The Vortex Æther Model: Æther Vortex Field Model

Omar Iskandarani[1]

[1]*Independent Researcher, Groningen, The Netherlands**

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Abstract

The Vortex Æther Model (VAM) introduces a unified, non-relativistic framework wherein gravity, electromagnetism, and quantum phenomena emerge from structured vorticity in an inviscid superfluid-like æther. Unlike General Relativity, which invokes spacetime curvature, VAM models stable vortex knots in a 3D Euclidean medium with absolute time. Observed time dilation results from vortex-induced local energy gradients. This paper derives time dilation analogs to GR, explores vortex-energetic time shifts, and presents experimental implications.

*ORCID: 0009-0006-1686-3961; Electronic address: info@omariskandarani.com

Core Assumptions

The Æther is modeled as an inviscid, incompressible superfluid governed by:

* Conservation of Absolute Vorticity
* A 3D Euclidean medium with absolute time
* Particles as vortex knots
* Irrotational outside vortex cores, but with conserved vorticity inside knots
* Gravity from vorticity-induced pressure gradients

Symbol	Description
$\vec{v}$	Æther velocity field
$\vec{\omega}$	Vorticity $\vec{\omega} = \nabla \times \vec{v}$
$\rho_{æ}$	Æther density (constant)
Φ	Vorticity-induced potential
κ	Circulation constant
$\mathcal{K}$	Knot topological class (Hopf link, torus knot, etc.)

Introduction to Fluid Dynamics and Vorticity Conservation

Euler Equation (Inviscid Flow)

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho_{æ}} \nabla p \tag{1}$$

Taking the curl to get the Vorticity Transport

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{v} \cdot \nabla)\vec{\omega} = (\vec{\omega} \cdot \nabla)\vec{v} \tag{2}$$

Vorticity-Induced Gravity

We define a Newtonian like vorticity-based gravitational potential Φ:

$$\vec{F}_g = -\nabla \Phi \tag{3}$$

Where Φ is the Vorticity Potential:

$$\Phi(\vec{r}) = \gamma \int \frac{|\vec{\omega}(\vec{r'})|^2}{|\vec{r} - \vec{r'}|} \, d^3r' \tag{4}$$

This mirrors the Newtonian potential but replaces mass density with vorticity intensity. This gives attractive force fields between vortex knots (like a particle).

I. TIME DILATION FROM VORTEX DYNAMICS

We consider an inviscid, irrotational superfluid æther with stable topological vortex knots. The Æther experiences absolute time t_{abs}, but local clocks experience slowed rates due to pressure gradients and knot energetics. The Vortex Æther Model posits that the rate at which time flows in the local frame (near the knot) depends on the internal angular frequency Ω_k. In this section, we derive time dilation analogues inspired by the predictions of general relativity (GR), based solely on pressure and vorticity gradients in the fluid.

