Swirl Clocks and Vorticity-Induced Gravity

Reformulating Relativity in a Structured Æther

A Topological Fluid Mechanics Approach to Time Dilation, Mass, and Gravitation

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Abstract

This paper presents a fluid-dynamic reformulation of General Relativity using the Vortex Æther Model (VAM), wherein gravity and time dilation arise from vorticity-induced pressure gradients in an incompressible, inviscid superfluid æther. Within a Euclidean 3D space governed by an absolute causal time $\mathcal{N}$ (Aithēr-Time), mass and inertia emerge as topologically stable vortex knots. Geodesic motion is replaced by alignment along vortex streamlines with conserved circulation, and gravitational force is modeled as a Bernoulli pressure potential:

$$\nabla^2 \Phi_v(\vec{r}) = -\rho_{æ} \, \|\boldsymbol{\omega}(\vec{r})\|^2$$

Time dilation is reinterpreted as an energetic effect of swirl phase and vortex pressure gradients. The measurable proper time τ—termed Chronos-Time—is derived from vortex energetics as:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{C_e^2}{c^2} e^{-r/r_c} - \frac{2G_{\text{swirl}} M_{\text{eff}}(r)}{rc^2} - \frac{C_e^2}{r_c^2 c^2} e^{-r/r_c}}$$

The third term originates from circulation energy with $\Omega = \Gamma/(2\pi r^2)$ and $\Gamma = \oint \vec{v} \cdot d\vec{\ell}$, while the coupling factor $\beta = 1/c^2$ reflects ætheric inertial drag. This yields a rotational dilation term $\beta\Omega^2 \sim \frac{C_e^2}{r_c^2 c^2} e^{-r/r_c}$.

VAM introduces a multilayered temporal ontology, distinguishing absolute causal time ($\mathcal{N}$), local proper time (τ), and internal vortex phase time $S(t)$ (Swirl Clock). A scale-dependent æther density governs transitions between dense core regions and asymptotic vacuum, leading to testable predictions in rotating systems, gravitational redshift anomalies, and low-energy nuclear resonance (LENR).

The model reproduces Newtonian gravity and Lense–Thirring frame-dragging in the appropriate limits and establishes a physically grounded, topologically invariant theory of time, mass, and gravitation. VAM extends analogue gravity frameworks [? ?] by embedding them in a consistent, vortex-based æther ontology.

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Contents

A Derivation of the vorticity-based gravitational field

In the Vortex Æther Model (VAM), the æther is modeled as a stationary, incompressible, inviscid fluid with constant mass density ρ. The dynamics of such a medium are described by the stationary Euler equation:

$$(\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho}\nabla p, \tag{1}$$

where $\vec{v}$ is the velocity field and p is the pressure. To rewrite this expression we use a vector identity:

$$(\vec{v} \cdot \nabla)\vec{v} = \nabla \left(\frac{1}{2}v^2\right) - \vec{v} \times (\nabla \times \vec{v}) = \nabla \left(\frac{1}{2}v^2\right) - \vec{v} \times \vec{\omega}, \tag{2}$$

where $\vec{\omega} = \nabla \times \vec{v}$ is the local vorticity. Substitution yields:

$$\nabla \left(\frac{1}{2}v^2\right) - \vec{v} \times \vec{\omega} = -\frac{1}{\rho}\nabla p. \tag{3}$$

We now take the dot product with $\vec{v}$ on both sides:

$$\vec{v} \cdot \nabla \left(\frac{1}{2}v^2 + \frac{p}{\rho}\right) = 0. \tag{4}$$

This equation shows that the quantity

$$B = \frac{1}{2}v^2 + \frac{p}{\rho} \tag{5}$$

is constant along streamlines, a familiar form of the Bernoulli equation. In regions of high vorticity (such as in vortex cores), v is large and thus p is relatively low. This results in a pressure gradient that behaves as an attractive force—a gravitational analogy within the VAM framework.

We therefore define a vorticity-induced potential Φ_v such that:

$$\vec{F}_g = -\nabla\Phi_v, \tag{6}$$

where the potential is given by:

$$\Phi_v(\vec{r}) = \gamma \int \frac{\|\vec{\omega}(\vec{r}')\|^2}{\|\vec{r} - \vec{r}'\|} d^3r', \tag{7}$$

with γ the vorticity-gravity coupling. This leads to the Poisson-like equation:

$$\nabla^2\Phi_v(\vec{r}) = -\rho\|\vec{\omega}(\vec{r})\|^2, \tag{8}$$

where the role of mass density (as in Newtonian gravitational theory) is replaced by vorticity intensity. This confirms the core hypothesis of the VAM: gravity is not a consequence of spacetime curvature, but an emergent phenomenon resulting from pressure differences caused by vortical flow.

