a. The convergence rate, estimated exact solution, and discretization error are tabulated below for the coarse grids (n = 10, 20, 40). The coarse grid with CDS does a reasonably good job of estimating the exact solution - within 0.4%.

φANALYTIC @(x = 0.9) = 1.1353

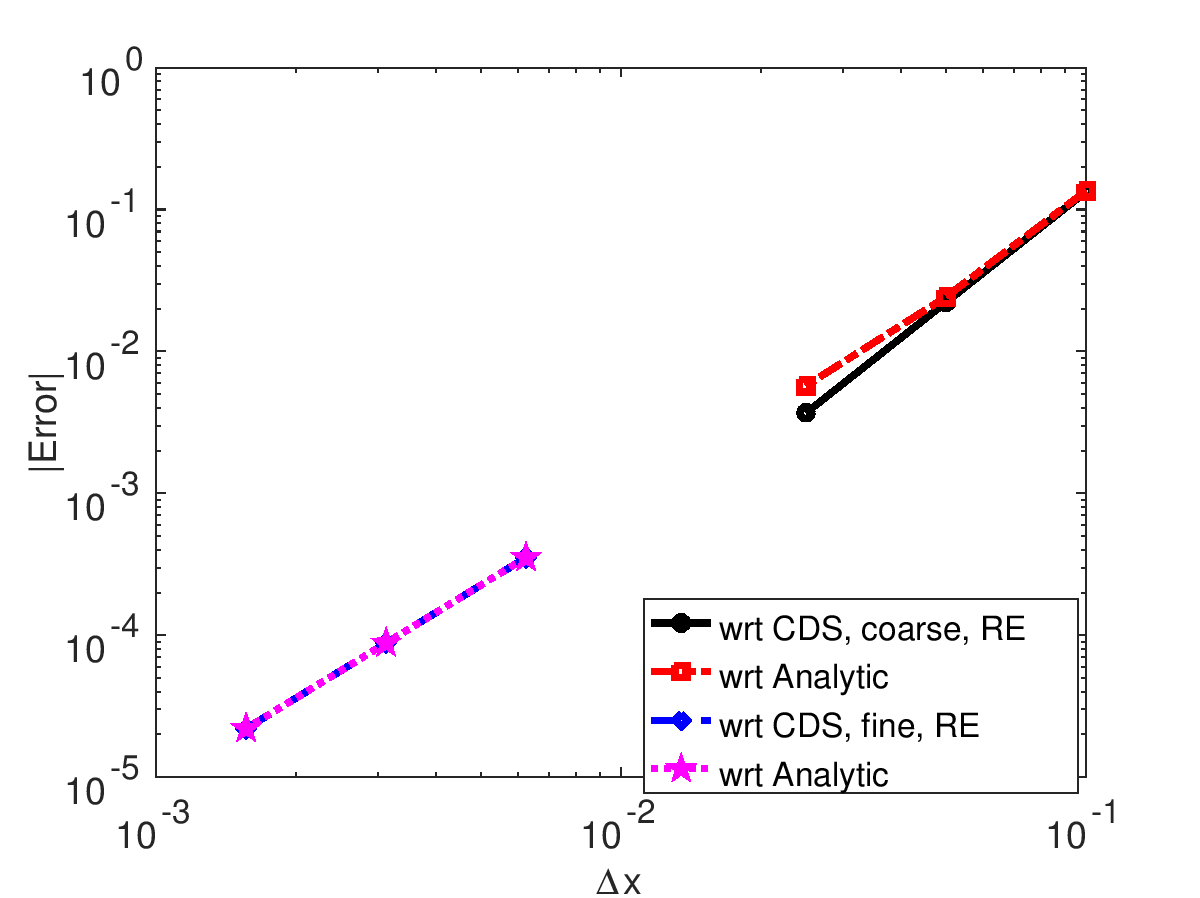
|  |  |
| --- | --- |
| PCDS,coarse | 2.5873 |
| φCDS,coarse,RE | 1.1333 |
| εhdCDS,coarse | 0.0036907 |

b. The convergence rate, estimated exact solution, and discretization error are tabulated below for the fine grids (n = 160, 320, 640). The estimated exact solution is very close to the analytic solution.

φANALYTIC @(x = 0.9) = 1.1353

|  |  |
| --- | --- |
| PCDS,fine | 2.0014 |
| φCDS,fine,RE | 1.1353 |
| εhdCDS,fine | 2.2006e-005 |

c. The errors in the numerical solutions with respect to the “exact” solutions at x = 0.9 are shown below. For the coarse grids, the estimated error values at the n = 10 and n = 20 are close to the exact error values, but the inaccuracy in slope (= convergence rate = P) results in an inaccurate estimation at n=40. For the fine meshes, the error is estimated extremely well; the curves are directly on top of each other and are indistinguishable.

