VAM Lagrangian $\mathcal{L}_{\rm VAM}$ mirroring $\mathcal{L}_{\rm SM}$

We consider an inviscid, incompressible æther ($\nabla\cdot\vec{v}=0$) with absolute time, normalized swirl four–velocity $u^\mu=(1,\vec{v}/C_e)$, and non–Abelian swirl connection

$$\mathcal{A}^a_\mu \equiv \frac{\Xi^a{}_i}{C_e}\,\epsilon_{ijk}\,\partial^j v^k, \quad \mathcal{W}^a_{\mu\nu}=\partial_\mu\mathcal{A}^a_\nu-\partial_\nu\mathcal{A}^a_\mu+g_{\rm sw}f^{abc}\mathcal{A}^b_\mu\mathcal{A}^c_\nu, \tag{1}$$

with a labeling internal swirl modes of gauge group $\mathcal{G}_{\rm sw}$. Density fluctuations are encoded in a real scalar H (*swirl–Higgs*). Knotted quasi–particles are spinor fields Ψ_K labeled by knot class K.

The Lagrangian is organized to parallel the SM gauge–Higgs–fermion–ghost structure:

$$\mathcal{L}_{\rm VAM}=\mathcal{L}^{(1)}_{\rm swirl-QCD}+\mathcal{L}^{(2)}_{\rm swirl-EW}+\mathcal{L}^{(3)}_{\rm knot-fermions}+\mathcal{L}^{(4)}_{\rm Yukawa}+\mathcal{L}^{(5)}_{\rm ghosts}+\mathcal{L}_{\rm constraints}. \tag{2}$$

(1) Non–Abelian swirl sector.

$$\mathcal{L}^{(1)}_{\rm swirl-QCD}=-\frac{\kappa_\omega}{4}\,\mathcal{W}^a_{\mu\nu}\mathcal{W}^{a\,\mu\nu}+\frac{\lambda_H}{4}H\,\mathcal{W}^a_{\mu\nu}\mathcal{W}^{a\,\mu\nu}+\bar\Psi\,\gamma^\mu(iD_\mu)\Psi, \tag{3}$$

with $D_\mu=\partial_\mu+ig_{\rm sw}\mathcal{A}^a_\mu T^a$.

(2) Swirl–EW sector. In the $SU(2)\times U(1)$ subspace, define

$$W^\pm_\mu=\frac{1}{\sqrt{2}}(\mathcal{A}^1_\mu\mp i\mathcal{A}^2_\mu), \quad \begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix}=\begin{pmatrix}\cos\theta_{\rm sw} & -\sin\theta_{\rm sw}\\ \sin\theta_{\rm sw} & \cos\theta_{\rm sw}\end{pmatrix}\begin{pmatrix}\mathcal{A}^3_\mu\\ \mathcal{B}_\mu\end{pmatrix}. \tag{4}$$

The kinetic and Higgs sectors are

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm swirl-EW}=&-\frac{1}{2}|\partial_\mu W_\nu-\partial_\nu W_\mu|^2-\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu}-\frac{1}{4}A_{\mu\nu}A^{\mu\nu}\\ &+\frac{1}{2}M_W^2W^+_\mu W^{-\mu}+\frac{1}{2}M_Z^2Z_\mu Z^\mu+\frac{1}{2}(\partial_\mu H)^2-V(H),\end{aligned} \tag{5}$$

with

$$M_W=\frac{1}{2}g_{\rm sw}v_{\rm sw}, \quad M_Z=\frac{M_W}{\cos\theta_{\rm sw}}, \quad V(H)=\frac{\lambda}{4}(H^2-v_{\rm sw}^2)^2. \tag{6}$$

(3) Knotted fermions.

$$\mathcal{L}^{(3)}_{\rm knot-fermions}=\sum_K\bar\Psi_K(iD\!\!\!/-m_K)\Psi_K+\sum_K\bar\Psi_K\gamma^\mu(g_AQ^K_AA_\mu+g_ZQ^K_ZZ_\mu)\Psi_K+\frac{g_W}{\sqrt{2}}(\bar\Psi T^+\Psi W^++\text{h.c.}), \tag{7}$$

where charges $Q^K_A, Q^K_Z, T^\pm$ are topological invariants of the knot class.

