

Photon as a Topological Vortex Ring: Torsion and the Geometry of Light in the Æther

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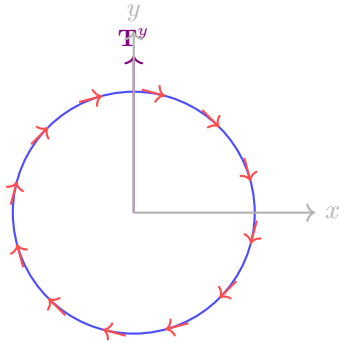
Abstract

We propose a reformulation of the photon as a quantized, massless vortex ring embedded in an incompressible, inviscid superfluid æther. Using Cartan’s geometric structure equations, we draw a formal correspondence between torsional and curvature defects in condensed matter (dislocations and disclinations) and the vortex-core features of ætheric flow. The Burgers and Frank vectors of defect theory find natural analogs in swirl density and angular vorticity. We show how the photon’s Lagrangian, Hamiltonian, and Jacobian structure follow directly from vortex ring energetics in this medium. By coupling this with a Biot–Savart swirl framework and time dilation due to swirl velocity, we reproduce the null proper time of the photon from first principles. This unification provides a geometric and fluid-mechanical basis for the photon’s quantized behavior and opens the door for a deeper topological reinterpretation of gauge bosons in the Standard Model.

I. Introduction

In the Vortex æther Model (VAM), gravitation and quantum phenomena are reformulated through the topology and dynamics of vorticity in an incompressible, inviscid fluid-like æther. This article derives a formal connection between classical defect theory—dislocations and disclinations in condensed matter physics—and VAM’s vortex core structures using the language of Cartan geometry. Inspired by recent developments [1], we reinterpret torsion and curvature within the Cartan structure equations in terms of swirl density and topological vortex curvature. Furthermore, we show that light itself—the photon—can be interpreted as a topologically stable vortex ring, unifying electromagnetic energy propagation with the mechanics of superfluid circulation.

II. Vortex Ring and Cartan Torsion



Vortex ring with tangential velocity C_e

This diagram shows a vortex ring in the x - y plane with tangential swirl velocity (red arrows) and central torsion aligned along the y -axis, representing \mathbf{T}^y from Cartan’s torsion 2-form.

III. Physical Interpretation of the Æther

In the Vortex æther Model (VAM), the æther is a physically real, incompressible, inviscid superfluid that fills all space and serves as the medium in which vortex structures propagate. Unlike the luminiferous æther of pre-relativistic physics, the VAM æther is not a rigid or preferred frame in the classical sense, but a relational fluid substrate whose dynamics obey local conservation laws and whose properties are encoded in geometric field variables such as vorticity $\vec{\omega}$, swirl potential χ , and pressure gradients ∇P .

A. Æther and Relativity

Contrary to historical objections (e.g., Michelson–Morley), the VAM æther is consistent with observed relativistic invariance. In VAM:

- The speed of light c emerges as the limiting swirl propagation velocity in the æther.
- Time dilation and length contraction are derived from local swirl energy density [2], not from coordinate transformations.
- There is no global rest frame detectable through mechanical means, since only local vorticity gradients affect physical observables.

This formulation parallels “emergent spacetime” scenarios in analogue gravity [3], where effective relativistic metrics arise from fluid microdynamics.

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B. Æther as a Relational Fluid

The VAM æther is defined by its core properties:

- **Incompressibility:** $\nabla \cdot \vec{v} = 0$
- **Inviscid Dynamics:** No intrinsic dissipation; supports persistent circulation.
- **Vortex-Centric Ontology:** All matter, radiation, and fields emerge as stable or propagating vortex excitations.

There is no absolute “grid” of space; instead, all physical quantities are encoded in the swirl structure of this medium. Thus, the æther is more akin to a quantum condensate than to a static frame, resembling ideas in superfluid vacuum theories [4].

C. Consistency with Experiments

Although VAM includes a real æther, it remains consistent with all null results from first-order ether-drift experiments:

- Light’s propagation is governed by local swirl—no frame-relative motion.
- The model predicts no Doppler shift or anisotropy in c detectable in co-moving swirl regions.
- Apparent Lorentz invariance emerges as a symmetry of the equations describing low-energy swirl excitations (e.g., vortex rings).

