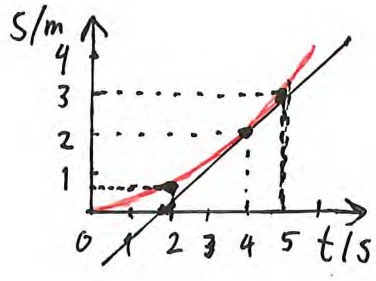
12.01



a) $s_1 = 0{,}6\,m$ for $t_1 = 2{,}0\,s$

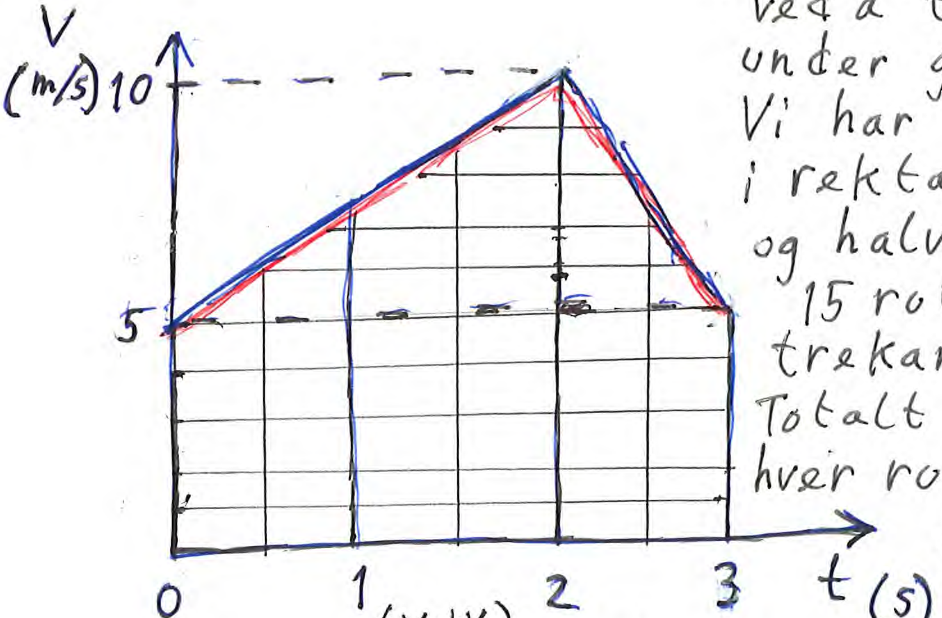
$s_2 = 3{,}0\,m$ for $t_2 = 5{,}0\,s$

$\Delta s = s_2 - s_1 = (3{,}0 - 0{,}6)\,m = \underline{2{,}4\,m}$

b) Tangenten for $t = 4{,}0\,s$ gir

$$v = \frac{\Delta s}{\Delta t} = \frac{(2{,}0 - 0{,}0)\,m}{(4{,}0 - 1{,}8)\,s} = \underline{0{,}9\,\tfrac{m}{s}}$$

12.02



a) $$s_1 = \bar{v}_1 \cdot t_1 = \frac{(\overset{v}{10} + \overset{v_0}{5})\,\frac{m}{s}}{2} \cdot 2\,s = 15\,m$$

$$s_2 = \bar{v}_2 \cdot t_2 = \frac{(\overset{v}{5} + \overset{v_0}{10})\,\frac{m}{s}}{2} \cdot 1\,s = 7{,}5\,m$$

$$s_1 + s_2 = (15 + 7{,}5)\,m \approx \underline{23\,m}$$

b) Vi finner arealet ved å telle ruter under grafen. Vi har 30 ruter i rektangulær del og halvparten, dvs. 15 ruter i trekantdelen. Totalt 45 ruter, og hver rute tilsvarer

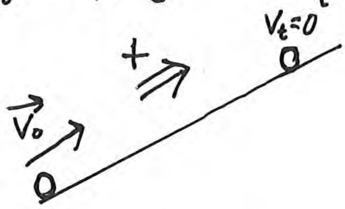
$\Delta s = \Delta v \cdot \Delta t$
$= 1{,}0\,\frac{m}{s} \cdot 0{,}50\,s$
$= 0{,}50\,m.$

