THE ULTIMATE SOLUTION TO RUBIK'S REVENGE

A scrambled Rubik's Revenge does indeed look formidable. And it is. Prof. Rubik did more than just increase the number of pieces on an edge from three to four or the number on a face from nine to 16. The center pieces did not simply increase from one to four on each side. Rather, these center pieces can now be moved relative to each other. Instead of 20 movable pieces of two kinds we have 56 movable pieces of three kinds.

But take heart. Things are not as bad as they seem. Rubik's Revenge can be solved using the same two series described in The Ultimate Solution to Rubik's Cube, namely The Edge Piece Series and The Corner Piece Series. Once the center pieces and the edge pieces are put together the cube can be assembled using the five steps described for Rubik's Cube.

Can this method be applied by the neophyte? Certainly. Recently (7-19-99) I received an e-mail message from a world-travelling businessman who claims never to have solved a Rubik puzzle of any kind. While in Tokyo he picked up a Rubik's Revenge at a toy store with the intent of spending the 12 hours returning to the states in an effort to solve it. He did try but, not surprisingly, he didn't succeed.

The puzzle was set aside. Periodically, over the next seven years, he would try again. The most recent time was just this month (July of 1999). Again he was unsuccessful but this time he thought to try the internet for help. The first site he found was this one. Although this method required that he first learn how to solve Rubik's Cube, he persevered and the restored Rubik's Revenge sits on his desk.

THE FACE CENTERS

If done first, the center pieces can be placed very easily. You must learn a simple process but there are no new series of turns to memorize. For example, consider the cube in Fig. 1a. We may arbitrarily set the color of the first face and, from the looks of things on these three faces, it appears that a good choice would be to make the right face orange.

Fig. 1a

Fig. 1b

Fig. 1c

During the first part of the solution, constructing cube faces, we may freely turn certain cube faces. We will turn the right face 90^{o} clockwise. This gives the cube in Fig. 1b. Now when the top half of the cube is rotated 90^{o} counterclockwise (giving Fig. 1c) the orange center piece from the front face moves to join the two orange pieces in the right face. If we had not made the first turn then one orange piece would have moved out of the right face as the third moved in.



Fig. 1d **Fig. 1e** **Fig. 1f**

If we rotate the entire cube 90^o clockwise (Fig. 1d) we see that the fourth orange center piece is in the next face (now the right face). How do we get it to join the other three in the front face? Note that if we turn the top half 90^o counterclockwise the two orange pieces in the right face would not line up but would be diagonal to each other. In order to get the two pieces to move together they must be adjacent to each other. To prepare for the turn of the top half of the cube we first turn the right face 90^o counterclockwise giving the cube in Fig. 1e. Now when we turn the top half of the cube 90^o counterclockwise (Fig. 1f) the two orange pieces in the right face will be adjacent to each other. Turn the right face 90^o counterclockwise (Fig. 1g) and then turn the top half 90^o clockwise and the orange face is complete.

Fig. 1g

Which face shall we do next? We know that red is opposite orange. We know that this is true on our Rubik's Cubes. It is also true for both the Rubik's Pocket Cube and Rubik's Revenge. It is true for all the cubes I own including the 5x5x5 "Professor's Cube".

If we choose to place the red face center pieces at this time some of our moves will break up the orange center and we will have to take precautions to restore it. On the other hand, if we choose one of the vertical faces in Fig. 2a we can turn the top and bottom halves of the cube freely. We can solve a vertical face so long as none of its pieces is now on the bottom of the cube. White appears to be a good choice.



Fig. 2a **Fig. 2b** **Fig. 2c**

The right face in Fig. 2a is turned 90^o counterclockwise giving Fig. 2b. The top half of the cube is now turned 90^o clockwise and we have three of the four white pieces in place. We need to

"go and get" the fourth white piece from the back side of the cube and the white face will be done (Fig. 2c). Now we have no choice but to work on a face which will cause faces which have already been completed to be temporarily broken up. Blue is opposite White. We will do it next.


Fig. 3a


Fig. 3b


Fig. 3c

The entire cube has been turned 90^{o} clockwise (about an axis from top to bottom) in Fig. 3a. White is on the left side so we must assemble the blue face center pieces on the right side. The top half of the cube is turned 90^{o} counterclockwise. Two blue face center pieces are adjacent to each other on the right side (Fig. 3b). Note that the top half of the white face center now appears on the front side of this cube. Turn the right side 90^{o} counterclockwise (Fig. 3c) and the two blue pieces are in the bottom half of the face center. They will be left behind in the right face when the top half of the cube is turned 90^{o} clockwise (Fig. 3d).


Fig. 3d

The most recent turn restored the white face. It will be alternately broken up and then restored as we move other face center pieces into position. For example, how do we move the blue face center piece in the front face of Fig. 3d into the right face where it belongs? Answer: rotate the top half of the cube in Fig. 3d by 90^{o} clockwise; rotate the front face 90^{o} counterclockwise; rotate the top half 90^{o} counterclockwise. The white face was broken up and then restored as the third blue piece was moved into place. When we get the fourth blue piece into place the blue center face will be complete.