Thus, the æther in VAM functions as a hidden substrate—locally observable only through second-order effects like swirl-induced time dilation, vortex pressure gradients, or geometric birefringence in extreme fields.

The VAM æther is a dynamic, relativistically compatible, vorticity-supporting fluid—not a classical rest frame.

IV. Geometric Framework: Cartan’s Structure Equations

Cartan’s geometric formalism provides two fundamental structure equations on a manifold \mathcal{M} equipped with coframe one-forms θ^i and connection one-forms ω^i_j :

$$\text{Torsion 2-form: } \mathbf{T}^i = d\theta^i + \omega^i_j \wedge \theta^j \quad (1)$$

$$\text{Curvature 2-form: } \mathbf{R}^i_j = d\omega^i_j + \omega^i_k \wedge \omega^k_j \quad (2)$$

In the Weitzenböck connection ($\omega^i_j = 0$), all geometric deformation arises from torsion: $\mathbf{T}^i = d\theta^i$, and $\mathbf{R}^i_j = 0$. Conversely, in the Levi-Civita connection (torsion-free), all deformation is encoded in curvature.

V. Æther Interpretation in VAM

In VAM, we associate:

- θ^i : Local æther displacement one-forms
- ω^i_j : Angular velocity of swirl (local rotational twist)
- \mathbf{T}^i : Core-induced torsion \Rightarrow vortex dislocation (line defect)
- \mathbf{R}^i_j : Swirl curvature \Rightarrow vortex disclination (rotational defect)

VI. Edge Dislocation in VAM as Torsion Source

Consider a single vortex line (edge dislocation) along the z -axis with Burgers vector $\vec{b} = b\hat{x}$. The coframe is:

$$\theta^1 = dx + \frac{b}{2\pi}d\theta, \quad \theta^2 = dy, \quad \theta^3 = dz \quad (3)$$

Using $d(d\theta) = 2\pi\delta(x)\delta(y)dx \wedge dy$, we compute:

$$\mathbf{T}^1 = d\theta^1 = b\delta(x)\delta(y)dx \wedge dy \quad (4)$$

$$\mathbf{T}^2 = 0, \quad \mathbf{T}^3 = 0 \quad (5)$$

The dual vortex density becomes:

$$\alpha^1 = *\mathbf{T}^1 = b\delta(x)\delta(y)dz \quad (6)$$

VII. Equivalence to Wedge Disclination Dipole

Following [1], we reinterpret the same geometry using the Levi-Civita connection:

$$\mathbf{R}^1_2 = \phi[\delta(x-L) - \delta(x+L)]\delta(y)dx \wedge dy \quad (7)$$

where $\phi = b\rho$ encodes the Frank vector.

Edge Dislocation \equiv Dipole of Wedge Disclinations

(8)

VIII. Biot–Savart Swirl Integral in æther

Using the Biot–Savart analogy:

$$\chi^i(\vec{x}) = \frac{1}{4\pi} \int \frac{\alpha^i(\vec{\xi}) \times (\vec{x} - \vec{\xi})}{|\vec{x} - \vec{\xi}|^3} d^3\xi \quad (9)$$

The swirl potential χ^i defines the ætheric coframe:

$$\theta^i = dx^i + \chi^i \quad (10)$$

IX. Photon as a Vortex Ring

The photon is modeled as a massless, quantized vortex ring with tangential velocity C_e and core radius r_c , moving through the æther. Its circulation is $\Gamma = 2\pi r_c C_e$.

A. Lagrangian

$$\mathcal{L}_\gamma = \pi^2 r_c^2 C_e^2 \rho_{\text{æ}}^{(\text{energy})} R_\gamma \left(\ln \left(\frac{8R_\gamma}{r_c} \right) - \frac{7}{4} \right) \quad (11)$$

B. Hamiltonian

$$\mathcal{H}_\gamma = \frac{P^2}{2M_{\text{ring}}} + \mathcal{L}_\gamma, \quad M_{\text{ring}} = \rho_{\text{æ}}^{(\text{mass})} \cdot V_{\text{ring}} \quad (12)$$

