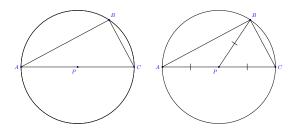
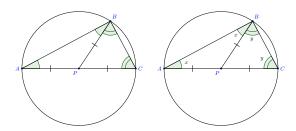
Inscribed Angles

Proofs updated.

Problem 1 In the figure below, \overline{AC} is a diameter of a circle with center P. Prove that $\angle ABC$ is a right angle.



(a) Beginning with the diagram on the left, Natalia draws \overline{PB} and marks the diagram to show segments that she knows to be congruent because each one is a \boxed{radius} of the circle.



(b) Natalia sees that $\triangle APB$ and $\triangle BPC$ are isosceles triangles, so she marks the figure to show angles that must be congruent.

Fix note: Do we need a statement or citation of the Isosceles Triangle Theorem?

(c) In order to do some algebra with these congruent angles, Natalia labels their measures x and y, as shown in the picture on the right.

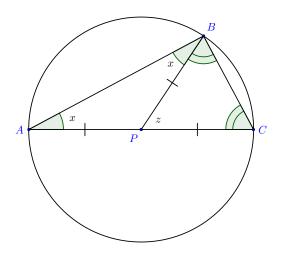
Learning outcomes: Author(s): Brad Findell (d) She writes an equation for the sum of the angles of $\triangle ABC$:

$$\boxed{x + (x+y) + y} = 180^{\circ}$$

(e) Since $m \angle ABC = \boxed{x+y}$, she divides that equation by $\boxed{2}$ to conclude that $m \angle ABC = \boxed{90}$ degrees.

Problem 2 A special case of the relationship between an inscribed angle and the corresponding central angle.

In the figure below, \overline{AC} is a diameter of a circle with center P. Prove that z=2x.



Because z is the measure of an angle exterior to $\triangle APB$, it is equal to the sum of the measures of the (opposite/adjacent/remote interior \checkmark / alternate interior) angles. In other words z=2x.

Alternatively, without using the exterior angle theorem, one might proceed as follows:

- (a) $\angle APB + x + x = 180^{\circ}$ because of the angle sum in $\triangle ABP$
- (b) $\angle APB + z = 180^{\circ}$ because they form a linear pair.
- (c) Then z = 2x by comparing the two equations.

Fix note: This handles the special case in which one side of the inscribed angle is a diameter. For the general result, consider two cases: (1) When the center of the circle is in the interior of the inscribed angle; and (2) When the center of the circle is not in the interior of the inscribed angle.