Kap.12. Bevegelse II

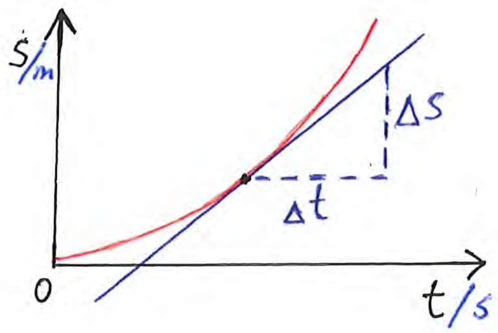
Bevegelse langs en rett linje

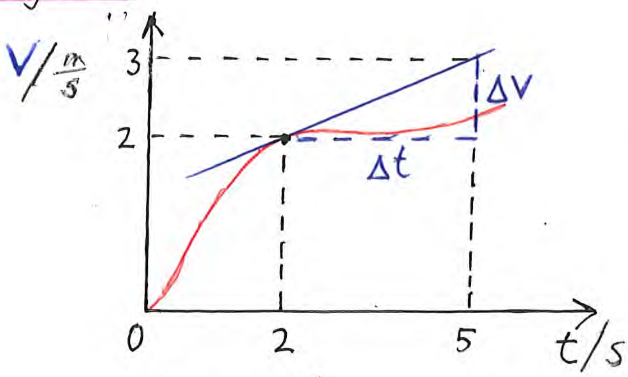
 som et punkt •

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = s'(t)$$

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = v'(t)$$

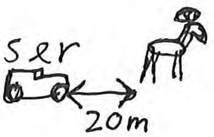
posisjons- og fartsgrafer





SI

eks. $a = \frac{\Delta v}{\Delta t} = \frac{1 \frac{m}{s}}{3\,s} = \underline{0{,}3 \frac{m}{s^2}}$ 12.01

Eks. 12.2 Bil bremser  og $s(t) = 25\frac{m}{s} \cdot t - 3{,}0\frac{m}{s^2} \cdot t^2$

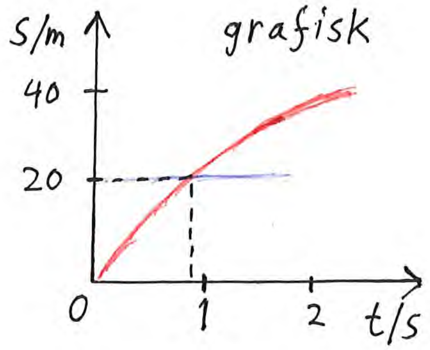
$(s = v_0 t + \frac{1}{2} a t^2)$

a) $v(t) = s'(t) = 25\frac{m}{s} - 2 \cdot 3{,}0\frac{m}{s^2} \cdot t = \underline{25\frac{m}{s} - 6{,}0\frac{m}{s^2} \cdot t}$ $(v = v_0 + at)$

$a(t) = v'(t) = 0 - 6{,}0\frac{m}{s^2} = \underline{-6{,}0\frac{m}{s^2}}$

d) Tid til kollisjon

grafisk



regning: $s(t) = 20\,m$

$25\frac{m}{s} \cdot t - 3{,}0\frac{m}{s^2} \cdot t^2 = 20\,m$

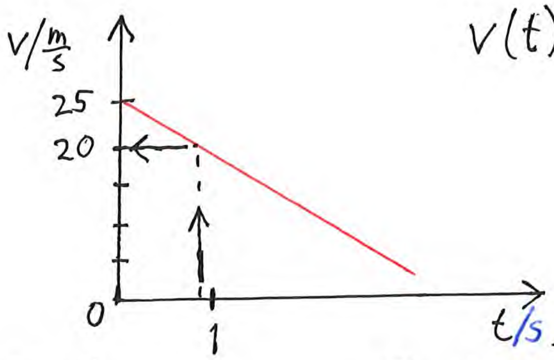
$3{,}0 \cdot t^2 - 25 \cdot t + 20 = 0$

$t = 0{,}8964\,s$ eller $\cancel{t = 7{,}436\,s}$

$\underline{t = 0{,}90\,s}$

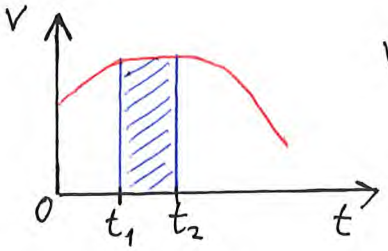
$(A = 3{,}0 \quad B = -25 \quad C = 20)$

e) Fart ved koll.



