# mcDSAM

## A monte carlo implementation for determining the lifetime from a doppler shifted gamma ray (DSAM experiment)

The idea of this code is to take a TRIM input of tracking data for nuclei in a backing material to determine the specific angle/motion of an ion before decay. At this point, a lifetime is selected via a Monte Carlo random sampling. At this given decay time, the position and velocity vector the decaying nucleus is calculated. Then, the gamma is generated and the shifted energies are determined/plotted for given angles. Finally, the shifts vs cos(angle) are plotted to determine the attenuation factor, F(tau). A special thanks to Jerry Hinnefeld, who originally came up with this idea and who's help was integral to this work and getting this working.

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In [1]:

```
import math
import random
import numpy as np
%matplotlib inline
import matplotlib.pyplot as plt
plt.style.use('seaborn-white')
import scipy.optimize as sopt
from scipy.interpolate import interp1d
from scipy.interpolate import UnivariateSpline
import codecs
import matplotlib.gridspec as gridspec
```

## 1. Open the TRIM output file and extract the collision/trajectory information

In [2]:

```
trajectoryData = np.zeros(((10000, 200, 16)))  # space for 10000 ion trajectories, 200 path segments (col
lisions) per ion, and 16 data items per segment
nSegments = np.zeros(10000)                    # number of segments for each ion track (for diagnostic pur
poses), segments are between collisions
currentIon = 0                                 # for tracking which ion is used in calculation and data im
port
massAMU = 15.0030656                           # TRIM uses the atomic weight, but we need the mass of the
 isotope in amu

# TRIM collision data output
f = codecs.open('./collision/COLLISON_O_PureTa_10000.txt', 'r', encoding='iso-8859-1')

#iSeg = 1
lineCounter = 0
ion = 1
for line in f:
    #line.decode("iso-8859-1").encode("utf-8")
    lineCounter += 1

    # Pull the ion energy from header lines, and calculate initial speed of the recoil
    if line[6:16] == 'Ion Energy':
        energy = float(line[21:29])
        print('\nIon energy = ', energy, 'keV')
        v0 = ((2 * energy / massAMU / 931500)**(0.5)) * 3000   # speed in A/fs, the 3000 is for convertin
g c
        print('Ion velocity = ', v0, 'A/fs')


    # Collect useful data elements from TRIM output for each ion segment
    if (line[0:1] == '³') and (line[1:2] != '='):
        ion = int(line[1:6])               # this pulls the ion number, which starts from 1
        if ion == currentIon:              # another segment for the current ion trajectory
            iSeg = iSeg + 1                # increment the segment counter
        else:
            if ion != 1:                   # if this is ion 0001, it's not the end of a prior ion traject
ory
                nSegments[ion-2] = iSeg    # nSegments[] index starts at 0, so ion-2 at this point to put
the value in the right elements of nSegments[]
            currentIon = ion               # reset this variable
            iSeg = 1                       # start with iSeg=1; later we'll add an iSeg=0 for the piece f
rom t=0 to the first collision, which is why we subtract 2 earlier

        trajectoryData[ion-1,iSeg,0] = float(line[7:16])   # E1 - energy at beginning of segment
        trajectoryData[ion-1,iSeg,1] = float(line[17:27])  # x1 - depth at beginning of segment
        trajectoryData[ion-1,iSeg,2] = float(line[28:38])  # y1 - horizontal lateral position at beginnin
g of segment
        trajectoryData[ion-1,iSeg,3] = float(line[39:49])  # z1 - vertical lateral position at beginning
 of segment
        trajectoryData[ion-1,iSeg,4] = float(line[50:57])  # Se1 - electronic stopping power, ev/A

# end of file behavior
nSegments[ion-1] = iSeg   # at end-of-file, save the no. of segments for the last ion

lastIon = ion
# print information for diagnostic purposes
print('\n')
print('Last ion = ',lastIon)
print('Minimum Segments = ',min(nSegments[0:]))
print('Maximum Segments = ',max(nSegments[0:]))
print('\n')

# close the TRIM file
f.close()
```

```
Ion energy =  106.2 keV
Ion velocity =  11.695453786604201 A/fs


Last ion =  10000
Minimum Segments =  1.0
Maximum Segments =  114.0
```

## 2. Calculate and store other data for each segment of the ion tracks

This only needs to be calculated once per different target input.

In [3]:

```
# Calculating things like distance travelled per segment, directional cosines, energy, velocity, electron
ic stopping powers, and time at the segments

# ion counter
i = 0

while i < lastIon:

    # Calculate initial segment 0
    trajectoryData[i,0,0] = energy        # specified initial energy
    trajectoryData[i,0,4] = trajectoryData[i,1,4]  # set Se equal to value at beginning of second segment

    # Distance travelled
    # Starts at (0,0,0)
    dist = (trajectoryData[i,1,1]**2 + trajectoryData[i,1,2]**2 + trajectoryData[i,1,3]**2)**(0.5)

    # directional cosines
    # initial value is 0 in all three directions
    xcos = trajectoryData[i,1,1] / dist
    ycos = trajectoryData[i,1,2] / dist
    zcos = trajectoryData[i,1,3] / dist

    E2 = energy - trajectoryData[i,1,4] * (dist/1000.0)  # Energy after loss from stopping (E2 = E - Se*d
ist) (divide by 1000 to convert from ev/A to keV/A)
    v1 = v0
    v2 = (2*E2/massAMU/931500)**(0.5)*3000    # calculated from new energy (in A/fs)
    timeSegment = dist / ((v1+v2)/2)    # in fs -- (v1+v2)/2 is avg v for this segment

    trajectoryData[i,0,5] = E2    # store these in the tdat (trajectory data) array
    trajectoryData[i,0,6] = dist  # ...for ion 0001 (index 0) and segment 0
    trajectoryData[i,0,7] = xcos
    trajectoryData[i,0,8] = ycos
    trajectoryData[i,0,9] = zcos
    trajectoryData[i,0,10] = v1
    trajectoryData[i,0,11] = v2
    trajectoryData[i,0,12] = timeSegment

    # Now calculate and store the same data items for other segments beginning with each atomic collision
    iSeg = 1
    while iSeg < nSegments[i]:

        # distance traveled
        # sqrt((x2-x1)^2 + (y2-y1)^2 + (z2-z1)^2)
        dist = ((trajectoryData[i,iSeg+1,1] - trajectoryData[i,iSeg,1])**2 + (trajectoryData[i,iSeg+1,2]
- trajectoryData[i,iSeg,2])**2 + (trajectoryData[i,iSeg+1,3] - trajectoryData[i,iSeg,3])**2)**(0.5)

        # directional cosines
        xcos = trajectoryData[i,iSeg,1] / dist
        ycos = trajectoryData[i,iSeg,2] / dist
        zcos = trajectoryData[i,iSeg,3] / dist

        # energy and velocity at segment start
        E2 = trajectoryData[i,iSeg,0] - (trajectoryData[i,iSeg,4] * (dist/1000.0)) # E = E0 - Se*dist (in
keV/A)
        v1 = (2*trajectoryData[i,iSeg,0]/massAMU/931500)**(0.5)*3000    # in A/fs
        v2 = (2*E2/massAMU/931500)**(0.5)*3000    # in A/fs
        timeSegment = dist / ((v1+v2)/2)    # in fs

        trajectoryData[i,iSeg,5] = E2
        trajectoryData[i,iSeg,6] = dist
        trajectoryData[i,iSeg,7] = xcos
        trajectoryData[i,iSeg,8] = ycos
        trajectoryData[i,iSeg,9] = zcos
        trajectoryData[i,iSeg,10] = v1
        trajectoryData[i,iSeg,11] = v2
        trajectoryData[i,iSeg,12] = timeSegment + trajectoryData[i,iSeg-1,12]   # cumulative time at end
 of this segment

        iSeg += 1
```

```
    i += 1


#DEBUG
# print out some random ion's values to see if they're reasonable...
#ion = random.randint(1, lastIon)
#segment = 0
#while segment < nSegments[ion]:
#    print('Ion ', ion+1, ', Segment ', segment, ', time at end = ', trajectoryData[ion,segment,12])
#    segment += 1
```

### 3. Generate Doppler Shifted Gamma rays

Change this block for adjusting the input lifetime or gamma energy.

The python module `random` contains several random number generators in addition to `random.random()`. In particular, the routine `random.expovariate()` generates random numbers distributed according to a negative exponential function, rather than uniformly between 0 and 1. This is a quicker and easier way to generate our desired sampling distribution instead of defining the normalized integral.

