Governing Equations of Vorticity in Æther Dynamics

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Abstract

This document presents a mathematical treatment of vorticity and time structure within the framework of the Vortex Æther Model (VAM), a fluid-dynamic reformulation of gravitation and temporal evolution. It introduces the core vorticity equation in natural coordinates, derived from the dynamics of an incompressible, inviscid æther medium. By interpreting vorticity as a measure of atomic clock delay, we couple swirl energy to experienced time and obtain expressions for vorticity in terms of local velocity gradients and flow curvature.

The appendices develop key substructures of VAM, including the topological and energetic conditions that trigger irreversible events, called Kairos moments, where the flow evolution becomes non-analytic. These singularities partition æther-time into epochs and correspond to phenomena such as vortex reconnection, swirl pressure rupture, or helicity discontinuities. Each temporal mode—Aithēr-Time ($\mathcal{N}$), Chronos-Time (τ), Swirl Clock ($S(t)$), and Vortex Proper Time (T_v)—is formally defined and related to physical vortex observables.

Altogether, the work establishes a temporally stratified æther theory with testable dynamical laws, offering an alternative to spacetime curvature by grounding time dilation and mass interactions in structured vorticity fields.

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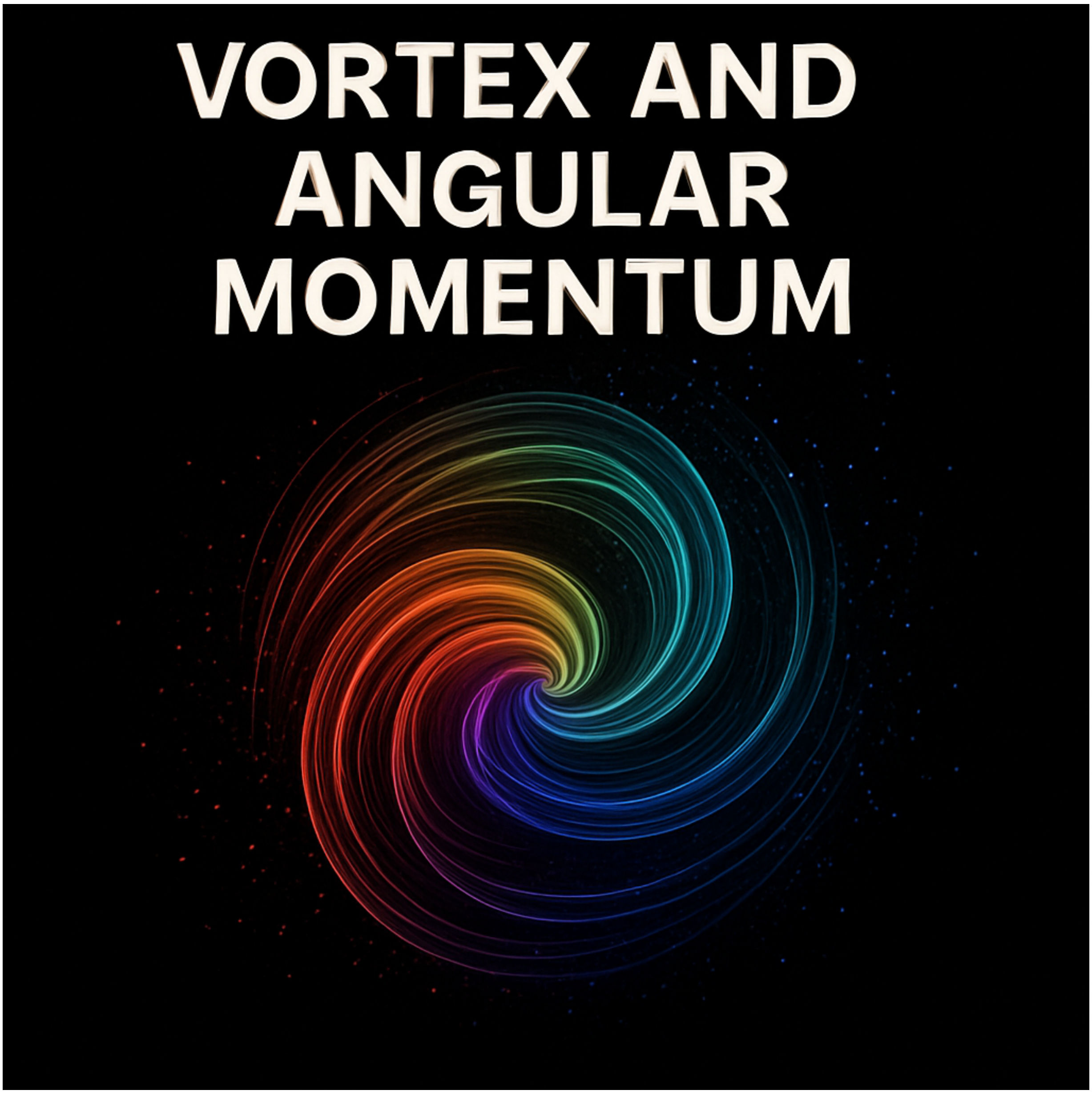


Figure 1: Universal time dilation formula in the Vortex Æther Model. The clock rate decreases with increasing relative velocity $|\vec{v}_{\text{rel}}|$ with respect to the æther. At $|\vec{v}_{\text{rel}}| = c$ time stops.