$$v(t) = 25\,\frac{m}{s} - 6{,}0\,\frac{m}{s^2}\cdot t$$
$$= 25\,\frac{m}{s} - 6{,}0\,\frac{m}{s^2}\cdot 0{,}8964\,s = 19{,}62\,\frac{m}{s}$$
$$\underline{= 20\,\frac{m}{s}}$$

Forflytning som areal



$$v = s'(t) \Rightarrow s(t) = \int v(t)\,dt$$
$$\text{og } \Delta s = s_2 - s_1 = s(t_2) - s(t_1) = \int_{t_1}^{t_2} v(t)\,dt$$

Bevegelseslikningene ved konstant akselerasjon:

$$v = v_0 + at$$
$$s = v_0 t + \tfrac{1}{2}at^2$$
$$s = \frac{(v+v_0)}{2}\cdot t$$
$$v^2 - v_0^2 = 2as$$

$$s'(t) = v(t) = v_0 + at$$
$$s(t) = \int_0^t (v_0 + at)\,dt$$
$$= (v_0 t + \tfrac{1}{2}at^2) - 0$$

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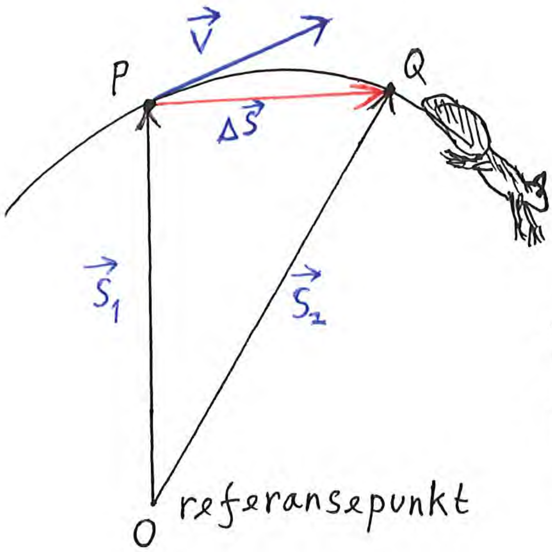
Vektorene fart og akselerasjon

Krumlinjet bevegelse



$\vec{s}$ posisjonsvektor

$\Delta\vec{s}$ forflytningsvektor

$$\overline{\vec{v}} = \frac{\Delta\vec{s}}{\Delta t} \quad \text{gj.sn. fart}$$

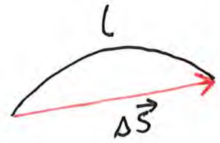
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{s}}{\Delta t} = s'(t)$$

fartsvektoren for momentanfart

tangent til banen i P

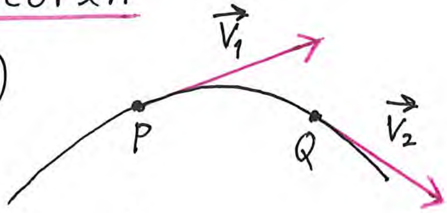
banefart er en skalar størrelse

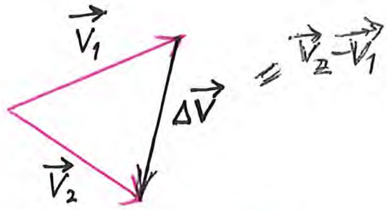
$$V_b = \frac{L}{\Delta t}$$



Akselerasjonsvektoren

$$a = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t} = \vec{V}'(t)$$





$\vec{V}$ endret retning $\Rightarrow$ aks.
endret |fart| $\Rightarrow$ aks.