In [4]:

```
nGammas = 50000                       # number of gammas to generate for each angle
eGamma0 = 6793.1                      # unshifted energy of transition gamma ray (in keV)
eGamma = np.zeros((7, nGammas))       # array to store Doppler-shifted gamma energies for each angle
decaySegment = np.zeros((7, nGammas)) # recording the segment in which the nucleus decays
genGammas = np.zeros(7)               # gammas generated for each angle

tau = 3.0                               # average lifetime of state, in fs
detectorResolution = 2.0                # 1-sigma detector resolution, in keV

expAngles = [0, 45, 60, 75, 90, 111, 135]    # angles used in measurement


# Loop over angles
for angle in range(len(expAngles)):
    detectorAngle = expAngles[angle]*(math.pi/180.0)      # detector angle, converted to radians

    # Gamma counters
    nGenerated = 0      # counter for number of gammas successfully generated

    # Generate gammas
    while nGenerated < nGammas:

        time = random.expovariate(1/tau)    # Monte Carlo generator for decreasing exponential
        ion = random.randint(0,9999)         # randomly select one of the 100 ion trajectories; CHANGE TH
IS IF THE TRIM FILE GENERATES A DIFFERENT NUMBER OF IONS
        #print('DEBUG: time = ',time)       # for debugging - time of decay
        #print('DEBUG: ion = ',ion)         # for debugging - ion

        iSeg = 0
        while iSeg < nSegments[ion]:

            # find the segment where the decay occurs
            if time < trajectoryData[ion,iSeg,12]:

                # Interpolate the ion speed between the values at the beginning and end of this segment
                # Assuming a constant slow-down
                decayVelocity = trajectoryData[ion,iSeg,10] + (time - trajectoryData[ion,iSeg-1,12]) * (t
rajectoryData[ion,iSeg,11] - trajectoryData[ion,iSeg,10])
                beta = decayVelocity/3000.0     # c = 3000 A/fs

                # Find the angle between ion direction and detector direction
                cosDetector = trajectoryData[ion,iSeg,7]*math.cos(detectorAngle) + trajectoryData[ion,iSe
g,8]*math.sin(detectorAngle)

                # Determines shifted gamma energy
                # Doppler formula +/- detector resolution
                eGamma[angle][nGenerated] = eGamma0*(1 + beta*cosDetector) + random.gauss(0,detectorResol
ution)

                # record in which segment the decay occurs
                decaySegment[angle][nGenerated] = iSeg

                # counters
                nGenerated += 1
                iSeg = nSegments[ion]

            else:

                #next segment
                iSeg += 1

                # If decay takes longer than all segments
                # generate another time and try again
                #if iSeg == nSegments[ion]:
                    #print('Time ',time,' fs not found in segments 1 - ', nSegments[ion],' for ion ', ion
+1,', i = ', i)
                    #print('Trying this one again.')

    genGammas[angle] = nGenerated
```

```
print('\n')
print('Gammas generated for each angle = ', genGammas)   # should be equal to nGammas
print('\n')
print('Angle\tMean Energy\t\tMedian Energy\t\tEnergy Range')
for angle in range(len(expAngles)):
    eGammaMean = np.mean(eGamma[angle][:nGenerated-1])
    eGammaMedian = np.median(eGamma[angle][:nGenerated-1])
    eGammaMin = np.min(eGamma[angle][:nGenerated-1])
    eGammaMax = np.max(eGamma[angle][:nGenerated-1])
    print(expAngles[angle], '\t', round(eGammaMean,3), '\t\t', round(eGammaMedian,3), '\t\t(', round(eGam
maMin,3), ',', round(eGammaMax,3), ')')
print('\n')

print('Median Energy:')
for angle in range(len(expAngles)):
    eGammaMedian = np.median(eGamma[angle][:nGenerated-1])
    print(round(eGammaMedian,3))
```

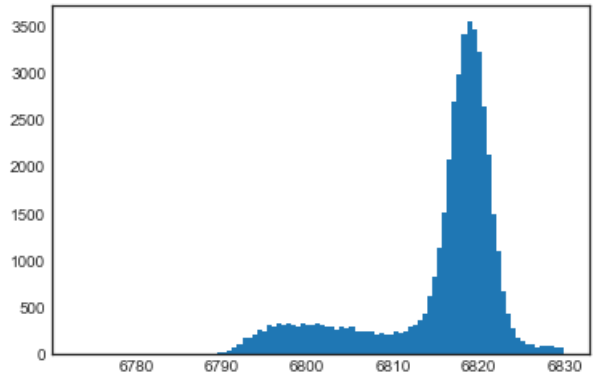
```
Gammas generated for each angle =  [50000. 50000. 50000. 50000. 50000. 50000. 50000.]


Angle    Mean Energy             Median Energy           Energy Range
0         6818.828                6818.421               ( 6786.693 , 7115.734 )
45        6811.302                6810.924               ( 6764.997 , 7016.225 )
60        6805.877                6805.558               ( 6713.684 , 6980.874 )
75        6799.691                6799.363               ( 6713.038 , 6918.852 )
90        6793.085                6793.087               ( 6697.605 , 6905.8 )
111       6783.934                6784.274               ( 6654.808 , 6862.785 )
135       6774.915                6775.309               ( 6578.624 , 6817.738 )


Median Energy:
6818.421
6810.924
6805.558
6799.363
6793.087
6784.274
6775.309
```

## 4. Plot the histograms

Here I plot various histograms for the calculation. First is a specific angle to check the validity of the method. Second is a plot of the segment in which the gamma decay occurs. All plots after that are to show all of the spectra for the given angle.

In [5]:

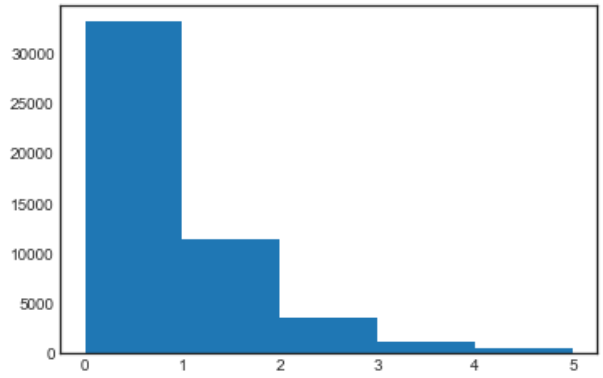
```
plt.hist(eGamma[0][:nGenerated],bins=100,range=(6773,6830))
plt.show()
```



In these spectra, there is a clear tail and bump around 6794 keV. This is due to gammas that are completely stopped before decaying.

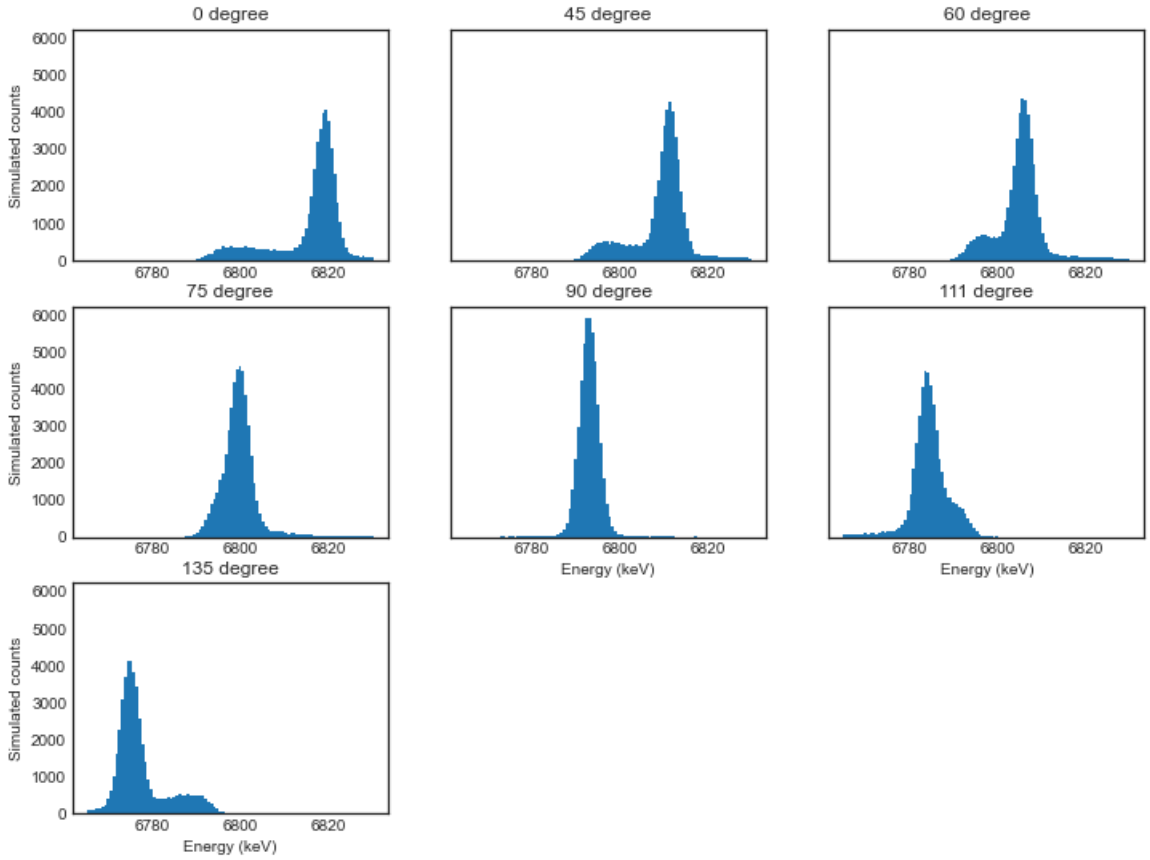
In [6]:

```
plt.hist(decaySegment[0][:nGenerated],bins=5,range=(0,5))
plt.show()
```



