Løsningsforslag tentamen vår 2019 MA-015

Oppgave 1

a)

$$\frac{(a^2b)^{-1}\cdot\sqrt[3]{b^4a}}{(ab)^{-\frac{2}{3}}}=\frac{a^{-2}b^{-1}b^{\frac{4}{3}}a^{\frac{1}{3}}}{a^{-\frac{2}{3}}b^{-\frac{2}{3}}}=a^{-2+\frac{1}{3}+\frac{2}{3}}b^{-1+\frac{4}{3}+\frac{2}{3}}=a^{-1}b^{1}=\frac{b}{\underline{\underline{a}}}$$

b)

$$\sin x+\sqrt{3}\cos x=0 \qquad\qquad x\in\left[0,2\pi\right\rangle$$

$$\frac{\sin x}{\cos x}+\frac{\sqrt{3}\cos x}{\cos x}=\frac{0}{\cos x}$$

$$\tan x+\sqrt{3}=0$$

$$\tan x=-\sqrt{3}$$

$$x=\frac{2\pi}{3}+n$$

$$\underline{\underline{x=\frac{2\pi}{3}\ \vee\ x=\frac{5\pi}{3}}}$$

c)

$$3e^{4x}-2e^{x}=0$$

$$e^{x}(3e^{3x}-2)=0$$

$$e^{x}\neq 0$$

$$3e^{3x}-2=0$$

$$e^{3x}=\frac{2}{3}$$

$$\ln e^{3x}=\ln\frac{2}{3}$$

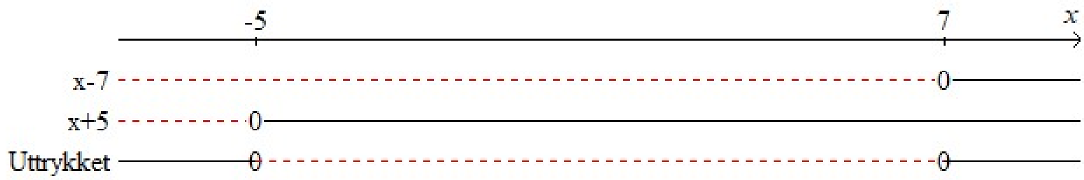
$$3x=\ln 2-\ln 3$$

$$\underline{\underline{x=\frac{\ln 2-\ln 3}{3}}}$$

d)

$$x^2-2x-35\leq 0$$

$$(x-7)(x+5)\leq 0$$



Løsning: $\underline{\underline{-5 \le x \le 7}}$

e)

$$f(x) = 2\cos(x^2)$$
$$f'(x) = 2(-\sin(x^2)) \cdot 2x$$
$$\underline{\underline{f'(x) = -4x\sin(x^2)}}$$

f)

$$g(x) = x^{\frac{3}{2}} - 2xe^{-x}$$
$$g'(x) = \frac{3}{2}x^{\frac{1}{2}} - (2e^{-x} + 2xe^{-x} \cdot (-1))$$
$$g'(x) = \frac{3}{2}x^{\frac{1}{2}} - 2e^{-x} + 2xe^{-x}$$
$$\underline{\underline{g'(x) = \frac{3}{2}x^{\frac{1}{2}} - 2e^{-x}(1-x)}}$$

g)

$$\int\left(-\frac{\sin x}{2}\right)dx = \int -\frac{1}{2}\sin x \; dx = \underline{\underline{\frac{1}{2}\cos x + C}}$$

h)

$$\int_0^2 \frac{4x-2}{x^2-x+2}dx$$
$$u = x^2 - x + 2$$
$$du = (2x-1)dx$$
$$2du = (4x-2)dx$$
$$\int \frac{4x-2}{x^2-x+2}dx = \int \frac{2}{u}du = 2\ln|u| + C = 2\ln|x^2-x+2| + C$$
$$\int_0^2 \frac{4x-2}{x^2-x+2}dx = \left[2\ln|x^2-x+2|\right]_0^2$$
$$\left(2\ln|2^2-2+2|\right) - \left(2\ln|0^2-0+2|\right) = 2\ln 4 - 2\ln 2 = 4\ln 2 - 2\ln 2 = \underline{\underline{2\ln 2}}$$

i)

$$y' - 4y = 2$$
$$y' = 4y + 2$$
$$y' = 2(2y+1)$$
$$\frac{1}{2y+1}\frac{dy}{dx} = 2$$
$$\int \frac{1}{2y+1}dy = \int 2dx$$
$$\frac{1}{2}\ln|2y+1| = 2x + C_1$$
$$\ln|2y+1| = 4x + C_2$$
$$e^{\ln|2y+1|} = e^{4x+C_2}$$
$$2y+1 = \pm e^{C_2} \cdot e^{4x}$$
$$2y = C_3 \cdot e^{4x} - 1$$
$$\underline{y = C \cdot e^{4x} - \frac{1}{2}}$$

