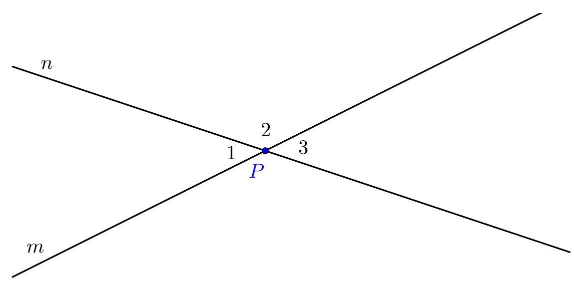
Vertical Angles

Proofs updated.

Below are three different proofs that vertical angles are congruent. Please consider them separately.

Problem 1 *Point P is the intersection of lines m and n. Prove that $\angle 1 \cong \angle 3$.*



Fix note: When students write equations about linear pairs, they often write two equations for non-overlapping linear pairs—which doesn't help. The figure above is intended to help avoid that dead end, but it would be worthwhile to discuss that dead end anyway.

Proof *Using adjacent angles, $\angle 1 \cong \angle 3$ because they are both (complementary / supplementary ✓/ opposite/ congruent) to $\angle 2$.* ■

Feedback(`correct`): *Additional detail: First write down equations about linear pairs of angles:*

$$m\angle 1 + m\angle 2 = \boxed{180} \text{ degrees}$$

$$m\angle 3 + m\angle 2 = \boxed{180} \text{ degrees}$$

By comparing the two equations, one might see that $m\angle 1 = m\angle 3$. Alternatively, one may do some algebra to conclude that $m\angle 1 = 180^\circ - m\angle 2 = m\angle 3$, which is essentially what the one-sentence proof says.

Problem 2 *Point P is the intersection of lines m and n. Prove that $\angle 1 \cong \angle 3$.*

Learning outcomes:
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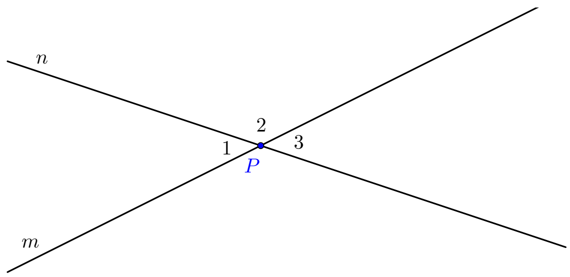
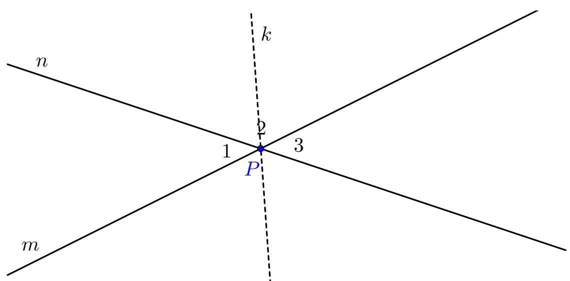


Proof *A rotation of* ($90°$ / $180°$ ✓/ $360°$) *about* P *maps* m *onto itself, maps* n *onto itself, and swaps* $\angle 1$ *and* ($\angle 1$/ $\angle 2$/ $\angle 3$ ✓). *Because rotations preserve angle measures, it must be that* $\angle 1 \cong \angle 3$. ■

Feedback(`correct`): *Additional detail: Line* m *is the union of two opposite* $\boxed{\textit{rays}}$ *with endpoint* P*. The rotation about* P *swaps these opposite rays, and the same idea holds for line* n*. That rotation maps the sides of* $\angle 1$ *onto the sides of* $\angle 3$ *and vice versa.*

Problem 3 *Point* P *is the intersection of lines* m *and* n*. Prove that* $\angle 1 \cong \angle 3$.



Proof *Reflecting about the* (bisector ✓/ supplement/ opposite) *of* $\angle 2$ *swaps the sides of* $\angle 2$ *and therefore lines* m *and* n*. Thus, that reflection swaps* $\angle 1$ *and* ($\angle 1$/ $\angle 2$/ $\angle 3$ ✓). *Because reflections preserve angle measures, it follows that* $\angle 1 \cong \angle 3$. ■

Feedback(`correct`): *Additional detail: Because reflections take lines to* $\boxed{\textit{lines}}$, *the reflection that swaps the sides of* $\angle 2$ *must swap not just the rays but lines* m *and* n*, which contain the rays.*