Dvs. totalt blir det

$s = 45 \cdot 0{,}50\,m$
$\approx \underline{23\,m}$

12.04 $v_0 = 3{,}0\,\frac{m}{s}$ $\qquad t_t = 1{,}2\,s$

a)



b) $s_t = \frac{(v_0 + v_t)}{2} \cdot t_t = \frac{(3{,}0+0)\frac{m}{s}}{2} \cdot 1{,}2s$

$= \underline{1{,}8m}$

c) $2as = v^2 - v_0^2$ $\qquad t = 0{,}40s$

$$a = \frac{v^2 - v_0^2}{2s} = \frac{0 - (3{,}0\frac{m}{s})^2}{2 \cdot 1{,}8m} = \frac{-9{,}0}{3{,}6}\frac{m}{s^2} = -2{,}5\frac{m}{s^2}$$

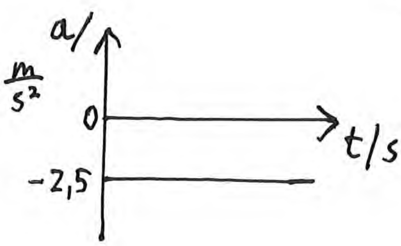
$$s = v_0 t + \tfrac{1}{2}at^2 = 3{,}0\tfrac{m}{s} \cdot 0{,}40s + \tfrac{1}{2}\cdot\left(-2{,}5\tfrac{m}{s^2}\right)\cdot(0{,}40s)^2$$

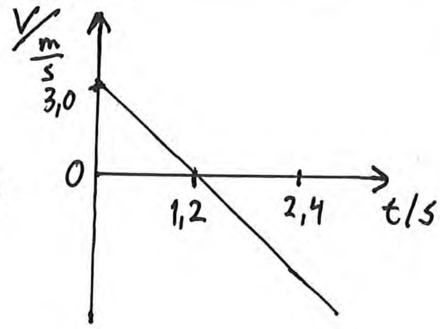
$$= 1{,}2m - 1{,}25 \cdot 0{,}16m = \underline{1{,}0m}$$

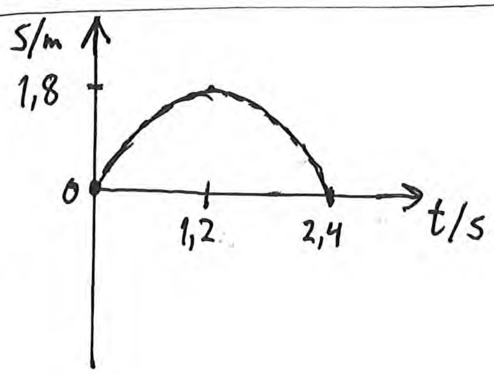
d) $s = 3{,}0 \cdot t - 1{,}25 \cdot t^2$ dvs. $s(t) = 3{,}0\frac{m}{s}\cdot t - 1{,}25\frac{m}{s^2}\cdot t^2$

$v = s' = 3{,}0 - 2{,}5 \cdot t$ dvs. $v(t) = 3{,}0\frac{m}{s} - 2{,}5\frac{m}{s^2}\cdot t$

$a = v' = -2{,}5$ dvs. $a(t) = -2{,}5\frac{m}{s^2}$







Casio: Velg GRAPH $\quad Y1 = 3X - 1.25X^2$ $\quad$ EXE

V-Window

X_{min} : 0

X_{max} : 2,4

DRAW (F6)

12.06

og $v_0 = 90\,\frac{km}{h} = 25\,\frac{m}{s}$

$v = \int a(t)\,dt$

$= \int ct\,dt = \frac{1}{2}ct^2 + k$

dvs $v = \frac{1}{2}\cdot\left(-2{,}5\,\frac{m}{s^3}\right)\cdot t^2 + 25\,\frac{m}{s}$

