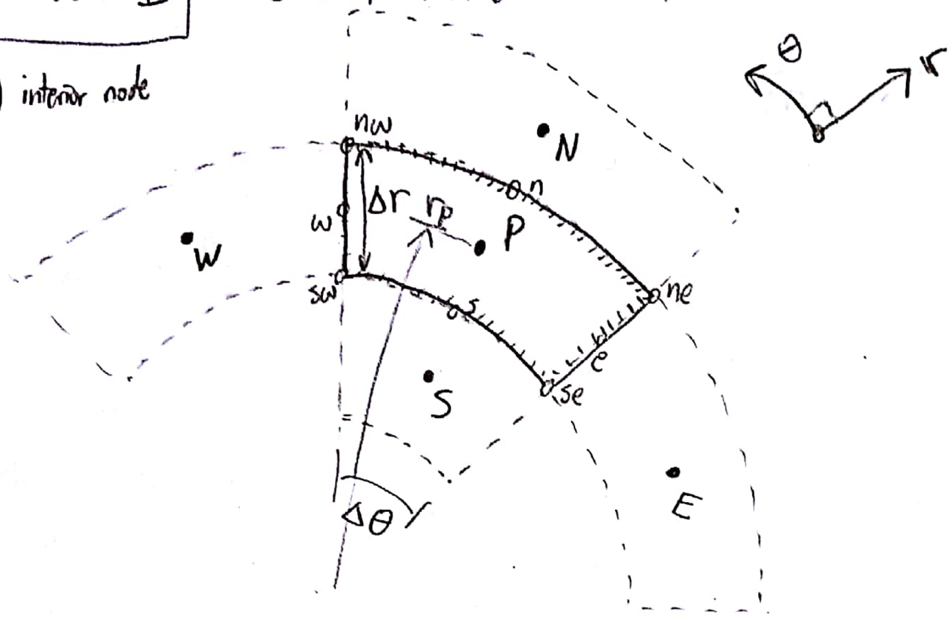
Brian Knisely ME523 HW3

Problem 1 Control Volume formulation

a) interior node



Steady 2D heat conduction: $k\nabla^2 T + q = 0$

$$\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}$$

$$\hookrightarrow k\nabla^2 T + q = 0 = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{k}{r^2}\frac{\partial^2 T}{\partial \theta^2} + q = 0$$

$$= k\frac{\partial^2 T}{\partial r^2} + \frac{k}{r}\frac{\partial T}{\partial r} + \frac{k}{r^2}\frac{\partial^2 T}{\partial \theta^2} + q = 0$$

Use F∀M

First integrate the PDE over the volume which has P at the centroid

$$\int_{\forall} k\nabla^2 T \, d\forall + \int_{\forall} q \, d\forall = 0$$

Use divergence theorem to convert the first volume integral to a surface integral

$$\int_{\forall} k\nabla^2 T \, d\forall = \int_{S} k\nabla T \cdot \hat{n} \, dS$$

Brian Knisely ME523 HW3

Problem 1 (cont.) Now we have:

$$\int_S k\nabla T\cdot\hat{n}\,dS + \int_{\forall} q\,d\forall = 0.$$

Divide the surface integral into four distinct surfaces (n, s, e, w)

$$\int_{S_n} k\nabla T\cdot(\hat{u}_r)dS + \int_{S_s} k\nabla T\cdot(-\hat{u}_r)dS + \int_{S_e} k\nabla T\cdot(-\hat{u}_\theta)dS + \int_{S_w} k\nabla T\cdot(\hat{u}_\theta)dS + \int_{\forall} q\,d\forall = 0$$

Simplify by expanding $\vec{\nabla}T = \frac{\partial T}{\partial r}\hat{u}_r + \frac{1}{r}\frac{\partial T}{\partial\theta}\hat{u}_\theta$ and evaluating dot products.

