

MATH 1100

Wk2 Discussions 9/8/20

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Before we start

- There's a good chance some of us will have to quarantine at some point...
you can attend any discussion section online in this case -

Discussion Outline:

- Why study math?
- Mathematics Mythbusting
 - Published math is always correct
 - Being correct is everything
 - Mathematics is done by solitary geniuses
- Math support groups
 - ↳ Plagiarism & Cheating
 - Letting Cornell know about horrible people.

Why Study Math?

Why Study Math?

Some reasons: (not all!)

- It's used throughout science
- It's a CAS prereq. (valid reason)
- Problem solving is fun
- You can use it to prove things about the world that are impossible to prove without math
- Once something's proved it's true forever

Question

Prove that at any given time, there are two opposite points on the surface of the Earth with the same temperature and pressure.

(2 min)

Proof:

Consider the xy -plane. Let the x -axis denote temperature, and the y -axis denote pressure. Then at each point on the Earth, the temperature and pressure at that point is represented by a point in the xy plane. The Earth is roughly spherical, so we have a function

$$f: S^2 \rightarrow \mathbb{R}^2,$$

where S^2 is the sphere and \mathbb{R}^2 is the plane, defined by taking a point to its temperature & pressure.

Now since temperature and pressure vary continuously over the surface of the Earth,

$$f: S^2 \rightarrow \mathbb{R}^2$$

A 3rd-year theorem,
you don't need to
know this yet

is a continuous function. The Borsuk-Ulam theorem states that for any continuous function

$$g: S^2 \rightarrow \mathbb{R}^2$$

there are at least two antipodal (opposite) points $x, y \in S^2$ such that $g(x) = g(y)$.

Hence for some pair of opposite points x, y on the Earth, $f(x) = f(y)$, proving the claim. \square

Mathematical Mythbusting

Everything published in mathematics
is true. In particular, everything in
a mathematics textbook is true.

Which one?

True / False

?

Fermat's Last Theorem

$x^n + y^n = z^n$ has no
integer solutions for $n \geq 3$.

- Unproven for 300+ years
- Theorem proved by Andrew Wiles in the 90s.

Wiles first announced his proof on 23 June 1993 at a lecture in Cambridge entitled "Modular Forms, Elliptic Curves and Galois Representations".^[2]

However, in September 1993 the proof was found to contain an error. One year later on 19 September 1994, in what he would call "the most important moment of [his] working life", Wiles stumbled upon a revelation that allowed him to correct the proof to the satisfaction of the mathematical community.

The corrected proof was published in 1995.^[3]

From Wikipedia, "Wiles's Proof of Fermat's Last Theorem"

Types / Errors in Textbooks

NOT in the course you don't need to know this.
But note there's an error every couple of pages.

- Page 144: In the third paragraph of the proof of the rising sun lemma (Lemma 1.6.17), b should be b_n in the definition of A and in the next two occurrences (i.e. " t but not b " should be t but not b_n ", and " $t_* \in [t, b)$ " should be $t_* \in [t, b_n)$ ".
- Page 145, bottom: " $f'(x)$ exists" should be " $F'(x)$ exists". After Exercise 1.6.52, "ensure the almost everywhere existence" should be "ensure the absolute integrability of the derivative".
- Pages 149-152: In Section 1.7.1, "Caratheodory extension theorem" should be "Caratheodory lemma" throughout.
- Page 150, Exercise 1.7.2: "Lebesgue outer measurable" should be "the Lebesgue outer measure"
- Page 151: In the last two displays, and in the first display on the next page, $E_{N+1} \setminus \bigcup_{n=1}^N E_n$ may be simplified to E_{N+1} . In the second paragraph, "a disjoint sequence of" should be "a sequence of disjoint".
- Page 156: In Theorem 1.7.9, $-\infty < b < a < \infty$ should be $-\infty < a < b < \infty$. In the second paragraph of proof of this theorem, before ", adopting the obvious conventions", add "to be the required value of $\mu_F(I)$ given by (1.33) (e.g., $\|[a, b]\|_F = F_+(b) - F_-(a)$)".