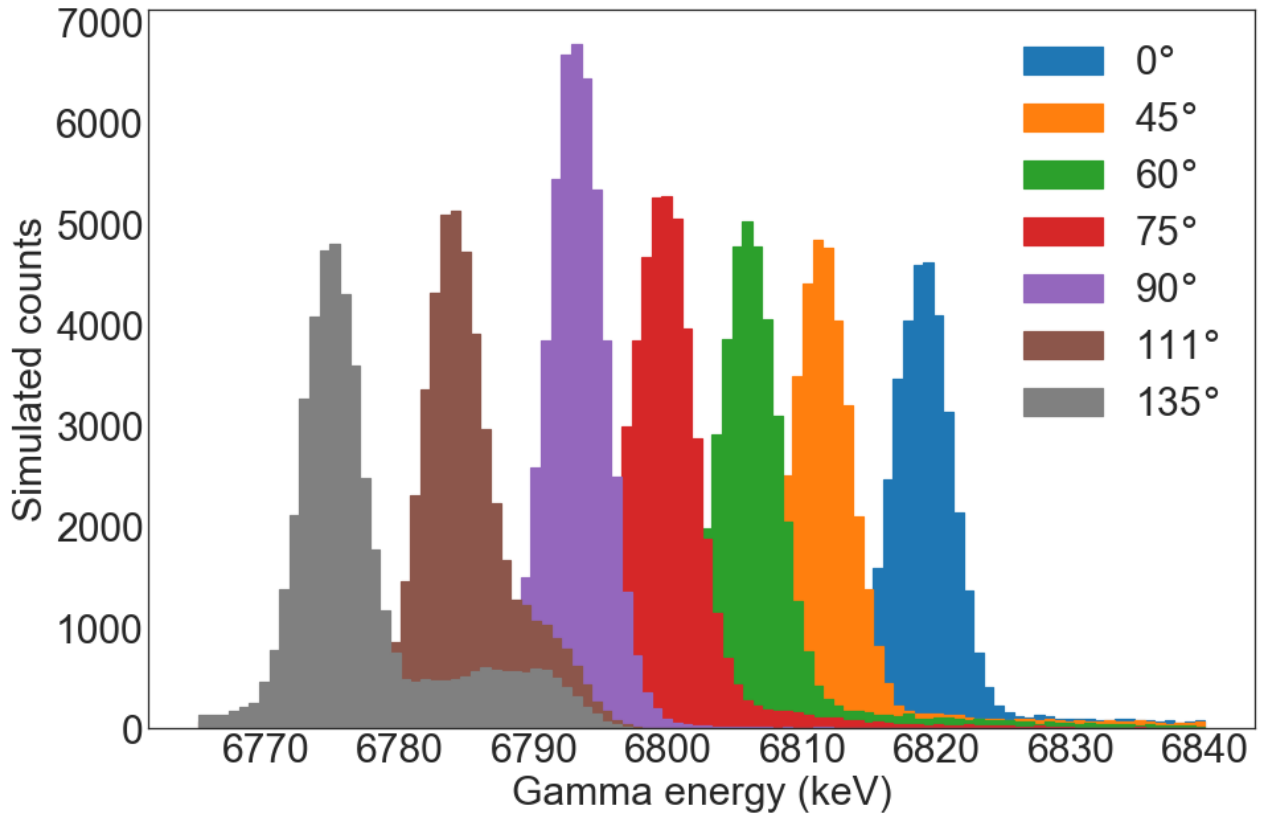
In [7]:

```
# Create nine subplots sharing y axis
fig_size = plt.rcParams["figure.figsize"]
fig_size[0] = 12
fig_size[1] = 9
plt.rcParams["figure.figsize"] = fig_size
fig, axs = plt.subplots(3, 3, sharey = True, sharex = False)

axs[0, 0].hist(eGamma[0][:nGenerated-1],bins=100,range=(6765,6830))
axs[0, 0].set_title('0 degree')
axs[0, 0].set(ylabel = 'Simulated counts')
axs[0, 1].hist(eGamma[1][:nGenerated-1],bins=100,range=(6765,6830))
axs[0, 1].set_title('45 degree')
axs[0, 2].hist(eGamma[2][:nGenerated-1],bins=100,range=(6765,6830))
axs[0, 2].set_title('60 degree')
axs[1, 0].hist(eGamma[3][:nGenerated-1],bins=100,range=(6765,6830))
axs[1, 0].set_title('75 degree')
axs[1, 0].set(ylabel = 'Simulated counts')
axs[1, 1].hist(eGamma[4][:nGenerated-1],bins=100,range=(6765,6830))
axs[1, 1].set_title('90 degree')
axs[1, 1].set(xlabel = 'Energy (keV)')
axs[1, 2].hist(eGamma[5][:nGenerated-1],bins=100,range=(6765,6830))
axs[1, 2].set_title('111 degree')
axs[1, 2].set(xlabel = 'Energy (keV)')
axs[2, 0].hist(eGamma[6][:nGenerated-1],bins=100,range=(6765,6830))
axs[2, 0].set_title('135 degree')
axs[2, 0].set(ylabel = 'Simulated counts')
axs[2, 0].set(xlabel = 'Energy (keV)')

axs[2, 1].remove()
axs[2, 2].remove()

#axs[2, 2].hist(expAngles[:7],bins=100,range=(0,180))
#axs[2, 2].set_title('Histogram of Angles')

#for ax in axs.flat:
    #ax.set(ylabel='y-label')

# Hide x labels and tick labels for top plots and y ticks for right plots.
#for ax in axs.flat:
    #ax.label_outer()

#fig.suptitle('Test stacking spectra')
```


0 degree
45 degree
60 degree
75 degree
90 degree
111 degree
135 degree
Simulated counts
Energy (keV)
0
1000
2000
3000
4000
5000
6000
6780
6800
6820

In [8]:

```
# All angles on one plot

#plt.style.use('bmh')
plt.style.use('seaborn-white')

# Create the plot and add the different datasets
fig, ax = plt.subplots(figsize=(15,10))
l0 = ax.hist(eGamma[0][:nGenerated-1], label='0$\degree$',   bins=100, range=(6765,6840), histtype='step'
, stacked=True, fill=True, linewidth=1.2)
l1 = ax.hist(eGamma[1][:nGenerated-1], label='45$\degree$',  bins=100, range=(6765,6840), histtype='step'
, stacked=True, fill=True, linewidth=1.2)
l2 = ax.hist(eGamma[2][:nGenerated-1], label='60$\degree$',  bins=100, range=(6765,6840), histtype='step'
, stacked=True, fill=True, linewidth=1.2)
l3 = ax.hist(eGamma[3][:nGenerated-1], label='75$\degree$',  bins=100, range=(6765,6840), histtype='step'
, stacked=True, fill=True, linewidth=1.2)
l4 = ax.hist(eGamma[4][:nGenerated-1], label='90$\degree$',  bins=100, range=(6765,6840), histtype='step'
, stacked=True, fill=True, linewidth=1.2)
l5 = ax.hist(eGamma[5][:nGenerated-1], label='111$\degree$', bins=100, range=(6765,6840), histtype='step'
, stacked=True, fill=True, linewidth=1.2)
l6 = ax.hist(eGamma[6][:nGenerated-1], label='135$\degree$', bins=100, range=(6765,6840), histtype='step'
, stacked=True, fill=True, linewidth=1.2, color='gray')

# Formatting
title = 'Monte Carlo Generated Gamma Spectrum for all angles, tau = '+str(tau)+' fs'
#ax.set_title(title, fontsize=30)
ax.set_ylabel('Simulated counts', fontsize=30)
ax.set_xlabel('Gamma energy (keV)', fontsize=30)
ax.tick_params(labelsize=30)
ax.legend(loc='best', shadow=True, fontsize=30)
#plt.xlim(6760, 6855)

plt.show()
```



## 5. Aggregate shift data

For this, I repeat the earlier cell where I generate the doppler shifted gammas (step 3) for each of the given lifetimes. With each, I copy the energy array to this cell to store the shifted lifetimes. Therefore, I have all of the data in one place to continue the analysis. Each code block represents a calculation for a different target.