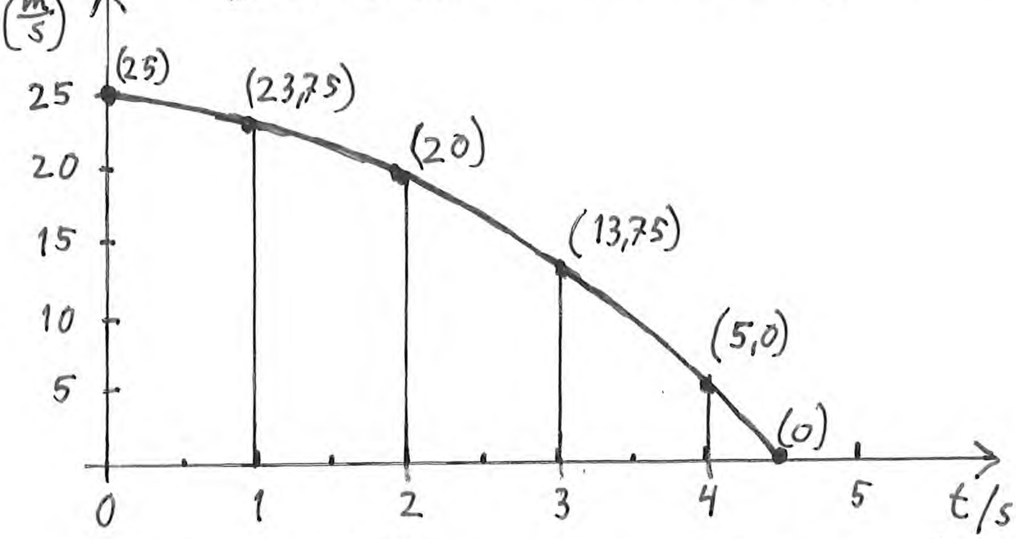
$v = 0 \Rightarrow 1{,}25\,\frac{m}{s^3}\cdot t^2 - 25\,\frac{m}{s} = 0$

$t^2 = \frac{25}{1{,}25}\,s^2$

$t = \sqrt{\frac{25}{1{,}25}}\,s = \underline{4{,}5\,s}$

$(4{,}472\,s)$

b) Bruk kalkulator som hjelp til tegning.



Kan f.eks. tilnærme arealet med rektangler:

$$S_{tot} = \bar{v}_1\cdot\Delta t_1 + \bar{v}_2\cdot\Delta t_2 + \bar{v}_3\cdot\Delta t_3 + \vec{v}_4\cdot\Delta t_4$$

$$= \left(\frac{25 + 23{,}75}{2}\cdot 1{,}0\right)m + \left(\frac{23{,}75 + 20}{2}\cdot 1{,}0\right)m + \left(\frac{20 + 13{,}75}{2}\cdot 1{,}0\right)m$$

$$+ \left(\frac{13{,}75 + 5{,}0}{2}\cdot 1{,}0\right)m + \left(\frac{5{,}0 + 0}{2}\cdot 0{,}472\right)m =$$

$$(24{,}375 + 21{,}875 + 16{,}875 + 9{,}375 + 1{,}18)\,m = 73{,}68\,m$$

$$\approx \underline{74\,m}$$

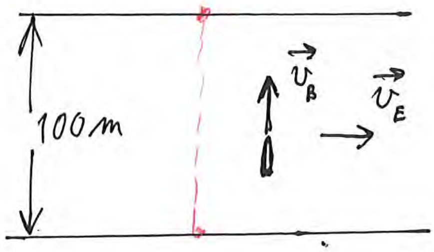
Ved integrering ville vi fått:

$$S_{tot} = \int_0^{4{,}472} v(t)\,dt = \int_0^{4{,}472}(25 - 1{,}25t^2)\,dt = \left[25t - \frac{1{,}25}{3}t^3\right]_0^{4{,}472}$$

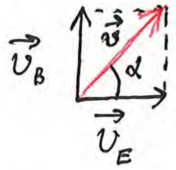
$$= \left[25\cdot 4{,}472 - \frac{1{,}25}{3}\cdot 4{,}472^3\right]m$$

$$= 74{,}53\,m \approx \underline{75\,m}$$

12.07



a)

