Løsningsforslag tentamen høst 2018 – MA015

Oppgave 1

Regn ut

a)
$$\frac{\sqrt[3]{a^2}\cdot b^2\cdot a^{-1}\cdot a^3\cdot\sqrt{b}}{a^2\cdot(b^2)^{-1}\cdot\sqrt[4]{b^3}}=\frac{a^{\frac{2}{3}}\cdot b^2\cdot a^{-1}\cdot a^3\cdot b^{\frac{1}{2}}}{a^2\cdot b^{-2}\cdot b^{\frac{3}{4}}}=$$
$$a^{\frac{2}{3}-1+3-2}b^{2+\frac{1}{2}+2-\frac{3}{4}}=a^{\frac{2}{3}}\cdot b^{\frac{8+2+8-3}{4}}=a^{\frac{2}{3}}\cdot b^{\frac{15}{4}}=\underline{\underline{\sqrt[3]{a^2}\cdot\sqrt[4]{b^{15}}}}=\underline{\underline{\sqrt[3]{a^2}\cdot b^3\cdot\sqrt[4]{b^3}}}$$

b)
$$\lg(a^2\cdot b^3)+\lg\left(\frac{1}{b^2}\right)-\lg\left(\frac{b}{a}\right)=\lg a^2+\lg b^3+\lg 1-\lg b^2-(\lg b-\lg a)$$
$$2\lg a+3\lg b+0-2\lg b-\lg b+\lg a=\underline{\underline{3\lg a}}$$

Oppgave 2

Forkort/forenkle følgende uttrykk

a) $$\frac{x^2-3x-10}{x^2-4}=\frac{(x-5)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}=\frac{x-5}{\underline{\underline{x-2}}}$$

b) $$(p^3-2)\cdot\frac{p^2}{2p^6-4p^3}=\frac{\cancel{(p^3-2)}\cdot\cancel{p^2}}{2p^{\cancel{3}}\cancel{(p^3-2)}}=\frac{1}{\underline{\underline{2p}}}$$

Oppgave 3

Løs likningene ved regning

a)

$$\frac{x}{x-1}-\frac{2}{x+1}=\frac{2}{x^2-1}\ |\cdot(x+1)(x-1) \qquad x\neq\pm1$$
$$x(x+1)-2(x-1)=2$$
$$x^2+x-2x+2-2=0$$
$$x^2-x=0$$
$$x(x-1)=0$$
$$\underline{x=0}$$
$$x-1=0$$
$$\underline{x=1}$$
$$Løsning:\underline{\underline{x=0}}$$

b)

$$\ln\frac{1}{2}+\ln 4x=2\ln 8$$
$$\ln(\frac{1}{2}\cdot 4x)=\ln 8^2$$
$$2x=64$$
$$\underline{\underline{x=32}}$$

c)

$$x-\sqrt{2x^2-8}=2$$
$$\sqrt{2x^2-8}=x-2$$
$$2x^2-8=(x-2)^2$$
$$2x^2-8=x^2-4x+4$$
$$2x^2-x^2+4x-8-4=0$$
$$x^2+4x-12=0$$
$$equq-2.grad$$
$$x=2 \ \vee \ x=-6$$
$$\text{Pr}\,øve$$
$$x=2$$
$$vs:2-\sqrt{2\cdot 2^2-8}=2$$
$$hs:2$$
$$x=-6$$
$$vs:2-\sqrt{2(-6)^2-8}=2-8=-6$$
$$hs:2$$

$$Løsning:\underline{\underline{x=2}}$$

d)

$$-3+\frac{4}{e^x}=0\mid\cdot e^x$$
$$-3e^x+4=0$$
$$-3e^x=-4$$
$$e^x=\frac{4}{3}$$
$$\ln e^x=\ln\frac{4}{3}$$
$$\underline{\underline{x=\ln 4-\ln 3}}$$