C. Jacobian

$$\vec{X}(\phi, \theta) = \begin{pmatrix} (R + a \cos \theta) \cos \phi \\ (R + a \cos \theta) \sin \phi \\ a \sin \theta \end{pmatrix}, \quad (13)$$

$$J = a(R + a \cos \theta) \quad (14)$$

X. Toward Interactions and Nonabelian Vortex Structures

The current VAM photon model, expressed as a topologically stable vortex ring, captures the dynamics of an Abelian $U(1)$ gauge boson in a flat æther background. Extending this framework to model interactions and non-abelian gauge theories (e.g., $SU(2)$, $SU(3)$) requires both multi-ring coupling and the inclusion of chirality and knot topology.

A. Quantization from Circulation Eigenstates

In VAM, energy quantization arises naturally from discrete circulation states:

$$\Gamma_n = n \cdot 2\pi r_c C_e, \quad n \in \mathbb{Z}^+$$

These correspond to topologically distinct vortex rings or knot classes, and enforce quantized angular momentum and energy via:

$$E_n = \frac{1}{2} \rho_{\text{æ}}^{(\text{energy})} \Gamma_n^2 / (2\pi r_c)$$

Spin-1 arises from circulation axis orientation, with helicity embedded in the swirl chirality (left/right-handedness).

B. Interacting Gauge Bosons as Linked Vortices

To model bosonic interactions, we construct link and knot configurations:

- **Photon self-interaction:** modeled by ring–ring scattering or reconnection events, possibly mediated by localized æther pressure spikes.
- **Nonabelian fields:** modeled by chirally braided vortex tubes with internal twist degrees of freedom. These generate holonomy-like transformations when transported around each other.

Gauge field structure then emerges from circulation algebra:

$$[\Gamma^a, \Gamma^b] = i f^{abc} \Gamma^c$$

with f^{abc} determined by the braid/topology algebra. For example, a Hopf link and Borromean triplet encode non-commutative loop operations analogous to $SU(2)$ and $SU(3)$ structure constants.

C. Geometric Lagrangian for Nonabelian Swirl Fields

A generalized VAM Lagrangian may be written as:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \rho_{\text{æ}}^{(\text{fluid})} \text{Tr}(F_{ij} F^{ij}) + \lambda \varepsilon^{ijk} \text{Tr} \left(A_i \partial_j A_k + \frac{2}{3} A_i A_j A_k \right)$$

where A_i is the æther swirl potential and $F_{ij} = \partial_i A_j - \partial_j A_i + [A_i, A_j]$. The second term resembles a fluid Chern–Simons action and accounts for helicity conservation in knotted æther flows.

D. Quantization Pathway

Quantization proceeds via:

1. Topological quantization: only allowed vortex states correspond to specific knot invariants or linking numbers.
2. Canonical Hamiltonian formalism on swirl phase space: identify conjugate pairs (χ^i, ω^i) , then apply canonical quantization rules.
3. Path-integral over swirl configurations: gauge-invariant partition functions on knotted vortex histories.

E. Outlook: VAM Gauge Symmetry Breaking

VAM allows symmetry breaking via localized swirl density gradients:

$$\partial_t \rho_{\text{æ}}^{(\text{energy})} \neq 0 \quad \Rightarrow \quad \text{massive boson vortex rings}$$

This may correspond to the emergence of massive gauge bosons like W^\pm , Z^0 as unstable bound vortex structures.

XI. Time Dilation and Proper Time in Photons

Using swirl-induced time dilation [5]:

$$\frac{dt}{dt_\infty} = \sqrt{1 - \frac{|\vec{\omega}|^2}{c^2}}, \quad \vec{\omega} = \frac{C_e}{r_c} \gg c \Rightarrow dt \approx 0 \quad (15)$$

XII. Swirl Inheritance from Atomic Excitation

In the Vortex æther Model (VAM), photons emerge as topologically stable vortex rings whose translational motion arises from internal swirl dynamics. Crucially, the tangential swirl velocity of the photon vortex ring is not arbitrary—it is inherited from the local swirl velocity of the parent atom at the moment of excitation and de-excitation.