$$\int_{S_n} k\frac{\partial T}{\partial r}dS + \int_{S_s} -k\frac{\partial T}{\partial r}dS + \int_{S_e} -\frac{k}{r_e}\frac{\partial T}{\partial\theta}dS + \int_{S_w}\frac{k}{r_w}\frac{\partial T}{\partial\theta}dS + \int_{\forall} q\,d\forall = 0$$

<u>Quadrature</u>: use midpoint rule to approximate surface integrals and the volume integral (with 2nd order accuracy)

$$\approx k\left.\frac{\partial T}{\partial r}\right|_n r_n\Delta\theta - k\left.\frac{\partial T}{\partial r}\right|_s r_s\Delta\theta - \frac{k}{r_e}\left.\frac{\partial T}{\partial\theta}\right|_e\Delta r + \frac{k}{r_w}\left.\frac{\partial T}{\partial\theta}\right|_w\Delta r + q_P\,\Delta\forall = 0$$

Extract common k and plug in $\Delta\forall = \Delta r\cdot r_P\Delta\theta\cdot(1) = r_P\Delta r\Delta\theta$, $\quad r_e = r_w = r_P$

$$k\left[\left.\frac{\partial T}{\partial r}\right|_n r_n\Delta\theta - \left.\frac{\partial T}{\partial r}\right|_s r_s\Delta\theta - \frac{1}{r_P}\left.\frac{\partial T}{\partial\theta}\right|_e\Delta r + \frac{1}{r_P}\left.\frac{\partial T}{\partial\theta}\right|_w\Delta r\right] + q_P r_P\Delta r\Delta\theta = 0$$

<u>Interpolation</u>: use CDS2 to define $\left.\frac{\partial T}{\partial r}\right|_n, \left.\frac{\partial T}{\partial r}\right|_s, \left.\frac{\partial T}{\partial\theta}\right|_e, \left.\frac{\partial T}{\partial\theta}\right|_w$ in terms of cell-centered values (with 2nd order accuracy)

$$\left.\frac{\partial T}{\partial r}\right|_n = \frac{T_N - T_P}{\Delta r} + \mathcal{O}((\Delta r)^2) \qquad \left.\frac{\partial T}{\partial r}\right|_s = \frac{T_S - T_P}{\Delta r} + \mathcal{O}((\Delta r)^2)$$

$$\left.\frac{\partial T}{\partial\theta}\right|_e = \frac{T_E - T_P}{\Delta\theta} + \mathcal{O}((r_P\Delta\theta)^2) \qquad \left.\frac{\partial T}{\partial\theta}\right|_w = \frac{T_W - T_P}{\Delta\theta} + \mathcal{O}((r_P\Delta\theta)^2)$$

Brian Knisely ME523 HW3

$\boxed{\text{Problem 1 (cont.)}}$ Substitute terms and drop higher-order terms $\left[\mathcal{O}((\Delta r)^2), \mathcal{O}((r_p\Delta\theta)^2)\right]$

$$k\left[\frac{T_N-T_P}{\Delta r}r_n\Delta\theta - \frac{T_S-T_P}{\Delta r}r_s\Delta\theta - \frac{T_E-T_P}{r_p\Delta\theta}\Delta r + \frac{T_W-T_P}{r_p\Delta\theta}\Delta r\right] + q_P r_p \Delta r \Delta\theta = 0$$

Move term with q_P to RHS and multiply all terms by $r_p \Delta r \Delta\theta / k$

$$(T_N-T_P)r_n r_p(\Delta\theta)^2 - (T_S-T_P)r_s r_p(\Delta\theta)^2 - (T_E-T_P)(\Delta r)^2 + (T_W-T_P)(\Delta r)^2 = \frac{-q_P}{k} r_p^2(\Delta r)^2(\Delta\theta)^2$$

