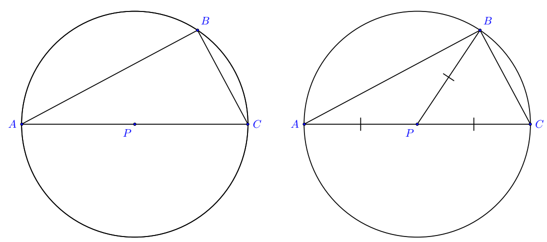
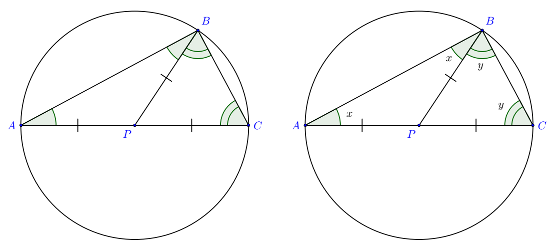
Inscribed Angles

Proofs updated.

Problem 1 *In the figure below, $\overline{AC}$ is a diameter of a circle with center P. Prove that $\angle ABC$ is a right angle.*



(a) *Beginning with the diagram on the left, Natalia draws $\overline{PB}$ and marks the diagram to show segments that she knows to be congruent because each one is a* $\boxed{\textit{radius}}$ *of the circle.*



(b) *Natalia sees that $\triangle APB$ and $\triangle BPC$ are* $\boxed{\textit{isosceles}}$ *triangles, so she marks the figure to show angles that must congruent.*

> *Fix note: Do we need a statement or citation of the Isosceles Triangle Theorem?*

(c) *In order to do some algebra with these congruent angles, Natalia labels their measures x and y, as shown in the picture on the right.*

Learning outcomes:
Author(s): Brad Findell

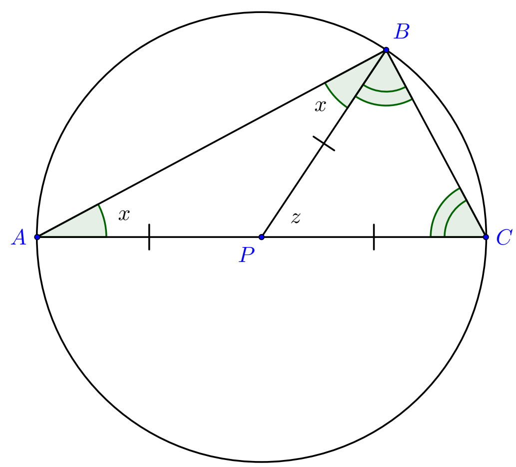
(d) *She writes an equation for the sum of the angles of* $\triangle ABC$:

$$\boxed{x + (x + y) + y} = 180^\circ$$

(e) *Since* $m\angle ABC = \boxed{x + y}$, *she divides that equation by* $\boxed{2}$ *to conclude that* $m\angle ABC = \boxed{90}$ *degrees.*

Problem 2 *A special case of the relationship between an inscribed angle and the corresponding central angle.*

In the figure below, $\overline{AC}$ *is a diameter of a circle with center* P. *Prove that* $z = 2x$.



Because z *is the measure of an angle exterior to* $\triangle\boxed{APB}$, *it is equal to the sum of the measures of the (opposite/ adjacent/ remote interior ✓/ alternate interior) angles. In other words* $z = 2x$.

Alternatively, without using the exterior angle theorem, one might proceed as follows:

(a) $\angle APB + x + x = 180^\circ$ *because of the angle sum in* $\triangle\boxed{ABP}$.

(b) $\angle APB + z = 180^\circ$ *because they form a linear pair.*

(c) *Then* $z = 2x$ *by comparing the two equations.*

Fix note: This handles the special case in which one side of the inscribed angle is a diameter. For the general result, consider two cases: (1) When the center of the circle is in the interior of the inscribed angle; and (2) When the center of the circle is not in the interior of the inscribed angle.