

# On the Vortex Interpretation of Kerr Geometry in the Vortex æther Model (VAM)

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## Abstract

We reinterpret the Kerr solution of general relativity through the framework of the Vortex æther Model (VAM), replacing spacetime curvature and singularities with structured swirl fields in an inviscid, incompressible medium. The Kerr frame-dragging term  $g_{t\phi}$  is shown to emerge naturally from the circulation  $\Gamma$  of an axisymmetric vortex ring, yielding a new expression:  $g_{t\phi}^{(\text{VAM})} \sim -\frac{\Gamma}{C_e^2}$

This correspondence allows us to model the rotating gravitational field of a compact object as a physical æther vortex with well-defined energy, swirl, and pressure gradients. We compare the VAM tensor formulation to the Einstein tensor of the Kerr metric and establish the conditions under which they produce similar observational effects. This work extends prior VAM formulations [1?–4].

## I. Introduction

The Kerr metric [5] is the exact, stationary, axisymmetric solution to Einstein’s vacuum field equations describing the spacetime geometry around a rotating mass. It has long been celebrated for its elegance, predicting phenomena such as frame dragging, the ergosphere, and event horizons without requiring any explicit matter distribution. However, the vacuum nature of the solution ( $G_{\mu\nu} = 0$ ) poses an ontological challenge: how can angular momentum, energy, and causality structure arise from an empty geometry?

The Vortex æther Model (VAM) [1–3] offers an alternative: gravity emerges not from spacetime curvature, but from swirl dynamics in an incompressible, inviscid æther. In this framework, frame dragging is interpreted as the physical result of a structured vorticity field—specifically, a quantized circulation of the æther surrounding a knotted vortex core. VAM replaces differential geometry with fluid dynamics, and the Ricci-flat Kerr spacetime with a pressure-balanced vortex soliton of finite energy.

In this work, we present a formal correspondence between the Kerr solution and the vortex structures permitted in VAM. We demonstrate that the off-diagonal metric term  $g_{t\phi}$  arises directly from the circulation  $\Gamma$  of an axisymmetric swirl, following the relation:

$$g_{t\phi}^{(\text{VAM})} \sim -\frac{\Gamma}{C_e^2},$$

where  $C_e$  is the maximum vortex-core tangential velocity of the æther. This recovers the correct frame-dragging signature in the weak-field limit and matches the angular momentum profile of the Kerr solution.

We further analyze the event horizon as a breakdown radius of æther pressure coherence, rather than a coordinate singularity. The absence of geometric singularities

in VAM enables a regular, topological interpretation of the Kerr geometry as a finite-energy, helicity-conserving configuration.

This paper is part of the broader VAM series [1–4, 6?], aiming to reconstruct gravity, time dilation, quantum structure, and electromagnetism from first principles using structured swirl fields in a superfluid medium. Here, we focus on showing that even one of the most celebrated exact solutions in GR can be reinterpreted as a manifestation of fluid mechanics and vortex topology.

## II. Swirl Circulation and the Off-Diagonal Kerr Term

The Kerr metric, expressed in Boyer–Lindquist coordinates  $(t, r, \theta, \phi)$ , contains a nonzero off-diagonal term  $g_{t\phi}$  that encodes the phenomenon of frame dragging. This term couples time and azimuthal angular displacement, and is given by:

$$g_{t\phi} = -\frac{2GMa r \sin^2 \theta}{\Sigma}, \quad \Sigma = r^2 + a^2 \cos^2 \theta, \quad (1)$$

where  $a = J/M$  is the specific angular momentum. In general relativity, this term is a consequence of spacetime curvature induced by rotation. However, within the VAM framework, it arises naturally from the swirl velocity field of an axisymmetric vortex.

In particular, we identify the ætheric analog of this off-diagonal term with the azimuthal circulation:

$$\Gamma_{\text{swirl}} = \oint v_\phi(r \sin \theta) d\phi = 2\pi r \sin \theta \cdot v_\phi. \quad (2)$$

Using this, we propose the leading-order correspondence:

$$g_{t\phi}^{(\text{VAM})} \sim -\frac{\Gamma_{\text{swirl}}}{C_e^2}, \quad (3)$$

where  $C_e$  is the vortex-core tangential velocity constant in VAM [1].

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To derive this, consider the momentum flux tensor in the æther due to angular swirl. The effective metric component associated with this flux takes the form:

$$g_{t\phi}^{(\text{VAM})} = -\kappa \cdot \rho_{\text{æ}}^{(\text{mass})} \cdot v_{\phi} \cdot (r \sin \theta) \cdot \Delta V, \quad (4)$$

where  $\kappa$  is a dimensionless geometric factor and  $\rho_{\text{æ}}^{(\text{mass})}$  is the mass-equivalent æther density [? ]. Normalizing by the maximum æther energy per unit volume,  $\rho_{\text{æ}}^{(\text{mass})} C_e^2$ , and integrating over the azimuthal loop yields:

$$g_{t\phi}^{(\text{VAM})} \sim -\frac{1}{\rho_{\text{æ}}^{(\text{mass})} C_e^2} \cdot \rho_{\text{æ}}^{(\text{mass})} \cdot \Gamma_{\text{swirl}} = -\frac{\Gamma_{\text{swirl}}}{C_e^2}, \quad (5)$$

which recovers the correct physical dimension and weak-field behavior expected from the Kerr frame-dragging term.

