

Parallel Lines

Proofs updated.

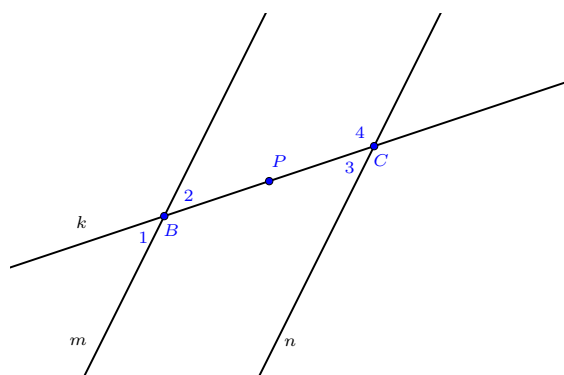
Parallel postulate: Given a line and a point not on the line, there is exactly one line through the given point parallel to the given line.

Theorems to prove:

- (a) A 180° rotation about a point on a line takes the line to itself.
- (b) A 180° rotation about a point not on a line takes the line to a parallel line.
- (c) If two parallel lines are cut by a transversal alternate interior and corresponding angles are congruent.
- (d) If two lines are cut by a transversal so that alternate interior (or corresponding) angles are congruent, then the lines are parallel.

Fix note: Below are two different proofs. Please consider them separately. And in each proof, which of the details should be included, and which should be omitted?

Problem 1 Given that parallel lines m and n are cut by transversal k , prove that alternate interior angles are congruent.



First proof:

Learning outcomes:
Author(s): Brad Findell

- (a) Let A and B be the intersections of transversal k with lines m and n , respectively.
- (b) Let P be the midpoint of \overline{BC}
- (c) Rotate 180° about P , which takes k to itself.
- (d) The rotation maps B to C and C to B because distances are preserved.
- (e) The rotation maps m to a parallel line through C , which must be n by the uniqueness of parallels.
- (f) The rotation maps n to m by the same reasoning.
- (g) The rotation swaps $\angle 2$ and $\angle 3$. These alternate interior angles must be congruent because the rotation preserves angle measures.

Second proof:

- (a) Let A and B be the intersections of transversal k with lines m and n , respectively.
 - (b) Translate line m along line k to the right by distance BC .
 - (c) The translation maps B to C , and it maps m to n because the translation maintains parallels.
 - (d) The translation maps $\angle 1$ to $\angle 3$. These corresponding angles must be congruent because the translation preserves angle measures.
-