B Newtonian limit and time dilation validation

To confirm the physical validity of the Vortex Æther Model (VAM), we analyze the limit $r \gg r_c$, in which the gravitational field is weak and the vorticity is far away from the source. We show that in this limit the vorticity potential Φ_v and the time dilation formula of VAM transform into classical Newtonian and relativistic forms.

B.1 Large distance vorticity potential

The vorticity-induced potential is defined in VAM as:

$$\Phi_v(\vec{r}) = \gamma \int \frac{\|\vec{\omega}(\vec{r}')\|^2}{\|\vec{r} - \vec{r}'\|} d^3r', \tag{9}$$

where $\gamma = G\rho_{æ}^2$ is the vorticity-gravity coupling. For a strongly localized vortex (core radius $r_c \ll r$), we can approximate the integration outside the core as coming from an effective point mass:

$$\Phi_v(r) \to -\frac{GM_{\mathrm{eff}}}{r}, \tag{10}$$

where $M_{\mathrm{eff}} = \int \rho_{æ}\|\vec{\omega}(\vec{r}')\|^2 d^3r'/\rho_{æ}$ acts as equivalent mass via vortex energy. This approximation exactly reproduces Newton's law of gravity.

B.2 Time dilation in the weak field limit

For $r \gg r_c$ we have $e^{-r/r_c} \to 0$ and $\Omega^2 \approx 0$ for non-rotating objects. The time dilation formula then reduces to:

$$\frac{d\tau}{dt} \approx \sqrt{1 - \frac{2G_{\mathrm{swirl}}M_{\mathrm{eff}}}{rc^2}}. \tag{11}$$

If we assume $G_{\mathrm{swirl}} \approx G$ (in the macroscopic limit), it exactly matches the first-order approximation of the Schwarzschild solution in general relativity:

$$\frac{d\tau}{dt}_{\mathrm{GR}} \approx \sqrt{1 - \frac{2GM}{rc^2}}. \tag{12}$$

This shows that VAM shows consistent transition to GR in weak fields.

B.3 Example: Earth as a vortex mass

Consider Earth as a vortex mass with mass $M = 5.97 \times 10^{24}$ kg and radius $R = 6.371 \times 10^6$ m. The Newtonian gravitational acceleration at the surface is:

$$g = \frac{GM}{R^2} \approx \frac{6.674 \times 10^{-11} \cdot 5.97 \times 10^{24}}{(6.371 \times 10^6)^2} \approx 9.8\,\mathrm{m/s}^2. \tag{13}$$

In the VAM, this acceleration is taken to be the gradient of the vorticity potential:

$$g = -\frac{d\Phi_v}{dr} \approx \frac{GM_{\mathrm{eff}}}{R^2}. \tag{14}$$

As long as $M_{\mathrm{eff}} \approx M$, the VAM reproduces exactly the known gravitational acceleration on Earth, including the correct redshift of time for clocks at different altitudes (as observed in GPS systems).

C Validation with the Hafele–Keating clock experiment

An empirical test for time dilation is the famous Hafele–Keating experiment (1971), in which atomic clocks in airplanes circled the Earth in easterly and westward directions. The results showed significant time differences compared to Earth-based clocks, consistent with predictions from both special and general relativity. In the Vortex Æther Model (VAM), these differences are reproduced by variations in local æther rotation and pressure fields.

C.1 Experiment summary

In the experiment, four cesium clocks were placed on board commercial aircraft orbiting the Earth in two directions:

- **Eastward** (with the Earth's rotation): increased velocity $\Rightarrow$ kinetic time dilation.
- **Westward** (against the rotation): decreased velocity $\Rightarrow$ less kinetic deceleration.

In addition, the aircraft were at higher altitudes, which led to lower gravitational acceleration and thus a gravitational *acceleration* of the clock frequency (blueshift).

The measured deviations were:

- Eastward: $\Delta\tau \approx -59$ ns (deceleration)
- Westward: $\Delta\tau \approx +273$ ns (acceleration)

C.2 Interpretation within the Vortex Æther Model

In VAM, both effects are reproduced via the time dilation formula:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{C_e^2}{c^2} e^{-r/r_c} - \frac{2G_{\text{swirl}} M_{\text{eff}}(r)}{rc^2} - \beta\Omega^2} \tag{15}$$

- The **gravity term** $-\frac{2G_{\text{swirl}} M_{\text{eff}}(r)}{rc^2}$ decreases at higher altitudes $\Rightarrow \tau$ accelerates (clock ticks faster).
- The **rotation term** $-\beta\Omega^2$ grows with increasing tangential velocity of the aircraft $\Rightarrow \tau$ slows down (clock ticks slower).

For eastward moving clocks, both effects reinforce each other: lower potential and higher velocity slow the clock. For westward moving clocks, they partly compensate each other, resulting in a net acceleration of time.

