13. Kraft og bevegelse II

Newtons tre lover

N's 2. lov: $\sum \vec{F} = m\vec{a}$ dvs. $\sum F_x = ma_x$ og $\sum F_y = ma_y$

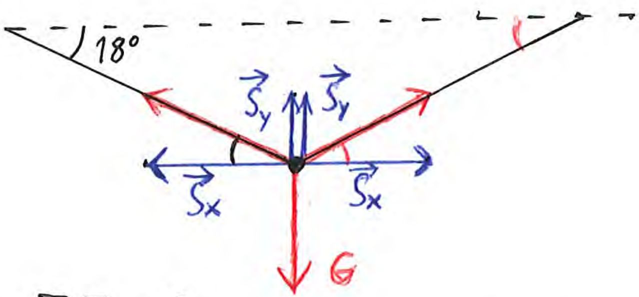
uavhengighetsprinsippet gjelder

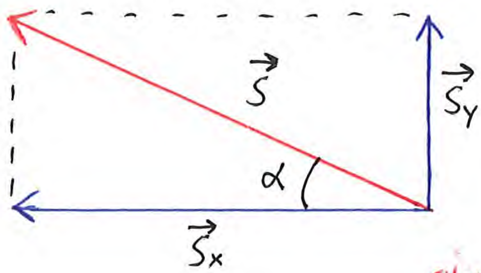
N's 1. lov: $\sum F_x = 0$ hvis $v_x = \text{konst.}$

$\sum F_y = 0$ hvis $v_y = \text{konst.}$

N's 3. lov: $\vec{F'} = -\vec{F}$

Eks 13.1 Finn snordraget S. $m = 85\,\text{kg}$





$\sin\alpha = \frac{S_y}{S}$

$\sum F_y = 0$

$S_y + S_y - G = 0$

$2S_y = mg$

$2 \cdot S \cdot \sin\alpha = mg$

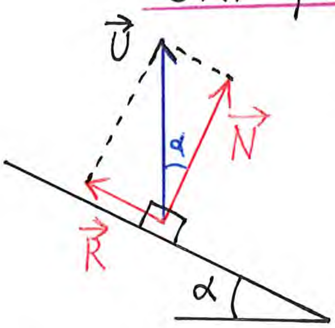
$G = mg$

$S_y = S \cdot \sin\alpha$

$$S = \frac{mg}{2\sin\alpha} = \frac{85\,\text{kg} \cdot 9{,}81\,\frac{\text{N}}{\text{kg}}}{2 \cdot \sin 18°} = \underline{1{,}3\,\text{kN}}$$

