MATH2210

Wk 1 Discussion 8/23/2023

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Slides Location

- These slides can be found at bgthompson.com:
  - → Teaching [bar]
  - → Linear Algebra (MATH 2210) Fall 2023 [link]
  - → Wk1 Slides (Wed) [link]

Discussion Outline

- What is the format of the discussion sections? Why come to them?
- What is linear algebra?
- Admin
- Mathematics mythbusting
- Math support groups
- Plagiarism & Cheating
- D&I Bias reporting

What is the format of these discussions?

- Half problem solving / new material
- Half revising homework / lecture material / etc.

Today's a bit different though, the latter half will be course admin. (Yay!)

Why bother coming to discussion sections?

Why bother coming to discussion sections?

- Collaborative — It's significantly easier to collaboratively learn in person than online.
- Easier to learn math from people who are also learning it
- Diversity of perspectives — everyone has different understandings of mathematics / ways of thinking
- Easy to ask questions / easy to get answers [sometimes :)]
- A consistent time / space to learn math
- Social!
- Many others…

Question

- What is linear algebra?

Question

- What is linear algebra?
- Many different answers...

Question

- What is linear algebra?
- Many different answers...
- Mine: "The study of linear functions"

Problem 1. (10min)

- Characterize all functions f that take real numbers to real numbers, and which satisfy the following two properties for all real numbers x, y:
  - $f(xy) = x * f(y)$
  - $f(x + y) = f(x) + f(y)$

(Characterize means provide an alternative, and hopefully simpler description of all such functions.)

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(Characterize means provide an alternative, and hopefully simpler description of all such functions.)

- Idea: Create an example of such a function
- Idea: What are the possible values of f(0)?

Problem 2. (5 min)

- Instead of adding real numbers, we can also add pairs of real numbers:
  - Define (a,b) + (c,d) := (a + b, c + d)
- We can also multiply pairs of real numbers by a single real number:
  - Define a * (b,c) := (a*b, a*c)
- Characterize all functions f that take pairs of real numbers to pairs of real numbers, and which satisfy the following two properties for all (a,b), (c,d) and e:
  - f(c*(a,b)) = c * f((a,b))
  - f((a,b) + (c,d)) = f((a,b)) + f((c,d))

Problem 2.

- Problem 2 is much harder than Problem 1!
- We'll solve this problem later on in the course.

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- Functions are called <u>linear</u> if they satisfy for scalars* c and vectors* x, y:
  - f(c * x) = c * f(x)
  - f(x + y) = f(x) + f(y)

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- My description of linear algebra: "The study of linear functions"
- It may not be apparent yet in the course how the lecture material so far is related, but we'll see the connections soon.

Assessment in the Course

- 2 Prelims (40%)
- Final exam (30%)
- Written homework (20%)
- Online (WeBWorK) homework (10%)

Where is the course textbook?

only one course in the following group: MATH 2210, 2230, 2310, 2940.

Course Materials

Textbook. W. Keith Nicholson, Linear Algebra with Applications, Version 2019-A or 2021-A, Lyryx–Open Texts, available as a pdf for free download from Lyryx. Printed copies in two volumes, Volume I: Ch 1–5 & Appendices, ISBN-10 : 1717015220 and Volume II: Ch 6–11, ISBN-10 : 1717015220, can be purchased from major book sellers for under $10 per volume. (The revision from Version 2019-A to 2021-A Typos is that "other minor fixes have been implemented throughout.")

We recommend that you build on what you learn in class by studying the corresponding textbook sections.

Class Notes. Please bring a *printed* copy or a *tablet version* of the Class Notes to every class. ↻ A pdf copy can be downloaded here. ↓

Non-publication of course materials. Students must not publish course materials or recordings.

- ● Online!
- ● Go to the course page on Canvas
- ● Link in "Syllabus"

Where is the course textbook?

Fall 2

Hom

Anno

Syll

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Quizzes

Linear Algebra with Applications

W. Keith Nicholson

2023-A-D
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downloaded here.

Students must not publish course materials or

A math professor at Cornell helped make it

Where are the course notes?

