

NOTES ON THE SOUND HORIZON

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1. DEFINITION OF R_S

We just want to see how far a sound wave travels from the beginning of time to the surface of last scattering,

which occurs at $t = t^*$:

$$\begin{aligned} r_s &= \int_0^{t^*} c_s dt \\ &= - \int_{z^*}^{\infty} c_s \frac{dt}{dz} dz \\ &= - \int_{z^*}^{\infty} \frac{c_s}{H(z)} \frac{da}{dz} dz \\ &= \int_{z^*}^{\infty} \frac{c_s}{H(z)(1+z)^2} dz \end{aligned}$$

2. CALCULATING THE SOUND SPEED

The sound speed of any fluid is the adiabatic derivative

$$c_s^2 = \frac{\partial p}{\partial \rho}$$

In the background, we have

$$\begin{aligned} \partial p &= \partial \left[\frac{\Omega_\gamma}{3} H(z)^2 M_p^2 \right] \\ \partial \rho &= \partial [(\Omega_\gamma + \Omega_b) H(z)^2 M_p^2] \end{aligned}$$

If the background is smooth, then the *Omegas*