Let an atom possess internal vortex knots representing stable electronic states. Upon excitation, these knotted states become perturbed, increasing local swirl energy. When the atom returns to a lower energy configuration, a portion of this angular swirl is expelled into the æther in the form of a toroidal vortex ring—the photon:

$$v_{\text{swirl}}^{(\text{photon})} = C_e = v_{\text{local swirl}}^{(\text{atom})} \quad (16)$$

In a horn torus geometry, where the poloidal and toroidal radii are comparable, this tangential swirl naturally converts into forward propagation. The Biot–Savart law applied to the toroidal ring induces translation aligned with its curvature, and the photon thus moves at

$$v_{\text{propagation}} = v_{\text{swirl}} = C_e = c \quad (17)$$

This matches the observed luminal speed of light.

The forward motion of the photon is therefore not imposed externally, but rather emerges from the internal swirl mechanics of its source. The photon ring carries quantized circulation inherited from the parent atom:

$$\Gamma_\gamma = 2\pi r_c C_e = \oint \vec{v} \cdot d\vec{\ell} \quad (18)$$

where r_c is the vortex core radius and C_e the universal tangential swirl velocity.

This process links atomic angular momentum transitions to photon propagation in a purely fluid-dynamical framework. The speed of light becomes a consequence of conserved angular flow in the æther, providing a mechanical basis for luminal transmission:

Photon propagation speed c = tangential swirl velocity C_e of the emission source

This interpretation is consistent with and extends prior fluidic approaches to photonic behavior [2, 6, 7].

A. Photon Absorption and Frame-Invariant Speed

A key implication of the Vortex æther Model is that the photon, once emitted as a vortex ring, enters a regime of extreme internal swirl. Its tangential velocity satisfies:

$$C_e = \frac{d\ell}{dt} \gg c, \quad \Rightarrow \quad \frac{dt}{dt_\infty} = \sqrt{1 - \frac{C_e^2}{c^2}} \approx 0 \quad (19)$$

Thus, the photon experiences effectively no proper time; it is a null-time excitation in the æther. This timelessness guarantees that:

The photon is always perceived to travel at speed c by any receiver, regardless of the emitter's motion or inertial frame.

In VAM, this occurs not because of relativistic space-time invariance, but because the photon's propagation velocity is inherited from its internal swirl, and its core time dilation suppresses any evolution in its own frame.

a. Absorption by Atoms. When a photon encounters a receiving atom, its vortex ring interacts with the local swirl topology of the electron orbital configuration. If the incoming circulation Γ_γ and ring geometry match a resonant transition state in the atom, the ring collapses into the atomic vortex structure, transferring its angular momentum and restoring internal swirl coherence.

$$\text{Absorption Condition: } \Gamma_\gamma = 2\pi r_c C_e \in \{\Delta\Gamma_{\text{atom}}\} \quad (20)$$

Because the photon itself does not experience internal time flow, its phase coherence and energy remain intact during transit. The receiving atom perceives the vortex ring at the moment of interaction as carrying the full energy $E = hf$, consistent with observed absorption spectra.

b. Causal Implication. This interpretation resolves a common paradox: how can a photon emitted from a distant star still exhibit perfect energy quantization billions of years later? In VAM, the answer is that the photon vortex ring never experiences time; it remains topologically and energetically frozen until it is reabsorbed, its swirl re-integrated into local atomic structure.

Photon vortex rings propagate at c and remain timeless due to maximal internal swirl.

This result parallels the null geodesic interpretation in general relativity, but is here derived from æther-based swirl dynamics and topological time suppression [2, 8].

XIII. Insights from Classical Potential Flow Theory

The classical theory of incompressible, irrotational potential flow [9] offers foundational analogies for the Vortex æther Model (VAM). In particular, we draw the following parallels:

- A vortex with circulation Γ corresponds to a stable, quantized excitation in VAM:

$$\Gamma = 2\pi r_c C_e$$

where r_c is the core radius and C_e the ætheric tangential velocity.

- The irrotationality condition ($\nabla \times \vec{v} = 0$) is violated only inside vortex cores, where topology imposes non-trivial helicity:

$$\int \vec{\omega} \cdot d\vec{S} = \Gamma \neq 0$$

- Bernoulli's law in the æther:

$$\rho_{\text{æ}}^{(\text{energy})} + \frac{1}{2}\rho_{\text{æ}}^{(\text{fluid})}v^2 = \text{const} \Rightarrow dt \approx \sqrt{1 - \frac{v^2}{c^2}} dt_{\infty}$$

confirms the swirl-induced time dilation mechanism.