$$y = C \cdot e^{4x} - \frac{1}{2}$$
$$1 = C \cdot e^{4\cdot 0} - \frac{1}{2}$$
$$2 = 2C - 1$$
$$\underline{C = \frac{3}{2}}$$

$$\underline{\underline{y = \frac{3}{2} \cdot e^{4x} - \frac{1}{2}}}$$

Oppgave 2

a)

$$f(x) = \frac{x^2}{2x-1}$$

$Skjæring\ \ med\ \ y-aksen$

$$f(0)=\frac{0^2}{2\cdot 0-1}=0$$

$$\underline{\underline{y=0}}$$

$Skjæring\ \ med\ \ x-aksen$

$$\frac{x^2}{2x-1}=0$$

$$x^2=0$$

$$\underline{\underline{x=0}}$$

b)

$Vertikal\ \ asymptote:$

$$2x-1=0$$

$$2x=1$$

$$\underline{\underline{x=\frac{1}{2}}} \qquad\qquad \text{Sjekker teller}:\left(\frac{1}{2}\right)^2=\frac{1}{4}\neq 0$$

Det er ingen horisontal asymptote men en skrå asymptote fordi graden av x er størst i teller.

$$x^2:(2x-1)=\frac{1}{2}x+\frac{1}{4}+\frac{\frac{1}{4}}{2x-1}=\underline{\frac{1}{2}x+\frac{1}{4}+\frac{1}{8x-4}}$$

$$\underline{-x^2+\frac{1}{2}x} \qquad\qquad \lim_{x\to\infty}\frac{1}{8x-4}=0$$

$$\frac{1}{2}x$$

$$\underline{-\frac{1}{2}x+\frac{1}{4}}$$

$$\frac{1}{4}$$

$$Skrå\ \ asymptote:\underline{\underline{y=\frac{1}{2}x+\frac{1}{4}}}$$

c)

$$f(x) = \frac{x^2}{2x-1}$$

$$f'(x) = \frac{2x(2x-1) - x^2 \cdot 2}{(2x-1)^2}$$

$$f'(x) = \frac{4x^2 - 2x - 2x^2}{(2x-1)^2}$$

$$f'(x) = \frac{2x^2 - 2x}{(2x-1)^2}$$

$$\underline{\underline{f'(x) = \frac{2x(x-1)}{(2x-1)^2}}}$$

$$f'(x) = 0$$

$$\frac{2x(x-1)}{(2x-1)^2} = 0 \quad | (2x-1)^2$$

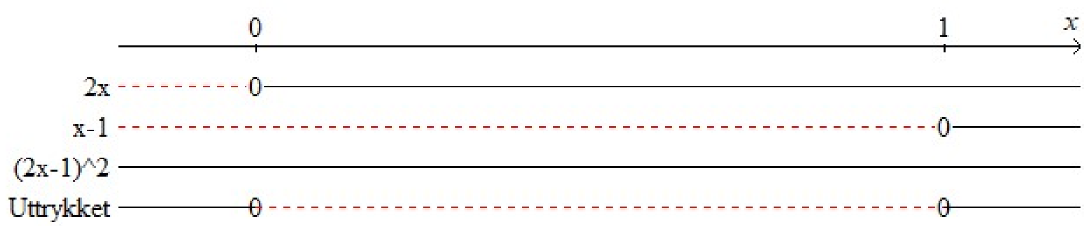
$$2x(x-1) = 0$$

$$2x = 0$$

$$\underline{x = 0}$$

$$x - 1 = 0$$

$$\underline{x = 1}$$



$$f(0) = \frac{0^2}{2 \cdot 0 - 1} = 0$$

$$f(1) = \frac{1^2}{2 \cdot 1 - 1} = 1$$

Toppunkt: $\underline{\underline{(0,0)}}$

Bunnpunkt: $\underline{\underline{(1,1)}}$

d)

