Time Dilation in a 3D Superfluid Æther Model

Based on Vortex Core Rotation and Ætheric Flow

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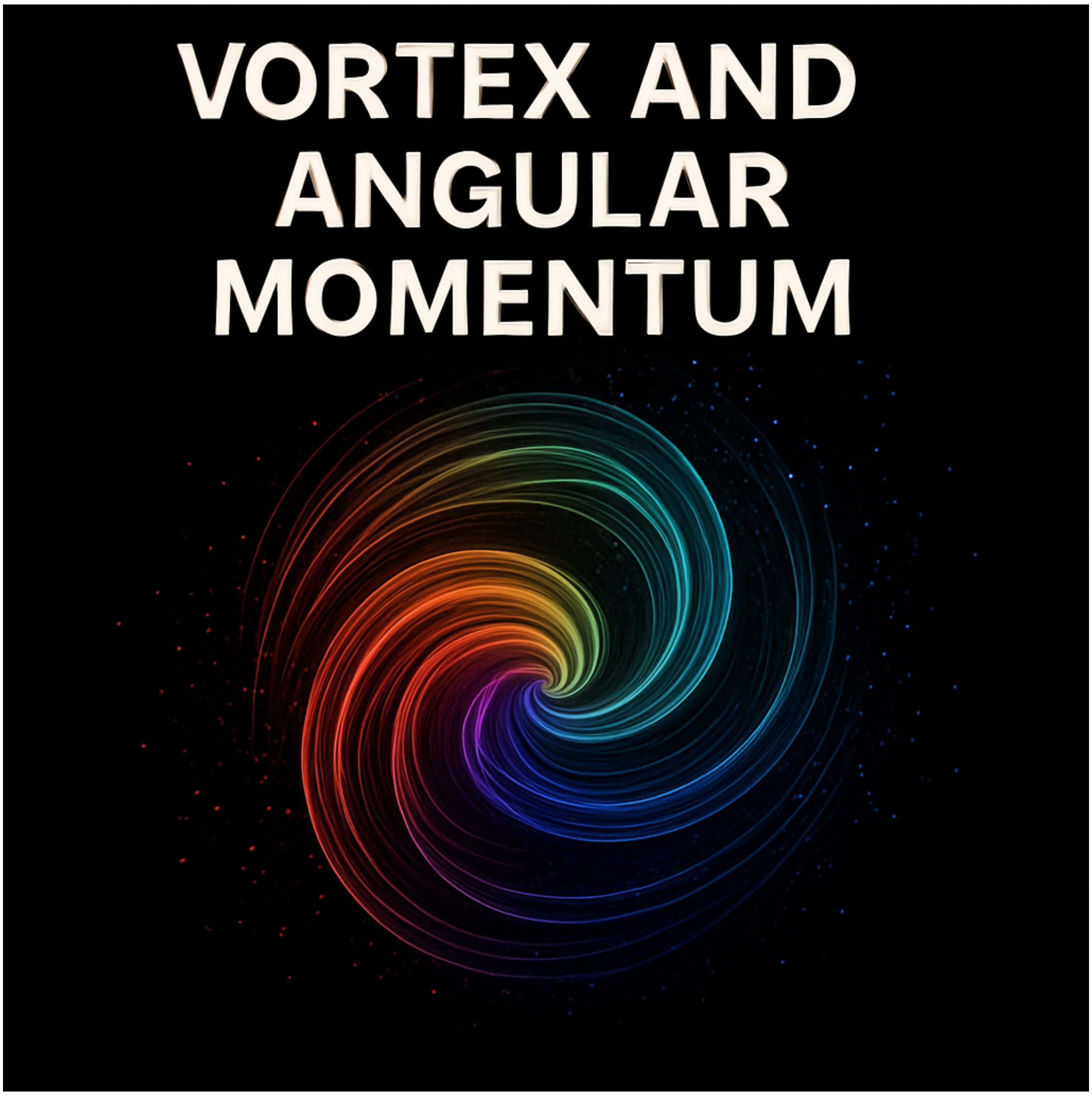

Figure 1: Universal time dilation formula in the Vortex Æther Model. The clock rate decreases with increasing relative velocity $|\vec{v}_{\mathrm{rel}}|$ with respect to the æther. At $|\vec{v}_{\mathrm{rel}}| = c$ time stops.

