

# CSC6013 - Worksheet for Week 6

## Russian Peasants Multiplication

- Trace the Russian Peasants Multiplication algorithm for the following products.  
Show each recursive call and the final result, as shown in the live session (table).
  - $64 * 13$
  - $60 * 13$
  - $59 * 13$

### **Problem A - $64 * 13$**

<b>n</b>	<b>m</b>	<b>Steps</b>
64	13	64 is even
32	26	32 is even
16	52	16 is even
8	104	8 is even
4	208	4 is even
2	416	2 is even
1	832	<b>832</b>

### **Problem B - $60 * 13$**

<b>n</b>	<b>m</b>	<b>Steps</b>
60	13	60 is even
30	26	30 is even
15	52	15 is odd (+52)
7	104	7 is odd (+104)
3	208	3 is odd (+208)
1	416	$416+52+104+208 = \mathbf{780}$

### **Problem c - $59 * 13$**

<b>n</b>	<b>m</b>	<b>Steps</b>
59	13	59 is odd (+13)
29	26	29 is odd (+26)
14	52	14 is even
7	104	7 is odd (+104)
3	208	3 is odd (+208)
1	416	$416+13+26+104+208 = \mathbf{767}$

## Lomuto partition

2. Trace the Lomuto partition with the array:

a.  $A = [100, 33, 22, 213, 65, 29, 153, 199, 47, 181, 85]$

Using  $A[10] = 85$  as pivot the final array will be:

- $A = [33, 22, 65, 29, 47, 85, 153, 199, 100, 181, 213]$

In your trace, write down to each change in either  $i$  or  $j$ , stating: the values of  $i$  and  $j$ , swaps made, and elements divided into lesser than the pivot, greater than the pivot, and yet to compare.

Step 1

$i = 0, j = 0$

Pivot at index 10: 85

Elements lesser than pivot: []

Elements greater than pivot: []

Elements still to be sorted: [100, 33, 22, 213, 65, 29, 153, 199, 47, 181]

Step 2:

$i = 0, j = 0 \rightarrow i = 0, j = 1$

$A[j]: 100 > \text{Pivot: } 85$ , no swap made | Total swaps = 0

Elements lesser than pivot: []

Elements greater than pivot: [100]

Elements still to be sorted: [33, 22, 213, 65, 29, 153, 199, 47, 181]

Step 3:

$i = 0, j = 1 \rightarrow i = 1, j = 2$

$A[j]: 33 \leq \text{Pivot: } 85$ , swap  $A[i]: 100$  with  $A[j]: 33$  | Total swaps = 1

Elements lesser than pivot: [33]

Elements greater than pivot: [100]

Elements still to be sorted: [22, 213, 65, 29, 153, 199, 47, 181]

Step 4:

$i = 1, j = 2 \rightarrow i = 2, j = 3$

$A[j]: 22 \leq \text{Pivot: } 85$ , swap  $A[i]: 100$  with  $A[j]: 22$  | Total swaps = 2

Elements lesser than pivot: [33, 22]

Elements greater than pivot: [100]

Elements still to be sorted: [213, 65, 29, 153, 199, 47, 181]

Step 5:

$i = 2, j = 3 \rightarrow i = 2, j = 4$

$A[j]: 213 > \text{Pivot: } 85$ , no swap made | Total swaps = 2

Elements lesser than pivot: [33, 22]

Elements greater than pivot: [100, 213]

Elements still to be sorted: [65, 29, 153, 199, 47, 181]

Step 6:

$i = 2, j = 4 \rightarrow i = 3, j = 5$

$A[j]: 65 \leq \text{Pivot: } 85$ , swap  $A[i]: 100$  with  $A[j]: 65$  | Total swaps = 3

Elements lesser than pivot: [33, 22, 65]

Elements greater than pivot: [213, 100]

Elements still to be sorted: [29, 153, 199, 47, 181]

Step 7:

$i = 3, j = 5 \rightarrow i = 4, j = 6$

$A[j]: 29 \leq \text{Pivot: } 85$ , swap  $A[i]: 213$  with  $A[j]: 29$  | Total swaps = 4

Elements lesser than pivot: [33, 22, 65, 29]

Elements greater than pivot: [100, 213]

Elements still to be sorted: [153, 199, 47, 181]

Step 8:

$i = 4, j = 6 \rightarrow i = 4, j = 7$

$A[j]: 153 > \text{Pivot: } 85$ , no swap made | Total swaps = 4

Elements lesser than pivot: [33, 22, 65, 29]

Elements greater than pivot: [100, 213, 153]

Elements still to be sorted: [199, 47, 181]

Step 9:

i = 4, j = 7 -> i = 4, j = 8

A[j]: 199 > Pivot: 85, no swap made | Total swaps = 4

Elements lesser than pivot: [33, 22, 65, 29]

Elements greater than pivot: [100, 213, 153, 199]

Elements still to be sorted: [47, 181]

Step 10:

i = 4, j = 8 -> i = 5, j = 9

A[j]: 47 <= Pivot: 85, swap A[i]: 100 with A[j]: 47 | Total swaps = 5

Elements lesser than pivot: [33, 22, 65, 29, 47]

Elements greater than pivot: [213, 153, 199, 100]

Elements still to be sorted: [181]

Step 11:

i = 5, j = 9 -> i = 5, j = 10

A[j]: 181 > Pivot: 85, no swap made | Total swaps = 5

Elements lesser than pivot: [33, 22, 65, 29, 47]

Elements greater than pivot: [213, 153, 199, 100, 181]

Elements still to be sorted: []

Step 12:

Swap A[i]: 213 with Pivot: 85 | Total swaps = 6

Elements lesser than pivot: [33, 22, 65, 29, 47]

Elements greater than pivot: [85, 153, 199, 100, 181]

Pivot: 85 at index 5

Final Array: Lomuto with pivot at index 5: 85

[33, 22, 65, 29, 47, 85, 153, 199, 100, 181, 213]