C.3 Numerical agreement

Using realistic values for r_c, C_e, and β derived from æther density and core structure (see Table **??**), the VAM can predict reproducible deviations of the same order of magnitude as measured within the measurement accuracy of the experiment. Hereby, the model shows not only conceptual agreement with GR, but also experimental compatibility.

Table 1: Typical parameters in the VAM model

Symbol	Meaning	Value
C_e	Tangential velocity of core	$\sim 1.09 \times 10^6$ m/s
r_c	Vortex core radius	$\sim 1.4 \times 10^{-15}$ m
β	Time dilation coupling	$\sim 1.66 \times 10^{-42}$ s^2
G_{swirl}	VAM gravitational constant	$\sim G$ (macro)

D Dynamics of vortex circulation and quantization

A central building block of the Vortex Æther Model (VAM) is the dynamics of circulating flow around a vortex core. The amount of rotation in a closed loop around the vortex is described by the circulation Γ, a fundamental quantity in classical and topological fluid dynamics.

D.1 Kelvin's circulation theorem

According to Kelvin's circulation theorem, the circulation Γ is preserved in an ideal, inviscid fluid in the absence of external forces:

$$\Gamma = \oint_{\mathcal{C}(t)} \vec{v} \cdot d\vec{l} = \text{const.} \tag{16}$$

Here $\mathcal{C}(t)$ is a closed loop that moves with the fluid. In the case of a superfluid æther, this means that vortex structures are stable and topologically protected — they cannot easily deform or disappear without breaking conservation.

D.2 Circulation around the vortex core

For a stationary vortex configuration with core radius r_c and maximum tangential velocity C_e, it follows from symmetry:

$$\Gamma = \oint \vec{v} \cdot d\vec{l} = 2\pi r_c C_e. \tag{17}$$

This expression describes the total rotation of the æther field around a single vortex particle, such as an electron.

D.3 Quantization of circulation

In superfluids such as helium II, it has been observed that circulation occurs only in discrete units. This principle is adopted in VAM by stating that circulation quantizes in integer multiples of a base unit κ:

$$\Gamma_n = n \cdot \kappa, \quad n \in \mathbb{Z}, \tag{18}$$

where

$$\kappa = C_e r_c \tag{19}$$

is the elementary circulation constant. This value is analogous to h/m in the context of quantum fluids and is coupled to vortex core parameters in VAM.

D.4 Physical interpretation

- The circulation Γ determines the rotational content of a vortex node and is coupled to the mass and inertia of the corresponding particle.
- The constant κ determines the "spin"-unit or vortex helicity of an elementary vortex particle.
- The vortex circulation is a conserved quantity and leads to intrinsically stable and discrete states — a direct analogy with quantization in particle physics.

VAM thus provides a formal framework in which classical flow laws — via Kelvin and Euler — transform into topologically quantized field structures describing fundamental particles.

E Time dilation from vortex energy and pressure gradients

In the Vortex Æther Model (VAM), time dilation is considered an energetic phenomenon arising from the rotational energy of local æther vortices. Instead of depending on spacetime curvature as in general relativity, the clock frequency in VAM is coupled to the vortex kinetics in the surrounding æther.

E.1 Formula: clock delay due to rotational energy

The eigenfrequency of a vortex-based clock depends on the total energy stored in local core rotation. For a clock with moment of inertia I and angular velocity Ω, we have:

$$\frac{d\tau}{dt} = \left(1 + \frac{1}{2}\beta I \Omega^2\right)^{-1}, \tag{20}$$

where β is a time-dilation coupling derived from æther parameters (e.g., r_c, C_e). This formula implies:

- The larger the local rotational energy, the stronger the clock delay.
- For weak rotation ($\Omega \to 0$), we have $\tau \approx t$ (no dilation).

This expression is analogous to relativistic dilation formulas, but has its roots in vortex mechanics.

E.2 Alternative derivation via pressure difference (Bernoulli approximation)

The same effect can be derived via Bernoulli's law in a stationary flow:

$$\frac{1}{2}\rho v^2 + p = \text{const.} \tag{21}$$

Around a rotating vortex holds:

$$v = \Omega r, \quad \Rightarrow \quad \Delta p = -\frac{1}{2}\rho(\Omega r)^2$$

This leads to a local pressure deficit around the vortex axis. In the VAM, it is assumed that the clock frequency ν increases at higher pressure (higher æther density), and decreases at low pressure. The clock delay then follows via enthalpy:

$$\frac{d\tau}{dt} \sim \frac{H_{\mathrm{ref}}}{H_{\mathrm{loc}}} \approx \frac{1}{1 + \frac{\Delta p}{\rho}}, \tag{22}$$

whatever small Δp leads to an approximation of the form:

$$\frac{d\tau}{dt} \approx \left(1 + \frac{1}{2}\beta I \Omega^2\right)^{-1}. \tag{23}$$

E.3 Physical interpretation

- **Mechanical**: Time dilation is a measure of the energy stored in core rotation; faster rotating nodes slow down the local clock.
- **Hydrodynamic**: Pressure reduction due to swirl slows down time — according to Bernoulli.
- **Thermodynamic**: Entropy increase in vortex expansion correlates with time delay.

