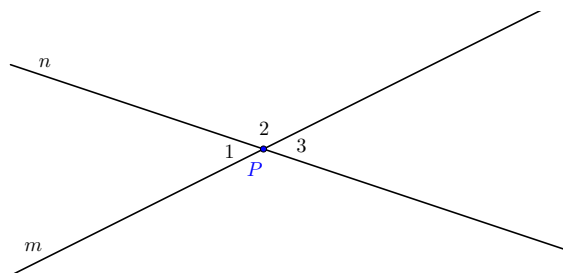


# Vertical Angles

Three proofs.

Below are three different proofs that vertical angles are congruent. Please consider them separately.

**Problem 1** Point  $P$  is the intersection of lines  $m$  and  $n$ . Prove that  $\angle 1 \cong \angle 3$ .



*Fix note: When students write equations about linear pairs, they often write two equations for non-overlapping linear pairs—which doesn't help. The figure above is intended to help avoid that dead end, but it would be worthwhile to discuss that dead end anyway.*

**Proof** Using adjacent angles,  $\angle 1 \cong \angle 3$  because they are both (complementary / supplementary ✓ / opposite / congruent) to  $\angle 2$ . ■

**Feedback(correct):** Additional detail: First write down equations about linear pairs of angles:

$$m\angle 1 + m\angle 2 = \boxed{180} \text{ degrees}$$

$$m\angle 3 + m\angle 2 = \boxed{180} \text{ degrees}$$

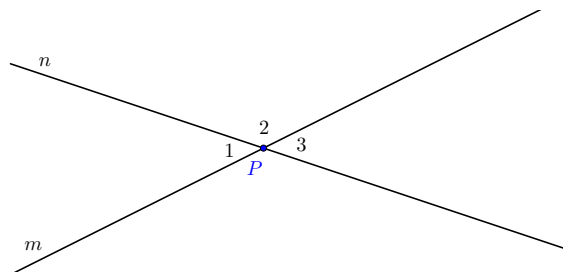
By comparing the two equations, one might see that  $m\angle 1 = m\angle 3$ . Alternatively, one may do some algebra to conclude that  $m\angle 1 = 180^\circ - m\angle 2 = m\angle 3$ , which is essentially what the one-sentence proof says.

**Problem 2** Point  $P$  is the intersection of lines  $m$  and  $n$ . Prove that  $\angle 1 \cong \angle 3$ .

Learning outcomes:

Author(s): Brad Findell

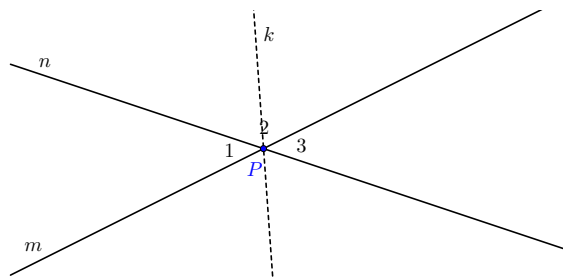
## Vertical Angles



**Proof** A rotation of  $(90^\circ / 180^\circ \checkmark / 360^\circ)$  about  $P$  maps  $m$  onto itself, maps  $n$  onto itself, and swaps  $\angle 1$  and  $(\angle 1 / \angle 2 / \angle 3 \checkmark)$ . Because rotations preserve angle measures, it must be that  $\angle 1 \cong \angle 3$ . ■

**Feedback(correct):** Additional detail: Line  $m$  is the union of two opposite rays with endpoint  $P$ . The rotation about  $P$  swaps these opposite rays, and the same idea holds for line  $n$ . That rotation maps the sides of  $\angle 1$  onto the sides of  $\angle 3$  and vice versa.

**Problem 3** Point  $P$  is the intersection of lines  $m$  and  $n$ . Prove that  $\angle 1 \cong \angle 3$ .



**Proof** Reflecting about the (bisector  $\checkmark$  / supplement / opposite) of  $\angle 2$  swaps the sides of  $\angle 2$  and therefore lines  $m$  and  $n$ . Thus, that reflection swaps  $\angle 1$  and  $(\angle 1 / \angle 2 / \angle 3 \checkmark)$ . Because reflections preserve angle measures, it follows that  $\angle 1 \cong \angle 3$ . ■

**Feedback(correct):** Additional detail: Because reflections take lines to lines, the reflection that swaps the sides of  $\angle 2$  must swap not just the rays but lines  $m$  and  $n$ , which contain the rays.