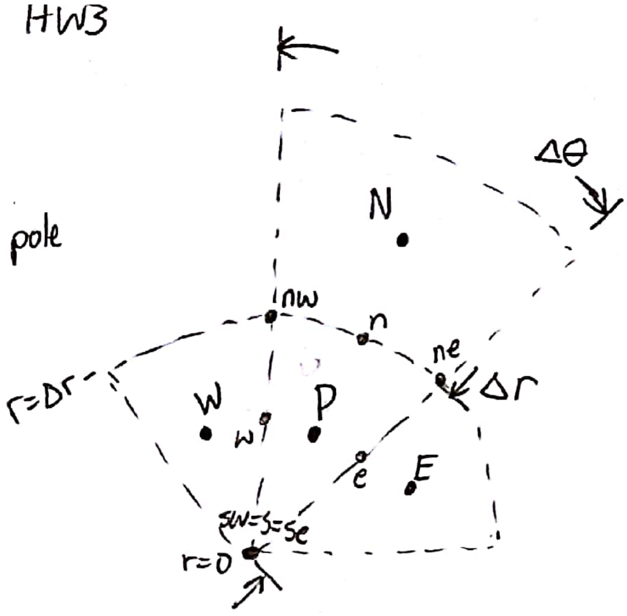
Rearrange in terms of coefficients A_N, A_S, A_P, A_E A_W, substitute $r_n = r_p + \frac{\Delta r}{2}$ and $r_s = r_p - \frac{\Delta r}{2}$

$$\underbrace{\left[\left(r_p+\frac{\Delta r}{2}\right)r_p(\Delta\theta)^2\right]}_{A_N}T_N + \underbrace{\left[\left(r_p-\frac{\Delta r}{2}\right)r_p(\Delta\theta)^2\right]}_{A_S}T_S + \underbrace{\left[-(\Delta r)\cdot r_p(\Delta\theta)^2\right]}_{A_P}T_P + \underbrace{\left[-(\Delta r)^2\right]}_{A_E}T_E + \underbrace{\left[(\Delta r)^2\right]}_{A_W}T_W = \underbrace{\left[\frac{-q_P}{k}(r_p\Delta r\Delta\theta)^2\right]}_{Q_P}$$

Brian Knisely ME523 HW3

Problem 1 (cont.)

b) $C\forall$ in the inner ring, adjacent to pole



$$k\nabla^2 T + q = 0$$

Integrate over the volume

$$\int_\forall k\nabla^2 T\, d\forall + \int_\forall q\, d\forall = 0$$

Invoke Gauss' Divergence Theorem to convert first volume integral to surface integral

$$k\int_S \nabla T \cdot \hat{n}\, dS + \int_\forall q\, d\forall$$

Separate surface integral into the three discrete surfaces

$$\int_{S_n} \nabla T \cdot \hat{u}_r\, dS + \int_{S_e} \nabla T \cdot (-\hat{u}_\theta)\, dS + \int_{S_w} \nabla T \cdot (\hat{u}_\theta)\, dS + \frac{1}{k}\int_\forall q\, d\forall = 0$$

Evaluate dot products with $\nabla T = \frac{\partial T}{\partial r}\hat{u}_r + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{u}_\theta$

$$\int_{S_n} \frac{\partial T}{\partial r}\, dS + \int_{S_e} -\frac{1}{r}\frac{\partial T}{\partial r}\, dS + \int_{S_w} \frac{1}{r}\frac{\partial T}{\partial r}\, dS + \frac{1}{k}\int_\forall q\, d\forall = 0$$

<u>Quadrature</u>: approximate integrals with midpoint rule (2^{nd} order accurate)

$$\left.\frac{\partial T}{\partial r}\right|_n r_n \Delta\theta - \frac{1}{r_e}\left.\frac{\partial T}{\partial \theta}\right|_e \Delta r + \frac{1}{r_w}\left.\frac{\partial T}{\partial \theta}\right|_w \Delta r + \frac{1}{k} q_P\, \Delta r\, r_p\, \Delta\theta\, (1) = 0$$

Brian Knisely ME523 HW3

Problem 1 (cont.)