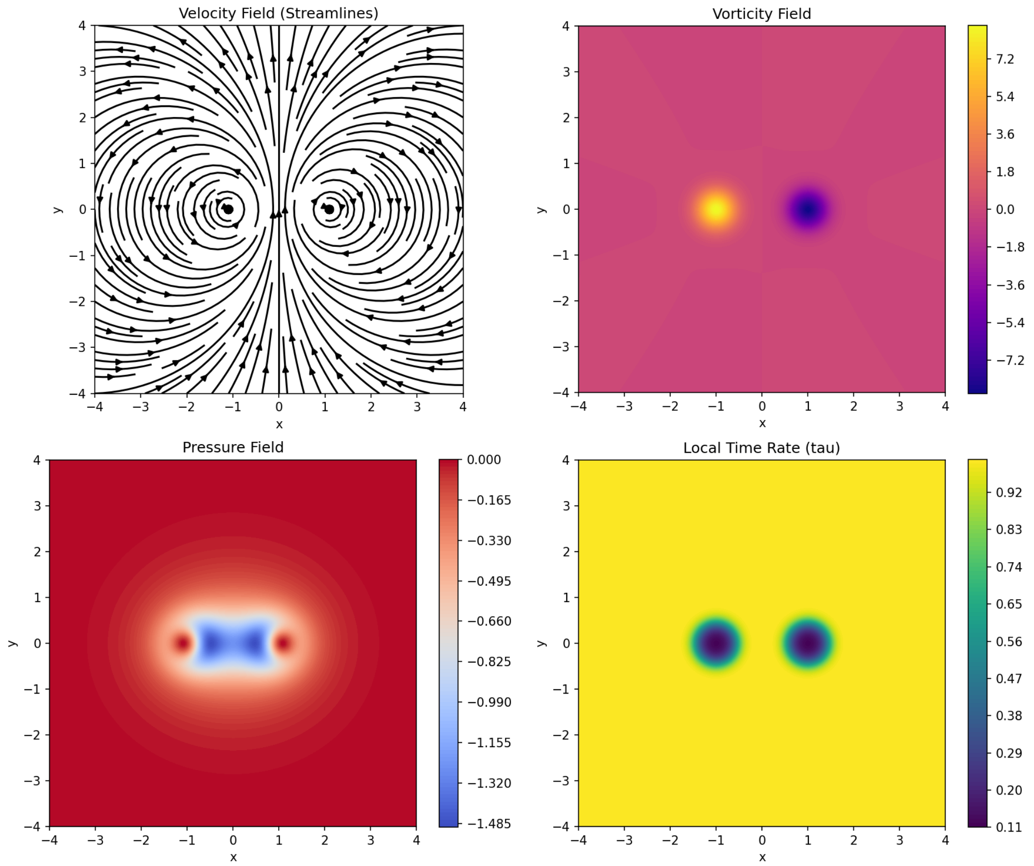

FIG. 1: Velocity streamlines, vorticity, pressure, and local time rate τ for a simulated vortex pair. The pressure minimum and time slow-down clearly align with the regions of high vorticity. This directly illustrates the æther model's central claim: time dilation follows from vortex energetics and pressure depletion.

A. Bernoulli Flow and Local Time Depletion

In a classical, inviscid, incompressible fluid, Bernoulli's equation describes the conservation of energy in a flow:

$$\frac{1}{2}\rho_{\text{æ}v^2+p=p_0\Rightarrow p=p_0-\frac{1}{2}\rho_{\text{æ}v^2}}(5)$$

Here:

- p_0 is the background reference pressure,
- $\rho_{\text{æ}}$ is the constant æther density,
- v is the local velocity of the æther near the vortex.

Assuming that clock rate is proportional to pressure (i.e., time slows in low-pressure regions), we relate the local clock frequency to the background as:

$$\frac{f_{\text{local}}}{f_0} = 1 - \frac{\rho_{\text{æ}v^2}}{2p_0} \tag{6}$$

Hence, time dilation is:

$$\frac{t_{\text{local}}}{t_0} = \left(1 - \frac{\rho_{\text{æ}v^2}}{2p_0}\right)^{-1} \tag{7}$$

For rotational flow, with $v = \Omega r$,

$$\frac{t_{\text{local}}}{t_0} = \left(1 - \frac{\rho_{\text{æ}\Omega^2 r^2}}{2p_0}\right)^{-1} \approx 1 + \frac{\rho_{\text{æ}\Omega^2 r^2}}{2p_0} \tag{8}$$

This expression recovers the first-order time dilation analog if we define the dimensionless coupling:

$$\frac{\rho_{\text{æ}}}{p_0} \sim \frac{1}{c^2} \tag{9}$$

This motivates the analogy to relativistic time dilation:

$$\frac{t_{\text{moving}}}{t_{\text{rest}}} \approx 1 + \frac{v^2}{2c^2} \tag{10}$$

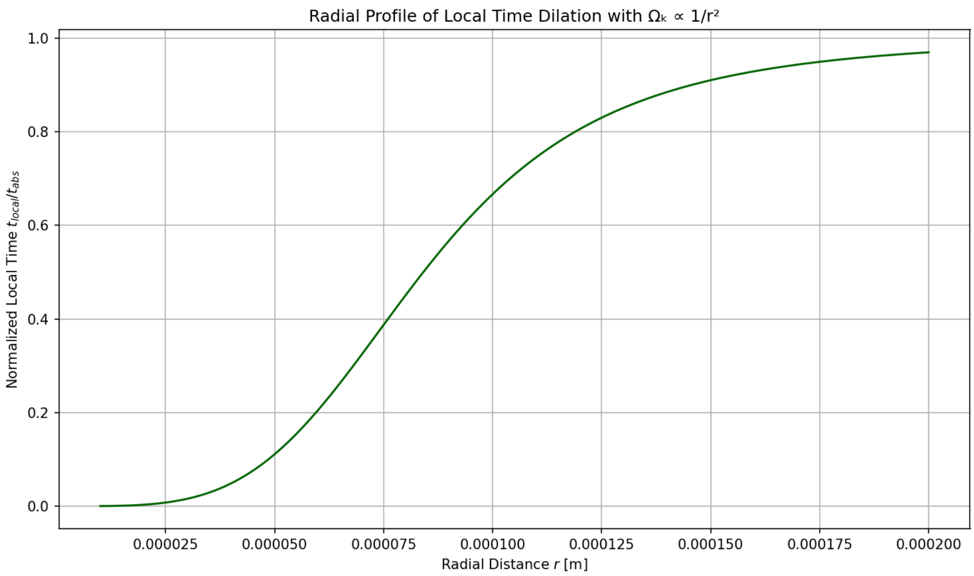

FIG. 2: Radial profile of normalized local time $t_{\mathrm{local}}/t_{\mathrm{abs}}$ as a function of distance r from the vortex core, assuming $\Omega_k \propto 1/r^2$. Time slows significantly near the vortex center and recovers to background values with distance.

