DeePoly - High-Order Accuracy Neural Network Framework for Function Approximation and PDE Solving

Introduction

DeePoly is a novel general-purpose platform for function approximation and equation solving algorithms. Core theorem and algorithm: DeePoly: A High-Order Accuracy and Efficiency Scientific Machine Learning Framework

Key Features

- **Universal PDE Solver**: Fit for all kinds of PDE.
- **Mesh-Free**: Sampling points can be randomly generated with no logical relationships, suitable for complex geometries.
- **High Accuracy**: Achieves high-order convergence.
- **Scheme-Free**: Handles derivative relationships using automatic differentiation.
- **Efficient**: Computational efficiency comparable to traditional finite difference methods.
- **GPU Accelerated**: Supports CPU parallelism and GPU acceleration.
- **Applicable to Complex and Discontinuous Problems**: Accurately approximates discontinuous and high-gradient functions.
- **Suitable for Inverse Problems**: Solves inverse problems with higher accuracy than PINNs.

Version Information

Previous version: v0.1 (Beta). The `cases` directory includes both function approximation and PDE solving test cases. The core algorithm in `src` contains all derivative computation code with comprehensive testing.

Current version: v0.2:

- High-accuracy solving for arbitrary-dimensional linear PDEs.
- Auto code of the PDE.
- English-commented version.
- Other corrections of output and visualize.

Usage

Develop your own problems in the `cases` directory, including data generation, output, and configuration files.

```
# Run function fitting example
python src/main_solver.py --case_path cases/func_fitting_cases/case_2d

# Run PDE solving example
python src/main_solver.py --case_path
cases/linear_pde_cases/poisson_2d_sinpixsinpiy
# First time run, you need set auto-code=true, and then rerun it.
```

Installation Requirements

- Python
- CUDA
- torch

Test Cases & Results

The following test cases demonstrate DeePoly's capabilities across function approximation and PDE solving:

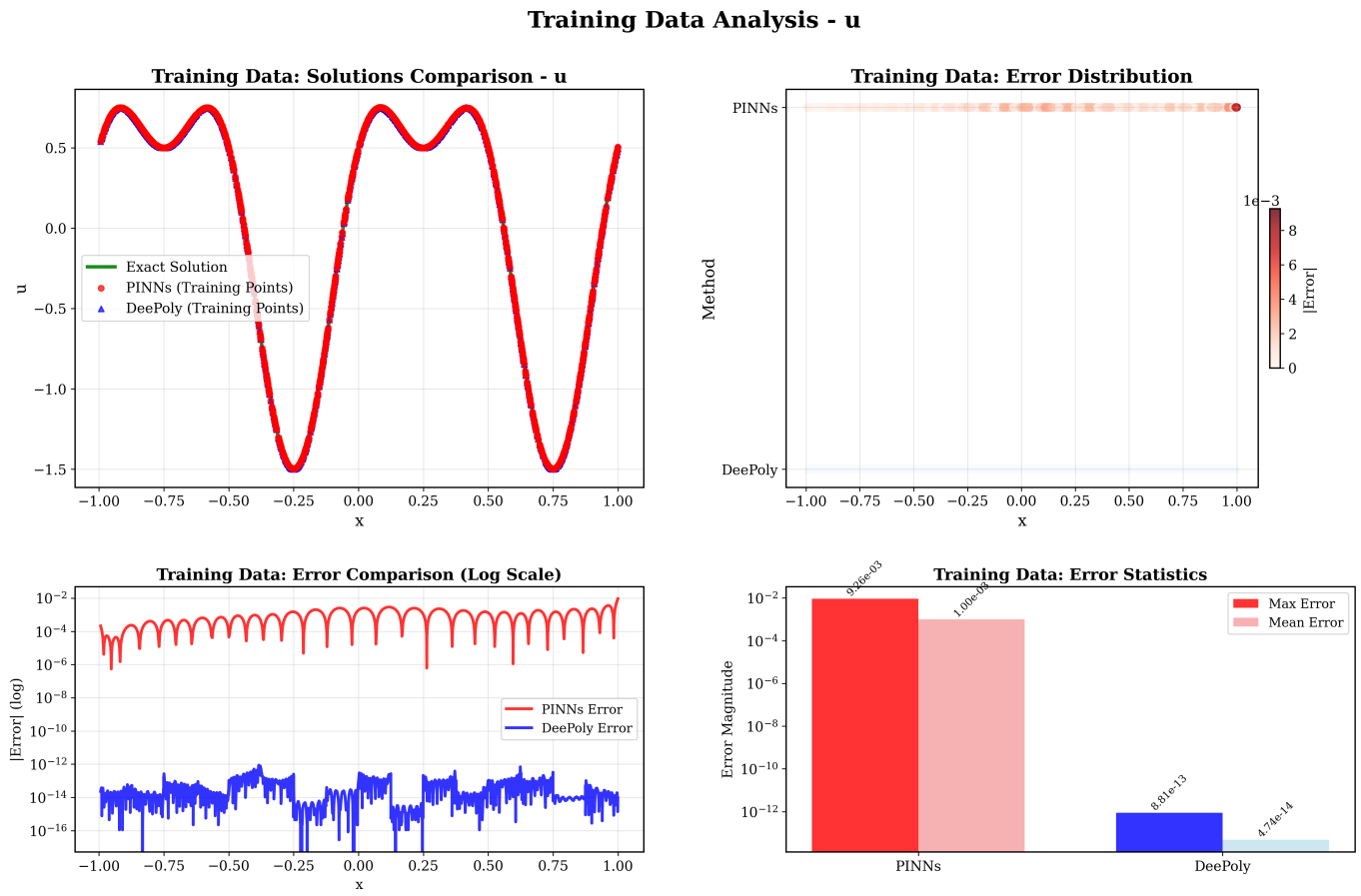
Function Approximation Cases

1. 1D Sine Function

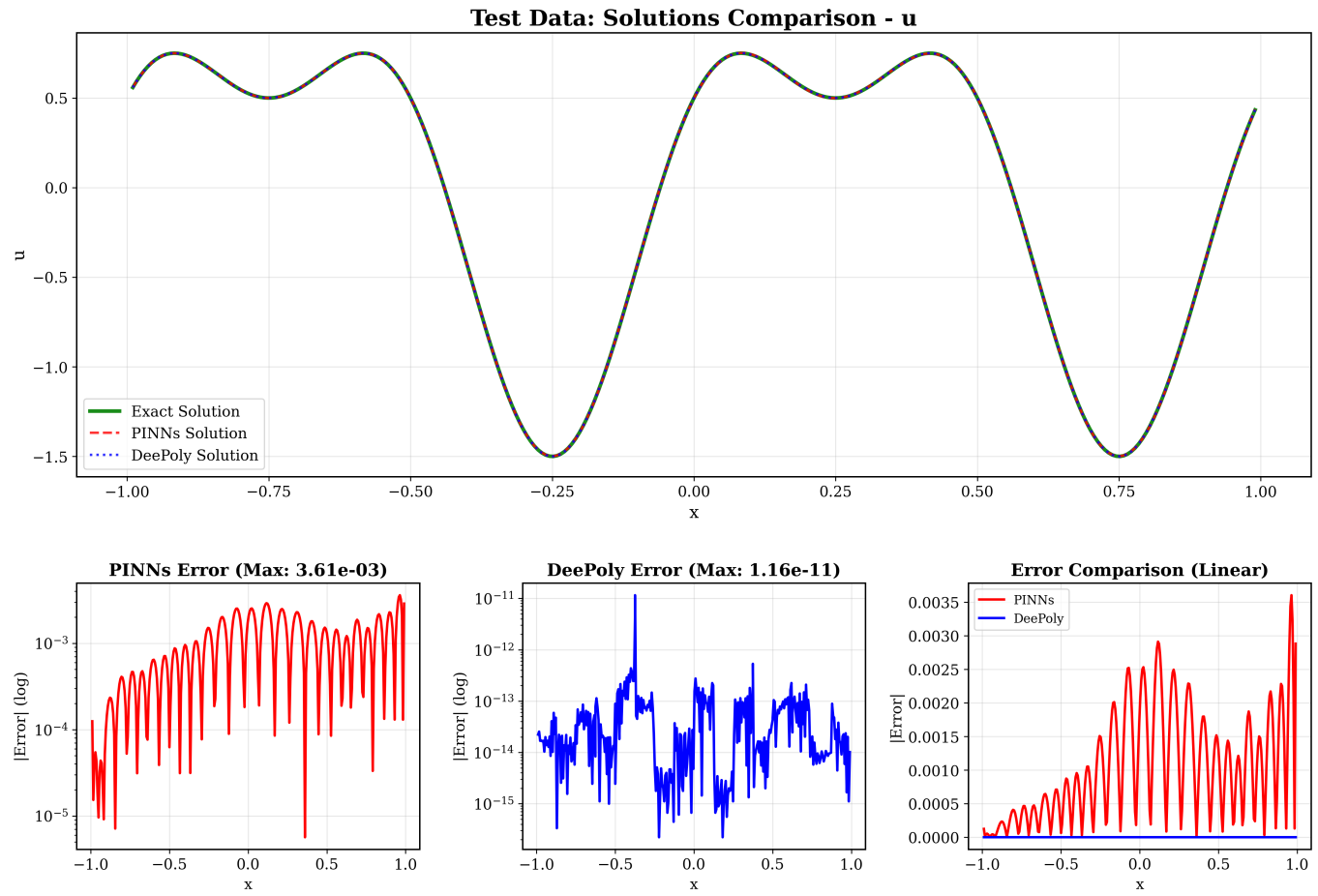
Case Directory: `func_fitting_cases/test_sin`
Target Function: `f(x) = sin(2πx) + 0.5cos(4πx)`
Problem Type: Baseline function approximation test case

Results

Training Data Analysis



Test Data Analysis



Performance Metrics

Component	Time Cost
Scoper/DNN	7.2049 seconds
Sniper	0.1630 seconds
Total	7.3679 seconds

Additional Information

- **Error Analysis**: Detailed metrics available in `cases/func_fitting_cases/test_sin/results/error_analysis_report.txt`
- **Configuration**: See `cases/func_fitting_cases/test_sin/config.json`

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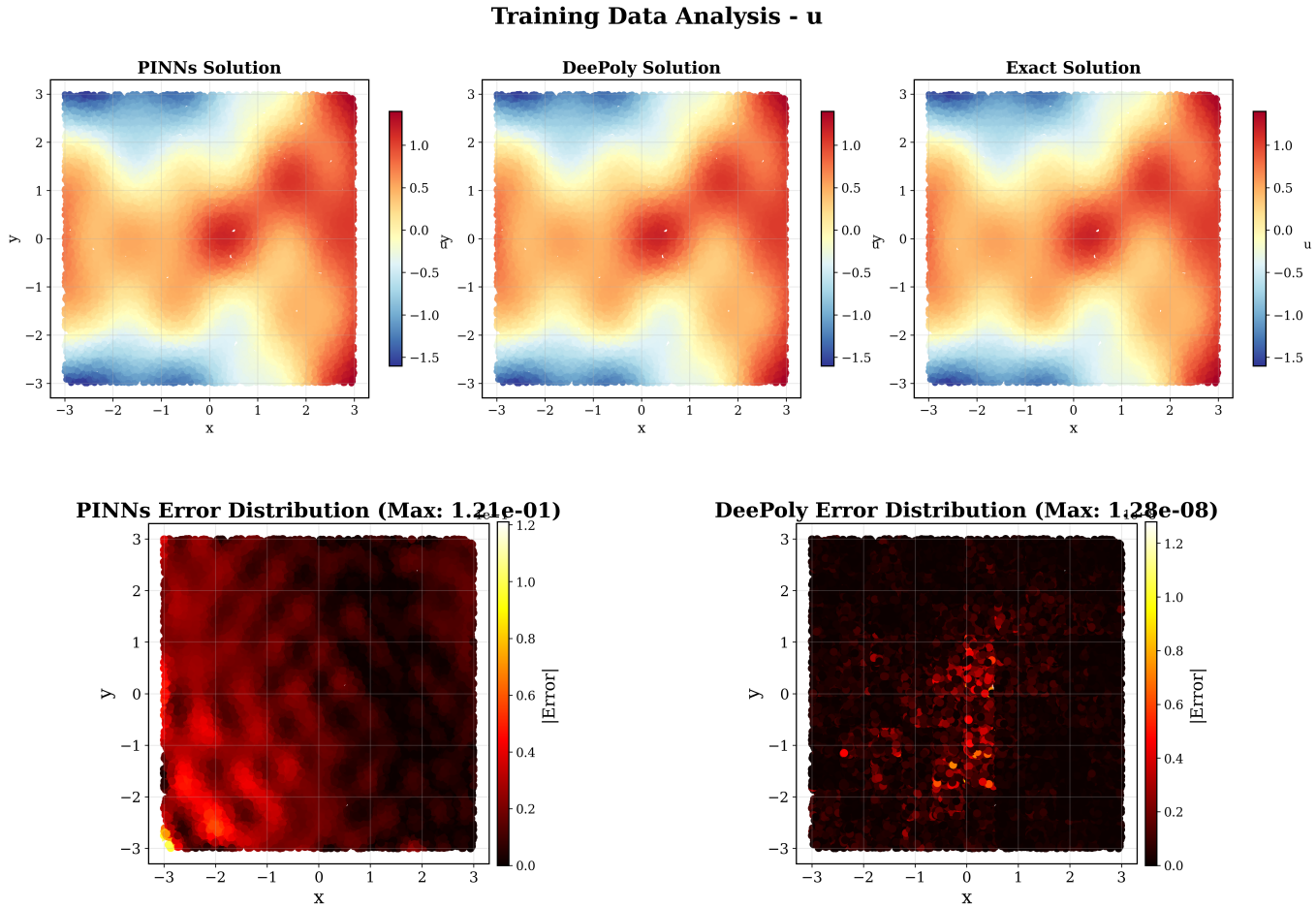
2. Complex 2D Function

