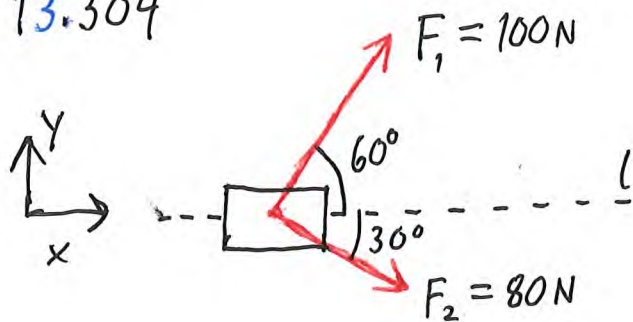


13.304



$$a) \quad \sum \vec{F} = 0 \text{ i } y\text{-r tning}$$

$$\vec{F}_{1y} + \vec{F}_{2y} + \vec{F}_G = 0$$

$$100\text{ N} \cdot \sin 60^\circ - 80\text{ N} \cdot \sin 30^\circ + F_G = 0$$

$$F_G = -46,60\text{ N} = -47\text{ N}$$

dvs 47 N i negativ y-r tning.
(normalt p  l)

$$b) \quad \sum F = 0 \text{ i } x\text{-r tning}$$

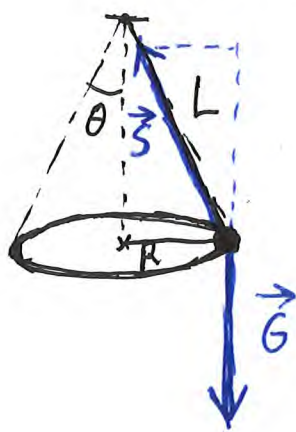
$$F_{1x} + F_{2x} + R = 0$$

$$R = -F_{1x} - F_{2x}$$

$$R = -100\text{ N} \cdot \cos 60^\circ - 80\text{ N} \cdot \cos 30^\circ = -119,2\text{ N}$$

$$= -0,12\text{ kN}$$

dvs. i negativ x-r tning.

13.311 $m = 0,090\text{ kg}$ $L = 0,75\text{ m}$ $R = 0,30\text{ m}$ 

a) Snordrag S og tyngde G virker p  kula.

$$b) \quad y: \quad \sum F = 0$$

$$S_y = G$$

$$S \cdot \cos \theta = mg$$

$$S = \frac{mg}{\cos \theta}$$

$$S = \frac{0,090\text{ kg} \cdot 9,81 \frac{\text{m}}{\text{s}^2}}{\cos 23,57^\circ} = 0,9632\text{ N}$$

$$= \underline{0,96\text{ N}}$$

$$\sin \theta = \frac{R}{L}$$

$$\theta = \sin^{-1}\left(\frac{R}{L}\right)$$

$$\theta = \sin^{-1}\left(\frac{0,30}{0,75}\right)$$

$$\theta = 23,57^\circ$$

$$c) \quad x: \quad \sum F = m \cdot a$$

$$S_x = m \cdot a$$

$$S \cdot \sin \theta = ma$$

$$a = \frac{S \cdot \sin \theta}{m} = \frac{0,9632\text{ N} \cdot \sin 23,57^\circ}{0,090\text{ kg}} = 4,279 \frac{\text{m}}{\text{s}^2}$$

$$= \underline{4,3 \frac{\text{m}}{\text{s}^2}}$$

13.307 +

$$F_L = kv^2$$

$$m = 8,0 \cdot 10^{-3} \text{ Kg}$$

$$K = 8,7 \cdot 10^{-4} \frac{\text{Ns}^2}{\text{m}^2}$$

a)



$$\Sigma F = 0$$

$$F_L = G$$

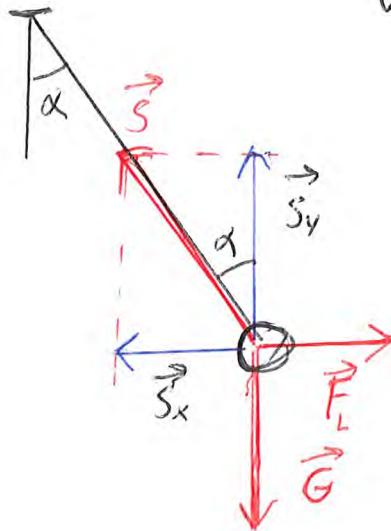
$$kv^2 = mg$$

$$v^2 = \frac{mg}{K}$$

$$v = \sqrt{\frac{mg}{K}} = \sqrt{\frac{8,0 \cdot 10^{-3} \text{ Kg} \cdot 9,81 \frac{\text{m}}{\text{s}^2}}{8,7 \cdot 10^{-4} \frac{\text{Ns}^2}{\text{m}^2}}}$$