The blue face of the cube in Fig. 4 is complete and the front face already has three green face center pieces. It should be fairly easy to complete that face by retrieving the fourth green face center piece. But that would be an error. It was permissible to arbitrarily locate the white face as we did. Then the blue face had to be opposite the white face. But the front face in Fig. 4 could be either green or yellow (which are opposite each other on my cubes). How do we know which?

Fig. 4

We can either memorize the order of the appearance of colors as we rotate a cube or we can rely on very clear indicators. Look at the corner piece at right/front/top in Fig. 4. It shows that, when orange is on the top, the green face is to the right of the blue face. Hence, green must be on the right side of the blue face, not on the left. This means that the front face must be yellow.

The green, yellow and red face centers are completed using the method outlined above. Note that the last two faces are completed simultaneously. When it comes to the final parts of the solution of Rubik's Revenge these face centers will act as a single piece and will not be moved relative to each other.

THE EDGE PIECES

As opposed to Rubik's Cube, the edge pieces are in two parts. Most of the edge pieces have been split in two and are scattered about the cube. Now we must pair up these edge pieces so the edge pieces as well as the center pieces may be considered as single units in the final parts of the solution.

In the illustration given here the two halves of the orange/blue edge piece are brought together to make the "whole". One of the orange/blue edge piece halves is shown on the right/front edge of the cube in Fig. 5a. It is in the third horizontal layer of the cube numbering from bottom to top. The other orange/blue edge piece is on the right/back edge of the cube and is in the second layer. The bottom half of the cube is rotated 90^{o} counterclockwise giving the cube in Fig. 5b.

We have joined the orange/blue edge pieces to make the "whole". Note that each of the white, yellow, blue and green faces has been split. You cannot turn front or right halves of the cube without making a mess. But you can turn cube faces providing you restore any vertical face before again turning the bottom half of the cube. We want to preserve the orange/blue edge piece which means that we must remove it from the vertical edge where it now resides.

Fig. 5a

Fig. 5b

This is done with an Edge Piece Series. One can use either the full four turns or only the first three turns omitting a final turn of the top face. In this case only three turns were used. They were:

one	**right face clockwise**
two	**top face clockwise**
three	**right face counterclockwise**

Following the Edge Piece Series the orange/blue piece is on the front/top edge (Fig. 5c) and the turn of the bottom half of the cube does not affect it (Fig. 5d).

Fig. 5c

Fig. 5d

If the two edge piece halves you are trying to recombine are across a cube face and on the same layer of the cube how can you bring them together? Rotate the face containing one of the edge pieces by 180^{o}. The two pieces are now on different layers and may be brought together by a 180^{o} rotation of half of the cube.

One thing you do not have to worry about is getting two edge piece halves together but not aligned. This can never happen. Although it has other problems the mechanics of Rubik's Revenge are such that, whenever the two halves of a given edge piece are brought together they are always in proper alignment.

If you carry out this rejoining of edge pieces as described above you will have to do it eight or nine times as you prepare for the final recombination. However, you should note that you introduced two edge piece halves just prior to the turn which restored the cube face centers. If you selected properly at least one of them would be in proper color alignment with the edge piece half which it joins. In that case you would be creating two "whole" edge pieces (and possibly three) each time you carry out this process.

Fig. 6a

Fig. 6b

How do you complete the process? You do it with three unaligned units remaining. The cube in Fig. 6a shows unmatched pairs at right/front, right/back and right/top. The two colors you cannot see at right/back are both white. A 90^{o} counterclockwise rotation of the bottom half of the cube brings the two yellow/white halves together on the right/front edge (Fig. 6b). The yellow/white edge piece is replaced by the unmatched edge piece pair at right/top. It is done in such a way that orange/white is in the second layer (Fig. 6c). Now when the opening turn of the bottom half of the cube is reversed the two orange/white halves are together and in proper alignment (Fig. 6d). So are the orange/yellow halves at right/front but you don't have to worry about them. They have no other choice.

Fig. 6c

Fig. 6d

You might make a mistake and overshoot the mark leaving only two unmatched edge piece pairs. How do you treat this situation? Arrange these unmatched pairs so that they are on facing cube edges with the identical edge piece halves on the same layer of the cube. If the two identical halves are on different layers then rotate a face containing one of the unmatched pairs by 180^{o}. The identical halves are now in the same layer. Rotate a cube half to bring two edge piece halves together. Remove this unmatched pair with an Edge Piece Series. Then replace this pair but invert it in the process. Now when the original turn of half a cube is reversed all edge pieces will be properly paired.

FINISHING THE CUBE

With the face centers in place and all edge pieces matched, Rubik's Revenge may be treated as a Rubik's Cube (3x3x3). This means that you turn the faces of the cube, never half a cube. Even though there are two parts to each edge piece they will respond to the Edge Piece Series in exactly the same way as the edge pieces in Rubik's Cube. And even though there are two rows of cubes between corner pieces (rather than one) the corners react to the Corner Piece Series in the same way.

