Glossary of Vorticity, Helicity, and Rotational Flow Terms

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Introduction

This glossary summarizes key concepts in rotational fluid dynamics, both in classical fluid mechanics and within the framework of the Vortex Æther Model (VAM). Each term is defined with relevant physical context and mathematical expressions.

Glossary

Local Rotation Concepts (Directly related to Vorticity)

Angular Velocity Vector $\boldsymbol{\Omega}$
Describes the instantaneous rotation rate of a fluid element.
Related to vorticity by:

$$\boldsymbol{\omega} = 2\boldsymbol{\Omega}$$

Role: Connects fluid kinematics to rigid-body rotation analogy.

Curl of Velocity $\boldsymbol{\omega} = \nabla \times \vec{v}$
This is the formal definition of vorticity.
Tensorial Form: Antisymmetric part of the velocity gradient.

Twisting and Linking (Helicity-related Quantities)

Kinetic Helicity $H = \int \vec{v} \cdot \boldsymbol{\omega} \, dV$
Scalar, conserved in ideal flows (incompressible, inviscid).
Measures alignment between flow velocity and vorticity.
Positive/Negative values indicate right/left-handed twist.

Magnetic Helicity $H_m = \int \vec{A} \cdot \vec{B} \, dV$
Analogue of kinetic helicity for magnetohydrodynamics (MHD).
Where $\vec{B} = \nabla \times \vec{A}$ is the magnetic field.

Cross Helicity $H_c = \int \vec{v} \cdot \vec{B} \, dV$
Measures alignment of velocity and magnetic field in MHD.
Shows dynamic coupling between flow and magnetic field lines.

Relative Helicity
Normalized form of helicity to compare twistedness independent of flow strength:

$$H_{\text{rel}} = \frac{\int \vec{v} \cdot \boldsymbol{\omega} \, dV}{\left(\int |\vec{v}|^2 dV\right)^{1/2} \left(\int |\boldsymbol{\omega}|^2 dV\right)^{1/2}}$$

Flow Structure Types (Swirl & Vorticity-related)

Spiral Vortex / Swirl Flow
Flow pattern with both radial and tangential components.
Described often in cylindrical coordinates as:

$$\vec{v}(r, \theta, z) = v_r(r, z)\,\hat{r} + v_\theta(r, z)\,\hat{\theta} + v_z(r, z)\,\hat{z}$$

Appears in tornadoes, hurricanes, and swirl combustors.

Rankine Vortex
A model vortex with solid-body rotation inside a core and irrotational flow outside:

$$\omega(r) = \begin{cases} \text{constant}, & r < r_c \\ 0, & r > r_c \end{cases}$$

Burgers Vortex
A steady solution to Navier-Stokes that balances vorticity diffusion and stretching:

$$\omega(r) = \omega_0 e^{-ar^2}$$

Useful in modeling turbulence and vortex stretching.

Vortex Sheet
Surface with discontinuous tangential velocity but continuous normal component.
Idealization of shear layers.

Vortex Filament / Line
Line-like structure where vorticity is concentrated (mathematically a delta function).
Central to Biot–Savart law and Kelvin's circulation theorem.

Derived Quantities & Diagnostics

Q-Criterion
Identifies vortices by comparing strain and rotation:

$$Q = \frac{1}{2}\left(\|\mathbf{\Omega}\|^2 - \|\mathbf{S}\|^2\right)$$

where $\mathbf{S}$ is the rate-of-strain tensor and $\mathbf{\Omega}$ is the rotation tensor.

Vorticity Magnitude $|\boldsymbol{\omega}|$
Scalar field used to visualize rotation intensity.

Enstrophy $E = \frac{1}{2}\int |\boldsymbol{\omega}|^2\,dV$
Analogous to energy, but for rotation.
Important in 2D turbulence where it is approximately conserved.

Circulation $\Gamma = \oint_C \vec{v} \cdot d\vec{l}$
Line integral of velocity around a closed curve.
Related to vorticity via Stokes' theorem:

$$\Gamma = \iint_S \boldsymbol{\omega} \cdot \hat{n}\,dA$$

In VAM-Specific Context

Swirl Clock Time Rate
Local time evolution determined by:

$$d\tau = dt\sqrt{1 - \frac{|\boldsymbol{\omega}|^2}{C_e^2}}$$

Corresponds to time dilation via rotation in VAM.