B. Heuristic Knot-Based Time Modulation

Topological vortex knots have intrinsic angular frequency Ω_k, conserved due to vorticity confinement. We introduce a first-principles motivated time dilation expression:

$$\frac{t_{\mathrm{local}}}{t_{\mathrm{abs}}} = \left(1 + \alpha\Omega_k^2\right)^{-1} \tag{11}$$

where α is a coupling parameter with dimensions $[\alpha] = \mathrm{s}^2$. Expanding for small Ω_k:

$$\frac{t_{\mathrm{local}}}{t_{\mathrm{abs}}} \approx 1 - \alpha\Omega_k^2 + \mathcal{O}(\Omega_k^4) \tag{12}$$

This form mirrors the expansion of the Lorentz factor:

$$\frac{t_{\mathrm{moving}}}{t_{\mathrm{rest}}} \approx 1 - \frac{v^2}{2c^2} \tag{13}$$

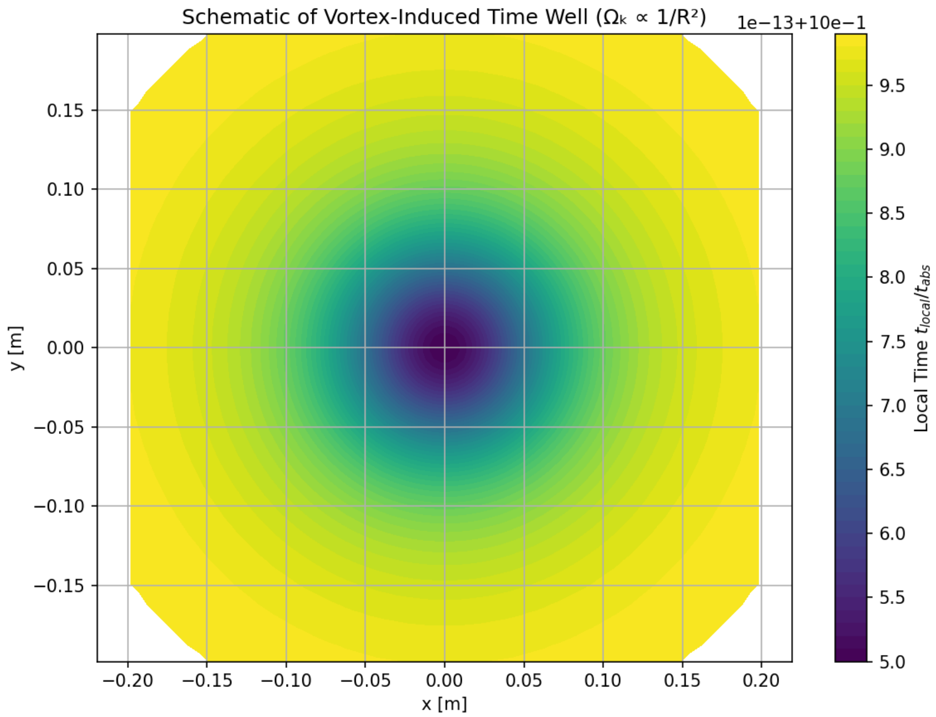

FIG. 3: Schematic of a vortex-induced time well in the æther. Local time $t_{\text{local}}/t_{\text{abs}}$ is shown as a color gradient in 2D space. The central vortex region exhibits the most time slowing due to high Ω_k, forming a well-like structure.

C. Time Dilation from Rotational Inertia

We now ground the heuristic form in physical energetics. For a knot with moment of inertia I, the rotational energy is:

$$E_{\text{rot}} = \frac{1}{2} I \Omega_k^2 \tag{14}$$

Thus, the time dilation becomes:

$$\frac{t_{\text{local}}}{t_{\text{abs}}} = (1 + \alpha E_{\text{rot}})^{-1} = \left(1 + \frac{1}{2}\alpha I \Omega_k^2\right)^{-1} \tag{15}$$

This boxed equation is the core result of this section:

$$\boxed{\frac{t_{\text{local}}}{t_{\text{abs}}} = \left(1 + \frac{1}{2}\alpha I \Omega_k^2\right)^{-1}} \tag{16}$$

D. Summary of Model Hierarchy

- Pressure-Based (Bernoulli): Time slows in low-pressure zones due to vortex velocity.
- Heuristic Angular Model: Time slows as a function of Ω_k^2.
- Energetic Model: Time flow depends on stored rotational energy in the knot.