A Governing Equations of Vorticity in Æther Dynamics

Fundamental Assumptions

The Æther is modeled as a homogeneous, incompressible, and inviscid fluid. This implies constant density:

$$\frac{d\rho}{dt} = 0. \tag{1}$$

We adopt a Cartesian coordinate system (x, y, z) fixed in absolute space. The velocity field $\vec{u} = (u, v, w)$ represents the local Æther flow:

$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}, \quad w = \frac{dz}{dt}. \tag{2}$$

Let P denote pressure, and (X, Y, Z) the external force per unit volume acting in each direction.

Stress Equilibrium in Free Æther

The general stress equilibrium equations are:

$$X = \frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{xy}}{\partial y} + \frac{\partial P_{xz}}{\partial z}, \tag{3}$$

$$Y = \frac{\partial P_{yx}}{\partial x} + \frac{\partial P_{yy}}{\partial y} + \frac{\partial P_{yz}}{\partial z}, \tag{4}$$

$$Z = \frac{\partial P_{zx}}{\partial x} + \frac{\partial P_{zy}}{\partial y} + \frac{\partial P_{zz}}{\partial z}. \tag{5}$$

Assuming irrotational flow, all shear stresses vanish:

$$P_{xy} = P_{xz} = P_{yz} = 0. \tag{6}$$

Hence the simplified stress force equations become:

$$X = \frac{\partial P_{xx}}{\partial x}, \quad Y = \frac{\partial P_{yy}}{\partial y}, \quad Z = \frac{\partial P_{zz}}{\partial z}. \tag{7}$$

The total force differential satisfies:

$$X\,dx + Y\,dy + Z\,dz = dV, \tag{8}$$

where V is a scalar potential.

Normal stresses are related to flow and pressure:

$$P_{xx} = \rho u^2 - P, \tag{9}$$

$$P_{yy} = \rho v^2 - P, \tag{10}$$

$$P_{zz} = \rho w^2 - P. \tag{11}$$

Substituting into the momentum equation yields:

$$X = \frac{Du}{Dt} + \frac{1}{\rho}\frac{\partial P}{\partial x}, \tag{12}$$

$$Y = \frac{Dv}{Dt} + \frac{1}{\rho}\frac{\partial P}{\partial y}, \tag{13}$$

$$Z = \frac{Dw}{Dt} + \frac{1}{\rho}\frac{\partial P}{\partial z}, \tag{14}$$

where the material derivative is:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u} \cdot \nabla. \tag{15}$$

Continuity Equation

For an incompressible fluid:

$$\nabla \cdot \vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{16}$$

Velocity Potential and Irrotational Flow

If the flow is irrotational, a scalar potential φ exists such that:

$$\vec{u} = \nabla \varphi. \tag{17}$$

This leads to the Laplace equation:

$$\nabla^2 \varphi = 0. \tag{18}$$

Vorticity and Circulation

In irrotational flow, the vorticity vector vanishes:

$$\vec{\omega} = \nabla \times \vec{u} = 0. \tag{19}$$

In rotational flow, the components become:

$$\omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \tag{20}$$

$$\omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \tag{21}$$

$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \tag{22}$$

The circulation Γ over an infinitesimal closed loop is:

$$\Gamma = \oint \vec{u} \cdot d\vec{l} = \iint (\nabla \times \vec{u}) \cdot \hat{n} \, dA. \tag{23}$$

Energy of a Vortex

The kinetic energy of a rotating vortex region is:

$$E = \frac{1}{2} \rho \iiint |\vec{u}|^2 \, dV. \tag{24}$$

Assuming solid-body rotation and edge tangential velocity c, the vorticity magnitude is:

$$|\vec{\omega}| = \frac{c}{r}. \tag{25}$$

Then the vortex energy simplifies to:

$$E = \frac{1}{2} M c^2, \tag{26}$$

where M is the effective mass of the rotating Æther parcel.

Conclusion

This appendix establishes the fluid-dynamic foundations of Æther theory under assumptions of incompressibility and inviscidity. It introduces the vorticity vector, velocity potential, and circulation as key constructs, which serve as a basis for gravitational analogs, time dilation, and vortex dynamics in the broader VAM framework.

B Governing Equations of Vorticity in Æther Dynamics

Vortex Pressure Relations

In a steadily rotating vortex tube, let the core pressure be P_0. The pressure at the vortex edge P_1 is:

$$P_1 = P_0 + \frac{1}{2}\rho c^2, \tag{27}$$

where ρ is the Æther density and c the tangential velocity at the vortex edge.

The axial pressure parallel to the vortex tube is:

$$P_2 = P_0 + \frac{1}{4}\rho c^2. \tag{28}$$

The transverse pressure difference becomes:

$$P_1 - P_2 = \frac{1}{4}\rho c^2. \tag{29}$$

For irrotational vortices where pressure arises from distributed angular momentum (e.g. as in swirl clocks or quantized vortices), this generalizes to:

$$P_1 - P_2 = N\rho c^2, \tag{30}$$

with N a coefficient dependent on angular profile and vortex density.

Stress Tensor Components

Define vortex orientation via direction cosines l, m, n with respect to the (x, y, z) axes.

