Acoustics

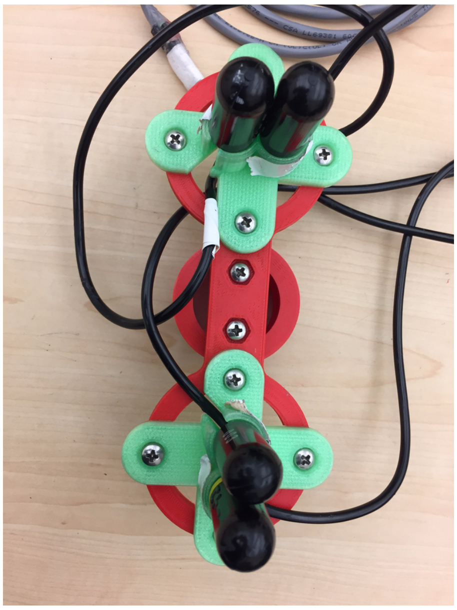
Section 1: System Overview

Introduction

The acoustics system is designed to locate a set of pingers in the water emitting ultrasonic frequencies and guide the robot towards them to complete various tasks. To do this, it uses an array of hydrophones (underwater microphones) and measures the time difference between each pair to determine the relative location of the pinger. Each hydrophone is connected to a channel on a USB oscilloscope, which captures the raw voltage data from the hydrophones and sends it to the onboard computer for processing.

Current Setup

Our acoustics setup currently uses an array of four HTI 96 Min hydrophones. Each hydrophone takes 9V power from the power distribution board and returns a voltage signal based on the intensity of the sound wave. These hydrophones are arranged as shown below



The top two hydrophones are placed at the front of the robot and measure the relative azimuth (yaw) to the pinger, and the bottom two measure the relative elevation (pitch). The signal and ground wires from each hydrophone are connected to a 4-channel Picotech 2600A series USB oscilloscope, which is set to trigger and capture on each ping. Once a signal is received, the main computer pulls the data from the Picotech via Python and C libraries. The computer will calculate the time difference of arrivals for each pair of hydrophones, and the azimuth and elevation from that data. It uses the azimuth calculation to determine the correct angle to turn towards the pinger, and the elevation calculation to determine when the robot is directly above the pinger.

Section 2: Theory of Operation

Hydrophone Geometry

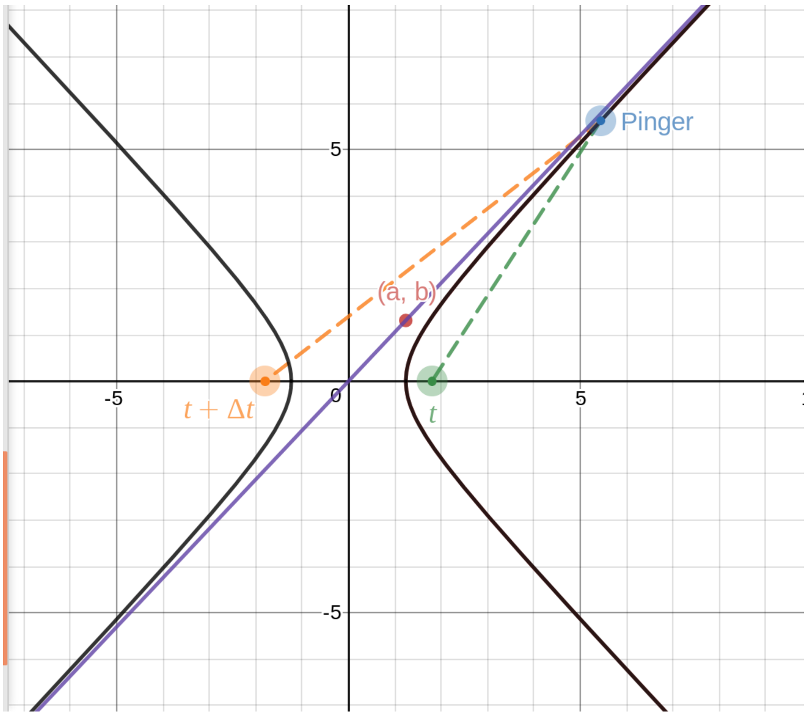
For simplicity's sake, we shall assume a two-dimensional system - that is, where the pinger and hydrophones are all on the same horizontal plane. Let the location of the pinger be (P_x, P_y) and the location of each of the hydrophones be (x_i, y_i). Take the first pair of hydrophones at (x_1, y_1) and (x_2, y_2). The theoretical time difference of arrival will be given by:

$$\Delta t = \frac{\sqrt{(P_x - x_1)^2 + (P_y - y_1)^2} - \sqrt{(P_x - x_2)^2 + (P_y - y_2)^2}}{v_{\text{sound}}}$$