This is the tedious part, as you are essentially running the earlier cell over and over and over and over again - each time copying the energies generated earlier into the appropriate locations in their distribution down here. However, when you're done, all the data is here in this one place. For a backup I'm also copying the data to an excel file.

**After this has been entered, don't change the data in this section**

In [9]:

```
# Ta30 target


# Calculate the full shift gamma ray energies
fullShift = np.zeros(7)

for angle in range(len(expAngles)):
    detectorAngle = expAngles[angle]*(math.pi/180.0)
    shift = eGamma0 * (1 + (v0/3000)*math.cos(detectorAngle))
    fullShift[angle] = round(shift,3)

    #DEBUG
    #print(round(shift,3))


# Energies for lifetimes and angles
# lifetimes = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.5, 2.0, 5.0
# angles = 0, 45, 60, 75, 90, 111, 135


energiesTa30 = np.zeros((15, 7))
energiesTa30[0][:] = fullShift[:]     # full shift
energiesTa30[1][:] = [6819.529, 6811.788, 6806.303, 6799.951, 6793.088, 6783.635, 6774.404]    # tau = 0.
1 fs
energiesTa30[2][:] = [6819.463, 6811.720, 6806.273, 6799.923, 6793.090, 6783.685, 6774.452]    # tau = 0.
2 fs
energiesTa30[3][:] = [6819.432, 6811.707, 6806.242, 6799.853, 6793.093, 6783.690, 6774.497]    # tau = 0.
3 fs
energiesTa30[4][:] = [6819.376, 6811.649, 6806.204, 6799.843, 6793.074, 6783.730, 6774.541]    # tau = 0.
4 fs
energiesTa30[5][:] = [6819.321, 6811.619, 6806.144, 6799.803, 6793.060, 6783.753, 6774.594]    # tau = 0.
5 fs
energiesTa30[6][:] = [6819.268, 6811.560, 6806.106, 6799.771, 6793.083, 6783.782, 6774.612]    # tau = 0.
6 fs
energiesTa30[7][:] = [6819.223, 6811.530, 6806.068, 6799.746, 6793.098, 6783.814, 6774.662]    # tau = 0.
7 fs
energiesTa30[8][:] = [6819.190, 6811.473, 6806.030, 6799.705, 6793.096, 6783.855, 6774.704]    # tau = 0.
8 fs
energiesTa30[9][:] = [6819.106, 6811.438, 6806.022, 6799.693, 6793.106, 6783.910, 6774.757]    # tau = 0.
9 fs
energiesTa30[10][:] = [6819.088, 6811.400, 6805.960, 6799.680, 6793.104, 6783.904, 6774.788]   # tau = 1.
0 fs
energiesTa30[11][:] = [6818.967, 6811.313, 6805.890, 6799.625, 6793.084, 6783.977, 6774.871]   # tau = 1.
2 fs
energiesTa30[12][:] = [6818.840, 6811.176, 6805.820, 6799.572, 6793.130, 6784.080, 6774.990]   # tau = 1.
5 fs
energiesTa30[13][:] = [6818.616, 6811.053, 6805.695, 6799.454, 6793.096, 6784.170, 6775.192]   # tau = 2.
0 fs
energiesTa30[14][:] = [6818.081, 6810.639, 6805.392, 6799.359, 6793.110, 6784.395, 6775.545]   # tau = 5.
0 fs

# DEBUG
#print(energiesTa30)
```

In [10]:

```
# Ta10 target


# Calculate the full shift gamma ray energies
fullShift = np.zeros(7)

for angle in range(len(expAngles)):
    detectorAngle = expAngles[angle]*(math.pi/180.0)
    shift = eGamma0 * (1 + (v0/3000)*math.cos(detectorAngle))
    fullShift[angle] = round(shift,3)

    #DEBUG
    #print(round(shift,3))


# Energies for lifetimes and angles
# lifetimes = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.5, 2.0, 5.0
# angles = 0, 45, 60, 75, 90, 111, 135


energiesTa10 = np.zeros((15, 7))
energiesTa10[0][:] = fullShift[:]     # full shift
energiesTa10[1][:] = [6819.536, 6811.766, 6806.323, 6799.936, 6793.092, 6783.627, 6774.415]    # tau = 0.
1 fs
energiesTa10[2][:] = [6819.478, 6811.740, 6806.267, 6799.891, 6793.094, 6783.647, 6774.458]    # tau = 0.
2 fs
energiesTa10[3][:] = [6819.419, 6811.697, 6806.213, 6799.856, 6793.093, 6783.731, 6774.521]    # tau = 0.
3 fs
energiesTa10[4][:] = [6819.355, 6811.635, 6806.188, 6799.843, 6793.118, 6783.736, 6774.551]    # tau = 0.
4 fs
energiesTa10[5][:] = [6819.314, 6811.577, 6806.145, 6799.809, 6793.088, 6783.776, 6774.591]    # tau = 0.
5 fs
energiesTa10[6][:] = [6819.249, 6811.562, 6806.123, 6799.791, 6793.103, 6783.802, 6774.639]    # tau = 0.
6 fs
energiesTa10[7][:] = [6819.214, 6811.509, 6806.049, 6799.731, 6793.082, 6783.831, 6774.672]    # tau = 0.
7 fs
energiesTa10[8][:] = [6819.174, 6811.472, 6806.035, 6799.727, 6793.103, 6783.893, 6774.739]    # tau = 0.
8 fs
energiesTa10[9][:] = [6819.111, 6811.423, 6805.994, 6799.687, 6793.077, 6783.916, 6774.783]    # tau = 0.
9 fs
energiesTa10[10][:] = [6819.033, 6811.394, 6805.959, 6799.643, 6793.083, 6783.922, 6774.826]   # tau = 1.
0 fs
energiesTa10[11][:] = [6818.954, 6811.322, 6805.896, 6799.604, 6793.109, 6783.992, 6774.908]   # tau = 1.
2 fs
energiesTa10[12][:] = [6818.834, 6811.199, 6805.807, 6799.557, 6793.090, 6784.087, 6775.003]   # tau = 1.
5 fs
energiesTa10[13][:] = [6818.635, 6811.014, 6805.659, 6799.439, 6793.095, 6784.202, 6775.152]   # tau = 2.
0 fs
energiesTa10[14][:] = [6818.287, 6810.778, 6805.516, 6799.387, 6793.109, 6784.263, 6775.456]   # tau = 5.
0 fs

# DEBUG
#print(energies)
```

