**Løsningsforslag til rep oppgaver i integrasjon:**

**Oppgave 1 Bestem integralene**

a.

$$\int\left(3\sqrt{x}+2x^4-\frac{1}{x}\right)dx=\int\left(3x^{\frac{1}{2}}+2x^4-\frac{1}{x}\right)dx$$

$$=3\cdot\frac{2}{3}x^{\frac{3}{2}}+2\frac{1}{5}x^5-\ln|x|+C=\underline{\underline{2x\sqrt{x}+\frac{2}{5}x^5-\ln|x|+C}}$$

b.

$\underline{Alt\ Variabelskifte}$

$$\int\frac{x}{x^2-4}dx \Rightarrow \text{ Setter } u=x^2-4, \text{ som gir } du=2xdx$$

$$\frac{1}{2}du=xdx$$

$$\int\frac{x}{x^2-4}dx=\frac{1}{2}\int\frac{1}{u}du=\frac{1}{2}\ln|u|+C=\ln\sqrt{u}+C=\underline{\underline{\ln\sqrt{x^2-4}+C}}$$

$\underline{Alt\ Delbrøkoppspalting}$

$$\frac{x}{x^2-4}=\frac{A}{x-2}+\frac{B}{x+2}\quad |\cdot(x-2)(x+2)$$

$$x=A(x+2)+B(x-2)$$

$$x=2 \text{ gir}: 2=A\cdot4+B\cdot0 \Rightarrow \underline{A=\frac{1}{2}}$$

$$x=-2 \text{ gir}: -2=A\cdot0+B\cdot(-4) \Rightarrow \underline{B=\frac{1}{2}}$$

$$\int\frac{x}{x^2-4}dx=\frac{1}{2}\int\frac{1}{x-2}dx+\frac{1}{2}\int\frac{1}{x+2}dx$$

$$=\frac{1}{2}\left(\ln|x-2|+\ln|x+2|\right)+C=\frac{1}{2}\ln\left|x^2-4\right|+C=\underline{\underline{\ln\sqrt{x^2-4}+C}}$$

c.

$$\int_1^4\frac{x^2-4}{x}dx=\int_1^4\left(x-\frac{4}{x}\right)dx=\int_1^4 xdx-4\int_1^4\frac{1}{x}dx$$

$$=\left[\frac{1}{2}x^2-4\ln x\right]_1^4=\frac{1}{2}\cdot16-4\ln4-\left(\frac{1}{2}\cdot1-4\ln1\right)$$

$$\underline{\underline{\int_1^4\frac{x^2-4}{x}dx=\frac{15}{2}-8\ln2\approx1{,}95}}$$

d.

$$\int_0^{\ln 3} (e^{2x} - e^{-x})\,dx = \left[\frac{1}{2}e^{2x} + e^{-x}\right]_0^{\ln 3}$$

$$= \frac{1}{2}e^{2\ln 3} + e^{-\ln 3} - (\frac{1}{2}e^{0} + e^{0})$$

$$= \frac{1}{2}e^{\ln 3^2} + e^{\ln 3^{-1}} - \left(\frac{1}{2}+1\right)$$

$$= \frac{1}{2}\cdot 9 + 3^{-1} - \frac{3}{2}$$

$$= \frac{6}{2} + \frac{1}{3} = \frac{6\cdot 3 + 1\cdot 2}{6} = \frac{20}{6} = \underline{\underline{\frac{10}{3}}}$$

e.

$$y' = -\frac{1}{x}(y-2) \qquad \text{separerer variablene, og integrerer}$$

$$\int \frac{1}{y-2}\,dy = \int \frac{-1}{x}\,dx$$

$$\ln|y-2| = -\ln|x| + C_1$$

$$e^{\ln|y-2|} = e^{\ln|x|^{-1} + C_1} = e^{\ln|x|^{-1}} \cdot e^{C_1}$$

$$|y-2| = \frac{1}{|x|}\cdot e^{C_1}$$

$$y - 2 = \frac{C}{x} \qquad \underline{\underline{y = \frac{C}{x} + 2}}$$

f.

