a. The convergence rate, estimated exact solution, and discretization error are tabulated below for the coarse grids (n = 10, 20, 40). The coarse grid with CDS does a reasonably good job of estimating the exact solution - within 0.4%.

$\phi_{ANALYTIC}$ @(x = 0.9) = 1.1353

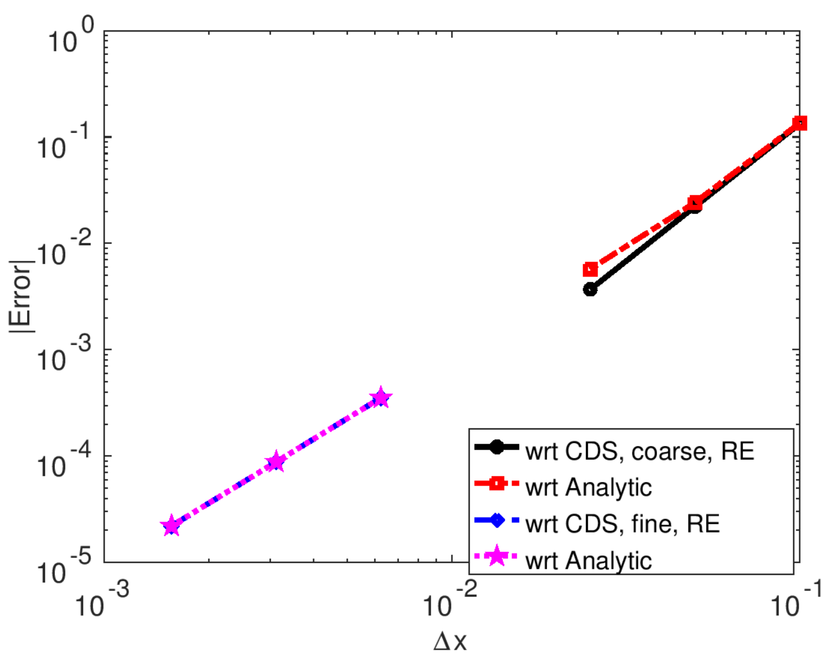
$P_{CDS,coarse}$	2.5873
$\phi_{CDS,coarse,RE}$	1.1333
$\varepsilon_h{}^d{}_{CDS,coarse}$	0.0036907

b. The convergence rate, estimated exact solution, and discretization error are tabulated below for the fine grids (n = 160, 320, 640). The estimated exact solution is very close to the analytic solution.

$\phi_{ANALYTIC}$ @(x = 0.9) = 1.1353

$P_{CDS,fine}$	2.0014
$\phi_{CDS,fine,RE}$	1.1353
$\varepsilon_h{}^d{}_{CDS,fine}$	2.2006e-005

c. The errors in the numerical solutions with respect to the “exact” solutions at x = 0.9 are shown below. For the coarse grids, the estimated error values at the n = 10 and n = 20 are close to the exact error values, but the inaccuracy in slope (= convergence rate = P) results in an inaccurate estimation at n=40. For the fine meshes, the error is estimated extremely well; the curves are directly on top of each other and are indistinguishable. It follows that in practice, one would want to find a sufficiently small grid spacing that allows for the “exact” solution to be very closely estimated.

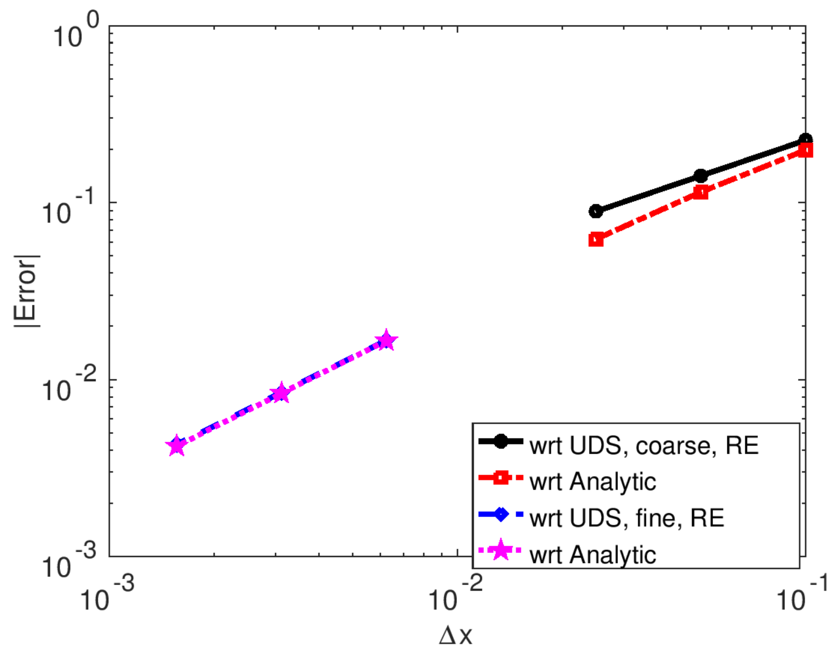


d. The convergence rates, estimated exact solutions, and discretization errors are tabulated below for the coarse grids (n = 10, 20, 40) and for the fine grids (n = 160, 320, 640).

$\phi_{ANALYTIC}$ @(x = 0.9) = 1.1353

$P_{UDS,coarse}$	0.66726
$\phi_{UDS,coarse,RE}$	1.1083
$\varepsilon_h{}^d{}_{UDS,coarse}$	-0.089224
$P_{UDS,fine}$	0.97704
$\phi_{UDS,fine,RE}$	1.1352
$\varepsilon_h{}^d{}_{UDS,fine}$	-0.0042989

The errors in the numerical solutions with respect to the "exact" solutions are shown below. Since UDS is only first-order accurate unlike CDS, the estimation for the three coarse grids is worse than for the CDS case. The slope (=convergence rate) is off significantly; it should be 1.00 while the estimation at coarse grids is only 0.667. For the finer grids, the convergence rate is estimated much better and as a result the estimated error is much closer to the exact error. When the exact analytic solution is not known, one could use the known order of accuracy of the numerical method to determine when the mesh is sufficiently fine to allow for accurate estimation of the exact solution. Decreasing the spacing to smaller and smaller values to directly determine ϕ would be computationally expensive compared to using Richardson extrapolation.



Summary of Results:

$\phi_{ANALYTIC}$ @(x = 0.9) = 1.1353

n	ϕ_{CDS} @ x = 0.9	ϕ_{UDS} @ x = 0.9
10	1.00000	1.3333
20	1.1111	1.2500
40	1.1296	1.1975
160	1.1350	1.1519
320	1.1352	1.1437
640	1.1353	1.1395

$P_{CDS,coarse}$	2.5873
$\phi_{CDS,coarse,RE}$	1.1333
$\varepsilon_h{}^d{}_{CDS,coarse}$	0.0036907
$P_{CDS,fine}$	2.0014
$\phi_{CDS,fine,RE}$	1.1353
$\varepsilon_h{}^d{}_{CDS,fine}$	2.2006e-005
$P_{UDS,coarse}$	0.66726
$\phi_{UDS,coarse,RE}$	1.1083
$\varepsilon_h{}^d{}_{UDS,coarse}$	-0.089224
$P_{UDS,fine}$	0.97704
$\phi_{UDS,fine,RE}$	1.1352
$\varepsilon_h{}^d{}_{UDS,fine}$	-0.0042989