Homework 5

SABIC: Physics

Due April 14, 2016

Reading:

Review chapters 1 and 4.

Problem 1: practice with errors

(a) A race car travels around a circular track of radius 130 ± 3m. A radar gun measures its velocity at 98 ± 5m/s. (i) How long does it take for the car to travel around the track? (ii) What is the acceleration of the car?

(b) A group of 10 physics students climb to the top of a 20m tower and drop stones. Using a stopwatch, each student times how long it takes for their stones to fall. The results are shown in the table below:

Stone Drop	
Student	Measured time (s)
Ali	2.508
Ben	2.619
Carlina	2.590
Dan	2.425
Emily	2.426
Francis	2.367
Gia	2.463
Hassan	2.426
Injit	2.627
Julia	2.388

The average time $\bar{t}$is given by

$$\bar{t} = \frac{1}{N}\sum_i t_i = 2.48\text{s}. \tag{1}$$

The Standard deviation is given by

$$\sqrt{\frac{1}{N-1}\sum_i (t_i - \bar{t})^2} = 0.09. \tag{2}$$

The students thus report the time for the stone to fall as $t = (2.48 \pm 0.09)$s. The strength of gravity is given

$$g = \frac{2\delta x}{t^2}. \tag{3}$$

Everything but the time is known exactly. So we have an average constraint on g of

$$\bar{g} = \frac{2(20\text{m})}{(2.48\text{s})^2} = 6.48\text{m/s}^2. \tag{4}$$

Now we need to find the uncertainty. First find the total uncertainty in the numerator (0m) and denominator $\sqrt{2\Delta t^2} = 0.13$s. Now, using the rule for combining errors in a ratio:

$$\Delta g = \bar{g}\sqrt{\left(\frac{0}{40}\right)^2 + \left(\frac{0.13}{2.48^2}\right)^2} = 0.94\text{m/s}^2. \tag{5}$$

The students would report their measurment as $(6.48 \pm 0.94)\text{m/s}^2$. Since we know the acceleration due to gravity is really 9.8m/s^2, the students measured a systematically lower acceleration than expected, due to a systematic error. Sources of error could be measuring times that were too long (by not properly using the stopwatch) or incorrectly measuring the height fallen (with a bad ruler, or forgetting to include the height of the person dropping the stone).

Problem 2: Unit vectors

(a) As was done in class for the unit vector $\hat{r}$, show that for a particle moving counterclockwise about a point with position $r(t)$, the change $\delta\hat{\theta}$ over a short time δt is given by

$$\Delta\hat{\theta} = \Delta\theta\hat{r}.$$

Problem 3: Kepler's laws

(a) Haley's comet orbits the sun with a period of 75 years. (i) What is the semi-major axis of the orbit? (ii) The eccentricity $\epsilon = f/a$ of Haley's orbit is 0.967, where $f = \sqrt{a^2 - b^2}$ is the distance from the center of the orbit to the focus. What is the average speed of the orbit? (iii) At what distance is the comet from the sun when its instantaneous speed matches the average speed? (iv) The comet moves fastest when it's closest to the sun. Using Kepler's second law

$$r^2\frac{d\theta}{dt} = rv_\perp = \text{ constant}, \tag{6}$$

what speed is the comet moving at that time?