This identification not only reproduces the metric structure of the Kerr solution, but also provides a physical mechanism grounded in æther dynamics: angular momentum is not an abstract geometric quantity, but the integral of real swirl in a structured, incompressible fluid.

#### A. Breakdown Radius and Horizon Analogy in VAM

In general relativity, the outer event horizon of the Kerr black hole occurs at:

$$r_+ = M + \sqrt{M^2 - a^2}$$

This surface marks a coordinate singularity, where  $g_{tt} \rightarrow 0$  and  $g_{rr} \rightarrow \infty$ , and is often interpreted as the causal boundary of a rotating black hole.

In VAM, no singularity occurs. Instead, we identify a breakdown radius  $r_c$ , where the swirl velocity reaches its peak and pressure drops to a minimum:

$$P(r) = P_{\infty} - \frac{1}{2} \rho_{\text{æ}}^{(\text{mass})} v_{\phi}^2(r)$$

As  $r \rightarrow 0$ , the pressure and swirl saturate smoothly, and the energy density remains finite. There is no horizon, but rather a stable vortex core with smooth topological transition. We define the mass and angular momentum as:

$$M = \frac{1}{\varphi} \cdot \frac{4}{\alpha} \cdot \left( \frac{1}{2} \rho_{\text{æ}}^{(\text{mass})} C_e^2 V \right) \quad (6)$$

$$J = n_t \hbar, \quad a = \frac{J}{M} = \frac{\hbar n_t}{M} \quad (7)$$

where  $V$  is the vortex core volume and  $n_t$  the twist number of the knotted vortex ring.

This provides a direct correspondence between Kerr parameters  $(M, a)$  and topological, energetic features of the vortex in VAM.

### III. Helicity Balance and Vacuum Structure

In general relativity, the vacuum condition  $G_{\mu\nu} = 0$  implies the absence of matter-energy sources. In VAM, this condition is reinterpreted:

The æther is not empty, but in a state of perfect dynamical balance, where all internal stresses cancel: swirl-induced pressure, centrifugal gradients, helicity tension, and energy flux divergence sum to zero.

This implies that the vortex configuration is a solitonic equilibrium, governed by:

$$\nabla_{\mu} T_{(\text{vortex})}^{\mu\nu} = 0$$

with  $T_{(\text{vortex})}^{\mu\nu}$  representing a tensor of internal æther dynamics. The stress-energy of the æther may include:

- Swirl pressure:  $-\rho v_{\phi}^2$
- Helicity flux:  $\vec{v} \cdot \vec{\omega}$
- Anisotropic tension along vortex lines

In this reinterpretation, the vacuum geometry is not devoid of physical content but is a topologically and energetically constrained vorticity configuration.

#### A. Toward a VAM Field Equation

Following [3], we propose:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}^{(\text{vortex})}, \quad (8)$$

with  $T_{\mu\nu}^{(\text{vortex})}$  including terms for swirl pressure, helicity flux, and anisotropic vorticity stress [6]. The vacuum limit  $G_{\mu\nu} = 0$  then represents a self-balanced topological soliton in the æther.

The Kerr geometry is reborn as a stable, rotating, helicity-conserving vortex configuration. Its structure emerges not from abstract curvature, but from measurable swirl, pressure, and vortex tension. This paves the way for experimental analogs, finite-energy alternatives to black holes, and a unified treatment of gravity and quantum structure within a fluidic topological framework.

### IV. Einstein Tensor from a Rotating Æther Vortex Mass

To compare with the Kerr geometry, we now construct the full Einstein tensor  $G_{\mu\nu}$  for a rotating æther vortex mass with finite energy density and swirl profile.

We assume axial symmetry and adopt a cylindrical coordinate system  $(t, r, \phi, z)$ , where the swirl is confined to the azimuthal direction:

$$\vec{v}(r) = v_{\phi}(r) \hat{\phi}, \quad \vec{\omega}(r) = \nabla \times \vec{v} = \frac{1}{r} \frac{d}{dr} (r v_{\phi}) \hat{z}.$$

Let the metric ansatz take the form:

$$ds^2 = -f(r)dt^2 - 2\chi(r)dt d\phi + h(r) d\phi^2 + \frac{1}{g(r)}dr^2 + dz^2,$$

where  $\chi(r) \sim \Gamma(r)/C_e^2$  encodes the swirl-induced frame dragging.

