1 Symmetry Classification of Knot-based Vortex Structures in the Vortex Æther Model (VAM)

In VAM, these symmetries classify the invariance properties of knotted ætheric filaments, constraining their physical stability, energy quantization, and possible transformation pathways. Remarks.

Any D_{2k} symmetry ($k \geq 2$) implies $D_2(r)$ symmetry; if k is even, period 2 is also present. D_{2k} symmetry further implies D_{2j} for divisors j of k. Z_{2k} symmetry entails positive amphichirality; $D_2(r)$ guarantees reversibility. I_2 symmetry implies negative amphichirality. These properties map directly to constraints on vortex knot energy spectra, fusion/interconversion rules, and topological charge conservation in VAM. The "full symmetry group" (FSG) is tabulated for comparison, though it may not capture all VAM-relevant invariances.

Note.

Knots such as 8_{10}, 8_{16}, 8_{17}, and 8_{20}, for which period 2 is absent, also uniquely have FSG D_2 among prime knots with 8 or fewer crossings, reflecting special restrictions on allowable vortex periodicities and energy levels in the æther. Exceptional knots $12a_{1202}$ and 15331 are included for their rare Z_2 symmetry, potentially corresponding to novel or unanticipated ætheric field states.

2 Glossary of Symmetry Table Symbols

$D_2(r)$ **Order-2 Dihedral (Reflectional) Symmetry.** The knot (or vortex structure) admits a dihedral symmetry of order 2, meaning it is invariant under a 180° rotation and a reflection; this often guarantees *reversibility* (the knot is equivalent to itself with reversed orientation).

D_{2k} **Higher-Order Dihedral Symmetry.** The knot is invariant under the full dihedral group of order $2k$, i.e., all rotations by $2\pi/k$ and reflections. In VAM, this corresponds to invariance under both cyclic flows and chirality-reversing operations.

Z_{2k} **Cyclic Symmetry of Order $2k$.** The knot admits rotational symmetry by $2\pi/(2k)$ (and its multiples), but not necessarily reflection symmetry. In VAM, such symmetry is associated with periodic vortex phase cycling and often positive amphichirality.

I **Icosahedral Symmetry or Inversion.** I often indicates additional point group symmetries (such as icosahedral, dodecahedral, or inversion symmetries), depending on the context. In tables, it may specify inversion axes or particular symmetry orders, e.g., I_8, I_4.

reversible Reversible Knot (Vortex). The knot is topologically equivalent to itself with the orientation reversed; in VAM, this reflects invariance under reversal of circulation or "vortex time".

amphichiral Amphichiral (Mirror-Image) Symmetry. The knot is equivalent to its mirror image:

- *Positive amphichirality* usually corresponds to cyclic (Z_{2k}) symmetry.
- *Negative amphichirality* is sometimes indicated by special inversion (I_2).

Table 1: **Known Symmetries of Prime Knots as VAM Vortex Structures.** This table catalogs the discrete symmetries of low-crossing-number prime knots, interpreted as possible stable knotted vortex configurations in the Vortex Æther Model (VAM). Columns show the principal symmetry groups ($D_2(r)$, D_{2k}, Z_{2k}, I), reversibility, amphichirality, allowed periods, and the full symmetry group (FSG).

	$\mathrm{D}_2(r)$	D_{2k}	Z_{2k}	I	reversible	amphichiral	periods	FSG
3_1	✓	D_4, D_6			✓		$2, 3$	Z_2
4_1	✓	D_4	Z_4	I_8	✓	✓	2	D_8
5_1	✓	D_4, D_{10}			✓		$2, 5$	Z_2
$5_2, 6_1, 6_2$	✓	D_4			✓		2	D_4
6_3	✓	D_4	Z_4		✓	✓	2	D_8
7_1	✓	D_4, D_{14}			✓		$2, 7$	Z_2
$7_2, 7_3$	✓	D_4			✓		2	D_4
7_4	✓	D_4			✓		2	D_8
$7_5, 7_6$	✓	D_4			✓		2	D_4
7_7	✓	D_4			✓		2	D_8
$8_1, 8_2$	✓	D_4			✓		2	D_4
8_3	✓	D_4	Z_4	I_8	✓	✓	2	D_8
$8_4, 8_5, 8_6, 8_7, 8_8$	✓	D_4			✓		2	D_4
8_9	✓	D_4		I_4	✓	✓	2	D_8
8_{10}	✓				✓		none	D_2
8_{11}	✓	D_4			✓		2	D_4
8_{12}	✓	D_4	Z_4		✓	✓	2	D_8
$8_{13}, 8_{14}, 8_{15}$	✓	D_4			✓		2	D_4
8_{16}	✓				✓		none	D_2
8_{17}						✓	none	D_2
8_{18}	✓	D_4, D_8	Z_8		✓	✓	$2, 4$	D_{16}
8_{19}	✓	D_4, D_6, D_8			✓		$2, 3, 4$	Z_2
8_{20}	✓				✓		none	D_2
8_{21}	✓	D_4			✓		2	D_4
$12a_{1202}$	✓		Z_2, Z_6		✓	✓		D_{12}
15331			Z_2			✓		

periods Periods of Symmetry. Lists the possible orders of cyclic symmetry—i.e., the integer n for which the knot is invariant under a $2\pi/n$ rotation. In VAM, this relates to allowed quantum/vortex mode numbers.

FSG Full Symmetry Group (FSG). The maximal discrete symmetry group of the knot, encoding all rotational, reflectional, and inversion symmetries. In VAM, the FSG constrains the topological conservation laws and fusion/annihilation selection rules for knotted vortex states.