

Theorem 5.6.1 in the course notes is quite confusing on a first read. Let's work through some examples to get a better understanding of it.

Question 1a

Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation which projects point in the plane to the y-axis. Let

$$b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

1. What is the matrix A corresponding to T (using the standard basis of \mathbb{R}^2)?
2. Geometrically, describe the set of vectors $\{b - Av : v \in \mathbb{R}^2\}$.
3. Geometrically, what is the smallest possible value of $\|b - Av\|$?
4. Give a geometric description of vectors v which attain the minimum possible value above.

Question 1b

Let

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

1. Is the equation $Av = b$ consistent?
2. What is the normal equation in z corresponding to $Av = b$?
3. Substitute it A, b into the normal equation in the previous step, and solve for z .
4. How do your z values correspond to your geometric description in Question 1a?

Let's use Theorem 5.6.1 to calculate the distance between a point and a plane.

Question 2

Consider the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by matrix

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix},$$

and let

$$b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

1. What is the equation of the plane given by $\text{Im} A$?
2. Geometrically, what is the closest point to b on the plane?
3. Is the equation $Ax = b$ consistent?
4. What is the normal equation in z corresponding to this equation?
5. Solve the normal equation for z .
6. What is the value of Az ? Is this value what you expected?

Question 3

Let $(a, b), (c, d)$ be two points in R^2 with $a \neq c$, and let

$$A = \begin{bmatrix} 1 & a \\ 1 & c \end{bmatrix}, \quad q = \begin{bmatrix} b \\ d \end{bmatrix}.$$

1. What is the normal equation corresponding to $Ax = q$?
2. Solve the normal equation above.
3. What is the equation of the line through (a, b) and (c, d) .
4. How is the line you found above related to your solutions of the normal equation?