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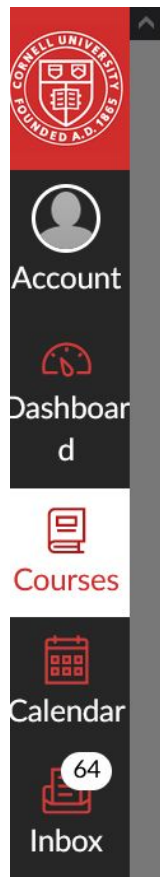
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Homework deadlines

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Files ⊘

Deadlines and Extensions

Written homework, WeBWorK homework, and projects are due 10pm on Wednesdays. We ask that you always try to submit before that deadline. But life happens: you miss the deadline because of other courses, illness, events in your personal life, and so forth. Rather than the burden of you having to ask for an extension in those circumstances, we have configured Gradescope and WeBWorK to accept late submissions up to 48 hours after the deadline. There is no penalty for these late submissions. They are an automatic extension you may choose to grant yourself. We trust you to do so wisely—making use of them will result in you having less time to work on the next assignment. Automatic extensions give you a tool to manage the demands of life, including routine illness, minor injury, travel, job fairs, job interviews, large workloads in other courses, and extra-curricular activities. After the submission deadline has passed—even 1 second past it—no further submissions will be accepted. Gradescope and WeBWorK do not provide grace periods on late submissions.

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Accessibility

Cornell University is committed to ensuring access to learning opportunities for all students. Student Disability Services (SDS) is the campus office that collaborates with students who have disabilities to provide and/or arrange reasonable accommodations.

If you are registered with SDS and have a faculty notification letter dated for this semester, please ask SDS to send a copy to Nicolas Templier (npt27@cornell.edu) early in the semester and we will discuss how the accommodations will be applied in the course.

If you have, or think you have, a disability in any area such as, mental health, attention, learning, chronic health, sensory, or physical, please contact the SDS office to arrange a confidential discussion regarding equitable access and reasonable accommodations.

Mathematical Mythbusting

Everything published in mathematics is true. In particular, everything in mathematical textbooks is true.

Which one ?   True   /   False

Fermat's Last Theorem

Theorem:

$X^n + Y^n = Z^n$ has <u>no</u> non-zero integer solutions for $n \geq 3$.

- Unproven for 300+ years
- Proved by Andrew Wiles in the 90s.

Wiles first announced his proof on 23 June 1993 at a lecture in Cambridge entitled "Modular Forms, Elliptic Curves and Galois Representations".[2] However, in September 1993 the proof was found to contain an error. One year later on 19 September 1994, in what he would call "the most important moment of [his] working life", Wiles stumbled upon a revelation that allowed him to correct the proof to the satisfaction of the mathematical community. The corrected proof was published in 1995.[3]

From Wikipedia, "Wiles' Proof of Fermat's Last Theorem"

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From Wikipedia, "Wiles' Proof of Fermat's Last Theorem"

NOT in the course, you don't need to know this (obviously). But note there's an error every couple of pages.

- Page 144: In the third paragraph of the proof of the rising sun lemma (Lemma 1.6.17), b should be b_n in the definition of A and in the next two occurrences (i.e. "t but not b" should be t but not b_n", and "$t_* \in [t, b)$" should be $t_* \in [t, b_n)$".
- Page 145, bottom: "$f'(x)$ exists" should be "$F'(x)$ exists". After Exercise 1.6.52, "ensure the almost everywhere existence" should be "ensure the absolute integrability of the derivative".
- Pages 149-152: In Section 1.7.1, "Caratheodory extension theorem" should be "Caratheodory lemma" throughout.
- Page 150, Exercise 1.7.2: "Lebesgue outer measurable" should be " the Lebesgue outer measure"
- Page 151: In the last two displays, and in the first display on the next page, $E_{N+1} \setminus \bigcup_{n=1}^{N} E_n$ may be simplified to E_{N+1}. In the second paragraph, "a disjoint sequence of" should be "a sequence of disjoint".
- Page 156: In Theorem 1.7.9, $-\infty < b < a < \infty$ should be $-\infty < a < b < \infty$. In the second paragraph of proof of this theorem, before ", adopting the obvious conventions", add "to be the required value of $\mu_F(I)$ given by (1.33) (e.g., $|[a,b]|_F = F_+(b) - F_-(a)$)".

From "An introduction to measure theory" by Terence Tao on Tao's website.

Takeaways

- Mathematicians aren't perfect (obviously), sometimes we make mistakes!
  - Never assume all mathematics is true! It is <u>essential</u> that *you* verify / check for *yourself* why something is true.
  - If something doesn't make sense in a textbook, it could be a typo. <u>Sometimes even solutions have typos.</u>

More Mythbusting

When mathematics is communicated, the only thing you need to worry about is whether or not it is correct.

