Skyrmionic Photon Emission from Knotted Swirl Sources: A Vortex Æther Model (VAM) Synthesis and Predictions

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Abstract

Recent demonstrations of *single-photon skyrmions* in spin–orbit–engineered semiconductor microcavities and per-photon conservation of orbital angular momentum (OAM) in nonlinear down-conversion provide a rigorous template for a Vortex Æther Model (VAM) theory of photon emission from knotted swirl sources. We (i) map optical skyrmion topology to VAM vortex charge, (ii) define a projection law from normalized vorticity to single-photon Stokes fields, (iii) formulate OAM selection rules for multi-photon channels as a VAM "radiative vertex," and (iv) anchor the emission frequency scale to C_e/r_c with numerical validation using the user's constants. The resulting framework yields falsifiable predictions: topological spectroscopy of knotted emitters, chirality–helicity control at fixed OAM additivity, robustness of skyrmion number in propagation, and Purcell-tunable topological textures.

1 From optical skyrmions to VAM topology

Optical skyrmions synthesized by superposing Laguerre–Gaussian (LG) cavity modes with opposite circular polarizations carry an integer topological charge (skyrmion number) computed from the local Stokes vector field [1, 2, 3]. Denote the normalized Stokes vector by $\hat{\vec{S}}(\mathbf{k}$

2 Projection law: from swirl to Stokes

Let the (transverse) swirl-aligned polarization source on Σ be

p(r) = $\mathrm{p}_0(\mathbf{r})\,\hat{\boldsymbol{\omega}}_\perp(\mathbf{r})$(1)

where $\hat{\boldsymbol{\omega}}\perp$ is the component orthogonal to the radiation direction. The far-field Jones vector in mode space is given by a projected Huygens–Kirchhoff integral (dimensionally: field amplitude)

E(k) $\propto \int_\Sigma d^2r\, \mathbf{P}_\perp(\hat{\mathbf{k}})\, \mathbf{p}(\mathbf{r})\, e^{-i\mathbf{k}\cdot\mathbf{r}}, \qquad \mathbf{P}_\perp = \mathbf{I} - \hat{\mathbf{k}}\hat{\mathbf{k}}^\top (2)$

Expanding $\mathbf{E}$ in the LG basis $u_{p,\ell}(\mathbf{r})$ [3], with circular unit vectors $\hat{e}_\sigma$ ($\sigma = \pm 1$),

$$A_{p\ell\sigma} = \int_\Sigma d^2r \;\; (\hat{e}_\sigma^* \cdot \mathbf{P}_\perp \hat{\boldsymbol{\omega}}_\perp)\, u_{p,\ell}(\mathbf{r})\, e^{i\Phi(\mathbf{r})} \tag{3}$$

and the Stokes field $S_i(\mathbf{k}) = \mathbf{E}^\dagger \sigma_i \mathbf{E}/(\mathbf{E}^\dagger\mathbf{E})$ produces $N_{\text{sk}}^{(\text{ph})}$ via (**??**).

Conclusion 2 (LG synthesis in VAM). The same SAM–OAM mixing (spin–orbit coupling) used to build optical skyrmions in microcavities [1, 2] is reproduced by (3): knotted swirl geometry sets the LG superposition $A_{p\ell\sigma}$, hence the emitted photon's skyrmionic Stokes texture.

Dimensional check. $\mathbf{P}_\perp$ is dimensionless; $u_{p,\ell}$ is normalized mode amplitude; $e^{-i\mathbf{k}\cdot\mathbf{r}}$ dimensionless; d^2r gives area. $\mathbf{p}$ carries the source amplitude. Thus $A_{p\ell\sigma}$ has the correct field-amplitude dimension (arbitrary global normalization fixed by radiometry).

3 VAM radiative vertex: OAM additivity and chirality

Experiments show per-photon OAM conservation in SPDC: $\ell_p = \ell_s + \ell_i$ via the azimuthal integral and a commuting OAM operator [4, 5]. VAM adopts the same symmetry logic for a knotted source of azimuthal index ℓ_{src}:

$$\boxed{\ell_{\text{src}} = \sum_{j=1}^{n} \ell_j \quad \text{for an } n\text{-photon channel}} \tag{4}$$

with no strict conservation law on the radial indices p_j (set by overlap waists and geometry), exactly as in SPDC [5].

Conclusion 3 (chirality–helicity rule). In VAM, ccw (matter) swirl $\Rightarrow \sigma = +1$ photon helicity; cw (antimatter) swirl $\Rightarrow \sigma = -1$. External control that flips swirl chirality flips photon helicity while preserving (4)—mirroring polarity control of single-photon skyrmions in cavities [1].