Oppgave 4

Deriver følgende uttrykk

a)

$$f(x) = x^3 - 4x + \frac{1}{x^2}$$

$$f(x) = x^3 - 4x + x^{-2}$$

$$f'(x) = 3x^2 - 4 - 2x^{-3}$$

$$\underline{\underline{f'(x) = 3x^2 - 4 - \frac{2}{x^3}}}$$

b)

$$g(x) = x \ln 2x$$

$$g'(x) = 1 \cdot \ln 2x + x \cdot \frac{1}{2x} \cdot 2$$

$$\underline{\underline{g'(x) = \ln 2x + 1}}$$

c)

$$h(x) = \frac{e^{2x}}{\ln x}$$

$$h'(x) = \frac{e^{2x} \cdot 2 \cdot \ln x - e^{2x} \cdot \frac{1}{x}}{(\ln x)^2}$$

$$\underline{\underline{h'(x) = \frac{e^{2x} \cdot 2 \cdot \ln x - \frac{e^{2x}}{x}}{(\ln x)^2}}}$$

$$eller$$

$$h'(x) = \frac{2xe^{2x} \ln x - e^{2x}}{x(\ln x)^2}$$

$$\underline{\underline{h'(x) = \frac{e^{2x}(2x \ln x - 1)}{x(\ln x)^2}}}$$

Oppgave 5

Løs likningssettet ved regning

$$2x^2 - y - 4 = 0$$
$$-x + y = 2 \quad \Rightarrow y = x + 2$$
$$2x^2 - (x+2) - 4 = 0$$
$$2x^2 - x - 2 - 4 = 0$$
$$2x^2 - x - 6 = 0$$
$$x = 2 \;\vee\; x = -\frac{3}{2}$$
$$y = 2 + 2 = 4$$
$$y = -\frac{3}{2} + 2 = \frac{1}{2}$$
$$Løsning: \underline{\underline{(2,4) \vee \left(-\frac{3}{2}, \frac{1}{2}\right)}}$$

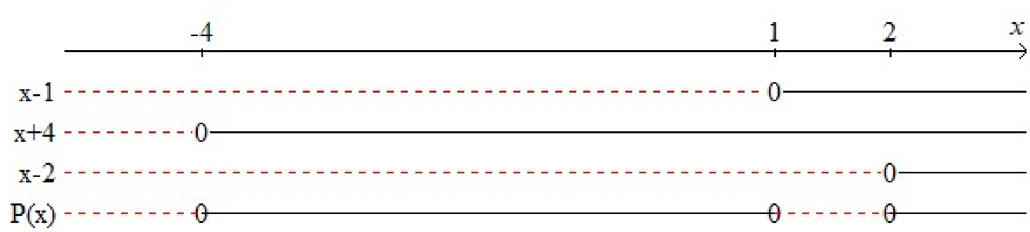
Oppgave 6

Gitt funksjonen $P(x) = x^3 + x^2 - 10x + 8, \quad D_P = \mathbb{R}$

a) Et av nullpunktene er $x = 2$. Faktoriser $P(x)$ til førstegradsfaktorer .

$$(x^3 + x^2 - 10x + 8) : (x - 2) = x^2 + 3x - 4$$
$$\underline{-x^3 + 2x^2}$$
$$3x^2 - 10x$$
$$\underline{-3x^2 + 6x}$$
$$-4x + 8$$
$$\underline{4x - 8}$$
$$0$$
$$x^2 + 3x - 4 = (x-1)(x+4)$$
$$\underline{\underline{P(x) = (x-1)(x+4)(x-2)}}$$

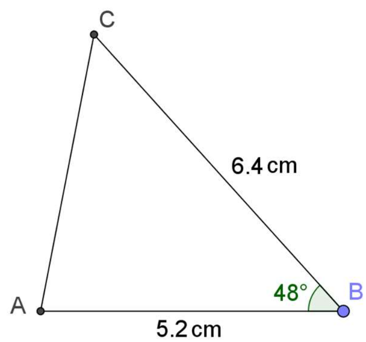
b) Løs ulikheten $P(x) \le 0$ ved regning