Interpolation: approximate derivatives with CDS2 (2nd-order accurate)

$$\left.\frac{\partial T}{\partial r}\right|_n = \frac{T_N - T_P}{\Delta r} + \mathcal{O}((\Delta r)^2) \qquad \left.\frac{\partial T}{\partial \theta}\right|_e = \frac{T_E - T_P}{\Delta \theta} + \mathcal{O}((\Delta\theta)^2)$$

$$\left.\frac{\partial T}{\partial \theta}\right|_w = \frac{T_W - T_P}{\Delta \theta} + \mathcal{O}((\Delta\theta)^2)$$

Plug into quadrature equation; also substitute $r_n = r_p + \frac{\Delta r}{2}$, $r_e = r_p$, $r_w = r_p$

$$\frac{T_N - T_P}{\Delta r}\left(r_p + \frac{\Delta r}{2}\right)\Delta\theta - \frac{1}{r_p}\frac{T_E - T_P}{\Delta\theta}\Delta r + \frac{1}{r_p}\frac{T_W - T_P}{\Delta\theta}\Delta r = \frac{-1}{k} q_p \, r_p \Delta r \Delta\theta$$

Multiply all terms by $r_p \Delta r \Delta\theta$

$$(T_N - T_P)\, r_p\left(r_p + \frac{\Delta r}{2}\right)(\Delta\theta)^2 - (T_E - T_P)(\Delta r)^2 + (T_W - T_P)(\Delta r)^2 = \frac{-q_p}{k}(r_p \Delta r \Delta\theta)^2$$

Rearrange terms into coefficient form with A_N, A_P, A_E, A_W, Q_P:

$$\underbrace{\left[r_p\left(r_p + \frac{\Delta r}{2}\right)(\Delta\theta)^2\right]}_{A_N} T_N + \underbrace{\left[-r_p\left(r_p + \frac{\Delta r}{2}\right)(\Delta\theta)^2\right]}_{A_P} T_P + \underbrace{\left[-(\Delta r)^2\right]}_{A_E} T_E + \underbrace{\left[(\Delta r)^2\right]}_{A_W} T_W = \underbrace{\left[\frac{-q}{k}(r_p \Delta r \Delta\theta)^2\right]}_{Q_P}$$

$\boxed{A_S = 0}$

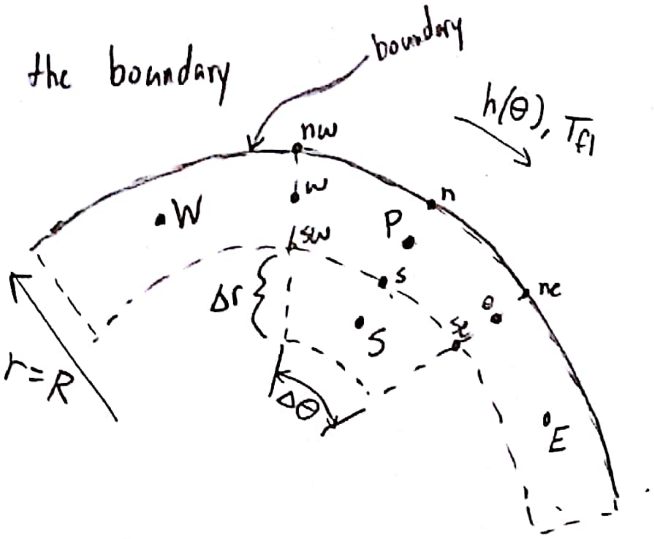
(in this case, $r_p = \frac{\Delta r}{2}$)

Scanned by CamScanner

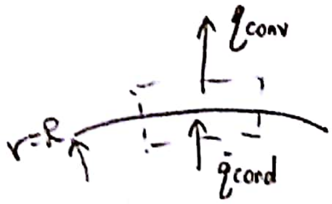
Brian Knisely ME523 HW3

Problem 1 (cont.)

