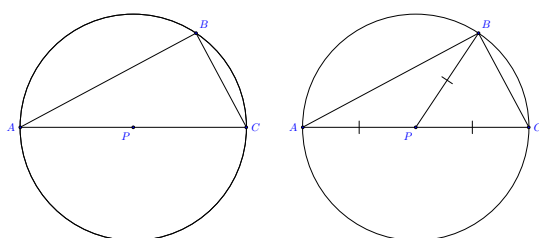


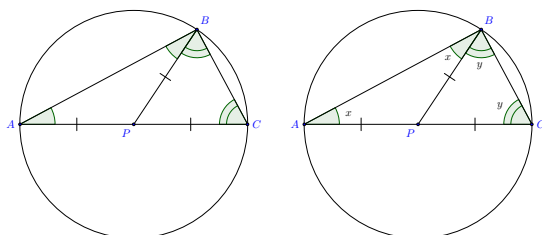
# Inscribed Angles

*Proofs updated.*

**Problem 1** In the figure below,  $\overline{AC}$  is a diameter of a circle with center  $P$ . Prove that  $\angle ABC$  is a right angle.



- (a) Beginning with the diagram on the left, Natalia draws  $\overline{PB}$  and marks the diagram to show segments that she knows to be congruent because each one is a radius of the circle.



- (b) Natalia sees that  $\triangle APB$  and  $\triangle BPC$  are isosceles triangles, so she marks the figure to show angles that must congruent.

*Fix note: Do we need a statement or citation of the theorem?*

- (c) In order to do some algebra with these congruent angles, Natalia labels their measures  $x$  and  $y$ , as shown in the picture on the right.
- (d) She writes an equation for the sum of the angles of  $\triangle ABC$ :

$$\boxed{x + (x + y) + y} = 180^\circ$$

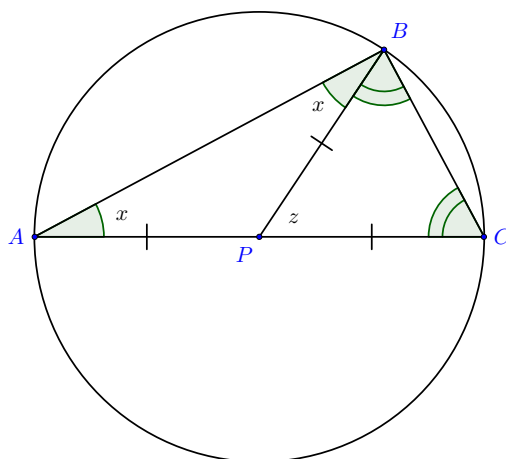
Learning outcomes:  
Author(s): Brad Findell

*Fix note: Need a prompt about dividing the equation by 2.*

(e) Since  $m\angle ABC = \boxed{x + y}$ , she concludes that  $m\angle ABC = 90^\circ$ .

**Problem 2** A special case of the relationship between an inscribed angle and the corresponding central angle.

In the figure below,  $\overline{AC}$  is a diameter of a circle with center  $P$ . Prove that  $z = 2x$ .



Because  $z$  is the measure of an angle exterior to  $\triangle APB$ , it is equal to the sum of the measures of the remote interior angles. In other words  $z = 2x$ .

Alternatively, without using the exterior angle theorem, one might proceed as follows:

- (a)  $\angle ABP + x + x = 180^\circ$  because of the angle sum in  $\triangle ABP$ .
- (b)  $\angle ABP + z = 180^\circ$  because they form a linear pair.
- (c) Then  $z = 2x$  by comparing the two equations.

*Fix note: This handles the special case in which one side of the inscribed angle is a diameter. For the general result, consider two cases: (1) When the center of the circle is in the interior of the inscribed angle; and (2) When the center of the circle is not in the interior of the inscribed angle.*