LØSNINGSFORSLAG – ORDINÆR EKSAMEN, MATEMATIKK 3-TERMIN, VÅREN 2019

OPPGAVE 1

a) $\dfrac{x^2 \cdot \cancel{x^{-1}}}{x^3} \cdot \cancel{x} = \dfrac{x^2}{x^3} = x^{2-3} = \underline{\underline{x^{-1}}} \quad \left(= \dfrac{1}{x}\right)$

b) $\dfrac{\ln(x^a) \cdot \ln(e^2) \cdot \ln(e^{1/a})}{\ln(x)} = \dfrac{\cancel{a}\,\cancel{\ln(x)} \cdot 2\,\overbrace{\ln(e)}^{=1} \cdot \frac{1}{\cancel{a}}\,\overbrace{\ln(e)}^{=1}}{\cancel{\ln(x)}}$

$$\underline{\underline{= 2}}$$

OPPGAVE 2

a) $f(x) = \ln(x^2) - e^2$

$f'(x) = \dfrac{1}{x^2} \cdot 2x - 0 = \underline{\underline{\dfrac{2}{x}}}$

b) $g(x) = x \cdot \sin(2x+1)$

$g'(x) = 1 \cdot \sin(2x+1) + x \cdot \cos(2x+1) \cdot 2$

$\underline{\underline{= \sin(2x+1) + 2x\cos(2x+1)}}$

OPPGAVE 3

a) $\displaystyle\int 4x \cdot e^{2x}\,dx \overset{\text{DELVIS INTEGRASJON}}{=} 4x \cdot \frac{1}{2}e^{2x} - \int 4 \cdot \frac{1}{2}e^{2x}\,dx$

$= 2xe^{2x} - 2 \cdot \frac{1}{2}e^{2x} + C$

$\underline{\underline{= e^{2x}(2x-1) + C}}$

b) $\displaystyle\int 4x \cdot e^{2x^2}\,dx \qquad u = 2x^2 \qquad \frac{du}{dx} = 4x \qquad du = 4x\,dx$

$= \displaystyle\int e^u\,du = e^u + C = \underline{\underline{e^{2x^2} + C}}$

OPPGAVE 4

a) $15 \cdot 1{,}2^t = 30$

$$1{,}2^t = 2$$

$$\ln(1{,}2^t) = \ln(2)$$

$$t \ln(1{,}2) = \ln(2)$$

$$t = \frac{\ln(2)}{\ln(1{,}2)} \qquad (\approx 3{,}80)$$

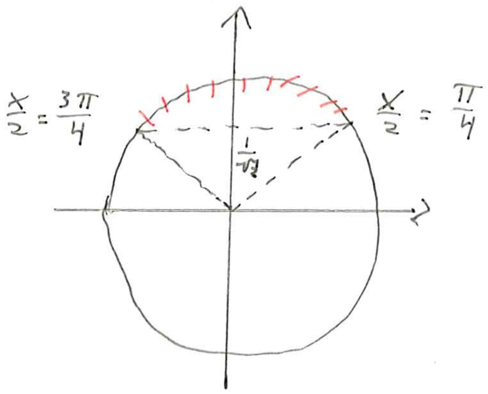
b) $\sin\left(\frac{x}{2}\right) - \frac{1}{\sqrt{2}} > 0$

$$\sin\left(\frac{x}{2}\right) > \frac{1}{\sqrt{2}}$$

LØSER LIKNING $\sin\left(\frac{x}{2}\right) = \frac{1}{\sqrt{2}}$, $\frac{x}{2} \in [0, \pi\rangle$