c) $C\forall$ in the outer ring, adjacent to the boundary



Handle convection BC by looking at energy balance at boundary



$$q''_{conv} = q''_{cond}$$

$$h(T_s - T_f) = k \left.\frac{\partial T}{\partial r}\right|_{r=R}$$

$$\underbrace{h\left[T(R) - T_f\right] = k \left.\frac{\partial T}{\partial r}\right|_{R}}_{\text{This equation must be solved at the boundary}}$$

Start with steady 2D conduction equation $k\nabla^2 T + q = 0$

Integrate over the $C\forall$

$$\int_{\forall} \nabla^2 T \, d\forall + \int_{\forall} q \, d\forall = 0$$

Invoke Divergence Theorem for first integral

$$\int_{S} \nabla T \cdot \hat{n} \, dS + \frac{1}{k}\int_{\forall} q \, d\forall = 0$$

Separate surface integral into four parts (one for each surface)

$$\int_{S_n} \nabla T \cdot (\hat{u}_r) \, dS + \int_{S_s} \nabla T \cdot (-\hat{u}_r) \, dS + \int_{S_e} \nabla T \cdot (-\hat{u}_\theta) \, dS + \int_{S_w} \nabla T \cdot (\hat{u}_\theta) \, dS + \frac{1}{k}\int_{\forall} q \, d\forall = 0$$

Evaluate dot products

$$\int_{S_n} \frac{\partial T}{\partial r} \, dS + \int_{S_s} \frac{-\partial T}{\partial r} \, dS + \int_{S_e} \frac{-1}{r}\frac{\partial T}{\partial \theta} \, dS + \int_{S_w} \frac{1}{r}\frac{\partial T}{\partial \theta} \, dS + \frac{1}{k}\int_{\forall} q \, d\forall = 0$$

Brian Knisely ME523 HW3

Problem 1 (cont.) <u>Quadrature</u>: approximate integrals with midpoint rule (2nd-order accurate)

$$\left.\frac{\partial T}{\partial r}\right|_n R\Delta\theta + -\left.\frac{\partial T}{\partial r}\right|_s (R-\Delta r)\Delta\theta + \frac{-1}{r_e}\left.\frac{\partial T}{\partial \theta}\right|_e \Delta r + \frac{1}{r_w}\left.\frac{\partial T}{\partial \theta}\right|_w \Delta r = \frac{-q_P}{k} r_P \Delta r \Delta\theta$$

Substitute $r_e = r_w = r_p = R - \frac{\Delta r}{2}$

$$\left.\frac{\partial T}{\partial r}\right|_n R\Delta\theta - \left.\frac{\partial T}{\partial r}\right|_s (R-\Delta r)\Delta\theta - \frac{1}{R-\frac{\Delta r}{2}}\left.\frac{\partial T}{\partial \theta}\right|_e \Delta r + \frac{1}{R-\frac{\Delta r}{2}}\left.\frac{\partial T}{\partial \theta}\right|_w \Delta r = \frac{-q_P}{k}\left(R-\frac{\Delta r}{2}\right)\Delta r\Delta\theta \qquad (\star)$$

<u>Interpolation</u>: use CDS2 to approximate derivatives at s, e, w faces (both 2nd-order accurate)
use BDS2 to approximate derivative at n face

$$\left.\frac{\partial T}{\partial r}\right|_s = \frac{T_S - T_P}{\Delta r} + \mathcal{O}((\Delta r)^2) \qquad \left.\frac{\partial T}{\partial \theta}\right|_e = \frac{T_E - T_P}{\Delta\theta} + \mathcal{O}((\Delta\theta)^2) \qquad \left.\frac{\partial T}{\partial \theta}\right|_w = \frac{T_W - T_P}{\Delta\theta} + \mathcal{O}((\Delta\theta)^2)$$