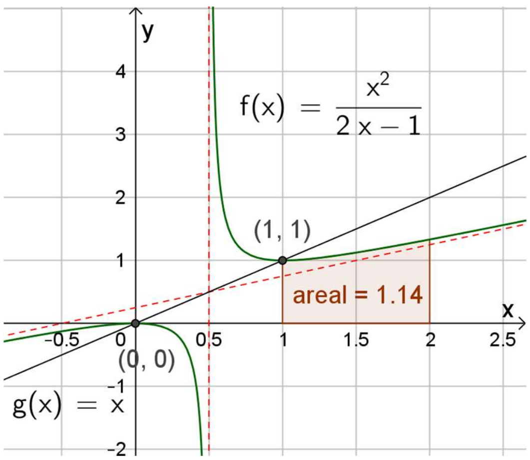
$\frac{x^2}{2x-1} = \frac{1}{2}x + \frac{1}{4} + \frac{1}{8x-4}$ er vist i b)

$$\int_1^2 \frac{x^2}{2x-1}dx = \int_1^2 \left(\frac{1}{2}x+\frac{1}{4}+\frac{1}{8x-4}\right)dx = \left[\frac{1}{4}x^2+\frac{1}{4}x+\frac{1}{8}\ln|8x-4|\right]_1^2$$

$$\left(\frac{1}{4}\cdot 2^2+\frac{1}{4}\cdot 2+\frac{1}{8}\ln|8\cdot 2-4|\right)-\left(\frac{1}{4}\cdot 1^2+\frac{1}{4}\cdot 1+\frac{1}{8}n|8\cdot 1-4|\right)=$$

$$1+\frac{1}{2}+\frac{1}{8}\ln 12-\frac{1}{4}-\frac{1}{4}-\frac{1}{8}\ln 4=$$

$$1+\frac{1}{8}\ln 3+\frac{1}{8}\ln 4-\frac{1}{8}\ln 4=\underline{\underline{1+\frac{\ln 3}{8}}}\approx 1{,}14$$



e)

$$g(x)=x$$
$$f(x)=g(x)$$
$$\frac{x^2}{2x-1}=x$$
$$x^2=2x^2-x$$
$$x^2-x=0$$
$$x(x-1)=0$$
$$\underline{x=0 \ \vee \ x=1}$$
$$g(1)=1$$
$$Skjæringspunkt: \underline{\underline{(0,0)\ \vee(1.1)}}$$

Oppgave 3

$$A(3,2,2) \quad B(6,1,-1) \quad D(3,4,0)$$

a)

$$\overrightarrow{AB} = [6-3, 1-2, -1-2] = \underline{\underline{[3, -1, -3]}}$$

$$\overrightarrow{AD} = [3-3, 4-2, 0-2] = \underline{\underline{[0, 2, -2]}}$$

$$|\overrightarrow{AB}| = \sqrt{3^2 + (-1)^2 + (-3)^2} = \sqrt{9+1+9} = \underline{\underline{\sqrt{19}}}$$

$$|\overrightarrow{AD}| = \sqrt{0^2 + 2^2 + (-2)^2} = \sqrt{0+4+4} = \underline{\underline{\sqrt{8}}}$$

$$\cos v = \frac{[3, -1, -3] \cdot [0, 2, -2]}{\sqrt{19} \cdot \sqrt{8}}$$

$$\cos v = \frac{3 \cdot 0 + (-1) \cdot 2 + (-3) \cdot (-2)}{\sqrt{19} \cdot \sqrt{8}}$$

$$\cos v = \frac{4}{\sqrt{19} \cdot \sqrt{8}}$$

$$\underline{\underline{v = 71{,}1^\circ}}$$

b)

Parallellogram ABCD

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{AD} = [6, 1, -1] + [0, 2, -2]$$

$$\overrightarrow{OC} = [6, 3, -3]$$

$$\underline{\underline{C(6, 3, -3)}}$$

c)

Areal parallellogram ABCD

$$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} 3 & -1 & -3 \\ 0 & 2 & -2 \end{vmatrix} = \left[(-1)\cdot(-2) - (-3)\cdot 2, -\left(3\cdot(-2) - (-3)\cdot 0\right), 3\cdot 2 - (-1)\cdot 0\right] = \underline{[8, 6, 6]}$$

$$A = \sqrt{8^2 + 6^2 + 6^2} = \sqrt{64+36+36} = \sqrt{136} = \underline{\underline{2\sqrt{34}}} \approx 11{,}66$$

d)

Linje gjennom A og C.

$$\vec{r} = \overrightarrow{AC} = [3, 1, -5]$$

$$\underline{\underline{l: \begin{cases} x = 3 + 3t \\ y = 2 + t \\ z = 2 - 5t \end{cases}}}$$

e)

Vis at midtpunkt M på AC også er midtpunkt N på BD.