Consequently, the difference in distance between the pinger and each hydrophone is $v_{sound}\Delta t$. There are an infinite number of positions in the xy plane that would give the measured Δt, and this relationship is given by finding all of those locations by the following formula, where (x, y) are the possible measured locations of the hydrophones.

$$v_{\text{sound}}\Delta t = \sqrt{(x - x_1)^2 + (y - y_1)^2} - \sqrt{(x - x_2)^2 + (y - y_2)^2}$$

From conic sections, a hyperbola can be defined as the set of points where the absolute difference between the distance from each point to both foci is the same. We don't know the exact time of arrival between when the ping starts and when we receive it in either hydrophone, but we do know the time difference between the hydrophone pair. Therefore, the set of points that satisfies the above equation is a hyperbola with foci at the hydrophones and a constant difference of $v_{\text{sound}}\Delta t$.



This equation can be converted into the standard form for a hyperbola centered at the origin $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. We do this because the standard form allows us to use linear estimations using the asymptote of the hyperbola. The standard form and relevant parameters are shown below, and are derived in the appendix.

$$\frac{4x^2}{(v_{\text{sound}}\Delta t)^2} - \frac{4y^2}{\Delta x^2 - (v_{\text{sound}}\Delta t)^2} = 1$$

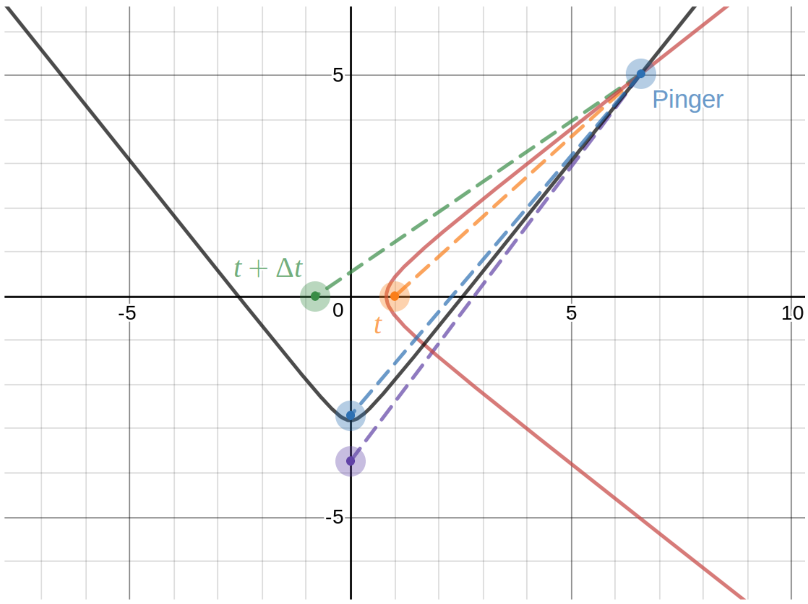
$$a = \frac{v_{sound}\Delta t}{2}$$

$$b = \frac{\sqrt{\Delta x^2 - a^2}}{2}$$

$$\theta_{\text{asymptote}} = \tan^{-1}\frac{a}{b}$$

Because the distance from the pinger to the robot is much greater than the hydrophone separation Δx , we calculate our azimuth and elevation just using the angle of the asymptote of the hyperbola, which is the line passing through the origin and (a, b).

A single time difference calculation with one pair of hydrophones gives an infinite number of possible locations for the pinger. To find the absolute location in 2-dimensional space, another time difference between another hydrophone pair is needed. This gives another hyperbola, which intersects with the first one at the pinger's location. Note that in our current system, we never calculate absolute position. We first calculate our relative yaw with the first pair of hydrophones, and navigate until we are directly above the pinger with the second pair of hydrophones.



You can play more with this acoustics simulator here.

To expand to 3-dimensional space, four hydrophones are needed for a total of three time-difference-of-arrival calculations. The math remains the same, except each hydrophone pair gives a hyperboloid, and an additional curve is needed to absolutely determine position.

Time Difference of Arrival Calculation

In order to determine the position of the pinger, we first have to look at the signals from the hydrophones and determine the time difference of arrival. There are two methods that we have explored for doing this: Fast-Fourier Transform phase difference calculation, and cross-correlation. There are more methods available, and this is an area we will explore further.