These form a continuum of physical justification, culminating in a replacement of spacetime curvature with rotational æther mechanics. This establishes the VAM time dilation framework as a fluidic, topologically-conserved analog to GR.

Next, we will explore how these models correspond to GR-like metrics and rotational observers in Section II.

Section II: Time Modulation by Vortex Knot Rotation

Building upon the previous section's treatment of time dilation via pressure and Bernoulli dynamics, we now focus on the intrinsic rotation of topological vortex knots. In the Vortex Æther Model (VAM), particles are modeled as stable, topologically conserved vortex knots embedded in an incompressible, inviscid superfluid medium. Each knot possesses a characteristic internal angular frequency Ω_k, and this internal motion induces local time modulation relative to the absolute time of the æther.

Rather than curving spacetime, we propose that internal rotational energy and helicity conservation induce temporal slowdowns analogous to gravitational redshift. This section develops these ideas through heuristic and energetic arguments consistent with the hierarchy introduced in Section I.

A. Heuristic and Energetic Derivation

We begin by proposing a rotationally-induced time dilation formula based on the knot's internal angular frequency:

$$\frac{t_{\text{local}}}{t_{\text{abs}}} = \left(1 + \alpha\Omega_k^2\right)^{-1} \tag{17}$$

where:

- t_{local} is the proper time near the knot,
- t_{abs} is the absolute time of the background æther,
- Ω_k is the average core angular frequency,
- α is a coupling coefficient with dimensions $[\alpha] = \text{s}^2$.

For small angular velocities, we obtain a first-order expansion:

$$\frac{t_{\text{local}}}{t_{\text{abs}}} \approx 1 - \alpha\Omega_k^2 + \mathcal{O}(\Omega_k^4) \tag{18}$$

This form parallels the Lorentz factor at low velocities in special relativity:

$$\frac{t_{\text{moving}}}{t_{\text{rest}}} \approx 1 - \frac{v^2}{2c^2} \tag{19}$$

This establishes an important analogy: internal rotational motion in VAM induces temporal slowing, similar to how translational velocity induces time dilation in SR.

To strengthen the physical foundation of this expression, we now relate time dilation to the energy stored in vortex rotation. Let the vortex knot have an effective moment of inertia I. Its rotational energy is given by:

$$E_{\text{rot}} = \frac{1}{2} I \Omega_k^2 \tag{20}$$

Assuming time slows due to this energy density, we write:

$$\frac{t_{\text{local}}}{t_{\text{abs}}} = (1 + \alpha E_{\text{rot}})^{-1} = \left(1 + \frac{1}{2}\alpha I \Omega_k^2\right)^{-1} \tag{21}$$

This expression serves as the energetic analog of the pressure-based Bernoulli model from Section I. It supports the interpretation of vortex-induced time wells via energy storage rather than geometric deformation.

We highlight this key result with a boxed formulation:

$$\boxed{\frac{t_{\text{local}}}{t_{\text{abs}}} = \left(1 + \frac{1}{2}\alpha I \Omega_k^2\right)^{-1}} \tag{22}$$

B. Topological and Physical Justification

Topological vortex knots are not only characterized by rotation but also by helicity:

$$H = \int \vec{v} \cdot \vec{\omega} \, d^3x \tag{23}$$

Helicity is a conserved quantity in ideal (inviscid, incompressible) fluids, encoding the linkage and twisting of vortex lines. The rotational frequency Ω_k becomes a topologically meaningful indicator of the knot's identity and dynamical state.

Higher Ω_k implies greater rotational energy and stronger localized pressure depletion, forming a "temporal well" in the æther. These wells naturally mimic gravitational redshift effects in curved spacetime, but arise here purely from classical fluid mechanics.

This model:

- Attributes time modulation to conserved, intrinsic rotational energy,
- Requires no external reference frames (absolute æther time is universal),
- Preserves temporal isotropy outside the vortex core,
- Provides a natural replacement for GR's spacetime curvature.

Therefore, this vortex-energetic time dilation principle provides a powerful alternative to relativistic time modulation by anchoring all temporal effects in rotational energetics and topological invariants.

In the next section, we will show how these ideas reproduce metric-like behavior for rotating observers, including a direct fluid-mechanical analog to the Kerr metric of General Relativity.

Section III: Proper Time for a Rotating Observer in Æther Flow

Having established time dilation in the Vortex Æther Model (VAM) through pressure, angular velocity, and rotational energy, we now extend our formalism to rotating observers. This section demonstrates that fluid-dynamic time modulation in VAM can reproduce expressions structurally similar to those derived in General Relativity (GR), particularly in axisymmetric rotating spacetimes like the Kerr geometry. However, VAM achieves this without invoking spacetime curvature. Instead, time modulation is governed entirely by kinetic variables in the æther field.

