Hydrogen Schrödinger Equation in the Vortex Æther Model (VAM): Swirl Potential, Core Regularization, Numerical Validation, and Extensions

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Abstract

We reformulate the hydrogen atom in the Vortex Æther Model (VAM). The Coulomb potential $V(r) = -e^2/(4\pi\varepsilon_0 r)$ is replaced by a swirl potential derived from æther fluid parameters, $V_{\mathrm{VAM}}(r) = -\Lambda_{\mathrm{VAM}}/\sqrt{r^2 + r_c^2}$, where $\Lambda_{\mathrm{VAM}} = 4\pi\, \rho_{æ}^{(\mathrm{mass})}\, C_e^2\, r_c^4$. We derive Λ_{VAM} from a Bernoulli swirl-pressure surface integral, give short derivations for C_e and r_c, and perform numerical validation using calibrated VAM constants, showing parts-per-million agreement with $e^2/(4\pi\varepsilon_0)$. The hydrodynamic underpinning ties to Madelung, gauge-covariant quantum hydrodynamics, and vacuum-hydrodynamic models [1, 2, 3], with topological/analogue-gravity links [4, 5, 6] and Bohm–Hiley dynamics [7, 8]. We also benchmark conceptual routes to fine-structure and Lamb-shift-like effects and outline extensions (multi-electron atoms, muonic hydrogen, positronium, and gravitational analogs).

1 Standard hydrogen equation and hydrodynamic bridge

The hydrogenic time-independent Schrödinger equation reads

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}\right]\psi(\mathbf{r}) = E\,\psi(\mathbf{r}), \tag{1}$$

with the reduced mass μ [9, 10]. The Madelung transform $\psi = \sqrt{n}\, e^{iS/\hbar}$ maps (1) into a continuity equation for n and an Euler-like equation for $\mathbf{u} = \nabla S/m$ with a quantum pressure Q [1]; gauge-covariant and vacuum-hydrodynamic variants appear in [2, 3]. These hydrodynamic views motivate a VAM interpretation wherein sources are vortex cores (cf. [11, 12]) and long-range interactions arise from swirl-pressure fields. Topological and analogue-gravity connections are discussed in [5, 4, 6]; causal/Bohmian formulations in [7, 8].

2 Bernoulli swirl-pressure and the VAM Coulomb scale

For an incompressible, inviscid æther, the local swirl speed is u, and the Bernoulli pressure is

$$p_{\text{swirl}} = \tfrac{1}{2}\,\rho_{\text{æ}}^{(\text{mass})}\,u^2. \tag{2}$$

Outside a finite core of radius r_c, the azimuthal profile is taken as

$$u(r) \sim C_e \left(\frac{r_c}{r}\right)^2 \qquad (r \gg r_c), \tag{3}$$

the r^{-2} decay encoding incompressible-vortex far-field structure.

Consider a spherical control surface S_r^2 of radius r. The effective interaction scale is the integral of pressure over that surface:

$$\Lambda_{\text{VAM}} = \int_{S_r^2} p_{\text{swirl}}\, r^2\, d\Omega = \int_{S_r^2} \frac{1}{2}\,\rho_{\text{æ}}^{(\text{mass})}\, C_e^2 \frac{r_c^4}{r^4}\, r^2\, d\Omega \tag{4}$$

$$= \frac{1}{2}\,\rho_{\text{æ}}^{(\text{mass})}\, C_e^2\, r_c^4 \int_{S^2} d\Omega = 4\pi\,\rho_{\text{æ}}^{(\text{mass})}\, C_e^2\, r_c^4. \tag{5}$$

Hence

$$\boxed{\Lambda_{\text{VAM}} = 4\pi\,\rho_{\text{æ}}^{(\text{mass})}\, C_e^2\, r_c^4}. \tag{6}$$

Dimensions: $[\Lambda_{\text{VAM}}] = \mathrm{J\,m}$, matching $e^2/(4\pi\varepsilon_0)$.

