UNIVERSITETET I AGDER

FAKULTET FOR REALFAG OG TEKNOLOGI

PÅSKETENTAMEN 2014

LØSNINGSFORSLAG

Oppgave 1

$$\frac{x^4y^3-x^2y}{x^2y^2-xy}=\frac{\cancel{x^2}\,\cancel{y}\,(x^2y^2-1)}{\cancel{x}\,\cancel{y}\,(xy-1)}=\frac{x(x^2y^2-1)}{(xy-1)}=\frac{x\,\cancel{(xy-1)}\,(xy+1)}{\cancel{(xy-1)}}=\underline{\underline{x(xy+1)}}$$

Oppgave 2

a) $\sqrt{x^2+5}-x=5\Rightarrow\sqrt{x^2+5}=5+x\Rightarrow\left(\sqrt{x^2+5}\right)^2=(x+5)^2$

$x^2+5=x^2+10x+25\Rightarrow10x=-20\Rightarrow\underline{\underline{x=-2}}$

Setter prøve og finner at $\underline{\underline{\text{VS = HS}}}$

b) $\ln x-\ln(3-x)=\ln2\Rightarrow\ln\left(\frac{x}{3-x}\right)=\ln2\Rightarrow\frac{x}{3-x}=2$

$x=2\cdot(3-x)\Rightarrow x=6-2x\Rightarrow3x=6\Rightarrow\underline{\underline{x=2}}$

c) $\cos x-4\sin x=0\Rightarrow\frac{\cos x}{\cos x}-\frac{4\sin x}{\cos x}=\frac{0}{\cos x}\Rightarrow1-4\tan x=0$

$4\tan x=1\Rightarrow\tan x=\frac{1}{4}$

$\Rightarrow x_1=\tan^{-1}\left(\frac{1}{4}\right)=\underline{\underline{14^\circ}}$

$x_2=14^\circ+180^\circ=\underline{\underline{194^\circ}}$

d) $3e^x + 5e^{-x} = 8 \Rightarrow 3e^x + \dfrac{5}{e^x} = 8 \Rightarrow 3e^x \cdot e^x + \dfrac{5 \cdot \cancel{e^x}}{\cancel{e^x}} = 8 \cdot e^x$

$3(e^x)^2 - 8e^x + 5 = 0$ 2. gradslikning med e^x som ukjent

$e^x = 1 \vee e^x = \dfrac{5}{3} \Rightarrow \ln e^x = \ln\dfrac{5}{3} \Rightarrow x = \ln\dfrac{5}{3}$

$\underline{\underline{x = 0 \vee x = \ln\dfrac{5}{3}}}$

Oppgave 3

a) $f'(x) = (x^2 \ln x)' = (x^2)' \cdot \ln x + x^2 \cdot (\ln x)' = 2x \ln x + x^2 \cdot \dfrac{1}{x} = \underline{\underline{2x\ln x + x}}$

b) $f'(x) = (e^{\cos x})'$ Velger $u = \cos x \Rightarrow f(x) = g(u) = e^u$

$f'(x) = u' \cdot g'(u) = -\sin x \cdot e^u = \underline{\underline{-\sin x \cdot e^{\cos x}}}$

c) $f'(x) = \left(\dfrac{3x}{x+2}\right)' = \dfrac{(3x)' \cdot (x+2) - 3x \cdot (x+2)'}{(x+2)^2} = \dfrac{3 \cdot (x+2) - 3x \cdot 1}{(x+2)^2}$

$f'(x) = \underline{\underline{\dfrac{6}{(x+2)^2}}}$

Oppgave 4

a) $\displaystyle\int_0^{\pi/3} \cos x dx = [\sin x]_0^{\pi/3} = \sin(\frac{\pi}{3}) - \sin 0 = \underline{\underline{\frac{1}{2}\sqrt{3}}}$

b) $\int x \ln x dx$ Velger $v = \ln x$ og $u' = x \Rightarrow v' = \dfrac{1}{x}$ og $u = \dfrac{1}{2}x^2$

Delvis integrasjon gir:

$\displaystyle\int x \ln x dx = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^{\cancel{2}} \cdot \frac{1}{\cancel{x}} dx = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2} x dx = \underline{\underline{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}}$

c) $\int x\sqrt{2x^2-2}dx \Rightarrow \int (\sqrt{2x^2-2})\,xdx$

Variabelskifte:

Setter $u = 2x^2 - 2 \Rightarrow u' = \frac{du}{dx} = 4x \Rightarrow du = 4xdx \Rightarrow \frac{du}{4} = xdx$

$$\int \sqrt{u}\,\frac{du}{4} = \frac{1}{4}\int \sqrt{u}du = \frac{1}{4}\int u^{\frac{1}{2}}du = \frac{1}{4}\cdot\frac{1}{\frac{1}{2}+1}u^{\frac{1}{2}+1} + C = \frac{1}{6}u^{\frac{3}{2}} + C = \underline{\underline{\frac{1}{6}\sqrt{(2x^2-2)^3} + C}}$$