We compute the Christoffel symbols, Ricci tensor, and Einstein tensor components from this ansatz. To leading order in  $\chi(r)$ , the nonzero components of the Einstein tensor are:

$$G_{tt} \approx \frac{g}{2r} \left( \frac{d^2 h}{dr^2} - \frac{1}{r} \frac{dh}{dr} \right), \quad (9)$$

$$G_{t\phi} \approx \frac{1}{2} \left( \frac{d^2 \chi}{dr^2} + \frac{1}{r} \frac{d\chi}{dr} \right), \quad (10)$$

$$G_{\phi\phi} \approx -\frac{1}{2} \left( \frac{df}{dr} \frac{d}{dr} \left( \frac{1}{g} \right) + \dots \right). \quad (11)$$

These components correspond respectively to: - Local energy density from azimuthal swirl tension - Angular momentum density via vorticity flux - Centrifugal balancing via pressure gradients

When matched with a structured æther stress-energy tensor  $T_{(\text{vortex})}^{\mu\nu}$ , we recover the analog of the Kerr metric outside the core region.

## Einstein Tensor $G_{\mu\nu}$ for a Rotating Vortex Mass in VAM

### 1. Objective

To compute the full Einstein tensor  $G_{\mu\nu}$  for a rotating mass configuration in the Vortex æther Model (VAM), where the source is a knotted, helicity-conserving vortex structure. Unlike the vacuum Kerr solution, we consider a nonzero pressure and vorticity distribution.

### 2. VAM Metric Ansatz for Rotating Mass

We define a generalized stationary, axisymmetric metric in coordinates  $(t, r, \theta, \phi)$  adapted to the æther swirl flow:

$$ds^2 = -(1 - \Phi(r, \theta)) dt^2 - 2\Omega(r, \theta) r^2 \sin^2 \theta dt d\phi + A(r, \theta) dr^2 + \overset{\text{geometry}}{B(r, \theta)} d\theta^2 + C(r, \theta) d\phi^2 \quad (12)$$

where:

- $\Phi(r, \theta)$ : gravitational potential from swirl-induced pressure drop
- $\Omega(r, \theta)$ : local frame-dragging rate = swirl angular velocity
- $A, B, C$ : radial and angular metric functions determined by æther compressibility and swirl field

### 3. Æther Stress-Energy Tensor

In VAM, the effective energy-momentum tensor is:

$$T_{\mu\nu}^{(\text{vortex})} = \rho_{\text{æ}}^{(\text{mass})} u_\mu u_\nu + P_{\text{swirl}} (g_{\mu\nu} + u_\mu u_\nu) + \tau_{\mu\nu} \quad (13)$$

with:

- $u^\mu$ : 4-velocity field aligned with æther flow
- $P_{\text{swirl}} = \frac{1}{2} \rho v_\phi^2$ : swirl pressure
- $\tau_{\mu\nu}$ : vorticity-induced anisotropic stress (nonzero off-diagonal terms)

### 4. Christoffel Symbols and Ricci Tensor

Using the metric ansatz, we compute all nonzero Christoffel symbols:

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} g^{\lambda\sigma} (\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu})$$

From this, construct  $R_{\mu\nu}$  and then the scalar curvature  $R = g^{\mu\nu} R_{\mu\nu}$ .

### 5. Einstein Tensor Components

The Einstein tensor is:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

We expect:

- $G_{tt} \approx 8\pi G \rho_{\text{æ}}^{(\text{mass})}$ : swirl energy density
- $G_{t\phi} \propto \Gamma_{\text{swirl}}(r, \theta)$ : frame dragging
- $G_{rr}, G_{\theta\theta}, G_{\phi\phi}$ : gradients of swirl pressure and centrifugal tension

### 6. Limiting Behavior

In the limit  $\Omega(r, \theta) \rightarrow 0$  and  $\Phi \rightarrow 2GM/r$ , the metric reduces to the Schwarzschild case. In the limit of vanishing pressure gradients,  $G_{\mu\nu} \rightarrow 0$  recovers the Kerr

### V. Stress-Energy Tensor from Æther Vorticity

We now define the stress-energy tensor in the VAM framework, incorporating real physical sources from structured vortex flow:

$$T_{(\text{vortex})}^{\mu\nu} = \rho_{\text{æ}}^{(\text{mass})} u^\mu u^\nu + P_{\text{swirl}} g^{\mu\nu} + \lambda_1 \omega^\mu \omega^\nu + \lambda_2 H^{\mu\nu},$$

where:

- $u^\mu = (1, v^i)$ : swirl velocity 4-vector
- $\omega^\mu = \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha u_\beta$ : vorticity 4-vector
- $H^{\mu\nu} = u^\mu \omega^\nu + u^\nu \omega^\mu$ : helicity tensor (symmetric)
- $P_{\text{swirl}} = -\frac{1}{2} \rho v_\phi^2$ : pressure drop due to rotational velocity

This tensor accounts for: - Compressional stress from vortex energy density - Bernoulli-type swirl pressure - Helicity coupling (similar to magnetohydrodynamics) - Tension along vortex lines via  $\omega^\mu \omega^\nu$

Substituting into:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}^{(\text{vortex})},$$

we can match the swirl-induced geometry to a real, conserved source. This redefines the gravitational source: not static mass, but swirl-coherent energy flux in a knotted æther.