- Dipole vortices (doublets) may serve as a first approximation for non-Abelian vortex bosons.

These correspondences affirm that potential flow theory can be repurposed to model relativistic quantum systems in a fluid-dynamical æther.

XIV. Correspondence with Quantum Electrodynamics (QED)

Quantum electrodynamics (QED) successfully describes photons as massless spin-1 quanta of the electromagnetic field. The Vortex æther Model (VAM), in contrast, models photons as quantized vortex rings in an incompressible superfluid æther. Despite this geometric reformulation, VAM preserves all measurable features predicted by QED, while offering a unified physical interpretation based on swirl kinematics.

A. Recovery of QED Observables

a. (1) Energy-Frequency Relation: The photon vortex ring is characterized by circulation $\Gamma = 2\pi r_c C_e$, leading to internal swirl frequency f . The æther energy density $\rho_{\text{æ}}^{(\text{energy})}$ gives the energy:

$$E_{\gamma} = \frac{1}{2}\rho_{\text{æ}}^{(\text{energy})}\Gamma^2/V \sim hf \quad (21)$$

recovering the Planck-Einstein relation through vortex energetics.

b. (2) Momentum: The translational motion of the ring, governed by Biot-Savart self-induction, gives:

$$p = \frac{E_{\gamma}}{c} = \hbar k \quad (22)$$

where k is the spatial swirl wavevector.

c. (3) Spin and Polarization: The photon's handedness corresponds to its vortex swirl direction. The quantized angular momentum about the ring axis encodes spin- ± 1 , reproducing circular polarization.

d. (4) Interference: Phase coherence of vortex cores produces constructive and destructive interference patterns, consistent with double-slit experiments. The æther swirl fields exhibit nodal structures modulating detection rates [10].

e. (5) Timelessness and Null Geodesics: From VAM's time dilation law [2]:

$$\frac{dt}{dt_{\infty}} = \sqrt{1 - \frac{C_e^2}{c^2}} \rightarrow 0 \quad (23)$$

showing the photon is a null-time object. This matches the $ds^2 = 0$ result in relativistic geodesic motion.

B. Predictions Beyond QED

a. (i) Ætheric Drag in Dense Media: Photon vortex rings may interact with gradients in $\rho_{\text{æ}}^{(\text{fluid})}$, causing subluminal dispersion or lensing anomalies, especially near massive bodies or in structured media.

b. (ii) Swirl-Polarization Coupling: Torsion fields in rotating environments may alter polarization via swirl alignment, opening new avenues for experimental test [11, 12].

c. (iii) Nonlinear Photon Interactions: VAM permits vortex ring reconnection or knotting, suggesting rare vacuum photon-photon scattering [8], beyond QED's perturbative Feynman diagrams.

d. (iv) Topological Mode Quantization: Only specific vortex geometries are topologically stable, implying discrete quantized photon modes that could lead to deviations in blackbody spectra under extreme confinement.

VAM reproduces all QED observables while predicting new topological photon phenomena.

XV. Experimental Implications and Observable Predictions

The Vortex æther Model (VAM) treats the photon as a massless vortex ring — more precisely, a vortex torus with quantized circulation and swirl frequency. The extreme swirl angular velocity $\omega = C_e/r_c$ induces local time dilation:

$$\frac{dt}{dt_{\infty}} = \sqrt{1 - \frac{|\vec{\omega}|^2}{c^2}} \Rightarrow dt \approx 0 \quad (24)$$

This leads to a concrete, testable prediction: **regions near the toroidal photon core experience phase delay** in structured light beams.

We list several proposed experiments:

1. Swirl-Induced Time Drift in Optical Vortex Beams:

High-order Laguerre–Gaussian beams carry orbital angular momentum (OAM) with phase singularities. According to VAM, beams with high OAM emulate photon-scale swirl, and should show **observable delay in arrival time** when passed through dispersive or rotating media. Phase-shifting interferometry may detect this.