4 Frequency and energy scale from VAM constants

Define the fundamental swirl eigenfrequency

$$\boxed{\Omega_0 \equiv \frac{C_e}{r_c}}, \qquad [\Omega_0] = \text{s}^{-1} \tag{5}$$

with quantized emission lines $\omega_{m\ell} = \Omega_0 \chi_{m\ell}$, $E_{m\ell} = \hbar\omega_{m\ell}$, where $\chi_{m\ell} \in (0, 1]$ are dimensionless eigenvalues fixed by geometry/cavity overlap.

Numerical validation (user constants).

$$C_e = 1.09384563\times10^{6}\,\mathrm{m\,s^{-1}}, \quad r_c = 1.40897017\times10^{-15}\,\mathrm{m} \Rightarrow \Omega_0 = \frac{C_e}{r_c} = 7.763440655\times10^{20}\,\mathrm{s^{-1}}$$

Hence

$$E_0 = \hbar\Omega_0 = (1.054571817\times10^{-34}\,\mathrm{J\,s})\times(7.763440655\times10^{20}\,\mathrm{s^{-1}}) = 8.18710565\times10^{-14}\,\mathrm{J} = 5.109989\times10^{5}\,\mathrm{eV}$$

This anchors the fundamental scale; higher modes satisfy $E_{m\ell} = \chi_{m\ell} \times 0.510999$ MeV (e.g. $\chi = 10^{-2} \Rightarrow 5.11$ keV). Units are consistent: $[C_e/r_c] = \mathrm{s}^{-1}$, $[\hbar\omega] = \mathrm{J}$.

5 Mapped source classes (examples)

- **Trefoil ($T(2,3)$)-class**: dominant $\ell \simeq 3$ along axis; single-photon Stokes texture with $N_{\text{sk}} = \pm2$ from the $\mathrm{LG}^{\sigma\pm}_{0,\mp2} + \mathrm{LG}^{\sigma\mp}_{1,0}$-type superposition (as realized in cavity skyrmions [1]); multiphoton channels respect (4).
- **Higher hyperbolic knots (5_2, 7_1)**: skyrmionium-like textures (multi-ring Stokes structure) accessible by shifting cavity aspect ratios, consistent with observed $k\pi$ optical skyrmionia at higher cavity orders [1].

6 Falsifiable predictions

(P1) Topological spectroscopy. A trefoil-class emitter yields a biphoton OAM correlation matrix concentrated on $\ell_s + \ell_i = \ell_{\text{src}} \simeq 3$, with radial-index spread governed by waists/overlaps (no p selection) [5, 4].

(P2) Chirality–helicity flip at fixed OAM sum. Reversing swirl chirality flips photon helicity ($\sigma \to -\sigma$) and the sign of the skyrmion polarity in the Stokes texture, while the OAM sum rule (4) remains intact (cavity-polarity flip analog [1]).

(P3) Robustness of $N^{(\text{ph})}_{\text{sk}}$. Attenuators, polarizers, and phase shifters perturb intensity/polarization locally but preserve the integer $N^{(\text{ph})}_{\text{sk}}$ (topological protection) [1].

(P4) Purcell-tunable topology. Changing the cavity aspect ratio/defect radius moves the system between single-ring and skyrmionium Stokes textures by reweighting $A_{p\ell\sigma}$ via the Purcell-enhanced overlap [7, 1].

7 Minimal experimental roadmap

1. **Source:** a skyrmionic microcavity (or RF/optical metacavity) supporting two near-degenerate LG modes with opposite σ; drive a knotted swirl (trefoil) near $\omega_{m\ell}$.

2. **Readout:** reconstruct single-photon Stokes maps $S_{1,2,3}(x, y)$ and OAM spectra; extract $N_{\rm sk}^{\rm (ph)}$ and verify (4).
3. **Control:** flip swirl chirality (field reversal) and confirm helicity flip with conserved OAM sum; scan cavity aspect ratio to toggle skyrmion $\leftrightarrow$ skyrmionium textures.

8 Conclusions

The two external pillars—single-photon skyrmions in spin–orbit cavities and per-photon OAM conservation—close a consistent loop with VAM: (i) topology of the emitter maps to topology of the photon's Stokes field; (ii) OAM additivity furnishes a clean radiative selection rule; (iii) the VAM scale C_e/r_c fixes emission energies with dimensional clarity; (iv) robustness and cavity control provide direct levers for experiments. This synthesis upgrades VAM from static knotted ontology to a predictive, testable photon-emission theory.

Acknowledgment of non-original elements

Equations (**??**), (2)–(3) rely on standard Stokes/OAM/LG-mode optics and cavity skyrmion constructions [1, 3, 2, 8, 9] and on OAM selection results in SPDC [4, 5]. The topological integral form parallels classical vortex-helicity ideas [6].

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