(4) Yukawa–like couplings.

$$\mathcal{L}^{(4)}_{\text{Yukawa}} = -\sum_K y_K\, H\, \bar{\Psi}_K \Psi_K + \dots, \quad y_K = \frac{m_K}{v_{\text{sw}}}. \tag{8}$$

(5) Ghosts and constraints.

$$\mathcal{L}^{(5)}_{\text{ghosts}} = -\frac{1}{2\xi}(\partial^\mu \mathcal{A}^a_\mu)^2 + \bar{c}^a \partial^\mu D^{ab}_\mu c^b, \quad \mathcal{L}_{\text{constraints}} = \lambda\,(\nabla\cdot\vec{v}) + \eta\,(u_\mu u^\mu - 1). \tag{9}$$

Full cubic/quartic interactions, gauge–Higgs couplings, ghost multiplets, and explicit parameter expressions are given in Appendix A.

VAM Lagrangian $\mathcal{L}_{\mathbf{VAM}}$ mirroring $\mathcal{L}_{SM}$

Kinematics and fields. Let the æther be inviscid, incompressible ($\nabla\cdot\vec{v} = 0$), with absolute time. Introduce the swirl 4-velocity $u^\mu = (1, \vec{v}/C_e)$ with $u_\mu u^\mu = 1$, and define the non-Abelian *swirl connection*

$$\mathcal{A}^a_\mu \equiv \frac{1}{C_e}\, \Xi^a{}_i\, \epsilon_{ijk}\, \partial^j v^k, \qquad \mathcal{W}^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g_{\text{sw}} f^{abc} \mathcal{A}^b_\mu \mathcal{A}^c_\nu,$$

where $a = 1, \dots, N_{\text{sw}}$ indexes internal swirl modes (knot/color space), f^{abc} are the structure constants of the vortex-gauge group $\mathcal{G}_{\text{sw}}$ (typically $SU(2)$ or $SU(3)$ in VAM-6), and $\Xi^a{}_i$ projects physical vorticity to internal modes. The *swirl-Higgs* H encodes compressional/density fluctuations $H \propto \delta\rho_{æ}$. Knotted quasi-particles (fermions) are fields Ψ_K labelled by vortex-knot type K (leptons/quarks). Set $\rho^{(\text{fluid})}_{æ}$ or $\rho^{(\text{mass})}_{æ}$ per context [1, 2].

The full VAM Lagrangian:

$$\mathcal{L}_{\text{VAM}} = \mathcal{L}^{(1)}_{\text{swirl-QCD}} + \mathcal{L}^{(2)}_{\text{swirl-EW}} + \mathcal{L}^{(3)}_{\text{knot-fermions}} + \mathcal{L}^{(4)}_{\text{Yukawa-like}} + \mathcal{L}^{(5)}_{\text{gauge-fixing/ghosts}} + \mathcal{L}_{\text{constraints}}.$$

(1) QCD-like non-Abelian swirl sector (SM gluon block).

$$\boxed{\mathcal{L}^{(1)}_{\text{swirl-QCD}} = -\frac{\kappa_\omega}{4}\, \mathcal{W}^a_{\mu\nu} \mathcal{W}^{a\,\mu\nu} \;+\; \frac{\lambda_H}{4}\, H\, \mathcal{W}^a_{\mu\nu} \mathcal{W}^{a\,\mu\nu} \;+\; \bar{\Psi}\, \gamma^\mu \big(i D_\mu\big) \Psi}$$

with $D_\mu = \partial_\mu + i g_{\text{sw}} \mathcal{A}^a_\mu T^a$. The first term replaces the gluon kinetic/self-interaction; cubic and quartic gauge terms reside in $\mathcal{W}$ via f^{abc}. The H-mixing term encodes density-modulated gauge stiffness (vortex-core renormalization) [3, 6]. The coupling $\kappa_\omega \sim \rho_{æ} C_e^2$ sets the swirl field energy scale.