From "An introduction to measure theory" by Terence Tao on Tao's website.

Takeaways:

- Mathematicians aren't perfect, sometimes we make mistakes
 - Never blindly trust mathematics. It is essential that you verify / check for yourself why something is true
 - If something doesn't make sense in a textbook, it could be a typo.
Sometimes even solutions have typos.

More Mythbusting

When Mathematics is communicated, the only thing you need to worry about is whether or not it is correct. In particular, correct numerical solutions in exams automatically get 100%.

Which one: True / False

?

Question: Let $a \leq b \leq c$ be positive integers
(i.e. $1, 2, 3, 4, \dots$).

which satisfy

$$a + b + c = abc.$$

What is the maximum possible value of abc ? Explain your reasoning.

(3 min)

Solution: We first show that if a, b, c

(Note: this
is hard
to figure
out in 5
min.)

satisfy the conditions of the
question, then $ab \leq 3$.

NOTE:

There is
no expectation
in this course
that you can
figure out a
solution like
this. It takes
years of practice.

Since $a \leq c$,

$$b \leq c,$$

$$c \leq c,$$

$$a+b+c \leq c+c+c = 3c$$

Since $abc = a+b+c$, this means

$$abc \leq 3c. \text{ Since } c \text{ is positive, we can}$$

divide both sides by c and get $ab \leq 3$.

Solution:

Now we know $ab \leq 3$,
try and solve the problem.

(What's the maximum possible value of abc
if $abc = a+bc$, $a \leq b \leq c$ are positive integers?)

NOTE: This part is
far less scary
than you may
think it is. You can all
do this. Really.

(2 min)

Expectations around Solutions

- Solutions should primarily convince the reader why something is true
 - Most questions will be graded as such
- In most 1000 / 2000 level math courses @ Cornell, solutions are graded based on
 - Completeness
 - Correctness

- Completeness

Did the student have a good go at solving the problem?

- Correctness

How convincing is the solution?

Let's grade some example solutions to the previous problem with these criteria.

Example Solution 1

We first show $ab \leq 3$.

Since $abc = a+b+c \leq 3c$, and $c > 0$,
 $ab \leq 3$. Hence $(a,b) = (1,1), (1,2), (1,3)$.

Substituting these into $abc = a+b+c$,
the only possible value of c is 3.
Hence the max is $1 \cdot 2 \cdot 3 = 6$.

Completeness: /3

(Good go?)

Correctness: /3

(How convincing?)

Example Solution 2

It's possible to show that
 $abc = a+b+c$ implies $abc \leq 3$.

$$\therefore ab = 1, 2, 3.$$

$$\therefore (a,b) = (1,1), (1,2), (1,3).$$

$$\text{If } (a,b) = (1,1), c = c+1$$

\therefore no solution

Completeness: /3

(Good go?)

If $(a,b) = (1,2)$, $2c = c+3$
 $\therefore c=3 \therefore abc=6$.
If $(a,b) = (1,3)$, $3c = c+4$
 $\therefore c=2$. but then $b > c$.
 \therefore Only solution is $(1,2,3)$
 \therefore maximum is 6.

Correctness: /3

(How convincing?)

Example Solution 3

6

Completeness: /3

(Good go?)

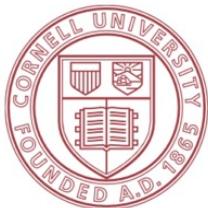
Correctness: /3

(How convincing?)

Takeaways

- Solutions in mathematics without an explanation usually have no value
- If asked to explain your reasoning, be sure to do so
- It's okay to write down partial ideas (e.g. "I tried to come up with a bound for a or b") if you're unable to come up with a solution

- You will need to explain solutions...