$$v = 9,497 \frac{\text{m}}{\text{s}} = \underline{9,5 \frac{\text{m}}{\text{s}}}$$

b)



$$\alpha = 34^\circ$$

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$S_x = F_L$$

$$S_y = G$$

$$S \cdot \cos \alpha = mg$$

$$S \cdot \sin \alpha = kv^2 \quad \leftarrow S = \frac{mg}{\cos \alpha}$$

$$\frac{mg \cdot \sin \alpha}{\cos \alpha} = kv^2$$

$$mg \cdot \tan \alpha = kv^2$$

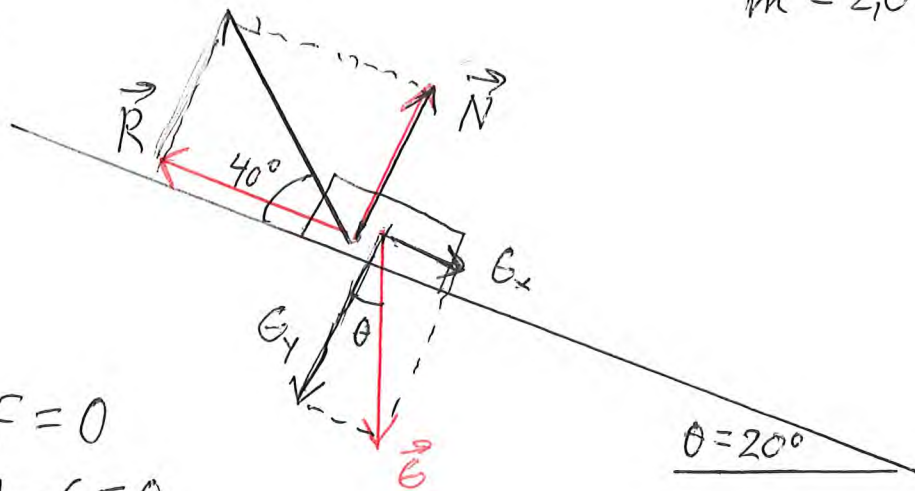
$$\frac{mg \cdot \tan \alpha}{K} = v^2$$

$$v = \sqrt{\frac{mg \cdot \tan \alpha}{K}}$$

$$v = \sqrt{\frac{8,0 \cdot 10^{-3} \text{ Kg} \cdot 9,81 \frac{\text{N}}{\text{Kg}} \cdot \tan 34^\circ}{8,7 \cdot 10^{-4} \frac{\text{Ns}^2}{\text{m}^2}}} = \underline{7,8 \frac{\text{m}}{\text{s}}}$$

$$13.315 +$$

$$m = 2,0 \text{ kg}$$



$$Y: \sum F = 0$$

$$N - G_y = 0$$

$$N = G_y$$

$$N = mg \cdot \cos 20^\circ$$

$$N = 2,0 \text{ kg} \cdot 9,81 \frac{\text{N}}{\text{kg}} \cdot \cos 20^\circ = \underline{18 \text{ N}} \quad (18,43 \text{ N})$$

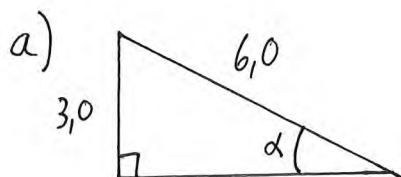
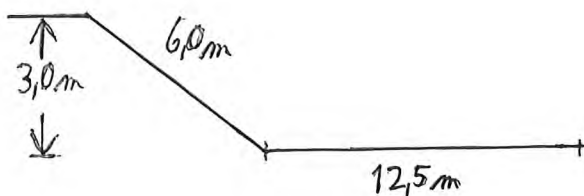
$$\tan 40^\circ = \frac{N}{R}$$