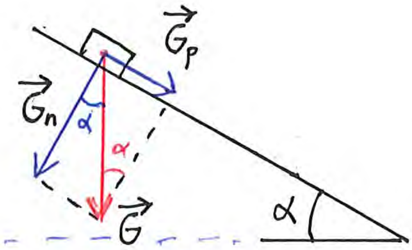
13.01

Skråplan



$U = \sqrt{N^2 + R^2}$

$R = \mu N$



$\sin\alpha = \frac{G_p}{G} \quad | \cdot G$

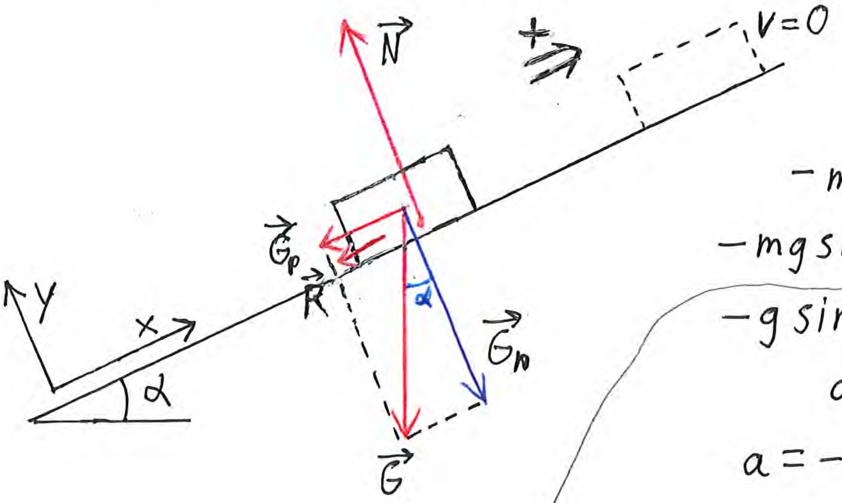
$G_p = mg\sin\alpha$

$\cos\alpha = \frac{G_n}{G}$

$G_n = mg\cos\alpha$

Eks. 13.4 Kloss med $v_0 = 3{,}6\,\frac{m}{s}$ oppover, $\alpha = 20°$, $\mu = 0{,}40$

a) Hvor langt opp planet kommer klossen?



$$\sum F_x = ma_x$$
$$-G_p - R = ma$$
$$-mg\sin\alpha - \mu N = ma$$
$$-mg\sin\alpha - \mu mg\cos\alpha = ma$$
$$-g\sin\alpha - \mu g\cos\alpha = a$$
$$a = -g(\sin\alpha + \mu\cos\alpha)$$
$$a = -9{,}81\,\frac{m}{s^2}(\sin 20° + 0{,}40\cos 20°)$$
$$= -7{,}042\,\frac{m}{s^2}$$

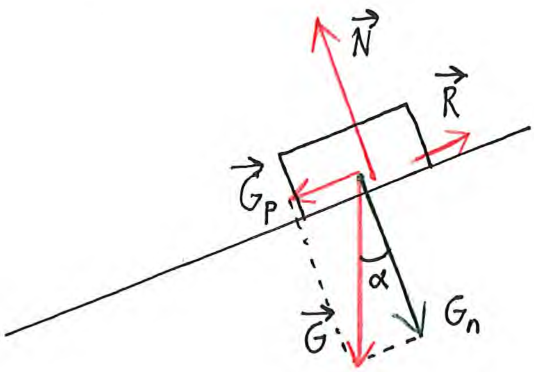
$$\sum F_y = 0$$
$$N - G_n = 0$$
$$N = mg\cos\alpha$$

og $2as = v^2 - v_0^2$ med $v = 0$

gir $2as = -v_0^2$

$$s = \frac{-v_0^2}{2a} = \frac{-(3{,}6\,\frac{m}{s})^2}{2\cdot(-7{,}042\,\frac{m}{s^2})}$$
$$= \underline{0{,}92\,m}$$

b) Ligger klossen nå i ro?



$m = 1{,}0\,kg$

$G_p = mg\sin\alpha = 3{,}4\,N$

$R_{gli} = \mu N = \mu mg\cos\alpha = 3{,}7\,N$

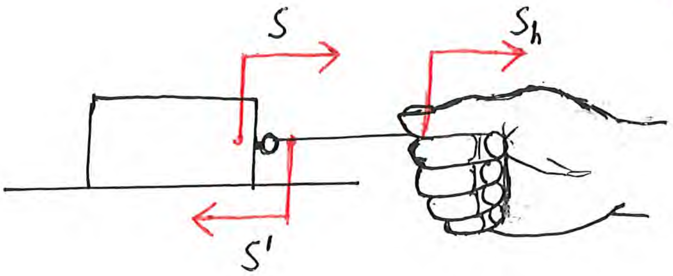
$G_p < R_{gli} \Rightarrow$ i ro

c) v = konst
ned $\alpha = ?$

$$\sum F_x = 0$$
$$R = G_p$$
$$\mu mg\cos\alpha = mg\sin\alpha$$
$$\mu = \tan\alpha$$
$$\alpha = \tan^{-1}\mu = \tan^{-1} 0{,}40 = \underline{22°}$$