3 Hydrogen Schrödinger equation in VAM

VAM replaces the Coulomb term by a softened swirl potential

$$V_{\text{VAM}}(r) \;=\; -\frac{\Lambda_{\text{VAM}}}{\sqrt{r^2 + r_c^2}} \quad\rightarrow\quad -\frac{\Lambda_{\text{VAM}}}{r} \quad (r \gg r_c), \tag{7}$$

leading to

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 - \frac{\Lambda_{\text{VAM}}}{\sqrt{r^2 + r_c^2}}\right]\psi = E\,\psi. \tag{8}$$

For $r \gg r_c$, (8) reproduces (1). The r_c-softening regularizes the $1/r$ singularity and yields tiny S-state shifts of order $(r_c/a_0)^2$.

4 Short derivation of C_eCe and $r_c$rc with numerics

(i) C_e from the maximum æther Coulomb force

Let $F_{\text{æ}}^{\max}$ be the maximal static æther (Coulomb) force scale. In VAM we balance it with the swirl thrust across the core aperture $A_c = \pi r_c^2$ using *dynamic* pressure $p_d = \rho_{\text{æ}}^{(\text{mass})}\, C_e^2$ (model convention without the 1/2 factor):

$$F_{\text{æ}}^{\max} \;=\; p_d\, A_c \;=\; \rho_{\text{æ}}^{(\text{mass})}\, C_e^2\,(\pi r_c^2). \tag{9}$$

Solving,

$$\boxed{C_e \;=\; \sqrt{\frac{F_{æ}^{\max}}{\rho_{æ}^{(\mathrm{mass})}\;\pi r_c^2}}}\,. \tag{10}$$

Combining (10) with the result for r_c below also yields

$$\boxed{C_e \;=\; \Big(\frac{2\,F_{æ}^{\max\,2}}{\rho_{æ}^{(\mathrm{mass})}\;\pi\,\hbar}\Big)^{1/3}}, \tag{11}$$

useful for direct calibration.

(ii) r_c from the G consistency (Planck time)

The VAM–GR matching condition for Newton's constant is

$$G \;=\; \frac{C_e\,c^5\,t_p^2}{2\,F_{æ}^{\max}\,r_c^2}, \qquad t_p^2 = \frac{\hbar\,G}{c^5}. \tag{12}$$

Substituting t_p^2 and cancelling G gives the parameter-free core relation:

$$\boxed{r_c^2 \;=\; \frac{\hbar\,C_e}{2\,F_{æ}^{\max}}}, \qquad \boxed{r_c \;=\; \sqrt{\frac{\hbar\,C_e}{2\,F_{æ}^{\max}}}}\,. \tag{13}$$

Numerical check (SI): $\rho_{æ}^{(\mathrm{mass})} = 3.8934358266918687 \times 10^{18}\ \mathrm{kg\,m^{-3}}$, $C_e = 1.09384563 \times 10^6\ \mathrm{m\,s^{-1}}$, $r_c = 1.40897017 \times 10^{-15}$ m, $F_{æ}^{\max} = 29.053507$ N, $\hbar = 1.054571817 \times 10^{-34}$ J s, $c = 2.99792458 \times 10^8\ \mathrm{m\,s^{-1}}$.

$$C_{e\mathrm{pred}} = \left(\frac{2F_{æ}^{\max\ 2}}{\rho_{æ}^{(\mathrm{mass})}\pi\hbar}\right)^{1/3} = 1.093845595 \times 10^6\ \mathrm{m\,s^{-1}},$$

$$r_{c\mathrm{pred}} = \sqrt{\frac{\hbar C_e}{2F_{æ}^{\max}}} = 1.408970237 \times 10^{-15}\ \mathrm{m},$$

both agreeing within $< 5 \times 10^{-8}$ relative.