In [11]:

```
# Mo30 target


# Calculate the full shift gamma ray energies
fullShift = np.zeros(7)

for angle in range(len(expAngles)):
    detectorAngle = expAngles[angle]*(math.pi/180.0)
    shift = eGamma0 * (1 + (v0/3000)*math.cos(detectorAngle))
    fullShift[angle] = round(shift,3)

    #DEBUG
    #print(round(shift,3))


# Energies for lifetimes and angles
# lifetimes = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.5, 2.0, 5.0
# angles = 0, 45, 60, 75, 90, 111, 135


energiesMo30 = np.zeros((15, 7))
energiesMo30[0][:] = fullShift[:]     # full shift
energiesMo30[1][:] = [6819.539, 6811.773, 6806.310, 6799.934, 6793.095, 6783.648, 6774.425]    # tau = 0.
1 fs
energiesMo30[2][:] = [6819.450, 6811.722, 6806.233, 6799.900, 6793.101, 6783.675, 6774.468]    # tau = 0.
2 fs
energiesMo30[3][:] = [6819.386, 6811.695, 6806.202, 6799.849, 6793.104, 6783.725, 6774.547]    # tau = 0.
3 fs
energiesMo30[4][:] = [6819.324, 6811.628, 6806.160, 6799.817, 6793.123, 6783.750, 6774.569]    # tau = 0.
4 fs
energiesMo30[5][:] = [6819.272, 6811.579, 6806.144, 6799.787, 6793.103, 6783.795, 6774.650]    # tau = 0.
5 fs
energiesMo30[6][:] = [6819.194, 6811.516, 6806.093, 6799.748, 6793.094, 6783.821, 6774.692]    # tau = 0.
6 fs
energiesMo30[7][:] = [6819.154, 6811.432, 6806.030, 6799.730, 6793.092, 6783.876, 6774.754]    # tau = 0.
7 fs
energiesMo30[8][:] = [6819.077, 6811.425, 6805.984, 6799.675, 6793.105, 6783.930, 6774.789]    # tau = 0.
8 fs
energiesMo30[9][:] = [6819.039, 6811.354, 6805.950, 6799.650, 6793.115, 6783.944, 6774.842]    # tau = 0.
9 fs
energiesMo30[10][:] = [6818.954, 6811.330, 6805.919, 6799.623, 6793.083, 6783.986, 6774.891]   # tau = 1.
0 fs
energiesMo30[11][:] = [6818.866, 6811.200, 6805.832, 6799.585, 6793.098, 6784.051, 6774.985]   # tau = 1.
2 fs
energiesMo30[12][:] = [6818.707, 6811.083, 6805.730, 6799.488, 6793.092, 6784.120, 6775.113]   # tau = 1.
5 fs
energiesMo30[13][:] = [6818.498, 6810.908, 6805.561, 6799.371, 6793.093, 6784.245, 6775.277]   # tau = 2.
0 fs
energiesMo30[14][:] = [6818.017, 6810.573, 6805.316, 6799.334, 6793.113, 6784.413, 6775.647]   # tau = 5.
0 fs

# DEBUG
#print(energies)
```

In [12]:

```
# W30 target


# Calculate the full shift gamma ray energies
fullShift = np.zeros(7)

for angle in range(len(expAngles)):
    detectorAngle = expAngles[angle]*(math.pi/180.0)
    shift = eGamma0 * (1 + (v0/3000)*math.cos(detectorAngle))
    fullShift[angle] = round(shift,3)

    #DEBUG
    #print(round(shift,3))


# Energies for lifetimes and angles
# lifetimes = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.5, 2.0, 5.0
# angles = 0, 45, 60, 75, 90, 111, 135


energiesW30 = np.zeros((15, 7))
energiesW30[0][:] = fullShift[:]     # full shift
energiesW30[1][:] = [6819.539, 6811.779, 6806.313, 6799.914, 6793.096, 6783.630, 6774.400]    # tau = 0.1
fs
energiesW30[2][:] = [6819.466, 6811.743, 6806.263, 6799.916, 6793.096, 6783.674, 6774.482]    # tau = 0.2
fs
energiesW30[3][:] = [6819.429, 6811.688, 6806.218, 6799.858, 6793.085, 6783.711, 6774.515]    # tau = 0.3
fs
energiesW30[4][:] = [6819.356, 6811.643, 6806.179, 6799.807, 6793.097, 6783.753, 6774.542]    # tau = 0.4
fs
energiesW30[5][:] = [6819.281, 6811.579, 6806.142, 6799.805, 6793.101, 6783.788, 6774.608]    # tau = 0.5
fs
energiesW30[6][:] = [6819.220, 6811.532, 6806.068, 6799.746, 6793.085, 6783.833, 6774.670]    # tau = 0.6
fs
energiesW30[7][:] = [6819.196, 6811.469, 6806.039, 6799.722, 6793.084, 6783.858, 6774.701]    # tau = 0.7
fs
energiesW30[8][:] = [6819.140, 6811.429, 6806.000, 6799.680, 6793.111, 6783.899, 6774.749]    # tau = 0.8
fs
energiesW30[9][:] = [6819.062, 6811.401, 6805.958, 6799.661, 6793.097, 6783.933, 6774.801]    # tau = 0.9
fs
energiesW30[10][:] = [6819.005, 6811.326, 6805.923, 6799.628, 6793.082, 6783.957, 6774.859]   # tau = 1.0
fs
energiesW30[11][:] = [6818.918, 6811.250, 6805.832, 6799.566, 6793.103, 6784.030, 6774.925]   # tau = 1.2
fs
energiesW30[12][:] = [6818.758, 6811.148, 6805.753, 6799.499, 6793.105, 6784.119, 6775.042]   # tau = 1.5
fs
energiesW30[13][:] = [6818.566, 6810.995, 6805.639, 6799.425, 6793.105, 6784.206, 6775.231]   # tau = 2.0
fs
energiesW30[14][:] = [6818.300, 6810.862, 6805.591, 6799.443, 6793.134, 6784.269, 6775.382]   # tau = 5.0
fs

# DEBUG
#print(energies)
```

In [13]:

```
# Ta2N3 Target


# Calculate the full shift gamma ray energies
fullShift = np.zeros(7)

for angle in range(len(expAngles)):
    detectorAngle = expAngles[angle]*(math.pi/180.0)
    shift = eGamma0 * (1 + (v0/3000)*math.cos(detectorAngle))
    fullShift[angle] = round(shift,3)

    #DEBUG
    #print(round(shift,3))


# Energies for lifetimes and angles
# lifetimes = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.5, 2.0, 5.0
# angles = 0, 45, 60, 75, 90, 111, 135


energiesTa2N3 = np.zeros((15, 7))
energiesTa2N3[0][:] = fullShift[:]     # full shift
energiesTa2N3[1][:] = [6819.507, 6811.762, 6806.303, 6799.914, 6793.123, 6783.652, 6774.428]    # tau =
 0.1 fs
energiesTa2N3[2][:] = [6819.464, 6811.707, 6806.237, 6799.887, 6793.102, 6783.703, 6774.474]    # tau =
 0.2 fs
energiesTa2N3[3][:] = [6819.381, 6811.653, 6806.200, 6799.834, 6793.098, 6783.733, 6774.528]    # tau =
 0.3 fs
energiesTa2N3[4][:] = [6819.325, 6811.615, 6806.143, 6799.788, 6793.099, 6783.777, 6774.609]    # tau =
 0.4 fs
energiesTa2N3[5][:] = [6819.263, 6811.539, 6806.083, 6799.752, 6793.107, 6783.841, 6774.648]    # tau =
 0.5 fs
energiesTa2N3[6][:] = [6819.180, 6811.490, 6806.022, 6799.704, 6793.085, 6783.865, 6774.715]    # tau =
 0.6 fs
energiesTa2N3[7][:] = [6819.109, 6811.422, 6806.006, 6799.663, 6793.104, 6783.913, 6774.776]    # tau =
 0.7 fs
energiesTa2N3[8][:] = [6819.068, 6811.381, 6805.943, 6799.648, 6793.091, 6783.983, 6774.828]    # tau =
 0.8 fs
energiesTa2N3[9][:] = [6819.017, 6811.344, 6805.945, 6799.599, 6793.102, 6783.999, 6774.855]    # tau =
 0.9 fs
energiesTa2N3[10][:] = [6818.929, 6811.309, 6805.865, 6799.578, 6793.118, 6784.027, 6774.918]   # tau =
 1.0 fs
energiesTa2N3[11][:] = [6818.934, 6811.305, 6805.875, 6799.598, 6793.079, 6784.059, 6774.927]   # tau =
 1.2 fs
energiesTa2N3[12][:] = [6818.728, 6811.070, 6805.717, 6799.472, 6793.100, 6784.186, 6775.112]   # tau =
 1.5 fs
energiesTa2N3[13][:] = [6818.583, 6810.990, 6805.610, 6799.403, 6793.094, 6784.223, 6775.233]   # tau =
 2.0 fs
energiesTa2N3[14][:] = [6818.643, 6811.161, 6805.805, 6799.590, 6793.121, 6784.024, 6775.016]   # tau =
 5.0 fs

# DEBUG
#print(energies)
```