Phase Difference

$$y = A \sin 2\pi f(t - \Delta t)$$

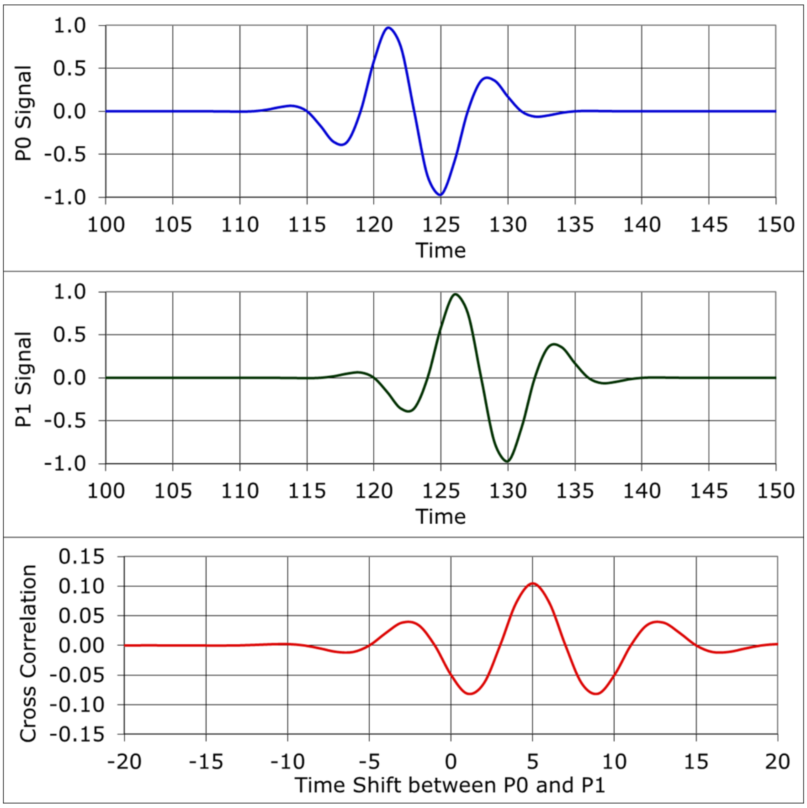
Remember that a sinusoid has a few different parameters:

- An amplitude(A) telling the power level
- A frequency(f) telling how often the wave repeats
- A shift (Δt) telling how far the signal is shifted in the time domain

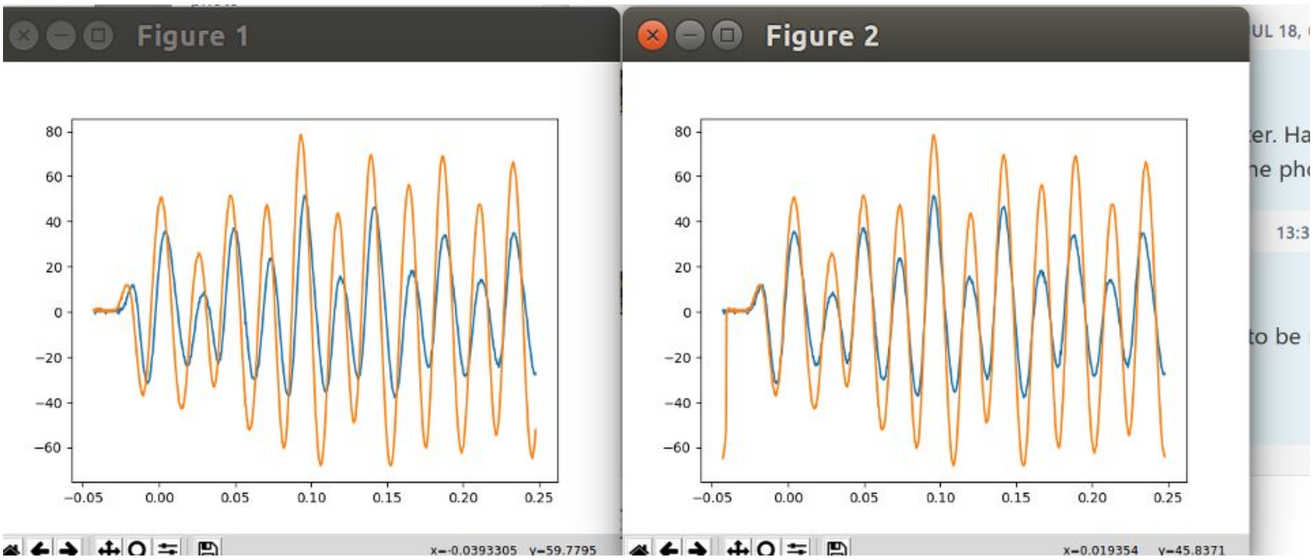
Out of these, the critical one is the phase shift. In theory, the frequency measured by the hydrophones will be equivalent and the amplitude will be nearly identical. We can measure the phase shift of each of our signals with a Fast Fourier Transform (FFT).

