Midsegment Theorem

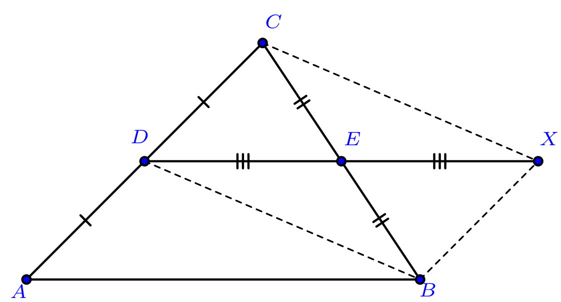
Proofs updated.

Theorem 1. *Midsegment Theorem: The segment joining the midpoints of two sides of a triangle is parallel to and half the length of the third side.*

Fix note: The typical proof uses similarity, which is suitable for Math 2. This one uses parallelograms, so that it is suitable for Math 1.

In preparation for the midsegment theorem, the class proved several useful theorems about parallelograms.

Problem 1 *To prove the midsegment theorem for $\triangle ABC$ with midpoints D and E of sides AC and BC, respectively, Mitch extended $\overline{DE}$ to a point X such that $EX = DE$, as shown in the marked figure. Then he added dotted lines to the figure to show parallelograms.*



Mitch organized his reasoning in the following flow chart:

Fix note: The flowchart omits reasons to reduce clutter. The most significant steps are green whereas the details are blue.

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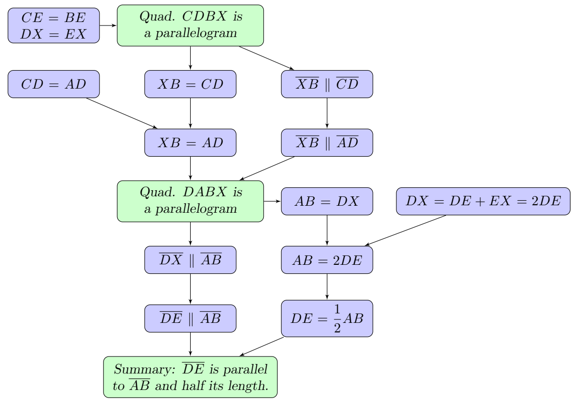

In the proof above, which theorem may Mitch use to conclude that quadrilateral $CDBX$ *a parallelogram?*

Multiple Choice:

(a) *If a pair of sides of a quadrilateral are congruent and parallel, then it is a parallelogram.*

(b) *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.* ✓

(c) *If opposite sides of a quadrilateral are congruent, then it is a parallelogram.*

(d) *If opposite angles of a quadrilateral are congruent, then it is a parallelogram.*

(e) *The Pythagorean Theorem.*

(f) *None of these.*

In the proof above, which theorem may Mitch use to conclude that quadrilateral $DABX$ *a parallelogram?*

Multiple Choice:

(a) *If one pair of sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.* ✓

(b) *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.*

(c) *If opposite sides of a quadrilateral are congruent, then it is a parallelogram.*

(d) *If opposite angles of a quadrilateral are congruent, then it is a parallelogram.*

(e) *The Pythagorean Theorem.*

(f) *None of these.*

Detail: Paragraph proof:
$CE = BE$ and $DX = EX$, *as given.*
Quadrilateral $CDBX$ *is a parallelogram because the diagonals bisect each other.*
$XB = CD$ *because opposite sides of a parallelogram are congruent.*
$XB = AD$ *because they are both equal to* CD*.*
$\overline{XB} \parallel \overline{CD}$ *because* $CDBX$ *is a parallelogram.*
$\overline{XB} \parallel \overline{AD}$ *because* A, C, and D *are collinear.*
Quadrilateral $DABX$ *is a parallelogram because a pair of sides is congruent and parallel.*
$\overline{DX} \parallel \overline{AB}$ *because* $DABX$ *is a parallelogram.*
$\overline{DE} \parallel \overline{AB}$ *because* D, E, and X *are collinear.*
$AB = DX = DE + EX = 2DE$
$DE = \frac{1}{2}AB$
Summary: DE *is parallel to* AB *and half its length.*