Color Gauge Coupling in Dirac Bilinear Form

$$\mathcal{L}_{\text{int}} = -g\,\bar{\Psi}\gamma^{\mu}G_{\mu}\Psi$$

$$\Psi = \begin{bmatrix} \psi_{r1} & \psi_{r2} & \psi_{r3} & \psi_{r4} \\ \psi_{g1} & \psi_{g2} & \psi_{g3} & \psi_{g4} \\ \psi_{b1} & \psi_{b2} & \psi_{b3} & \psi_{b4} \end{bmatrix}^{T}$$

$$\bar{\Psi} = \Psi^{\dagger}\gamma^{0} = \begin{bmatrix} \psi_{r1}^{*} & \psi_{r2}^{*} & -\psi_{r3}^{*} & -\psi_{r4}^{*} \\ \psi_{g1}^{*} & \psi_{g2}^{*} & -\psi_{g3}^{*} & -\psi_{g4}^{*} \\ \psi_{b1}^{*} & \psi_{b2}^{*} & -\psi_{b3}^{*} & -\psi_{b4}^{*} \end{bmatrix}$$

$$\gamma^{\mu} = \begin{bmatrix} \gamma_{11}^{\mu} & \gamma_{12}^{\mu} & \gamma_{13}^{\mu} & \gamma_{14}^{\mu} \\ \gamma_{21}^{\mu} & \gamma_{22}^{\mu} & \gamma_{23}^{\mu} & \gamma_{24}^{\mu} \\ \gamma_{31}^{\mu} & \gamma_{32}^{\mu} & \gamma_{33}^{\mu} & \gamma_{34}^{\mu} \\ \gamma_{41}^{\mu} & \gamma_{42}^{\mu} & \gamma_{43}^{\mu} & \gamma_{44}^{\mu} \end{bmatrix}, \quad G_{\mu} = \begin{bmatrix} G_{\mu}^{11} & G_{\mu}^{12} & G_{\mu}^{13} \\ G_{\mu}^{21} & G_{\mu}^{22} & G_{\mu}^{23} \\ G_{\mu}^{31} & G_{\mu}^{32} & G_{\mu}^{33} \end{bmatrix}$$

$$\mathcal{L}_{\text{int}} = -g\,\bar{\Psi}_{i}^{\alpha}(\gamma^{\mu})_{\alpha\beta}(G_{\mu})^{ij}\Psi_{j}^{\beta}$$

$$\text{where } i, j \in \{\mathrm{r}, \mathrm{g}, \mathrm{b}\}, \quad \alpha, \beta \in \{1, 2, 3, 4\}$$

This can be written as a **color block matrix**:

$$\bar{\Psi}\gamma^{\mu}G_{\mu}\Psi = \begin{bmatrix} \bar{\psi}_{r\alpha} & \bar{\psi}_{g\alpha} & \bar{\psi}_{b\alpha} \end{bmatrix} \cdot \begin{bmatrix} \gamma_{\alpha\beta}^{\mu}G_{\mu}^{11} & \cdots & \gamma_{\alpha\beta}^{\mu}G_{\mu}^{13} \\ \vdots & \ddots & \vdots \\ \gamma_{\alpha\beta}^{\mu}G_{\mu}^{31} & \cdots & \gamma_{\alpha\beta}^{\mu}G_{\mu}^{33} \end{bmatrix} \cdot \begin{bmatrix} \psi_{r\beta} \\ \psi_{g\beta} \\ \psi_{b\beta} \end{bmatrix}$$

Color interactions mix flavors within and across color channels using $SU(3)$ gauge couplings G_{μ}^{ij} modulated by Lorentz γ^{μ} structure.