MATH 1110

Name: _____

Homework 1

NetID: _____

Friday, Sept 11

Worked with: _____

If you have a printer or tablet computer, answer the questions in the spaces provided by printing this handout or annotating the pdf. If you do not have a printer or tablet, label and write each answer on a separate sheet of paper. In either case, your solutions should be hand-written (unless that is not possible). Please include full explanations and write your answers using complete sentences. When you have completed the assignment, upload all the pages to Gradescope by 11:59PM on the due date. Please make your grader's life easier by writing legibly, and make sure to "select pages" when uploading to Gradescope!!

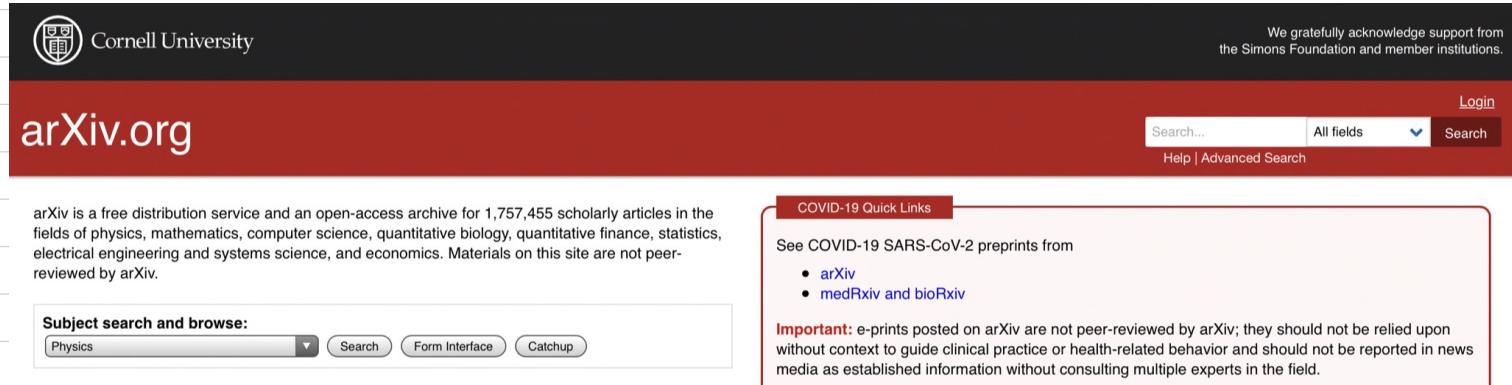
Final Myth

Mathematics is done by geniuses,
usually alone. In particular, mathematicians
know how to solve most problems
quickly.

Which one?
?

True / False

- Cutting-edge mathematics research is free: pretty much all mathematics papers are posted to arXiv.org.



The screenshot shows the arXiv.org homepage. At the top, there's a dark header with the Cornell University logo and the text "We gratefully acknowledge support from the Simons Foundation and member institutions." Below the header is a red navigation bar with the "arXiv.org" logo. On the right side of the red bar are links for "Login", "Search...", "All fields", a dropdown menu, and "Search". Underneath the red bar, there's a white section containing a paragraph about arXiv being a free distribution service for scholarly articles in various fields. Below this paragraph is a "Subject search and browse" form with dropdown menus for "Physics", "Search", "Form Interface", and "Catchup". To the right of the search form is a red-bordered box containing COVID-19 Quick Links and information about SARS-CoV-2 preprints, along with a note about the reliability of e-prints.

We gratefully acknowledge support from
the Simons Foundation and member institutions.

arXiv.org

Search... All fields Search

Help | Advanced Search

Login

COVID-19 Quick Links

See COVID-19 SARS-CoV-2 preprints from

- arXiv
- medRxiv and bioRxiv

Important: e-prints posted on arXiv are not peer-reviewed by arXiv; they should not be relied upon without context to guide clinical practice or health-related behavior and should not be reported in news media as established information without consulting multiple experts in the field.

- It's actually hosted by Cornell!