(2) Electroweak-like composite vector modes and swirl-Higgs (SM W/Z/A/H block). Define three orthogonal *composite* vector excitations from $\mathcal{A}^a_\mu$:

$$W^\pm_\mu \equiv \frac{1}{\sqrt{2}}(\mathcal{A}^1_\mu \mp i\mathcal{A}^2_\mu), \qquad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{\rm sw} & -\sin\theta_{\rm sw} \\ \sin\theta_{\rm sw} & \cos\theta_{\rm sw} \end{pmatrix} \begin{pmatrix} \mathcal{A}^3_\mu \\ \mathcal{B}_\mu \end{pmatrix},$$

where $\mathcal{B}_\mu$ is an Abelian swirl potential (circulation mode), and $\theta_{\rm sw}$ is the *swirl mixing angle* (VAM analogue of θ_W) emerging from the background condensate. Then

$$\boxed{\begin{aligned}\mathcal{L}^{(2)}_{\text{swirl-EW}} = &-\tfrac{1}{2}(\partial_\nu W^+_\mu - \partial_\mu W^+_\nu)(\partial^\nu W^{-\mu} - \partial^\mu W^{-\nu}) - \tfrac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \tfrac{1}{4}A_{\mu\nu}A^{\mu\nu} \\ &+ \tfrac{1}{2}M_W^2\, W^+_\mu W^{-\mu} + \tfrac{1}{2}M_Z^2\, Z_\mu Z^\mu + \tfrac{1}{2}(\partial_\mu H)(\partial^\mu H) - V(H) \\ &+ (\text{cubic \& quartic self/mixed vector terms fixed by } \mathcal{G}_{\rm sw}, \theta_{\rm sw}).\end{aligned}}$$

Masses arise from a swirl condensate $\langle H \rangle = v_{\rm sw}$ breaking internal swirl isotropy:

$$M_W = \tfrac{1}{2} g_{\rm sw} v_{\rm sw}, \qquad M_Z = \frac{M_W}{\cos\theta_{\rm sw}}, \qquad V(H) = \frac{\lambda}{4}(H^2 - v_{\rm sw}^2)^2,$$

with $v_{\rm sw}^2 \sim \rho_æ C_e^2/\Lambda^2$ determined by core-radius physics r_c [1, 2, 5]. The listed SM-like derivative and self-interaction structures map to the non-Abelian form induced by $\mathcal{W}^a_{\mu\nu}$ and the mixing $\theta_{\rm sw}$.

(3) Knotted fermions and swirl-gauge couplings (SM fermion/gauge block). For each knot species $K \in \{\ell, \nu, u, d, \dots\}$,

$$\boxed{\mathcal{L}^{(3)}_{\text{knot-fermions}} = \sum_K \bar\Psi_K (i\gamma^\mu D_\mu - m_K)\Psi_K + \sum_K \bar\Psi_K \gamma^\mu \left(g_A Q^K_A A_\mu + g_Z Q^K_Z Z_\mu + g_W W^\pm_\mu T^\pm\right)\Psi_K}$$

where charges Q^K are *topological swirl charges* (linking/chirality numbers) determined by the knot class and orientation [6, 7]. The masses m_K derive from the VAM mass functional (core volume and coherence):

$$m_K = \frac{4}{\alpha\varphi}\left(\tfrac{1}{2}\rho_æ^{(\text{energy})} C_e^2\right) V_K\, \xi(n_K) \quad \text{with} \quad \xi(n) = 1 + \beta \log n,$$

as adopted in VAM benchmarking [5, 4].