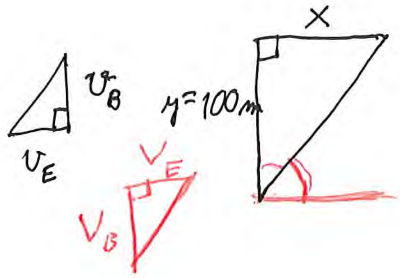


$$\vec{v} = \vec{v}_B + \vec{v}_E$$

$$v = \sqrt{v_B^2 + v_E^2} = \sqrt{4{,}0^2 + 3{,}0^2}\ \frac{m}{s}$$
$$= \underline{5{,}0\ \frac{m}{s}}$$

$$\tan\alpha = \frac{v_B}{v_E}$$
$$\alpha = \tan^{-1}\left(\frac{v_B}{v_E}\right)$$
$$\alpha = \tan^{-1}\left(\frac{4{,}0}{3{,}0}\right) = \underline{53^\circ} \text{ med elvebredden}$$

b)

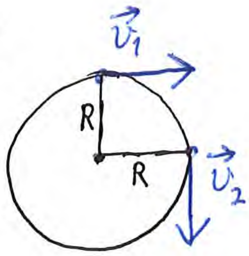


Farts- og posisjonstrekantene har samme form.
Vi får derfor $\frac{x}{100\,m} = \frac{v_E}{v_B}$

$$x = \frac{3{,}0}{4{,}0} \cdot 100\,m = \underline{75\,m}$$

c) $s = v \cdot t$
$$t = \frac{s}{v} = \frac{100\,m}{4{,}0\,\frac{m}{s}} = \underline{25\,s}$$

12.09

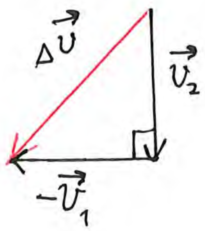


$R = 3{,}0\ cm$

a) $v_b = \frac{2\pi R}{T} = \frac{2\pi \cdot 3{,}0\,cm}{60\,s} = 3{,}1\ \frac{mm}{s}\ \left(3{,}141\ \frac{mm}{s}\right)$

b) $3{,}1\ \frac{mm}{s}$ øst (mot høyre)
$3{,}1\ \frac{mm}{s}$ sør (nedover)

c) $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

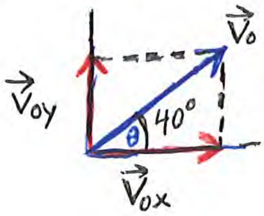


$$\Delta v = \sqrt{v_2^2 + v_1^2} = \sqrt{3{,}141^2 + 3{,}141^2}\ \frac{mm}{s} = 4{,}442\ \frac{mm}{s}$$
$$= \underline{4{,}4\ \frac{mm}{s}}$$
mot sørvest

$$\bar{\vec{a}} = \frac{\Delta\vec{v}}{t} = \frac{4{,}442\ \frac{mm}{s}}{15\,s} = \underline{0{,}30\ \frac{mm}{s^2}}$$

mot sørvest

12.10 $v_0 = 12\,\frac{m}{s}$



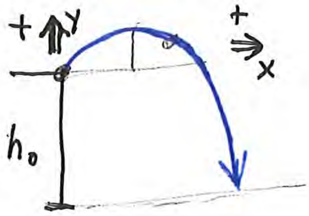
a) $v_{ox} = v_0 \cdot \cos\theta = 12\,\frac{m}{s} \cdot \cos 40° = 9{,}1925\,\frac{m}{s}$

$= \underline{9{,}2\,\frac{m}{s}}$

$v_{oy} = v_0 \cdot \sin\theta = 12\,\frac{m}{s} \cdot \sin 40°$

$= 7{,}713\,\frac{m}{s} = \underline{7{,}7\,\frac{m}{s}}$

b) $t = 1{,}5\,s$ $\quad h_0 = 2{,}0\,m$



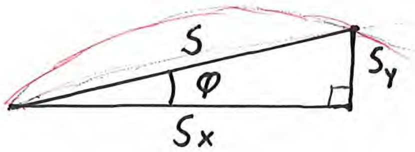
$s_y = v_{oy}t + \frac{1}{2}a_y t^2$ $\qquad a_y = g = -9{,}81\,\frac{m}{s^2}$

$s_y = 7{,}713\,\frac{m}{s} \cdot 1{,}5\,s + \frac{1}{2} \cdot \left(-9{,}81\,\frac{m}{s^2}\right) \cdot (1{,}5\,s)^2$