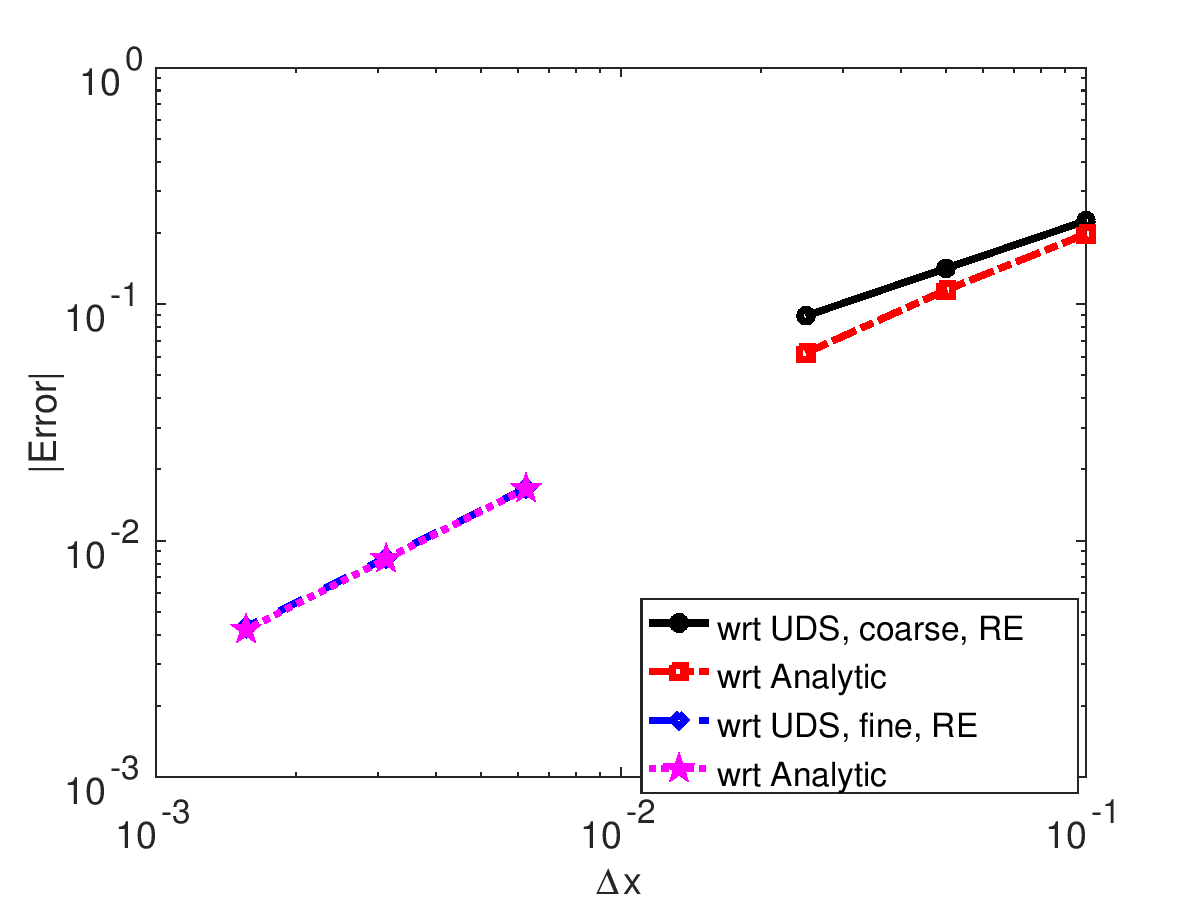


d. The convergence rates, estimated exact solutions, and discretization errors are tabulated below for the coarse grids (n = 10, 20, 40) and for the fine grids (n = 160, 320, 640).

φANALYTIC @(x = 0.9) = 1.1353

|  |  |
| --- | --- |
| PUDS,coarse | 0.66726 |
| φUDS,coarse,RE | 1.1083 |
| εhdUDS,coarse | -0.089224 |
| PUDS,fine | 0.97704 |
| φUDS,fine,RE | 1.1352 |
| εhdUDS,fine | -0.0042989 |

The errors in the numerical solutions with respect to the “exact” solutions are shown below. Since UDS is only first-order accurate unlike CDS, the estimation for the three coarse grids is worse than for the CDS case. The slope (=convergence rate) is off significantly; it should be 1.00 while the estimation at coarse grids is only 0.667. For the finer grids, the convergence rate is estimated much better and as a result the estimated error is much closer to the exact error.



**Summary of Results:**

φANALYTIC @(x = 0.9) = 1.1353

|  |  |  |
| --- | --- | --- |
| n | φCDS @ x = 0.9 | φUDS @ x = 0.9 |
| 10 | 1.00000 | 1.3333 |
| 20 | 1.1111 | 1.2500 |
| 40 | 1.1296 | 1.1975 |
|  |  |  |
| 160 | 1.1350 | 1.1519 |
| 320 | 1.1352 | 1.1437 |
| 640 | 1.1353 | 1.1395 |

|  |  |
| --- | --- |
| PCDS,coarse | 2.5873 |
| φCDS,coarse,RE | 1.1333 |
| εhdCDS,coarse | 0.0036907 |
| PCDS,fine | 2.0014 |
| φCDS,fine,RE | 1.1353 |
| εhdCDS,fine | 2.2006e-005 |
|  |  |
| PUDS,coarse | 0.66726 |
| φUDS,coarse,RE | 1.1083 |
| εhdUDS,coarse | -0.089224 |
| PUDS,fine | 0.97704 |
| φUDS,fine,RE | 1.1352 |
| εhdUDS,fine | -0.0042989 |