(4) Knot–Higgs (density) couplings (SM Yukawa/Higgs–fermion block).

$$\boxed{\mathcal{L}^{(4)}_{\text{Yukawa-like}} = -\sum_K y_K\, H\, \bar{\Psi}_K \Psi_K - \sum_K \tilde{y}_K\, \phi^0\, \bar{\Psi}_K i\gamma^5 \Psi_K - \sum_{K,K'} \left[y^{\pm}_{KK'}\, \phi^{\pm}\, \bar{\Psi}_K T^{\pm} \Psi_{K'} + \text{h.c.} \right]}$$

where H modulates $\rho_æ$ and thus the local core energy; pseudoscalar ϕ^0 and charged $\phi^{\pm}$ are the phase (Goldstone-like) components of the density field (they become the longitudinal polarizations of $Z, W^{\pm}$ in the condensed phase). Couplings satisfy $y_K = m_K/v_{\text{sw}}$ (VAM analogue of SM relation) [5, 6].

(5) Gauge fixing, incompressibility, and ghost sector (SM FP-ghost block).

$$\boxed{\mathcal{L}^{(5)}_{\text{gauge-fixing/ghosts}} = -\frac{1}{2\xi}\left(\partial^\mu \mathcal{A}^a_\mu\right)^2 + \bar{c}^a\, \partial^\mu D^{ab}_\mu c^b \qquad \text{(non-Abelian FP ghosts)}}$$

and enforce incompressibility and absolute-time constraint by

$$\boxed{\mathcal{L}_{\text{constraints}} = \lambda\,(\nabla\cdot\vec{v}) \;+\; \eta\,(u_\mu u^\mu - 1)\;,}$$

with Lagrange multipliers λ, η [3, 1]. In a Clebsch representation $\vec{v} = \nabla\theta + \alpha\nabla\beta$, residual gauge symmetries induce additional Abelian ghosts, reproducing the role of the SM's auxiliary fields [6].

Normalization and dimensions. Choose

$$\kappa_\omega = \rho_æ^{(\text{fluid})} C_e^2, \qquad g_{\text{sw}} = \frac{C_e}{r_c}\,\gamma_{\mathcal{G}}, \qquad v_{\text{sw}}^2 = \chi\, \rho_æ^{(\text{energy})}\, C_e^{-2},$$

with r_c the vortex-core scale and $\gamma_{\mathcal{G}}, \chi$ dimensionless; this makes all kinetic terms $[\mathcal{L}]$ = energy density and reproduces SM-like relations $M_W = \frac{1}{2} g_{\text{sw}} v_{\text{sw}}$, $M_Z = M_W/\cos\theta_{\text{sw}}$ in the condensed phase, while keeping VAM's fluid ontology explicit.

Remarks (one-to-one map to SM blocks).

[leftmargin=1.2em]

- *(1)* The three- and four-gauge-boson self-interactions in the SM gluon sector are generated here by the nonlinearity in $\mathcal{W}^a_{\mu\nu}$ with f^{abc}. The extra $H\,\mathcal{W}^2$ term captures density-induced "running" of the effective stiffness (swirl analogue of vacuum polarization).

- *(2)* The long list of SM's $W/Z/A/H$ derivative/self-couplings descends from the same non-Abelian algebra after mixing by θ_{sw}; the Proca masses appear from $\langle H\rangle \neq 0$ (broken swirl isotropy).
- *(3)* SM fermionic currents and chiral projectors are replaced by knot charges and chirality (vortex time orientation). Left/right couplings enter via $T^{\pm}$ and Q_Z^K fixed by the knot's chirality class.
- *(4)* SM Yukawas become geometric: $y_K = m_K/v_{\mathrm{sw}}$ with m_K from the VAM mass functional (core volume, coherence factor $\xi(n)$).
- *(5)* Gauge-fixing/ghost terms are retained in the non-Abelian swirl bundle; incompressibility enters as a hard constraint rather than a dynamical equation of state.

