DeePoly - High-Order Accuracy Neural Network Framework for Function Approximation and PDE Solving

Introduction

DeePoly is a novel general-purpose platform for function approximation and equation solving algorithms. Core theorem and algorithm: DeePoly: A High-Order Accuracy and Efficiency Scientific Machine Learning Framework

Key Features

- **Universal PDE Solver**: Fit for all kinds of PDE.
- **Mesh-Free**: Sampling points can be randomly generated with no logical relationships, suitable for complex geometries.
- **High Accuracy**: Achieves high-order convergence.
- **Scheme-Free**: Handles derivative relationships using automatic differentiation.
- **Efficient**: Computational efficiency comparable to traditional finite difference methods.
- **GPU Accelerated**: Supports CPU parallelism and GPU acceleration.
- **Applicable to Complex and Discontinuous Problems**: Accurately approximates discontinuous and high-gradient functions.
- **Suitable for Inverse Problems**: Solves inverse problems with higher accuracy than PINNs.

Version Information

Previous version: v0.1 (Beta). The `cases` directory includes both function approximation and PDE solving test cases. The core algorithm in `src` contains all derivative computation code with comprehensive testing.

Current version: v0.2: - High-accuracy solving for arbitrary-dimensional linear PDEs. - Auto code of the PDE. - English-commented version. - Other corrections of output and visualize.

Usage

Develop your own problems in the `cases` directory, including data generation, output, and configuration files.

```
# Run function fitting example
python src/main_solver.py --case_path cases/func_fitting_cases/case_2d

# Run PDE solving example
python src/main_solver.py --case_path cases/linear_pde_cases/poisson_2d_sinpixsinpiy
# First time run, you need set auto-code=true, and then rerun it.
```

Installation Requirements

- Python
- CUDA
- torch

Test Cases & Results

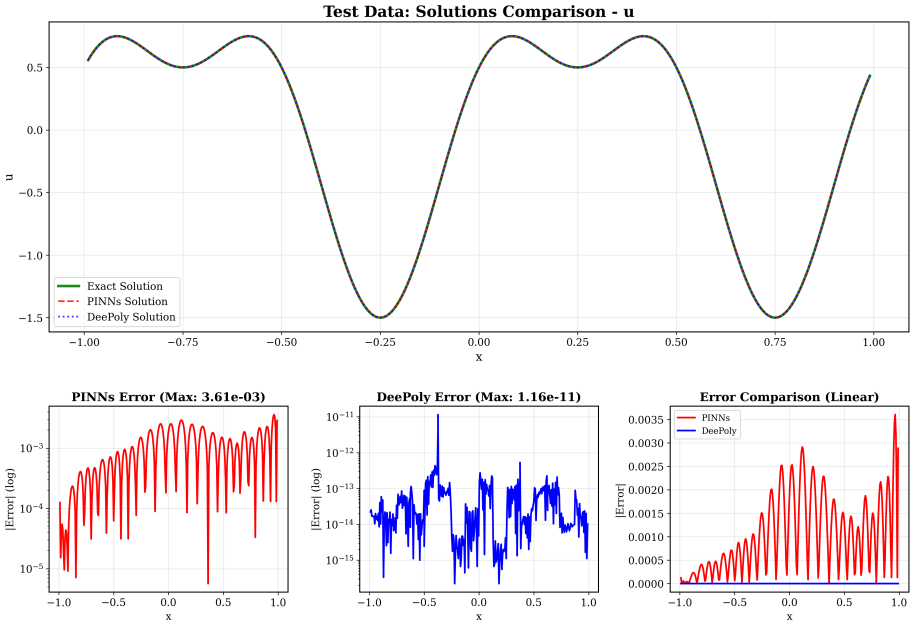
The following test cases demonstrate DeePoly's capabilities across function approximation and PDE solving:

Function Approximation Cases

1. 1D Sine Function Case Directory: `func_fitting_cases/test_sin`
Target Function: `f(x) = sin(2 x) + 0.5cos(4 x)`
Domain: [0, 1] (1D)
Problem Type: Baseline function approximation test case