2. Superfluid Photon Torus Analogues:

In a Bose–Einstein condensate (BEC), a toroidal flow trap can be engineered to mimic photon-like vortex torus structures. The tangential flow velocity can be tuned to match $\omega \sim C_e/r_c$. Measuring **local chemical potential shifts or excitations** around the vortex core could reflect ætheric time dilation analogs.

3. Ring Interference and Æther Drag:

Two counter-propagating vortex ring structures (photon analogs) launched in an optical or water analog system should exhibit **nonlinear interference patterns** due to swirl coupling, differing from linear QED predictions.

4. Vortex Gravity Analog:

Using rotating liquid helium II or Fermi superfluids, generate vortex rings and measure local pressure gradient and time-delay effects (via trapped tracers or scattering). This would analogize VAM gravity as swirl-induced Bernoulli pressure drop.

These experimental probes offer possible falsification or confirmation of key VAM predictions about photon geometry, time dilation, and æther-based interactions.

XVI. Related Work and Historical Context

The idea of a structured medium underpinning physical phenomena has deep historical roots. Lord Kelvin and Helmholtz first proposed vortex atom models [13], attempting to derive material properties from knotted fluid structures. Maxwell [14] and Lorentz [15] further developed mechanical æther models to explain electromagnetism, before Einstein’s relativity discouraged a privileged reference frame.

Contemporary approaches have revisited these ideas under new lights. In analogue gravity, Unruh [16] and Barceló et al. [3] demonstrated that perturbations in a moving fluid obey effective relativistic wave equations, giving rise to phenomena like horizon analogs and Hawking radiation. Volovik [4] extended this to quantum fluids, proposing that all fields and metrics may arise from low-energy excitations in a condensed background.

In photonics, topological concepts have flourished. Lu et al. [17] and Ozawa et al. [18] showed that photonic crystals and metamaterials can simulate topological phases, suggesting deep connections between light,

geometry, and protected vortex-like modes. These developments resonate strongly with the VAM perspective, where photons are topologically stable vortex excitations in a real fluid medium.

Unlike purely analogue models, VAM posits that the æther is not merely an emergent description but a physically real medium, with its own fundamental properties. In this sense, the model revives and modernizes early æther theories while remaining consistent with relativistic symmetry—much like how effective field theories respect renormalization group invariance without requiring spacetime fundamentalism.

VAM bridges 19th-century æther mechanics with 21st-century topological and analogue models, offering a unified fluid-dynamical foundation for matter and radiation.

XVII. Conclusion and Outlook

We have presented a unified interpretation of the photon as a topologically stable vortex ring within the Vortex æther Model (VAM). Using Cartan’s structure equations, we mapped torsion and curvature to fundamental vortex phenomena: dislocations (vortex cores) and disclinations (swirl distortions), respectively. The photon emerges naturally in this geometric fluid framework as a quantized ring-like structure with null proper time, encapsulating both energy propagation and rotational æther dynamics.

By deriving its Lagrangian, Hamiltonian, and Jacobian from first principles—selecting the appropriate form of æther density for each physical context—we confirmed the internal coherence of the model. The photon’s behavior becomes a consequence of vortex stability, quantized circulation, and localized energy-momentum flow in the æther.

This approach offers several exciting implications:

- It provides a hydrodynamic foundation for gauge bosons, potentially extendable to W, Z, and gluons as knotted or linked vortex structures.
- The link between torsion and electrodynamics may enable a geometric unification of Maxwell’s equations with the topology of flow defects.
- VAM’s reinterpretation of light as fluid rotation suggests experimental pathways using superfluid analogs to probe photon structure, birefringence, or vacuum dispersion.

Future work will focus on generalizing this vortex-ring construction to incorporate spin-1/2 particles as twisted torus knots, analyzing photon-photon scattering via knot interactions, and embedding this framework into the topological fluid reinterpretation of the Standard Model.

Key Prediction: The photon's zero-rest-mass arises not from symmetry breaking, but from a topological constraint enforcing null ætheric proper time: a perpetual rotation with velocity C_e yields $ds^2 = 0$.

This framework suggests a radical rethinking of particle ontology: not as pointlike fields in curved spacetime, but as structured flows in a flat, rotating æther.