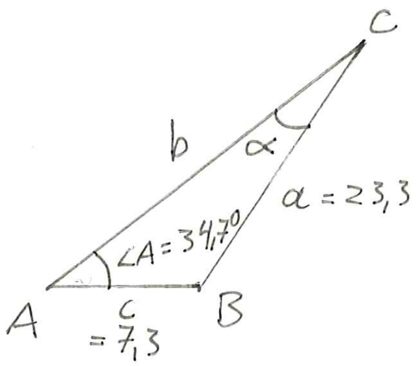
$$\frac{x}{2} = \frac{\pi}{4} + n \cdot 2\pi \quad \vee \quad \frac{x}{2} = \frac{3\pi}{4} + n \cdot 2\pi$$

$$\frac{x}{2} = \frac{\pi}{4} \qquad \frac{x}{2} = \frac{3\pi}{4}$$



$$x \in \left\langle \frac{\pi}{2}, \frac{3\pi}{2} \right\rangle$$

OPPGAVE 5



Sinussetningen:

$$\frac{\sin \angle A}{a} = \frac{\sin \alpha}{c}$$

$$\sin \alpha = \frac{c}{a} \sin \angle A = \frac{7{,}3}{23{,}3} \cdot \sin(34{,}7^\circ)$$

$$= 0{,}17835\ldots$$

$$\Rightarrow \alpha = 10{,}27^\circ \approx 10^\circ$$

OPPGAVE 6

a) i. $f'(a) = 0$

ii. $f'(b) = 0$

iii. $f''(a) > 0$

iv. $f''(b) < 0$

b) $\int_0^b f(x)\,dx < 0$ fordi arealet under x-aksen er større enn arealet over x-aksen i integrasjonsintervallet.

OPPGAVE 7

$$\frac{dy}{dx} - 0{,}2y = 1{,}2$$

$$dy - 0{,}2y\,dx = 1{,}2\,dx$$

$$dy = (0{,}2y + 1{,}2)\,dx$$

$$\int \frac{dy}{0{,}2y+1{,}2} = \int dx$$

$$\frac{1}{0{,}2} \ln|0{,}2y + 1{,}2| = x + C'$$

$$|0{,}2y + 1{,}2| = e^{0{,}2x + C''}$$

$$0{,}2y = C'''e^{0{,}2x} - 1{,}2$$

$$\underline{\underline{y = Ce^{0{,}2x} - 6}}$$

OPPGAVE 8

a) $\vec{AB} = [2-0,\ 3-1,\ 3-2] = [2, 2, 1]$

$\vec{AC} = [2-0,\ 2-1,\ 0-2] = [2, 1, -2]$

$|\vec{AB}| = \sqrt{2^2+2^2+1^2} = 3$ $\quad |\vec{AC}| = \sqrt{2^2+1^2+(-2)^2} = 3$

$\vec{AB} \cdot \vec{AC} = [2,2,1] \cdot [2,1,-2] = 2\cdot 2 + 2\cdot 1 - 2\cdot 1 = 4$

$$\vec{AB} \cdot \vec{AC} = |\vec{AB}| \cdot |\vec{AC}| \cdot \cos(\angle BAC)$$

$$\cos(\angle BAC) = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|} = \frac{4}{3 \cdot 3} = \frac{4}{9}$$

$$\angle BAC = \cos^{-1}\left(\frac{4}{9}\right) = 63{,}6122^\circ \approx \underline{\underline{63{,}6^\circ}}$$

(Kan også løses f.eks. ved å finne lengden til alle sidene og bruke cosinussetningen)

b) Kan løses ved f.eks. å bruke vektorprodukt for å finne en normalvektor til planet og sette inn i likn. s. 572 i Sinus.
Eller:
Sjekker om alle tre punkter tilfredsstiller likning:

A: $-5 \cdot 0 + 6 \cdot 1 - 2 \cdot 2 - 2 = 0$ $\quad \underline{\underline{\text{OK}}}$

B: $-5 \cdot 2 + 6 \cdot 3 - 2 \cdot 3 - 2 = 0$ $\quad \underline{\underline{\text{OK}}}$

C: $-5 \cdot 2 + 6 \cdot 2 - 2 \cdot 0 - 2 = 0$ $\quad \underline{\underline{\text{OK}}}$

c) $-5 \cdot (-2+t) + 6 \cdot (1+2t) - 2 \cdot 0 - 2 = 0$

$10 - 5t + 6 + 12t - 2 = 0$

$7t + 14 = 0$

$\underline{t = -2}$

$x = -2 + t = -2 - 2 = -4$

$y = 1 + 2t = 1 - 2 \cdot 2 = -3$

$z = 0$

<u>SKJÆRINGSPUNKT: $(-4, -3, 0)$</u>