$$-g \begin{bmatrix} \psi_{r1}^{*} \\ \psi_{r2}^{*} \\ \psi_{r3}^{*} \\ \psi_{r4}^{*} \\ \psi_{g1}^{*} \\ \psi_{g2}^{*} \\ \psi_{g3}^{*} \\ \psi_{g4}^{*} \\ \psi_{b1}^{*} \\ \psi_{b2}^{*} \\ \psi_{b3}^{*} \\ \psi_{b4}^{*} \end{bmatrix}^{T} \begin{bmatrix} \gamma_{11}^{\mu}G_{\mu}^{11} & \cdots & \gamma_{14}^{\mu}G_{\mu}^{11} & \gamma_{11}^{\mu}G_{\mu}^{12} & \cdots & \gamma_{14}^{\mu}G_{\mu}^{12} & \gamma_{11}^{\mu}G_{\mu}^{13} & \cdots & \gamma_{14}^{\mu}G_{\mu}^{13} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \gamma_{41}^{\mu}G_{\mu}^{11} & \cdots & \gamma_{44}^{\mu}G_{\mu}^{11} & \gamma_{41}^{\mu}G_{\mu}^{12} & \cdots & \gamma_{44}^{\mu}G_{\mu}^{12} & \gamma_{41}^{\mu}G_{\mu}^{13} & \cdots & \gamma_{44}^{\mu}G_{\mu}^{13} \\ \gamma_{11}^{\mu}G_{\mu}^{21} & \cdots & \gamma_{14}^{\mu}G_{\mu}^{21} & \gamma_{11}^{\mu}G_{\mu}^{22} & \cdots & \gamma_{14}^{\mu}G_{\mu}^{22} & \gamma_{11}^{\mu}G_{\mu}^{23} & \cdots & \gamma_{14}^{\mu}G_{\mu}^{23} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \gamma_{44}^{\mu}G_{\mu}^{21} & \cdots & \gamma_{44}^{\mu}G_{\mu}^{21} & \gamma_{44}^{\mu}G_{\mu}^{22} & \cdots & \gamma_{44}^{\mu}G_{\mu}^{22} & \gamma_{44}^{\mu}G_{\mu}^{23} & \cdots & \gamma_{44}^{\mu}G_{\mu}^{23} \\ \gamma_{11}^{\mu}G_{\mu}^{31} & \cdots & \gamma_{14}^{\mu}G_{\mu}^{31} & \gamma_{11}^{\mu}G_{\mu}^{32} & \cdots & \gamma_{14}^{\mu}G_{\mu}^{32} & \gamma_{11}^{\mu}G_{\mu}^{33} & \cdots & \gamma_{14}^{\mu}G_{\mu}^{33} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \gamma_{44}^{\mu}G_{\mu}^{31} & \cdots & \gamma_{44}^{\mu}G_{\mu}^{31} & \gamma_{44}^{\mu}G_{\mu}^{32} & \cdots & \gamma_{44}^{\mu}G_{\mu}^{32} & \gamma_{44}^{\mu}G_{\mu}^{33} & \cdots & \gamma_{44}^{\mu}G_{\mu}^{33} \end{bmatrix} \begin{bmatrix} \psi_{r1} \\ \psi_{r2} \\ \psi_{r3} \\ \psi_{r4} \\ \psi_{g1} \\ \psi_{g2} \\ \psi_{g3} \\ \psi_{g4} \\ \psi_{b1} \\ \psi_{b2} \\ \psi_{b3} \\ \psi_{b4} \end{bmatrix}$$

$$\begin{bmatrix}
\gamma^{\mu}_{11}G^{11}_{\mu} & \gamma^{\mu}_{12}G^{11}_{\mu} & \gamma^{\mu}_{13}G^{11}_{\mu} & \gamma^{\mu}_{14}G^{11}_{\mu} & \gamma^{\mu}_{11}G^{12}_{\mu} & \gamma^{\mu}_{12}G^{12}_{\mu} & \gamma^{\mu}_{13}G^{12}_{\mu} & \gamma^{\mu}_{14}G^{12}_{\mu} & \gamma^{\mu}_{11}G^{13}_{\mu} & \gamma^{\mu}_{12}G^{13}_{\mu} & \gamma^{\mu}_{13}G^{13}_{\mu} & \gamma^{\mu}_{14}G^{13}_{\mu} \\
\gamma^{\mu}_{21}G^{11}_{\mu} & \gamma^{\mu}_{22}G^{11}_{\mu} & \gamma^{\mu}_{23}G^{11}_{\mu} & \gamma^{\mu}_{24}G^{11}_{\mu} & \gamma^{\mu}_{21}G^{12}_{\mu} & \gamma^{\mu}_{22}G^{12}_{\mu} & \gamma^{\mu}_{23}G^{12}_{\mu} & \gamma^{\mu}_{24}G^{12}_{\mu} & \gamma^{\mu}_{21}G^{13}_{\mu} & \gamma^{\mu}_{22}G^{13}_{\mu} & \gamma^{\mu}_{23}G^{13}_{\mu} & \gamma^{\mu}_{24}G^{13}_{\mu} \\
\gamma^{\mu}_{31}G^{11}_{\mu} & \gamma^{\mu}_{32}G^{11}_{\mu} & \gamma^{\mu}_{33}G^{11}_{\mu} & \gamma^{\mu}_{34}G^{11}_{\mu} & \gamma^{\mu}_{31}G^{12}_{\mu} & \gamma^{\mu}_{32}G^{12}_{\mu} & \gamma^{\mu}_{33}G^{12}_{\mu} & \gamma^{\mu}_{34}G^{12}_{\mu} & \gamma^{\mu}_{31}G^{13}_{\mu} & \gamma^{\mu}_{32}G^{13}_{\mu} & \gamma^{\mu}_{33}G^{13}_{\mu} & \gamma^{\mu}_{34}G^{13}_{\mu} \\
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\end{bmatrix}$$