$$(x+1)\,y' = 2y$$

$$\frac{dy}{dx} = \frac{2y}{x+1}$$

$$\int \frac{1}{y}\,dy = \int \frac{2}{x+1}\,dx$$

$$\ln|y| = 2\ln|x+1| + C_1 = \ln|x+1|^2 + C_1$$

$$e^{\ln|y|} = e^{\ln|x+1|^2 + C_1}$$

$$|y| = C_2\,(x+1)^2$$

$$\underline{\underline{y = C\,(x+1)^2}}$$

**Oppgave 2 Bestem integralene**

a. $\int \frac{1-x^4-x^6}{x^2}\,dx = \int \frac{1}{x^2} - x^2 - x^4\,dx = \underline{\underline{-\frac{1}{x} - \frac{1}{3}x^3 - \frac{1}{5}x^5 + C}}$

b.

$$\int_0^{\frac{\pi}{3}} \sin x \cdot \cos^2 x \, dx \qquad\qquad u = \cos x$$

$$du = -\sin x \, dx$$

$$-du = \sin x \, dx$$

$$= \int_1^{\frac{1}{2}} -u^2 \, du = -\frac{1}{3}\left[u^3\right]_1^{\frac{1}{2}} = -\frac{1}{3}\left(\frac{1}{8}-1\right) = -\frac{1}{3}\left(-\frac{7}{8}\right) = \underline{\underline{\frac{7}{24}}}$$

c. $\int \frac{x-2}{x^2+x} dx$

$$\frac{x-2}{x^2+x} = \frac{x-2}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}.$$

Ganger vi så med $x(x+1)$, får vi

$$x - 2 = A(x+1) + Bx.$$

Ved å sette $x = 0$ får vi $-2 = A$, og $x = -1$ gir $-3 = -B$, dvs. $B = 3$. Altså er

$$\int \frac{x-2}{x^2+x} dx = \int \left(\frac{3}{x+1} - \frac{2}{x}\right) dx = \underline{\underline{3\ln|x+1| - 2\ln|x| + C = \ln(|x+1|^3) - \ln(x^2) + C.}}$$

d.

$$f(x) = 2x\sqrt{x} = r \qquad dV = \pi r^2 dx$$

$$V = \pi \int_0^1 \left(2x\sqrt{x}\right)^2 dx = \pi \int_0^1 4x^3 dx = \pi\left[x^4\right]_0^1 = \underline{\underline{\pi}}$$

Oppgave 2 e  Bestem arealet av flaten avgrenset av $f(x) = x^2 e^{-x}$ , $x$-aksen og linjen $x$=2.

Løsning:

Sjekker først grafen og ser at arealet ligger over x- aksen

$$A = \int_0^2 x^2 e^{-x}\, dx =$$

Løser først det ubestemte integralet ved ubestemt integrasjon.

$$\int \overset{v}{x^2}\, \overset{u'}{e^{-x}}\, dx \qquad\qquad v = x^2 \;, \qquad v' = 2x$$

$$u' = e^{-x} \;, \qquad u = -e^{-x}$$

$$= \overbrace{-x^2 e^{-x}}^{uv} - \int \overbrace{-2x \cdot e^{-x}}^{uv'} dx$$

$$= -x^2 e^{-x} + \int 2x \cdot e^{-x} dx$$ ny runde m formelen

$$= -x^2 e^{-x} - 2xe^{-x} + \int 2e^{-x} \qquad\qquad v = 2x \;, \qquad v' = 2$$

$$= \underline{-x^2 e^{-x} - 2xe^{-x} + 2e^{-x} + C}$$

$$u' = e^{-x} \;, \qquad u = -e^{-x}$$

$$A = \int_0^2 x^2 e^{-x}\, dx = \left[ -e^{-x} \left( x^2 + 2x + 2 \right) \right]_0^2$$

$$= -e^{-2} \left( 4 + 4 + 2 \right) - \left( -e^0 \cdot 2 \right)$$

$$= -\frac{10}{e^2} + 2 = \underline{\underline{2 - \frac{10}{e^2}}} \approx 0,65$$