$= 0{,}5332\,m$

$h_{tot} = h_0 + s_y = 2{,}0\,m + 0{,}5332\,m = \underline{2{,}5\,m}$

$s_x = v_{ox} \cdot t = 9{,}1925\,\frac{m}{s} \cdot 1{,}5\,s = 13{,}78\,m = \underline{14\,m}$



$s = \sqrt{s_x^2 + s_y^2} = \sqrt{0{,}5332^2 + 13{,}78^2}\,m$

$= 13{,}79\,m = \underline{14\,m}$

$\tan\varphi = \frac{s_y}{s_x}$

$\varphi = \tan^{-1}\left(\frac{s_y}{s_x}\right) = \tan^{-1}\left(\frac{0{,}5332}{13{,}78}\right) = 2{,}215° = \underline{2{,}2°}$

over horisonten.

c) $v_x = v_{ox} = 9{,}1925\,\frac{m}{s}$

$+\Uparrow$ $v_y = v_{oy} + a_y \cdot t = v_{oy} + gt = 7{,}713\,\frac{m}{s} - 9{,}81\,\frac{m}{s^2} \cdot 1{,}5\,s$

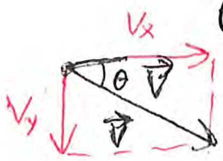
$= -7{,}002\,\frac{m}{s}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{9{,}1925^2 + (-7{,}002)^2}\,\frac{m}{s}$

$= 11{,}55\,\frac{m}{s} = \underline{12\,\frac{m}{s}}$

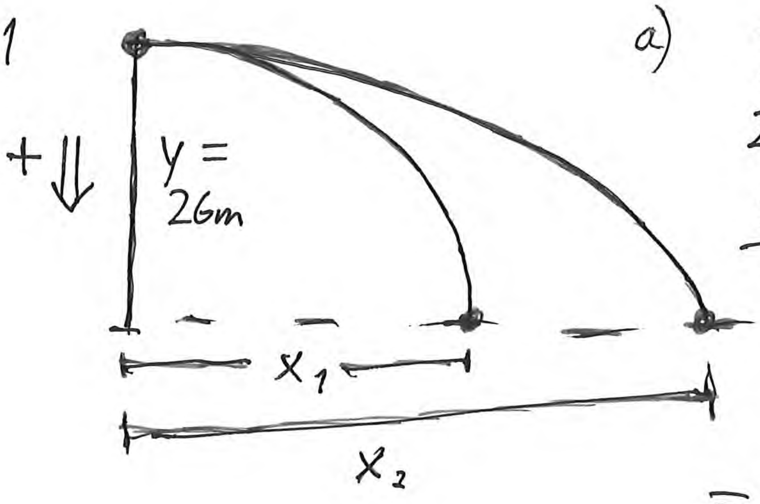
$\tan\theta = \frac{v_y}{v_x}$

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-7{,}002}{9{,}1925}\right) = -37{,}29° = \underline{-37°}$



dvs 37° under horisonten.

12.11



$s = \cancel{v_0 t} + \frac{1}{2} a t^2$

a) $y = \frac{1}{2} a_y t^2$

$2y = a_y t^2$

$\frac{2y}{a_y} = t^2$

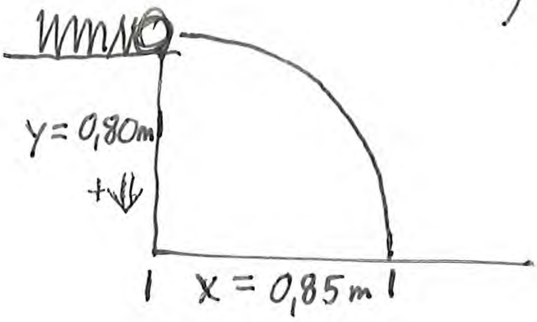
$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2 \cdot 26m}{9{,}81 \frac{m}{s^2}}}$

$= 2{,}302 s = \underline{2{,}3 s}$

b) $x_1 = v_{0x_1} \cdot t = 0{,}050 \frac{m}{s} \cdot 2{,}302 s = \underline{0{,}12 m}$ (0,115 m)