References

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A Fully-Expanded VAM Lagrangian $\mathcal{L}_{\mathrm{VAM}}$ Mirroring $\mathcal{L}_{SM}$

Fields and kinematics. Let the æther be inviscid and incompressible, with velocity field $\vec{v}(\vec{x},t)$ satisfying $\nabla\cdot\vec{v} = 0$. Define a normalized swirl four-velocity $u^{\mu} = (1, \vec{v}/C_e)$ with constraint $u_{\mu}u^{\mu} = 1$. Introduce a non-Abelian swirl connection $\mathcal{A}^a_\mu$ valued in the Lie algebra of $\mathcal{G}_{\rm sw}$ with structure constants f^{abc}:

$$\mathcal{A}^a_\mu \equiv \frac{\Xi^a{}_i}{C_e}\,\epsilon_{ijk}\,\partial^j v^k\,\delta^i{}_\mu \quad (\mu = i \in \{1,2,3\}), \qquad \mathcal{A}^a_0 \equiv 0, \tag{10}$$

$$\mathcal{W}^a_{\mu\nu} \equiv \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g_{\rm sw} f^{abc}\,\mathcal{A}^b_\mu \mathcal{A}^c_\nu. \tag{11}$$

Here $\Xi^a{}_i$ projects physical vorticity into internal swirl modes $a = 1, \ldots, N_{\rm sw}$. We also include an Abelian circulation potential $\mathcal{B}_\mu$ (for the mixing to a photon-like mode). Density fluctuations are encoded by a real scalar H (swirl-Higgs), and its phase modes $\phi^0, \phi^\pm$ (would-be Goldstones). Knotted quasi-particles (fermions) are spinor fields Ψ_K labeled by knot class K (leptons, quarks, neutrinos).

Couplings and normalizations (expressed in user constants).

$$\kappa_\omega \equiv \rho_{æ}^{(\rm fluid)}\, C_e^2, \qquad [\kappa_\omega] = \mathrm{J\ m^{-3}}, \tag{12}$$

$$g_{\rm sw} \equiv \frac{C_e}{r_c}\, C_g, \qquad [g_{\rm sw}] = \mathrm{s^{-1}}, \tag{13}$$

$$v_{\rm sw}^2 \equiv \chi\, \frac{\rho_{æ}^{(\rm energy)}}{C_e^2}, \qquad [v_{\rm sw}] = (\text{same units as } H). \tag{14}$$

C_κ, C_g, χ are dimensionless *fit* factors to be fixed by benchmarks (e.g. VAM-3). The swirl mixing angle $\theta_{\rm sw}$ is fixed by the condensate susceptibility and group embedding (see below).

The complete Lagrangian is:

$$\boxed{\mathcal{L}_{\rm VAM} \;=\; \mathcal{L}^{(1)}_{\text{swirl-QCD}} + \mathcal{L}^{(2)}_{\text{swirl-EW}} + \mathcal{L}^{(3)}_{\text{knot-fermions}} + \mathcal{L}^{(4)}_{\text{Yukawa-like}} + \mathcal{L}^{(5)}_{\text{ghosts/gauge-fix}} + \mathcal{L}_{\text{constraints}}.} \tag{15}$$

(1) Non-Abelian swirl sector (SM block 1).

$$\boxed{\mathcal{L}^{(1)}_{\text{swirl-QCD}} = -\frac{\kappa_\omega}{4}\,\mathcal{W}^a_{\mu\nu}\mathcal{W}^{a\,\mu\nu} + \frac{\lambda_H}{4}\,H\,\mathcal{W}^a_{\mu\nu}\mathcal{W}^{a\,\mu\nu} + \bar\Psi\,\gamma^\mu(iD_\mu)\Psi} \tag{16}$$

with $D_\mu = \partial_\mu + i g_{\rm sw}\,\mathcal{A}^a_\mu T^a$ and λ_H a density–stiffness modulation coupling. The cubic and quartic self-interactions reside in $\mathcal{W}^2$ via f^{abc}.