A. GR Proper Time in Rotating Frames

In General Relativity, the proper time $d\tau$ for an observer with angular velocity Ω_{eff} in a stationary, axisymmetric spacetime is given by:

$$\left(\frac{d\tau}{dt}\right)^2_{\text{GR}} = -\left[g_{tt} + 2g_{t\varphi}\Omega_{\text{eff}} + g_{\varphi\varphi}\Omega_{\text{eff}}^2\right] \tag{18}$$

where $g_{\mu\nu}$ are components of the spacetime metric (e.g., in Boyer–Lindquist coordinates for Kerr spacetime). This formulation accounts for both gravitational redshift and rotational (frame-dragging) effects.

B. Æther-Based Analog: Velocity-Derived Time Modulation

In VAM, spacetime is not curved. Instead, observers reside within a dynamically structured æther whose local flow velocities determine time dilation. Let the radial and tangential components of æther velocity be:

- v_r: radial velocity,
- $v_\varphi = r\Omega_k$: tangential velocity due to local vortex rotation,
- $\Omega_k = \frac{\kappa}{2\pi r^2}$: local angular velocity (with κ as circulation).

We postulate a correspondence between GR metric components and æther velocity terms:

$$\begin{aligned} g_{tt} &\to -\left(1 - \frac{v_r^2}{c^2}\right), \\ g_{t\varphi} &\to -\frac{v_r v_\varphi}{c^2}, \\ g_{\varphi\varphi} &\to -\frac{v_\varphi^2}{c^2 r^2} \end{aligned} \tag{19}$$

Substituting these into the GR expression for proper time, we obtain the VAM-based analog:

$$\left(\frac{d\tau}{dt}\right)^2_{æ} = 1 - \frac{v_r^2}{c^2} - \frac{2v_r v_\varphi}{c^2} - \frac{v_\varphi^2}{c^2} \tag{20}$$

Combining the terms:

$$\left(\frac{d\tau}{dt}\right)^2_{æ} = 1 - \frac{1}{c^2}(v_r + v_\varphi)^2 \tag{21}$$

This formulation reproduces gravitational and frame-dragging time effects purely from Æther dynamics: $\langle\omega^2\rangle$ plays the role of gravitational redshift, and circulation κ encodes rotational drag. This approach aligns with recent fluid-dynamic interpretations of gravity and time [8], [1]. This model currently assumes irrotational flow outside knots and neglects viscosity, turbulence, and quantum compressibility. Future extensions may include quantized circulation spectra or boundary effects in confined Æther systems.

$$\boxed{\left(\frac{d\tau}{dt}\right)^2_{æ} = 1 - \frac{1}{c^2}(v_r + r\Omega_k)^2} \qquad \text{(Æther-Based Proper Time for Rotating Observer)}$$

C. Physical Interpretation and Model Consistency

This boxed result mirrors the GR expression for rotating observers but arises strictly from classical fluid dynamics. It shows that as the local æther speed approaches the speed of light—due to either radial inflow or rotational motion—the proper time slows. This implies the existence of "time wells" where kinetic energy density dominates.

Key observations:

- In the absence of radial flow ($v_r = 0$), time slowing arises entirely from vortex rotation.
- When both v_r and Ω_k are present, the cumulative velocity reduces local time rate.
- This expression agrees with Section II's energetic model if we interpret $v_r + r\Omega_k$ as contributing to the local energy density.

Thus, in the VAM framework, the structure of the observer's proper time emerges from ætheric flow fields. This confirms that GR-like temporal behavior can emerge in a flat, Euclidean 3D space with absolute time, governed entirely by structured vorticity and circulation.

In the next section, we explore how VAM extends this correspondence to gravitational potentials and frame-dragging effects via circulation and vorticity intensity, forming an analog to the Kerr time redshift formula.

II. KERR-LIKE TIME ADJUSTMENT FROM VORTICITY AND CIRCULATION

To complete the analogy between General Relativity (GR) and the Vortex Æther Model (VAM), we now derive a time modulation formula that mirrors the redshift and frame-dragging structure found in the Kerr solution. In GR, the Kerr metric describes the spacetime geometry around a rotating mass, predicting both gravitational time dilation and frame-dragging due to angular momentum. VAM captures similar phenomena through the dynamics of structured vorticity and circulation in the æther, without requiring spacetime curvature.

A. General Relativistic Kerr Redshift Structure

In the GR Kerr metric, the proper time $d\tau$ for an observer near a rotating mass is affected by both mass-energy and angular momentum. A simplified approximation for the time dilation factor near a rotating body is:

$$t_{\text{adjusted}} = \Delta t \cdot \sqrt{1 - \frac{2GM}{rc^2} - \frac{J^2}{r^3c^2}} \tag{22}$$

where:

- M: mass of the rotating body,
- J: angular momentum,
- r: radial distance from the source,
- G: Newton's gravitational constant,
- c: speed of light.