Løsning: $\underline{\underline{x \le -4 \;\vee\; 1 \le x \le 2}}$

Oppgave 7

I trekanten ΔABC er det to kjente lengder AB = 5,2 cm og BC = 6,4 cm, samt er $\angle B$=48°.



a) Regn ut arealet til trekanten.

$$A_{ABC} = \frac{1}{2} AB \cdot BC \cdot \sin B$$

$$A_{ABC} = \frac{1}{2} 5,2 \cdot 6,4 \cdot \sin 48 = 12,366$$

$$\underline{\underline{A_{ABC} = 12,4 cm^2}}$$

b) Finn lengden til siden AC ved regning.

$$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos B$$

$$AC^2 = 5,2^2 + 6,4^2 - 2 \cdot 5,2 \cdot 6,4 \cdot \cos 48$$

$$AC = 4,844$$

$$\underline{\underline{AC = 4,8 cm}}$$

c) Regn ut vinklene $\angle A$ og $\angle C$

$$\frac{\sin A}{BC} = \frac{\sin B}{AC}$$

$$\frac{\sin A}{6,4} = \frac{\sin 48}{4,844}$$

$$\sin A = \frac{6,4 \cdot \sin 48}{4,844}$$

$$\underline{\underline{\angle A = 79,1^\circ}}$$

$$\angle C = 180^\circ - 48^\circ - 79,1^\circ$$

$$\underline{\underline{\angle C = 52,9^\circ}}$$

Oppgave 8

Gitt funksjonen $f(x) = e^{x^2-1} - e$

a) Regn ut nullpunktene.

$e^{x^2-1} - e = 0$

$e^{x^2-1} = e$

$x^2 - 1 = 1$

$x^2 = 2$

$\underline{\underline{x = \pm\sqrt{2}}}$

b) Regn ut eventuelle topp- eller bunnpunkter.

$f(x) = e^{x^2-1} - e$

$f'(x) = e^{x^2-1} \cdot 2x$

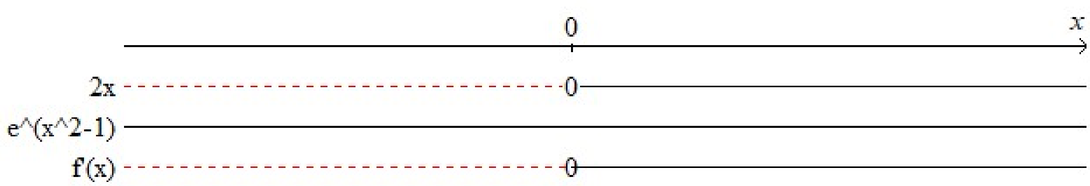
$f'(x) = 2xe^{x^2-1}$

$2xe^{x^2-1} = 0$

$2x = 0$

$\underline{x = 0}$

$e^{x^2-1} \neq 0$



$f(0) = e^{0^2-1} - e = e^{-1} - e = \frac{1}{e} - e = \underline{\frac{1-e^2}{e}}$

$Bunnpunkt: \underline{\underline{\left(0, \frac{1-e^2}{e}\right) = (0, -2.35)}}$

c) Finn likningen til tangenten i $(1, f(1))$.

$x_1 = 1$

$y_1 = f(1) = e^{1^2-1} - e = 1 - e$

$a = f'(1) = 2 \cdot 1 \cdot e^{1^2-1} = 2$

$y - y_1 = a(x - x_1)$

$y - (1 - e) = 2(x - 1)$

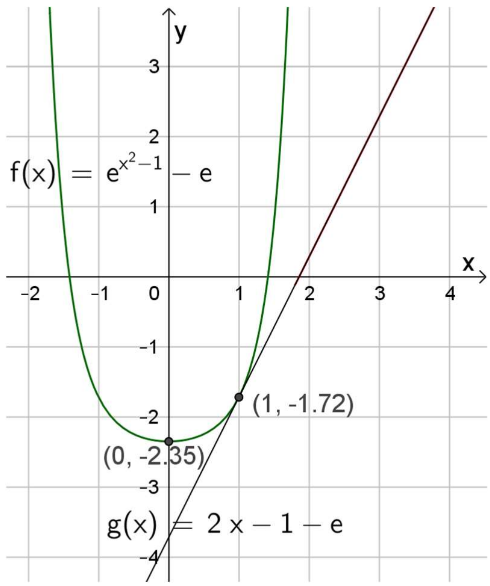
$y - 1 + e = 2x - 2$

$y = 2x - 2 + 1 - e$

$y = 2x - 1 - e$

$\underline{\underline{y = 2x - 3,72}}$

d) Tegn grafen og tangenten.