The full stress tensor becomes:

$$P_{xx} = \rho c^2 l^2 - P_1, \qquad P_{xy} = \rho c^2 lm, \qquad P_{xz} = \rho c^2 ln, \tag{31}$$
$$P_{yy} = \rho c^2 m^2 - P_1, \qquad P_{yz} = \rho c^2 mn, \qquad P_{yx} = P_{xy}, \tag{32}$$
$$P_{zz} = \rho c^2 n^2 - P_1, \qquad P_{zx} = \rho c^2 nl, \qquad P_{zy} = P_{yz}. \tag{33}$$

If velocity components are defined by:

$$u = cl, \quad v = cm, \quad w = cn, \tag{34}$$

then the stress tensor rewrites as:

$$P_{xx} = \rho u^2 - P_1, \qquad P_{xy} = \rho uv, \qquad P_{xz} = \rho uw, \tag{35}$$
$$P_{yy} = \rho v^2 - P_1, \qquad P_{yz} = \rho vw, \qquad P_{yx} = \rho vu, \tag{36}$$
$$P_{zz} = \rho w^2 - P_1, \qquad P_{zx} = \rho wu, \qquad P_{zy} = \rho wv. \tag{37}$$

Equilibrium of Stresses and Force Components

The force per unit volume follows the momentum balance:

$$X = \frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{xy}}{\partial y} + \frac{\partial P_{xz}}{\partial z}, \tag{38}$$

$$Y = \frac{\partial P_{yx}}{\partial x} + \frac{\partial P_{yy}}{\partial y} + \frac{\partial P_{yz}}{\partial z}, \tag{39}$$

$$Z = \frac{\partial P_{zx}}{\partial x} + \frac{\partial P_{zy}}{\partial y} + \frac{\partial P_{zz}}{\partial z}. \tag{40}$$

Using the velocity substitution, the identity:

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x} = \frac{1}{2}\frac{\partial}{\partial x}(u^2 + v^2 + w^2)$$

leads to:

$$X = \frac{1}{2}\rho\frac{\partial}{\partial x}(c^2) + u\rho(\nabla \cdot \vec{u}) - \rho v(2\zeta) + \rho w(2\eta) - \frac{\partial P_1}{\partial x}, \tag{41}$$

$$Y = \frac{1}{2}\rho\frac{\partial}{\partial y}(c^2) + v\rho(\nabla \cdot \vec{u}) - \rho w(2\xi) + \rho u(2\zeta) - \frac{\partial P_1}{\partial y}, \tag{42}$$

$$Z = \frac{1}{2}\rho\frac{\partial}{\partial z}(c^2) + w\rho(\nabla \cdot \vec{u}) - \rho u(2\eta) + \rho v(2\xi) - \frac{\partial P_1}{\partial z}. \tag{43}$$

Connection to Vorticity and Coriolis Acceleration

The terms involving:

$$\frac{1}{2}\rho\frac{\partial}{\partial x}(u^2 + v^2 + w^2) \tag{44}$$

can be interpreted as a **Coulomb-like acceleration** from local kinetic energy gradients.

The cross terms:

$$-v(2\zeta) + w(2\eta), \tag{45}$$

represent **Coriolis-type accelerations** in the rotating æther due to vorticity components in the transverse plane.

Conclusion

This derivation exposes the deeper link between internal vortex pressure gradients and inertial forces resulting from vorticity. The decomposition of stresses into axial and transverse components reveals how Coriolis accelerations and pressure anisotropies arise naturally in rotating æther systems. These structures provide the mechanical foundation for VAM's time dilation, mass generation, and vortex-induced gravitational fields.

Example: Vortex Pressure Drop

Assuming:

$$\rho = 7 \times 10^{-7}\,\mathrm{kg/m^3}, \quad c = C_e = 1.09384563 \times 10^6\,\mathrm{m/s}$$

We compute:

$$P_1 - P_0 = \frac{1}{2}\rho c^2 = \boxed{418{,}774.39\,\mathrm{Pa}}, \qquad P_1 - P_2 = \frac{1}{4}\rho c^2 = \boxed{209{,}387.20\,\mathrm{Pa}}$$

This anisotropic pressure difference supports the vortex stability and creates a radial pressure gradient consistent with centripetal balance.

C Governing Equations of Vorticity in Æther Dynamics

C.1 Vorticity in Natural Coordinates

We define $d\omega$ as the experienced time rate for atoms moving along natural coordinates in a vorticity field. Let us consider a central æther particle located at the core of a vortex, having no velocity potential. It satisfies:

$$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 2\xi, \quad \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 2\eta, \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 2\zeta. \tag{46}$$

Since the æther particle remains fixed at the center, its local rotation is only about the Z-axis, leading to:

$$\xi = 0, \quad \eta = 0, \quad \zeta = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right). \tag{47}$$

We interpret this component of vorticity as the **rate of experienced time**, i.e., the swirl clock rate. It corresponds to the rotation of the vortex core.

