1. Brian Knisely ME 523 HW 7

Basic form, steady:

$$\underbrace{\left(\frac{\dot m_n}{2} - \frac{\Gamma \Delta x}{\Delta y}\right)}_{A_N}\phi_N + \underbrace{\left(\frac{-\dot m_s}{2} - \frac{\Gamma \Delta x}{\Delta y}\right)}_{A_S}\phi_S + \underbrace{\left(\frac{\dot m_e}{2} - \frac{\Gamma \Delta y}{\Delta x}\right)}_{A_E}\phi_E + \underbrace{\left(\frac{-\dot m_w}{2} - \frac{\Gamma \Delta y}{\Delta x}\right)}_{A_W}\phi_W + \underbrace{2\left(\frac{\Gamma \Delta y}{\Delta x} + \frac{\Gamma \Delta x}{\Delta y}\right)}_{A_P}\phi_P = \underbrace{q_p \Delta x \Delta y}_{Q_P}$$

Basic form, unsteady:

$$\rho \Delta x \Delta y \frac{\phi_P^{n+1} - \phi_P^{n}}{\Delta t} + A_W \bar\phi_W + A_S \bar\phi_S + A_P \bar\phi_P + A_N \bar\phi_N + A_E \bar\phi_E = Q_P$$

Implicit Euler: $\bar\phi_W = f\,\phi_W^{n+1} + (1-f)\,\phi_W^{n}$, $f = 1$

$\bar\phi_W = \phi_W^{n+1}$ and similar for other coefficients

$$\rho \Delta x \Delta y \frac{\phi_P^{n+1}}{\Delta t} - \rho \Delta x \Delta y \frac{\phi_P^{n}}{\Delta t} + A_W \phi_W^{n+1} + A_S \phi_S^{n+1} + A_P \phi_P^{n+1} + A_N \phi_N^{n+1} + A_E \phi_E^{n+1} = Q_P$$

$$\underbrace{A_W}_{A_{W_u}} \phi_W^{n+1} + \underbrace{A_S}_{A_{S_u}} \phi_S^{n+1} + \underbrace{\left(\frac{\rho \Delta x \Delta y}{\Delta t} + A_P\right)}_{A_{P_u}} \phi_P^{n+1} + \underbrace{A_N}_{A_{N_u}} \phi_N^{n+1} + \underbrace{A_E}_{A_{E_u}} \phi_E^{n+1} = \underbrace{\rho \frac{\phi_P^{n}}{\Delta t} \Delta x \Delta y + Q_P}_{Q_{P_u}}$$

Implicit Euler Unsteady coefficients in terms of steady coefficients:

$$\boxed{A_{W_u} = A_{W_s} = \frac{-\dot m_w}{2} - \frac{\Gamma \Delta y}{\Delta x}}$$

$$\boxed{A_{S_u} = A_{S_s} = \frac{-\dot m_s}{2} - \frac{\Gamma \Delta x}{\Delta y}}$$

$$\boxed{A_{P_u} = \frac{\rho \Delta x \Delta y}{\Delta t} + A_{P_s} = \frac{\rho \Delta x \Delta y}{\Delta t} + \frac{2\Gamma \Delta y}{\Delta x} + \frac{2\Gamma \Delta x}{\Delta y}}$$

$$\boxed{A_{N_u} = A_{N_s} = \frac{\dot m_n}{2} - \frac{\Gamma \Delta x}{\Delta y}}$$

$$\boxed{A_{E_u} = A_{E_s} = \frac{\dot m_e}{2} - \frac{\Gamma \Delta y}{\Delta x}}$$

$$\boxed{Q_{P_u} = Q_{P_s} + \rho \Delta x \Delta y \frac{\phi_P^{n}}{\Delta t} = \rho \frac{\phi_P^{n}}{\Delta t} \Delta x \Delta y + q_p \Delta x \Delta y}$$