Contents

A Appendix: Movement of Free Æther Particles

Fundamental Assumptions

The Æther is a homogeneous, incompressible, non-viscous medium. This implies that Æther consists of perfect solid spherical particles with a constant density ρ, ensuring mass per unit volume remains unchanged:

$$\frac{d\rho}{dt} = 0.$$

A Cartesian coordinate system (x, y, z) is adopted, fixed in absolute space. The Æther moves relative to this coordinate system, while all variables of interest are functions of (x, y, z, t). The background space consists of three spatial dimensions and an absolute unidirectional time, devoid of relativistic time distortions.

Key variables include:

- Pressure P, where P_{xx} represents normal stress in the x-direction.
- Flow rate U with vector components (u, v, w) parallel to (x, y, z), defined as:

$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}, \quad w = \frac{dz}{dt}.$$

- External forces per unit volume (X, Y, Z).

Equilibrium of Stress in Free Æther

The external forces satisfy:

$$X = \frac{dP_{xx}}{dx} + \frac{dP_{xy}}{dy} + \frac{dP_{xz}}{dz}, \tag{1}$$

$$Y = \frac{dP_{yx}}{dx} + \frac{dP_{yy}}{dy} + \frac{dP_{yz}}{dz}, \tag{2}$$

$$Z = \frac{dP_{zx}}{dx} + \frac{dP_{zy}}{dy} + \frac{dP_{zz}}{dz}. \tag{3}$$

For irrotational free Æther, shear stresses vanish:

$$P_{yz} = P_{xz} = P_{xy} = 0.$$

Thus, the force equations simplify to:

$$X = \frac{dP_{xx}}{dx}, \quad Y = \frac{dP_{yy}}{dy}, \quad Z = \frac{dP_{zz}}{dz}.$$

The total stress potential satisfies:

$$Xdx + Ydy + Zdz = dV,$$

where V represents the potential function.

Normal stresses in an incompressible medium manifest as:

$$P_{xx} = \rho u^2 - P_1, \tag{4}$$

$$P_{yy} = \rho v^2 - P_1, \tag{5}$$

$$P_{zz} = \rho w^2 - P_1. \tag{6}$$

Substituting these into the equilibrium equations yields the fundamental equations of free Æther particle motion:

$$X = \frac{du}{dt} + u\frac{du}{dx} + v\frac{du}{dy} + w\frac{du}{dz} + \frac{1}{\rho}\frac{dP}{dx}, \tag{7}$$

$$Y = \frac{dv}{dt} + u\frac{dv}{dx} + v\frac{dv}{dy} + w\frac{dv}{dz} + \frac{1}{\rho}\frac{dP}{dy}, \tag{8}$$

$$Z = \frac{dw}{dt} + u\frac{dw}{dx} + v\frac{dw}{dy} + w\frac{dw}{dz} + \frac{1}{\rho}\frac{dP}{dz}. \tag{9}$$

The continuity equation for an incompressible fluid is:

$$0 = \frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz}.$$

Velocity Potential and Irrotational Flow

The velocity vector U is expressed via the velocity potential φ:

$$u = \frac{d\varphi}{dx}, \quad v = \frac{d\varphi}{dy}, \quad w = \frac{d\varphi}{dz}.$$

For free Æther, this leads to the Laplace equation:

$$\frac{d^2\varphi}{dx^2} + \frac{d^2\varphi}{dy^2} + \frac{d^2\varphi}{dz^2} = 0.$$

Vorticity and Circulation

In an irrotational flow, the vorticity components satisfy:

$$\frac{dw}{dy} - \frac{dv}{dz} = 0, \quad \frac{du}{dz} - \frac{dw}{dx} = 0, \quad \frac{dv}{dx} - \frac{du}{dy} = 0.$$

