samba 公式推导

运动方程：

$$\frac{d(r,z)}{dt} = (u,v) \tag{1}$$

质量守恒：

$$\frac{dm}{dt} = 0 \tag{2}$$

动量守恒方程：

$$\begin{aligned} \rho\frac{du}{dt} &= \frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial s_{rz}}{\partial z} \\ \rho\frac{dv}{dt} &= \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial s_{rz}}{\partial r} \end{aligned} \tag{3}$$

能量守恒方程：

$$\begin{aligned} &\rho\frac{de}{dt} = \sigma_{rr}\frac{\partial u}{\partial r} + \sigma_{zz}\frac{\partial v}{\partial z} + s_{rz}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right) \\ &= -p\frac{dV}{dt} + s_{rr}\frac{\partial u}{\partial r} + s_{zz}\frac{\partial v}{\partial z} + s_{rz}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right) \end{aligned} \tag{4}$$

本构方程：

$$\begin{aligned} \frac{ds_{rr}}{dt} &= 2\mu\left(\frac{\partial u}{\partial r} - \frac{1}{3}\Delta\cdot\vec{u}\right) + s_{rz}\left(\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}\right) \\ \frac{ds_{zz}}{dt} &= 2\mu\left(\frac{\partial v}{\partial z} - \frac{1}{3}\Delta\cdot\vec{u}\right) - s_{rz}\left(\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}\right) \\ \frac{ds_{rz}}{dt} &= \mu\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right) + \frac{s_{rr} - s_{zz}}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}\right) \end{aligned} \tag{5}$$