In [14]:

```
# Pure Ta Target


# Calculate the full shift gamma ray energies
fullShift = np.zeros(7)

for angle in range(len(expAngles)):
    detectorAngle = expAngles[angle]*(math.pi/180.0)
    shift = eGamma0 * (1 + (v0/3000)*math.cos(detectorAngle))
    fullShift[angle] = round(shift,3)

    #DEBUG
    #print(round(shift,3))


# Energies for lifetimes and angles
# lifetimes = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.5, 2.0, 5.0
# angles = 0, 45, 60, 75, 90, 111, 135


energiesPureTa = np.zeros((15, 7))
energiesPureTa[0][:] = fullShift[:]     # full shift
energiesPureTa[1][:] = [6819.524, 6811.783, 6806.310, 6799.940, 6793.092, 6783.632, 6774.420]    # tau =
 0.1 fs
energiesPureTa[2][:] = [6819.467, 6811.730, 6806.269, 6799.908, 6793.092, 6783.665, 6774.476]    # tau =
 0.2 fs
energiesPureTa[3][:] = [6819.437, 6811.692, 6806.215, 6799.868, 6793.096, 6783.734, 6774.515]    # tau =
 0.3 fs
energiesPureTa[4][:] = [6819.360, 6811.634, 6806.169, 6799.831, 6793.085, 6783.747, 6774.560]    # tau =
 0.4 fs
energiesPureTa[5][:] = [6819.292, 6811.612, 6806.151, 6799.773, 6793.124, 6783.794, 6774.613]    # tau =
 0.5 fs
energiesPureTa[6][:] = [6819.249, 6811.552, 6806.120, 6799.764, 6793.100, 6783.836, 6774.672]    # tau =
 0.6 fs
energiesPureTa[7][:] = [6819.169, 6811.493, 6806.044, 6799.740, 6793.091, 6783.850, 6774.696]    # tau =
 0.7 fs
energiesPureTa[8][:] = [6819.123, 6811.438, 6806.010, 6799.697, 6793.107, 6783.909, 6774.755]    # tau =
 0.8 fs
energiesPureTa[9][:] = [6819.075, 6811.395, 6805.969, 6799.687, 6793.094, 6783.939, 6774.810]    # tau =
 0.9 fs
energiesPureTa[10][:] = [6819.021, 6811.351, 6805.951, 6799.623, 6793.090, 6783.982, 6774.847]   # tau =
 1.0 fs
energiesPureTa[11][:] = [6818.934, 6811.305, 6805.875, 6799.598, 6793.079, 6784.059, 6774.927]   # tau =
 1.2 fs
energiesPureTa[12][:] = [6818.758, 6811.148, 6805.753, 6799.499, 6793.105, 6784.119, 6775.042]   # tau =
 1.5 fs
energiesPureTa[13][:] = [6818.613, 6811.025, 6805.678, 6799.445, 6793.090, 6784.199, 6775.174]   # tau =
 2.0 fs
energiesPureTa[14][:] = [6818.461, 6810.925, 6805.606, 6799.478, 6793.103, 6784.163, 6775.273]   # tau =
 5.0 fs

# DEBUG
#print(energies)
```

## 6. Plotting shifts to dermine the attenuation factors

In [15]:

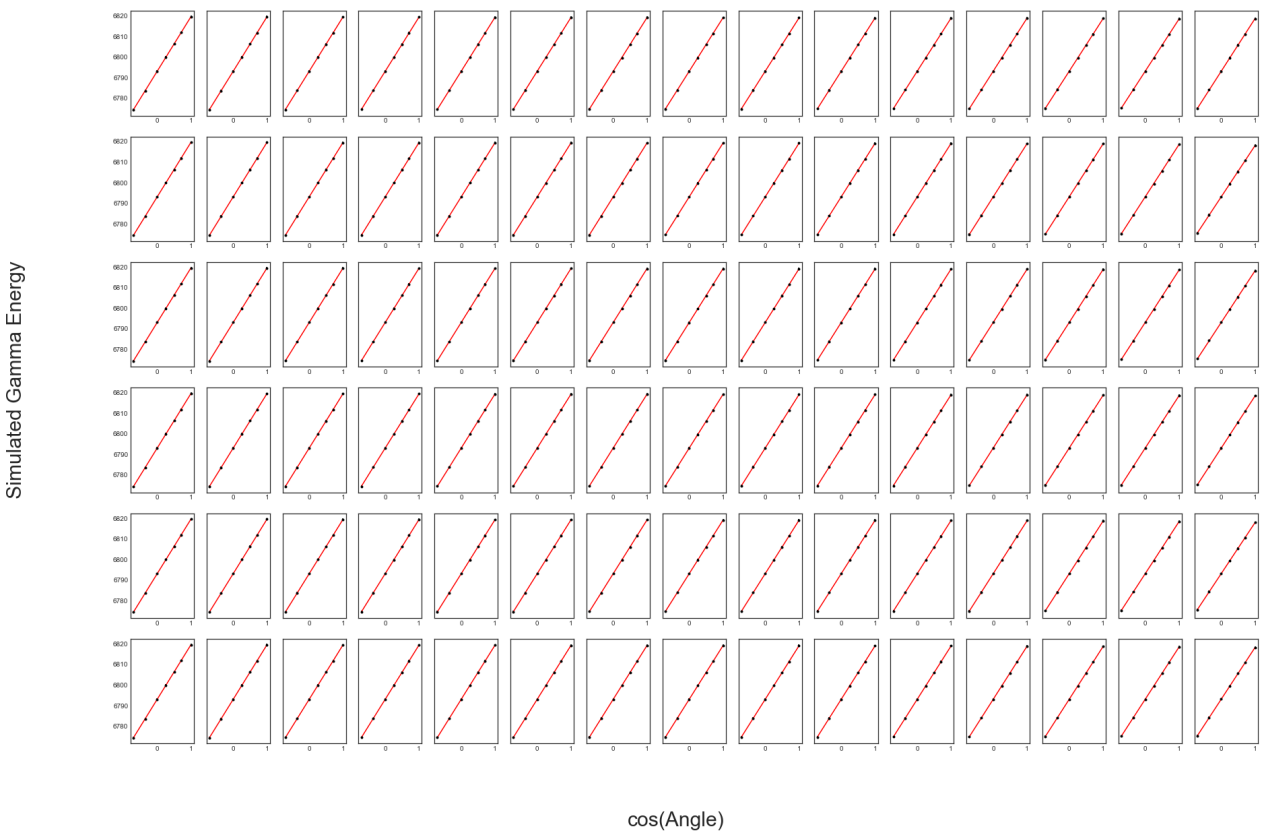
```
# Some preliminary definition of data structures

slopes = np.zeros((6, 15))      # Slopes for doppler shift plots
slopeUncs = np.zeros((6,15))    # uncertainties on doppler shift slopes
attFactors = np.zeros((6, 15))  # F(tau) for different lifetime/target combo
attUncs = np.zeros((6, 15))     # Uncertainties on F(tau)

cosAngles = np.zeros(7)         # cos(angle)
for angle in range(len(expAngles)):
    cosAngles[angle] = math.cos(expAngles[angle]*(math.pi/180.0))

# The generated gamma energies for each of the targets (entered manually in section 5)
targets = [energiesTa2N3, energiesTa30, energiesTa10, energiesPureTa, energiesMo30, energiesW30]

# Lifetimes in a list
lifetimes = [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.5, 2.0, 5.0]

# DEBUG
#for backing in targets:
#    print(backing[1][1])
```

In [16]:

```
# Pretty self explanatory
def line(x, m, b):
    return m*x + b
```

In [17]:

```
# Create and fit all doppler shift plots

# 6 target materials and 15 lifetimes tested
fig, axs = plt.subplots(6, 15, figsize=(30,20), sharex=False, sharey=True)


yerr = 0.5              # Representative error on centroid positioning from experiment


for i in range(0, 6):            # 6 targets
    for j in range(0, 15):       # 15 lifetimes

        y = targets[i][j][:]     # Get the current lifetime/target combination of energies
        #DEBUG
        #print(y)

        axs[i][j].errorbar(cosAngles, y, yerr, fmt='.k')   # Plot

        dy = [0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]                                # Sigma for the calculati
on
        parameters, covariance = sopt.curve_fit(line, cosAngles, y, sigma=dy)   # Fit the line to each da
taset
        fitX = np.linspace(-0.7,1.0,20)
        fitY = line(fitX, parameters[0], parameters[1])
        axs[i][j].plot(fitX,fitY,'r-')                     # Plot the fit result

        paramUncs = np.sqrt(np.diag(covariance))           # Calculate the uncertainties in fit parameter
s
        slopes[i][j] = parameters[0]
        slopeUncs[i][j] = paramUncs[0]

fig.suptitle('Doppler Shift for different lifetime/target combinations', fontsize=40)
fig.text(0.5, 0.04, 'cos(Angle)', ha='center', fontsize=30)
fig.text(0.04, 0.5, 'Simulated Gamma Energy', va='center', rotation='vertical', fontsize=30)
plt.show()

#DEBUG
#print(slopes)
#print(slopeUncs)
```