## VI. VAM Interpretation of the Kerr Solution

### 1. From Frame Dragging to Swirl Flow

The Kerr metric, a solution to the Einstein field equations in vacuum, describes the geometry surrounding a rotating mass. Traditionally, it is derived from purely geometric and relativistic assumptions: axial symmetry, stationarity, and asymptotic flatness. However, within the framework of the Vortex æther Model (VAM), this same geometry can be interpreted as the manifestation of a physically real, helicity-conserving, rotating vortex structure in an inviscid, incompressible æther. Frame dragging, rather than arising from spacetime curvature, emerges from circulation of the æther itself.

### 2. Kerr Metric Overview

In Boyer–Lindquist coordinates, the Kerr line element is:

$$ds^2 = - \left( 1 - \frac{2GMr}{\Sigma} \right) dt^2 - \frac{4GMa r \sin^2 \theta}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left( r^2 + a^2 + \frac{2GMa^2 r \sin^2 \theta}{\Sigma} \right) \sin^2 \theta d\phi^2, \quad (14)$$

where

$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 - 2GMr + a^2,$$

and  $a = J/M$  is the angular momentum per unit mass.

The Einstein tensor  $G_{\mu\nu} = 0$  implies a Ricci-flat space-time, with curvature fully encoded in the Weyl tensor.

### 3. VAM Interpretation: Tangential Swirl and Helicity Balance

We reinterpret the off-diagonal metric component as arising from the swirl of the æther:

$$g_{t\phi} = - \frac{2GMa r \sin^2 \theta}{\Sigma}. \quad (15)$$

This corresponds to the ætheric circulation:

$$\Gamma_{\text{swirl}} = \oint v_\phi r \sin \theta d\phi = 2\pi r \sin \theta v_\phi. \quad (16)$$

Hence,

$$g_{t\phi}^{(\text{VAM})} \sim - \frac{\Gamma_{\text{swirl}}}{C_e^2}, \quad (17)$$

where  $C_e$  is the characteristic tangential velocity of the vortex core.

### 4. Helicity-Conserved Vacuum as Vortex Equilibrium

Although  $G_{\mu\nu} = 0$ , this does not imply emptiness in VAM. Rather, it signifies that the æther is in a state of perfect dynamical balance:

A topologically stable vortex configuration in which all ætheric pressure gradients and swirl-induced inertial forces cancel, yielding zero net energy-momentum divergence.

The vortex energy is stored in rotational motion:

$$E_{\text{vortex}} = \int \frac{1}{2} \rho_{\text{æ}}^{(\text{mass})} v_\phi^2 dV. \quad (18)$$

### 5. Mass and Spin from Knot Geometry

We define mass and spin in terms of vortex core properties:

$$M = \frac{1}{\varphi} \cdot \frac{4}{\alpha} \cdot \left( \frac{1}{2} \rho_{\text{æ}}^{(\text{mass})} C_e^2 V \right), \quad (19)$$

$$J = n_t \hbar, \quad a = \frac{J}{M} = \frac{\hbar n_t}{\rho_{\text{æ}}^{(\text{mass})} C_e^2 V} \cdot \frac{\alpha \varphi}{4}, \quad (20)$$

where  $n_t$  is the twist number and  $V$  the volume of the vortex core.

### 6. Consequences and Testable Predictions

- **Frame dragging** is real æther circulation; testable in rotating superfluid or electromagnetic systems.

- **Kerr singularity** is replaced by a smooth core of radius  $r_c$ , finite energy, and continuous pressure.
- **Vacuum solutions** in GR correspond to helicity-neutral vortex equilibria in VAM.
- **Time dilation** follows from rotational energy:

$$dt = dt_\infty \sqrt{1 - \frac{v_\phi^2}{C_e^2}}.$$

## 7. Conclusion

The Kerr geometry arises not from empty curvature, but from structured swirl in a physical æther. This interpretation offers a smooth, finite-energy foundation for gravity, where frame dragging, mass, and rotation emerge from vortex dynamics.

### Stress-Energy Tensor in VAM from Vorticity Flow

#### 1. Objective

To define the full stress-energy tensor  $T_{(\text{vortex})}^{\mu\nu}$  within the Vortex æther Model (VAM), expressed in terms of swirl velocity  $v^\mu$ , vorticity  $\omega^\mu$ , pressure gradients, and topological structure. This tensor serves as the physical source for Einstein's equations in the VAM reinterpretation of gravity.

#### 2. Æther as an Inviscid, Incompressible Fluid

The æther is modeled as a relativistically consistent, inviscid and incompressible superfluid with absolute time and flat spatial geometry. The 4-velocity is:

$$u^\mu = \gamma \left( 1, \frac{\vec{v}}{C_e} \right), \quad \gamma = \frac{1}{\sqrt{1 - \vec{v}^2/C_e^2}} \quad (21)$$

Vorticity is defined as:

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta \quad (22)$$

#### 3. General Form of the Energy-Momentum Tensor

We define:

$$T_{(\text{vortex})}^{\mu\nu} = \rho_{\text{æ}}^{(\text{mass})} u^\mu u^\nu + P g^{\mu\nu} + \pi^{\mu\nu} \quad (23)$$

where:

- $\rho_{\text{æ}}^{(\text{mass})} \approx 3.89 \times 10^{18} \text{ kg/m}^3$  is the æther mass density [2]

- $P$  is the isotropic pressure (background or swirl-induced)
- $\pi^{\mu\nu}$  is the anisotropic stress tensor due to helicity, swirl gradients, and knot tension

#### 4. Swirl-Induced Pressure and Anisotropic Stress

We model the swirl pressure as:

$$P_{\text{swirl}} = \frac{1}{2} \rho_{\text{æ}}^{(\text{mass})} v_\phi^2 \quad (24)$$

The anisotropic stress is:

$$\pi^{\mu\nu} = \lambda_1 \omega^\mu \omega^\nu + \lambda_2 H^{\mu\nu} + \lambda_3 \nabla^{(\mu} v^{\nu)} \quad (25)$$

with:

- $H^{\mu\nu} = \epsilon^{\mu\alpha\beta\sigma} u_\alpha \nabla_\beta \omega_\sigma$ : helicity flux tensor
- $\lambda_i$ : coupling constants encoding vortex core structure, chirality, and swirl coherence

#### 5. Conservation and Dynamical Balance

The stress-energy tensor is conserved:

$$\nabla_\mu T_{(\text{vortex})}^{\mu\nu} = 0 \quad (26)$$

This condition ensures Bernoulli equilibrium and global helicity conservation for stable knotted vortex structures.

#### 6. Example: Toroidal Axisymmetric Vortex Knot

For a rotating toroidal knot with dominant azimuthal swirl:

$$\begin{aligned} T^{tt} &\approx \rho_{\text{æ}}^{(\text{mass})} + P_{\text{swirl}} \\ T^{t\phi} &\approx \rho_{\text{æ}}^{(\text{mass})} v_\phi \\ T^{rr} &\approx -P + \lambda_1 (\omega^r)^2 \\ T^{\phi\phi} &\approx P + \frac{1}{2} \lambda_1 (\omega^\phi)^2 \end{aligned}$$

#### 7. Toward a VAM-Based Gravitational Field Equation

We substitute this vortex-derived tensor into the field equation:

$$G^{\mu\nu} = 8\pi T_{(\text{vortex})}^{\mu\nu} \quad (27)$$

This replaces abstract spacetime curvature with concrete, swirl-supported energy flow — a tangible source grounded in knotted fluid dynamics.

A natural next step is to construct a Lagrangian density  $\mathcal{L}_{\text{vortex}}$  whose variation with respect to  $u^\mu$  and  $\omega^\mu$  reproduces the above stress-energy tensor via the Euler–Lagrange formalism.

## Vortex Lagrangian Density for Structured æther Stress

### 1. Objective

To construct a Lagrangian density  $\mathcal{L}_{\text{vortex}}$  that generates the vortex stress-energy tensor in the Vortex æther Model (VAM) via the Euler–Lagrange equations applied to the 4-velocity field  $u^\mu$  and the vorticity field  $\omega^\mu$ .

### 2. Governing Fields

We define:

- $u^\mu = \gamma(1, \vec{v}/C_e)$ ,  $\gamma = (1 - \vec{v}^2/C_e^2)^{-1/2}$ : æther swirl 4-velocity
- $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$ : vorticity pseudovector
- $H^{\mu\nu} = \epsilon^{\mu\alpha\beta\sigma}u_\alpha\nabla_\beta\omega_\sigma$ : helicity flux tensor

### 3. Lagrangian Density Form

We propose the following vortex-field Lagrangian:

$$\mathcal{L}_{\text{vortex}} = -\frac{1}{2}\rho_{\text{æ}}^{(\text{mass})}u^\mu u_\mu + \lambda_1\omega^\mu\omega_\mu + \lambda_2 H^{\mu\nu}H_{\mu\nu} + \lambda_3 \nabla^{(\mu}u^{\nu)}\nabla_{(\mu}u_{\nu)} + \text{topological helicity terms} \quad (28)$$

This expression encodes:

- Ætheric rest energy and swirl kinetic energy
- Topological tension via vorticity self-interaction
- Helicity transport and conservation
- Anisotropic stress from swirl shear gradients
- Background pressure from swirl confinement

### 4. Stress Tensor via Variation

Applying:

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\text{vortex}})}{\delta g_{\mu\nu}}$$

reproduces:

$$T_{(\text{vortex})}^{\mu\nu} = \rho_{\text{æ}}^{(\text{mass})}u^\mu u^\nu + P g^{\mu\nu} + \lambda_1 \omega^\mu \omega^\nu + \lambda_2 H^{\mu\nu} + \lambda_3 \nabla^{(\mu}u^{\nu)}$$

### 5. Interpretation

This Lagrangian formalism shows that the gravitational source in VAM is a structured, topologically stabilized energy field — not an abstract mass point. The Euler–Lagrange dynamics reflect the conservation of vortex helicity, swirl momentum, and ætheric coherence.

## Topological Conservation Law from Vortex Helicity

### 1. Objective

To identify the topologically conserved quantity associated with the vortex Lagrangian  $\mathcal{L}_{\text{vortex}}$ , and express its conservation via a divergence-free helicity current.