Oppgave 9

Gitt følgende uttrykk: $f(x) = \dfrac{x^2-4}{x^2-5x+4}$

a) Finn de eventuelle nullpunktene til f(x).

$$\frac{x^2-4}{x^2-5x+4} = 0 \quad | \cdot x^2-5x+4$$

$$x^2-4=0$$

$$\underline{\underline{x=\pm 2}}$$

b) Finn de eventuelle asymptotene til f(x).

Vertikale asymptoter:

$$x^2-5x+4=0$$

$$equa-2.grad$$

$$\underline{x=4 \ \vee \ x=1}$$

sjekker teller:

$$4^2-4=16-4=12 \neq 0$$

$$1^2-4=-3 \neq 0$$

Vertikale asymptoter:

$$\underline{\underline{x=4 \ \vee \ x=1}}$$

Horisontal asymptote:

$$(x^2-4):(x^2-5x+4)=1+\frac{5x-8}{x^2-5x+4}$$
$$\frac{-x^2+5x-4}{\quad 5x-8}$$

$$\lim_{x\to\pm\infty}\frac{5x-8}{x^2-5x+4}=0$$

Horisontal asymptote: $\underline{\underline{y=1}}$

Oppgave 10

Gitt punktene $A(1,3),\ B(5,-1)\ og\ C(4,4)$

a) Regn ut $\overrightarrow{BA}\ og\ |\overrightarrow{BA}|$.

$$\overrightarrow{BA}=\left[1-5,3-(-1)\right]=\underline{\underline{\left[-4,4\right]}}$$
$$|\overrightarrow{BA}|=\sqrt{(-4)^2+4^2}=\sqrt{16+16}=\sqrt{32}=\underline{\underline{4\sqrt{2}}}$$

b) Regn ut $\angle ABC$.

$$\cos ABC=\frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}|\cdot|\overrightarrow{BC}|}=\frac{\left[-4,4\right]\cdot\left[-1,5\right]}{4\sqrt{2}\cdot\sqrt{(-1)^2+5^2}}=\frac{(-4)\cdot(-1)+4\cdot 5}{4\sqrt{2}\cdot\sqrt{26}}=\frac{24}{4\sqrt{2}\cdot\sqrt{26}}$$
$$\underline{\underline{\angle ABC=33,7^\circ}}$$

c) Bestem et punkt D på y-aksen slik at $\overrightarrow{CD}\parallel\overrightarrow{BA}$.

$D(0,y)$

$\overrightarrow{CD}\parallel\overrightarrow{BA}$

$\overrightarrow{CD}=t\cdot\overrightarrow{BA}$

$\left[-4,y-4\right]=t\left[-4,4\right]$

$-4=-4t \qquad y-4=4t$

$t=1 \qquad y-4=4\cdot 1$

$\qquad y=4+4=8$

$\underline{\underline{D(0,8)}}$

d) La M være midtpunktet på BC. Bestem ved regning koordinatene til M.

$$\overrightarrow{OM} = \overrightarrow{OB} + \frac{1}{2}\overrightarrow{BC}$$

$$\overrightarrow{OM} = [5,-1] + \frac{1}{2}[-1,5]$$

$$\overrightarrow{OM} = \left[5 - \frac{1}{2}, -1 + \frac{5}{2}\right]$$

$$\overrightarrow{OM} = \left[\frac{9}{2}, \frac{3}{2}\right]$$

$$\underline{\underline{M\left(\frac{9}{2}, \frac{3}{2}\right)}}$$