We now introduce stream-aligned coordinates with tangent and normal directions denoted by unit vectors:

$$\hat{s} = (\cos\theta, \sin\theta), \quad \hat{n} = (-\sin\theta, \cos\theta),$$

so that:

$$\hat{s}_x = \cos\theta, \quad \hat{n}_x = -\sin\theta, \tag{48}$$

$$\hat{s}_y = \sin\theta, \quad \hat{n}_y = \cos\theta. \tag{49}$$

If the velocity vector has magnitude V, then:

$$u = V\cos\theta, \quad v = V\sin\theta. \tag{50}$$

Differentiating u and v gives:

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(V\sin\theta) = \frac{\partial V}{\partial x}\sin\theta + V\frac{\partial \sin\theta}{\partial x}, \tag{51}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(V\cos\theta) = \frac{\partial V}{\partial y}\cos\theta + V\frac{\partial \cos\theta}{\partial y}. \tag{52}$$

Then the vorticity in the z-direction is:

$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{53}$$

$$= \frac{\partial V}{\partial x}\sin\theta - \frac{\partial V}{\partial y}\cos\theta + V\left(\frac{\partial \sin\theta}{\partial x} - \frac{\partial \cos\theta}{\partial y}\right). \tag{54}$$

Transforming to streamline coordinates, we get:

$$\vec{\omega} = -\frac{\partial V}{\partial \eta} + V\frac{\partial \theta}{\partial s}. \tag{55}$$

Using the definition of the radius of curvature:

$$R = \frac{ds}{d\theta} = \left(\frac{d\theta}{ds}\right)^{-1}, \tag{56}$$

and assuming constant velocity $dV = 0$, the vorticity simplifies to:

$$\boxed{\vec{\omega} = \frac{V}{R}}. \tag{57}$$

This shows that **vorticity is proportional to the curvature of the flow path**, and hence the local rotation experienced by atoms. In VAM, this angular rotation governs the **rate of experienced time**, making $\vec{\omega}$ the physical clock hand of the æther medium.

D Governing Equations of Vorticity in Æther Dynamics

The governing equations of vortex dynamics in an idealized fluid system constitute a fundamental framework in contemporary theoretical and applied physics. These equations, rigorously derived from foundational principles in classical mechanics and continuum physics, provide profound insights into a broad spectrum of physical phenomena. By integrating vorticity fields, energy dissipation mechanisms, and entropy dynamics, these formulations extend beyond conventional applications, enabling high-fidelity analyses of macroscopic fluid behaviors and their microscopic analogs within the context of Æther Physics. This synthesis offers an unparalleled theoretical foundation for examining complex interactions, bridging domains from geophysical fluid dynamics to quantum mechanical interpretations of turbulence.

Symbol	Description	Unit	VAM Interpretation
u, v, w	Velocity components in x, y, z directions	m/s	Æther flow vector field
ζ	Relative vorticity $= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$	s^{-1}	Local fluid rotation rate
ζ_a	Absolute vorticity $= \zeta + f$	s^{-1}	Total vorticity (includes Coriolis)
f	Coriolis parameter $= 2\omega \sin(\theta)$	s^{-1}	Rotation due to background frame
ω	Planetary or core angular velocity	rad/s	Frame or vortex rotation rate
$\vec{\omega}$	Vorticity vector field	s^{-1}	Curl of the velocity field: $\nabla \times \vec{v}$
R	Radius of curvature of streamlines	m	Local geometric curvature of vortex
V	Tangential swirl velocity	m/s	Clock-hand velocity around vortex
Π	Potential vorticity $= \frac{f_a + \zeta_r}{h}$	s^{-1}	Conserved in barotropic VAM flows
h	Column height (layer thickness)	m	Local æther depth in height-varying flows
ψ	Streamfunction ($\zeta = \nabla^2 \psi$)	m^2/s	Encodes flow via level curves
ϕ	Scalar potential	m^2/s^2	Source-based potential (e.g. gravity)
$\mathcal{R}_x, \mathcal{R}_y$	Forcing terms (e.g., turbulence, friction)	m/s^2	External interaction effects
$J(\psi, \nabla^2 \psi)$	Jacobian term	m^2/s^3	Nonlinear advection in 2D GFD
$H = \int \vec{v} \cdot \vec{\omega} \, dV$	Helicity	m^4/s^2	Topological twist/linkage of vortex lines

Table 1: Glossary of symbols used in VAM vortex dynamics equations.

Fundamental Equations of Vortex Dynamics

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

This equation enforces the incompressibility constraint in ideal fluid dynamics, ensuring conservation of mass. The divergence-free condition of the velocity field is essential for characterizing both naturally occurring and engineered fluid flows, preserving volumetric consistency throughout the domain.

Momentum Conservation

$$\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x} = \frac{1}{2}\frac{\partial (u^2 + v^2 + w^2)}{\partial x}$$

This equation delineates the redistribution of momentum within a dynamic fluid system, elucidating the interplay between velocity gradients and pressure variations.

Definition of Vorticity

$$u = x\omega, \quad \nu = 0 \tag{58}$$

$$f = 2\omega, \quad \zeta = -\alpha \tag{59}$$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{60}$$

Vorticity quantifies the local rotational characteristics of a fluid element and serves as a fundamental diagnostic parameter for analyzing turbulence, circulation, and eddy formation.

