# Golden Rapidity and Tangential Velocity in VAM

## Setup

Let the golden ratio be

$$\varphi \equiv \frac{1+\sqrt{5}}{2}, \qquad \varphi^2 = \varphi + 1. \tag{1}$$

Define the *golden rapidity*

$$\xi_g \;\equiv\; \frac{3}{2}\ln\varphi. \tag{2}$$

We use the standard hyperbolic functions (definitions in [**?**]).

## Identity: $\tanh(\xi_g) = 1/\varphi$

Using $\tanh y = \frac{e^{2y}-1}{e^{2y}+1}$, substitute $y = \xi_g$ to obtain

$$\tanh(\xi_g) \;=\; \frac{e^{3\ln\varphi}-1}{e^{3\ln\varphi}+1} \;=\; \frac{\varphi^3-1}{\varphi^3+1}. \tag{3}$$

From $\varphi^2 = \varphi + 1$ it follows $\varphi^3 = \varphi(\varphi+1) = 2\varphi + 1$. Hence

$$\tanh(\xi_g) \;=\; \frac{(2\varphi+1)-1}{(2\varphi+1)+1} \;=\; \frac{2\varphi}{2(\varphi+1)} \;=\; \frac{\varphi}{\varphi+1} \;=\; \frac{\varphi}{\varphi^2} \;=\; \frac{1}{\varphi}. \tag{4}$$

Therefore

$$\boxed{\tanh\left(\tfrac{3}{2}\ln\varphi\right) = \frac{1}{\varphi}} \qquad \Longleftrightarrow \qquad \boxed{\coth\left(\tfrac{3}{2}\ln\varphi\right) = \varphi}. \tag{5}$$

## VAM Mapping to Tangential Velocity

In a rapidity parametrization, the dimensionless speed is

$$\beta \;\equiv\; \frac{v}{C_e} \;=\; \tanh \xi. \tag{6}$$

Setting $\xi = \xi_g$ gives the *golden* tangential fraction

$$\beta_g \;=\; \tanh(\xi_g) \;=\; \frac{1}{\varphi}, \tag{7}$$

and thus a characteristic tangential velocity and swirl frequency

$$v_g \;=\; \frac{C_e}{\varphi}, \qquad \Omega_g \;=\; \frac{v_g}{r_c} \;=\; \frac{1}{\varphi}\,\frac{C_e}{r_c}. \tag{8}$$

Both are dimensionally consistent: $v_g$ has units of m/s and $\Omega_g$ of $\mathrm{s}^{-1}$.

## Numerical Validation (User Constants)

Using $C_e = 1\,093\,845.63\,\mathrm{m/s}$ and $r_c = 1.408\,970\,17 \times 10^{-15}\,\mathrm{m}$,

$$\varphi \approx 1.618033988749895, \tag{9}$$
$$\xi_g = \tfrac{3}{2}\ln\varphi \approx 0.721817737589405, \tag{10}$$
$$\beta_g = \tanh\xi_g \approx 0.618033988749895 \;=\; \frac{1}{\varphi}, \tag{11}$$
$$v_g = \frac{C_e}{\varphi} \approx 6.760\,337\,777\,855\,416 \times 10^{5}\,\mathrm{m/s}, \tag{12}$$
$$\Omega = \frac{C_e}{r_c} \approx 7.763\,440\,655\,383\,073 \times 10^{20}\,\mathrm{s}^{-1}, \tag{13}$$
$$\Omega_g = \frac{\Omega}{\varphi} \approx 4.798\,070\,194\,669\,498 \times 10^{20}\,\mathrm{s}^{-1}. \tag{14}$$

**Consistency checks.** Since $\beta_g = 1/\varphi$, we have $v_g = C_e/\varphi$ and $\Omega_g = \Omega/\varphi$ exactly, up to machine precision in floating-point arithmetic.

## Discussion

This construction supplies a natural, dimensionless benchmark ($1/\varphi$) for tangential speeds in VAM. If swirl states quantize in hyperbolic angle $\xi$, the golden rapidity $\xi_g$ defines a preferred scaling layer where tangential velocity and core swirl frequency are reduced by a factor $\varphi$ relative to their maxima, potentially useful for defining stable vortexknot operating points or resonance bands in the spectrum.