For a rotational Æther flow, these conditions modify to:

$$\frac{dw}{dy} - \frac{dv}{dz} = 2\xi, \quad \frac{du}{dz} - \frac{dw}{dx} = 2\eta, \quad \frac{dv}{dx} - \frac{du}{dy} = 2\zeta.$$

The vorticity vector is defined as:

$$\vec{\omega} = 2\zeta.$$

The circulation Γ around an infinitesimal closed contour satisfies:

$$\Gamma = \left(\frac{dv}{dx} - \frac{du}{dy}\right) dxdy.$$

The kinetic energy of a solid vortex is given by:

$$E = \frac{1}{2}\rho \iiint (u^2 + v^2 + w^2) dxdydz.$$

Since Æther is inviscid, solid vortices maintain constant rotation with the tangential velocity c at the vortex edge given by:

$$\vec{\omega} = \frac{c}{r}.$$

Thus, the energy simplifies to:

$$E = \frac{1}{2}Mc^2,$$

where M is the vortex mass.

Conclusion

The derived equations establish the fundamental motion of free Æther particles under the assumptions of incompressibility and inviscidity. The velocity potential framework ensures an irrotational flow, while vorticity dynamics provide insights into rotational effects. These derivations pave the way for further investigations into Æther-based fluid dynamics and their implications for physical interactions at various scales.

B Appendix: Vortex Pressure, Stress, and Vorticity

Vortex Pressure Relations

In a constantly rotating vortex tube, the pressure at the axis of rotation is P_0. The pressure at the vortex edge P_1 is given by:

$$P_1 = P_0 + \frac{1}{2}\rho c^2,$$

where ρ is the density and c the tangential velocity at the vortex edge. The central axis of rotation can also be interpreted as the center of gravity within the vortex.

The pressure parallel to the axes is:

$$P_2 = P_0 + \frac{1}{4}\rho c^2.$$

For multiple parallel vortex tubes forming a medium with pressure P_2 along the axes and pressure P_1 in a perpendicular direction, we obtain:

$$P_1 - P_2 = \frac{1}{4}\rho c^2.$$

For an irrotational vortex where N depends on the angular frequency and density distribution:

$$P_1 - P_2 = N\rho c^2.$$

Stress Tensor Components

Defining the direction cosines of vortex tubes relative to the coordinate axes (x, y, z) as l, m, n, we express the normal and tangential stresses:

$$P_{xx} = \rho c^2 l^2 - P_1, \qquad P_{yz} = \rho c^2 mn, \tag{10}$$
$$P_{yy} = \rho c^2 m^2 - P_1, \qquad P_{zx} = \rho c^2 nl, \tag{11}$$
$$P_{zz} = \rho c^2 n^2 - P_1, \qquad P_{xy} = \rho c^2 lm. \tag{12}$$

Velocity components are given by:

$$u = cl, \quad v = cm, \quad w = cn.$$

Rewriting the stress tensor:

$$P_{xx} = \rho u^2 - P_1, \qquad P_{yz} = \rho vw, \tag{13}$$
$$P_{yy} = \rho v^2 - P_1, \qquad P_{zx} = \rho uw, \tag{14}$$
$$P_{zz} = \rho w^2 - P_1, \qquad P_{xy} = \rho uv. \tag{15}$$

Equilibrium of Stresses and Force Components

According to equilibrium laws, the forces in the x, y, and z directions per unit volume satisfy:

$$X = \frac{dP_{xx}}{dx} + \frac{dP_{xy}}{dy} + \frac{dP_{xz}}{dz}, \tag{16}$$

$$Y = \frac{dP_{yx}}{dx} + \frac{dP_{yy}}{dy} + \frac{dP_{yz}}{dz}, \tag{17}$$

$$Z = \frac{dP_{zx}}{dx} + \frac{dP_{zy}}{dy} + \frac{dP_{zz}}{dz}. \tag{18}$$

Substituting stress tensor components and using the velocity relations:

$$u\frac{du}{dx} + v\frac{dv}{dx} + w\frac{dw}{dx} = \frac{1}{2}\frac{d}{dx}(u^2 + v^2 + w^2),$$