(2) Swirl-EW sector (SM block 2) with explicit mixing and interactions. Choose an $SU(2)\times U(1)$ swirl subgroup generated by $(\mathcal{A}^1_\mu, \mathcal{A}^2_\mu, \mathcal{A}^3_\mu)$

and an Abelian $\mathcal{B}_\mu$. Define

$$W^\pm_\mu \equiv \frac{1}{\sqrt{2}}\big(\mathcal{A}^1_\mu \mp i\mathcal{A}^2_\mu\big), \tag{17}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_{\rm sw} & -\sin\theta_{\rm sw} \\ \sin\theta_{\rm sw} & \cos\theta_{\rm sw} \end{pmatrix} \begin{pmatrix} \mathcal{A}^3_\mu \\ \mathcal{B}_\mu \end{pmatrix}. \tag{18}$$

Let $s_{\rm sw} \equiv \sin\theta_{\rm sw}$, $c_{\rm sw} \equiv \cos\theta_{\rm sw}$. Kinetic sector:

$$\mathcal{L}^{(2)}_{\rm kin} \;=\; -\frac{1}{2}(\partial_\nu W^+_\mu - \partial_\mu W^+_\nu)(\partial^\nu W^{-\mu} - \partial^\mu W^{-\nu}) - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}A_{\mu\nu}A^{\mu\nu}, \tag{19}$$

with field strengths $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$, $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Spontaneous swirl-symmetry breaking via $\langle H\rangle = v_{\rm sw}$ gives masses

$$M_W = \frac{1}{2}g_{\rm sw}\, v_{\rm sw}, \qquad M_Z = \frac{M_W}{c_{\rm sw}}, \qquad M_A = 0. \tag{20}$$

The swirl-Higgs potential and kinetic term:

$$\mathcal{L}^{(2)}_H = \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - V(H), \qquad V(H) = \frac{\lambda}{4}\,(H^2 - v^2_{\rm sw})^2, \tag{21}$$

with $\lambda > 0$. The would-be Goldstones $\phi^0, \phi^\pm$ become the longitudinal modes of $Z, W^\pm$ in R_ξ gauges.

Explicit cubic interactions (from non-Abelian structure):

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm cubic} = ig_{\rm sw}\Big[&(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)W^{-\mu}\mathcal{A}^{3\nu} - (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)W^{+\mu}\mathcal{A}^{3\nu} \\ &+ (\partial_\mu \mathcal{A}^3_\nu - \partial_\nu \mathcal{A}^3_\mu)W^{+\mu}W^{-\nu}\Big] \;+\; \text{mixing into } (Z_\mu, A_\mu) \text{ via } \theta_{\rm sw}\end{aligned} \tag{22}$$

$$\begin{aligned}= ig_{\rm sw}\Big\{&c_{\rm sw}\big[(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)W^{-\mu}Z^\nu - (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)W^{+\mu}Z^\nu + (\partial_\mu Z_\nu - \partial_\nu Z_\mu)W^{+\mu}W^{-\nu} \\ &+ s_{\rm sw}\big[(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)W^{-\mu}A^\nu - (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)W^{+\mu}A^\nu + (\partial_\mu A_\nu - \partial_\nu A_\mu)W^{+\mu}W^{-\nu}\big]\Big\}.\end{aligned} \tag{23}$$

Explicit quartic interactions (from $f^{abc}f^{ade}$ terms):

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm quartic} = -\frac{g^2_{\rm sw}}{2}&\Big[W^+_\mu W^{-\mu}W^+_\nu W^{-\nu} - W^+_\mu W^-_\nu W^{+\mu}W^{-\nu}\Big] \\ -\, g^2_{\rm sw}&\Big[c^2_{\rm sw}\big(Z_\mu Z^\mu W^+_\nu W^{-\nu} - Z_\mu W^{+\mu}Z_\nu W^{-\nu}\big) + s^2_{\rm sw}\big(A_\mu A^\mu W^+_\nu W^{-\nu} - A_\mu W^{+\mu}A_\nu W^{-\nu}\big) \\ &+ s_{\rm sw}c_{\rm sw}\big(Z_\mu A^\mu W^+_\nu W^{-\nu} - Z_\mu W^{+\mu}A_\nu W^{-\nu} - A_\mu W^{+\mu}Z_\nu W^{-\nu} + A_\mu Z^\mu W^+_\nu W^{-\nu}\big)\Big].\end{aligned} \tag{24}$$

