

## First part

- Given a line, a circle, and a point on the circle, the line is said to be *tangent to the circle at the point* if it intersects the circle at the point and nowhere else. Such a line is called a *tangent line* to the circle.
- Let  $S^1$  refer to the circle in the Euclidean plane centered at the origin with radius 1.

**Main question:** Characterize all tangent lines to  $S^1$ .

- Specifically, any point on  $S^1$  can be described in terms of an angle  $\theta$ .<sup>1</sup> What is an equation describing the tangent line to  $S^1$  at the point corresponding to  $\theta$ ?

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<sup>1</sup>E.g. The right-most point of  $S^1$  corresponds to  $\theta = 0$ , the top-most point of  $S^1$  corresponds to  $\theta = \pi/2$ .

## Second part

1. Characterize all matrices in row-echelon form with the following property: If any entry is changed, the matrix is no longer in row-echelon form.
2. Create a system of three linear equations in three variables whose solution set geometrically corresponds to:
  - (a) The empty set
  - (b) A point
  - (c) A line
  - (d) A plane
  - (e) Euclidean 3-space.
3. In the example above whose solution set is a plane, describe the solution set in terms of a linear combination of vectors.