Case Directory: `func_fitting_cases/case_2d`
Target Function: `f(x,y) = Multi-Gaussian peaks + trigonometric + polynomial terms on [-3,3]²`
Problem Type: Complex multi-modal function approximation

Results

Training Data Analysis



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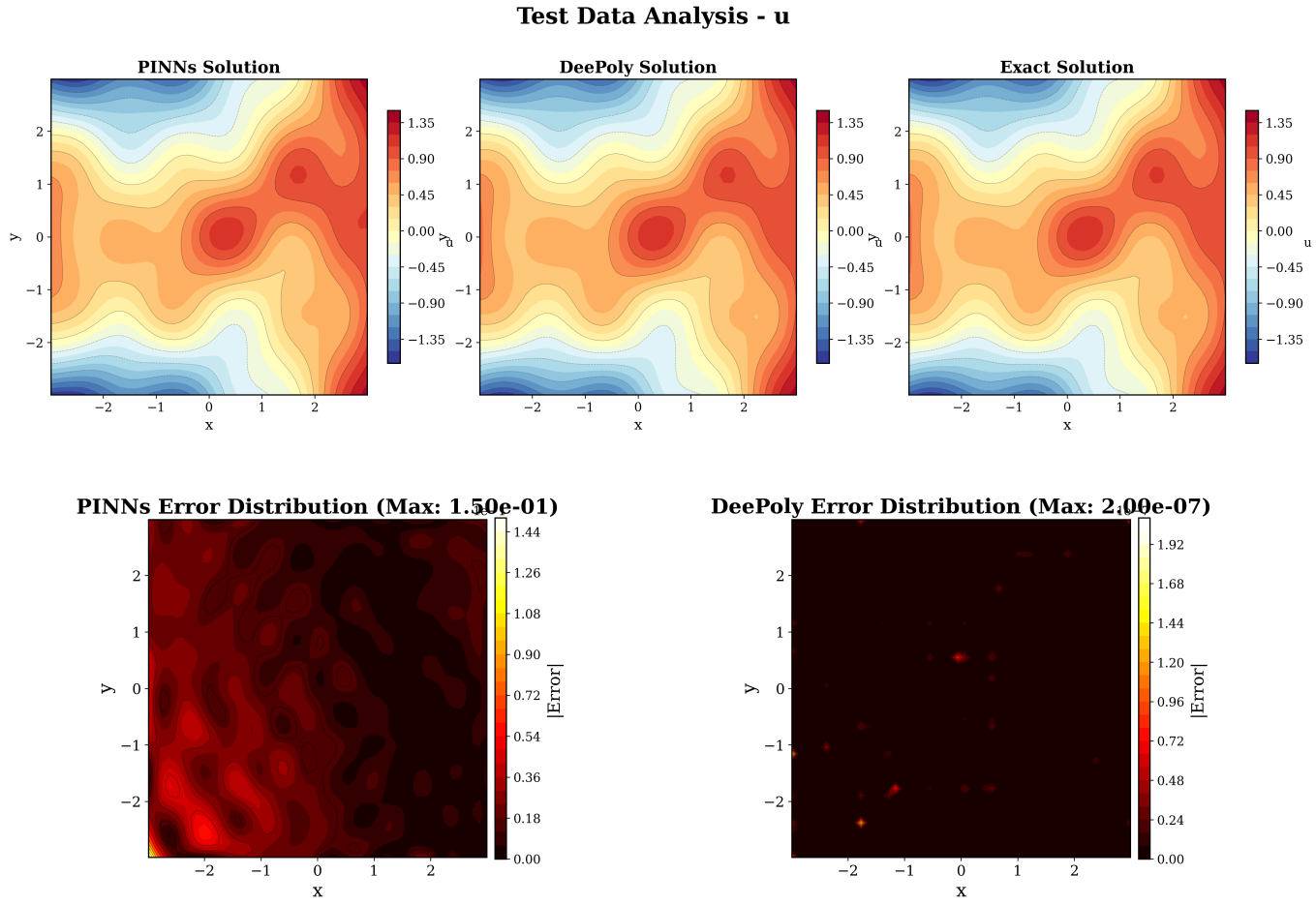

Performance Metrics

Component	Time Cost
Scoper/DNN	36.7667 seconds
Sniper	6.2360 seconds
Total	43.0027 seconds

Additional Information

- **Error Analysis**: Detailed metrics available in `cases/func_fitting_cases/case_2d/results/error_analysis_report.txt`
- **Configuration**: See `cases/func_fitting_cases/case_2d/config.json`

