Homework 2

SABIC: Physics

Due January 28, 2016

Reading (Due April 1, 2015):

Read Chapter 2.

Problem 1: practice with estimation

(a) **How many times the acceleration due to gravity does a drag racer experience when accelerating?** A drag racer goes 1/4 of a mile, which is about 1km. To do that in the typical 5second times, you have to travel at an average velocity of 1km/5seconds, or 200m/s. Assuming constant acceleration, that means the final velocity is around $v_f = 400\text{m/s}$, and that gives a typical acceleration of $a = v_f/t = 100\text{m/s}^2 = 10\text{g}$.

Problem 2: conceptual

(a) Can an object with constant acceleration change tis direction of motion? Is there a maximum number of times this can happen? Yes, for example a cart initially going uphill will change direction and go downhill under the influence of gravity, a source of constant acceleration. For it to change direction again, though, the acceleration would have to change direction, and therefore would not be constant.

(b) Can you have a zero displacement and a nonzero average velocity? A nonzero velocity? Illustrate your answers on an xt graph. No, you can't, and yes, you can, respectively. The average velocity is zero, because $\Delta x = 0$. But the same is not true for the instantaneous velocities. For instance, if you throw a ball directly up in the air, the instantaneous velocity is only zero at the highest point of the trajectory, but the ball comes back exactly where it started.

(c) **Argue for the following statements: (i) Neglecting air resistance, anything thrown vertically upward with some speed v will return to the point at which it is thrown with that same speed; and (ii) again neglecting air resistance, the amount of time it takes to return will be twice the time it takes to get to its highest point.**

(d) **An object is thrown straight up into the air and feels no air resistance. Give the acceleration and velocity at its highest point.** $a = 9.8\text{m/s}^2$, $v = 0$.

(e) **Dropping a ball from some height d without air resistance causes it to hit the ground in time T. How long does it take (in terms of T) for an object to fall that's dropped at a height $3d$?** x goes as t^2, so $3x$ goes as $3t^2 = (\sqrt{3}t)^2$. So the answer is $\sqrt{3}T$.

Problem 3: Velocity

(a) **A car is stopped at a traffic light. IT then travels along a straight road so that its distance from the light is given by $x(t) = bt^2 - ct^3$, where $b = 2.40\text{m/s}^2$ and $c = 1.20\text{m/s}^3$. (i) Calculate the average velocity of the car between $t = 0$s and $t = 10.0$s.** At $t = 0$, $x(0) = 0$. At $t = 10$s, $x(10) = 240 - 1200 = -960$m. That means the average velocity is $v_{\text{ave}} = -960/10 = -96$m/s (That doesn't make sense, because there was a typo, whoops.) **(ii) Calculate the instantaneous velocity of the care at $t = 0$s, $t = 5.0$s, and $t = 10.0$s.** The instantaneous velocity is the derivative, which is $v(t) = 2bt - 3ct^2$. Plugging in, we get $v(0) = 0$, $v(5) = -66$m/s, and $v(10) = -312$m/s. **(iii) How long does it take for the car to return to being at rest?** We want the time where $v(t) = 0$, which happens at $t = 0$, but also at $t = \frac{2b}{3c} = 4/3$s

(b) **A lunar lander is descending toward the moon's surface. Until the lander reaches the surface, its height above the surface of the moon is given by $y(t) = b - ct + dt^2$, with $b = 800m$ the initial height of the lander, $c = 60.0$m/s, and $d = 1.05\text{ms}^2$. (i) What is the initial velocity of the lander?** The initial velocity is $c = 60$m/s downward. **What is the velocity of the lander just before it reaches the lunar surface?** We need to find when $y(t) = 0$. That occurs at $t = \frac{c \pm \sqrt{c^2 - 4bd}}{2d}$, or $t = 21.2$s and $t = 35.9$s. By the latter time, the lander has already crashed, so we throw that out. The velocity is then $v(21.2s) = -c + 2dt \simeq 15.5$m/s downwards (towards the surface). Ouch.