$$R = \frac{N}{\tan 40^\circ}$$

$$R = \frac{18,43 \text{ N}}{\tan 40^\circ} = \underline{22 \text{ N}}$$

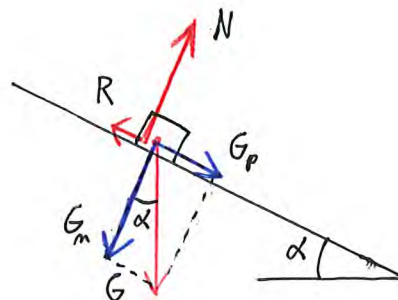
13.316 +

$$\mu = 0,15$$



$$\sin \alpha = \frac{3,0}{6,0}$$

$$\alpha = 30^\circ$$



$$\sum_p F = ma$$

$$G_p - R = ma$$

$$mg \sin \alpha - \mu N = ma$$

$$\cancel{mg} \sin \alpha - \mu \cancel{mg} \cos \alpha = \cancel{ma}$$

$$a = g(\sin \alpha - \mu \cos \alpha) = 9,81 \frac{m}{s^2} (\sin 30^\circ - 0,15 \cdot \cos 30^\circ)$$

$$= 3,630 \frac{m}{s^2}$$

$$\sum_n F = 0$$

$$N = G_m$$

$$N = mg \cos \alpha$$

$$2as = v^2 - v_0^2 \quad \text{og } v_0 = 0$$

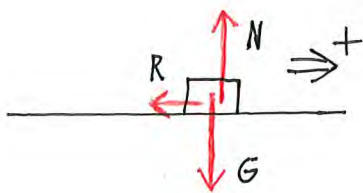
$$v = \sqrt{2as} = \sqrt{2 \cdot 3,630 \frac{m}{s^2} \cdot 6,0 m} = 6,600 \frac{m}{s} = \underline{6,6 \frac{m}{s}}$$

b)

$$2as = v^2 - v_0^2 \quad \text{og } v = 0$$

$$2as = -v_0^2$$

$$a = \frac{-v_0^2}{2s} = \frac{-(6,600 \frac{m}{s})^2}{2 \cdot 12,5 m} = -1,7424 \frac{m}{s^2}$$



$$\sum F = ma$$

$$-R = ma$$

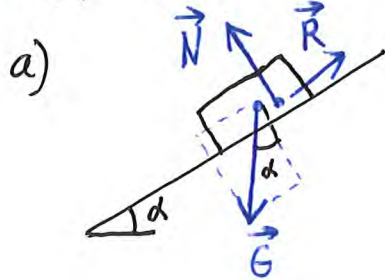
$$-\mu N = ma$$

$$-\mu \cancel{mg} = \cancel{ma}$$

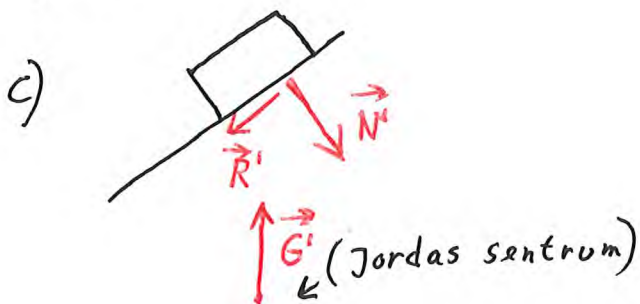
$$\mu = \frac{-a}{g} = \frac{-(-1,7424 \frac{m}{s^2})}{9,81 \frac{m}{s^2}} = 0,1776 = 0,18$$

da μ er noe større enn før

13.319 $\alpha = 20^\circ$ $m = 0,150 \text{ kg}$



b) $\sum \vec{F} = 0$



d) $\alpha = 0^\circ$ $\sum F = 0$ fordi \vec{G} ikke har noen komponent parallelt med planet.
Dette gir $R = 0$

$\alpha = 20^\circ$ $\sum F = 0$ fordi klossen er i ro,

$$R = G \cdot \sin \alpha$$

$$R = mg \cdot \sin \alpha = 0,150 \text{ kg} \cdot 9,81 \frac{\text{m}}{\text{s}^2} \cdot \sin 20^\circ$$

$$= 0,5032 \text{ N} = \underline{0,50 \text{ N}}$$

opp skråplanet.