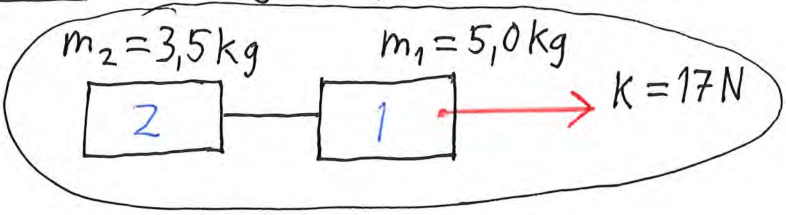
13.07

(Ta før skråplan)



Snordraget er like stort i begge ender av snora $(m_{snor} \approx 0)$

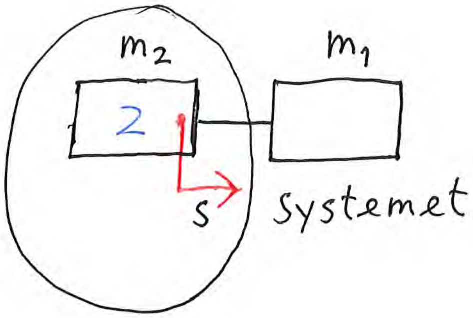
Eks. 13.5 To vogner, tau, lik aks.



systemet

Finn snordraget på vogn 2.

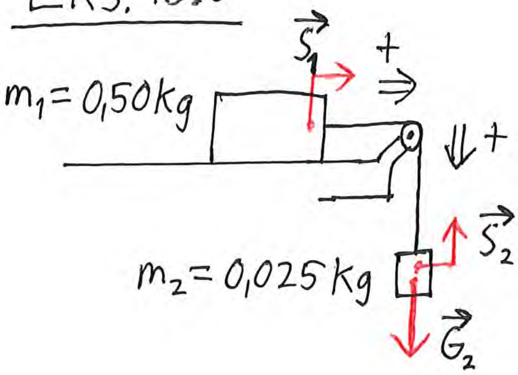
$$\sum F = ma$$
$$K = (m_1 + m_2)a$$
$$a = \frac{K}{(m_1+m_2)} = \frac{17\,N}{(5{,}0+3{,}5)\,kg} = 2{,}000\,\frac{m}{s^2}$$



$$\sum F = ma$$
$$S = m_2 a = 3{,}5\,kg \cdot 2{,}000\,\frac{m}{s^2} = \underline{7{,}0\,N}$$

13.05

Eks. 13.6



$$S_1 = S_2 = S$$
$$a_1 = a_2 = a$$

Finn a og S.

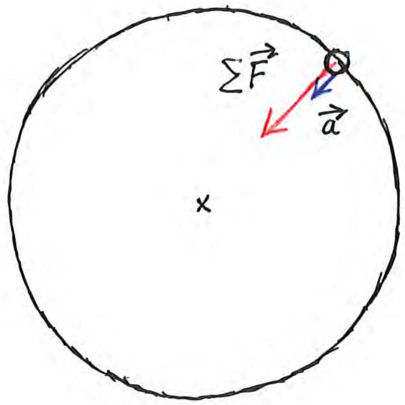
$$\sum F_1 = m_1 a \qquad \sum F_2 = m_2 a$$
$$S = m_1 a \qquad G_2 - S = m_2 a$$
$$G_2 - m_1 a = m_2 a$$
$$m_2 g = (m_1 + m_2) a$$
$$a = \frac{m_2 g}{(m_1+m_2)}$$
$$a = \frac{0{,}025 \cdot 9{,}81\,N}{(0{,}50 + 0{,}025)\,kg}$$
$$a = 0{,}4671\,\frac{m}{s^2} = \underline{0{,}47\,\frac{m}{s^2}}$$

$$S = m_1 a$$
$$S = 0{,}50\,kg \cdot 0{,}4671\,\frac{m}{s^2}$$
$$= \underline{0{,}23\,N}$$