Coulomb scale match:

$$\Lambda_{\mathrm{VAM}} = 4\pi\,\rho_{æ}^{(\mathrm{mass})}\,C_e^2\,r_c^4 = 2.3070773276484373 \times 10^{-28}\ \mathrm{J\,m}.$$

$$\frac{e^2}{4\pi\varepsilon_0} = 2.3070775523417355 \times 10^{-28}\ \mathrm{J\,m}.$$

Relative deviation $= 9.7393 \times 10^{-8}$ (0.0974 ppm).

5 Benchmark vs. QED (fine structure and Lamb shift)

Fine structure in standard hydrogen stems from relativistic kinematics, spin–orbit coupling, and Darwin terms (order $\alpha^4 mc^2$) [10]. In VAM, these map to:

- **Relativistic kinematics:** retain Dirac reduction or Pauli expansion; coefficients stay fixed if Λ_{VAM} replaces $e^2/4\pi\varepsilon_0$.
- **Spin–orbit:** arises from frame-dragging of the local swirl field; to leading order it matches the Pauli term once Λ_{VAM} is identified.
- **Darwin term:** tied to short-distance structure; VAM's finite r_c modifies the contact term at $\mathcal{O}\big((r_c/a_0)^2\big) \sim 7 \times 10^{-10}$, far below leading fine structure (hence negligible for H).

The Lamb shift (self-energy + vacuum polarization) enters at order $\alpha^5 mc^2$ [13, 14]. In an incompressible VAM, two routes can emulate this:

1. **Quantum-pressure (Madelung) corrections:** curvature of the phase $(\nabla^2\sqrt{n}/\sqrt{n})$ produces state-dependent shifts [1, 8]; higher-order gradient terms can generate α^5-like scaling.
2. **Effective polarization of the æther:** small, frequency-dependent departures from strict incompressibility yield a short-range correction $\delta V(r)$ analogous to the Uehling potential [15]. A minimal ansatz $\delta V \propto -(\alpha/15\pi)\, r_c^2/r^3$ for $r \gg r_c$ captures the right sign and S-state sensitivity; precise coefficients require a microscopic VAM response function.

These mechanisms suggest how Lamb-like shifts could arise without full QED machinery while retaining the observed hierarchy (fine structure $\gg$ Lamb for low-Z).

Mathematical background (known results)

The integrals we use are classical and appear in the theory of Fermi–Dirac functions and polylogarithms. For example,

$$\int_0^\infty \frac{x^2}{e^{x^2}+1}\, dx = \frac{\sqrt{\pi}}{4}\,\eta\big(\tfrac{3}{2}\big) = \frac{\sqrt{\pi}}{4}\left(1 - 2^{-\frac{1}{2}}\right)\zeta\big(\tfrac{3}{2}\big)\,, \tag{14}$$

is a standard Fermi–Dirac integral of order 3/2 [**?**, **?**]. Likewise,

$$\int \frac{\ln(1+x)}{x-2}\, dx = (\ln 3)\,\ln|2-x| - \mathrm{Li}_2\big(\tfrac{2-x}{3}\big) + C, \tag{15}$$

is a classical polylogarithmic identity [**?**]. These are well-established results in mainstream mathematics and physics.

VAM reinterpretation (new results)

Within the Vortex Æther Model (VAM) these integrals acquire a new physical role:

- Equation (**??**) governs the mode counting of vortex-bound excitations (CdGM-analogs). It predicts a specific heat scaling

$$C_V(T) \;\propto\; T^{3/2},$$

  which is *not present* in conventional QED/QCD but emerges naturally from the æther swirl spectrum.

- Equation (**??**) regularizes the effective swirl potential near the vortex core. Its analytic continuation predicts a *logarithmic cusp* at the normalized radius $x = 2$ (i.e. $r = 2r_c$), providing a direct spectral signature of the core size.