VAM thus shows that time dilation is an emergent phenomenon of vortex energy and flow pressure, and reproduces the classical relativistic behavior from fluid dynamics principles.

F Parameter tuning and limit behavior

To make the equations of the Vortex Æther Model (VAM) consistent with classical gravity, the model parameters must be tuned to reproduce known physical constants in the appropriate limits. In this section, we derive the effective gravitational constant G_{swirl} and analyze the behavior of the gravitational field for $r \to \infty$.

F.1 Derivation of G_{swirl} from vortex parameters

The VAM potential is given by:

$$\Phi_v(\vec{r}) = G_{\mathrm{swirl}} \int \frac{\|\vec{\omega}(\vec{r}')\|^2}{\|\vec{r} - \vec{r}'\|} \, d^3r', \tag{24}$$

where G_{swirl} must satisfy a dimensionally and physically consistent relationship with fundamental vortex parameters. In terms of:

- C_e: tangential velocity at the vortex core,
- r_c: vortex core radius,
- t_p: Planck time,
- $F_{æ}^{\max}$: maximum force in æther interactions,

we derive:

$$G_{\text{swirl}} = \frac{C_e c^5 t_p^2}{2F_{\text{æ}}^{\max} r_c^2}. \tag{25}$$

This expression follows from dimension analysis and matching of the VAM field equations with the Newtonian limit (see also [Iskandarani, 2025]).

F.2 Limit $r \to \infty$: classical gravity

For large distances outside a compact vortex configuration, we have:

$$\Phi_v(r) = G_{\text{swirl}} \int \frac{\|\vec{\omega}(\vec{r}')\|^2}{|\vec{r} - \vec{r}'|} d^3r' \approx \frac{G_{\text{swirl}}}{r} \int \|\vec{\omega}(\vec{r}')\|^2 d^3r'. \tag{26}$$

Define the **effective mass** of the vortex object as:

$$M_{\text{eff}} = \frac{1}{\rho_{\text{æ}}} \int \rho_{\text{æ}} \|\vec{\omega}(\vec{r}')\|^2 d^3r' = \int \|\vec{\omega}(\vec{r}')\|^2 d^3r'. \tag{27}$$

This means:

$$\Phi_v(r) \to -\frac{G_{\text{swirl}} M_{\text{eff}}}{r}, \tag{28}$$

which is identical to the Newtonian potential provided $M_{\text{eff}} \approx M_{\text{grav}}$ and $G_{\text{swirl}} \approx G$.

F.3 Relationship between M_{eff} and observed mass

The effective mass M_{eff} is not a direct mass content as in classical physics, but reflects the integrated vorticity energy in the æther:

$$M_{\text{eff}} \propto \int \frac{1}{2} \rho_{\text{æ}} \|\vec{v}(\vec{r})\|^2 d^3r. \tag{29}$$

In VAM, this mass is associated with a topologically stable vortex knot (like a trefoil for the electron) and thus quantitatively:

$$M_{\text{eff}} = \alpha \cdot \rho_{\text{æ}} C_e r_c^3 \cdot L_k, \tag{30}$$

where L_k is the linking number of the knot and α is a shape factor. By tuning C_e, r_c and $\rho_{\text{æ}}$ to known masses (e.g. of the electron or the earth), VAM can reproduce the classical mass exactly:

$$M_{\text{eff}} \stackrel{!}{=} M_{\text{obs}}. \tag{31}$$

F.4 Conclusion

By parameter tuning, G_{swirl} satisfies classical limits and VAM yields a gravitational field that is similar to Newtonian gravity at large distances. The effective mass M_{eff} acts as a source term, analogous to the role of M in Newton and GR.

G Fundamentals of velocity fields and energies in a vortex system.

G.1 Introduction

Velocity dynamics is a core component of many fluid and plasma systems, including tornado-like flows, knotted vortices in classical or superfluid turbulence, and various complex topological fluid systems. A better understanding of the energy balances associated with these flows can shed light on processes such as vortex stability, reconnection, and global flow organization. We begin with a motivation for how velocity fields can be decomposed to capture the total energy (i.e., self- plus cross-energy), and how this approach aids in tracing flows in both 2D and 3D.

G.2 Foundations: Velocity Fields and Total (Self- + Transverse) Energy

In an incompressible fluid, the velocity field $\mathbf{u}(\mathbf{x}, t)$ is usually determined by the Navier-Stokes or Euler equations. For inviscid analyses, the Euler equations for incompressible flow are:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p, \quad \nabla \cdot \mathbf{u} = 0. \tag{32}$$

We also consider the vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$, which can be used to characterize vortex structures.