13.12

Sirkelbevegelse

Konstant banefart

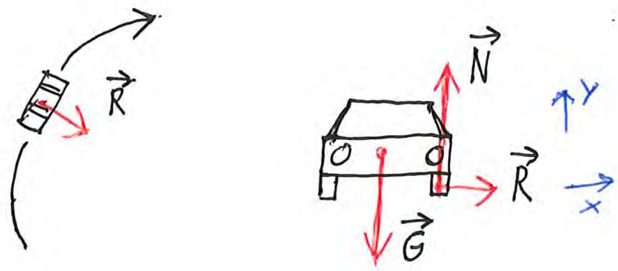


$$\underbrace{\Sigma F}_{} = ma = \frac{mv^2}{r}$$

(S392)

kalles sentripetalkraften
(kan f.eks. være et snordrag)

Eks. 13.7 Bil svinger



d) $\Sigma F_x = ma_x$ $\Sigma F_y = 0$

$R = m\frac{v^2}{r}$ $N = G$

$\mu N = m\frac{v^2}{r}$

$\mu \cancel{m} g = \cancel{m}\frac{v^2}{r}$

$\mu g r = v^2 \Rightarrow v = \sqrt{\mu g r}$ er maks fart.

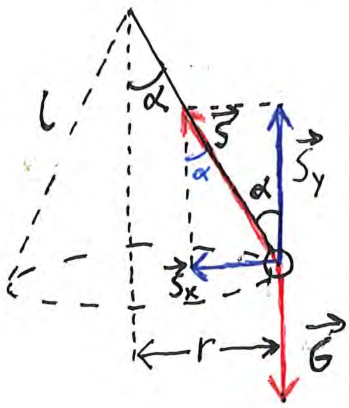
13.14

Kjeglependel

Eks. 13.8 Lekefly

$v_b = \text{konst.}$, $\alpha = 28°$, $m = 0{,}25\,kg$, $r = 0{,}60\,m$

a) Tegn krefter



b) Regn ut krefter.

$G = mg = 0{,}25\,kg \cdot 9{,}81\,\frac{N}{kg} = 2{,}452\,N = \underline{2{,}5\,N}$

$\Sigma F_y = 0$

$S_y = G$

$S\cos\alpha = G$

$S = \frac{G}{\cos\alpha} = \frac{2{,}452\,N}{\cos 28°} = 2{,}777\,N = \underline{2{,}8\,N}$

c) Finn v_b.

$\Sigma F_x = ma_x$

$S_x = m\frac{v^2}{r}$

$S \cdot \sin\alpha = m\frac{v^2}{r}$

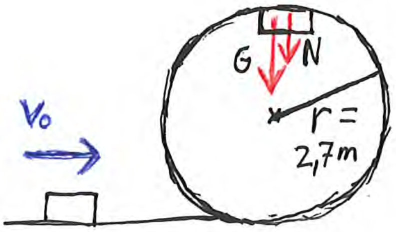
$v^2 = r \cdot S \cdot \sin\alpha / m$

13.18

$$v = \sqrt{\frac{r \cdot S \cdot \sin\alpha}{m}} = \sqrt{\frac{0{,}60\,m \cdot 2{,}777\,N \cdot \sin 28°}{0{,}25\,kg}} = \underline{1{,}8\,\frac{m}{s}}$$

(kunstig tyngdekraft 5g betyr $5 \cdot mg$)

Vertikal loop



Eks. 13.9 Sykkel. Fart i toppen minst lik...?