Swirl Lagrangian
Includes helicity or knottedness:

$$\mathcal{L}_{\text{swirl}} = \lambda \vec{v} \cdot (\nabla \times \vec{v}) = \lambda \vec{v} \cdot \boldsymbol{\omega}$$

Topological Charge / Linking Number
Quantifies the entanglement of vortex tubes:

$$H = \sum_i \Gamma_i^2 \, \mathrm{Lk}_i$$

Related to the Hopf invariant in field theory.

Vorticity $\boldsymbol{\omega}$

The curl of the velocity field:

$$\boldsymbol{\omega} = \nabla \times \mathbf{v}$$

It describes the local spinning motion of a fluid parcel. In VAM, vorticity modulates local time and governs pressure differentials.

Relative Vorticity

Vorticity measured relative to a rotating frame. Important in geophysical flows and rotating reference systems in æther dynamics.

Angular Velocity Vector $\boldsymbol{\Omega}$

Describes the solid-body rotation rate of a fluid element, related to vorticity via:

$$\boldsymbol{\omega} = 2\boldsymbol{\Omega}$$

Vortex Tube

A bundle of vortex lines forming a tubular region of concentrated vorticity. Vortex tubes can evolve into knotted structures in VAM.

Vortex Line

A curve everywhere tangent to the vorticity vector field. These lines illustrate the direction and connectivity of local rotational motion.

Vortex Stretching

Occurs when a vortex tube is elongated by the flow. In 3D, this process increases the magnitude of vorticity, conserving angular momentum.

Helicity H

A scalar defined as:

$$H = \int \mathbf{v} \cdot \boldsymbol{\omega} \, dV$$

It measures the extent to which velocity and vorticity are aligned. It is a topological invariant in ideal fluids and central to vortex knot theory in VAM.

Kinetic Helicity Density

The pointwise scalar product:

$$h = \mathbf{v} \cdot \boldsymbol{\omega}$$

Gives the local density of helicity, whose integral over a volume is total helicity.

Cross Helicity

Used in magnetohydrodynamics (MHD):

$$H_c = \int \mathbf{v} \cdot \mathbf{B} \, dV$$

Measures the alignment between fluid velocity and magnetic field.

Magnetic Helicity

Another MHD quantity:

$$H_m = \int \mathbf{A} \cdot \mathbf{B} \, dV$$

where $\mathbf{B} = \nabla \times \mathbf{A}$. It quantifies the linkage of magnetic field lines.

Swirl

A qualitative term indicating rotational flow. In cylindrical coordinates, the azimuthal component v_θ is often referred to as the swirl velocity.

Swirl Clock

In VAM, a clock governed by the internal angular frequency ω_0 of a vortex core. Time dilation is given by:

$$d\tau = dt\sqrt{1 - \frac{\omega^2}{C_e^2}}$$

Swirl Lagrangian

A field Lagrangian incorporating helicity terms:

$$\mathcal{L}_{\text{swirl}} = \lambda(\mathbf{v} \cdot \boldsymbol{\omega})$$

This encapsulates the topological and dynamical behavior of swirl fields in VAM.

Rotation

A general term describing angular motion in fluids, encompassing both local (vorticity) and global (rigid body) rotations.

Circulation Γ

Defined by:

$$\Gamma = \oint_C \mathbf{v} \cdot d\mathbf{l}$$

Measures the total rotation around a closed loop. Stokes' theorem connects it to surface-integrated vorticity.

Enstrophy $\mathcal{E}$

A quadratic measure of rotational intensity:

$$\mathcal{E} = \frac{1}{2} \int |\boldsymbol{\omega}|^2 \, dV$$

Important in turbulence analysis and vortex conservation.

Q-Criterion

A scalar diagnostic for vortex identification:

$$Q = \frac{1}{2}(||\boldsymbol{\Omega}||^2 - ||\mathbf{S}||^2)$$

where $\mathbf{S}$ is the strain-rate tensor. Positive Q indicates vortex-dominated regions.

Rankine Vortex

A model vortex with solid-body rotation inside a core ($r < r_c$) and irrotational flow outside ($r > r_c$). Simplified but useful for teaching.

$$\omega(r) = \begin{cases} \text{constant}, & r < r_c \\ 0, & r > r_c \end{cases}$$

Burgers Vortex

A steady-state vortex model balancing stretching and viscous diffusion:

$$\omega(r) = \omega_0 e^{-ar^2}$$

Frequently used in studies of turbulent vortex dynamics.

Topological Charge

Quantifies the entanglement or linkage of vortex lines. In VAM, this relates to conserved quantities such as helicity and can be described using the Hopf invariant.

References

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