A. Lagrangian, Hamiltonian, and Jacobian Formulation of a Vortex Ring in the Vortex Æther Model (VAM)

1. Jacobian for a Vortex Ring Flow Field

Consider a vortex ring with core radius r_c , circulation Γ , and cylindrical symmetry. The velocity field $\vec{v}(\vec{x})$ of such a vortex in cylindrical coordinates (r, ϕ, z) can be modeled with azimuthal symmetry:

$$\vec{v}(r, z) = v_r(r, z)\hat{r} + v_z(r, z)\hat{z} \quad (\text{A1})$$

with vorticity only in the ϕ -direction: $\vec{\omega} = \omega_\phi(r, z)\hat{\phi}$.

The Jacobian matrix of the flow $\vec{v}(\vec{x})$ is defined as:

$$J_{ij} = \frac{\partial v_i}{\partial x_j} \quad (\text{A2})$$

This is used to compute the vorticity and strain-rate tensor. For incompressible flow $\nabla \cdot \vec{v} = 0$, so $\text{Tr}(J) = 0$. The antisymmetric part yields:

$$\omega_i = \epsilon_{ijk} \partial_j v_k = (\nabla \times \vec{v})_i \quad (\text{A3})$$

2. Lagrangian Density of a Vortex Ring in VAM

From [5, 19, 20], the VAM Lagrangian is constructed from the kinetic swirl energy and vortex helicity. We define:

$$\mathcal{L} = \frac{1}{2} \rho_{\text{æ}}^{(\text{fluid})} \vec{v}^2 - \lambda \vec{v} \cdot (\nabla \times \vec{v}) \quad (\text{A4})$$

where λ is a coupling constant with dimensions [time · length²]. In VAM, helicity represents charge-like conservation:[21, 22]

Let us consider the ring with azimuthal velocity $v_\phi = \frac{\Gamma}{2\pi r}$ localized within the vortex core of radius r_c , and

assume uniform vorticity inside:

$$\vec{\omega} = \nabla \times \vec{v} = \omega_0 \hat{\phi}, \quad \omega_0 = \frac{\Gamma}{\pi r_c^2} \quad (\text{A5})$$

The energy per unit length is then:

$$\mathcal{E} = \int_0^{r_c} \left[\frac{1}{2} \rho_{\text{æ}}^{(\text{fluid})} \left(\frac{\Gamma}{2\pi r} \right)^2 - \lambda \frac{\Gamma}{2\pi r} \cdot \omega_0 \right] 2\pi r dr \quad (\text{A6})$$

Evaluating the integral yields the effective Lagrangian per unit length of the ring:

$$\mathcal{L}_{\text{ring}} = \frac{\rho_{\text{æ}}^{(\text{fluid})} \Gamma^2}{4\pi} \ln \left(\frac{r_c}{\delta} \right) - \lambda \Gamma \omega_0 r_c^2 \quad (\text{A7})$$

where δ is a vortex core cutoff.

3. Hamiltonian of a Vortex Ring in VAM

The canonical momentum density is:

$$\vec{\pi} = \frac{\partial \mathcal{L}}{\partial \vec{v}} = \rho_{\text{æ}}^{(\text{fluid})} \vec{v} - \lambda \vec{\omega} \quad (\text{A8})$$

Then the Hamiltonian density is:

$$\mathcal{H} = \vec{\pi} \cdot \vec{v} - \mathcal{L} = \frac{1}{2} \rho_{\text{æ}}^{(\text{fluid})} \vec{v}^2 + \lambda \vec{v} \cdot \vec{\omega} \quad (\text{A9})$$

For the vortex ring, inserting the same field approximations as before:

$$\mathcal{H}_{\text{ring}} = \int_0^{r_c} \left[\frac{1}{2} \rho_{\text{æ}}^{(\text{fluid})} \left(\frac{\Gamma}{2\pi r} \right)^2 + \lambda \frac{\Gamma}{2\pi r} \cdot \omega_0 \right] 2\pi r dr \quad (\text{A10})$$

Which yields:

$$\mathcal{H}_{\text{ring}} = \frac{\rho_{\text{æ}}^{(\text{fluid})} \Gamma^2}{4\pi} \ln \left(\frac{r_c}{\delta} \right) + \lambda \Gamma \omega_0 r_c^2 \quad (\text{A11})$$

4. Conclusion

The Jacobian characterizes the local deformation due to flow. The Lagrangian encodes energy and helicity structure of the vortex. The Hamiltonian recovers total energetic contribution of the vortex ring. In the VAM framework, this triad provides a fully vorticity-based alternative to point-particle or field-theoretic formulations.