Absolute and Relative Vorticity

$$\zeta_{\text{Absolute}} = f_{\text{atom}} + \zeta_{\text{relative}} \tag{61}$$
$$f_{\text{atom}} = 2\omega \sin(\theta) \tag{62}$$
$$\zeta_{\text{relative}} = \frac{dv}{dx} - \frac{du}{dy} \tag{63}$$

Absolute vorticity incorporates planetary rotation effects through the Coriolis parameter and integrates them with local vorticity contributions.

Energy-Entropy Relationship

$$\Pi = \frac{f_a + \zeta_r}{h}$$

This formulation establishes a bridge between vorticity dynamics and thermodynamic fluxes, providing a robust mechanism for quantifying entropy generation.

Poisson's Equation for Scalar Potential

$$\nabla^2 \phi = -4\pi\rho \tag{64}$$
$$\frac{\delta^2 \phi}{\delta x^2} + \frac{\delta^2 \phi}{\delta y^2} + \frac{\delta^2 \phi}{\delta z^2} = -4\pi\rho \tag{65}$$

This equation governs the scalar potential arising from mass density distributions.

Energy and Momentum Conservation in Vortical Systems

$$\rho \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - \zeta_{\text{atom}} v \right) = -\frac{\partial p}{\partial x} + r_x \tag{66}$$
$$p = \rho g(\eta z) \tag{67}$$

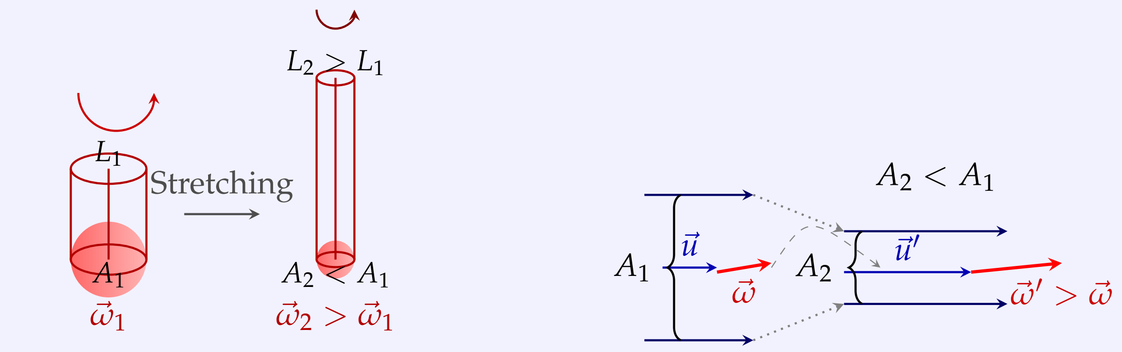
These equations encapsulate the intricate force and momentum interactions within vortex-dominated regimes.

Helicity and Topological Constraints

$H = \int \vec{v} \cdot \vec{\omega} \, dV$ Helicity, a measure of the linkage and knottedness of vortex lines, serves as a conserved quantity in idealized flows. This conservation underpins the study of topological invariants in fluid mechanics and their extensions into quantum fluids and plasmas.

Vortex Stretching Derivation

Vortex Stretching in Inviscid Æther Flow



(a) Cylindrical representation of vortex stretching. **(b)** Streamline-based view of vortex filament stretching.

Figure 2: Two perspectives on vortex stretching in incompressible flow: (a) cylindrical vortex tube conservation, (b) streamline deformation and induced vorticity growth.

In incompressible, inviscid flow, the evolution of vorticity $\vec{\omega} \equiv \nabla \times \vec{u}$ is governed by deformation of vortex lines due to velocity gradients. Starting from the inviscid Navier–Stokes equation:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}\nabla p,$$

we take the curl of both sides:

$$\frac{\partial}{\partial t}(\nabla \times \vec{u}) + \nabla \times [(\vec{u} \cdot \nabla)\vec{u}] = 0.$$

Recognizing the vorticity $\vec{\omega} = \nabla \times \vec{u}$, and applying the identity:

$$\nabla \times [(\vec{u} \cdot \nabla)\vec{u}] = (\vec{\omega} \cdot \nabla)\vec{u} - (\vec{u} \cdot \nabla)\vec{\omega},$$

we derive the vorticity transport equation:

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{u} \cdot \nabla)\vec{\omega} = (\vec{\omega} \cdot \nabla)\vec{u},$$

which simplifies using the material derivative:

$$\boxed{\frac{D\vec{\omega}}{Dt} = (\vec{\omega} \cdot \nabla)\vec{u}} \tag{68}$$

Interpretation: The right-hand side represents the *vortex stretching term*. It governs the increase in vorticity magnitude when a vortex filament is stretched by the flow. When a velocity gradient aligns with the direction of vorticity, angular momentum conservation causes the rotation rate to intensify.

Analogy: Pulling on a spinning elastic band makes it spin faster. Vortex filaments behave similarly under æther flow deformation.

Reference: G. K. Batchelor, *An Introduction to Fluid Dynamics*, Cambridge University Press, 2000.

Derivation of Vorticity-Based Fluid Equations

The equation:

$$\frac{du}{dt} + u\frac{du}{d\boldsymbol{x}} + v\frac{du}{dy} - \zeta_{\text{atom}}v = -g\frac{d\eta}{d\boldsymbol{x}} + \mathcal{R}_x$$

is a form of the **momentum equation** for the velocity component u, incorporating vorticity, gravity effects, and external forcing terms.