To understand the total kinetic energy, we can decompose it as follows:

$$E_{\text{total}} \ = \ E_{\text{self}} \ + \ E_{\text{cross}}. \tag{33}$$

Here, E_{self} is the part of the energy that each vortex or substream element contributes independently (e.g., by local vortex motions), while E_{cross} encodes the contributions that arise from the interaction of different vortex elements. In a multi-vortex scenario, such a decomposition helps to isolate the direct interaction between two (or more) vortex filaments or layers.

G.3 Considerations on momentum and self-energy

A starting point is to remember that for a single vortex Γ, with an azimuthally symmetric core, the induced velocity is sometimes approximated by classical results such as

$$V \ = \ \frac{\Gamma}{4\pi R}\left(\ln \tfrac{8R}{a} - \beta\right), \tag{34}$$

where R is the radius of the main vortex loop, $a \ll R$ is a measure of the core thickness, and β depends on the details of the core model [**?**]. The *self-energy* associated with that vortex, E_{self}, can be cast in a similar form that depends on $\ln(R/a)$, illustrating how the energies of thin-core vortices scale with geometry.

In more general fluid or vortex-lattice models, we can follow E_{self} as the sum of the individual core energies. Furthermore, the presence of multiple filaments modifies the total energy by the cross terms of the velocity fields (the cross energy). This cross energy is often the driving force behind important phenomena such as vortex merging or the 'recoil' effects in wave-vortex interactions.

G.4 Defining and tracking cross energy

When multiple vortices (or partial velocity distributions) coexist, the total velocity field $\mathbf{u}$ can be superposed:

$$\mathbf{u} \;=\; \mathbf{u}_1 \;+\; \mathbf{u}_2, \tag{35}$$

where $\mathbf{u}_1$ and $\mathbf{u}_2$ come from different subsystems. In that scenario is the kinetic energy for a fluid volume V

$$E_{\text{total}} = \frac{\rho}{2}\int_V \mathbf{u}^2\, dV = \frac{\rho}{2}\int_V (\mathbf{u}_1 + \mathbf{u}_2)^2\, dV \tag{36}$$

$$= \frac{\rho}{2}\int_V \mathbf{u}_1^2\, dV \;+\; \frac{\rho}{2}\int_V \mathbf{u}_2^2\, dV \;+\; \rho\int_V \mathbf{u}_1 \cdot \mathbf{u}_2\, dV, \tag{37}$$

disclosure of an interaction or *cross energy* term

$$E_{\text{cross}} \;=\; \rho\int_V \mathbf{u}_1 \cdot \mathbf{u}_2\, dV. \tag{38}$$

Much of the interesting physics comes from (**??**), because it grows or shrinks depending on the geometry of the vortices and the distance between them. Its dynamic evolution can lead to, for example, merging or rebounding. An important point is that the eigenvelocity of each vortex can significantly affect the mutual velocities and thus create net forces or torque.

G.5 Applications to helicity and topological flows

A related concept is helicity, which measures the topological complexity (knots or connections) of vortex tubes. Classically, helicity H is given by

$$H \;=\; \int_V \mathbf{u} \cdot \boldsymbol{\omega}\, dV, \tag{39}$$

which can remain constant or be partially lost during reconnection events. In certain dissipative flows, the cross-energy terms in (**??**) can affect the effective rate of helicity change. Understanding E_{cross} is important for analyzing reconnection paths in classical or superfluid turbulence.

G.6 Derivation scheme for cross-energy

Finally, we give a concise scheme for deriving the expression for cross-energy. Starting with the total velocity field $\mathbf{u} = \sum_{n=1}^{N} \mathbf{u}_n$ for N eddy or partial velocity fields the total kinetic energy is:

$$E_{\text{total}} = \frac{\rho}{2}\int_V \left(\sum_{n=1}^{N} \mathbf{u}_n\right)^2 dV = \frac{\rho}{2}\sum_{n=1}^{N}\int_V \mathbf{u}_n^2\, dV \;+\; \rho\sum_{n<m}\int_V \mathbf{u}_n \cdot \mathbf{u}_m\, dV. \tag{40}$$

One obtains N self-energy terms plus pairwise cross-energy integrals. The cross energy for a pair (i, j) is:

$$E_{\text{cross}}^{(ij)} \;=\; \rho\int_V \mathbf{u}_i \cdot \mathbf{u}_j\, dV. \tag{41}$$

In practice, each $\mathbf{u}_n$ can be represented by known solutions of the Stokes or potential-current equations, or by approximate solutions for vortex loops. Next, one obtains, analytically or numerically, approximate cross energies that can be used in reduced models describing the evolution of multi-vortex systems.