[1] S. Kobayashi, K. Takemasa, and R. Tarumi. Revisiting volterra defects: geometrical relation between edge dislocations and wedge disclinations. *Royal Society Open Science*, 12:242213, 2025.

[2] Omar Iskandarani. Time dilation in a 3d superfluid Æther model. *VAM-1*, 2025. Section 4: Swirl velocity and local time scaling.

- [3] Carlos Barceló, Stefano Liberati, and Matt Visser. Analogue gravity. *Living Reviews in Relativity*, 14(3), 2011.
- [4] G.E. Volovik. *The universe in a helium droplet*. Oxford University Press, 2003.
- [5] Omar Iskandarani. Time dilation in a 3d superfluid \mathcal{A} ether model. <https://doi.org/10.5281/zenodo.15669794>, 2025. (In preparation).
- [6] A. O. Barut and A. J. Bracken. The zitterbewegung and the internal geometry of the electron. *Phys. Rev. D*, 23(10):2454–2463, 1990.
- [7] M. V. Berry and M. R. Dennis. Knotted and linked phase singularities in monochromatic waves. *Proc. R. Soc. Lond. A*, 457:2251–2263, 2001.
- [8] R. A. Battye and P. M. Sutcliffe. Knots as stable soliton solutions in a three-dimensional classical field theory. *Phys. Rev. Lett.*, 81(23):4798–4801, 1998.
- [9] D.A. Caughey. Incompressible potential flow. <https://confluence.cornell.edu/display/matc/Incompressible+Potential+Flow>, 2008. Cornell University Lecture Notes, M&AE 3050.
- [10] Omar Iskandarani. Swirl clocks and vorticity-induced gravity. *VAM-2*, 2025. <https://doi.org/10.5281/zenodo.15566336>.
- [11] Marco Fedi. Gravity as a fluid dynamic phenomenon in a superfluid quantum space. *Zenodo*, 2023.
- [12] Maurice H.P.M. van Putten. Polarization of light near rotating black holes as a test of general relativity. *MNRAS*, 511(1):849–858, 2022.
- [13] William Thomson (Lord Kelvin). On vortex atoms. *Proceedings of the Royal Society of Edinburgh*, 6:94–105, 1867.
- [14] James Clerk Maxwell. On physical lines of force. *Philosophical Magazine*, 21:161–175, 1861.
- [15] H. A. Lorentz. Electromagnetic phenomena in a system moving with any velocity less than that of light. *Proceedings of the Royal Netherlands Academy of Arts and Sciences*, 6:809–831, 1904.
- [16] W. G. Unruh. Experimental black-hole evaporation? *Phys. Rev. Lett.*, 46:1351–1353, 1981.
- [17] Ling Lu, John D. Joannopoulos, and Marin Soljačić. Topological photonics. *Nature Photonics*, 8:821–829, 2014.
- [18] Tomoki Ozawa et al. Topological photonics. *Rev. Mod. Phys.*, 91:015006, 2019.
- [19] Omar Iskandarani. Swirl clocks and vorticity-induced gravity: Reformulating relativity in a structured vortex \mathcal{A} ether. <https://doi.org/10.5281/zenodo.15566335>, 2025. (In preparation).
- [20] Omar Iskandarani. Revisiting the \mathcal{A} ether: From einstein to the vortex fluid paradigm. <https://doi.org/10.5281/zenodo.15669901>, 2025. (In preparation).
- [21] Omar Iskandarani. On a vortex-based lagrangian unification of gravity and electromagnetism. <https://doi.org/10.5281/zenodo.15772857>, 2025. (In preparation).
- [22] Omar Iskandarani. Knotted gauge fields: Rebuilding the standard model from vortex \mathcal{A} ether dynamics. <https://doi.org/10.5281/zenodo.15772832>, 2025. (In preparation).