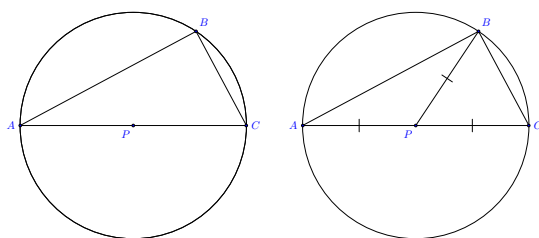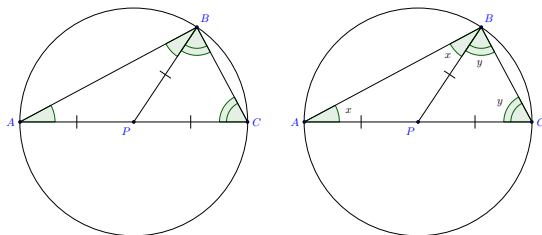# Inscribed Angles

Proofs updated.

Problem 1 *In the figure below, $\overline{AC}$ is a diameter of a circle with center P. Prove that $\angle ABC$ is a right angle.*

(a) *Beginning with the diagram on the left, Natalia draws $\overline{PB}$ and marks the diagram to show segments that she knows to be congruent because each one is a* ? *of the circle.*

(b) *Natalia sees that $\triangle APB$ and $\triangle BPC$ are* ? *triangles, so she marks the figure to show angles that must be congruent.*

> *Fix note: Do we need a statement or citation of the Isosceles Triangle Theorem?*

(c) *In order to do some algebra with these congruent angles, Natalia labels their measures x and y, as shown in the picture on the right.*

(d) *She writes an equation for the sum of the angles of $\triangle ABC$:*

$$\boxed{?} = 180°$$

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(e) Since $m\angle ABC = \boxed{?}$, she divides that equation by $\boxed{?}$ to conclude that $m\angle ABC = \boxed{?}$ degrees.

Problem 2 *A special case of the relationship between an inscribed angle and the corresponding central angle.*

In the figure below, $\overline{AC}$ is a diameter of a circle with center P. Prove that $z = 2x$.
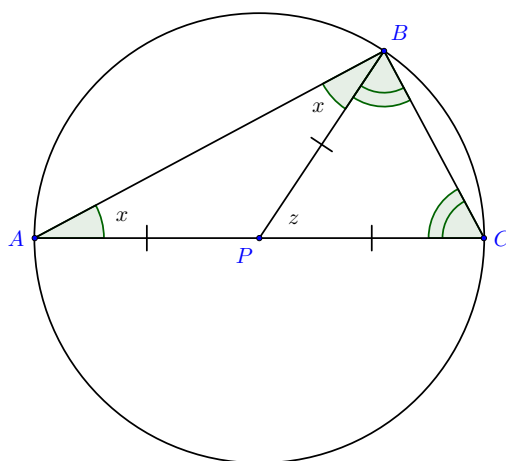
Because z is the measure of an angle exterior to $\triangle\boxed{?}$, it is equal to the sum of the measures of the (opposite / adjacent / remote interior / alternate interior) angles. In other words $z = 2x$.

Alternatively, without using the exterior angle theorem, one might proceed as follows:

(a) $\angle APB + x + x = 180°$ *because of the angle sum in $\triangle\boxed{?}$.*

(b) $\angle APB + z = 180°$ *because they form a linear pair.*

(c) *Then $z = 2x$ by comparing the two equations.*

Fix note: This handles the special case in which one side of the inscribed angle is a diameter. For the general result, consider two cases: (1) When the center of the circle is in the interior of the inscribed angle; and (2) When the center of the circle is not in the interior of the inscribed angle.