### 2. Helicity Current Definition

We define the helicity 4-current as:

$$J_{\text{helicity}}^\mu = \epsilon^{\mu\nu\alpha\beta}u_\nu\omega_\alpha v_\beta \quad (29)$$

This current reflects the intertwining of vorticity and æther flow, i.e., the degree to which the swirl field forms knotted or linked structures.

### 3. Conservation Law

Variation of the Lagrangian with respect to  $u^\mu$  and  $\omega^\mu$ , together with the Bianchi identities, yields the conservation law:

$$\nabla_\mu J_{\text{helicity}}^\mu = 0 \quad (30)$$

This equation encodes \*\*topological invariance of total helicity\*\*.

$$\mathcal{H} = \int J_{\text{helicity}}^0 d^3x = \text{constant} \quad (31)$$

### 4. Physical Interpretation in VAM

In the Vortex æther Model:

- $\mathcal{H}$  corresponds to the linking and twisting of vortex tubes in 3D æther space.
- Quantized particles (e.g., electrons, protons) correspond to knotted structures with nonzero conserved helicity.
- Helicity conservation ensures stability of vortex cores and suppresses topological transitions (unless external torque or swirl violation occurs).

### 5. Connection to VAM Gravitational Sources

In this formulation:

$$T^{\mu\nu} \Rightarrow \text{depends functionally on } \mathcal{H}$$

Thus, gravitational effects in VAM are not sourced by mass-energy alone, but by the \*topological class of the swirl structure\*. Gravitational lensing, time dilation, and inertial effects emerge from the conserved helicity content.

## 6. Relation to Hopf Invariant and Knot Class

The total helicity  $\mathcal{H}$  is formally equivalent to the Hopf invariant  $H \in \pi_3(S^2) = \mathbb{Z}$ , classifying mappings from the compactified spatial 3-sphere to the unit vorticity sphere:

$$\mathcal{H} \sim \int_{\mathbb{R}^3} \vec{v} \cdot \vec{\omega} d^3x \longleftrightarrow \text{Hopf}(u^\mu) \quad (32)$$

This links the VAM helicity current to the topological winding of vortex lines — where distinct particle species correspond to distinct knotted or linked configurations. For example:

- Trefoil knot:  $H = \pm 1$ , fermionic soliton
- Hopf link:  $H = 0$ , neutrino-like structure
- Higher-order torus knots:  $|H| > 1$ , hadronic bound states

**Conclusion:** In VAM, helicity conservation is not merely a dynamical symmetry, but a topological invariant encoding particle identity, gravitational coupling, and quantum coherence.

## VII. Stress-Energy Tensor and Helicity Conservation in VAM

### 1. Objective

To define the complete stress-energy tensor  $T_{(\text{vortex})}^{\mu\nu}$  within the Vortex æther Model (VAM), incorporating swirl velocity  $u^\mu$ , vorticity  $\omega^\mu$ , pressure gradients, and topological structure — and to show how this leads to helicity conservation as a fundamental topological law.

### 2. Æther as an Inviscid, Incompressible Fluid

The æther is modeled as a relativistically consistent, incompressible, inviscid fluid with absolute time. The 4-velocity of the swirl flow is:

$$u^\mu = \gamma \left( 1, \frac{\vec{v}}{C_e} \right), \quad \gamma = \frac{1}{\sqrt{1 - \vec{v}^2/C_e^2}} \quad (33)$$

Vorticity is defined via the relativistic extension:

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta \quad (34)$$

### 3. Stress-Energy Tensor Formulation

We propose the total energy-momentum tensor of a vortex configuration:

$$T_{(\text{vortex})}^{\mu\nu} = \rho_{\text{æ}}^{(\text{mass})} u^\mu u^\nu + P g^{\mu\nu} + \pi^{\mu\nu} \quad (35)$$

where:

- $\rho_{\text{æ}}^{(\text{mass})}$  is the æther mass density (e.g.  $\sim 3.89 \times 10^{18} \text{ kg/m}^3$ )
- $P$  is isotropic Bernoulli pressure from tangential vortex tension
- $\pi^{\mu\nu}$  is an anisotropic stress term encoding swirl directionality and topology

## 4. Swirl Pressure and Anisotropic Structure

The tangential swirl pressure is:

$$P_{\text{swirl}} = \frac{1}{2} \rho_{\text{æ}}^{(\text{mass})} v_\phi^2 \quad (36)$$

The anisotropic stress is modeled as:

$$\pi^{\mu\nu} = \lambda_1 \omega^\mu \omega^\nu + \lambda_2 H^{\mu\nu} + \lambda_3 \nabla^{(\mu} u^{\nu)} \quad (37)$$

with:

- $H^{\mu\nu} = \epsilon^{\mu\alpha\beta\sigma} u_\alpha \nabla_\beta \omega_\sigma$ : helicity flux tensor
- $\lambda_i$ : coefficients that encode vortex topology (e.g., trefoil, link, torus knots)

## 5. Conservation Laws

The covariant conservation equation:

$$\nabla_\mu T_{(\text{vortex})}^{\mu\nu} = 0 \quad (38)$$

ensures dynamic stability and embodies Bernoulli balance within structured swirl fields.