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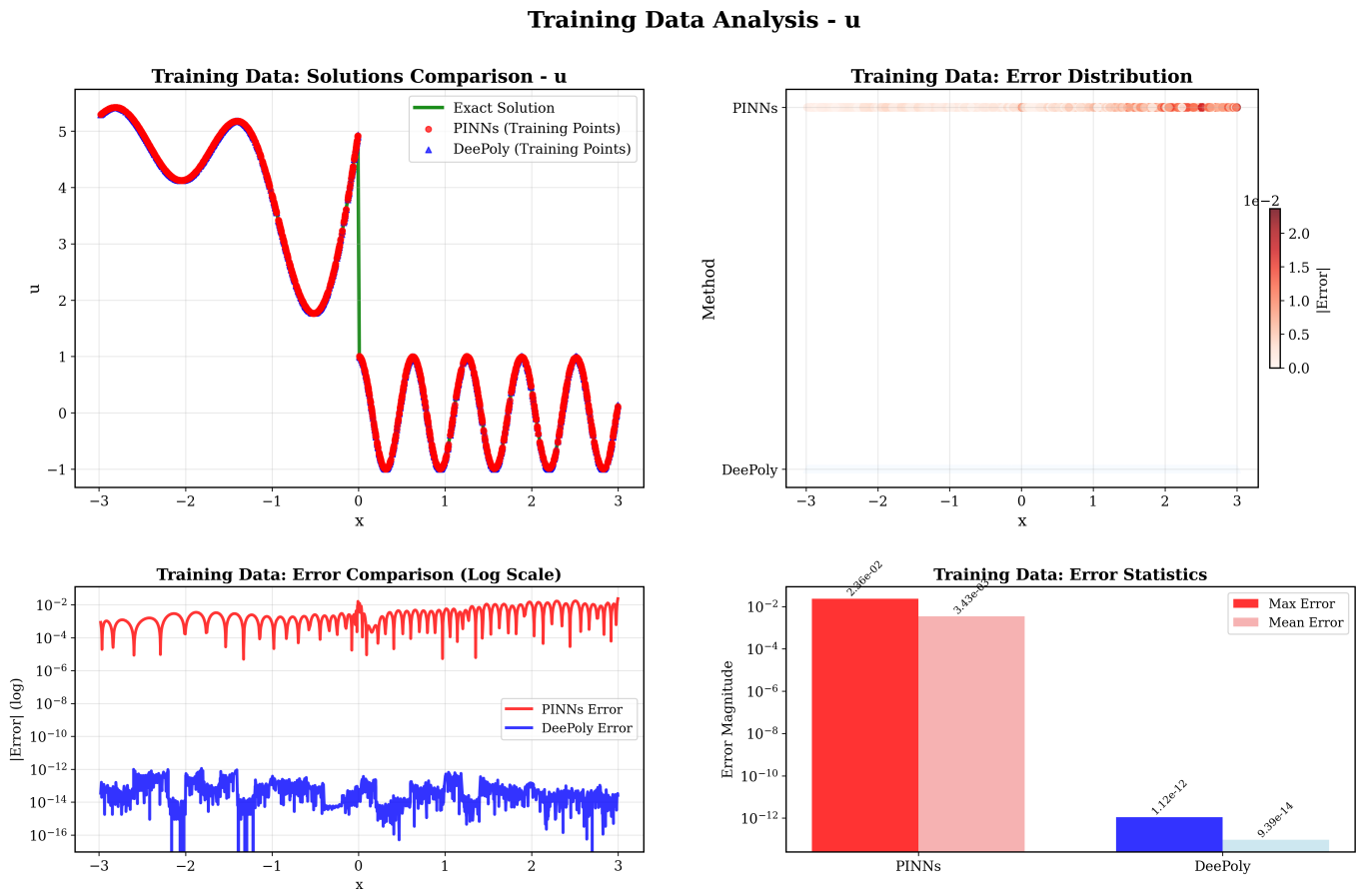
3. Discontinuous Function

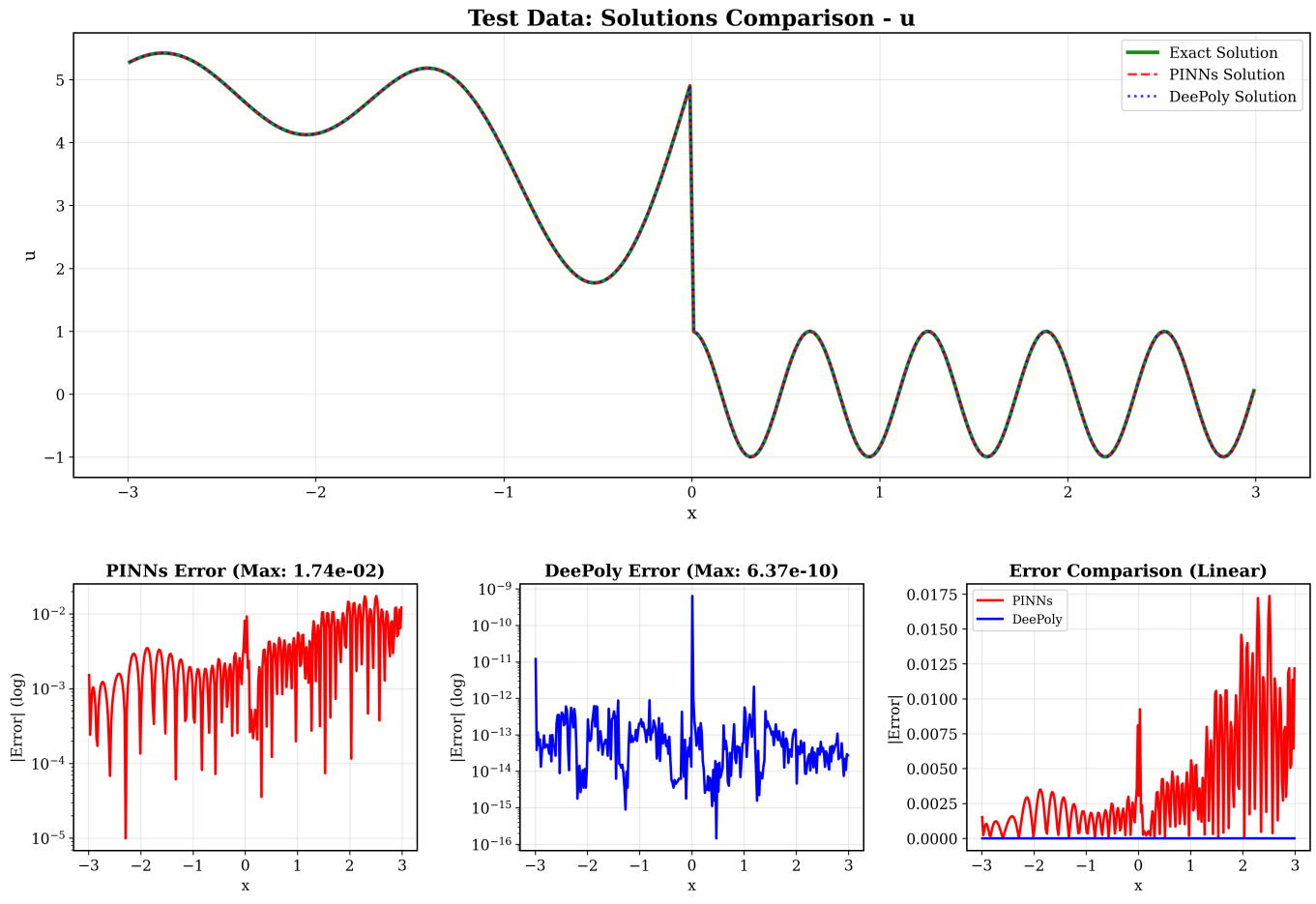
Case Directory: `func_fitting_cases/discontinuous_case1`
Target Function: `f(x,y) = Near-discontinuous function with sharp transitions`
Problem Type: Challenging discontinuous function approximation