In particular, correct numerical solutions in exams automatically get 100%

Which one   True   /   False

    ?

Question

Let $A \le B \le C$ be positive integers (i.e. 1,2,3,...)

which satisfy

$$A + B + C = ABC$$

What is the maximum possible value of ABC?

Explain your reasoning.

(2 min)

Solution

We first show that if A,B,C satisfy the conditions of the question, then AB ≤ 3.

Since

NOTE: There is no expectation in this course that you can figure out a solution like this in a few minutes. It takes years of practice.

$$A \le C,$$

$$B \le C,$$

$$C \le C,$$

$$ABC = A + B + C \le C + C + C = 3C.$$

Since C is positive, we divide both sides by C and get AB ≤ 3.

Solution (continued)

Now that we know AB ≤ 3, try and solve the problem:

What's the maximum possible value of ABC if ABC = A + B + C and A ≤ B ≤ C are positive integers?

(2 min)

Expectations around solutions

- Solutions should primarily <u>convince</u> the reader why something is true.
  - Most questions will be graded as such.
- In most 1000 / 2000 level math courses at Cornell, solutions are graded based on:
  - Completeness
  - Correctness

Expectations around solutions

- Completeness:
  - Did the student have a good go at solving the problem?
- Correctness:
  - How convincing is the solution?

Let's grade some example solutions to the previous problem with these criteria.

Example Solution 1

We first show AB ≤ 3.

Since $ABC = A + B + C \leq 3C$ and $C \geq 0$, $AB \leq 3$.

Hence $(A,B) = (1,1), (1,2), (1,3)$.

Substituting these into $ABC = A + B + C$, the only possible value of C is 3. Hence the max is $1*2*3 = 6$.

Completeness: / 3   Correctness   / 3

(Good go?)   (How convincing?)

Example Solution 2

It's possible to show that ABC = A + B + C implies AB ≤ 3.

So AB = 1,2,3.

So (A,B) = (1,1),(1,2),(1,3).

If (A,B) = (1,1), C = C + 2

→ no solution.

If (A,B) = (1,2), 2C = C + 3

So C = 3, so ABC = 6.

If (A,B) = (1,3), 3C = C + 4

So C = 2. But then B > C, which

Is not allowed.

→ Only solution is (1,2,3)

→ Maximum is 6.

Completeness: / 3

(Good go?)

Correctness   / 3

(How convincing?)

6

Completeness: / 3

(Good go?)

Correctness / 3

(How convincing?)

Takeaways

- Solutions in mathematics without an explanation *are usually difficult to understand.*
- If asked to explain your reasoning, be sure to do so!
- It's okay to write down partial ideas (e.g. "I tried to come up with a bound for a or b") if you're unable to come up with a solution.

Final Myth

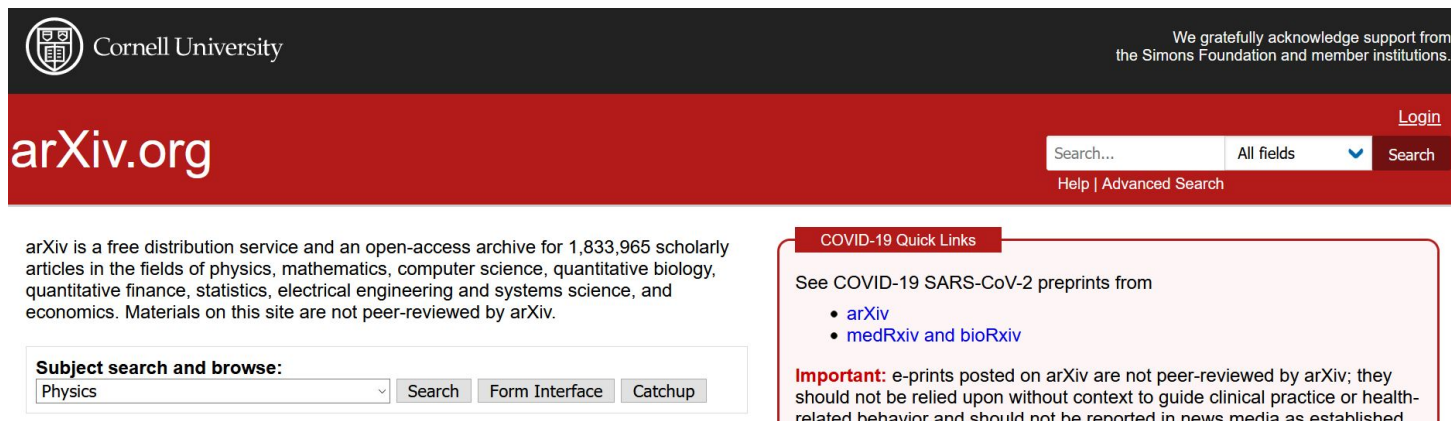Mathematics is done by geniuses, usually alone. In particular, mathematicians know how to solve most problems quickly.