1. arXiv:2007.09828 [pdf] math.GT

Over then Under Tangles

Authors: Dror Bar-Natan, Zsuzsanna Dancso, Roland van der Veen

Abstract: Brilliant wrong ideas should not be buried and forgotten. In underneath the layer of wrong. In this paper we explain how "over the and under the surface, also to some valid mathematics: a separation with an overview of other instances wh... ▽ More

Submitted 21 July, 2020; v1 submitted 19 July, 2020; originally announced July 2020

2. arXiv:1910.00979 [pdf, ps, other] math.AG math.CO

Deletion-contraction triangles for Hause-Proudfoot

Authors: Zsuzsanna Dancso, Michael McBreen, Vivek Shende

Abstract: To a graph, Hause and Proudfoot associate two complex moduli of local systems on a Riemann surface, and moduli of Higgs bundles, which generalize local systems, and D carries the structure of a complex of these varieties count spanning s... ▽ More

Submitted 2 October, 2019; originally announced October 2019.

3. arXiv:1905.03067 [pdf, other] math.CO math.RA math.RT

Koszul Algebras and Flow Lattices

Authors: Zsuzsanna Dancso, Anthony M. Licata

Abstract: We provide a homological algebraic realization of the lattice edge-connected graph Γ with a spanning tree T , we associate a finite construction, planar dual graphs with dual spanning trees are associated category of finitely-generated... ▽ More

Submitted 28 May, 2019; v1 submitted 8 May, 2019; originally announced May 2019

4. arXiv:1811.08558 [pdf, ps, other] math.QA math.GT

Ribbon 2-Knots, $1 + 1 = 2$, and Duflo's Theorem for

Authors: Dror Bar-Natan, Zsuzsanna Dancso, Nancy Scherich

Abstract: We explain a direct topological proof for the multiplicativity

- When searching Mathematics papers on there, the majority of papers have several authors.

→ i.e. most mathematicians work on solving problems together.

- It's best to start early
→ find a classmate to work on problems with. (e.g. via Piazza)

You don't
need to
understand
these words!

- Almost all of mathematics research is about solving problems no one has been able to solve yet

→ the idea that mathematicians know how to solve most problems quickly is false, simply because there are always more unsolved problems!

Math is a language

- Just as no one is born being able to speak a given language, no one is born being able to do math
- Reading a dictionary of another language will not make you fluent in that language... You need to practice!

- If you encounter words / symbols you don't know the meaning of, it's ultimately your responsibility to look them up.

$\ln(x)$

Derivative

e

$\sin(x)$

function

Integral

continuous

differentiable

π

Warning

For native speakers of English, translating another language into English is usually much easier than going in the opposite direction.

Well give you loads of practice to do this

→ Don't fall into the same trap with math!! Understanding a solution is easier than coming up with it yourself. You will be expected to come up with solutions. This only comes about with practice.

Math is hard

(Anyone who says otherwise is lying!)

- Are you expected to be able to solve every problem on your own without help?

Which one? Yes / No

- Where can you get help? 

Math Support (not exhaustive)

- Other students
 - Piazza
 - Office hours
 - Math support centre
 - Online Q & A boards (e.g. Math Exchange)
 - Student support groups
 - Tutorials / examples on YouTube
- 

Some student STEM support groups

Association for Women in Mathematics (AWM)

Society of Women Engineers at Cornell (SWE)

Women In Computing at Cornell (WICC)

Association for Women in Science (ASWC)

Underrepresented Minorities in Computing (URMC)

ColorStack

The National Society of Black Engineers (NSBE)

Society of Hispanic Professional Engineers (SHPE)

American Indian Science & Engineering Society (AISES)

Cornell First Generation Students Union

Women's Resource Center

LGBT Resource Center

If those other options fail...

- Is it okay to search the internet for a solution to an assignment problem?

Yes / No

- Is it okay to use a solution to a math problem on the internet in an assignment?

Which one: Yes / No
?

• PROVIDED you:

- Cite the webpage you got the solution from
- Rewrite the solution in your own words
- Build on the solution to make a better solution

... it's okay for assignments.

It's never okay for exams.

Maximum value of abc where abc = a + b + c and a,b,c positive integers

Example

I actually got
the question
before from
Math Exchange.