Results

Training Data Analysis



Test Data Analysis



Performance Metrics

Component	Time Cost
Scoper/DNN	17.8730 seconds
Sniper	0.2645 seconds
Total	18.1374 seconds

Additional Information

- **Error Analysis**: Detailed metrics available in `cases/func_fitting_cases/discontinuous_case1/results/error_analysis_report.txt`
- **Configuration**: See `cases/func_fitting_cases/discontinuous_case1/config.json`

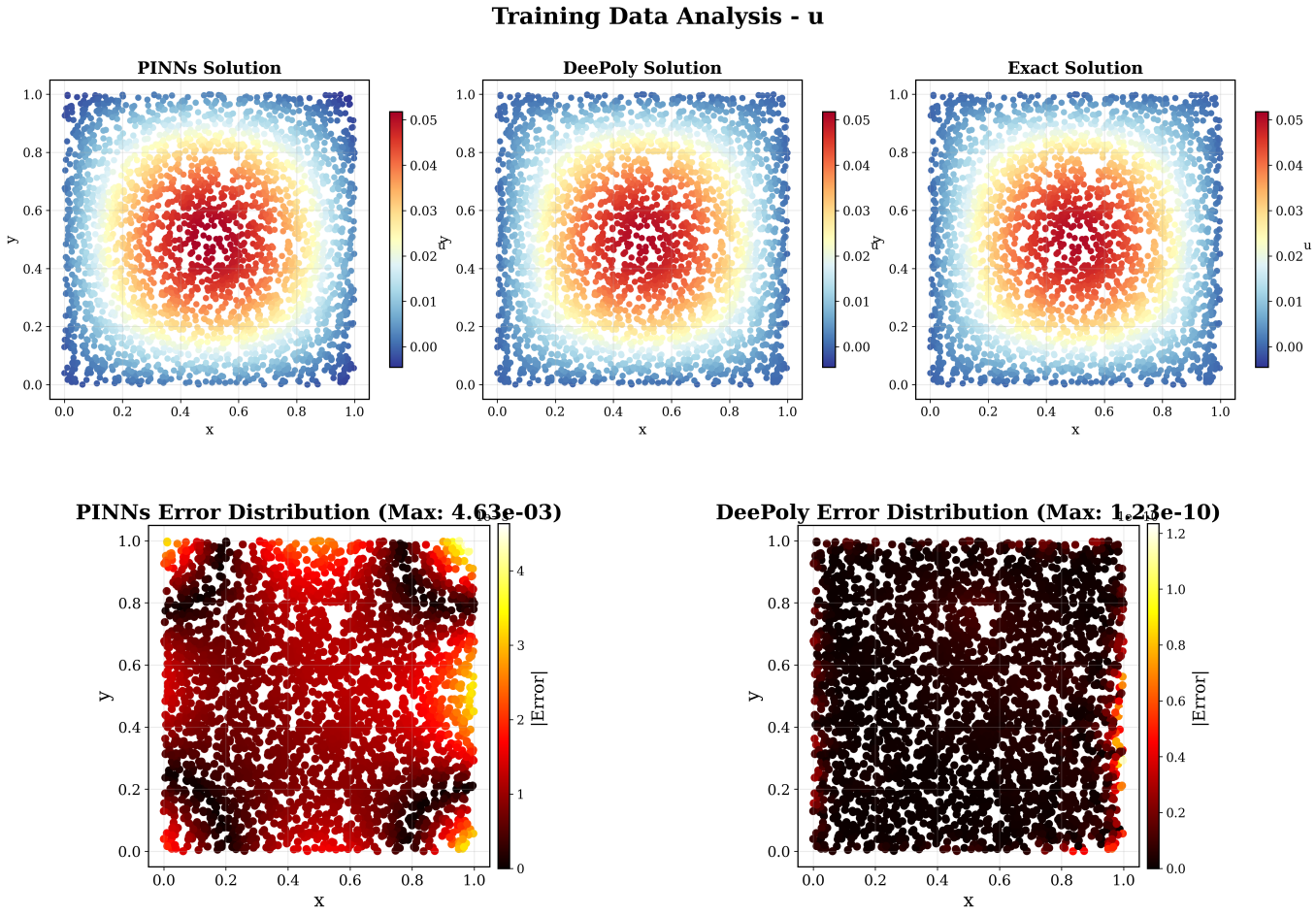
PDE Solving Cases

4. 2D Poisson Equation

Case Directory: `linear_pde_cases/poisson_2d_sinpixsinpiy`
PDE Equation: $\nabla^2 u = -\sin(\pi x)\sin(\pi y)$ on $[0,1]^2$
Problem Type: Linear elliptic PDE with analytical solution