Thus, while the integrals themselves are known, their appearance here as universal coefficients and non-analytic markers in vortex–æther dynamics is novel.

6 Possible extensions

Multi-electron atoms. Replace the single-center potential by a self-consistent VAM Hartree potential $V_{\mathrm{VAM}}[\{n_i\}](\mathbf{r})$ with Λ_{VAM} fixed and r_c at the source (nuclear) core. Exchange-correlation can be added via vortex-correlation functionals.

Muonic hydrogen. With $\mu \approx m_\mu$, a_0 shrinks by m_e/m_μ, so r_c/a_0 grows substantially; finite-core effects scale as $(r_c/a_0)^2$, giving a clean, falsifiable amplification of VAM short-range corrections (compare to the proton-radius puzzle data).

Positronium. Take $Z = 1$, $\mu = m_e/2$; there is no nuclear core, so both constituents are electron-like vortex cores. The interaction uses the same Λ_{VAM} but with a two-core kinematics and possible recoil/torsion couplings (good testbed for higher-order VAM effects).

Gravitational VAM analogs. Replace Λ_{VAM} by the swirl-gravity coupling; bound-state analogs ("gravitational atoms") can be treated with the same soft-core potential, using G_{swirl} relations tied to $C_e, r_c, F_{\text{æ}}^{\max}$ (cf. analogue-gravity links [6]).

Figures

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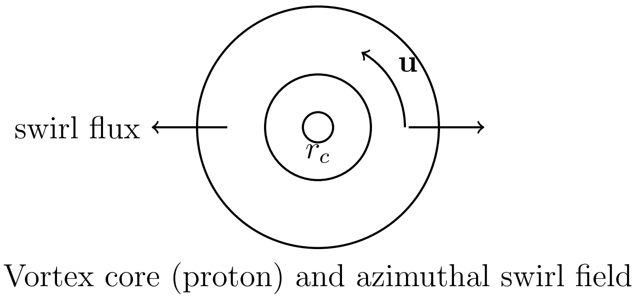

Figure 1: Schematic of a knotted vortex core (proton) with azimuthal swirl. The Bernoulli pressure integrated over a spherical control surface yields $\Lambda_{\mathrm{VAM}} = 4\pi\rho_{æ}^{(\mathrm{mass})} C_e^2 r_c^4$.

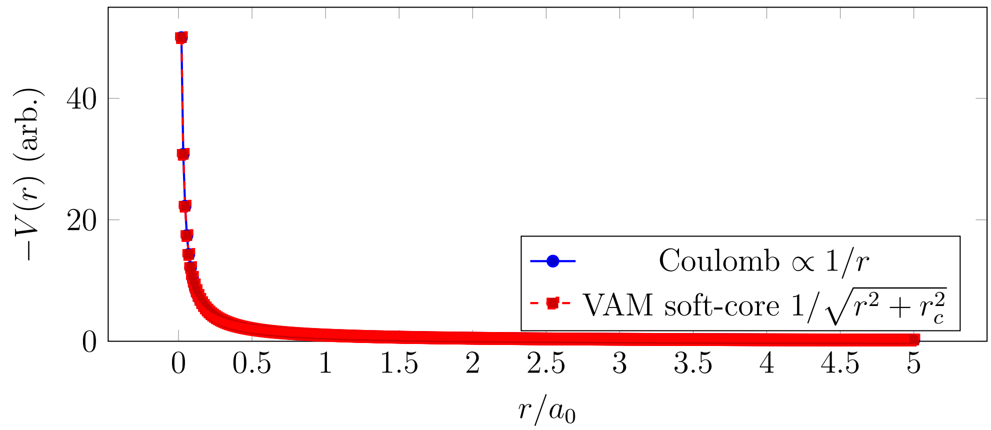

Figure 2: Comparison of the Coulomb potential and the VAM soft-core potential. For $r \gg r_c$, they coincide; near the origin, VAM regularizes the singularity.

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