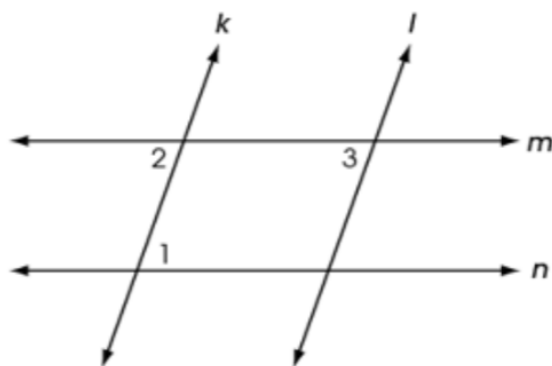


Quadrilaterals

Proof.

Problem 1 Adapted from Ohio's 2017 Geometry released item 13.

Two pairs of parallel lines intersect to form a parallelogram as shown.



Complete the following proof that opposite angles of a parallelogram are congruent:

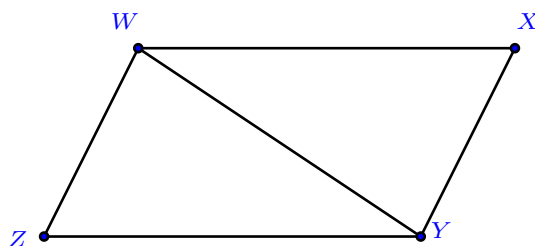
- (a) $\angle 1 \cong \angle 2$ as (opposite angles / alternate interior angles ✓ / corresponding angles) for parallel lines (m and n ✓ / k and l).
- (b) $\angle 3 \cong \angle 2$ as (opposite angles / alternate interior angles / corresponding angles ✓) for parallel lines (m and n / k and l ✓).
- (c) Then $\angle 1 \cong \angle 3$ because they are both congruent to $\angle 2$.

Problem 2 Adapted from Ohio's 2018 Geometry released item 21.

Given the parallelogram $WXYZ$, prove that $\overline{WX} \cong \overline{YZ}$.

Learning outcomes:

Author(s): Brad Findell



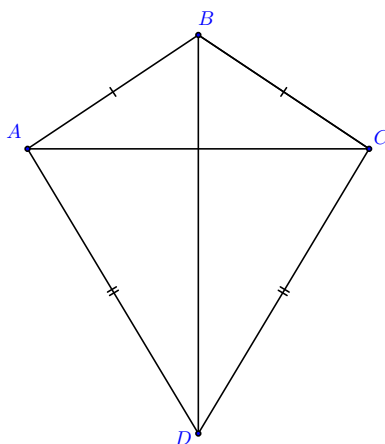
Fix note: It really would help to have an online environment that allows students to mark diagrams.

Complete the proof below:

- (a) $\angle ZWY \cong \angle XYW$ as (alternate interior angles ✓/ corresponding angles/ opposite angles) for parallel segments (\overline{WZ} and \overline{XY} ✓/ \overline{WX} and \overline{YZ}).
- (b) $\angle ZYW \cong \angle XWY$ for the same reason, this time for parallel segments (\overline{WZ} and \overline{XY} / \overline{WX} and \overline{YZ} ✓).
- (c) $\overline{WY} \cong \overline{YW}$ because a segment is congruent to itself.
- (d) $\triangle WYZ \cong \triangle YWX$ by (SAS/ ASA ✓/ SSS).
- (e) Then $\overline{YZ} \cong \overline{WX}$ as corresponding parts of congruent triangles.

Fix note: Maybe number the angles.

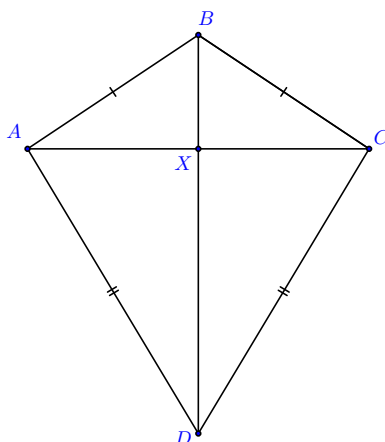
Problem 3 Quadrilateral $ABCD$ is a kite as marked. Prove that \overleftrightarrow{BD} is the perpendicular bisector of \overline{AC} .



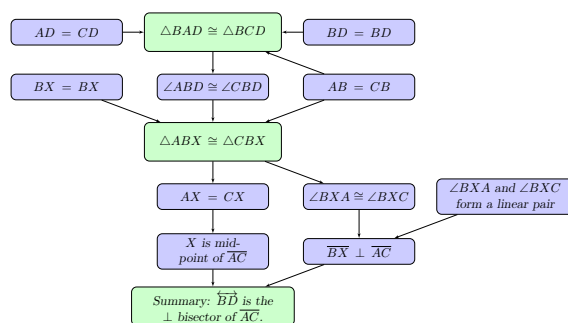
Key theorem: The points on a perpendicular bisector are exactly those that are equidistant from the endpoints of a segment.

Proof: Because B and D are each equidistant from A and C , they each must lie on the perpendicular bisector of segment AC , which implies that \overleftrightarrow{BD} is its perpendicular bisector.

Problem 4 Quadrilateral $ABCD$ is a kite as marked. Prove that \overleftrightarrow{BD} is the perpendicular bisector of \overline{AC} .



A proof that makes use of triangle congruence:



Fix note: Do we need a step about \overleftrightarrow{BX} and \overleftrightarrow{BD} being the same line?

In the proof above, $\triangle BAD \cong \triangle BCD$ by \boxed{SSS} , and $\triangle ABX \cong \triangle CBX$ by \boxed{SAS} .

Detail: Paragraph proof:

$\overline{BD} \cong \overline{BD}$, so that $\triangle BAD \cong \triangle BCD$ by SSS.

$\angle ABD \cong \angle CBD$ by CPCTC.

$\overline{BX} \cong \overline{BX}$, so that $\triangle ABX \cong \triangle CBX$ by SAS.

$\angle BXA \cong \angle BXC$ by CPCTC, and they are a linear pair, so $\overline{BX} \perp \overline{AC}$.

$\overline{AX} \cong \overline{CX}$ by CPCTC, so X is the midpoint of \overline{AC} .

Thus, \overleftrightarrow{BD} is the perpendicular bisector of \overline{AC} .

