Midsegment Theorem

Proofs updated.

Definition 1. *In a triangle, a* midsegment *is a line joining the midpoints of two sides.*

Theorem 1. *Midsegment Theorem: A midsegment in a triangle is parallel to and half the length of the corresponding side.*

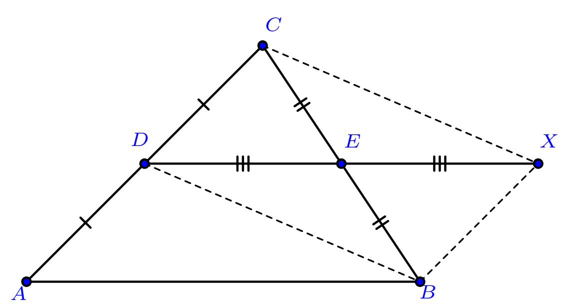
Fix note: The typical proof uses similarity, which is suitable for Math 2. This one uses parallelograms, so that it is suitable for Math 1.

In preparation for the midsegment theorem, the class proved some useful theorems about parallelograms.

Theorem 2. *Antonia's Theorem: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.*

Theorem 3. *Lu's Theorem: If one pair of sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.*

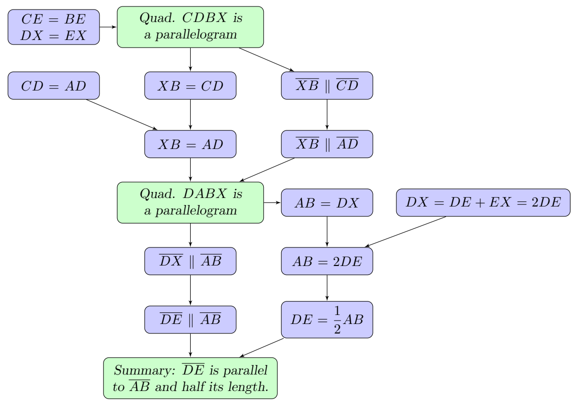
Problem 1 *To prove the midsegment theorem for $\triangle ABC$ with midpoints D and E of sides AC and BC, respectively, Jesse extends $\overline{DE}$ to a point X such that $EX = DE$, as shown in the marked figure.*



Jesse organizes her reasoning in the following flow chart:

Author(s): Brad Findell

Fix note: The flowchart omits reasons to reduce clutter. The most significant steps are green whereas the details are blue.



Fix note: For the following questions, does it help to name the theorems? Would theorem numbers be better? Or maybe we should write each theorem in words.

In the proof above, why may Jesse conclude that quadrilateral $CDBX$ a parallelogram?

Multiple Choice:

(a) *By the Pythagorean Theorem.*

(b) *By Lu's Theorem.*

(c) *By Antonia's Theorem.* ✓

(d) *By the Parallelogram Theorem.*

In the proof above, why may Jesse conclude that quadrilateral $DABX$ a parallelogram?

Multiple Choice:

(a) *By the Pythagorean Theorem.*

(b) *By Lu's Theorem.* ✓

(c) *By Antonia's Theorem.*

(d) *By the Parallelogram Theorem.*

Detail: Paragraph proof:
$CE = BE$ and $DX = EX$, *as given.*
Quadrilateral $CDBX$ *is a parallelogram by Antonia's Theorem.*
$XB = CD$ *because opposite sides of a parallelogram are congruent.*
$XB = AD$ *because they are both equal to* CD.
$\overline{XB} \parallel \overline{CD}$ *because CDBX is a parallelogram.*
$\overline{XB} \parallel \overline{AD}$ *because* A, C, *and* D *are collinear.*
Quadrilateral $DABX$ *is a parallelogram by Lu's Theorem.*
$\overline{DX} \parallel \overline{AB}$ *because* $DABX$ *is a parallelogram.*
$\overline{DE} \parallel \overline{AB}$ *because* D, E, *and* X *are collinear.*
$AB = DX = DE + EX = 2DE$
$DE = \dfrac{1}{2}AB$
Summary: DE *is parallel to* AB *and half its length.*