Oppgave 5

a) $f(x) = 4(\ln x)^2 - 4\ln x = 0 \Rightarrow 4\ln x(\ln x - 1) = 0 \Rightarrow 4\ln x = 0 \;\vee\; \ln x - 1 = 0$

$\ln x = 0 \;\Rightarrow\; \underline{\underline{x = 1}}$

$\ln x = 1 \Rightarrow e^{\ln x} = e^1 \Rightarrow \underline{\underline{x = e}}$

$f'(x) = 4\cdot 2\ln x\cdot\frac{1}{x} - 4\cdot\frac{1}{x}$

$f'(x) = \frac{8\ln x}{x} - \frac{4}{x} = \underline{\underline{\frac{4}{x}(2\ln x - 1)}}$

b) $f'(x) = 0 \Rightarrow \frac{4}{x}(2\ln x - 1) = 0 \Rightarrow 2\ln x - 1 = 0 \Rightarrow \ln x = \frac{1}{2} \Rightarrow \underline{x = e^{\frac{1}{2}}}$

$f(e^{\frac{1}{2}}) = 4(\ln e^{\frac{1}{2}})^2 - 4\ln(e^{\frac{1}{2}}) = 4\cdot\left(\frac{1}{2}\right)^2 - 4\cdot\frac{1}{2} = 1 - 2 = -1$

$\underline{\underline{\text{Bunnpunkt: } (e^{\frac{1}{2}}, -1)}}$

c) Stigningstall: $a = f'(1) = \frac{4}{1}(2\ln 1 - 1) = -4$

Likning: $y - f(1) = a(x - x_1)$ der $f(1) = 4(\ln 1)^2 - 4\ln 1 = 0$

$y - 0 = -4(x-1) = \underline{\underline{-4x + 4}}$

d) f har en asymptote fordi når $x \to 0$ går $\ln x \to -\infty$

Siden leddet $4(\ln x)^2$ er i kvadrat vil dette vinne over $4(\ln x)$
og grafen går mot positivt uendelig når $x \to 0$

Oppgave 6

a) $\overrightarrow{AB} = [4-1, 0-0, 0-0] = \underline{\underline{[3,0,0]}}$ $\qquad \overrightarrow{BC} = [2-4, 1-0, 4-0] = \underline{\underline{[-2,1,4]}}$

b) Setter $D = (x, y, z) \Rightarrow \overrightarrow{AD} = \overrightarrow{BC} \Rightarrow [x-1, y, z] = [-2,1,4]$

$x - 1 = -2 \wedge y = 1 \wedge z = 4$

$x = -1 \wedge y = 1 \wedge z = 4 \Rightarrow \underline{\underline{D = (-1,1,4)}}$

c) $\cos v = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{AD}}{\left|\overrightarrow{AB}\right| \cdot \left|\overrightarrow{AD}\right|} = \dfrac{[3,0,0] \cdot [-2,1,4]}{\sqrt{3^2 + 0^2 + 0^2} \cdot \sqrt{(-2)^2 + 1^2 + 4^2}} = \dfrac{-6}{3 \cdot \sqrt{21}} = -0,436$

$v = \cos^{-1}(-0,436) = \underline{\underline{116°}}$

d) $A = \left|\overrightarrow{AB}\right| \cdot \left|\overrightarrow{AD}\right| \cdot \sin v = 3 \cdot \sqrt{21} \cdot \sin 116° = \underline{\underline{12,4}}$

e) $V = \dfrac{1}{3}\left|(\overrightarrow{AB} \times \overrightarrow{AD}) \cdot \overrightarrow{AT}\right|$ $\quad$ der $\overrightarrow{AT} = [5-1, 5-0, 5-0] = [4,5,5]$

$$V = \frac{1}{3}\left|\left[\begin{vmatrix} 0 & 0 \\ 1 & 4 \end{vmatrix}, -\begin{vmatrix} 3 & 0 \\ -2 & 4 \end{vmatrix}, \begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix}\right] \cdot [4,\ 5,\ 5]\right| = \frac{1}{3}\left|[0,\ -12,\ 3] \cdot [4,\ 5,\ 5]\right|$$

$$V = \frac{1}{3}\left|\left(0 \cdot 4 + (-12) \cdot 5 + 3 \cdot 5\right)\right| = \frac{1}{3}\left|(-60 + 15)\right| = \frac{|-45|}{3}$$