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→

Bevegelse med konstant akselerasjon

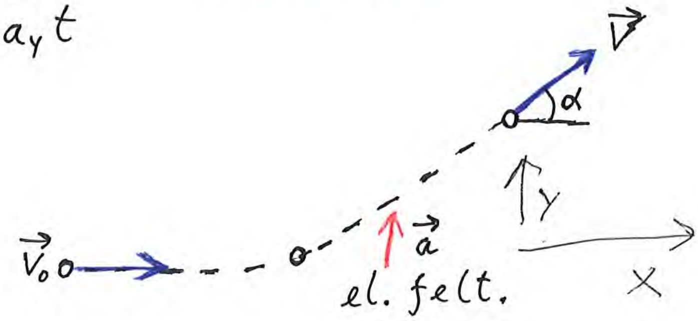
N's 2. lov er en vektorlov → uavhengighetsprinsippet

aks. konst. $\Rightarrow$

$$x = V_{0x} t + \tfrac{1}{2} a_x t^2 \qquad s = v_0 t + \tfrac{1}{2} a t^2$$
$$y = V_{0y} t + \tfrac{1}{2} a_y t^2$$
$$V_x = V_{0x} + a_x t \qquad V = V_0 + at$$
$$V_y = V_{0y} + a_y t$$

Eks. 12.5

Elektron $V_0 = 2{,}0 \cdot 10^7 \frac{m}{s}$
$a_y = 2{,}1 \cdot 10^{15} \frac{m}{s^2}$
$t = 5{,}0\,ns$



Posisjon: $x = V_{0x} t = 2{,}0 \cdot 10^7 \frac{m}{s} \cdot 5{,}0 \cdot 10^{-9} s = \underline{0{,}10m}$
$y = \frac{1}{2} a_y t^2 = \frac{1}{2} \cdot 2{,}1 \cdot 10^{15} \frac{m}{s^2} \cdot (5{,}0 \cdot 10^{-9} s)^2 = \underline{2{,}6cm}$

Fartsvektor: $V_x = V_{0x} = 2{,}0 \cdot 10^7 \frac{m}{s}$
$V_y = a_y t = 2{,}1 \cdot 10^{15} \frac{m}{s^2} \cdot 5{,}0 \cdot 10^{-9} s = 1{,}050 \cdot 10^7 \frac{m}{s}$

Absoluttverdi: $V = \sqrt{V_x^2 + V_y^2} = \sqrt{(2{,}0 \cdot 10^7)^2 + (1{,}050 \cdot 10^7)^2} \, \frac{m}{s}$
$= \underline{2{,}3 \cdot 10^7 \frac{m}{s}}$

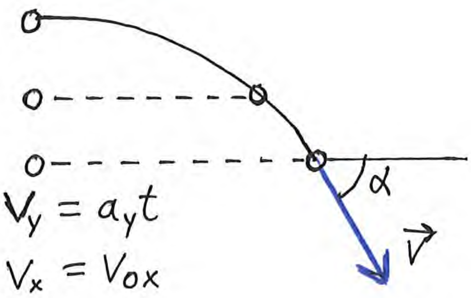
Retning: $\tan\alpha = \frac{V_y}{V_x} = \frac{1{,}050 \cancel{\ }}{2{,}0 \cancel{\ }}$
$\alpha = \tan^{-1}\left(\frac{1{,}050}{2{,}0}\right) = \underline{28^\circ}$

Kastebevegelse

kun tyngdekraft $\Rightarrow$ fritt fall $\qquad a_x = 0$

$$a_y = -9{,}81\,\frac{m}{s^2}$$

Horisontalt kast:



