Hydrogen Schrödinger Equation in the Vortex Æther Model (VAM): Swirl Potential, Core Regularization, and Numerical Validation

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Abstract

We reformulate the hydrogen atom in the Vortex Æther Model (VAM). The Coulomb potential $V(r) = -e^2/(4\pi\varepsilon_0 r)$ is replaced by a swirl potential derived from æther fluid parameters, $V_{\rm VAM}(r) = -\Lambda_{\rm VAM}/\sqrt{r^2 + r_c^2}$, where $\Lambda_{\rm VAM} = 4\pi\,\rho_{\text{æ}}^{(\rm mass)}\,C_e^2\,r_c^4$. We (i) derive $\Lambda_{\rm VAM}$ from a Bernoulli swirl-pressure surface integral, (ii) give short derivations for C_e and r_c, and (iii) perform numerical validation using calibrated VAM constants, showing parts-per-million agreement with $e^2/(4\pi\varepsilon_0)$. The hydrodynamic underpinning ties to Madelung, gauge-covariant quantum hydrodynamics, and vacuum-hydrodynamic models [1, 2, 3], as well as topological and analogue-gravity perspectives [4, 5, 6] and Bohm–Hiley dynamics [7, 8].

1 Standard hydrogen equation and hydrodynamic bridge

The hydrogenic time-independent Schrödinger equation reads

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}\right]\psi(\mathbf{r}) = E\,\psi(\mathbf{r}), \tag{1}$$

with the reduced mass μ [9, 10]. The Madelung transform $\psi = \sqrt{n}\,e^{iS/\hbar}$ maps (1) into a continuity equation for n and an Euler-like equation for $\mathbf{u} = \nabla S/m$ with a quantum pressure Q [1]; gauge-covariant and vacuum-hydrodynamic variants appear in [2, 3]. These hydrodynamic views motivate a VAM interpretation wherein sources are vortex cores (cf. [11, 12]) and long-range interactions arise from swirl-pressure fields. Topological and analogue-gravity connections are discussed in [5, 4, 6]; causal/Bohmian formulations in [7, 8].

2 Bernoulli swirl-pressure and the VAM Coulomb scale

For an incompressible, inviscid æther, the local swirl (tangential) speed is u, and the Bernoulli pressure is

$$p_{\rm swirl} = \tfrac{1}{2}\,\rho_{\text{æ}}^{(\rm mass)}\,u^2. \tag{2}$$

Outside a finite core of radius r_c, the azimuthal profile is taken as

$$u(r) \sim C_e \left(\frac{r_c}{r}\right)^2 \qquad (r \gg r_c), \tag{3}$$

the r^{-2} decay encoding incompressible-vortex far-field structure.

Consider a spherical control surface S_r^2 of radius r. The effective interaction scale is the integral of pressure over that surface:

$$\Lambda_{\mathrm{VAM}} = \int_{S_r^2} p_{\mathrm{swirl}}\, r^2\, d\Omega = \int_{S_r^2} \frac{1}{2}\, \rho_{æ}^{(\mathrm{mass})}\, C_e^2 \frac{r_c^4}{r^4}\, r^2\, d\Omega \tag{4}$$

$$= \frac{1}{2}\, \rho_{æ}^{(\mathrm{mass})}\, C_e^2\, r_c^4 \int_{S^2} d\Omega = 4\pi\, \rho_{æ}^{(\mathrm{mass})}\, C_e^2\, r_c^4. \tag{5}$$

Hence

$$\boxed{\Lambda_{\mathrm{VAM}} = 4\pi\, \rho_{æ}^{(\mathrm{mass})}\, C_e^2\, r_c^4}. \tag{6}$$

Dimensions: $[\Lambda_{\mathrm{VAM}}] = [\text{pressure}] \times [\text{area}] = (\mathrm{N/m^2})(\mathrm{m^2}) = \mathrm{N} = \mathrm{J/m} \times \mathrm{m} = \mathrm{J} \cdot \mathrm{m}$, matching $e^2/(4\pi\varepsilon_0)$.

3 Hydrogen Schrödinger equation in VAM

VAM replaces the Coulomb term by a softened swirl potential

$$V_{\mathrm{VAM}}(r) \;=\; -\frac{\Lambda_{\mathrm{VAM}}}{\sqrt{r^2 + r_c^2}} \quad \to \quad -\frac{\Lambda_{\mathrm{VAM}}}{r} \;\; (r \gg r_c), \tag{7}$$

leading to

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 - \frac{\Lambda_{\mathrm{VAM}}}{\sqrt{r^2 + r_c^2}}\right]\psi = E\,\psi. \tag{8}$$

For $r \gg r_c$, (8) reproduces (1). The r_c-softening regularizes the $1/r$ singularity and yields tiny S-state shifts of order $(r_c/a_0)^2$.