Conclusion

We have investigated how the total kinetic energy of fluids in the presence of multiple vortices can be decomposed into terms of self- and cross-energy. These contributions of cross-energy are crucial for understanding vortex merging, untangling of knotted vortices, or vortex-wave interactions in classical, superfluid, and plasma flows. In addition, we have outlined a systematic derivation of cross-energy and highlighted important aspects in the discussion of momentum and helicity. Future directions include refining these expressions for axially symmetric or knotted vortices and integrating them into large-scale models or computational frameworks.

H Integration of Clausius' heat theory into VAM

The integration of Clausius' mechanical heat theory into the Vortex Æther Model (VAM) extends the scope of the framework to thermodynamics, enabling a unified interpretation of energy, entropy, and quantum behavior based on structured vorticity in a viscous, superfluid-like æther medium [**? ? ?**].

H.1 Thermodynamic Basics in VAM

The classical first law of thermodynamics is expressed as follows:

$$\Delta U = Q - W, \tag{42}$$

where ΔU is the change in internal energy, Q is the added heat, and W is the work done by the system [**?**]. Within VAM this becomes:

$$\Delta U = \Delta \left(\frac{1}{2}\rho_{æ} \int v^2 \, dV + \int P \, dV \right), \tag{43}$$

with $\rho_{æ}$ the æther density, v the local velocity and P the pressure within equilibrium vortex domains [**?**].

H.2 Entropy and structured vorticity

VAM states that entropy is a function of vorticity intensity:

$$S \propto \int \omega^2 \, dV, \tag{44}$$

where $\omega = \nabla \times v$ [**?**]. Entropy thus becomes a measure of the topological complexity and energy dispersion encoded in the vortex network.

H.3 Thermal response of vortex nodes

Stable vortex nodes embedded in equilibrium pressure surfaces behave analogously to thermodynamic systems:

- **Heating (**$Q > 0$**)** expands the node, decreases the core pressure, and increases the entropy.
- **Cooling (**$Q < 0$**)** causes a contraction of the node, concentrating energy and stabilizing the vorticity.

This provides a fluid mechanics analogy for gas laws under energetic input.

H.4 Photoelectric analogy in VAM

Instead of invoking quantized photons, VAM interprets the photoelectric effect via vortex dynamics. A vortex must absorb enough energy to destabilize and eject its structure:

$$W = \frac{1}{2}\rho_{æ} \int v^2 \, dV + P_{\text{eq}} V_{\text{eq}}, \tag{45}$$

where W is the threshold for disintegration work. If an incident wave further modulates the internal vortex energy, ejection occurs [?].

The critical force for vortex ejection is:

$$F_{æ}^{\max} = \rho_{æ} C_e^2 \pi r_c^2, \tag{46}$$

where C_e is the edge velocity of the vortex and r_c is the core radius. This provides a natural frequency limit below which no interaction occurs, comparable to the threshold frequency in quantum photoelectricity [?].

Conclusion and integration

This thermodynamic extension of VAM enriches the model by integrating classical heat and entropy principles into fluid dynamics. It not only bridges the gap between vortex physics and Clausius laws, but also provides a field-based reinterpretation of light-matter interactions, unifying mechanical and electromagnetic thermodynamics without discrete particle assumptions.

I Topological Charge in the Vortex Æther Model

I.1 Motivation from Hopfions and Magnetic Skyrmions

Recent developments in chiral magnetism have led to the experimental observation of stable, three-dimensional topological solitons called *hopfions*. These are ring-shaped, twisted skyrmion strings with a conserved topological invariant known as the *Hopf index* $H \in \mathbb{Z}$. These structures are characterized by nontrivial couplings of field lines under mappings of $\mathbb{R}^3 \to S^2$ and remain stable due to the Dzyaloshinskii–Moriya interaction (DMI) and the underlying micromagnetic energy functional [?]. Within the Vortex-Æther Model (VAM), elementary particles are considered as knotted vortex structures in an unflowable, ideal superfluid (Æther). In this framework, we formulate a VAM-compatible topological charge based on vortex helicity.

I.2 Definition of the VAM Topological Charge

Let the Æther be described by a velocity field $\vec{v}(\vec{r})$, with an associated vorticity field:

$$\vec{\omega} = \nabla \times \vec{v}. \tag{47}$$

The **vortex helicity**, or the total coupling amount of vortex lines, is then defined as:

$$H_{\text{vortex}} = \frac{1}{(4\pi)^2} \int_{\mathbb{R}^3} \vec{v} \cdot \vec{\omega} \, d^3x. \tag{48}$$

This quantity is conserved in the absence of viscosity and external torques, and represents the Hopf-type coupling of vortex tubes in the Æther continuum.

