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OSCILLATING AXIONS NOTES

Brent Follin

1. THE BACKGROUND

Our model is the ΛCDM cosmology with cold, pressureless dark matter, photons, 3 massless neutrino species, the standard baryon sector, and a cosmological constant, extended with a axionic field χ. The χ field evolves according to the standard equation for a scalar field in an expanding FRW cosmology,

$$\ddot{\chi} = -3H\dot{\chi} - V'(\chi), \tag{1}$$

with H the Hubble rate

$$H = \frac{\dot{a}}{a} = -\frac{\dot{z}}{1+z}, \tag{2}$$

and potential $V(\chi) = m_\chi^2 \chi^2$ The Hubble rate obeys the Friedmann equation:

$$H^2 = H_{0*}^2(\Omega_m(1+z)^3 + \Omega_{rad}(1+z)^4 + \Omega_\Lambda) + \frac{\rho_\chi(z)}{3M_p^2}, \tag{3}$$

with M_p the reduced planck mass $\sqrt{8\pi G}$, $\rho_\chi = V(\chi) + \frac{1}{2}\dot{\chi}^2$ the total energy density of the χ field at redshift z, Ω_m the energy density fraction of pressureless fluids, Ω_{rad} the energy density fraction of relativistic fluids, and Ω_Λ the energy density fraction of the cosmological constant. The parameter H_{0*}^2 is the 'reduced' Hubble rate today, given (as is implied in equation 3) by

$$H_{0*}^2 = H_0^2 - \frac{\rho_\chi(z=0)}{3M_p^2} \tag{4}$$

The Ω_i's obey the summation rule

$$\sum_i \Omega_i = 1. \tag{5}$$

1.1. *Solving the Background*

The background metric is given by

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2, \tag{6}$$

whose evolution is completely specified by the equation of motion for the scale factor $a(t)$:

$$\dot{a} = aH. \tag{7}$$

Since the χ field is neither directly observable nor does it interact with other specie except through gravity, the background effects of the χ field are entirely specified by the Hubble rate $H(z)$. We can formulate the Hamiltonian dynamics of background by introducing a new field $\phi \equiv \dot{\chi}$, after which we can write equation 1 and equation 2 as

$$\begin{pmatrix} \dot{(1+z)} \\ \dot{\chi} \\ \dot{\phi} \end{pmatrix} = \begin{bmatrix} -H & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -m_\chi^2 & -3H \end{bmatrix} \begin{pmatrix} 1+z \\ \chi \\ \phi \end{pmatrix} \tag{8}$$

with H given by equation 3. Defining

$$\vec{x} = \begin{pmatrix} (1+z) \\ \chi \\ \phi \end{pmatrix}$$

$$\hat{D} = \begin{bmatrix} -H & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -m_\chi^2 & -3H \end{bmatrix}$$

$$\vec{x}_0 = \begin{pmatrix} (1+z_0) = 20\,(m_\chi/H_{0*})^{2/3} \\ \chi_0 \\ \phi_0 = 0 \end{pmatrix}$$

equation 8 has the formal solution

$$\vec{x}(t) = e^{\hat{D}t}\vec{x}_0 \tag{9}$$

Analytically, this equation is solved through the implicit Adams method, implemented in Fortran, and sampled in time at timesteps given by

$$t_i = t_{i-1} + \frac{0.005}{max\left(H(t_{i-1}), \sqrt{2}M_\chi\right)} \tag{10}$$

to obtain histories $z(t_i)$ and $H(t_i)$. The function $H(z)$ is then estimated as the linear interpolation of the ordered pairs $(H(t_i), z(t_i))$, with $H(z > z_0)$ estimated by

$$H^2(z) = H_{0*}^2(\Omega_m(1+z)^3 + \Omega_{rad}(1+z)^4 + \Omega_\Lambda) \tag{11}$$

The background cosmology is totally specified by the parameters Ω_m, H_{0*}, χ_0, and m_χ.

2. PERTURBATIONS

The Jeans length (deBroglie wavelength) of the axion field is nearly the horizon, so we can't have observable perturbations. We have

$$\lambda_J \sim (m_\chi v)^{-1} \sim mH\lambda_J \tag{12}$$

or $\lambda_J \sim \frac{1}{\sqrt{m_\chi H}}$. Since $m_\chi \lesssim H$ always, $\lambda_J \lesssim 1/H$, which is the horizon size.