Which one   True   /   False
    ?

Check out arXiv!

- Cutting-edge mathematics research is free: pretty much all mathematical papers are posted to arxiv.org

- It's actually hosted by Cornell!

Loads of authors...

- When searching mathematics papers on the arXiv, the majority of papers have several authors
  - I.e. most mathematicians work on solving problems together.
- It's best to start early!
  - Find a classmate(s) to work with on problems (e.g. via Ed Discussion)

Circuit algebras are wheeled props

Authors: Zsuzsanna Dancso, Iva Halacheva, Marcy Robertson

Abstract: Circuit algebras, introduced by Bar-Natan and the first author, are a gen Jones's planar algebras, in which one drops the planarity condition on "connection provide a useful language for the study of virtual and welded tangles in low-dimen this note, we present the circuit algebra analogue of the well-known classification as pivot... ▽ More

Submitted 21 September, 2020; originally announced September 2020.
Comments: 29 pages, many figures
MSC Class: 57M25; 18D50

2. arXiv:2007.09828 [pdf, other] math.GT

Over then Under Tangles

Authors: Dror Bar-Natan, Zsuzsanna Dancso, Roland van der Veen

Abstract: Over-then-Under (OU) tangles are oriented tangles whose strands trave over crossings before any under crossings. In this paper we discuss the idea of gli by which any tangle diagram could be brought to OU form. Unfortunately, the alg However, by analyzing cases in which it does succeed we obtain a braid classificat also e... ▽ More

Submitted 4 February, 2021; v1 submitted 19 July, 2020; originally announced July 2020.
Comments: 35 pages, lots of figures
MSC Class: 57M25

3. arXiv:1910.00979 [pdf, ps, other] math.AG math.CO

Deletion-contraction triangles for Hausel-Proudfoot varieties

Authors: Zsuzsanna Dancso, Michael McBreen, Vivek Shende

Abstract: To a graph, Hausel and Proudfoot associate two complex manifolds, B a

- Almost all of mathematics research is about solving problems no one has been able to solve yet.
  - The idea that mathematicians know how to solve most problems is false, simply because there are always more unsolved problems!

Math is a language

- Just as no one is born being able to speak a given language, no one is born being able to do math.
- Reading a dictionary of another language will not make you fluent in that language… you need to practice!

Math is a language (continued)

- If you encounter words / symbols you don't know the mean of in this course, ultimately it's your responsibility to look them up.

Warning

- For native speaker of English, translating another language into English is usually a lot easier than translating in the opposite direction.
  - <u>Don't fall into the same trap with math:</u>
  - Understanding a solution is easier than coming up with it yourself. *You will be expected to come up with solutions.*
  - This only comes about with practice.

Math is hard

Math is hard; anyone who says math is easy is a liar.

What are some places you can get help with math?

Getting support with math is not as hard as getting in here…

Math Support (not exhaustive)

- Other students
- Ed Discussion
- Office hours
- Math support center
- Online Q&A boards (e.g. math exchange)
- Student support groups
- Tutorials / examples on Youtube / other platforms

Some student STEM support groups

- [Association for Women in Mathematics Cornell (AWM)](#)
- [Society for Women Engineers at Cornell (SWE)](#)
- [Women in Computing at Cornell (WICC)](#)
- [Underrepresented Minorities in Computing Cornell (URMC)](#)
- [ColorStack](#)
- [National Society of Black Engineers Cornell (NSBE)](#)
- [Society of Hispanic Professional Engineers Cornell (SHPE)](#)
- [American Indian Science & Engineering Society (AISES)](#)
- [First Generation Students Union Cornell](#)
- [Haven // qStem (LGBTQI+ in STEM)](#)
- [LGBT Resource Center](#)

Plagiarism

- The consequences of plagiarism can be severe:
  - Failing a course
  - A permanent record of cheating on your transcript

- *In the case of an exam, using non-approved internet resources is strictly prohibited.*

Takeaways

- If you're stuck on an assignment problem, seek help! There are LOADS of ways to get help in the course.
- Highly, highly recommended way:

  Support course. MATH 1021: Academic Support for MATH 2210, which meets weekly to provide extra support, practice problems, problem-solving techniques, and prelim reviews. Everyone is welcome to participate in MATH 1021 and use its resources without commitment or obligation (if you are not enrolled, you may feel free to come any week you want extra support). You can access its Canvas page without enrolling in the class.

