# Parallel Lines

*Proofs updated.*

Parallel postulate (uniqueness of parallels): Given a line and a point not on the line, there is exactly one line through the given point parallel to the given line.

Theorems to prove:

(a) A $180°$ rotation about a point on a line takes the line to itself.

(b) A $180°$ rotation about a point not on a line takes the line to a parallel line.

(c) If two parallel lines are cut by a transversal alternate interior (and corresponding angles) are congruent.

(d) If two lines are cut by a transversal so that alternate interior (or corresponding) angles are congruent, then the lines are parallel.

**Problem    1**    *Given that parallel lines $m$ and $n$ are cut by transversal $k$, prove that alternate interior angles are congruent.*
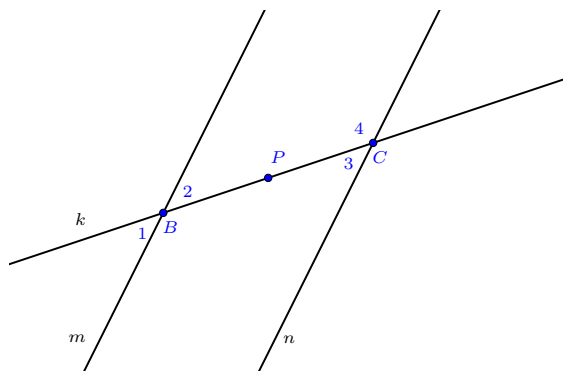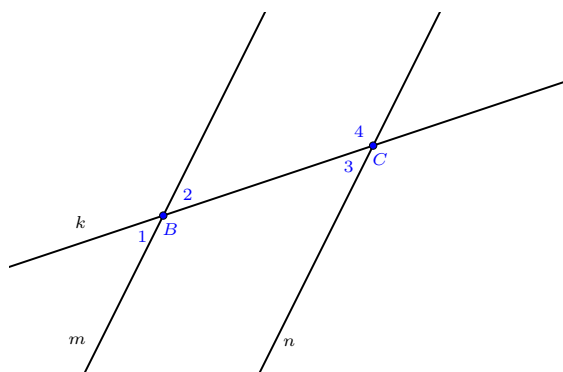


(a) *Let $B$ and $C$ be the intersections of transversal $k$ with lines $m$ and $n$, respectively. Let $P$ be the midpoint of $\overline{BC}$*

(b) *Rotate $180°$ about $P$, which takes $k$ to itself.*

(c) *The rotation maps $B$ to $C$ and $C$ to $B$ because distances are preserved.*

(d) *The rotation maps $m$ to a parallel line through $C$, which must be ($k$ / $m$ / $n$ ✓) by the uniqueness of parallels.*

Author(s): Brad Findell

(e) *The rotation maps $n$ to $(k/m \checkmark/n)$ by the same reasoning.*

(f) *The rotation swaps $\angle 2$ and $(\angle 1/\angle 2/\angle 3 \checkmark/\angle 4)$. These alternate interior angles must be congruent because the rotation preserves angle measures.*

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**Problem    2**   *Given that parallel lines $m$ and $n$ are cut by transversal $k$, prove that corresponding angles are congruent.*



(a) *Let $B$ and $C$ be the intersections of transversal $k$ with lines $m$ and $n$, respectively.*

(b) *Translate to the right along line $k$ by distance $BC$, which takes $k$ to itself.*

(c) *The translation maps $B$ to $C$, and it maps $m$ to $(k/m/n \checkmark)$ because the rotation maintains parallels, and there is a unique parallel to $m$ through $C$.*

(d) *The translation maps $\angle 1$ to $(\angle 1/\angle 2/\angle 3 \checkmark/\angle 4)$. These corresponding angles must be congruent because the translation preserves angle measures.*

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