Results

Training Data Analysis



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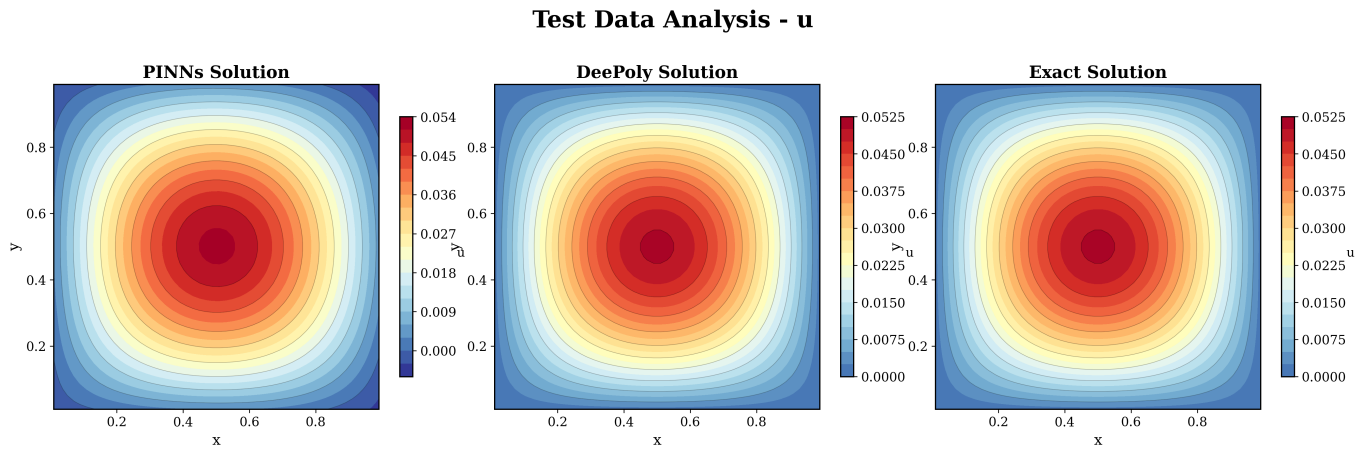

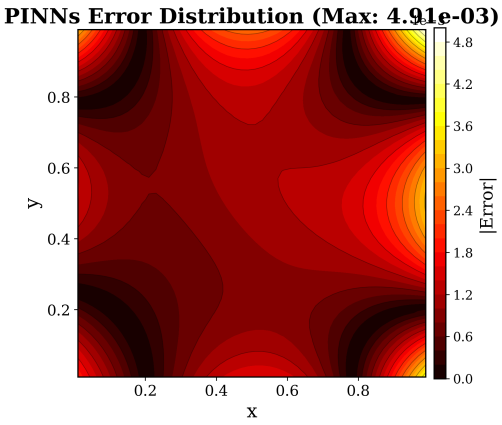

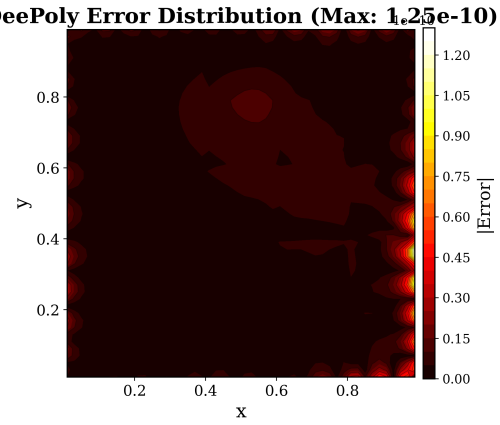

Performance Metrics

Component	Time Cost
Scoper/PINNs	10.2692 seconds
Sniper	2.0110 seconds
Total	12.2802 seconds

Additional Information

- **Error Analysis**: Detailed metrics available in `cases/linear_pde_cases/poisson_2d_sinpixsinpiy/results/error_analysis_report.txt`
- **Configuration**: See `cases/linear_pde_cases/poisson_2d_sinpixsinpiy/config.json`

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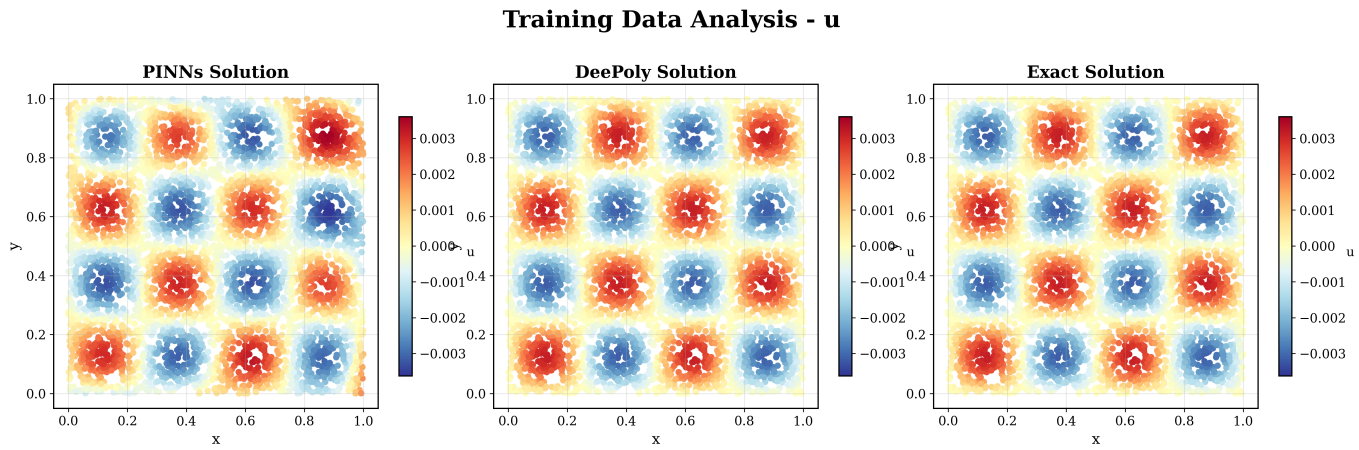
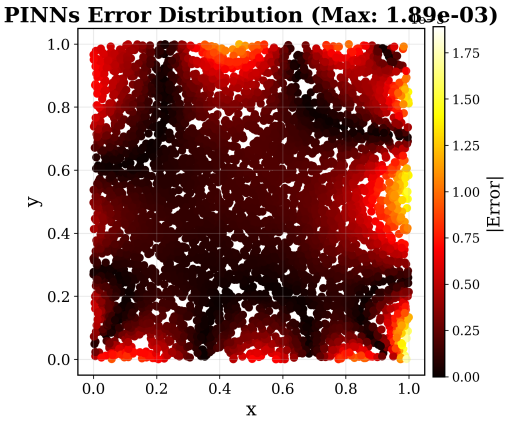
5. High-Frequency Poisson