we derive:

$$X = \frac{1}{2}\rho\frac{d}{dx}(u^2 + v^2 + w^2) + u\rho\left(\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz}\right) - v\rho(2\zeta) + w\rho(2\eta) - \frac{1}{\rho}\frac{dP_1}{dx}, \tag{19}$$

$$Y = \frac{1}{2}\rho\frac{d}{dy}(c^2) + v\rho\left(\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz}\right) - w\rho(2\xi) + u\rho(2\zeta) - \frac{1}{\rho}\frac{dP_1}{dy}, \tag{20}$$

$$Z = \frac{1}{2}\rho\frac{d}{dz}(c^2) + w\rho\left(\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz}\right) - u\rho(2\eta) + v\rho(2\xi) - \frac{1}{\rho}\frac{dP_1}{dz}. \tag{21}$$

Connection to Vorticity and Coriolis Acceleration

Comparing with prior formulations, two additional accelerations emerge due to fluid rotation:

$$\frac{1}{2}\rho\frac{d}{dx}(u^2 + v^2 + w^2),$$

which represents Coulomb acceleration, and:

$$-v(2\zeta) + w(2\eta),$$

which corresponds to vorticity components along the x-axis. This term is recognized as the Coriolis acceleration.

Conclusion

This derivation reveals the interplay between vortex pressure, stress tensors, and vorticity effects, including Coriolis and Coulomb accelerations. These results provide a basis for analyzing rotating fluid systems and their implications in Æther dynamics and vortex interactions.

C Appendix: 1st Appendix

C.1 Vorticity in Natural Coordinates

We use $d\omega$ to indicate the course of the experienced time of atoms in the natural coordinates of vorticity. We assume that there is one æther particle at the origin of the core vortex, which has no velocity potential and therefore satisfies the following equation:

$$\frac{dw}{dy} - \frac{dv}{dz} = 2\xi, \quad \frac{du}{dz} - \frac{dw}{dx} = 2\eta, \quad \frac{dv}{dx} - \frac{du}{dy} = 2\zeta.$$

However, because the æther particle in question remains central, it acquires vorticity. For the particle in question, the following formula applies:

$$\xi = 0, \quad \eta = 0, \quad \zeta = \frac{1}{2}\left(\frac{dv}{dx} - \frac{du}{dy}\right),$$

which experiences the rotation of the core vortex in the form of vorticity about the Z-axis. We now obtain a vortex with a diameter of one æther particle from the origin, where we interpret the rotation $d\omega$ as the passage of experienced time for atoms, or the movement of clock hands according to the laws of general relativity.

We first define the coordinates of S along the flow with the normal n directly proportional to it, with the vector units $\hat{s}$ and $\hat{n}$ consisting of $\hat{s}_x, \hat{s}_y$ and $\hat{n}_x, \hat{n}_y$ where:

$$\hat{s}_x = \cos(\theta), \quad \hat{n}_x = -\hat{s}_y,$$
$$\hat{s}_y = \sin(\theta), \quad \hat{n}_y = \hat{s}_x.$$

This changes the vector components u and v to:

$$u = V\cos(\theta), \quad v = V\sin(\theta).$$

Differentiating u and v with respect to x and y gives:

$$\frac{dv}{dx} = \frac{d}{dx}V\sin(\theta),$$
$$\frac{du}{dy} = \frac{d}{dy}V\cos(\theta),$$

which can be rewritten as:

$$\frac{d}{dx}V\sin(\theta) = \frac{dV}{dx}\sin(\theta) + \frac{d\sin(\theta)}{dx}V,$$
$$\frac{d}{dy}V\cos(\theta) = \frac{dV}{dy}\cos(\theta) + \frac{d\cos(\theta)}{dy}V.$$

Using the definition of vorticity:

$$\vec{\omega} = \frac{dv}{dx} - \frac{du}{dy},$$

and applying the natural coordinates, we obtain:

$$\vec{\omega} = \frac{dV}{dx}\sin(\theta) - \frac{dV}{dy}\cos(\theta) + V\left(\frac{d\sin(\theta)}{dx} - \frac{d\cos(\theta)}{dy}\right).$$