$x_2 = v_{0x_2} \cdot t = 5{,}0 \frac{m}{s} \cdot 2{,}302 s = \underline{12 m}$ (11,5 m)

12.12



a)

$$y = \overset{0}{\overset{\shortparallel}{v_{0y}}} \cdot t + \tfrac{1}{2} a_y \cdot t^2$$

$$y = \tfrac{1}{2} g t^2$$

$$2y = g t^2$$

$$\frac{2y}{g} = t^2$$

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \cdot 0{,}80\,\mathrm{m}}{9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2}}} = 0{,}4038\,\mathrm{s}$$

$$x = v_{0x} \cdot t + \tfrac{1}{2} \overset{0}{\overset{\shortparallel}{a_x}} \cdot t^2$$

$$x = v_0 \cdot t$$

$$v_0 = \frac{x}{t} = \frac{0{,}85\,\mathrm{m}}{0{,}4038\,\mathrm{s}} = \underline{2{,}1\,\frac{\mathrm{m}}{\mathrm{s}}} \qquad \left(2{,}105\,\frac{\mathrm{m}}{\mathrm{s}}\right)$$

b) $v_x = v_{0x} = v_0 = 2{,}105\,\frac{\mathrm{m}}{\mathrm{s}}$

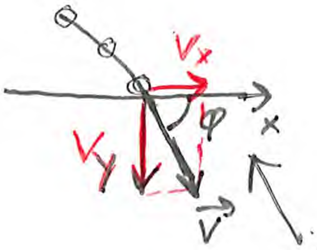
$v = v_0 + at$

$+\Downarrow$ $v_y = \overset{0}{\overset{\shortparallel}{v_{0y}}} + a_y \cdot t$

$$v_y = gt = 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 0{,}4038\,\mathrm{s} = 3{,}9612\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{2{,}105^2 + 3{,}9612^2}\,\frac{\mathrm{m}}{\mathrm{s}} = \underline{4{,}5\,\frac{\mathrm{m}}{\mathrm{s}}} \quad \left(4{,}485\,\frac{\mathrm{m}}{\mathrm{s}}\right)$$

verdi

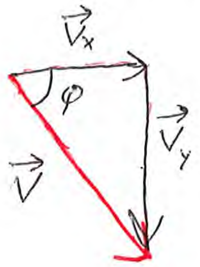


$$\tan\varphi = \frac{v_y}{v_x}$$

$$\varphi = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{3{,}9612}{2{,}105}\right) = \underline{62^\circ}$$

retning

φ måles nedover fra horisontalretningen.



12.13 $V_{0x} = 160\,\frac{m}{s}$ $\quad V_{0y} = 75\,\frac{m}{s}$

a) $V_{0y} = 0$ i høyeste punkt fordi det er snopunktet for bevegelsen i y-retning.

$V = V_x = V_{0x} = \underline{160\,\frac{m}{s}}$ fordi vi ikke har noen akselerasjon i x-retning etter at prosjektilet er skott ut.

b) $2a_y s_y = v_y^2 - V_{0y}^2$ og $V_y = 0$

$-2g s_y = -V_{0y}^2$ $\qquad \Uparrow +$

$$s_y = \frac{-V_{0y}^2}{-2g} = \frac{-(75\,\frac{m}{s})^2}{-2(+9{,}81\,\frac{m}{s^2})} = 286{,}6\,m = \underline{0{,}29\,km}$$

c)

$s_y = V_{0y} \cdot t + \frac{1}{2}(-g)t^2$ og $s_y = 0$

$0 = (V_{0y} - \frac{1}{2}gt) \cdot t$ og $t \neq 0$

Det vil si at $V_{0y} - \frac{1}{2}gt = 0$

$V_{0y} = \frac{1}{2}gt$ $\qquad \Uparrow +$

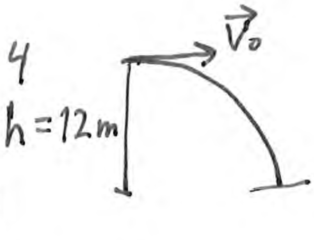
$2V_{0y} = gt$

$\frac{2V_{0y}}{g} = t$

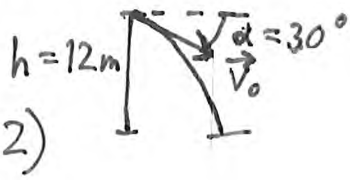
$$t = \frac{2 \cdot 75\,\frac{m}{s}}{9{,}81\,\frac{m}{s^2}} = 15{,}29\,s$$