$V_y = a_y t$

$V_x = V_{0x}$

Eks. 12.6

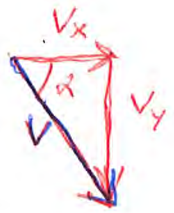
$V_{0x} = 1{,}2\,\frac{m}{s}$

$y_0 = 0$

$y = -0{,}80\,m$

$V_x = V_{0x} = 1{,}2\,\frac{m}{s}$

$V_y = a_y t = -9{,}81\,\frac{m}{s^2} \cdot t$



a) Fart ved gulvet:

$$y = \frac{1}{2} a_y t^2 \quad \leftarrow \quad s = v_0 t + \frac{1}{2} a t^2$$

$$2y = a_y t^2$$

$$\frac{2y}{a_y} = t^2$$

$$t = \sqrt{\frac{2y}{a_y}}$$

$$t = \sqrt{\frac{2 \cdot (-0{,}80\,m)}{-9{,}81\,\frac{m}{s^2}}} = 0{,}4038\,s \Rightarrow V_y = -9{,}81\,\frac{m}{s^2} \cdot 0{,}4038\,s = -3{,}961\,\frac{m}{s}$$

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{1{,}2^2 + (-3{,}961)^2}\,\frac{m}{s} = \underline{4{,}1\,\frac{m}{s}}$$

Retning: $\tan\alpha = \frac{|V_y|}{|V_x|} = \frac{3{,}961\,\frac{m}{s}}{1{,}2\,\frac{m}{s}}$

$$\alpha = \tan^{-1}\left(\frac{3{,}961}{1{,}2}\right) = \underline{73^\circ}$$ dvs. ned mot høyre.

b) $x = ?$

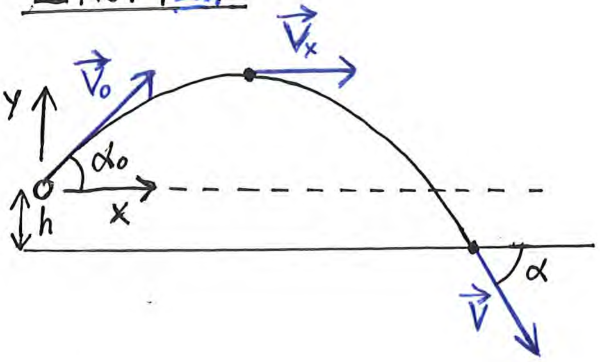
$$x = V_{0x} t = 1{,}2\,\frac{m}{s} \cdot 0{,}4038\,s = \underline{0{,}48\,m}$$

12.11

12.12

Skrått kast

Eks. 12.7



$v_0 = 12{,}6\,\frac{m}{s}$ $\quad \alpha_0 = 50^\circ$ $\quad h = 1{,}80\,m$

a) Fart i toppen:

$$v = v_x = v_{0x} = v_0 \cos\alpha_0$$
$$= 12{,}6\,\frac{m}{s} \cdot \cos 50^\circ = 8{,}099\,\frac{m}{s}$$
$$= \underline{8{,}1\,\frac{m}{s}}$$

b) Tid til toppen og høyde der:

$v_y = 0$ der, og $\quad v_y = v_{0y} + a_y t$ $\qquad v_{0y} = v_0 \sin 50^\circ$

$$0 = v_{0y} + a_y t \qquad = 12{,}6\,\frac{m}{s} \cdot \sin 50^\circ$$

$a_y = -9{,}81\,\frac{m}{s^2}$

$$-v_{0y} = a_y t \qquad = 9{,}652\,\frac{m}{s}$$

$$t = \frac{-v_{0y}}{a_y}$$

$s = v_0 t + \frac{1}{2} a t^2$

$$t = \frac{-9{,}652\,\frac{m}{s}}{-9{,}81\,\frac{m}{s^2}} = 0{,}9838\,s = \underline{0{,}98\,s}$$

$$y = v_{0y} t + \frac{1}{2} a_y t^2$$
$$= 9{,}652\,\frac{m}{s} \cdot 0{,}9838\,s + \frac{1}{2} \cdot \left(-9{,}81\,\frac{m}{s^2}\right)(0{,}9838\,s)^2 = 4{,}748\,m$$