The first term corresponds to gravitational redshift from mass, while the second accounts for rotational (frame-dragging) effects.

B. Æther Analog via Vorticity and Circulation

In VAM, we express gravitational-like influences through vorticity intensity $\langle \omega^2 \rangle$ and total circulation κ. These are interpreted as:

- $\langle \omega^2 \rangle$: mean squared vorticity over a region,
- κ: conserved circulation, encoding angular momentum.

We define the æther-based analog by making the replacements:

$$\begin{gathered} \frac{2GM}{rc^2} \to \frac{\gamma \langle \omega^2 \rangle}{rc^2}, \\ \frac{J^2}{r^3c^2} \to \frac{\kappa^2}{r^3c^2} \end{gathered} \tag{23}$$

Here, γ is a coupling constant relating vorticity to effective gravitational strength (analogous to G). Then the æther-based proper time becomes:

$$\boxed{t_{\text{adjusted}} = \Delta t \cdot \sqrt{1 - \frac{\gamma \langle \omega^2 \rangle}{rc^2} - \frac{\kappa^2}{r^3c^2}}}$$

(Kerr-Like Time Dilation from Vorticity and Circulation)

This formulation preserves the structure of Kerr's redshift and frame-dragging effects, now recast in terms of measurable fluid-dynamic quantities. In this picture:

- $\langle \omega^2 \rangle$ plays the role of energy density producing gravitational redshift,
- κ represents angular momentum generating temporal frame-dragging,
- The equation reduces to flat æther time ($t_{\text{adjusted}} \to \Delta t$) when both terms vanish.

C. Model Assumptions and Scope

This result depends on several assumptions:

- The flow is irrotational outside the vortex cores,
- Viscosity and turbulence are neglected,
- Compressibility is ignored (ideal incompressible superfluid),
- Vorticity fields are sufficiently smooth to define $\langle \omega^2 \rangle$.

These conditions mirror the assumptions of ideal fluid GR analog models. The formulation bridges the macroscopic flow dynamics of the æther with effective geometric predictions, reinforcing the possibility of replacing curved spacetime with structured vorticity fields.

For detailed derivations of cross-energy and vortex interaction energetics, see Appendix A 6.

In future work, corrections for boundary conditions, quantized vorticity spectra, and compressible effects may be added to refine the analogy. Next, we will summarize how these fluid-based time dilation mechanisms unify under the VAM framework and identify their experimental implications.

Section V: Unified Framework and Synthesis of Time Dilation in VAM

We now consolidate the various time dilation mechanisms explored throughout this manuscript into a unified framework under the Vortex Æther Model (VAM). By moving beyond geometric spacetime curvature, VAM provides a consistent and physically motivated model for temporal modulation grounded in classical fluid dynamics, rotational energetics, and topological vorticity structures.

Hierarchical Structure of Time Dilation Mechanisms

Each section of this work contributes a distinct yet interrelated mechanism for time dilation:

1. Bernoulli-Induced Time Depletion: Time slows near regions of low pressure resulting from vortex-induced kinetic velocity fields. This recovers a special relativistic time dilation form when $\rho_{æ/p_0 \sim 1/c^2}$.

2. Angular Frequency Heuristic Model: A quadratic dependence of time rate on local knot angular frequency Ω_k^2, mimicking the Lorentz factor expansion for small velocities.

3. Energetic Formulation via Rotational Inertia: The core result:

$$\frac{t_{\text{local}}}{t_{\text{abs}}} = \left(1 + \frac{1}{2}\alpha I \Omega_k^2\right)^{-1}$$

links time modulation directly to the rotational energy of vortex knots.

4. Velocity-Field Based Proper Time Flow: Time dilation derives from local flow velocities v_r and $v_\varphi = r\Omega_k$, providing a fluid analog to proper time in Kerr spacetimes:

$$\left(\frac{d\tau}{dt}\right)^2 = 1 - \frac{1}{c^2}(v_r + r\Omega_k)^2$$

5. Kerr-Like Redshift and Frame-Dragging: Vorticity intensity $\langle\omega^2\rangle$ and circulation κ replace gravitational mass and angular momentum in the Kerr redshift formula:

$$t_{\text{adjusted}} = \Delta t \cdot \sqrt{1 - \frac{\gamma\langle\omega^2\rangle}{rc^2} - \frac{\kappa^2}{r^3c^2}}$$

These five expressions form a self-consistent ladder, ranging from heuristic to rigorous, and establish a robust replacement for general relativistic time dilation based entirely on classical field variables.