$s_x = V_{0x} \cdot t = 160\,\frac{m}{s} \cdot 15{,}29\,s = 2446\,m = \underline{2{,}4\,km}$

12.14

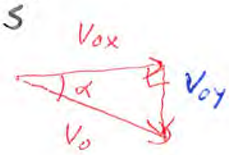


1)



2)

$V_0 = 17\,\frac{m}{s}$



a) 1) $V_x = V_{0x} = V_0$

$+\Downarrow$

$V_y = \underbrace{V_{0y}}_{=0} + gt$

$S_y = \cancel{V_{0y}\cdot t} + \frac{1}{2}gt^2$

$12\,m = \frac{1}{2}\cdot 9{,}81\,\frac{m}{s^2}\cdot t^2$

$\frac{2\cdot 12\,m}{9{,}81\,\frac{m}{s^2}} = t^2$

$t = \sqrt{\frac{24}{9{,}81}}\,s = 1{,}564\,s$

$V_y = 9{,}81\,\frac{m}{s^2}\cdot 1{,}564\,s = 15{,}34\,\frac{m}{s}$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{17^2 + 15{,}34^2}\,\frac{m}{s}$

$= \underline{23\,\frac{m}{s}} \quad (22{,}89\,\frac{m}{s})$

2) $V_x = V_{0x} = V_0\cdot\cos 30°$

$= 17\,\frac{m}{s}\cdot\cos 30° = 14{,}72\,\frac{m}{s}$

$+\Downarrow$

$V_y = V_{0y} + gt$

$V_{0y} = V_0\cdot\sin 30°$

$= 17\,\frac{m}{s}\cdot\frac{1}{2} = 8{,}500\,\frac{m}{s}$

$S_y = V_{0y}\cdot t + \frac{1}{2}gt^2$

$12\,m = 8{,}5\,\frac{m}{s}\cdot t + \frac{1}{2}\cdot 9{,}81\,\frac{m}{s^2}\cdot t^2$

$12 = 8{,}5\cdot t + 4{,}905\cdot t^2$

$4{,}905\cdot t^2 + 8{,}5\cdot t - 12 = 0$

$t = 0{,}9216\,s \lor \cancel{t = -2{,}65\,s}$ (falsk løsning)

$V_y = 8{,}500\,\frac{m}{s} + 9{,}81\,\frac{m}{s^2}\cdot 0{,}9216\,s$

$= 17{,}54\,\frac{m}{s}$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{14{,}72^2 + 17{,}54^2}\,\frac{m}{s}$

$= \underline{23\,\frac{m}{s}} \quad (22{,}89\,\frac{m}{s})$

Vi konne også broke prinsippet om bevaring av mekanisk energi i en kastbevegelse der kun tyngden virker til å vise at svarene må være like.

$\cancel{E_p} + E_K = E_{p0} + E_{K0}$

$\frac{1}{2}mv^2 = mgh + \frac{1}{2}mv_0^2$

$\frac{1}{2}v^2 = gh + \frac{1}{2}v_0^2$

$v^2 = 2gh + v_0^2$

$v = \sqrt{2gh + v_0^2}$

$v = \sqrt{2\cdot 9{,}81\cdot 12 + 17^2}\,\frac{m}{s} = \underline{23\,\frac{m}{s}} \quad (22{,}9\,\frac{m}{s})$

12.14 b) 1) $V_x = 17 \frac{m}{s}$

$V_y = 15{,}34 \frac{m}{s}$

$\tan\varphi = \frac{V_y}{V_x}$

$\varphi = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{15{,}34}{17}\right) = \underline{42^\circ}$

