent because each one is a* $\boxed{?}$ *of the circle.*



(b) *Natalia sees that $\triangle APB$ and $\triangle BPC$ are* $\boxed{?}$ *triangles, so she marks the figure to show angles that must congruent.*

Fixnote: Do we need a statement or citation of the theorem?

(c) *In order to do some algebra with these congruent angles, Natalia labels their measures x and y, as shown in the picture on the right.*

(d) *She writes an equation for the sum of the angles of $\triangle ABC$:*

$$\boxed{?} = 180°$$

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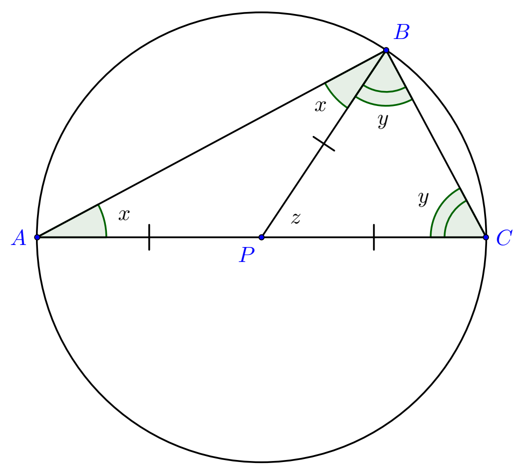
Fixnote: Need a prompt about dividing the equation by 2.

(e) *Since* $m\angle B = \boxed{?}$, *she concludes that* $m\angle B = 90^\circ$.

Fixnote: Should call it $\angle ABC$ *because of the new segment. Or maybe note this earlier.*

Problem 9 *Fixnote: New problem about relationship between inscribed angle and central angle.*



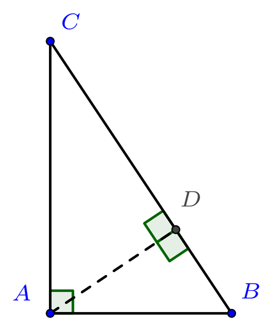
Part II

Math 2

Similar Right Triangles

Proofs.

Problem 10 *Adapted from Ohio's 2017 Geometry released item 17.*



Complete the following proof that $\triangle DAC$ *is similar to* $\triangle DBA$:

(a) $\triangle ABC \sim \triangle\boxed{?}$ *by AA because they share* $\angle B$ *and they each have a right angle.*

(b) $\triangle ABC \sim \triangle\boxed{?}$ *by AA because they share* $\angle C$ *and they each have a right angle.*

(c) $\triangle DAC \sim \triangle\boxed{?}$ *because they are both similar to* $\triangle ABC$.

> *Fixnote: Need to prompt for AA in the first two steps.*
> *The 2017 EOC item calls AA a postulate, which it is not.*
> *Should AA be called a criterion? a theorem? a condition?*

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