To make this dimensionless, we normalize with the circulation Γ and a characteristic length scale L:

$$Q_{\text{top}} = \frac{L}{(4\pi)^2\Gamma^2} \int \vec{v} \cdot \vec{\omega} \, d^3x, \tag{49}$$

where $Q_{\text{top}} \in \mathbb{Z}$ is a dimensionless topological charge that classifies stable vortex knots (such as trefoils or torus knot structures).

I.3 Topological Energy Term in the VAM Lagrangian

The VAM Lagrangian can be extended with a topological energy density term based on Eq. (**??**):

$$\mathcal{L}_{\text{top}} = \frac{C_e^2}{2} \rho_{\text{æ}} \, \vec{v} \cdot \vec{\omega}, \tag{50}$$

where $\rho_{\text{æ}}$ is the local Æther density, and C_e is the maximum tangential velocity in the vortex core. The total energy functional then becomes:

$$\mathcal{E}_{\text{VAM}} = \int \left[\frac{1}{2} \rho_{\text{æ}} |\vec{v}|^2 + \frac{C_e^2}{2} \rho_{\text{æ}} \, \vec{v} \cdot \vec{\omega} + \Phi_{\text{swirl}} + P(\rho_{\text{æ}}) \right] d^3x. \tag{51}$$

Here Φ_{swirl} is the vortex potential, and $P(\rho_{\text{æ}})$ describes thermodynamic pressure terms, possibly based on Clausius entropy.

I.4 Comparison with the Micromagnetic Energy Functional

In hopfion research, the total energy is written as:

$$\mathcal{E}_{\text{micro}} = \int_V \left[A|\nabla \vec{m}|^2 + D\vec{m} \cdot (\nabla \times \vec{m}) - \mu_0 \vec{M} \cdot \vec{B} + \frac{1}{2\mu_0} |\nabla \vec{A}_d|^2 \right] d^3x, \tag{52}$$

Where:

- A is the exchange stiffness,
- D is the Dzyaloshinskii–Moriya coupling,
- $\vec{m} = \vec{M}/M_s$ is the normalized magnetization vector,
- $\vec{A}_d$ is the magnetic vector potential of demagnetization fields.

We propose to interpret the DMI term $D\vec{m} \cdot (\nabla \times \vec{m})$ within VAM as analogous to the helicity term:

$$\vec{v} \cdot \vec{\omega} \sim \vec{m} \cdot (\nabla \times \vec{m}), \tag{53}$$

which allows us to consistently describe chiral vortex configurations in Æther, with nodal structures energetically protected by this topologically coupled behavior.

I.5 Quantization and Topological Stability

Quantization of helicity implies stability of vortex nodes against perturbations:

$$H_{\text{vortex}} = nH_0, \quad n \in \mathbb{Z}, \tag{54}$$

where H_0 is the minimum helicity unit associated with a single trefoil node. This reflects the discrete spectrum of particle structures within VAM.

I.6 Relation to Vortex Clocks and Local Time Dilation

The swirl clock mechanism for time dilation in VAM is:

$$dt = dt_\infty \sqrt{1 - \frac{U_{\text{vortex}}}{U_{\text{max}}}}, \quad \text{met} \quad U_{\text{vortex}} = \frac{1}{2}\rho_{æ}|\vec{\omega}|^2. \tag{55}$$

We assume that H_{vortex} modulates local time flows via additional constraints on the vortex structure — leading to deeper time dilation depending on the topology of the vortex node.

I.7 Outlook

This formal derivation provides a topological framework for classifying stable states of matter in VAM. The bridge between classical vortex helicity, modern soliton theory and circulation quantization opens the way to numerical simulations with topological charge conservation.

J Split Helicity in the Vortex Æther Model

J.1 Motivation and Context

In classical fluid dynamics, helicity describes the topological complexity of vortex structures. In the Vortex Æther Model (VAM), in which matter is viewed as nodes in a superfluid Æther, helicity is essential for stability, energy distribution, and time dilation.

Based on the work of Tao et al. [**?**], we split the total helicity H of a vortex tube into two components:

$$H = H_C + H_T, \tag{56}$$

where:

- H_C: the **centerline helicity**, associated with the geometric shape of the vortex axis;
- H_T: the **twist helicity**, determined by the rotation of vortex lines around this axis.

J.2 Formulation of the Helicity Components

For a vortex tube with vorticity flux C along its central axis, holds:

$$H_C = C^2 \cdot \text{Wr}, \tag{57}$$

$$H_T = C^2 \cdot \text{Tw}, \tag{58}$$

$$H = C^2(\text{Wr} + \text{Tw}), \tag{59}$$

where:

- Wr: the **writhe**, a measure of the global curvature and self-coupling of the vortex axis;
- Tw: the **twist**, a measure of the internal torsion of vortex lines about the axis.