- **Material Derivative** $\frac{du}{dt}$: Represents the total derivative (substantial derivative) following a fluid parcel.
- **Convective Terms** $u\frac{du}{dx} + v\frac{du}{dy}$: Describe how velocity gradients impact acceleration.
- **Vorticity Term** $-\zeta_{\text{atom}}v$: Arises from the influence of vorticity on velocity evolution.
- **Gravity-Induced Term** $-g\frac{d\eta}{dx}$: Represents pressure gradient due to gravity.
- **External Forcing Term** $\mathcal{R}_x$: Represents additional external forces such as resistive or turbulent effects.

This equation is derived from the **Navier-Stokes Equations** under the assumption of an inviscid, incompressible fluid with rotational effects.

Differentiation with Respect to y

Differentiating the equation with respect to y:

$$\frac{d}{dy}\left(\frac{du}{dt} + u\frac{du}{dx} + v\frac{du}{dy} - \zeta_{\text{atom}}v\right) = \frac{d}{dy}\left(-g\frac{d\eta}{dx} + \mathcal{R}_x\right) \tag{69}$$

Expanding this:

$$\frac{d^2u}{dtdy} + \frac{du}{dy}\frac{du}{dx} + u\frac{d^2u}{dxdy} + \frac{dv}{dy}\frac{du}{dy} + v\frac{d^2u}{dy^2} - \zeta_a\frac{dv}{dy} - \beta v = -g\frac{d^2\eta}{dxdy} + \frac{d\mathcal{R}_x}{dy} \tag{70}$$

Similarly, differentiating the equation for v:

$$\frac{dv}{dt} + u\frac{dv}{dx} + v\frac{dv}{dy} + \zeta_{\text{atom}}\, u = -g\frac{d\eta}{dy} + \mathcal{R}_v$$

Differentiating with respect to x:

$$\frac{d^2v}{dtdx} + \frac{du}{dx}\frac{dv}{dx} + u\frac{d^2v}{dx^2} + \frac{dv}{dx}\frac{dv}{dy} + v\frac{d^2v}{dxdy} + \zeta_a\frac{du}{dx} = -g\frac{d^2\eta}{dxdy} + \frac{d\mathcal{R}_w}{dx} \tag{71}$$

Combination of the Two Equations

By adding both derived equations, we get:

$$\frac{\delta\zeta}{\delta t} + \zeta\frac{du}{dx} + u\frac{\delta\zeta}{\delta x} + \zeta\frac{dv}{dy} + v\frac{\delta\zeta}{\delta y} + \zeta_a\left(\frac{du}{dx} + \frac{dv}{dy}\right) + \beta v = \frac{d\mathcal{R}_v}{dx} - \frac{d\mathcal{R}_x}{dy} \tag{72}$$

which is a vorticity-based formulation of the original momentum equations.

Representation of Forcing Terms

In the presence of external forcing and turbulence:

$$\mathcal{R}_x = \frac{1}{\rho}(\tau_x^w - \tau_x^v) \tag{73}$$

$$\mathcal{R}_y = \frac{1}{\rho}(\tau_y^w - \tau_y^b) \tag{74}$$

where $\tau_x^w, \tau_x^v, \tau_y^w, \tau_y^b$ represent the stress terms.

Final Vorticity Equation

$$\frac{D\zeta}{dt} - \frac{\zeta_r + \zeta_a}{h}\frac{Dh}{dt} + \frac{D\zeta_a}{dt} = \frac{dR_u}{dx} - \frac{dR_x}{dy} \tag{75}$$

This equation models higher-order vortex interactions, crucial for understanding turbulence, energy dissipation, and wave-vortex interactions.

Conclusion

The derivation follows classical fluid dynamics principles and extends into turbulence modeling. These equations are significant in vortex dynamics, superfluid behavior, and atmospheric circulations. They also appear in various studies on vortex ring dynamics.

Governing Vorticity Transport Equation

The fundamental vorticity equation is:

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + (\zeta_r + \zeta_a)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \beta v = \frac{\partial \mathcal{R}_v}{\partial x} - \frac{\partial \mathcal{R}_x}{\partial y}$$

where:

- ζ is the **relative vorticity**.
- ζ_r and ζ_a represent **relative and absolute vorticity contributions**.
- βv is the **beta-effect**, modeling the variation of planetary vorticity with latitude.
- $\mathcal{R}_x, \mathcal{R}_y$ are external forcing terms, such as frictional forces or turbulence-induced vorticity changes.

Vorticity in Height-Dependent Flow

$$\frac{D\zeta}{dt} - \frac{\zeta_r + \zeta_a}{h}\frac{Dh}{dt} + \frac{D\zeta_a}{dt} = \frac{\partial \mathcal{R}_y}{\partial x} - \frac{\partial \mathcal{R}_x}{\partial y}$$

This ensures vorticity conservation even in **variable-height flows**, such as oceanic or atmospheric circulations.

Barotropic Vorticity Equation and Potential Vorticity

$$D\left(\frac{\zeta_r + \zeta_a}{h}\right) = \frac{1}{h}\left(\frac{dR_y}{dx} - \frac{dR_x}{dy}\right)$$

The **Potential Vorticity (PV)** is conserved:

$$\Pi = \frac{f_a + \zeta_r}{h}$$

This is crucial for **understanding Rossby waves, planetary circulation, and stratified fluid dynamics**.

