

# Parallel Lines

*Proofs updated.*

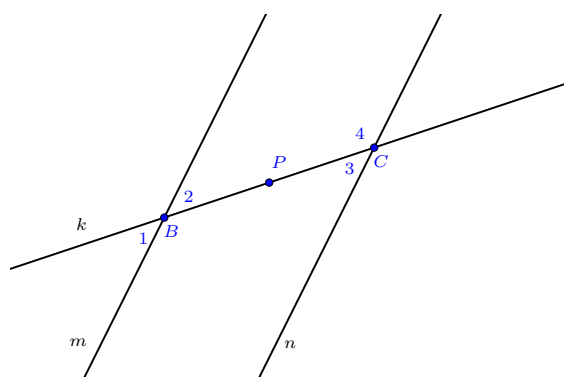
Parallel postulate: Given a line and a point not on the line, there is exactly one line through the given point parallel to the given line.

Theorems to prove:

- (a) A  $180^\circ$  rotation about a point on a line takes the line to itself.
- (b) A  $180^\circ$  rotation about a point not on a line takes the line to a parallel line.
- (c) If two parallel lines are cut by a transversal alternate interior and corresponding angles are congruent.
- (d) If two lines are cut by a transversal so that alternate interior (or corresponding) angles are congruent, then the lines are parallel.

Fix note: Below are two different proofs. Please consider them separately. And in each proof, which of the details should be included, and which should be omitted?

**Problem 1** Given that parallel lines  $m$  and  $n$  are cut by transversal  $k$ , prove that alternate interior angles are congruent.



*First proof:*

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- (a) Let  $A$  and  $B$  be the intersections of transversal  $k$  with lines  $m$  and  $n$ , respectively.
- (b) Let  $P$  be the midpoint of  $\overline{BC}$
- (c) Rotate  $180^\circ$  about  $P$ , which takes  $k$  to itself.
- (d) The rotation maps  $B$  to  $C$  and  $C$  to  $B$  because distances are preserved.
- (e) The rotation maps  $m$  to a parallel line through  $C$ , which must be  $n$  by the uniqueness of parallels.
- (f) The rotation maps  $n$  to  $m$  by the same reasoning.
- (g) The rotation swaps  $\angle 2$  and  $\angle 3$ . These alternate interior angles must be congruent because the rotation preserves angle measures.

Second proof:

- (a) Let  $A$  and  $B$  be the intersections of transversal  $k$  with lines  $m$  and  $n$ , respectively.
  - (b) Translate line  $m$  along line  $k$  to the right by distance  $BC$ .
  - (c) The translation maps  $B$  to  $C$ , and it maps  $m$  to  $n$  because the translation maintains parallels.
  - (d) The translation maps  $\angle 1$  to  $\angle 3$ . These corresponding angles must be congruent because the translation preserves angle measures.
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