$$\begin{matrix}
\gamma^{\mu}_{11}G^{11}_{\mu} & \gamma^{\mu}_{12}G^{11}_{\mu} & \gamma^{\mu}_{13}G^{11}_{\mu} & \gamma^{\mu}_{14}G^{11}_{\mu} & \gamma^{\mu}_{11}G^{12}_{\mu} & \gamma^{\mu}_{12}G^{12}_{\mu} & \gamma^{\mu}_{13}G^{12}_{\mu} & \gamma^{\mu}_{14}G^{12}_{\mu} & \gamma^{\mu}_{11}G^{13}_{\mu} & \gamma^{\mu}_{12}G^{13}_{\mu} & \gamma^{\mu}_{13}G^{13}_{\mu} & \gamma^{\mu}_{14}G^{13}_{\mu} \\
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\gamma^{\mu}_{11}G^{31}_{\mu} & \gamma^{\mu}_{12}G^{31}_{\mu} & \gamma^{\mu}_{13}G^{31}_{\mu} & \gamma^{\mu}_{14}G^{31}_{\mu} & \gamma^{\mu}_{11}G^{32}_{\mu} & \gamma^{\mu}_{12}G^{32}_{\mu} & \gamma^{\mu}_{13}G^{32}_{\mu} & \gamma^{\mu}_{14}G^{32}_{\mu} & \gamma^{\mu}_{11}G^{33}_{\mu} & \gamma^{\mu}_{12}G^{33}_{\mu} & \gamma^{\mu}_{13}G^{33}_{\mu} & \gamma^{\mu}_{14}G^{33}_{\mu} \\
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\end{matrix}$$

Vortex Velocity and Stream Function

$$V_\theta = \frac{\Gamma}{2\pi r}$$
$$V_\theta = -\frac{d\psi}{dr}$$
$$V_r = -\frac{1}{r}\frac{d\psi}{d\theta}$$

Stream function:

$$\psi = \frac{\Gamma}{2\pi}\ln(r)$$

Potential function:

$$\phi = \frac{\Gamma}{2\pi}\theta$$

Circulation:

$$\Gamma = \oint \mathbf{V} \cdot d\mathbf{s} = \iint_S (\boldsymbol{\nabla} \times \mathbf{v}) \cdot \hat{n}\, dS$$

$$\Gamma = \iint (u\,dx + v\,dy) = \int V\cos(\phi)\,ds$$

Boyle's Law for Ideal Gases (Kinetic Theory)

$$pV = \frac{1}{3}T - \frac{1}{6}\rho \iiint (u^2 + v^2 + w^2)(x\,dy\,dz + y\,dz\,dx + z\,dx\,dy)$$

Newton's Potential around the Sun

$$\Phi = -\frac{GM}{r} + \frac{1}{6}\Delta c^2 r^2$$

Vacuum Energy Density Estimate

$$\frac{1}{V}\int \frac{1}{2}\hbar\omega \approx \frac{\hbar}{2\pi^2 c^3}\int_0^{\omega_{\max}} \omega^3 d\omega = \frac{\hbar}{2\pi^2 c^3}\omega_{\max}^4$$

Cosmological constant:

$$\Lambda \approx \frac{G^2 m^6}{\hbar^4}$$

Triple Integral Volume Identity

$$\int_S (x\,dy\,dz + y\,dz\,dx + 2z\,dx\,dy) = \iiint (1+1+1)\,dx\,dy\,dz = 3\int_0^1 dx \int_0^1 dy \int_0^1 dz = \boxed{3}$$