Relationship to Streamfunction

$$\zeta = \nabla^2 \psi$$

The vorticity field is linked to the **streamfunction** through the Laplacian operator.

Absolute Vorticity and Coriolis Terms

$$f = 2\omega, \quad \zeta = -\alpha \tag{76}$$

$$f_{\text{atom}} = 2\omega \sin(\theta) \tag{77}$$

$$\zeta_{\text{Absolute}} = f_{\text{atom}} + \zeta_{\text{relative}} \tag{78}$$

Absolute vorticity is the sum of relative vorticity and the Coriolis parameter.

Conservation of Vorticity

$$\frac{D\zeta}{Dt} = 0 = \frac{\partial \zeta}{\partial t} + u \cdot \nabla \zeta$$

In an inviscid flow, vorticity is conserved along streamlines.

$$\frac{\partial \zeta}{\partial t} + u \cdot \nabla(\zeta + f) = 0$$

$$\frac{\partial \zeta}{\partial t} + J(\psi, \nabla^2 \psi) = 0$$

This is used in **geophysical fluid dynamics**, where the Jacobian term represents nonlinear advection of vorticity.

Conclusion

These equations describe the **evolution of vorticity in a rotating fluid with height variations and external forcing effects**. They are foundational for:

- Geophysical fluid dynamics (GFD).
- Turbulence modeling.
- Vortex dynamics in atmospheric and oceanic flows.

This framework allows for **wave-vortex interactions**, barotropic/baroclinic instabilities, and the development of **cyclonic systems**.

E Governing Equations of Vorticity in Æther Dynamics

Temporal Coupling Between Rotational and Translational Æther Dynamics

Rigid Rotor Dynamics: Each vortex knot is modeled as a rigidly rotating entity, maintaining a stable angular velocity throughout its core under Vortex Proper Time T_v. These cores are assumed to deform minimally, preserving their rotation under ideal conditions.

Vorticity as a Vector Field: The vorticity vector for each knot is aligned with the Z-axis:

$$\vec{\omega} = \omega \hat{z},$$

which simplifies analysis and reflects cylindrical symmetry in the ætheric vortex tube.

Kinematic Parameters:

- **Spatial positions:** Knots are located at z_1 and z_2 along the Z-axis.
- **Axial velocities (Chronos-Time):**

$$v_1 = \frac{dz_1}{d\tau_1}, \quad v_2 = \frac{dz_2}{d\tau_2}.$$

- **Relative velocity:**

$$v_{\text{rel}} = \frac{d(z_2 - z_1)}{d\mathcal{N}},$$

  which measures spatial separation rate in absolute Aithēr-Time $\mathcal{N}$.

Vortex Tube Structure: A connecting vortex tube with uniform vorticity transmits angular momentum along z, coupling the two knots dynamically.

Æther Properties: The surrounding æther is modeled as incompressible and inviscid, enabling conservative transmission of vorticity and swirl pressure without dissipative losses.

Derivation of Relative Vorticity

Vorticity Difference:

$$\Delta\omega = \omega_2 - \omega_1,$$

where each angular velocity evolves along its respective vortex in proper time:

$$\omega_1 = \frac{d\theta_1}{dT_{v1}}, \quad \omega_2 = \frac{d\theta_2}{dT_{v2}}.$$

Relative Angular Displacement:

$$\Delta\omega = \omega_{\text{rel}} = \frac{d(\theta_2 - \theta_1)}{d\mathcal{N}}.$$

This projects the time evolution of rotational disparity into the global causal frame $\mathcal{N}$, ensuring frame-independent vortex coupling.

Translational–Rotational Coupling

Vorticity–Velocity Mapping:

$$\omega_{\text{rel}}(d\mathcal{N}) = C\frac{v_2 - v_1}{|z_2 - z_1|},$$

where $v_i = \frac{dz_i}{d\tau_i}$ are measured in local Chronos-Time τ_i. The constant C encodes the vortex tube's inertial and elastic response properties.

Extended Interpretation in Temporal Ontology

- **Temporal Layering:** - Vortex rotations evolve in T_v, - Translational flow in τ, - Their interaction is projected onto $\mathcal{N}$, providing a unified causal metric.
- **Spatial Scaling:** The term $|z_2 - z_1|$ reflects inverse distance scaling typical of fluid vortex interactions.
- **Energetic Feedback:** Increases in v_{rel} (Chronos) drive higher ω_{rel} (Swirl Clock acceleration), redistributing kinetic energy within the tube.

Energy Transfer Implications

This time-mode-aware formulation supports a VAM-based energy transport mechanism where vortex-to-vortex coupling transmits angular information along $\mathcal{N}$, modulating Swirl Clocks $S(t)$ and altering time dilation rates in surrounding æther regions.

Conclusion

This refined derivation aligns the rotational-translational dynamics of vortex knots with the layered time framework of the Vortex Æther Model. By mapping proper times T_v, τ, and the global causal time $\mathcal{N}$ into a unified structure, we obtain a temporally coherent view of vortex interactions. Future research may explore non-linearities in C, and energy bifurcations (Kairos moments κ) as topological instabilities in the vortex chain.