Derive BDS2: set $A \neq 0$ and $B \neq 0$, $C=0$, $D=0$

$$A\,T_{i-2} + B\,T_{i-1} = (A+B)T_i + (-2A-B)\Delta x \left.\frac{\partial T}{\partial x}\right|_i + \left(2A+\tfrac{1}{2}B\right)(\Delta x)^2 \left.\frac{\partial^2 T}{\partial x^2}\right|_i + \left(-\tfrac{4}{3}A - \tfrac{1}{6}B\right)(\Delta x)^3 \left.\frac{\partial^3 T}{\partial x^3}\right|_i + \text{HOTs}$$

Set $2A + \frac{1}{2}B = 0 \rightarrow B = -4A$

$$A T_{i-2} - 4A T_{i-1} + 3T_i = 2A\,\Delta x \left.\frac{\partial T}{\partial x}\right|_i + \cancel{(0)(\Delta x)^2 \left.\frac{\partial^2 T}{\partial x^2}\right|_i}^{\,0} + \text{HOTs}$$

Divide by A and reorder

$$\underbrace{\left.\frac{\partial T}{\partial x}\right|_i = \frac{T_{i-2} - 4T_{i-1} + 3T_i}{2\Delta x} + \mathcal{O}((\Delta x)^2)}$$

Apply to this problem for $\left.\frac{\partial T}{\partial r}\right|_n$

lowercase "s"

$$\left.\frac{\partial T}{\partial r}\right|_n = \frac{T_s - 4T_P + 3T_n}{\Delta r} + \mathcal{O}((\Delta r)^2)$$

Now we need to interpolate for T_s

$$\underbrace{T_s = \frac{T_P + T_S}{2}}$$

$$\longrightarrow \left.\frac{\partial T}{\partial r}\right|_n \approx \frac{\frac{T_P + T_S}{2} - 4T_P + 3T_n}{\Delta r}$$

Brian Knisely ME523 HW3

Problem 1 (cont.) Now use the BC to solve for T_n

$h = h(\theta)$

$$\frac{h}{k}\left[T_n - T_f\right] = \frac{-\frac{7}{2}T_P + \frac{1}{2}T_S + 3T_n}{\Delta r} = \frac{h}{k}T_n - \frac{h}{k}T_f = \frac{-7T_P + T_S}{2\Delta r} + \frac{3T_n}{\Delta r}$$

$$\frac{h}{k}T_n - \frac{3}{\Delta r}T_n = \frac{-7T_P + T_S}{2\Delta r} + \frac{h}{k}T_f$$

$$T_n = \frac{\left(\frac{-7T_P + T_S}{2\Delta r} + \frac{h}{k}T_f\right)}{\left(\frac{h\Delta r - 3k}{k\Delta r}\right)} = \frac{\frac{k}{2}(T_S - 7T_P) + h\Delta r T_f}{h\Delta r - 3k}$$

$$\left.\frac{\partial T}{\partial r}\right|_n = \frac{-7}{2\Delta r}T_P + \frac{1}{2\Delta r}T_S + \frac{3}{\Delta r}\left[\frac{\frac{k}{2}(T_S - 7T_P) + h\Delta r T_f}{h\Delta r - 3k}\right]$$

$$\left.\frac{\partial T}{\partial r}\right|_n = \frac{-7}{2\Delta r}T_P + \frac{1}{2\Delta r}T_S + \frac{3}{\Delta r}\frac{k}{2}\frac{T_S}{h\Delta r - 3k} - \frac{21}{\Delta r}\frac{k}{2}\frac{T_P}{h\Delta r - 3k} + \frac{3hT_f}{h\Delta r - 3k}$$

$$\left.\frac{\partial T}{\partial r}\right|_n = T_P\underbrace{\left[\frac{-7}{2\Delta r} - \frac{21k}{2\Delta r(h\Delta r - 3k)}\right]}_{c_1} + T_S\underbrace{\left[\frac{1}{2\Delta r} + \frac{3k}{2\Delta r(h\Delta r - 3k)}\right]}_{c_2} + T_f\underbrace{\left[\frac{3h}{h\Delta r - 3k}\right]}_{c_3}$$