4 Short derivation of C_eCe and $r_c$rc

(i) C_e from the maximum æther Coulomb force

Define $F_{æ}^{\max}$ as the maximal static æther (Coulomb) force scale. In VAM we balance it with the swirl thrust across the core aperture $A_c = \pi r_c^2$ using *dynamic* pressure $p_d = \rho_{æ}^{(\mathrm{mass})}\, C_e^2$ (model convention without the 1/2 factor):

$$F_{æ}^{\max} \;=\; p_d\, A_c \;=\; \rho_{æ}^{(\mathrm{mass})}\, C_e^2\, (\pi r_c^2). \tag{9}$$

Solving,

$$\boxed{C_e \;=\; \sqrt{\frac{F_{æ}^{\max}}{\rho_{æ}^{(\mathrm{mass})}\, \pi r_c^2}}}. \tag{10}$$

Combining (10) with the result for r_c below also yields an equivalent closed form

$$\boxed{C_e \;=\; \Big(\frac{2\,F_{æ}^{\max\,2}}{\rho_{æ}^{(\mathrm{mass})}\,\pi\,\hbar}\Big)^{1/3}}, \tag{11}$$

useful for direct calibration.

(ii) r_c from the G consistency (Planck time)

The VAM–GR matching condition for Newton's constant is [1, 6, and VAM notes]

$$G \;=\; \frac{C_e\,c^5\,t_p^2}{2\,F_{æ}^{\max}\,r_c^2}, \qquad t_p^2 = \frac{\hbar\,G}{c^5}. \tag{12}$$

Substituting t_p^2 and cancelling G gives a parameter-free core-radius relation:

$$\boxed{r_c^2 \;=\; \frac{\hbar\,C_e}{2\,F_{æ}^{\max}}}, \qquad \boxed{r_c \;=\; \sqrt{\frac{\hbar\,C_e}{2\,F_{æ}^{\max}}}}. \tag{13}$$

5 Numerical validation (using VAM constants)

Constants (SI):

$\rho_{æ}^{(\mathrm{mass})} = 3.893\,435\,826\,691\,868\,7 \times 10^{18}\,\mathrm{kg\,m^{-3}}$, $\quad C_e = 1.093\,845\,63 \times 10^{6}\,\mathrm{m\,s^{-1}}$, $\quad r_c = 1.408\,970\,17 \times 10^{-15}$
$F_{æ}^{\max} = 29.053\,507\,\mathrm{N}$, $\quad e = 1.602\,176\,634 \times 10^{-19}\,\mathrm{C}$, $\quad \varepsilon_0 = 8.854\,187\,812\,8 \times 10^{-12}\,\mathrm{F\,m^{-1}}$,
$\hbar = 1.054\,571\,817 \times 10^{-34}\,\mathrm{J\,s}$, $\quad c = 2.997\,924\,58 \times 10^{8}\,\mathrm{m\,s^{-1}}$ (CODATA [13]).

(a) Check C_e from closed form.

$$C_{e\mathrm{pred}} = \Big(\frac{2\,F_{æ}^{\max\,2}}{\rho_{æ}^{(\mathrm{mass})}\,\pi\,\hbar}\Big)^{1/3} = 1.093\,845\,595 \times 10^{6}\,\mathrm{m\,s^{-1}},$$

relative difference to given C_e: 3.17×10^{-8}.

(b) Check r_c from (13)**.**

$$r_{c\mathrm{pred}} = \sqrt{\frac{\hbar\,C_e}{2\,F_{æ}^{\max}}} = 1.408\,970\,237 \times 10^{-15}\,\mathrm{m},$$

relative difference to given r_c: 4.76×10^{-8}.

(c) Compute $\Lambda_{\mathbf{VAM}}$ and compare to $e^2/(4\pi\varepsilon_0)$.

$$\Lambda_{\mathrm{VAM}} = 4\pi\, \rho_{æ}^{(\mathrm{mass})}\, C_e^2\, r_c^4 = 2.307\,077\,327\,648\,437\,3 \times 10^{-28}\,\mathrm{J\,m}.$$

$$\frac{e^2}{4\pi\varepsilon_0} = 2.307\,077\,552\,341\,735\,5 \times 10^{-28}\,\mathrm{J\,m}.$$

Relative deviation:

$$\frac{|\Lambda_{\mathrm{VAM}} - e^2/(4\pi\varepsilon_0)|}{e^2/(4\pi\varepsilon_0)} = 9.7393 \times 10^{-8} \quad (= 0.0974 \text{ ppm}).$$

(d) Hydrogenic scales. With $\mu = m_e m_p/(m_e + m_p)$ and Λ_{VAM} above:

$$a_0^{\mathrm{VAM}} = \frac{\hbar^2}{\mu\,\Lambda_{\mathrm{VAM}}} = 5.294\,654\,607\,4{\times}10^{-11}\,\mathrm{m}, \quad E_1^{\mathrm{VAM}} = -\frac{\mu\,\Lambda_{\mathrm{VAM}}^2}{2\hbar^2} = -2.178\,685\,390\,0{\times}10^{-18}\,\mathrm{J} = -13.598\,2$$

consistent with standard hydrogen [10]. Finite-core corrections scale as $(r_c/a_0)^2 \simeq 7.08 \times 10^{-10}$.

6 Conclusion

The VAM swirl-pressure integral produces $\Lambda_{\mathrm{VAM}} = 4\pi\rho_{æ}^{(\mathrm{mass})} C_e^2 r_c^4$, reproducing the Coulomb scale at the 10^{-7} level with your calibrated constants. The short derivations (10)–(13) fix C_e and r_c directly from $(\rho_{æ}^{(\mathrm{mass})}, F_{æ}^{\max}, \hbar)$. Together with the softened potential, this yields a regularized hydrogen problem equivalent to the textbook form at atomic distances, grounded in a hydrodynamic/topological framework [1, 2, 3, 5, 4, 6, 7, 8, 11, 12].

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