Physical Unification: Time as a Vorticity-Derived Observable

Across all formulations, a recurring theme emerges: time modulation in VAM is always reducible to local kinetic or rotational energy density within the æther. Whether encoded in pressure (Bernoulli), angular frequency (Ω_k), or field circulation (κ), the modulation of time is not geometric but energetic and topological.

Key unifying elements:

- Local Time Wells form due to high vorticity and circulation.
- Frame-Independence: Absolute time exists; only local rates are affected.
- No Need for Tensor Geometry: All time effects arise from scalar or vector fields.
- Topological Conservation: Vortex knots preserve helicity and circulation, ensuring temporal consistency.

This unification reinforces VAM's conceptual core: spacetime curvature is an emergent illusion produced by structured vorticity in an absolute, superfluid æther.

Experimental Implications and Outlook

Each time dilation formula introduced here can, in principle, be tested in laboratory analog systems:

- Rotating superfluid droplets (e.g., Helium-II, BECs) can simulate vortex-induced slowing of local clock signals.
- Electrohydrodynamic lifters and plasma vortex systems may reveal temporal energy gradients.
- Magneto-fluidic and optical analogs can encode vorticity fields mimicking $\langle\omega^2\rangle$ structures.

Future work includes:

- Deriving dynamic equations for temporal feedback in multi-knot systems.
- Measuring vortex-induced clock drift in rotating superfluids.
- Applying the model to astrophysical observations (e.g., neutron star precession, frame dragging, time delay).

In summary, the Vortex Æther Model offers a compelling, energetically grounded framework for gravitational time dilation that is logically and mathematically consistent, experimentally tractable, and conceptually elegant. By eliminating curvature and restoring absolute time, VAM provides a classical foundation for gravitational phenomena through the lens of vorticity and topological flow dynamics.

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Appendix A: Foundations of Velocity Fields and Energies in a Vortex System.

1. Introduction

Vortex dynamics are a core component of many fluid and plasma systems, including tornado-like flows, knotted vortices in classical or superfluid turbulence, and various complex topological fluid systems. A deeper understanding of the energy budgets associated with these flows can shed light on processes like vortex stability, reconnection, and global flow organization. We begin by motivating how velocity fields can be decomposed so as to capture the total energy (i.e. self- plus cross-energy), and how this approach helps track flows in both 2D and 3D.

2. Foundations: Velocity Fields and Total (Self + Cross) Energy

In an incompressible fluid, the velocity field $\mathbf{u}(\mathbf{x}, t)$ is typically governed by the Navier–Stokes or Euler equations. For inviscid analyses, the Euler equations for incompressible flow read

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p, \quad \nabla \cdot \mathbf{u} = 0. \tag{A1}$$

We also consider the vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$, which can be used to characterize vortex structures.

To understand the *total* kinetic energy, we can split it as follows:

$$E_{\text{total}} \ = \ E_{\text{self}} \ + \ E_{\text{cross}}. \tag{A2}$$

Here, E_{self} is that portion of energy which each vortex or partial flow element contributes independently (for instance, from local swirling motions), while E_{cross} encodes the contributions that arise from the interaction of different vortical elements. In a multi-vortex scenario, such a decomposition helps isolate the direct interaction between two (or more) vortex filaments or sheets.

3. Momentum and Self-Energy Considerations

A starting point is to recall that for a single vortex of circulation Γ, with an azimuthally symmetric core, the induced velocity is sometimes approximated by classical results such as

$$V \;=\; \frac{\Gamma}{4\pi R}\big(\ln \tfrac{8R}{a} - \beta\big), \tag{A3}$$

where R is the main vortex loop radius, $a \ll R$ is a measure of core thickness, and β depends on details of the core model [1]. The *self-energy* associated with that vortex, E_{self}, can be cast in a similar form that depends on $\ln(R/a)$, exemplifying how thin-core vortices' energies scale with geometry.

In more general fluid or vortex-lattice models, we can track E_{self} as the sum of individual core energies. Further, the presence of multiple filaments modifies the total energy by cross-terms of the velocity fields (the cross-energy). This cross-energy often drives key phenomena such as vortex merging or the 'recoil' effects in wave–vortex interactions.