$$\left.\frac{\partial T}{\partial r}\right|_n = c_1T_P + c_2T_S + c_3T_f$$

Substitute derivative approximations into original equation (*)

$$\left[c_1T_P + c_2T_S + c_3T_f\right]R\Delta\theta - \frac{T_S - T_P}{\Delta r}(R - \Delta r)\Delta\theta - \frac{1}{R - \frac{\Delta r}{2}}\frac{T_E - T_P}{\Delta\theta}\Delta r + \frac{1}{R - \frac{\Delta r}{2}}\frac{T_W - T_P}{\Delta\theta}\Delta r = \underbrace{\frac{-q_P}{k}\left(R - \frac{\Delta r}{2}\right)\Delta r\Delta\theta}_{(RHS)_a}$$

Start re-arranging to eventually find A_S, A_P, A_E, A_W, Q_P

Cancel

$$T_P\left[c_1 + \frac{(R - \Delta r)(\Delta\theta)}{\Delta r} + \cancel{\frac{\Delta r}{\left(R - \frac{\Delta r}{2}\right)\Delta\theta}} - \cancel{\frac{\Delta r}{\left(R - \frac{\Delta r}{2}\right)\Delta\theta}}\right] + T_S\left[c_2 - \frac{(R - \Delta r)\Delta\theta}{\Delta r}\right] + T_E\left[\frac{-\Delta r}{\left(R - \frac{\Delta r}{2}\right)\Delta\theta}\right] + T_W\left[\frac{\Delta r}{\left(R - \frac{\Delta r}{2}\right)\Delta\theta}\right]$$

$$+ R\Delta\theta c_3T_f = (RHS)_a$$

Brian Knisely ME523 HW3

Problem 1 (cont.) The Full coefficients are:

$$\left[\frac{-7}{2\Delta r} - \frac{21k}{2\Delta r(h\Delta r - 3k)} + \frac{(R-\Delta r)(\Delta\theta)}{\Delta r}\right] = A_P$$

$$\left[\frac{1}{2\Delta r} + \frac{3k}{2\Delta r(h\Delta r - 3k)} - \frac{(R-\Delta r)\Delta\theta}{\Delta r}\right] = A_S$$

$$0 = A_N$$

$$\left[\frac{-\Delta r}{\left(R-\frac{\Delta r}{2}\right)\Delta\theta}\right] = A_E$$

$$\left[\frac{\Delta r}{\left(R-\frac{\Delta r}{2}\right)\Delta\theta}\right] = A_W$$

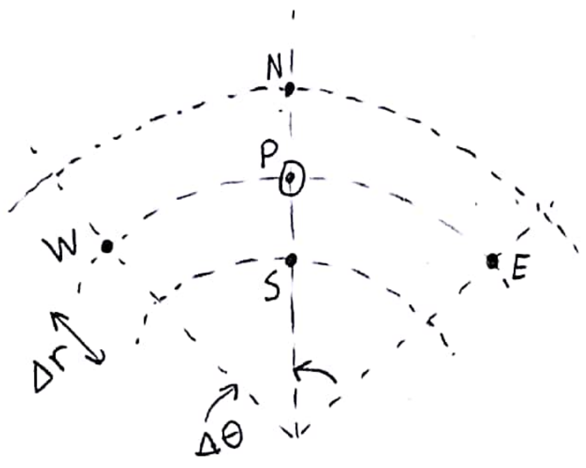
$$\left[\frac{-q_P}{k}\left(R-\frac{\Delta r}{2}\right)\Delta r\Delta\theta - \frac{3hR\Delta\theta T_{fluid}}{h\Delta r - 3k}\right] = Q_P$$

$$A_P T_P + A_S T_S + A_E T_E + A_W T_W = Q_P$$

Brian Knisely ME523 HW3

Problem 2

FDM for internal node



Find approximation to

$k\nabla^2 T + q = 0$

using Taylor Series expansions

PDE is $k\nabla^2 T + q = 0 \rightarrow \dfrac{\partial^2 T}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial T}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2 T}{\partial \theta^2} = \dfrac{-q}{k}$