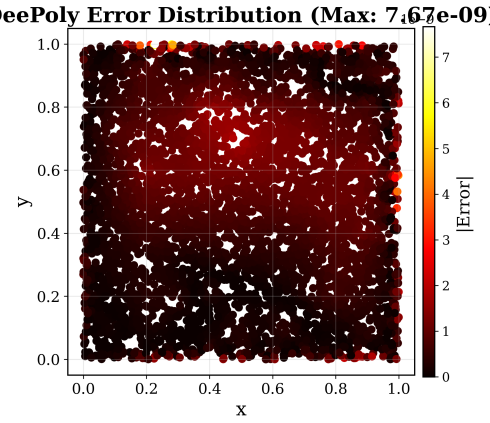
Case Directory: `linear_pde_cases/poisson_2d_sin4pixsin4piy`
PDE Equation: $\nabla^2 u = -32\pi^2 \sin(4\pi x)\sin(4\pi y)$ on $[0,1]^2$
Problem Type: High-frequency variant of Poisson equation

Results

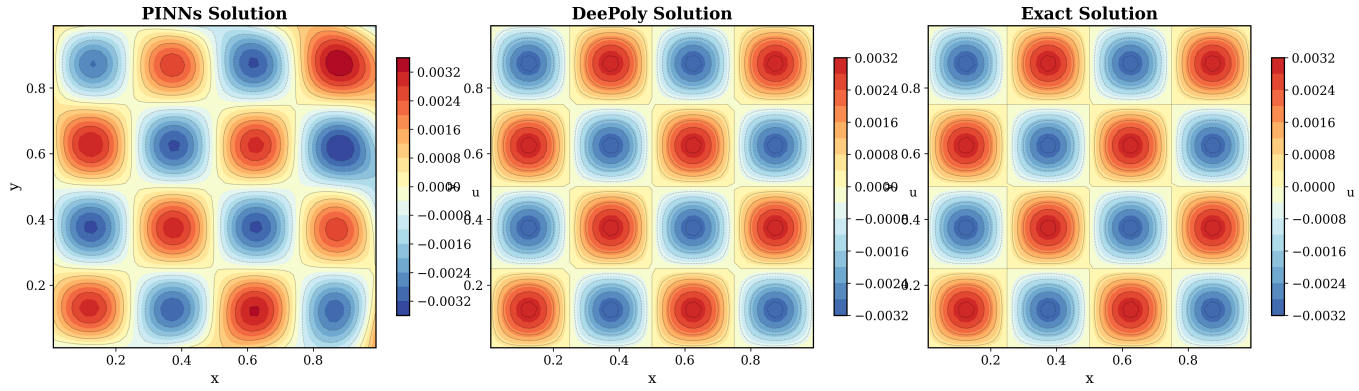
Training Data Analysis

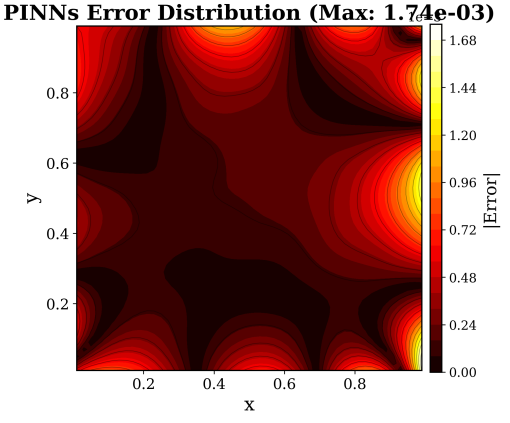


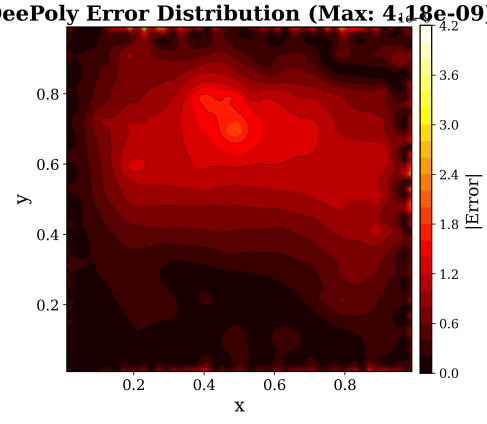




Test Data Analysis







Performance Metrics

Component	Time Cost
Scoper/PINNs	34.7317 seconds
Sniper	14.3702 seconds
Total	49.1018 seconds

Additional Information

- **Error Analysis**: Detailed metrics available in `cases/linear_pde_cases/poisson_2d_sin4pixsin4piy/results/error_analysis_report.txt`
- **Configuration**: See `cases/linear_pde_cases/poisson_2d_sin4pixsin4piy/config.json`

