Parallel Lines

Proofs updated.

Parallel postulate (uniqueness of parallels): Given a line and a point not on the line, there is exactly one line through the given point parallel to the given line.

Theorems to prove:

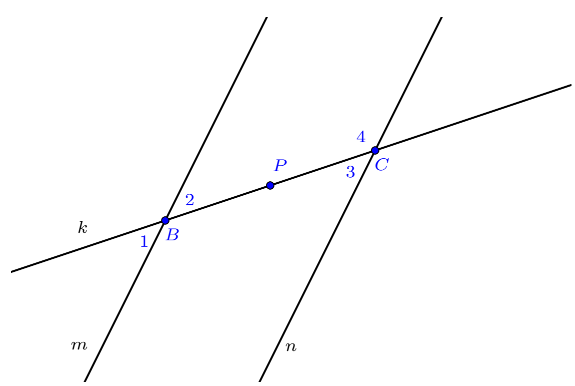
(a) A 180° rotation about a point on a line takes the line to itself.

(b) A 180° rotation about a point not on a line takes the line to a parallel line.

(c) If two parallel lines are cut by a transversal alternate interior (and corresponding angles) are congruent.

(d) If two lines are cut by a transversal so that alternate interior (or corresponding) angles are congruent, then the lines are parallel.

Problem 1 *Given that parallel lines m and n are cut by transversal k, prove that alternate interior angles are congruent.*



(a) *Let B and C be the intersections of transversal k with lines m and n, respectively. Let P be the midpoint of $\overline{BC}$*

(b) *Rotate 180° about P, which takes k to itself.*

(c) *The rotation maps B to C and C to B because distances are preserved.*

Learning outcomes:
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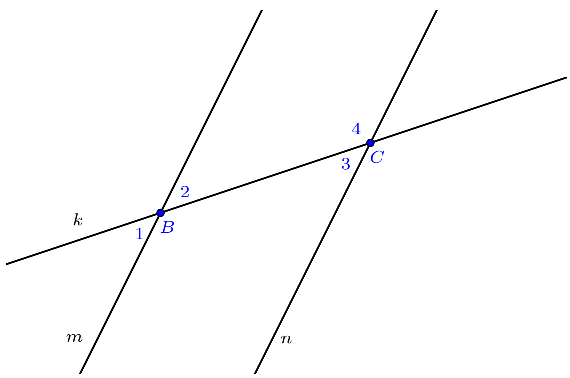
(d) *The rotation maps m to a parallel line through C, which must be (k / m / n ✓) by the uniqueness of parallels.*

(e) *The rotation maps n to (k / m ✓/ n) by the same reasoning.*

(f) *The rotation swaps $\angle 2$ and ($\angle 1$/ $\angle 2$/ $\angle 3$ ✓/ $\angle 4$). These alternate interior angles must be congruent because the rotation preserves angle measures.*

Problem 2 *Given that parallel lines m and n are cut by transversal k, prove that corresponding angles are congruent.*



(a) *Let B and C be the intersections of transversal k with lines m and n, respectively.*

(b) *Translate to the right along line k by distance BC, which takes k to itself.*

(c) *The translation maps B to C, and it maps m to (k / m / n ✓) because the translation maintains parallels, and there is a unique parallel to m through C.*

(d) *The translation maps $\angle 1$ to ($\angle 1$/ $\angle 2$/ $\angle 3$ ✓/ $\angle 4$). These corresponding angles must be congruent because the translation preserves angle measures.*