Asked yesterday Active yesterday Viewed 26 times

 I came across an exercise in an algebra book which is the question in the title of this post.

 1 We can assume WLOG that $a \leq b \leq c$. The book gave a hint: show that $ab \leq 3$.

 I was able to figure out the answer using the hint: it implies $(a, b) = (1, 1), (1, 2), (1, 3)$, and then after substitution you get that $(a, b, c) = (1, 2, 3)$ is the only option with $b \leq c$.

 I have no idea how to show $ab \leq 3$ though. How do I do this?

algebra-precalculus inequality

Featured

New

Hot
mod

Guid

Related

2 Ho

1 Answer

Active Oldest Votes

 With $a \leq b \leq c$, we have

3

$$abc = a + b + c \leq c + c + c = 3c$$

 share cite improve this answer follow

answered yesterday

 Hagen von Eitzen

How can this
solution be
improved?

- We all get stuck from time to time, solutions can point us in a good direction
- Provided all the previous criteria are met, you won't be penalized.
- A failure to cite a website, or a close collaboration with friend is an instance of plagiarism
- There is no need to cite course materials.

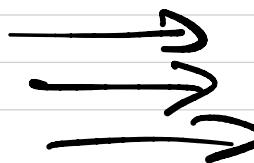
- The consequences for plagiarism can be severe.

- Failing a course
 - A permanent record of cheating on your transcript.

- In the case of an exam, using non-approved internet resources is strictly prohibited.

Takeaways

- If you're stuck on an assignment problem, were unable to get help and found a solution online:
 - Cite the url
 - Rewrite the solution
 - Improve the solution



Demonstrate you've thought about the problem.

From
the
Cornell
"cheating"
video



- Mayor Myrick's thoughts on cheating
- You should know who he is!

Interactions with horrible people

If you discover someone is a terrible person,
let Cornell know about it.



This guy
would absolutely
need to be
reported &
should be kicked
out of Cornell.

From Rina Sawayama's excellent "STFU"
Music Video available on YouTube.

Diversity and Inclusion



Our Story

Belonging at Cornell

Our Community

Our Commitments

Our Commitments

Diversity Leadership
at Cornell

DIWD

Inclusive Excellence
Network

[Home](#) / [Our Commitments](#) / Bias Reporting at Cornell

Bias Reporting at Cornell

[Report A Bias Incident](#)

Since 2000, Cornell University has had a program to track bias that is occurring on all campuses in an effort to be proactive in creating an inclusive climate for all. The [Department of Inclusion and Workforce](#)

Aug one
can use
this about
anyone
E

It's not just bias

- It doesn't necessarily need to be intentional

↳ e.g. a professor saying "him or her" instead of the more inclusive "they".

Bias
Criminal Activity
Discrimination
Harassment
Domestic and Dating Violence
Gender Based or Sexual Harassment
Hazing
Retaliation
Sexual Assault or Misconduct
Sexual Exploitation
Stalking
Violation of Interim Measure under Policy 6.4
Workplace Climate Concerns
Other/Not Sure Which Category to Select

Anonymity Options

Please remember that the timing and manner in which the University addresses this report will vary depending on the information provided and whether involved parties are available for further discussion.

- You may contact me
- To the extent possible, I would like to remain anonymous to involved individuals, but you may contact me
- Please do not contact me

Summary

- Why study math?
- Mathematics Mythbusting
 - Published math is always correct
 - Being correct is everything
 - Mathematics is done by solitary geniuses
- Math support groups
 - ↳ Plagiarism & Cheating
- Letting Cornell know about horrible people.
- SAWAYAMA is an amazing album. Listen to it!

Next Time:

- Limits
- Continuity of functions

Upcoming Assessment

- HW0 assignments from wk. 1 (Fri, 9/11)
- wk 2 Pre class activity (Thu, 9/10)
- Written HW (Fri, 9/11)
- online HW (Fri, 9/11)

Thanks for coming!

These slides are available
on Canvas:

Files → Thompson ⇒ Disslides-Wk2.pdf