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Explicit Euler: $f=0 \rightarrow \bar{\Phi}_W = \phi_W^n$

$$\rho \Delta x \Delta y \frac{\phi_P^{n+1} - \phi_P^n}{\Delta t} + A_W \phi_W^n + A_S \phi_S^n + A_P \phi_P^n + A_N \phi_N^n + A_E \phi_E^n = Q_P$$

$$\frac{\rho \Delta x \Delta y}{\Delta t} \phi_P^{n+1} = Q_P + \rho \Delta x \Delta y \frac{\phi_P^n}{\Delta t} - A_W \phi_W^n - A_S \phi_S^n - A_P \phi_P^n - A_N \phi_N^n - A_E \phi_E^n$$

Explicit Euler Unsteady Coefficients in terms of steady coefficients:

$$\boxed{A_{W_u} = A_{S_u} = A_{N_u} = A_{E_u} = 0}$$

$$\boxed{A_{P_u} = \frac{\rho \Delta x \Delta y}{\Delta t}}$$

$$\boxed{Q_{P_u} = Q_{P_s} + \frac{\rho \Delta x \Delta y}{\Delta t} \phi_P^n - A_{W_s} \phi_W^n - A_{S_s} \phi_S^n - A_{P_s} \phi_P^n - A_{N_s} \phi_N^n - A_{E_s} \phi_E^n}$$

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Crank-Nicolson : $f = 1/2 \rightarrow \overline{\Phi}_W = \frac{1}{2}\phi_W^{n+1} + \frac{1}{2}\phi_W^n$

$$\rho\Delta x\Delta y \frac{\phi_P^{n+1} - \phi_P^n}{\Delta t} + A_W\overline{\Phi}_W + A_S\overline{\Phi}_S + A_P\overline{\Phi}_P + A_N\overline{\Phi}_N + A_E\overline{\Phi}_E = Q_P$$

$$\rho\Delta x\Delta y \frac{\phi_P^{n+1}}{\Delta t} + \frac{1}{2}\left\{A_W\phi_W^{n+1} + A_S\phi_S^{n+1} + A_P\phi_P^{n+1} + A_N\phi_N^{n+1} + A_E\phi_E^{n+1}\right\} = Q_P + \frac{\rho\Delta x\Delta y}{\Delta t}\phi_P^n - \frac{1}{2}\left\{A_W\phi_W^n + A_S\phi_S^n + A_P\phi_P^n + A_N\phi_N^n + A_E\phi_E^n\right\}$$

$$\frac{1}{2}A_W\phi_W^{n+1} + \frac{1}{2}A_S\phi_S^{n+1} + \left(\frac{1}{2}A_P + \frac{\rho\Delta x\Delta y}{\Delta t}\right)\phi_P^{n+1} + \frac{1}{2}A_N\phi_N^{n+1} + \frac{1}{2}A_E\phi_E^{n+1} = Q_P - \frac{1}{2}A_W\phi_W^n - \frac{1}{2}A_S\phi_S^n + \left(\frac{\rho\Delta x\Delta y}{\Delta t} - \frac{1}{2}A_P\right)\phi_P^n - \frac{1}{2}A_N\phi_N^n - \frac{1}{2}A_E\phi_E^n$$

Crank-Nicolson Unsteady coefficients in terms of steady coefficients:

$\boxed{A_{W_u} = \frac{1}{2}A_{W_s}}$ $\boxed{A_{S_u} = \frac{1}{2}A_{S_s}}$ $\boxed{A_{N_u} = \frac{1}{2}A_{N_s}}$ $\boxed{A_{E_u} = \frac{1}{2}A_{E_s}}$ $\boxed{A_{P_u} = \frac{1}{2}A_{P_s} + \frac{\rho\Delta x\Delta y}{\Delta t}}$

$$\boxed{Q_{P_u} = Q_{P_s} - \frac{1}{2}A_{W_s}\phi_W^n - \frac{1}{2}A_{S_s}\phi_S^n + \left(\frac{\rho\Delta x\Delta y}{\Delta t} - \frac{1}{2}A_{P_s}\right)\phi_P^n - \frac{1}{2}A_{N_s}\phi_N^n - \frac{1}{2}A_E\phi_E^n}$$