From the vorticity and 4-velocity, we construct the helicity current:

$$\mathcal{J}^\mu = \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta = 2\omega^\mu \quad (39)$$

**Topological helicity conservation** then follows:

$$\nabla_\mu \mathcal{J}^\mu = 0 \quad (40)$$

ensuring that total helicity is conserved unless external forcing or vortex reconnection occurs.

## 6. Topological Interpretation via Hopf Invariant

The spatial integral of helicity recovers the Hopf invariant:

$$\mathcal{H} = \int \vec{v} \cdot \vec{\omega} d^3x \leftrightarrow H \in \pi_3(S^2) = \mathbb{Z} \quad (41)$$

This classifies the knot type:

- Trefoil knot:  $H = \pm 1$
- Hopf link:  $H = 0$
- Torus knots:  $H = n \in \mathbb{Z}$

The VAM interpretation is that each particle species corresponds to a different topological equivalence class in this homotopy group.

## 7. Toward a VAM Field Equation

Finally, we posit:

$$G^{\mu\nu} = 8\pi T_{(\text{vortex})}^{\mu\nu} \quad (42)$$

linking spacetime geometry not to abstract curvature, but to the real swirl energy of a structured æther field.

**Next:** Derive a Lagrangian density  $\mathcal{L}_{\text{vortex}}$  that yields this stress-energy tensor via Euler–Lagrange variation with respect to  $u^\mu$  and  $\omega^\mu$ , and guarantees topological conservation as a Noether current.

### Lagrangian Density for Vortex Fields in the Vortex æther Model (VAM)

We propose the following swirl-based Lagrangian density  $\mathcal{L}_{\text{vortex}}$ , capturing both dynamics and topology of the æther flow:

$$\mathcal{L}_{\text{vortex}} = \underbrace{-\rho_{\text{æ}}^{(\text{mass})} C_e^2 \sqrt{1 - \frac{v^2}{C_e^2}}}_{\text{relativistic fluid energy}} + \underbrace{\frac{\lambda_1}{2} \omega_\mu \omega^\mu}_{\text{swirl energy (vorticity norm)}} + \underbrace{\lambda_2 \epsilon^{\mu\nu\alpha\beta} u_\mu \omega_\nu \partial_\alpha u_\beta}_{\text{topological helicity term}} + \underbrace{\lambda_3 \nabla_\mu u_\nu \nabla^\mu u^\nu}_{\text{æther strain energy}} + \underbrace{P g^{\mu\nu}}_{\text{swirl pressure (isotropic)}} + \underbrace{\lambda_1 \omega^\mu \omega^\nu g_{\mu\nu}}_{\text{vorticity energy}} \quad (43)$$

### Euler–Lagrange Equation for the Æther 4-Velocity Field

We derive the Euler–Lagrange equation for the swirl 4-velocity  $u^\mu$  from the VAM Lagrangian density:

$$\mathcal{L}_{\text{vortex}} = \underbrace{\rho_{\text{æ}}^{(\text{mass})} u^\mu u^\nu g_{\mu\nu}}_{\text{Kinetic energy}} + \underbrace{P g^{\mu\nu}}_{\text{Swirl pressure (isotropic)}} + \underbrace{\lambda_1 \omega^\mu \omega^\nu g_{\mu\nu}}_{\text{Vorticity energy}} + \underbrace{\lambda_2 \epsilon^{\mu\nu\alpha\beta} u_\mu \omega_\nu \partial_\alpha u_\beta}_{\text{topological helicity term}} + \underbrace{\lambda_3 \nabla_\mu u_\nu \nabla^\mu u^\nu}_{\text{æther strain energy}} \quad (43)$$

We apply the Euler–Lagrange field equation to the vector field  $u^\mu$ :

$$\frac{\partial \mathcal{L}}{\partial u^\mu} - \nabla_\alpha \left( \frac{\partial \mathcal{L}}{\partial (\nabla_\alpha u^\mu)} \right) = 0 \quad (44)$$

#### Interpretation

- The **first term** gives the base energy of the moving æther parcel, mirroring perfect-fluid dynamics [? ].
- The **second term** models energy from rotation, responsible for centrifugal tension and swirl pressure.
- The **third term** captures topological linking of vortex lines — formally a helicity-related Chern–Simons analog [? ? ].
- The **fourth term** introduces internal æther strain from velocity shear (analogous to elastic tension).

#### Topological Conservation Law

Varying the helicity term yields a conserved current:

$$\mathcal{J}^\mu = \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta = 2\omega^\mu, \quad \nabla_\mu \mathcal{J}^\mu = 0$$

This guarantees conservation of vortex linkage class (Hopf charge) in the absence of dissipation.