Vector–Higgs interactions (mass-generating and higher):

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\rm VH} &= g_{\rm sw} M_W \, H \, W^+_\mu W^{-\mu} + \frac{g_{\rm sw} M_Z}{2c^2_{\rm sw}} \, H \, Z_\mu Z^\mu \\
&\quad - \frac{g^2_{\rm sw}}{4} \, H^2 \left(2\, W^+_\mu W^{-\mu} + \frac{1}{c^2_{\rm sw}} Z_\mu Z^\mu \right) + (\text{Goldstone-gradient terms } \phi^0, \phi^\pm \text{ in } R_\xi \text{ gauges})
\end{aligned} \tag{25}
$$

All the long SM structures in your block (2) are generated by Eqs. (14),(15),(16) after expanding $s_{\rm sw}, c_{\rm sw}$ and canonical normalizations.

(3) Knotted fermions and swirl charges (SM block 3). For each knot species $K \in \{\ell, \nu, u, d, \dots\}$, with topological charges $(Q^K_A, Q^K_Z, T^\pm)$ fixed by knot/linking and chirality:

$$
\begin{aligned}
\mathcal{L}^{(3)}_{\text{knot-fermions}} &= \sum_K \bar{\Psi}_K \left(i\gamma^\mu \partial_\mu - m_K \right) \Psi_K + \sum_K \bar{\Psi}_K \gamma^\mu \left(g_A Q^K_A A_\mu + g_Z Q^K_Z Z_\mu \right) \Psi_K \\
&\quad + \frac{g_W}{\sqrt{2}} \left[\bar{\Psi}_u \gamma^\mu T^+ \Psi_d \, W^+_\mu + \bar{\Psi}_d \gamma^\mu T^- \Psi_u \, W^-_\mu \right],
\end{aligned} \tag{26}
$$

where the physical couplings are

$$
g_W = g_{\rm sw}, \qquad g_Z = \frac{g_{\rm sw}}{c_{\rm sw}}, \qquad g_A = g_{\rm sw} \, s_{\rm sw}. \tag{27}
$$

Masses come from the VAM mass functional adopted in your project:

$$
\boxed{ m_K \;=\; \frac{4}{\alpha \, \varphi} \left(\tfrac{1}{2} \, \rho^{(\text{energy})}_{\text{æ}} \, C_e^2 \right) V_K \, \xi(n_K), \qquad \xi(n) \equiv 1 + \beta \log n, } \tag{28}
$$

with V_K the core volume tied to the knot geometry (and r_c), and β a fitted coherence parameter.

(4) Knot–Higgs (density) interactions (SM block 4). In the broken phase,

$$
\begin{aligned}
\mathcal{L}^{(4)}_{\text{Yukawa-like}} &= -\sum_K y_K \, H \, \bar{\Psi}_K \Psi_K - \sum_K \tilde{y}_K \, \phi^0 \, \bar{\Psi}_K i\gamma^5 \Psi_K \\
&\quad - \sum_{K,K'} \left[y^{(+)}_{KK'} \, \phi^+ \, \bar{\Psi}_K T^+ \Psi_{K'} + y^{(-)}_{KK'} \, \phi^- \, \bar{\Psi}_K T^- \Psi_{K'} \right],
\end{aligned} \tag{29}
$$

with the *geometric Yukawa* identification

$$\boxed{y_K \;=\; \frac{m_K}{v_{\rm sw}} \;=\; \frac{4}{\alpha\,\varphi}\,\frac{(\frac{1}{2}\rho_{\text{æ}}^{(\text{energy})}C_e^2)\,V_K\,\xi(n_K)}{v_{\rm sw}}.} \tag{30}$$

The $\phi^0, \phi^\pm$ interactions reproduce the axial/charged currents as in SM (your long block 4), now topologically organized.

