Midsegment Theorem

Proofs updated.

Definition 1. *In a triangle, a* midsegment *is a line joining the midpoints of two sides.*

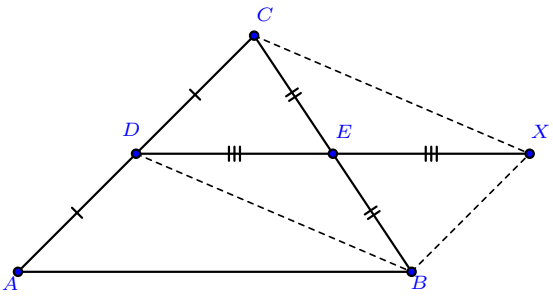
Theorem 1. *Midsegment Theorem: A midsegment in a triangle is parallel to and half the length of the corresponding side.*

Fix note: The typical proof uses similarity, which is suitable for Math 2. This one uses parallelograms, so that it is suitable for Math 1.

In preparation for the midsegment theorem, the class proved some useful theorems about parallelograms.

Fix note: Should the four parallelogram theorems be listed here?

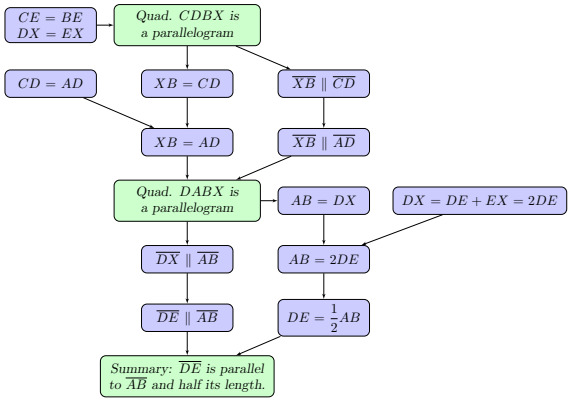
Problem 1 *To prove the midsegment theorem for $\triangle ABC$ with midpoints D and E of sides AC and BC, respectively, Jesse extended $\overline{DE}$ to a point X such that $EX = DE$, as shown in the marked figure. Then she added dotted lines to the figure to show parallelograms.*



Jesse organizes her reasoning in the following flow chart:

Learning outcomes:
Author(s): Brad Findell

Fix note: The flowchart omits reasons to reduce clutter. The most significant steps are green whereas the details are blue.



Fix note: Is this a helpful way to write the theorems in words? Another approach would be to list them as "conditions that guarantee a parallelogram."

In the proof above, which theorem may Jesse use to conclude that quadrilateral $CDBX$ a parallelogram?

Multiple Choice:

(a) *If a pair of sides of a quadrilateral are congruent and parallel, then it is a parallelogram.*

(b) *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.* ✓

(c) *If opposite sides of a quadrilateral are congruent, then it is a parallelogram.*

(d) *If opposite angles of a quadrilateral are congruent, then it is a parallelogram.*

(e) *The Pythagorean Theorem.*

(f) *None of these.*

In the proof above, which theorem may Jesse use to conclude that quadrilateral $DABX$ a parallelogram?

Multiple Choice:

(a) *If one pair of sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.* ✓

(b) *If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.*

(c) *If opposite sides of a quadrilateral are congruent, then it is a parallelogram.*

(d) *If opposite angles of a quadrilateral are congruent, then it is a parallelogram.*

(e) *The Pythagorean Theorem.*

(f) *None of these.*

Detail: Paragraph proof:
$CE = BE$ and $DX = EX$, as given.
Quadrilateral $CDBX$ is a parallelogram because the diagonals bisect each other.
$XB = CD$ because opposite sides of a parallelogram are congruent.
$XB = AD$ because they are both equal to CD.
$\overline{XB} \parallel \overline{CD}$ because $CDBX$ is a parallelogram.
$\overline{XB} \parallel \overline{AD}$ because A, C, and D are collinear.
Quadrilateral $DABX$ is a parallelogram because a pair of sides is congruent and parallel.
$\overline{DX} \parallel \overline{AB}$ because $DABX$ is a parallelogram.
$\overline{DE} \parallel \overline{AB}$ because D, E, and X are collinear.
$AB = DX = DE + EX = 2DE$
$DE = \frac{1}{2}AB$
Summary: DE is parallel to AB and half its length.