$$\sum F = ma$$

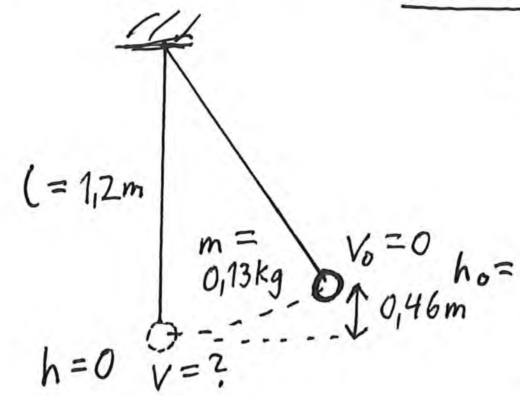
$$N + G = m\frac{v^2}{r} \quad \text{og } N = 0 \text{ for } v \text{ minimum}$$

$$\cancel{m}g = \cancel{m}\frac{v^2}{r}$$

$$v = \sqrt{gr} = \sqrt{9{,}81\tfrac{m}{s^2} \cdot 2{,}7m} = \underline{5{,}1\tfrac{m}{s}}$$

(18 km/h)

Planpendel



Eks. 13.10 $v = ?$

$$E = E_0$$

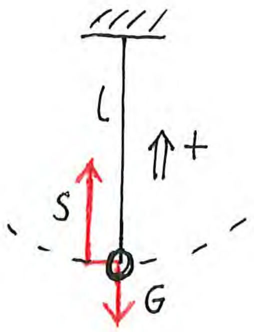
$$mgh + \tfrac{1}{2}mv^2 = mgh_0 + \tfrac{1}{2}mv_0^2 \quad \text{og } h = 0,\ v_0 = 0$$

$$\tfrac{1}{2}\cancel{m}v^2 = \cancel{m}gh_0$$

$$v^2 = 2gh_0$$

$$v = \sqrt{2gh_0}$$

$$v = \sqrt{2 \cdot 9{,}81\tfrac{m}{s^2} \cdot 0{,}46m} = 3{,}004\tfrac{m}{s} = \underline{3{,}0\tfrac{m}{s}}$$



$S = ?$

$$\sum F = ma$$

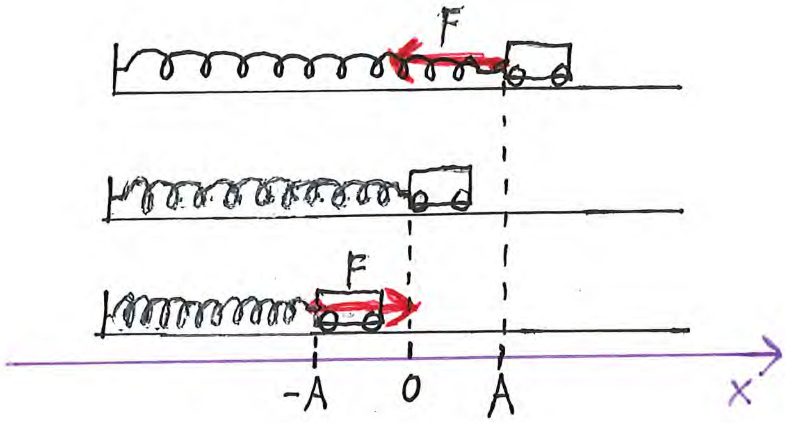
$$S - G = m\frac{v^2}{l}$$

$$S = mg + m\frac{v^2}{l}$$

$$S = 0{,}13kg \cdot \left[9{,}81\tfrac{N}{kg} + \frac{(3{,}004\tfrac{m}{s})^2}{1{,}2m}\right] = \underline{2{,}3N}$$

13.19

13.3 Elastisk pendel. Svingninger



A = amplitude

$$\Sigma F = ma \quad \text{og} \quad F = -kx$$

$$-kx = ma \quad \text{og} \quad a(t) = v'(t) = x''(t)$$

$$-kx = mx''$$

$$\Downarrow$$

$$x'' + \frac{k}{m}x = 0 \qquad \text{svingelikningen}$$

og $\boxed{x(t) = A\cos(2\pi f t) \quad \text{der } f = \text{frekvensen til svingebevegelsen}}$