Use CDS2 for all derivatives (2nd-order accurate)

$$\left.\frac{\partial T}{\partial r}\right|_P = \frac{T_N - T_S}{2\Delta r} - \frac{1}{6}(\Delta r)^2 \left.\frac{d^3 T}{dx^3}\right|_P + HOTs$$

$$\left.\frac{\partial^2 T}{\partial r^2}\right|_P = \frac{T_N - 2T_P + T_S}{(\Delta r)^2} - \frac{1}{12}(\Delta r)^2 \left.\frac{d^4 T}{dr^4}\right|_P + HOTs$$

$$\left.\frac{\partial^2 T}{\partial \theta^2}\right|_P = \frac{T_W - 2T_P + T_E}{(\Delta\theta)^2} - \frac{1}{12}(\Delta\theta)^2 \frac{d^4 T}{d\theta^4} + HOTs$$

Leading-order TE terms

Plug into PDE, evaluate at point P

$$\frac{T_N - 2T_P + T_S}{(\Delta r)^2} + \frac{1}{r_P}\frac{T_N - T_S}{2\Delta r} + \frac{1}{r_P^2}\frac{T_W - 2T_P + T_E}{(\Delta\theta)^2} = \frac{-q}{k}$$

Multiply all terms by $(r_P \Delta r \Delta\theta)^2$

$$(T_N - 2T_P + T_S)(r_P\Delta\theta)^2 + \tfrac{1}{2}(T_N - T_S) r_P \Delta r (\Delta\theta)^2 + (T_W - 2T_P + T_E)(\Delta r)^2 = \frac{-q}{k}(r_P \Delta r \Delta\theta)^2$$

Brian Knisely ME523 HW3

Problem 2 (cont.)

Now rearrange to show coefficients

$$\left[(r_P\Delta\theta)^2+\frac{1}{2}r_P\Delta r(\Delta\theta)^2\right]T_N+\left[(r_P\Delta\theta)^2-\frac{1}{2}r_P\Delta r(\Delta\theta)^2\right]T_S+\left[-2(r_P\Delta\theta)^2-2(\Delta r)^2\right]T_P$$
$$+\left[(\Delta r)^2\right]T_E+\left[(\Delta r)^2\right]T_W=\left[\frac{-q}{k}(r_P\Delta r\Delta\theta)^2\right]$$

Simplify expressions

$$\boxed{\underbrace{\left[\left(r_P+\frac{\Delta r}{2}\right)r_P(\Delta\theta)^2\right]}_{A_N}T_N+\underbrace{\left[\left(r_P-\frac{\Delta r}{2}\right)r_P(\Delta\theta)^2\right]}_{A_S}T_S+\underbrace{\left[-2\left\{(r_P\Delta\theta)^2+(\Delta r)^2\right\}\right]}_{A_P}T_P+\underbrace{\left[(\Delta r)^2\right]}_{A_E}T_E+\underbrace{\left[(\Delta r)^2\right]}_{A_W}T_W=\underbrace{\left[\frac{-q}{k}(r_P\Delta r\Delta\theta)^2\right]}_{Q_P}}$$

Problem 3

FVM vs FDM

The result for FDM is almost identical to the FVM result for an interior volume. The only difference is the A_P coefficient. Both results have the same A_n, A_s, A_E, A_W, and Q_P. They both form linear matrix equations. They are both second-order accurate. In applying boundary conditions, the FVM requires an expression for the temperature on a surface of a control volume, while the FDM requires an expression for temperature on a node itself. The FVM has more flexibility with boundary conditions, as corner points with discontinuous BCs can be handled, while only one value can be used at a corner node in FDM.