We may find a problem when all of the edge pieces have been placed. One of the edge pieces may be inverted. If we were dealing with Rubik's Cube we would be tempted to believe that our cube had been sabotaged. But with Rubik's Revenge this problem developed when we placed the center pieces. There appears to be nothing we can do to prevent it. We will get it 50% of the time. We just need to learn how to deal with it.

According to Glenn Rhoads (recent e-mail) the inverted edge piece will show up whenever, following the completion of the face centers, the number of permutations of edge pieces required to put all in place is odd. Since the edge piece permutations occur in pairs, if you try to correct the inverted edge piece you will invert some other edge piece.

To eliminate this situation you must change the arrangement of some of the center face pieces. This change must be such as to give an odd number of permutations to the edge pieces. Andy Olsen (e-mail on 7/29/99) suggests that I should use <u>one</u> turn of half the cube. For example:

turn the bottom half of the cube 90^{o} clockwise

If orange is on the top and green in front then white will be on the right side (look at the cube in Fig. 6d). The 90^{o} turn of the bottom half of the cube will disrupt the green, white, yellow and blue faces leaving the orange and red faces intact. We must restore the faces <u>one at a time</u> using the "go and get" method described above. (We <u>cannot</u> simply turn the bottom half 90^{o} counterclockwise.) We are looking for a process which has an even number of turns of half a cube. When these are added to the first turn we just made we will always get an odd number of turns of a cube half. This will invert the problem edge piece. For example, the green face is restored by:

turn the right face 180°
turn the bottom half 90° clockwise
turn the right face 180°
turn the bottom half 90° counterclockwise

Always remember the face color which is to be restored next. It might be wise, whenever you carry out the edge piece inversion, to hold the cube in the same way and match up the same color first. In this case orange is on top and green is restored first. The next color restored will then be white. Now restore the yellow face (this step also restores the blue face). When you have completed this process you will find that only two edge pieces have been broken up. These are restored simultaneously using the method given above for two remaining unmatched edge pieces.

Most cubers will say that either a 90° rotation or a 180° rotation of half a cube (or of a cube face) is a single move. But that does not mean that they are identical when considering permutations or parity. With a Rubik's Revenge a 90° rotation of half a cube will invert an edge piece. A 180° rotation of half a cube will not. The latter is the same as two 90° rotations which means that the first inversion is followed by a second inversion and you are right back where you started from. If you would like to see a further discussion of permutations and parity as applied to various Rubik puzzles refer to Alan Hensel's site, How to Solve Almost Any Rubik-like Puzzle.

There is another potential problem. The 4x4x4 cube, like the 2x2x2 cube, can have six corners in place and two out of place. Or perhaps five are in place while two of the remaining pieces are out of position with the third in position but not properly oriented. But unlike the 2x2x2 cube you cannot get out of this dilemma by simply turning the top face. We will have to be more innovative. Again someone has sabotaged your cube. Not really. That's just the way things are.

To correct this situation rotate the top half of the cube by 180° and remove one of the unmatched edge pieces in the middle section of the cube with an Edge Piece Series. Now return that pair to the same position but inverted by another Edge Piece Series. Do the same thing with the unmatched pair on the opposite side of the cube (which looks just like the first pair). Rotate the top half by 180°.

None of the face centers or edge pieces have been broken. And two corner pieces and half of the edge pieces are still in place. Complete the cube in the standard manner for a Rubik's Cube.

Each time the center pieces are moved into place you will have a 50-50 chance of completing the edge pieces without having one of them inverted. And each time you complete the matching of the edge pieces you will have a 50-50 chance of placing all corner pieces in proper position and alignment. Therefore, 25% of the time you will be able to complete the cube without incident. For 25% of your attempts you will have to change only the center pieces and for another 25% you will have to change only the edge piece pairs. Unfortunately, for another 25% you will have to go through both processes.

How does this method compare to others? In general other methods avoid the two problems described above by completing the edges before the centers and the corners before the edges. But these methods are extremely complex. One needs to learn 50 to 60 new series, (for example, Minh Thai's Rubik's Revenge (Dell, 1982) and Denny's Rubik's Revenge Solution), and the total number of moves far exceed the number needed here.

Thai's method required an average of about 400 moves. Denny's required 476 moves the first time I tried it and 500 the second. The method described here requires an average of about 140 moves provided that neither of the problems described above occur. The correction for the inverted edge piece requires about 50 moves and that for the misplaced corner piece about 45

moves. Hence, even if you have to make both corrections the average is about 235 moves, far below either of the other methods. And you don't have to memorize any new series!

Rubik's Revenge is also not well suited for racing. Although it is more substantial than Rubik's Pocket Cube, some of the pieces are attached to the turning mechanism by thin plastic pins. If you do not have the various layers properly lined up one or more of these pieces may become stuck and a pin may snap if you try to force it.

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