Problem 4: Acceleration

(a) **A world-class sprinter accelerates to his maximum speed in 4.0s. He then maintains this speed for the remainder of a 100m race, finishing with a total time of 9.1s. (i) What is the runner's average acceleration during hte first 4.0s?** We know for the last 5.1s of the race, the runner travels a distance $x = 5.1v_f$, where v_f is the final velocity of the runner after 4.0s of acceleration. The average acceleration is then $a_{\text{ave}} = \frac{v_f - v_i}{4.0s}$, which gives us $a_{\text{ave}} = a = \frac{v_f}{4.0s}$. The distance traveled in the first 4s is then $x = \frac{1}{2}at^2 = 2v_f$. Adding up the total distance, we have $3.1v_f + 2v_f = 10$m, which gives $v_f \simeq 14$m/s, and $a_{ave} = 3.5\text{m/s}^2$ **(ii) What is his average acceleration during hte last 5.1s?** Zero. **(iii) What is his average acceleration for the entire race?** $a_{ave} = \frac{v_f}{9.1} = 1.5\text{m}s^2$. **(iv) Explain why the answer to part (iii) isn't the average of parts (i) and (ii).** It's because the runner spends more time not accelerating than accelerating, so the not accelerating bit gets weighed more.

(b) **A 7500kg rocket blasts off vertically from the launch pad with a constant upward acceleration of $2.25\text{m}/s^2$ and feels no air resistance. When it has reached a height of 525m, its engines suddenly fail so that the only force acting on it is now gravity. (i) What is the maximum height this rocket will reach above the launch pad?** First, find the velocity when the engine cuts. We can use $v^2 = v_0^2 + 2as$, since $s = 525$m and acceleration is given (and $v_0 = 0$). We get that $v = 48.6$m/s. the additional height reached is again given by $v^2 = v_0^2 + 2as$, where now s is unknown, $a = 10\text{m/s}^2$ due to gravity, $v_0 = 48.6$m/s, and $v_f = 0$. We get a distance of $s = 121$m, which gives a total height of 646m **(ii) How much time after engine failure will elapse before the rocket crashlands on the launch pad, and how fast will it be moving just before it crashes?** We know that it will pass 525m on the way back down with a velocity of $v_0 = 58.6$m/s, by one of our answers above. Again using the kinematic equation $v^2 = v_0^2 + 2as$, we have $v_f = 112$m/s down into the ground. Ouch. The time is given by $v_f = 48.6 + at$, with $a = -10\text{m/s}^2$, which means that $t = 16.4$s. Eject!

Problem 5: Motion under constant acceleration

(a) You throw a glob of putty straight up toward the ceiling, which is 3.60m above the point where the putty leaves your hand. The initial speed of the putty is 9.50m/s. (i) What is the speed of the putty just before it strikes the ceiling? Again use $v^2 = v_0^2 + 2as$. Solving for v gives 4.44m/s. **(ii) How much time when it leaves your hand does it take the putty to reach the ceiling?** As the last problem, using $v = v_0 + at$ gives $t = 0.52$s.

(b) A jet fighter wishes to accelerate at $5g$ (g=9.8m/s^2) to escape a dogfight as quickly as possible. Experimental evidence shows that this acceleration will black out the pilot if it lasts for longer than 5.0s. (i) What is the greatest speed the pilot can reach before blacking out? $v = 50 * 5 = 250$m/s. **(ii) How far will the pilot travel?** $x = 1/2at^2 = 5^4 = 625$m.

(c) A basketball player jumping towards the basket seems to 'hang' in the air. Even the best athletes spend at most 1.00s in the air. Let $y_{\max}$ be the maximum height of the athlete off the ground. To see why they appear to hang in the air, calculate the ratio of the time he is above $y_{\max}/2$ to the total time the athlete is off the ground. Ignore air resistance. Explain your answer. If the total time in the air is 1s, then the player reaches his maximum height in 0.5s, and falls back down the next 0.5s. Consider the way down, when the initial velocity is zero. Since he's under constant acceleration, this means he travels a distance of $h = 5(.5)^2 = 1.25$m. We'll look for the time where that height is above $1.25/2 = 0.625$m. That height occurs at $0.625 = 5t^2$, or $t = .35$s. That means it take only .15s to travel the rest of the way down, so the player spends .70s total at heights above half his maximum height, and only .3s total at heights below this height.