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Overview

In the Vortex Æther Model (VAM), time is structured into distinct modes:

- **Aithēr-Time** ($\mathcal{N}$): universal causal background
- **Chronos-Time** (τ): proper time along macroscopic trajectories
- **Swirl Clock** ($S(t)$): local clock rate modulated by vorticity energy
- **Vortex Proper Time** (T_v): time along rotating core structures
- **Kairos Moment** (κ): irreversible topological or energetic bifurcation

Among these, κ marks critical transition points where smooth time evolution in $\mathcal{N}$ or τ cannot be maintained.

Definition of a Kairos Moment

A *Kairos Moment* κ is defined as a non-analytic point in the evolution of the vortex æther field, typically accompanied by a discontinuity in the temporal or topological structure:

$$\lim_{\epsilon \to 0} \left(\frac{d\vec{\omega}}{dt} \right)_{t=\kappa-\epsilon} \neq \left(\frac{d\vec{\omega}}{dt} \right)_{t=\kappa+\epsilon}. \tag{79}$$

Such moments correspond to irreversible events like:

- vortex reconnection,
- knot topological transitions ($\Delta Lk \in \mathbb{Z}$),
- swirl energy overloads,
- swirl clock rate rupture.

Trigger Conditions for Kairos Transitions

Type	Trigger Condition	Physical Interpretation
1. Vorticity Gradient Singularity	$\lvert\nabla\vec{\omega}\rvert \geq \frac{C_e}{r_c^2}$	Core rupture or instability onset
2. Helicity Discontinuity	$\Delta H \neq 0$, $\Delta Lk \in \mathbb{Z}$	Knottedness transition
3. Energy Threshold Exceeded	$U_{\text{swirl}} > U_{\text{max}} = \frac{1}{2}\rho_{æ} C_e^2$	Collapse from over-rotation
4. Vortex Collision	$\vec{\omega}_1 \cdot \vec{\omega}_2 < 0$, at $\lvert\vec{r}_1 - \vec{r}_2\rvert < \delta r_c$	Reconnection or annihilation
5. Swirl Clock Discontinuity	$\left.\frac{dS}{dt}\right\rvert_{t=\kappa^-} \neq \left.\frac{dS}{dt}\right\rvert_{t=\kappa^+}$	Local time rupture in vortex cores

Table 2: Trigger conditions for Kairos transitions in the vortex æther field.

Energetic Criterion from Swirl Potential

Kairos events are energetically triggered when local swirl energy exceeds the maximum sustainable value in the æther medium:

$$U_{\text{swirl}} = \frac{1}{2}\rho_{æ}\lvert\vec{\omega}\rvert^2, \quad U_{\text{max}} = \frac{1}{2}\rho_{æ} C_e^2. \tag{80}$$

Hence, the condition:

$$U_{\text{swirl}} > U_{\text{max}}$$

triggers structural realignment, swirl rupture, or reconnection.

Helicity and Knot Transitions

The total helicity H of a vortex system is given by:

$$H = \int \vec{v} \cdot \vec{\omega}\, dV, \tag{81}$$

which is conserved under smooth evolution. However, when:

$$\Delta H = H_{\text{after}} - H_{\text{before}} \neq 0,$$

a topological transformation has occurred, such as knot reconnection or linking number jump ($\Delta Lk \in \mathbb{Z}$) — a definitive marker of κ.

Temporal Discontinuity in Swirl Clocks

Swirl clocks $S(t)$ track local time using vorticity-based rates. A Kairos moment induces a discontinuity in the swirl time derivative:

$$\lim_{\epsilon \to 0} \left[\frac{dS}{dt}(t = \kappa - \epsilon) \right] \neq \left[\frac{dS}{dt}(t = \kappa + \epsilon) \right]. \tag{82}$$

This represents an irreversible reconfiguration of the internal clock rate, isolating vortex epochs before and after κ.

Interpretation Across Time Modes

Time Mode	Symbol	Effect at Kairos Moment
Aithēr-Time	$\mathcal{N}$	Globally continuous, but re-indexed at κ
Chronos-Time	τ	Broken derivative continuity ($d\tau/dt$ jump)
Swirl Clock	$S(t)$	Discontinuous rate: $\Delta \left(\frac{dS}{dt} \right) \neq 0$
Vortex Proper Time	T_v	Reset or bifurcation of local vortex clock phase
Kairos Marker	κ	Singular time point; cannot be evolved through

Table 3: Temporal ontology response to a Kairos transition.

Experimental Analogy

An accessible analogy to Kairos transitions exists in superfluid helium (^{4}He), where vortex reconnections are captured by tracer particles and Kelvin waves:

> Bewley, G. P., Paoletti, M. S., Sreenivasan, K. R., & Lathrop, D. P. (2006). *Superfluid helium: Visualization of quantized vortices*. **Nature**, 441(7093), 588. https://doi.org/10.1038/441588a

These observations reveal the discontinuous evolution of topological structures, paralleling the role of κ in VAM.

Conclusion

Kairos Moments κ encode the breakdown of smooth time evolution in the Vortex Æther Model. They delineate epochs of distinct topological and energetic configurations, demarcate irreversible transitions, and formalize events beyond the classical conservation paradigm. These singularities lie at the intersection of topology, dynamics, and temporality — and serve as central markers in vortex chronology.