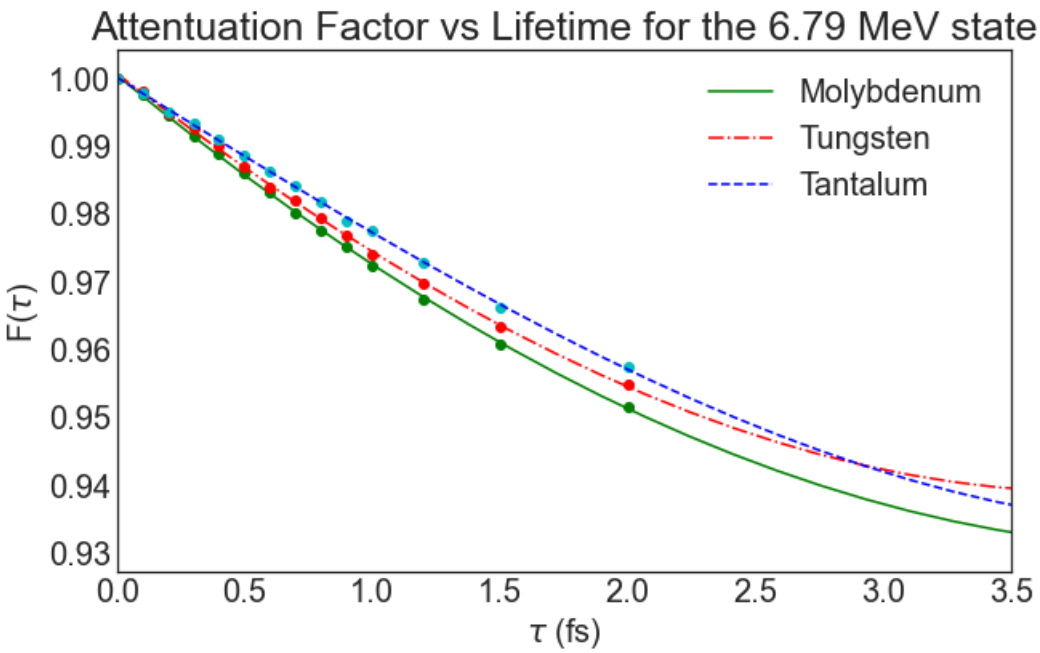



In [18]:

```
# Calculate the attenuation factors from the slopes

for i in range(0, 6):
    for j in range(0, 15):
        factor = slopes[i][j]/slopes[i][0]
        uncertainty = factor * np.sqrt((slopeUncs[i][j]/slopes[i][j])**2 + (slopeUncs[i][0]/slopes[i][0])
**2)
        attFactors[i][j] = round(factor, 4)
        attUncs[i][j] = round(uncertainty, 4)

# DEBUG
#print(attFactors)
#print(attUncs)
```

In [19]:

```
# Plot the attenuation factors
#plt.errorbar(lifetimes, attFactors[1][:], attUncs[1][:], marker='o', ls='none', ms=5, label='Ta30')#, fmt='.k')
#plt.errorbar(lifetimes, attFactors[2][:], attUncs[2][:], marker='o', ls='none', ms=5, label='Ta10')#, fmt='.k')
#plt.errorbar(lifetimes, attFactors[4][:], attUncs[4][:], marker='o', ls='none', ms=5, label='Mo30')#, fmt='.k')
#plt.errorbar(lifetimes, attFactors[5][:], attUncs[5][:], marker='o', ls='none', ms=5, label='W30')#, fmt='.k')

fig_size = plt.rcParams["figure.figsize"]
fig_size[0] = 10
fig_size[1] = 6
plt.rcParams["figure.figsize"] = fig_size

# Beautify
plt.title(r'Attentuation Factor vs Lifetime for the 6.79 MeV state', fontsize=25)
plt.xlabel(r'${\tau}$ (fs)', fontsize=20)
plt.ylabel(r'F(${\tau}$)', fontsize=20)
plt.tick_params(labelsize=20)


# Interpolation
x = np.linspace(0, 5, num=15, endpoint=True)
xnew = np.linspace(0, 5, 30)
splMo = UnivariateSpline(lifetimes, attFactors[4][:])
splMo.set_smoothing_factor(1)
splW = UnivariateSpline(lifetimes, attFactors[5][:])
splW.set_smoothing_factor(1)
splTa30 = UnivariateSpline(lifetimes, attFactors[1][:])
splTa30.set_smoothing_factor(1)
splTa10 = UnivariateSpline(lifetimes, attFactors[2][:])
splTa10.set_smoothing_factor(1)
plt.plot(lifetimes, attFactors[4][:], 'go')
plt.plot(xnew, splMo(xnew), 'g-', label='Molybdenum')
plt.plot(lifetimes, attFactors[5][:], 'ro')
plt.plot(xnew, splW(xnew), 'r-.', label='Tungsten')
plt.plot(lifetimes, attFactors[1][:], 'co')
plt.plot(xnew, splTa30(xnew), 'b--', label='Tantalum')
# plt.plot(lifetimes, attFactors[2][:], 'bo')
# plt.plot(xnew, splTa10(xnew), 'b--', label='Tantalum (low)')


plt.legend(loc='upper right', shadow=True, fontsize=20)

plt.xlim(0,3.5)
plt.show()
```




In [20]:

```
def lookup(F, err, spl):

    test = 0
    options = []                    # lifetimes in the correct range
    value = []                      # lifetime corresponding to F
    z = np.linspace(0, 5, 500)      # potential lifetimes
    for lifetime in z:
        if (spl(lifetime) < (F+err)) and (spl(lifetime) > (F-err)):
            options.append(lifetime)                                    # lifetimes in F+/-err
        if (spl(lifetime) < (F+0.0001) and (spl(lifetime) > (F-0.0001))):
            value.append(lifetime)                                      # lifetime for F
            test = 1

    if value == []:
        for lifetime in z:
            if (spl(lifetime) < (F+0.0005) and (spl(lifetime) > (F-0.0005))):
                value.append(lifetime)                                      # lifetime for F
                test = 2

    if value == []:
        for lifetime in z:
            if (spl(lifetime) < (F+0.01) and (spl(lifetime) > (F-0.0001))):
                value.append(lifetime)                                      # lifetime for F
                test = 3

    # DEBUG
    print(test)
    print(value)

    #print(options)
    tau = np.mean(value[:])
    print('Tau =    ', round(tau, 3))
    print('F(tau) = ', round(float(spl(tau)), 3))
    print('Range:    (', round(options[0], 3), ', ', round(options[len(options)-1], 3), ')\n', sep='')
```

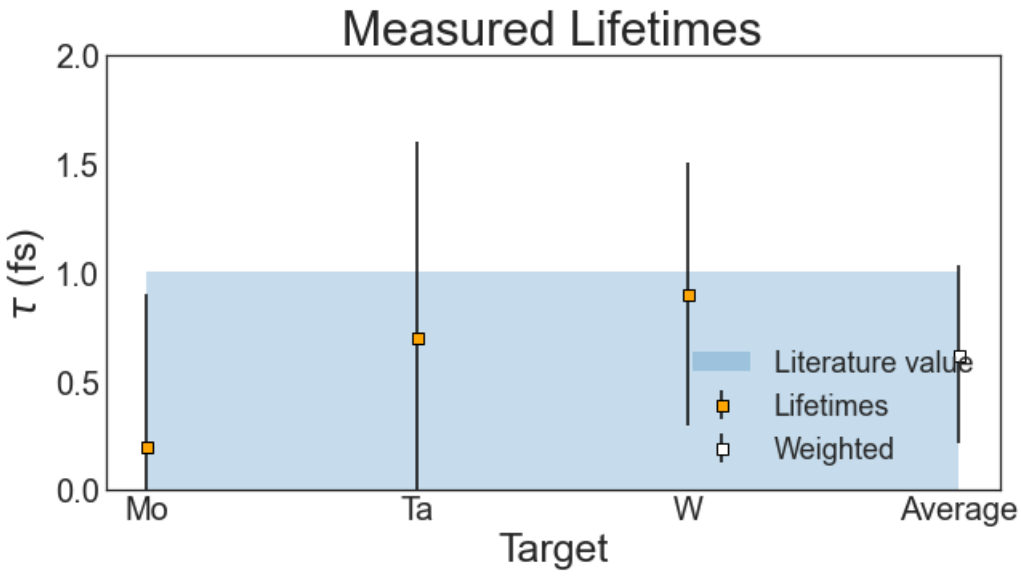
In [21]:

```
# Find values
lookup(0.995, 0.019, splMo)
lookup(0.978, 0.015, splW)
lookup(0.942, 0.04, splTa10)
lookup(0.983, 0.019, splTa30)
```

```
1
[0.18036072144288576]
Tau =     0.18
F(tau) =  0.995
Range:    (0.0, 0.862)

1
[0.8517034068136272]
Tau =     0.852
F(tau) =  0.978
Range:    (0.281, 1.523)

1
[3.3366733466933867, 3.346693386773547, 3.356713426853707, 4.719438877755511, 4.7294589178356
71, 4.739478957915831]
Tau =     4.038
F(tau) =  0.94
Range:    (0.752, 5.0)

1
[0.7414829659318637]
Tau =     0.741
F(tau) =  0.983
Range:    (0.0, 1.633)
```