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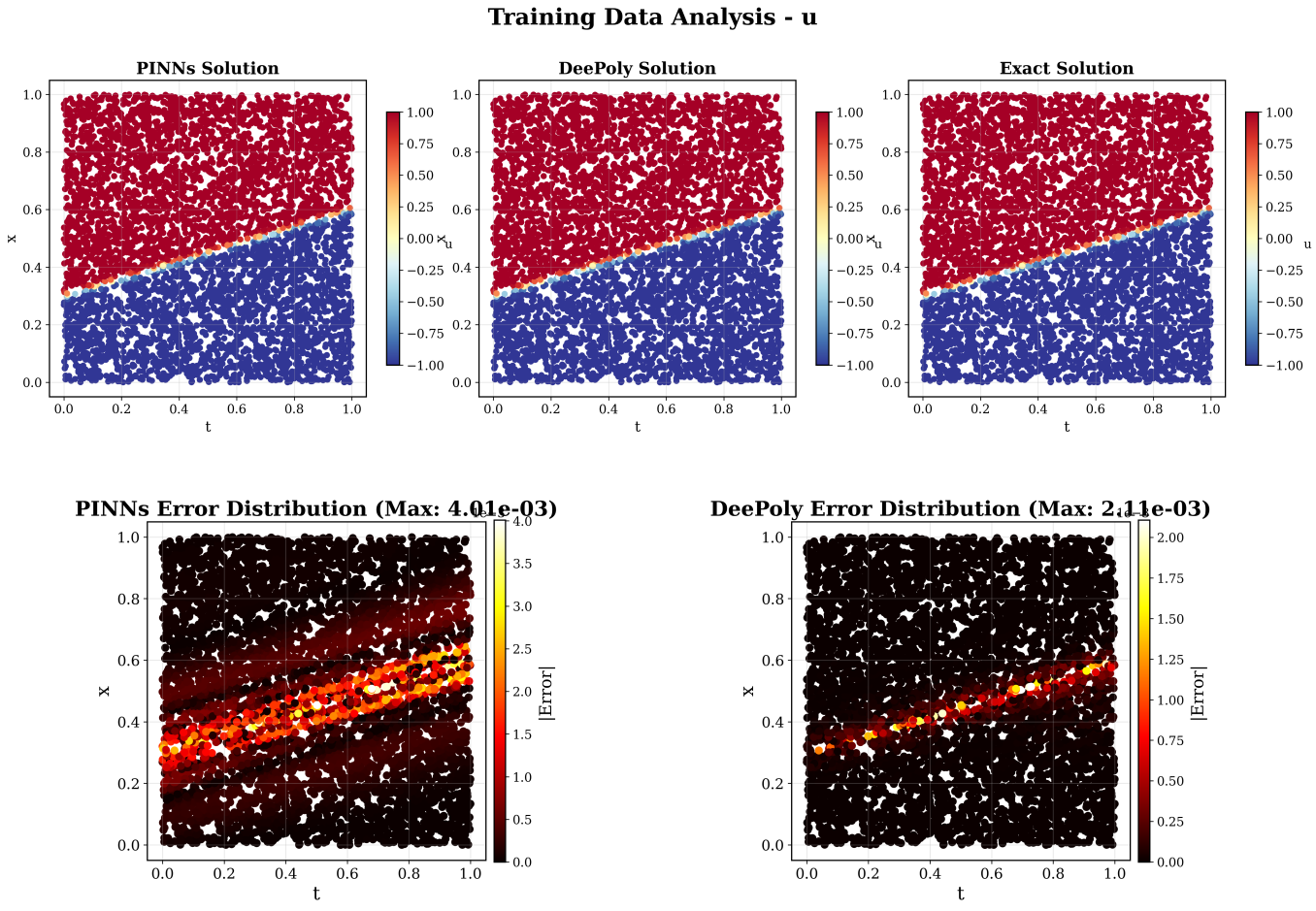
6. Linear Convection with Discontinuity

Case Directory: `linear_pde_cases/linear_convection_discontinuity`
PDE Equation: `du/dt + 0.3du/dx = 0` (time-dependent)
Problem Type: Hyperbolic PDE with discontinuous wave propagation

Results

Training Data Analysis



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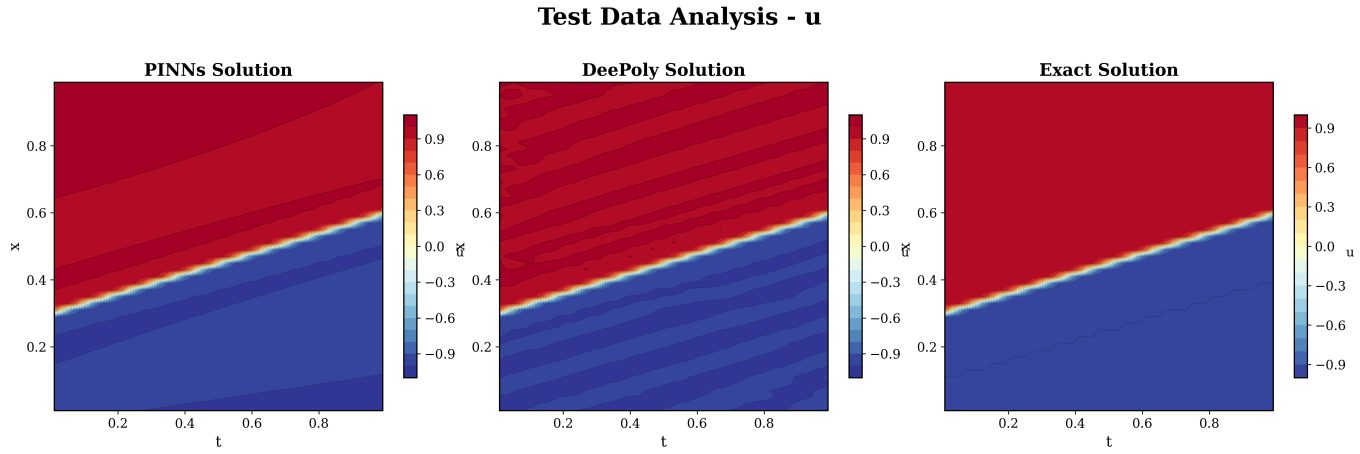

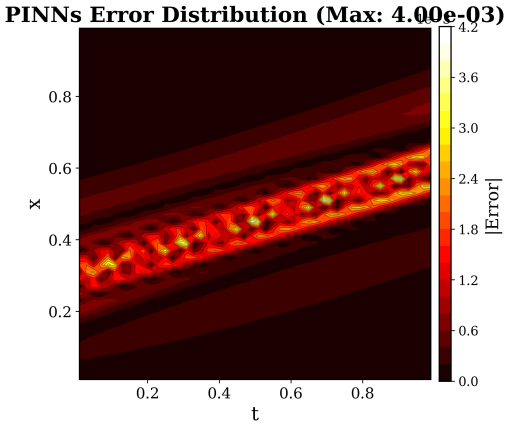

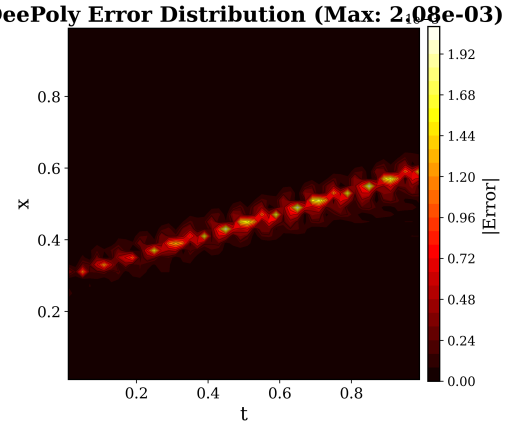

Performance Metrics

Component	Time Cost
Scoper/PINNs	16.8255 seconds
Sniper	0.6267 seconds
Total	17.4521 seconds

Additional Information

- **Error Analysis**: Detailed metrics available in `cases/linear_pde_cases/linear_convection_discontinuity/results/error_analysis_report.txt`
- **Configuration**: See `cases/linear_pde_cases/linear_convection_discontinuity/config.json`