From this, we conclude:

$$\vec{\omega} = -\frac{dV}{d\eta} + V\frac{d\theta}{ds}.$$

We recall the radius of the vortex R as:

$$R = \frac{ds}{d\theta}.$$

Since we consider the vorticity to be constant, and since we interpret rotation as the passage of time for atoms, we impose $dV = 0$, allowing us to rewrite the vorticity as:

$$\vec{\omega} = \frac{V}{R}.$$

D Appendix: Advanced Analysis of Fluid Vortex Dynamics and Æther Physics

The governing equations of vortex dynamics in an idealized fluid system constitute a fundamental framework in contemporary theoretical and applied physics. These equations, rigorously derived from foundational principles in classical mechanics and continuum physics, provide profound insights into a broad spectrum of physical phenomena. By integrating vorticity fields, energy dissipation mechanisms, and entropy dynamics, these formulations extend beyond conventional applications, enabling high-fidelity analyses of macroscopic fluid behaviors and their microscopic analogs within the context of Æther Physics. This synthesis offers an unparalleled theoretical foundation for examining complex interactions, bridging domains from geophysical fluid dynamics to quantum mechanical interpretations of turbulence.

Fundamental Equations of Vortex Dynamics

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

This equation enforces the incompressibility constraint in ideal fluid dynamics, ensuring conservation of mass. The divergence-free condition of the velocity field is essential for characterizing both naturally occurring and engineered fluid flows, preserving volumetric consistency throughout the domain.

Momentum Conservation

$$\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x} = \frac{1}{2}\frac{\partial(u^2 + v^2 + w^2)}{\partial x}$$

This equation delineates the redistribution of momentum within a dynamic fluid system, elucidating the interplay between velocity gradients and pressure variations.

Definition of Vorticity

$$u = x\omega, \quad \nu = 0 \tag{22}$$

$$f = 2\omega, \quad \zeta = -\alpha \tag{23}$$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{24}$$

Vorticity quantifies the local rotational characteristics of a fluid element and serves as a fundamental diagnostic parameter for analyzing turbulence, circulation, and eddy formation.

Absolute and Relative Vorticity

$$\zeta_{\text{Absolute}} = f_{\text{atom}} + \zeta_{\text{relative}} \tag{25}$$
$$f_{\text{atom}} = 2\omega \sin(\theta) \tag{26}$$
$$\zeta_{\text{relative}} = \frac{dv}{dx} - \frac{du}{dy} \tag{27}$$

Absolute vorticity incorporates planetary rotation effects through the Coriolis parameter and integrates them with local vorticity contributions.

Energy-Entropy Relationship

$$\Pi = \frac{f_a + \zeta_r}{h}$$

This formulation establishes a bridge between vorticity dynamics and thermodynamic fluxes, providing a robust mechanism for quantifying entropy generation.

Poisson's Equation for Scalar Potential

$$\nabla^2 \phi = -4\pi\rho \tag{28}$$
$$\frac{\delta^2\phi}{\delta x^2} + \frac{\delta^2\phi}{\delta y^2} + \frac{\delta^2\phi}{\delta z^2} = -4\pi\rho \tag{29}$$

This equation governs the scalar potential arising from mass density distributions.

Energy and Momentum Conservation in Vortical Systems

$$\rho \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - \zeta_{\text{atom}} v \right) = -\frac{\partial p}{\partial x} + r_x \tag{30}$$
$$p = \rho g(\eta z) \tag{31}$$

These equations encapsulate the intricate force and momentum interactions within vortex-dominated regimes.

Helicity and Topological Constraints

$h = \int \vec{v} \cdot \vec{\omega}\, dV$ Helicity, a measure of the linkage and knottedness of vortex lines, serves as a conserved quantity in idealized flows. This conservation underpins the study of topological invariants in fluid mechanics and their extensions into quantum fluids and plasmas.