4. Defining and Tracking Cross-Energy

When multiple vortices (or partial velocity distributions) co-exist, the total velocity field $\mathbf{u}$ can be superposed:

$$\mathbf{u} \;=\; \mathbf{u}_1 \;+\; \mathbf{u}_2, \tag{A4}$$

where $\mathbf{u}_1$ and $\mathbf{u}_2$ come from distinct sub-systems. In that scenario, the kinetic energy for a fluid volume V is

$$E_{\text{total}} = \frac{\rho}{2}\int_V \mathbf{u}^2\, dV = \frac{\rho}{2}\int_V \big(\mathbf{u}_1 + \mathbf{u}_2\big)^2\, dV \tag{A5}$$

$$= \frac{\rho}{2}\int_V \mathbf{u}_1^2\, dV \;+\; \frac{\rho}{2}\int_V \mathbf{u}_2^2\, dV \;+\; \rho\int_V \mathbf{u}_1\cdot\mathbf{u}_2\, dV, \tag{A6}$$

revealing an interaction or *cross-energy* term

$$E_{\text{cross}} \;=\; \rho\int_V \mathbf{u}_1\cdot\mathbf{u}_2\, dV. \tag{A7}$$

Much of the interesting physics arises from (A7), because it grows or shrinks depending on the vortex geometry and distance between them. Its dynamical evolution can lead to, e.g., merging or rebound. A main point is that each vortex's self-velocity can significantly affect the mutual velocities and thus create net forces or torque.

5. Applications to Helicity and Topological Flows

A related concept is helicity, measuring the topological complexity (knotting or linking) of vortex tubes. Classically, helicity H is given by

$$H \;=\; \int_V \mathbf{u} \cdot \boldsymbol{\omega} \, dV, \tag{A8}$$

which can remain constant or be partially lost during reconnection events. In certain dissipative flows, the cross-energy terms in (A7) can influence the effective rate of helicity change. Understanding E_{cross} is important for analyzing reconnection pathways in classical or superfluid turbulence.

6. Derivation Outline for Cross-Energy

Finally, we provide a succinct outline for deriving the cross-energy expression. Starting with the total velocity field $\mathbf{u} = \sum_{n=1}^{N} \mathbf{u}_n$ for N vortex or partial velocity fields, the total kinetic energy is:

$$E_{\text{total}} = \frac{\rho}{2} \int_V \left(\sum_{n=1}^{N} \mathbf{u}_n \right)^2 dV = \frac{\rho}{2} \sum_{n=1}^{N} \int_V \mathbf{u}_n^2 \, dV \;+\; \rho \sum_{n<m} \int_V \mathbf{u}_n \cdot \mathbf{u}_m \, dV. \tag{A9}$$

One obtains N self-energy terms plus pairwise cross-energy integrals. The cross-energy for a pair (i, j) is:

$$E_{\text{cross}}^{(ij)} \;=\; \rho \int_V \mathbf{u}_i \cdot \mathbf{u}_j \, dV. \tag{A10}$$

In practice, each $\mathbf{u}_n$ may be represented by known solutions of the Stokes or potential flow equations, or from approximate solutions for vortex loops. Then, either analytically or numerically, one obtains approximate cross-energies that can be used in reduced models describing the evolution of multi-vortex systems.

Conclusion

We have surveyed how the total fluid kinetic energy in the presence of multiple vortices can be split into self- and cross-energy terms. These cross-energy contributions are crucial for understanding vortex merging, knotted vortex untangling, or vortex–wave interactions in classical, superfluid, and plasma flows. In addition, we have sketched a systematic derivation of cross-energy and highlighted key aspects in discussing momentum and helicity. Future directions include refining these expressions for axisymmetric or knotted vortices and integrating them into large-scale models or computational frameworks.

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I. Vortex Knots as Particles

Each particle is a topological vortex knot:

- Charge twist or chirality of knot
- Mass integrated vorticity energy
- Spin knot helicity:

Helicity as Particle Identity

$$\mathcal{H} = \int \vec{v} \cdot \vec{\omega} \, d^3x \tag{A11}$$

Stability knot type (Hopf links, Trefoil, etc.) and energy minimization in the vortex core

II. Vortex Thread Interaction

Interactions arise from exchange of vorticity or reconnections between vortex filaments:

- Attractive if threads reinforce circulation (parallel)
- Repulsive if threads cancel (antiparallel)
- Interaction strength:

$$\vec{F}_{\text{int}} = \beta \cdot \kappa_1 \kappa_2 \cdot \frac{\vec{r}_{12} \times (\vec{v}_1 - \vec{v}_2)}{|\vec{r}_{12}|^3} \tag{A12}$$

Where κ_i are circulations of filaments and $\vec{r}_{12}$ is the vector between them.

III. Thermodynamic Quantum Behavior from Vorticity Fluctuations

- Entropy $\leftrightarrow$ volume of vortex expansion or knot deformation
- Quantum transitions $\leftrightarrow$ topological reconnection events
- Zero-point motion $\leftrightarrow$ background quantum turbulence of the Æther:

Quantum Vorticity Background

$$\langle \omega^2 \rangle \sim \frac{\hbar}{\rho_{\text{æ}} \xi^4} \tag{A13}$$

Where ξ is the coherence length between vortex filaments