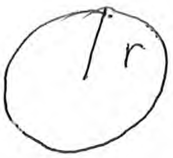
2) $V_x = 14{,}72 \frac{m}{s}$

$V_y = 17{,}54 \frac{m}{s}$

$\tan\varphi = \frac{V_y}{V_x}$

$\varphi = \tan^{-1}\left(\frac{17{,}54}{14{,}72}\right) = \underline{50^\circ}$

12.15

 $r = 400m$

$V = 22 \frac{m}{s}$

$a = \frac{V^2}{r} = \frac{(22 \frac{m}{s})^2}{400m} = \underline{1{,}2 \frac{m}{s^2}}$

12.16

 $r = 5{,}0m$

$a = 90 \frac{m}{s^2}$

$V = ?$

$a = \frac{V^2}{r}$

$ar = V^2$

$\sqrt{ar} = V$

$V = \sqrt{90 \frac{m}{s^2} \cdot 5{,}0m} = 21{,}21 \frac{m}{s}$

$= \underline{21 \frac{m}{s}}$

$T = ?$

$s = 2\pi r$

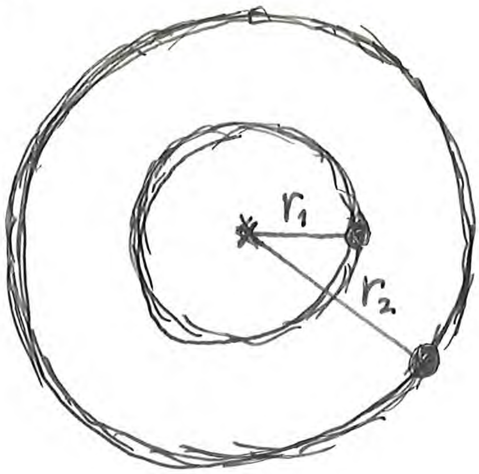
$s = V \cdot t$

$2\pi r = V \cdot T$

$\frac{2\pi r}{V} = T$

$T = \frac{2\pi \cdot 5{,}0m}{21{,}21 \frac{m}{s}} = \underline{\underline{1{,}5 s}}$

12.19



a) $V_1 = \frac{2\pi r_1}{T}$ $\quad V_2 = \frac{2\pi r_2}{T}$

$r_2 = 2r_1$

$$\frac{V_2}{V_1} = \frac{\frac{2\pi \cdot 2r_1}{T}}{\frac{2\pi r_1}{T}} = 2$$

dvs dobbel banefart.

b) $a_1 = \frac{V_1^2}{r_1}$ $\quad a_2 = \frac{V_2^2}{r_2}$

$$\frac{a_2}{a_1} = \frac{\frac{V_2^2}{r_2}}{\frac{V_1^2}{r_1}} = \frac{V_2^2}{r_2} \cdot \frac{r_1}{V_1^2}$$

$$= \frac{(2V_1)^2}{2r_1} \cdot \frac{r_1}{V_1^2} = \frac{4}{2} = \underline{2}$$

c) Nei, fordi $\sum F = ma$ gir

$F = m\frac{v^2}{r}$ og $F_1 = ma_1$ mens $F_2 = ma_2$
$= \underline{2ma_1}$

↑ friksjon med plata

dvs dobbel kraft

d) $T = \frac{60 s}{33} = \frac{20}{11} s$

$V_1 = \frac{2\pi r_1}{T} = \frac{2\pi \cdot 0{,}070 m}{\frac{20}{11} s} = \underline{0{,}24 \frac{m}{s}}$ $\quad (0{,}2419 \frac{m}{s})$

$V_2 = 2 \cdot V_1 = 2 \cdot 0{,}2419 \frac{m}{s} = \underline{0{,}48 \frac{m}{s}}$

$a_1 = \frac{V_1^2}{r_1} = \frac{(0{,}2419 \frac{m}{s})^2}{0{,}070 m} = 0{,}84 \frac{m}{s^2}$ $\quad (0{,}8359 \frac{m}{s^2})$

$a_2 = 2 \cdot a_1 = 2 \cdot 0{,}8359 \frac{m}{s^2} = \underline{1{,}7 \frac{m}{s^2}}$

e) $\frac{V_2}{V_1} = \underline{2{,}0}$ $\quad \frac{a_2}{a_1} = \underline{2{,}0}$ som forventet.