Results Training Data Analysis
Training Results

Test Data Analysis



Performance Metrics

Component	Time Cost
Scoper/DNN	7.2049 seconds
Sniper	0.1630 seconds
Total	7.3679 seconds

Additional Information

- **Error Analysis**: Detailed metrics available in `cases/func_fitting_cases/test_sin/results/error_`
- **Configuration**: See `cases/func_fitting_cases/test_sin/config.json`
- **Data Generation**: Automatic sampling with configurable density

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2. Complex 2D Function (`func_fitting_cases/case_2d`) **Function**: Multi-Gaussian peaks + trigonometric + polynomial terms
Domain: $[-3,3]^2$ | **Results**: MSE 4.85e-17, Max Error 2.00e-07, Time 42.9s

2D Function Results

Figure 1: 2D Function Results

3. Discontinuous Function (`func_fitting_cases/discontinuous_case1`)
Function: Near-discontinuous with sharp transitions
Domain: Variable | **Results**: MSE 2.11e-05, Max Error 1.83e-02, Time 14.0s

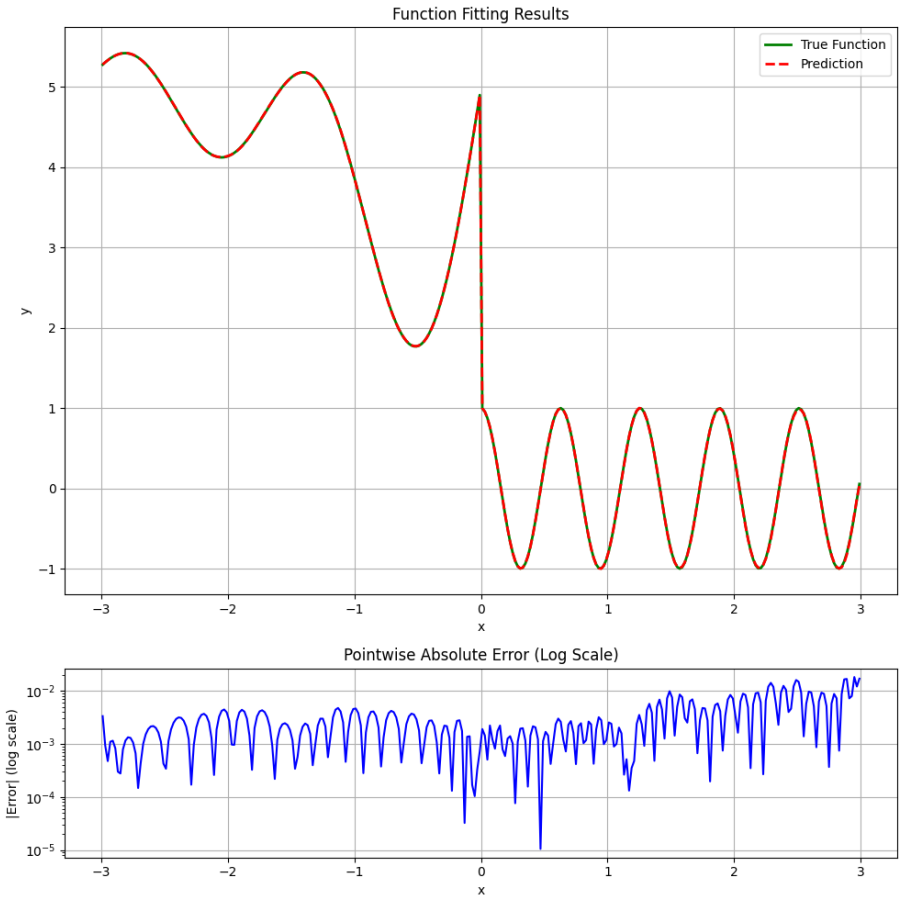
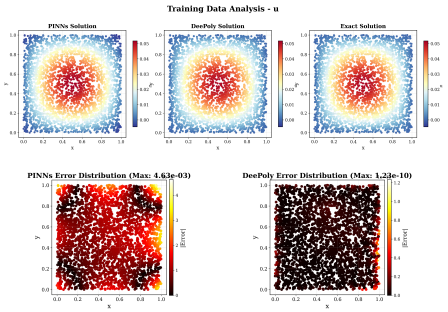
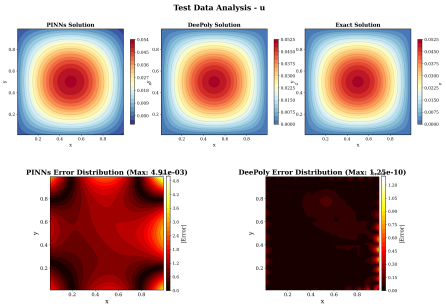