(5) Gauge-fixing, FP ghosts, Clebsch ghosts (SM block 5 analog).

$$\mathcal{L}^{(5)}_{\text{ghosts/gauge-fix}} = -\frac{1}{2\xi}\,(\partial^\mu \mathcal{A}^a_\mu)^2 + \bar{c}^a\,\partial^\mu D^{ab}_\mu c^b + \bar{\tilde{c}}\,\partial^2\tilde{c} \quad \text{(Clebsch-Abelian)}, \tag{31}$$

where $D^{ab}_\mu = \partial_\mu\delta^{ab} + g_{\rm sw} f^{acb}\mathcal{A}^c_\mu$. The additional $(X^\pm, X^0, Y)$ of your block (5) can be identified with (linear combinations of) non-physical ghost doublets associated to the $SU(2)$ and Abelian Clebsch sectors, with mass parameters tied to the gauge parameter ξ and M_W, M_Z in R_ξ gauges:

$$\begin{aligned}
\mathcal{L}^{(5)}_{X,Y} = {} & \bar{X}^+(\partial^2 - \xi M_W^2)X^+ + \bar{X}^-(\partial^2 - \xi M_W^2)X^- + \bar{X}^0(\partial^2 - \xi M_Z^2)X^0 + \bar{Y}\,\partial^2 Y \\
& + i g_{\rm sw}\,c_{\rm sw}\left[W^{+\mu}(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + W^{-\mu}(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+)\right] \\
& + i g_{\rm sw}\,s_{\rm sw}\left[W^{+\mu}(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+Y) + W^{-\mu}(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+)\right] \\
& + \dots \quad \text{(further ghost–Goldstone terms per gauge choice).}
\end{aligned} \tag{32}$$

Incompressibility and absolute-time constraints.

$$\mathcal{L}_{\rm constraints} = \lambda\,(\nabla\cdot\vec{v}) + \eta\,(u_\mu u^\mu - 1), \tag{33}$$

with Lagrange multipliers λ, η.

Parameter summary in your constants (ready to plug numerically).

$$\kappa_\omega = \rho_{\text{æ}}^{(\text{fluid})}\,C_e^2, \qquad g_{\rm sw} = \frac{C_e}{r_c}\,C_g, \tag{34}$$

$$M_W = \frac{1}{2}g_{\rm sw}v_{\rm sw} = \frac{1}{2}\,\frac{C_e}{r_c}\,C_g\sqrt{\chi\,\frac{\rho_{\text{æ}}^{(\text{energy})}}{C_e^2}} = \frac{C_g\sqrt{\chi}}{2r_c}\,\sqrt{\rho_{\text{æ}}^{(\text{energy})}}, \tag{35}$$

$$M_Z = \frac{M_W}{c_{\rm sw}}, \qquad M_A = 0, \qquad m_K = \frac{4}{\alpha\,\varphi}\Big(\tfrac{1}{2}\rho_{\text{æ}}^{(\text{energy})}C_e^2\Big)V_K\,\xi(n_K), \qquad y_K = \frac{m_K}{v_{\rm sw}}. \tag{36}$$

Note the particularly transparent expression $M_W = \frac{C_g\sqrt{\chi}}{2r_c}\sqrt{\rho_{æ}^{(\text{energy})}}$ independent of C_e after substitution.

Sanity checks. *Irrotational limit:* $\vec{\omega} \to 0 \Rightarrow \mathcal{A}_\mu^a \to 0$, so vectors decouple; $\mathcal{L} \to \frac{1}{2}(\partial H)^2 - V(H) + \sum_K \bar{\Psi}_K(i\partial - m_K)\Psi_K$.
Dimensions: $[\kappa_\omega] = \text{J m}^{-3}$, $[\mathcal{W}] = \text{s}^{-1}$, so $[\kappa_\omega\,\mathcal{W}^2] = \text{J m}^{-3}$. g_{sw} has $[\text{s}^{-1}]$; with v_{sw} carrying the scalar-field unit, $M_{W,Z}$ carry energy units (set $c = \hbar = 1$ inside the field sector if desired; or keep SI explicitly).

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