For a more complete introduction to the FFT read more here. For grasping the acoustics algorithm, all you need to know is that when you run a signal through an FFT, it tells you where there is the most power in a signal (so if you have a 22 KHz signal, there will be a peak in the FFT graph at 22 KHz), and it tells you the phase at each frequency. We calculate the phase difference between hydrophone pairs and use that to calculate the time difference by dividing by the frequency. This does limit the spacing of the hydrophones to half the wavelength, because of the periodic nature of the signal. For a pure sine wave, if the phase offset is 30°, any offset of $30^\circ \pm 360^\circ$ would align the signals. If the hydrophone spacing is half the wavelength or less, the time shift is unambiguous. One benefit of this method is that it is relatively resistant to signal noise. However, finding the right window of the signal to calculate the phase is difficult to do algorithmically.

Cross-Correlation



Cross-correlation is a method where you essentially slide one signal across the other to find out where they are the most similar. In the graphic above, there are two identical signals, but P1 is shifted by 5. The cross-correlation shows a peak at 5 in accordance with that. This method has shown to be more consistent in trial data and allows for hydrophones to be placed at larger distances from each other. The result of aligning the signals via cross-correlation on a test is shown below. The left is the raw time-domain data from a hydrophone pair, and the right is the data shifted by the calculated time difference.



Section 3: Path forward

While we have the building blocks of our acoustics system, it has not been tested in a mission and there are many areas for improvement. The main tasks we will be focusing on this year are:

- Verifying existing functionality
  - Experimentally calculating speed of sound to verify data integrity
  - More exhaustive testing of different pinger configurations, comparing actual data to expected, and resolving discrepancies
  - Test in a mission and navigate to pinger
- Improvement to algorithms
  - Find effective methods in finding time difference
    - Time and space effective
    - Tolerant to noise in the signal
  - Convert simulator to Python and make it more realistic
  - Implement multilateration, which will require change in hydrophone configuration
  - Algorithmically determine signal and data quality
- Physical and electrical setup
  - Simulate and test different configuration
    - See which ones are tolerant to noise and error
  - Improve signal integrity
    - Reduce noise in hydrophone signals from thrusters

- Amplify and filter signals to improve integrity
  - Find sponsors for new hydrophones and pingers

Section 4: Appendix

Hyperbola Derivation

For simplicity's sake, we assume that the hydrophones are placed with equal spacing at $(a, 0)$, and (a, -0). We also define $d = v_{sound}\Delta t$.

$$d = \sqrt{(x-a)^2 + y^2} - \sqrt{(x+a)^2 + y^2}$$
$$d + \sqrt{(x+a)^2 + y^2} = \sqrt{(x-a)^2 + y^2}$$
$$d^2 + x^2 + 2ax + a^2 + 2d\sqrt{(x+a)^2 + y^2} = x^2 - 2ax + a^2$$
$$d^2 + 4ax = -2d\sqrt{(x+a)^2 + y^2}$$
$$(d^2 + 4ax)^2 = 4d^2((x+a)^2 + y^2)$$

$$d^4 + 16a^2x^2 + 8d^2ax = 4d^2x^2 + 8d^2ax + 4d^2a^2 + 4d^2y^2$$
$$d^4 + 16a^2x^2 = 4d^2x^2 + 4d^2a^2 + 4d^2y^2$$
$$16a^2x^2 - 4d^2x^2 - 4d^2y^2 = 4d^2a^2 - d^4$$
$$\frac{4a^2x^2}{d^2} - x^2 - y^2 = a^2 - \frac{d^2}{4}$$
$$\frac{4a^2x^2 - d^2x^2}{d^2} - y^2 = a^2 - \frac{d^2}{4}$$
$$\frac{x^2(4a^2 - d^2)}{d^2} - y^2 = a^2 - \frac{d^2}{4}$$
$$\frac{4x^2(4a^2 - d^2)}{d^2} - 4y^2 = 4a^2 - d^2$$
$$\frac{4x^2}{d^2} - \frac{4y^2}{4a^2 - d^2} = 1$$

Here, if we take $2a = \Delta x$ (the hydrophone separation), we get

$$\frac{4x^2}{d^2} - \frac{4y^2}{\Delta x^2 - d^2} = 1$$

Therefore, from the standard form of a hyperbola centered at the origin $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we have

$$a = \frac{d}{2} = \frac{v\Delta t}{2}$$
$$b = \frac{\sqrt{\Delta x^2 - d^2}}{2}$$

The asymptotes of a hyperbola are the lines passing through the center of the hyperbola with a slope of $\frac{b}{a}$. Therefore, the angle to the pinger is given by

$$\theta = \tan^{-1} \frac{a}{b} = \tan^{-1} \frac{v\Delta t}{\sqrt{\Delta x^2 - d^2}}$$