gir

$$x'(t) = -2\pi f A \cdot \sin(2\pi f t) = v(t)$$

$$x''(t) = -4\pi^2 f^2 A \cdot \cos(2\pi f t) = a(t)$$

dvs. $x'' + \frac{k}{m}x = 0$

blir til $-4\pi^2 f^2 A\cos(2\pi f t) + \frac{k}{m}A\cos(2\pi f t) = 0$

$$-4\pi^2 f^2 + \frac{k}{m} = 0$$

$$f^2 = \frac{k}{4\pi^2 m}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

og fordi $f = \frac{1}{T}$ blir $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$

eks. 13.11 Elastisk pendel med $m = 0{,}10\,\text{Kg}$ og $k = 2{,}5\,\frac{N}{m}$ og $A = 0{,}050\,m$

a) Finn frekvens og periode.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2{,}5\,\frac{N}{m}}{0{,}10\,\text{Kg}}} = 0{,}7957\,s^{-1} = \underline{0{,}80\,\text{Hz}}$$

$$T = \frac{1}{f} = \frac{1}{0{,}7957\,s^{-1}} = 1{,}256\,s = \underline{1{,}3\,s}$$

b) Finn posisjon, fart og aks. ved $t = 2{,}0\,s$.

Still inn Kalkulator på radianer nå

$$x(t) = A\cos(2\pi f t)$$

$$x(2{,}0\,s) = 0{,}050\,m \cdot \cos(2\pi \cdot 0{,}7957\,s^{-1} \cdot 2{,}0\,s) = -0{,}04197\,m = \underline{-4{,}2\,cm}$$

$$v(t) = x'(t) = 2\pi f A \sin(2\pi f t)$$

$$v(2{,}0\,s) = -2\pi \cdot 0{,}7957\,s^{-1} \cdot 0{,}050\,m \cdot \sin(2\pi \cdot 0{,}7957\,s^{-1} \cdot 2{,}0\,s) = 0{,}1360\,\frac{m}{s} = \underline{0{,}14\,\frac{m}{s}}$$

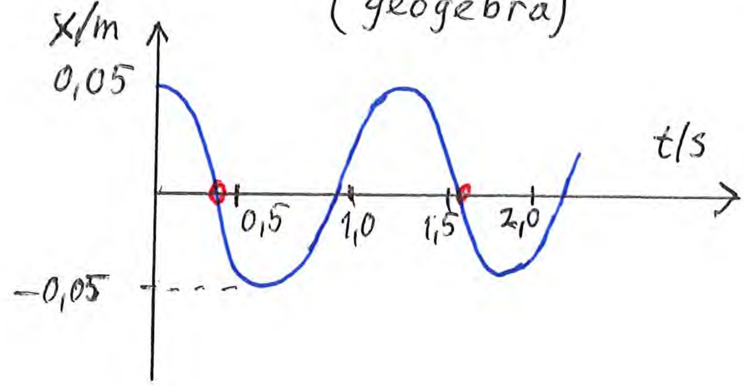
$$a(t) = v'(t) = -4\pi^2 f^2 A \cdot \cos(2\pi f t)$$

$$a(2{,}0\,s) = -4\pi^2 \cdot (0{,}7957\,s^{-1}) \cdot 0{,}050\,m \cdot \cos(2\pi \cdot 0{,}7957\,s^{-1} \cdot 2{,}0\,s) = 1{,}049\,\frac{m}{s^2} = \underline{1{,}0\,\frac{m}{s^2}}$$

c) Tegn grafen for svingningen.

$$x(t) = 0{,}050\,m \cdot \cos(2\pi \cdot 0{,}7957\,s^{-1} \cdot t)$$

(geogebra)



d) Bestem perioden fra grafen.

$$T = 1{,}5739\,s - 0{,}3147\,s = 1{,}2594\,s = \underline{1{,}3\,s}$$

Å løse sammensatte mekanikkoppgaver

Vanlige prinsipper:

1. Newtons 3 lover
2. $E_k + E_p$ bevart
3. $\sum mv$ bevart

13.27

* oversikt
* stor figur med alle opplysninger symboler og fortegn
* prøve og feile med 1,2,3