## References

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### Variation of Individual Terms

a. 1. *Kinetic Energy Term:*

$$\frac{\partial}{\partial u^\mu} (u^\nu u^\sigma g_{\nu\sigma}) = 2u_\mu \Rightarrow \frac{\partial \mathcal{L}}{\partial u^\mu} \supset 2\rho_{\text{æ}}^{(\text{mass})} u_\mu$$

b. 2. *Strain Energy Term:*

$$\frac{\delta}{\delta u^\mu} (\nabla_\lambda u_\nu \nabla^\lambda u^\nu) = -2\nabla_\lambda \nabla^\lambda u_\mu = -2\Box u_\mu \Rightarrow \frac{\partial \mathcal{L}}{\partial u^\mu} \supset -2\lambda_3 \Box u_\mu$$

c. 3. *Helicity Coupling Term:* The helicity interaction is:

$$\mathcal{L}_{\text{hel}} = \lambda_2 \epsilon^{\mu\nu\alpha\beta} u_\mu \omega_\nu \partial_\alpha u_\beta$$

Given  $\omega^\nu = \frac{1}{2} \epsilon^{\nu\sigma\lambda\kappa} u_\sigma \partial_\lambda u_\kappa$ , this term involves both  $u^\mu$  and its derivatives. Variation yields terms of the form:

$$\delta \mathcal{L}_{\text{hel}} \sim \lambda_2 [\epsilon_{\mu\nu\alpha\beta} \omega^\nu \partial^\alpha u^\beta + \text{nonlinear combinations of } u^\mu \text{ and } \partial_\lambda u^\kappa]$$

Exact evaluation requires symmetry assumptions (e.g., axisymmetry) or numerical evaluation.



## Resulting Equation of Motion

Combining the above:

$$\boxed{\rho_{\text{æ}}^{(\text{mass})} u_{\mu} - \lambda_3 \square u_{\mu} + \lambda_2 [\epsilon_{\mu\nu\alpha\beta} \omega^{\nu} \partial^{\alpha} u^{\beta} + \dots] = 0} \quad (45)$$

This is the effective equation of motion for the æther swirl field, governing the evolution of vortex-knotted configurations in the presence of helicity tension and internal swirl strain.

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## Euler–Lagrange Equation for the Vorticity Field $\omega^{\mu}$

We now vary the vortex Lagrangian density with respect to the vorticity 4-vector  $\omega^{\mu}$ , where:

$$\mathcal{L}_{\text{vortex}} = \rho_{\text{æ}}^{(\text{mass})} u^{\mu} u^{\nu} g_{\mu\nu} + P g^{\mu\nu} + \lambda_1 \omega^{\mu} \omega^{\nu} g_{\mu\nu} + \lambda_2 \epsilon^{\mu\nu\alpha\beta} u_{\mu} \omega_{\nu} \partial_{\alpha} u_{\beta} + \lambda_3 \nabla_{\mu} u^{\mu} \nabla_{\nu} u^{\nu} \quad (46)$$

We apply the Euler–Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial \omega^{\mu}} - \nabla_{\nu} \left( \frac{\partial \mathcal{L}}{\partial (\nabla_{\nu} \omega^{\mu})} \right) = 0 \quad (47)$$

## Variation of Terms Involving $\omega^{\mu}$

d. 1. *Vorticity Energy Term:*

$$\frac{\partial}{\partial \omega^{\mu}} (\omega^{\alpha} \omega^{\beta} g_{\alpha\beta}) = 2\omega_{\mu} \Rightarrow \frac{\partial \mathcal{L}}{\partial \omega^{\mu}} \supset 2\lambda_1 \omega_{\mu}$$

e. 2. *Helicity Coupling Term:*

$$\frac{\partial}{\partial \omega^{\mu}} (\epsilon^{\alpha\beta\gamma\delta} u_{\alpha} \omega_{\beta} \partial_{\gamma} u_{\delta}) = \epsilon^{\alpha\mu\gamma\delta} u_{\alpha} \partial_{\gamma} u_{\delta} \Rightarrow \frac{\partial \mathcal{L}}{\partial \omega^{\mu}} \supset \lambda_2 \epsilon^{\alpha\mu\gamma\delta} u_{\alpha} \partial_{\gamma} u_{\delta}$$

No derivatives of  $\omega^{\mu}$  appear in  $\mathcal{L}$ , so the second term of the Euler–Lagrange equation vanishes.

## Vorticity Evolution Equation

Putting it together:

$$\boxed{2\lambda_1 \omega_{\mu} + \lambda_2 \epsilon^{\alpha\mu\gamma\delta} u_{\alpha} \partial_{\gamma} u_{\delta} = 0} \quad (48)$$

**Interpretation**

- This equation equates the inertial vorticity field to a helicity-induced swirl curvature.
- The second term is proportional to the swirl helicity density:

$$\mathcal{H}^{\mu} = \epsilon^{\mu\nu\alpha\beta} u_{\nu} \partial_{\alpha} u_{\beta} = 2\omega^{\mu} \Rightarrow \omega^{\mu} \propto \mathcal{H}^{\mu}$$

- Hence, the dynamics of  $\omega^{\mu}$  encode the topological conservation of linked/swirl lines in the æther — a key feature of knotted vortex solitons in VAM.

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