Totalt: $1{,}80\,m + 4{,}748\,m = \underline{6{,}5\,m}$

c) Kastlengde:

$y = -h$ ved bakken og $y = v_{0y} t + \frac{1}{2} a_y t^2$

$$\frac{1}{2} a_y t^2 + v_{0y} t + h = 0$$

$Ax^2 + Bx + C = 0$

$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$t = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - 4 \cdot \frac{1}{2} a_y h}}{2 \cdot \frac{1}{2} a_y}$$

$$t = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - 2 a_y h}}{a_y}$$

$$t = \frac{-9{,}652 \pm \sqrt{9{,}652^2 - 2 \cdot (-9{,}81) \cdot 1{,}80}}{-9{,}81}\,s$$

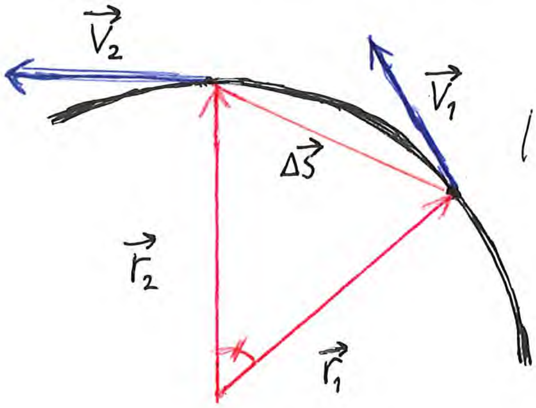
$t = \cancel{-0{,}1715\,s}$ eller $t = 2{,}139\,s$

$$x = v_{0x} t = 8{,}099\,\frac{m}{s} \cdot 2{,}139\,s = \underline{17\,m}$$

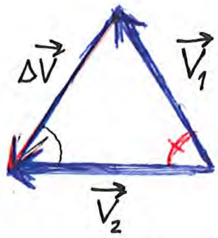
12.13

Akselerasjon i sirkelbevegelse

Sentripetalakselerasjonen



$|\vec{r}_1| = |\vec{r}_2| = r$



$|\vec{V}_1| = |\vec{V}_2| = V$

$|\vec{r}_1| = |\vec{r}_2| = r$
$|\vec{V}_1| = |\vec{V}_2| = V$

Formlike trekanter gir

$$\frac{\Delta V}{V} = \frac{\Delta S}{r}$$

$$\Delta V = \frac{V}{r} \cdot \Delta S$$

$$\frac{\Delta V}{\Delta t} = \frac{V}{r} \cdot \frac{\Delta S}{\Delta t}$$

med $\Delta t \to 0$

blir $\frac{\Delta V}{\Delta t} \to a$ og $\frac{\Delta S}{\Delta t} \to V$

dvs. $a = \frac{V}{r} \cdot V = \frac{V^2}{r}$

$$a = \frac{V^2}{r} \quad \left(\text{mot sentrum i sirkelen}\right)$$

Eks. 12.8 $\quad V = 20 \frac{km}{h} \quad r = 6{,}0 m$ $\quad$ Bil

$$V = 20 \cdot \frac{1000 m}{3600 s} = 5{,}555 \frac{m}{s}$$

$$a = \frac{V^2}{r} = \frac{(5{,}555 \frac{m}{s})^2}{6{,}0 m} = 5{,}1 \frac{m}{s^2} \quad \text{mot sentrum}$$

Eks. 12.9 a) Omløpstid og radius er kjent. Finn a.

$$V = \frac{2\pi r}{T}$$

$$a = \frac{V^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{\frac{4\pi^2 r^2}{T^2}}{r} = \frac{4\pi^2 r}{T^2}$$

b) Månens akselerasjon:

$$a = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 \cdot 3{,}84 \cdot 10^8 m}{(27{,}3 \cdot 24 \cdot 3600 s)^2} = \underline{2{,}72 \cdot 10^{-3} \frac{m}{s^2}}$$

mot jorda

12.15
12.16