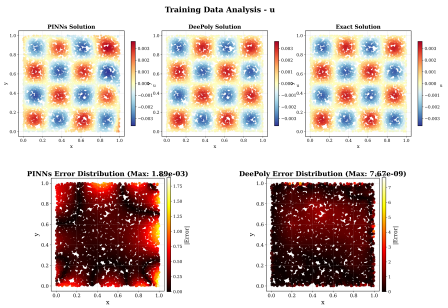
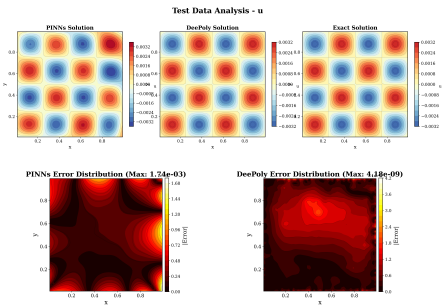
Figure 2: Discontinuous Results

PDE Solving Cases

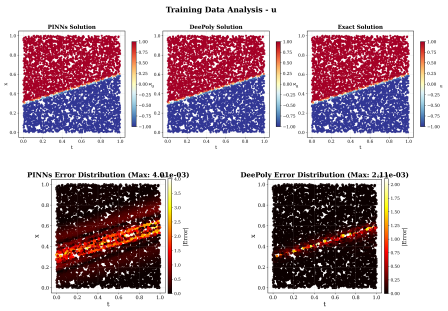
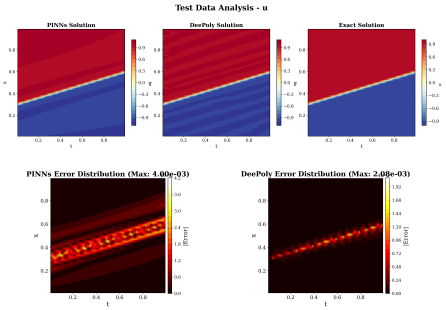
4. 2D Poisson Equation (`linear_pde_cases/poisson_2d_sinpixsinpiy`)
PDE: ^{2}`u = -sin(x)sin(y)` | **vs PINNs**: 10 × better accuracy (MSE 1.45e-14 vs 1.37e-06)

Training Results	Test Results
	

5. High-Frequency Poisson (`linear_pde_cases/poisson_2d_sin4pixsin4piy`)
PDE: `²u = -32 ²sin(4 x)sin(4 y)` | **Higher frequency variant**

Training Results	Test Results
	

6. Linear Convection with Discontinuity (`linear_pde_cases/linear_convection_discontinuity`)
PDE: `u/ t + 0.3 u/ x = 0` | **Discontinuous wave propagation**

Training Results	Test Results
	

Performance Summary

Case Type	Best MSE	Key Feature
Smooth Functions	4.85e-17	Multi-modal complexity
Discontinuous	2.11e-05	Sharp transitions
Linear PDEs	1.45e-14	vs PINNs: 10 $\times$ better
Time-dependent	-	Discontinuous propagation