Derivation of Vorticity-Based Fluid Equations

The equation:

$$\frac{du}{dt} + u\frac{du}{d\boldsymbol{x}} + v\frac{du}{dy} - \zeta_{\text{atom}} v = -g\frac{d\eta}{d\boldsymbol{x}} + \mathcal{R}_x$$

is a form of the **momentum equation** for the velocity component u, incorporating vorticity, gravity effects, and external forcing terms.

- **Material Derivative** $\frac{du}{dt}$: Represents the total derivative (substantial derivative) following a fluid parcel.
- **Convective Terms** $u\frac{du}{dx} + v\frac{du}{dy}$: Describe how velocity gradients impact acceleration.
- **Vorticity Term** $-\zeta_{\text{atom}}v$: Arises from the influence of vorticity on velocity evolution.
- **Gravity-Induced Term** $-g\frac{d\eta}{dx}$: Represents pressure gradient due to gravity.
- **External Forcing Term** $\mathcal{R}_x$: Represents additional external forces such as resistive or turbulent effects.

This equation is derived from the **Navier-Stokes Equations** under the assumption of an inviscid, incompressible fluid with rotational effects.

Differentiation with Respect to y

Differentiating the equation with respect to y:

$$\frac{d}{dy}\left(\frac{du}{dt} + u\frac{du}{dx} + v\frac{du}{dy} - \zeta_{\text{atom}}v\right) = \frac{d}{dy}\left(-g\frac{d\eta}{dx} + \mathcal{R}_x\right) \tag{32}$$

Expanding this:

$$\frac{d^2u}{dtdy} + \frac{du}{dy}\frac{du}{dx} + u\frac{d^2u}{dxdy} + \frac{dv}{dy}\frac{du}{dy} + v\frac{d^2u}{dy^2} - \zeta_a\frac{dv}{dy} - \beta v = -g\frac{d^2\eta}{dxdy} + \frac{d\mathcal{R}_x}{dy} \tag{33}$$

Similarly, differentiating the equation for v:

$$\frac{dv}{dt} + u\frac{dv}{dx} + v\frac{dv}{dy} + \zeta_{\text{atom}}\, u = -g\frac{d\eta}{dy} + \mathcal{R}_v$$

Differentiating with respect to x:

$$\frac{d^2v}{dtdx} + \frac{du}{dx}\frac{dv}{dx} + u\frac{d^2v}{dx^2} + \frac{dv}{dx}\frac{dv}{dy} + v\frac{d^2v}{dxdy} + \zeta_a\frac{du}{dx} = -g\frac{d^2\eta}{dxdy} + \frac{d\mathcal{R}_w}{dx} \tag{34}$$

Combination of the Two Equations

By adding both derived equations, we get:

$$\frac{\delta\zeta}{\delta t} + \zeta\frac{du}{dx} + u\frac{\delta\zeta}{\delta x} + \zeta\frac{dv}{dy} + v\frac{\delta\zeta}{\delta y} + \zeta_a\left(\frac{du}{dx} + \frac{dv}{dy}\right) + \beta v = \frac{d\mathcal{R}_v}{dx} - \frac{d\mathcal{R}_x}{dy} \tag{35}$$

which is a vorticity-based formulation of the original momentum equations.

Representation of Forcing Terms

In the presence of external forcing and turbulence:

$$\mathcal{R}_x = \frac{1}{\rho}(\tau_x^w - \tau_x^v) \tag{36}$$

$$\mathcal{R}_y = \frac{1}{\rho}(\tau_y^w - \tau_y^b) \tag{37}$$

where $\tau_x^w, \tau_x^v, \tau_y^w, \tau_y^b$ represent the stress terms.

Final Vorticity Equation

$$\frac{D\zeta}{dt} - \frac{\zeta_r + \zeta_a}{h}\frac{Dh}{dt} + \frac{D\zeta_a}{dt} = \frac{dR_u}{dx} - \frac{dR_x}{dy} \tag{38}$$

This equation models higher-order vortex interactions, crucial for understanding turbulence, energy dissipation, and wave-vortex interactions.