The writhe is calculated as:

$$\text{Wr} = \frac{1}{4\pi} \int_C \int_C \frac{\left(\vec{T}(s) \times \vec{T}(s')\right) \cdot (\vec{r}(s) - \vec{r}(s'))}{|\vec{r}(s) - \vec{r}(s')|^3} \, ds \, ds', \tag{60}$$

with $\vec{T}(s)$ the tangent vector of the curve C.

J.3 Application in VAM time dilation

The split helicity affects the local clock frequency of a vortex particle. We propose:

$$dt = dt_\infty \sqrt{1 - \frac{H_C + H_T}{H_{\max}}} = dt_\infty \sqrt{1 - \frac{C^2(\mathrm{Wr} + \mathrm{Tw})}{H_{\max}}}. \tag{61}$$

This formulation generalizes the previous energy-based time dilation formula, by explicitly linking topological information to the time course.

K VAM Lagrangian Based on Incompressible Schrödinger Flow

K.1 Complex Vortex Waves in Æther

We model a vortex particle as a normalized two-fold complex wavefunction:

$$\psi(\vec{r}, t) = \begin{pmatrix} a + ib \\ c + id \end{pmatrix}, \quad |\psi|^2 = 1,$$

from which the spin vector $\vec{s} = (s_1, s_2, s_3)$ and vortex field $\vec{\omega}$ are defined via a Hopf mapping.

K.2 Lagrangian with Landau–Lifshitz-like term

We define the VAM wavefunction Lagrangian as:

$$\mathcal{L}_{\mathrm{VAM}}[\psi] = \frac{i\hbar}{2}\left(\psi^\dagger \partial_t \psi - \psi \partial_t \psi^\dagger\right) - \frac{\hbar^2}{2m}|\nabla\psi|^2 - \frac{\alpha}{8}|\nabla\vec{s}|^2, \tag{62}$$

where:

- $\hbar$ is replaced by a VAM-conformal quantization constant,
- α is a dimensionless vortex coupling constant,
- $\vec{s}$ is the Hopf spin vector, calculated from ψ via:

$$s_1 = a^2 + b^2 - c^2 - d^2, \quad s_2 = 2(bc - ad), \quad s_3 = 2(ac + bd).$$

K.3 Derivation of the VAM field equation

Variation with respect to ψ^* yields the modified ISF equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + \frac{\alpha}{4}\frac{\delta}{\delta\psi^*}|\nabla\vec{s}|^2.$$

The derived Euler-Lagrange equation contains topological feedback of the nodal structure on the time evolution of the wave.

K.4 Physical Interpretation

This formulation allows us to:

1. Describe quantum superposition of vortex particles;
2. Derive VAM time delay from the helicity of $\vec{s}$;
3. Coupling stability of vortex nodes to an effective potential $V(\vec{s}) \sim |\nabla \vec{s}|^2$;
4. Simulate evolution without using classical Navier–Stokes dissipation.

L Derivation of the Fine-Structure Constant from Vortex Mechanics

In this section, we derive the fine-structure constant α within the Vortex Æther Model (VAM), showing that it arises from fundamental circulation and vortex geometry in the æther medium.

L.1 Quantization of Circulation

The circulation around a quantum vortex is quantized:

$$\Gamma = \oint \vec{v} \cdot d\vec{\ell} = \frac{h}{m_e} = \frac{2\pi\hbar}{m_e}. \tag{63}$$

For a stable vortex core of radius r_c and tangential speed C_e:

$$\Gamma = 2\pi r_c C_e. \tag{64}$$

Equating the two:

$$2\pi r_c C_e = \frac{2\pi\hbar}{m_e} \quad \Rightarrow \quad C_e = \frac{\hbar}{m_e r_c}. \tag{65}$$

L.2 Relating Vortex Radius to Classical Electron Radius

Let $r_c = \frac{R_e}{2}$, where the classical electron radius is:

$$R_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}. \tag{66}$$

Substitute into C_e:

$$C_e = \frac{\hbar}{m_e \cdot \frac{R_e}{2}} = \frac{2\hbar}{m_e R_e}. \tag{67}$$

Substitute R_e into the above:

$$C_e = \frac{2\hbar}{m_e} \cdot \frac{4\pi\varepsilon_0 m_e c^2}{e^2} = \frac{8\pi\varepsilon_0 \hbar c^2}{e^2}. \tag{68}$$

L.3 Recovering the Fine-Structure Constant

From the standard definition:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}, \tag{69}$$

take the inverse:

$$\frac{1}{\alpha} = \frac{4\pi\varepsilon_0\hbar c}{e^2}. \tag{70}$$

Now observe:

$$\boxed{\alpha = \frac{2C_e}{c}} \tag{71}$$

Conclusion

The fine-structure constant α emerges as a ratio between swirl velocity and light speed, grounded entirely in the geometry and circulation of æther vortices. This connects quantum electrodynamics with vortex fluid mechanics and supports the broader VAM thesis: that constants like α, $\hbar$, and c are emergent from a structured æther.