>> The world treats you by what you do,
not by who you think you are.

14:42 / 15:51
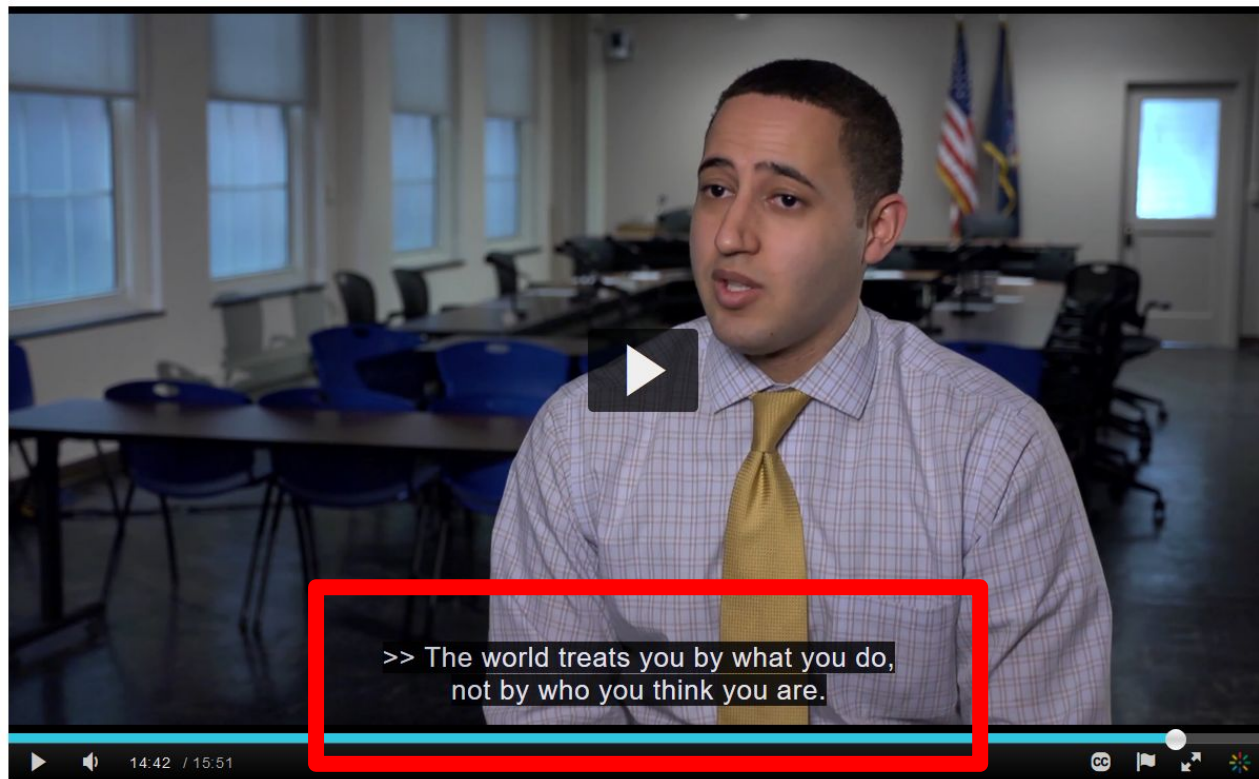
From the Cornell "cheating" video

- Former Mayor Myrick's thoughts on cheating.

Not all faculty think a lot about D&I...

Anyone can use this about anyone

It's not just bias

- It doesn't necessarily need to be intentional.
- E.g. a professor saying "him or her" instead of the more inclusive "they".

Bias
Criminal Activity
Discrimination
Harassment
Domestic and Dating Violence
Gender Based or Sexual Harassment
Hazing
Retaliation
Sexual Assault or Misconduct
Sexual Exploitation
Stalking
Violation of Interim Measure under Policy 6.4
Workplace Climate Concerns
Other/Not Sure Which Category to Select

Anonymity Options

Please remember that the timing and manner in which the University addresses this report will vary depending on the information provided and whether involved parties are available for further discussion.

○ You may contact me

○ To the extent possible, I would like to remain anonymous to involved individuals, but you may contact me

○ Please do not contact me

Summary

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