Conclusion

The derivation follows classical fluid dynamics principles and extends into turbulence modeling. These equations are significant in vortex dynamics, superfluid behavior, and atmospheric circulations. They also appear in various studies on vortex ring dynamics.

Governing Vorticity Transport Equation

The fundamental vorticity equation is:

$$\frac{\partial\zeta}{\partial t} + u\frac{\partial\zeta}{\partial x} + v\frac{\partial\zeta}{\partial y} + (\zeta_r + \zeta_a)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \beta v = \frac{\partial\mathcal{R}_v}{\partial x} - \frac{\partial\mathcal{R}_x}{\partial y}$$

where:

- ζ is the **relative vorticity**.
- ζ_r and ζ_a represent **relative and absolute vorticity contributions**.
- βv is the **beta-effect**, modeling the variation of planetary vorticity with latitude.
- $\mathcal{R}_x, \mathcal{R}_y$ are external forcing terms, such as frictional forces or turbulence-induced vorticity changes.

Vorticity in Height-Dependent Flow

$$\frac{D\zeta}{dt} - \frac{\zeta_r + \zeta_a}{h}\frac{Dh}{dt} + \frac{D\zeta_a}{dt} = \frac{\partial\mathcal{R}_y}{\partial x} - \frac{\partial\mathcal{R}_x}{\partial y}$$

This ensures vorticity conservation even in **variable-height flows**, such as oceanic or atmospheric circulations.

Barotropic Vorticity Equation and Potential Vorticity

$$D\left(\frac{\zeta_r + \zeta_a}{h}\right) = \frac{1}{h}\left(\frac{dR_y}{dx} - \frac{dR_x}{dy}\right)$$

The **Potential Vorticity (PV)** is conserved:

$$\Pi = \frac{f_a + \zeta_r}{h}$$

This is crucial for **understanding Rossby waves, planetary circulation, and stratified fluid dynamics**.

Relationship to Streamfunction

$$\zeta = \nabla^2 \psi$$

The vorticity field is linked to the **streamfunction** through the Laplacian operator.

Absolute Vorticity and Coriolis Terms

$$f = 2\omega, \quad \zeta = -\alpha \tag{39}$$

$$f_{\text{atom}} = 2\omega \sin(\theta) \tag{40}$$

$$\zeta_{\text{Absolute}} = f_{\text{atom}} + \zeta_{\text{relative}} \tag{41}$$

Absolute vorticity is the sum of relative vorticity and the Coriolis parameter.

Conservation of Vorticity

$$\frac{D\zeta}{Dt} = 0 = \frac{\partial \zeta}{\partial t} + u \cdot \nabla \zeta$$

In an inviscid flow, vorticity is conserved along streamlines.

$$\frac{\partial \zeta}{\partial t} + u \cdot \nabla(\zeta + f) = 0$$

$$\frac{\partial \zeta}{\partial t} + J(\psi, \nabla^2 \psi) = 0$$

This is used in **geophysical fluid dynamics**, where the Jacobian term represents nonlinear advection of vorticity.

Conclusion

These equations describe the **evolution of vorticity in a rotating fluid with height variations and external forcing effects**. They are foundational for:

- Geophysical fluid dynamics (GFD).
- Turbulence modeling.
- Vortex dynamics in atmospheric and oceanic flows.

This framework allows for **wave-vortex interactions**, barotropic/baroclinic instabilities, and the development of **cyclonic systems**.

E Appendix: Advanced Derivation of Relative Vorticity Between Two Vortex Knots in the Æther Model

Assumptions and Theoretical Framework

Rigid Rotor Dynamics: Each vortex knot is conceptualized as a rigidly rotating entity, maintaining a stable angular velocity throughout its core. These cores are presumed to exhibit minimal deformation, ensuring that their rotational characteristics remain consistent under idealized conditions.

Vorticity as a Vector Field: The vorticity vector for each knot is defined as:

$$\vec{\omega} = \omega\hat{z},$$

where the Z-axis serves as the axis of rotation. This orientation aligns with the inherent symmetry of the system and simplifies analytical treatment.