In [26]:

```
targs = ['Mo', 'Ta', 'W']
taus = [0.2, 0.7, 0.9]
tauerrs = [0.7, 0.9, 0.6]

weighted = ['Average']
weight = [1/(x**2) for x in tauerrs]
weightedAverage = np.average(taus, weights = weight)
weightedError = np.sqrt(1/(1/(tauerrs[0]**2) + 1/(tauerrs[1]**2) + 1/(tauerrs[2]**2)))

#lit = ['Lit value']
#litVal = [1]
#litErr = [1.5]

from pylab import *
# Create the plot, set some options, and add the datasets
fig, ax = plt.subplots(figsize=(10,5))
rcParams['xtick.labelsize'] = 20
rcParams['ytick.labelsize'] = 20
ax.errorbar(targs, taus, yerr=tauerrs, fmt='.k', label='Lifetimes',marker='s', markersize=7, markerfaceco
lor='orange', markeredgecolor='black')   # Plot
ax.errorbar(weighted, weightedAverage, yerr=weightedError, fmt='.k', label='Weighted',marker='s', markers
ize=7, markerfacecolor='w', markeredgecolor='black')   # Plot
#ax.errorbar(lit, litVal, yerr=litErr, fmt='.k', label='Lit. value',marker='s', markersize=7, markerfacec
olor='black', markeredgecolor='black')   # Plot

# Formatting
title = 'Measured Lifetimes'
ax.set_title(title, fontsize=30)
ax.set_ylabel(r'$\tau$ (fs)', fontsize=25)
ax.set_xlabel('Target', fontsize=25)
ax.tick_params(labelsize=20)

alltargs = targs+weighted#+lit
ax.fill_between(alltargs, 1.0, 0, alpha=.25, label='Literature value')

legend(loc='lower right',fontsize=18)
plt.ylim(0,2)
plt.show()
```

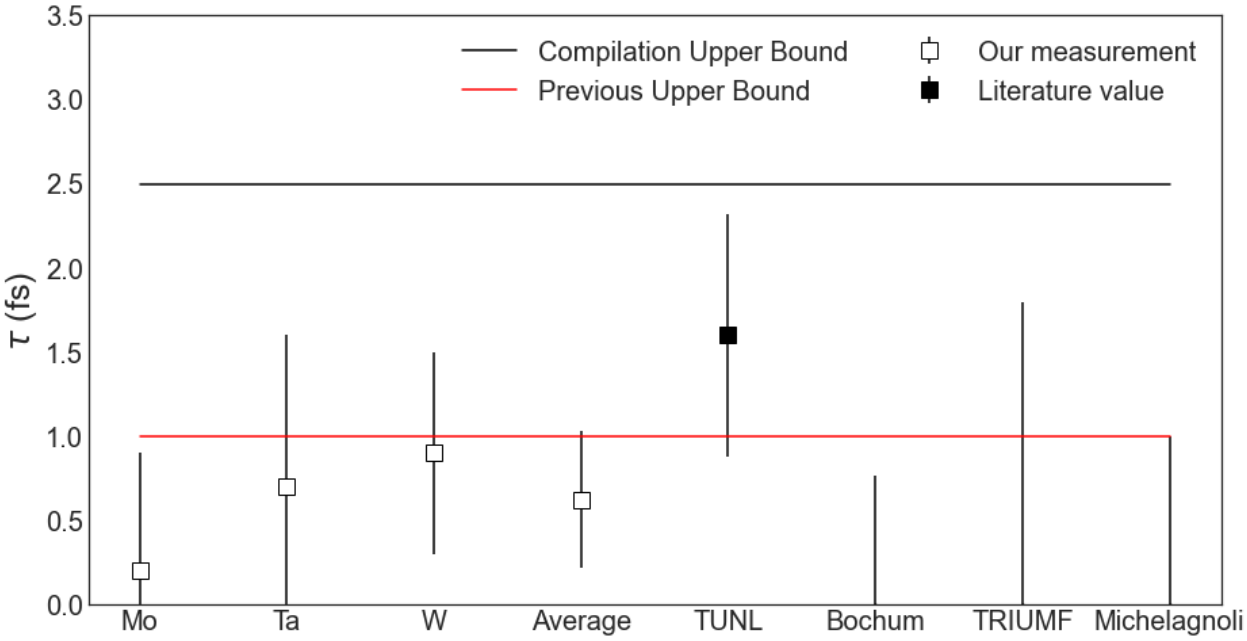


In [23]:

```
tunl = ['TUNL']
tunlVal = [1.6]
tunlErr = [0.72]

bochum = ['Bochum']
bochumVal = [0]
bochumErr = [0.77]

triumf = ['TRIUMF']
triumfVal = [0]
triumfErr = [1.8]

mich = ['Michelagnoli']
michVal = [0]
michErr = [1]
```

In [24]:

```
# Create the plot, set some options, and add the datasets
fig, ax = plt.subplots(figsize=(15,8))
rcParams['xtick.labelsize'] = 20
rcParams['ytick.labelsize'] = 20
ax.errorbar(targs, taus, yerr=tauerrs, fmt='.k', label='Our measurement',marker='s', markersize=12, marke
rfacecolor='w', markeredgecolor='black')   # Plot
ax.errorbar(weighted, weightedAverage, yerr=weightedError, fmt='.k',marker='s', markersize=12, markerface
color='w', markeredgecolor='black')   # Plot
ax.errorbar(tunl, tunlVal, yerr=tunlErr, fmt='.k',marker='s', label='Literature value', markersize=12, ma
rkerfacecolor='black', markeredgecolor='black')   # Plot
ax.errorbar(bochum, bochumVal, yerr=bochumErr, fmt='.k',marker='s', markersize=1, markerfacecolor='black'
, markeredgecolor='black')   # Plot
ax.errorbar(triumf, triumfVal, yerr=triumfErr, fmt='.k',marker='s', markersize=1, markerfacecolor='black'
, markeredgecolor='black')   # Plot
ax.errorbar(mich, michVal, yerr=michErr, fmt='.k',marker='s', markersize=1, markerfacecolor='black', mark
eredgecolor='black')   # Plot

# Formatting
title = 'Measured and Literature Lifetimes of the 6.79 MeV State in $^{15}$O'
#ax.set_title(title, fontsize=30)
ax.set_ylabel(r'$\tau$ (fs)', fontsize=25)
#ax.set_xlabel('Target', fontsize=25)
ax.tick_params(labelsize=20)

alltargs = targs+weighted+tunl+bochum+triumf+mich
ours = targs+weighted
#ax.fill_between(ours, 3.5, 0, alpha=0.25, color='g')
#ax.fill_between(alltargs, 2.5, 0, alpha=.25, label='Compilation')
plt.plot(alltargs, [2.5]*8, 'k-', label='Compilation Upper Bound')
plt.plot(alltargs, [1.0]*8, 'r-', label='Previous Upper Bound')

legend(loc='best',fontsize=20, ncol=2)
plt.ylim(0,3.5)
plt.show()
```




In [25]:

```
print(weightedAverage)
print(weightedError)
```

```
0.6232049947970862
0.4064516129032258
```

In [ ]:

```

```