

# Parallel Lines

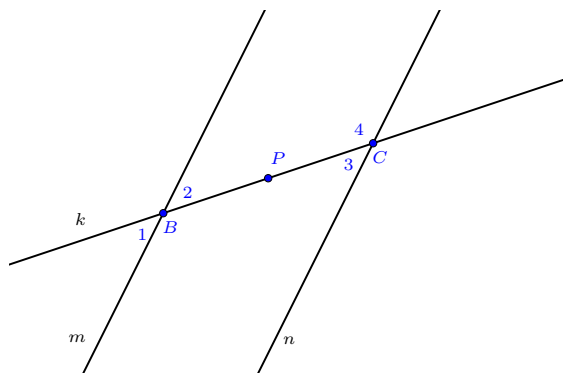
*Proofs updated.*

Parallel postulate (uniqueness of parallels): Given a line and a point not on the line, there is exactly one line through the given point parallel to the given line.

Theorems to prove:

- (a) A  $180^\circ$  rotation about a point on a line takes the line to itself.
- (b) A  $180^\circ$  rotation about a point not on a line takes the line to a parallel line.
- (c) If two parallel lines are cut by a transversal alternate interior (and corresponding angles) are congruent.
- (d) If two lines are cut by a transversal so that alternate interior (or corresponding) angles are congruent, then the lines are parallel.

**Problem 1** Given that parallel lines  $m$  and  $n$  are cut by transversal  $k$ , prove that alternate interior angles are congruent.



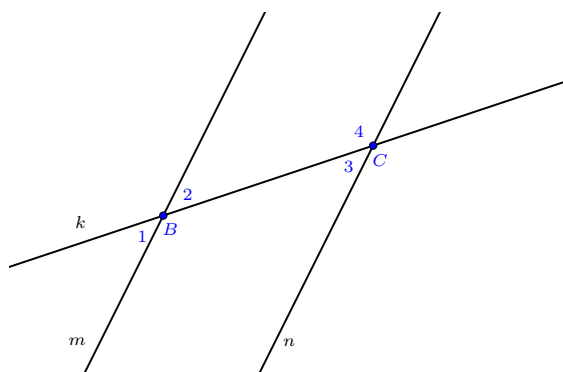
- (a) Let  $B$  and  $C$  be the intersections of transversal  $k$  with lines  $m$  and  $n$ , respectively. Let  $P$  be the midpoint of  $\overline{BC}$
- (b) Rotate  $180^\circ$  about  $P$ , which takes  $k$  to itself.
- (c) The rotation maps  $B$  to  $C$  and  $C$  to  $B$  because distances are preserved.

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Learning outcomes:  
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- (d) The rotation maps  $m$  to a parallel line through  $C$ , which must be  $(k/m/n)$  by the uniqueness of parallels.
- (e) The rotation maps  $n$  to  $(k/m/n)$  by the same reasoning.
- (f) The rotation swaps  $\angle 2$  and  $(\angle 1/\angle 2/\angle 3/\angle 4)$ . These alternate interior angles must be congruent because the rotation preserves angle measures.

**Problem 2** Given that parallel lines  $m$  and  $n$  are cut by transversal  $k$ , prove that corresponding angles are congruent.



- (a) Let  $B$  and  $C$  be the intersections of transversal  $k$  with lines  $m$  and  $n$ , respectively.
- (b) Translate to the right along line  $k$  by distance  $BC$ , which takes  $k$  to itself.
- (c) The translation maps  $B$  to  $C$ , and it maps  $m$  to  $(k/m/n)$  because the translation maintains parallels, and there is a unique parallel to  $m$  through  $C$ .
- (d) The translation maps  $\angle 1$  to  $(\angle 1/\angle 2/\angle 3/\angle 4)$ . These corresponding angles must be congruent because the translation preserves angle measures.