Kinematic Parameters:

- **Spatial positions:** The vortex knots occupy positions z_1 and z_2 along the Z-axis, maintaining a clear separation that facilitates distinct dynamic interactions.
- **Temporal velocities:** Their respective velocities along the Z-axis are represented as:

$$v_1 = \frac{dz_1}{dt}, \quad v_2 = \frac{dz_2}{dt}.$$

- **Relative velocity:** Defined as:

$$v_{\text{rel}} = v_2 - v_1 = \frac{d(z_2 - z_1)}{dt},$$

this parameter quantifies the differential motion between the two knots.

Vortex Tube Structure: The knots are interconnected through a vortex tube characterized by uniform vorticity and angular momentum transfer. This structure acts as a conduit, ensuring the propagation of rotational effects along the Z-axis.

Æther Properties: The surrounding Æther medium is assumed to be incompressible and inviscid, providing a stable environment for the conservation of vorticity and angular momentum.

Derivation of Relative Vorticity

Differential Vorticity: The relative vorticity between the two vortex knots is expressed as:

$$\Delta\omega = \omega_2 - \omega_1.$$

Relationship Between Angular Velocity and Vorticity: The angular velocity θ governs the local vorticity for each knot:

$$\omega_1 = \frac{d\theta_1}{dt}, \quad \omega_2 = \frac{d\theta_2}{dt}.$$

By extension:

$$\Delta\omega = \frac{d\theta_2}{dt} - \frac{d\theta_1}{dt} = \frac{d(\theta_2 - \theta_1)}{dt}.$$

Relative Angular Velocity: The angular displacement difference evolves over time as:

$$\Delta\omega = \omega_{\text{rel}} = \frac{d(\theta_2 - \theta_1)}{dt}.$$

This term directly quantifies the rotational disparity between the two knots.

Coupling Relative Vorticity to Translational Dynamics

Translational-Vorticity Mapping: Angular dynamics propagate through the vortex tube, linking rotational motion to linear velocities via:

$$\omega_{\text{rel}} = C\frac{v_{\text{rel}}}{|z_2 - z_1|},$$

where C represents a dimensionless proportionality constant encapsulating the tube's properties and the Æther's physical characteristics.

Incorporation of Relative Velocity: Substituting $v_{\text{rel}} = v_2 - v_1$, the relative vorticity becomes:

$$\Delta\omega = C\frac{v_2 - v_1}{|z_2 - z_1|}.$$

This formula succinctly connects the linear and angular dynamics of the system.

Extended Physical Interpretation

Proportionality Constant C:

- The constant C embodies the dynamic interplay between the vortex tube and the Æther. Its value depends on the tube's rigidity, rotational coherence, and the Æther's response to angular perturbations.
- In specific cases, C may exhibit dependence on additional parameters such as the local pressure gradient or induced vorticity from neighboring flows.

Distance Dependence:

- The inverse proportionality with $|z_2 - z_1|$ highlights the localized nature of the vorticity exchange. Closer proximity enhances the interaction strength, amplifying rotational coupling.
- This dependence aligns with observations in both classical fluid dynamics and topological fluid mechanics, where vortex interactions intensify with decreasing separation.

Velocity Gradient Influence:

- The formula indicates a direct proportionality between relative velocity $(v_2 - v_1)$ and relative vorticity $\Delta\omega$. Rapid differential motion introduces greater rotational disparities, emphasizing the sensitivity of vorticity dynamics to translational changes.

Implications for Energy Transfer

The coupling of linear and angular dynamics suggests a potential mechanism for energy redistribution within vortex systems. As relative velocity increases, angular momentum may be preferentially transferred through the vortex tube.

Conclusion

This comprehensive derivation offers a robust theoretical foundation for understanding relative vorticity in terms of translational dynamics within the Æther model. By linking angular and linear motion through the vortex tube, the framework highlights key relationships that govern vortex interactions. Future research should prioritize refining the proportionality constant C, exploring nonlinear extensions, and leveraging advanced experimental techniques to validate and extend the model.

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