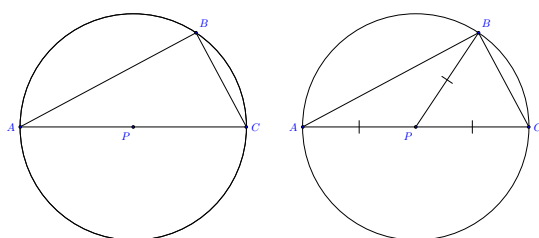


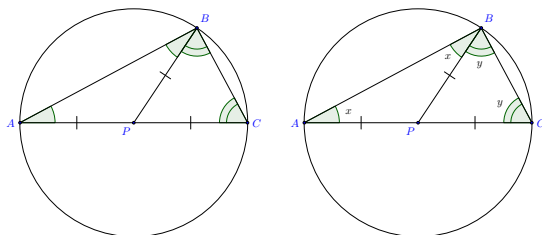
Inscribed Angles

Proofs updated.

Problem 1 In the figure below, \overline{AC} is a diameter of a circle with center P . Prove that $\angle ABC$ is a right angle.



- (a) Beginning with the diagram on the left, Natalia draws \overline{PB} and marks the diagram to show segments that she knows to be congruent because each one is a radius of the circle.



- (b) Natalia sees that $\triangle APB$ and $\triangle BPC$ are isosceles triangles, so she marks the figure to show angles that must congruent.

Fix note: Do we need a statement or citation of the Isosceles Triangle Theorem?

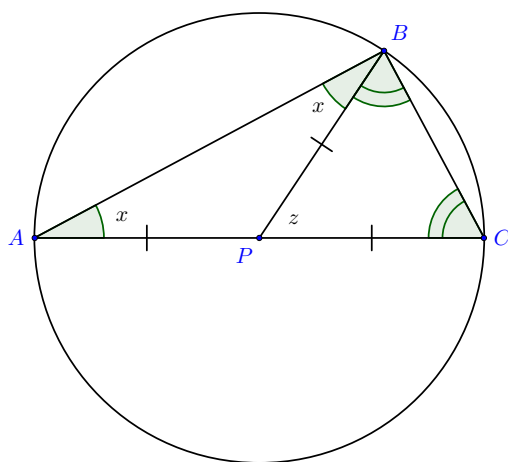
- (c) In order to do some algebra with these congruent angles, Natalia labels their measures x and y , as shown in the picture on the right.
- (d) She writes an equation for the sum of the angles of $\triangle ABC$:

$$\boxed{x + (x + y) + y} = 180^\circ$$

- (e) Since $m\angle ABC = x + y$, she divides that equation by 2 to conclude that $m\angle ABC = 90$ degrees.

Problem 2 A special case of the relationship between an inscribed angle and the corresponding central angle.

In the figure below, \overline{AC} is a diameter of a circle with center P . Prove that $z = 2x$.



Because z is the measure of an angle exterior to $\triangle APB$, it is equal to the sum of the measures of the (opposite / adjacent / remote interior / alternate interior) angles. In other words $z = 2x$.

Alternatively, without using the exterior angle theorem, one might proceed as follows:

- $\angle APB + x + x = 180^\circ$ because of the angle sum in $\triangle ABP$.
- $\angle APB + z = 180^\circ$ because they form a linear pair.
- Then $z = 2x$ by comparing the two equations.

Fix note: This handles the special case in which one side of the inscribed angle is a diameter. For the general result, consider two cases: (1) When the center of the circle is in the interior of the inscribed angle; and (2) When the center of the circle is not in the interior of the inscribed angle.