$$\overrightarrow{OM}=\overrightarrow{OA}+\frac{1}{2}\overrightarrow{AC}=[3,2,2]+\frac{1}{2}[3,1,-5]=\left[\frac{9}{2},\frac{5}{2},-\frac{1}{2}\right]$$

$$\overrightarrow{ON}=\overrightarrow{OB}+\frac{1}{2}\overrightarrow{BD}=[6,1,-1]+\frac{1}{2}[-3,3,1]=\left[\frac{9}{2},\frac{5}{2},-\frac{1}{2}\right]$$

$$M\left(\frac{9}{2},\frac{5}{2},-\frac{1}{2}\right)$$

$$N\left(\frac{9}{2},\frac{5}{2},-\frac{1}{2}\right)$$

$M=N$. M er midtpunktet på AC og på BD

f)

Høyden i pyramiden.

$$T(4,2,5)$$

$$V=\frac{1}{3}(\overrightarrow{AB}\times\overrightarrow{AD})\cdot\overrightarrow{AT}$$

$$V=\frac{1}{3}[8,6,6]\cdot[1,0,3]=\frac{1}{3}(8\cdot 1+6\cdot 0+6\cdot 3)=\underline{\frac{26}{3}}\approx 8,67$$

$$V=\frac{1}{3}G\cdot h$$

$$h=\frac{3V}{G}=\frac{3\cdot\frac{26}{3}}{2\sqrt{34}}=\underline{\underline{\frac{13}{\sqrt{34}}}}=\underline{\underline{2,23}}$$

Oppgave 4

a)

Rekka $1,5+1,5\cdot 0,80+1,5\cdot 0,80^2+1,5\cdot 0,80^3+\cdots\cdot$ uttrykker mengden virkestoff i kroppen til Emma fordi det brytes ned 20% per døgn, da er vekstfaktoren 0,80.

Det første leddet er dagens dose, det andre leddet er gårsdagens dose, det tredje leddet er dosen tatt for 3 dager siden osv.

b)

Finn k, a_6, S_6

$$k=\frac{a_2}{a_1}=\frac{1,5\cdot 0,80}{1,5}=\underline{\underline{0,80}}$$

$$a_n=a_1\cdot k^{n-1}$$

$$a_6=1,5\cdot 0,80^{6-1}=\underline{\underline{1,5\cdot 0,80^5}}=\underline{\underline{0,492}}$$

$$S_n=a_1\cdot\frac{k^n-1}{k-1}$$

$$S_6=1,5\cdot\frac{0,80^6-1}{0,80-1}=\underline{\underline{5,534}}$$

f)

Da $-1<k<1$ så er det en uendelig geometrisk rekke. Summen er:

$$S=\frac{a_1}{1-k}$$

$$S=\frac{1,5}{1-0,8}=\underline{7,5}$$

Emma vil maksimalt ha 7,5 mg virkestoff i kroppen og vil derfor ikke overskride maks-grense som kroppen tåler.

Oppgave 5

a)

Vis at volumet kan skrives som $V=\frac{\pi}{3}(9x-6x^2+x^3)$

$$V=\frac{1}{3}G\cdot h$$

$$V=\frac{1}{3}\pi r^2\cdot h$$

$$V=\frac{1}{3}\pi(3-x)^2\cdot x$$

$$V=\frac{1}{3}\pi(9-6x+x^2)\cdot x$$

$$\underline{\underline{V=\frac{\pi}{3}(9x-6x^2+x^3)}}$$

b)

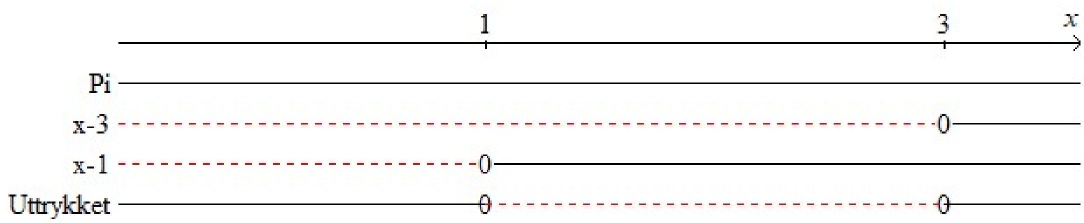
Koordinatene til C ved størst mulig volum,

$$V(x) = \frac{\pi}{3}(9x - 6x^2 + x^3)$$

$$V'(x) = \frac{\pi}{3}(9 - 12x + 3x^2)$$

$$V'(x) = \pi(3 - 4x + x^2)$$

$$V'(x) = \pi(x-3)(x-1)$$



Størst volum når $x = 1$. Punktet C er da:

$f(1) = 3 - 1 = 2$

$\underline{\underline{C(1,2)}}$

$$V(1) = \frac{\pi}{3}(9 \cdot 1 - 6 \cdot 1^2 + 1^3) = \underline{\frac{4\pi}{3}}$$

$\underline{\underline{\text{Størst volum er } \frac{4\pi}{3}}}$