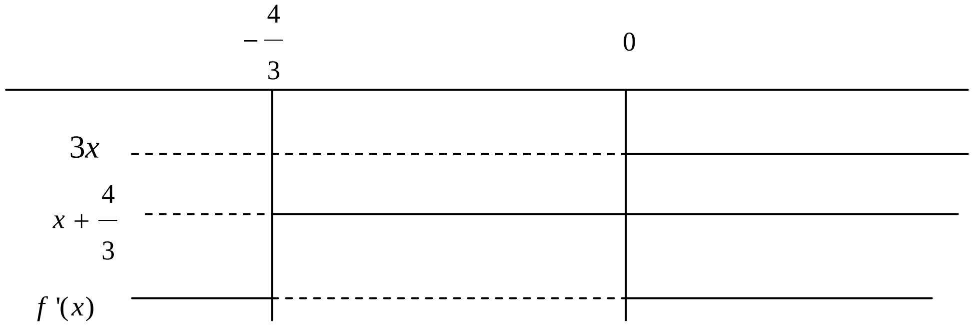
$\underline{\underline{V = 15}}$

Oppgave 7

a) $f(1) = 1^3 + 2 \cdot 1^2 - 3 = 3 - 3 = \underline{\underline{0}}$

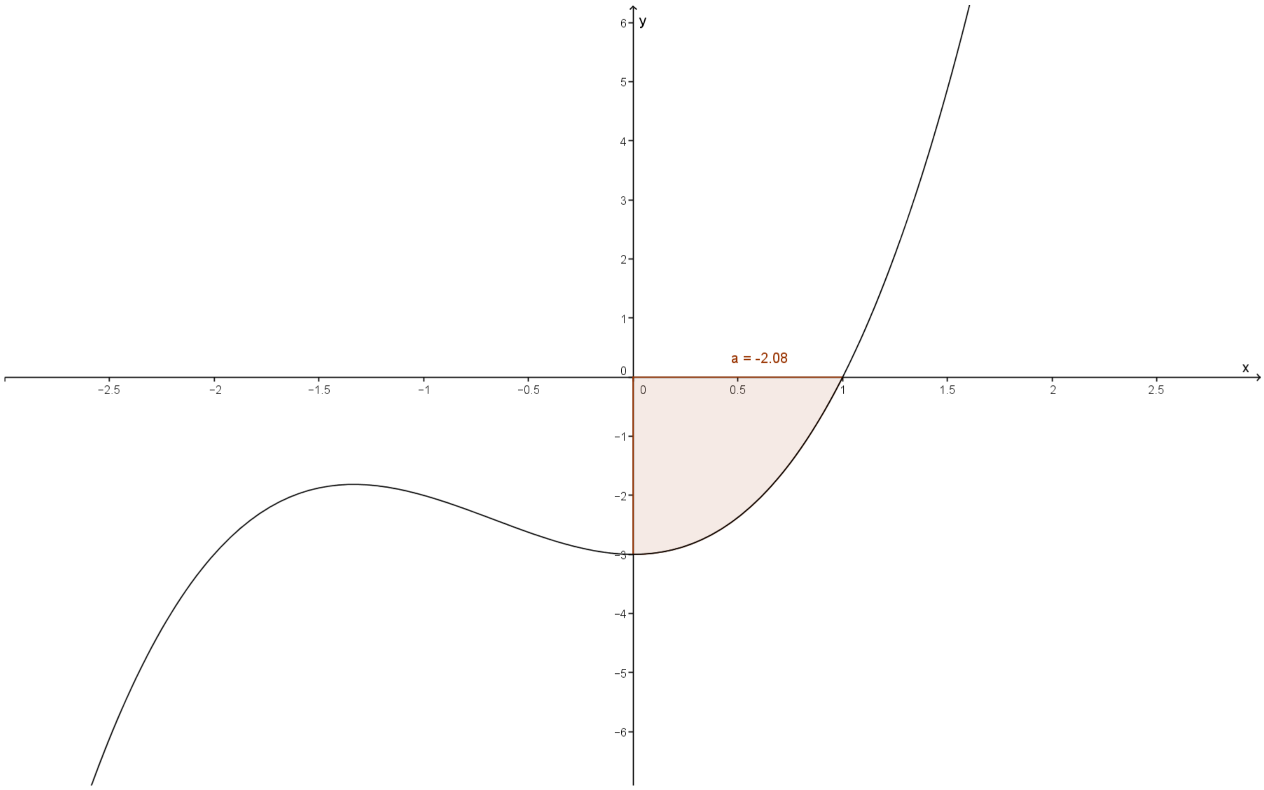
b) $f'(x) = 3x^2 + 4x \Rightarrow f'(x) = 0 \Rightarrow 3x^2 + 4x = 0$

$f'(x) = 0$ for $x = 0$ eller $x = -\dfrac{4}{3} \Rightarrow f'(x) = 3x(x + \dfrac{4}{3})$



Grafen stiger når $x \in \langle \leftarrow, -\dfrac{4}{3}\rangle$ og når $x \in \langle 0, \rightarrow\rangle$ $\qquad$ Grafen synker når $x \in \langle -\dfrac{4}{3}, 0\rangle$

c)



d) Nedre integralgrense er $x = 0$ og øvre grense er $x = 1$. Dermed

$$A = \left| \int_0^1 (x^3 + 2x^2 - 3)dx \right| = \left| \left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - 3x \right]_0^1 \right| = \left| \frac{1}{4}1^4 + \frac{2}{3}1^3 - 3 \cdot 1 \right| = \left| -\frac{25}{12} \right| = \underline{\underline{\frac{25}{12}}}$$