Golden Rapidity and Tangential Velocity in VAM

Let the golden ratio be

$$\varphi \equiv \frac{1+\sqrt{5}}{2}. \tag{1}$$

Recall the definition of the inverse hyperbolic sine [?]:

$$\operatorname{asinh}(x) = \ln\left(x + \sqrt{x^2+1}\right). \tag{2}$$

Substituting $x = \frac{1}{2}$ into (??) gives

$$\operatorname{asinh}\left(\tfrac{1}{2}\right) = \ln\left(\tfrac{1}{2} + \sqrt{\tfrac{1}{4}+1}\right) \tag{3}$$

$$= \ln\left(\tfrac{1+\sqrt{5}}{2}\right) \tag{4}$$

$$= \ln\varphi. \tag{5}$$

Exponentiating both sides yields the clean identity

$$\boxed{\varphi = \exp\left(\operatorname{asinh}\left(\tfrac{1}{2}\right)\right)}. \tag{6}$$

Numerical check. Using double precision, $\varphi \approx 1.618033988749895$ and $\exp(\operatorname{asinh}(1/2)) \approx 1.618033988749895$, matching to machine precision.

Setup

Let the golden ratio be

$$\varphi \equiv \frac{1+\sqrt{5}}{2}, \qquad \varphi^2 = \varphi + 1. \tag{7}$$

Define the *golden rapidity*

$$\xi_g \equiv \frac{3}{2}\ln\varphi. \tag{8}$$

We use the standard hyperbolic functions (definitions in [?]).

Identity: $\tanh(\xi_g) = 1/\varphi$

Using $\tanh y = \dfrac{e^{2y}-1}{e^{2y}+1}$, substitute $y = \xi_g$ to obtain

$$\tanh(\xi_g) \;=\; \frac{e^{3\ln\varphi}-1}{e^{3\ln\varphi}+1} \;=\; \frac{\varphi^3-1}{\varphi^3+1}. \tag{9}$$

From $\varphi^2 = \varphi + 1$ it follows $\varphi^3 = \varphi(\varphi+1) = 2\varphi + 1$. Hence

$$\tanh(\xi_g) \;=\; \frac{(2\varphi+1)-1}{(2\varphi+1)+1} \;=\; \frac{2\varphi}{2(\varphi+1)} \;=\; \frac{\varphi}{\varphi+1} \;=\; \frac{\varphi}{\varphi^2} \;=\; \frac{1}{\varphi}. \tag{10}$$

Therefore

$$\boxed{\tanh\!\left(\tfrac{3}{2}\ln\varphi\right) = \frac{1}{\varphi}} \qquad \Longleftrightarrow \qquad \boxed{\coth\!\left(\tfrac{3}{2}\ln\varphi\right) = \varphi}. \tag{11}$$

VAM Mapping to Tangential Velocity

In a rapidity parametrization, the dimensionless speed is

$$\beta \;\equiv\; \frac{v}{C_e} \;=\; \tanh\xi. \tag{12}$$

Setting $\xi = \xi_g$ gives the *golden* tangential fraction

$$\beta_g \;=\; \tanh(\xi_g) \;=\; \frac{1}{\varphi}, \tag{13}$$

and thus a characteristic tangential velocity and swirl frequency

$$v_g \;=\; \frac{C_e}{\varphi}, \qquad \Omega_g \;=\; \frac{v_g}{r_c} \;=\; \frac{1}{\varphi}\,\frac{C_e}{r_c}. \tag{14}$$

Both are dimensionally consistent: v_g has units of m/s and Ω_g of s^{-1}.

Numerical Validation (User Constants)

Using $C_e = 1\,093\,845.63\,\mathrm{m/s}$ and $r_c = 1.408\,970\,17 \times 10^{-15}\,\mathrm{m}$,

$$\varphi \approx 1.618033988749895, \tag{15}$$
$$\xi_g = \tfrac{3}{2}\ln\varphi \approx 0.721817737589405, \tag{16}$$
$$\beta_g = \tanh\xi_g \approx 0.618033988749895 \;=\; \frac{1}{\varphi}, \tag{17}$$
$$v_g = \frac{C_e}{\varphi} \approx 6.760\,337\,777\,855\,416 \times 10^{5}\,\mathrm{m/s}, \tag{18}$$
$$\Omega = \frac{C_e}{r_c} \approx 7.763\,440\,655\,383\,073 \times 10^{20}\,\mathrm{s}^{-1}, \tag{19}$$
$$\Omega_g = \frac{\Omega}{\varphi} \approx 4.798\,070\,194\,669\,498 \times 10^{20}\,\mathrm{s}^{-1}. \tag{20}$$

Consistency checks. Since $\beta_g = 1/\varphi$, we have $v_g = C_e/\varphi$ and $\Omega_g = \Omega/\varphi$ exactly, up to machine precision in floating-point arithmetic.

Discussion

This construction supplies a natural, dimensionless benchmark ($1/\varphi$) for tangential speeds in VAM. If swirl states quantize in hyperbolic angle ξ, the golden rapidity ξ_g defines a preferred scaling layer where tangential velocity and core swirl frequency are reduced by a factor φ relative to their maxima, potentially useful for defining stable vortexknot operating points or resonance bands in the spectrum.