$\alpha = 30^\circ$ $\sum F = 0$ $v = \text{konst.}$

$$R = mg \sin \alpha = 0,150 \text{ kg} \cdot 9,81 \frac{\text{m}}{\text{s}^2} \cdot \sin 30^\circ$$

$$= 0,7357 \text{ N} = \underline{0,74 \text{ N}}$$

opp skråplanet.

$\alpha = 45^\circ$ $\sum F = ma$

Vet at: $R = \mu N = 0,7357 \text{ N}$

$$R = \mu N$$

$$R = \mu \cdot mg \cos \alpha$$

$$R = 0,5773 \cdot 0,150 \text{ kg} \cdot 9,81 \frac{\text{m}}{\text{s}^2} \cdot \cos 45^\circ$$

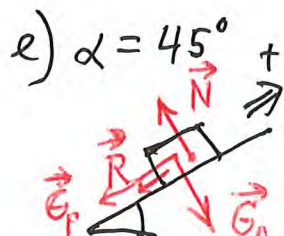
$$= 0,6006 \text{ N} = \underline{0,60 \text{ N}}$$

oppover skråplanet.

$$\mu \cdot mg \cdot \cos 30^\circ = mg \sin 30^\circ$$

$$\mu = \tan 30^\circ$$

$$\mu = 0,5773$$



$\sum F = ma$ i parallellretningen (langs planet)

$$-R - G_p = ma$$

$$\sum F = -R - mg \sin \alpha = -0,6006 \text{ N} - 0,150 \text{ kg} \cdot 9,81 \frac{\text{m}}{\text{s}^2} (\sin 45^\circ)$$

$$= -1,641 \text{ N} = \underline{-1,6 \text{ N}}$$

(ned planet)

13.319 e) $v_0 = 16 \frac{m}{s}$

$$s = v_0 t + \frac{1}{2} a t^2$$

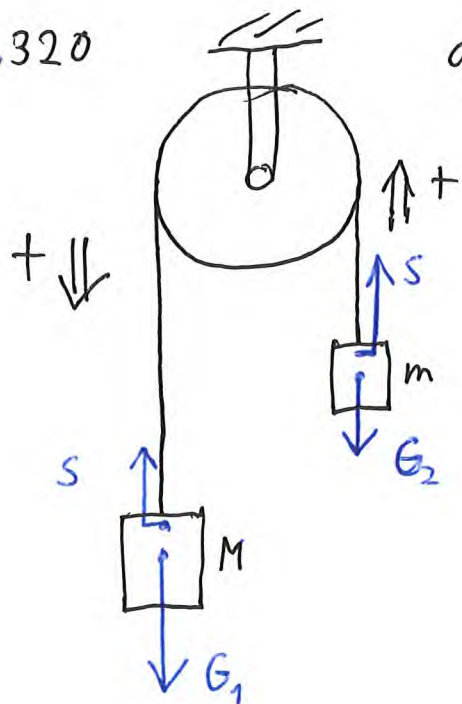
$$\Sigma F = m a$$

$$a = \frac{\Sigma F}{m} = \frac{-1,641 N}{0,150 kg} = -10,94 \frac{m}{s^2}$$

$$s = 16 \frac{m}{s} \cdot 1,0 s + \frac{1}{2} \cdot (-10,94 \frac{m}{s^2}) \cdot (1,0 s)^2 = 10,53 m = \underline{11 m}$$

13.320

a) $M = 8,0 kg$ $m = 2,0 kg$



$$\Sigma F = M a$$

$$G_1 - S = M a$$

$$\Sigma F = m a$$

$$S - G_2 = m a$$

$$\frac{S - m g}{m} = a$$

$$G_1 - S = M \cdot \frac{(S - m g)}{m}$$

$$M g - S = \frac{M}{m} \cdot S - M g$$

$$2 M g = \left(1 + \frac{M}{m}\right) \cdot S$$

$$\frac{2 M g}{\left(1 + \frac{M}{m}\right)} = S$$

$$S = \frac{2 \cdot 8,0 kg \cdot 9,81 \frac{m}{s^2}}{\left(1 + \frac{8,0 kg}{2,0 kg}\right)} = \frac{16,0 kg \cdot 9,81 \frac{m}{s^2}}{5,0} = 31,39 N = \underline{31 N}$$

b) $G_1 - S = M a$

$$S = G_1 - M a \quad \text{og} \quad S - G_2 = m a$$

gir:

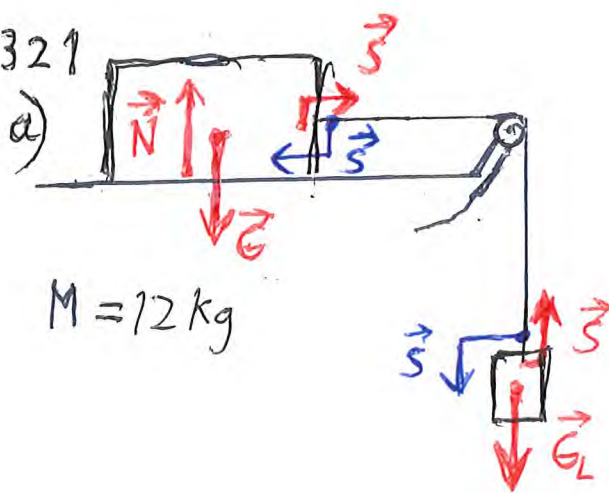
$$G_1 - M a - G_2 = m a$$

$$G_1 - G_2 = (M + m) \cdot a$$

$$a = \frac{G_1 - G_2}{(M + m)} = \frac{M g - m g}{(M + m)}$$

$$\underline{a = \frac{(M - m)}{(M + m)} \cdot g}$$

13.321



$$a = 1,4 \frac{\text{m}}{\text{s}^2}$$

$$b) \Sigma F = ma$$

$$S = 12 \text{ kg} \cdot 1,4 \frac{\text{m}}{\text{s}^2} = 16,8 \text{ N}$$

$$= \underline{17 \text{ N}}$$

$$\Sigma F = ma$$

$$G_L - S = ma$$

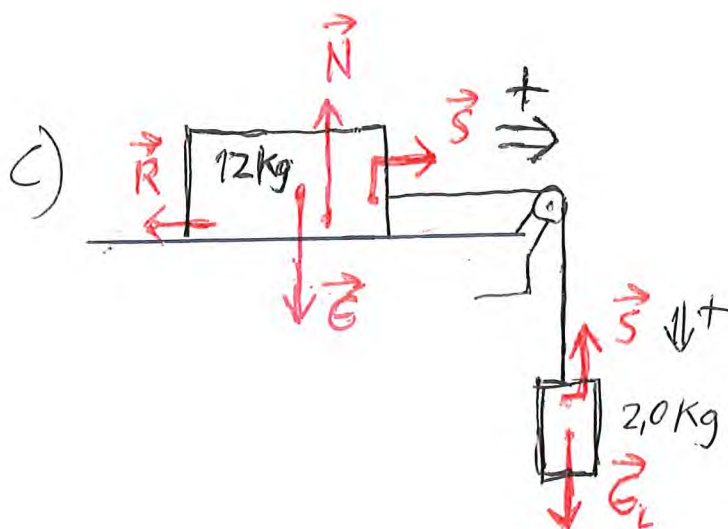
$$mg - S = ma$$

$$m(g - a) = S$$

$$m = \frac{S}{(g - a)} = \frac{16,8 \text{ N}}{(9,81 - 1,4) \frac{\text{m}}{\text{s}^2}}$$

$$= 1,997 \text{ kg}$$

$$= \underline{2,0 \text{ kg}}$$



$$\Sigma F = Ma$$

$$\Sigma F = ma$$

$$S - R = Ma$$

$$G_L - S = ma$$

$$S - \mu N = Ma$$

$$S - \mu G = Ma$$

$$S - \mu Mg = Ma$$

$$S = Ma + \mu Mg$$

$$S = M(a + \mu g)$$

$$mg - M\mu g = (M + m)a$$

$$(m - \mu M)g = (M + m)a$$

$$a = \frac{(m - \mu M)g}{(M + m)}$$

$$a = \frac{(2,0 - 0,10 \cdot 12) \text{ kg} \cdot 9,81 \frac{\text{m}}{\text{s}^2}}{(12 + 2,0) \text{ kg}}$$

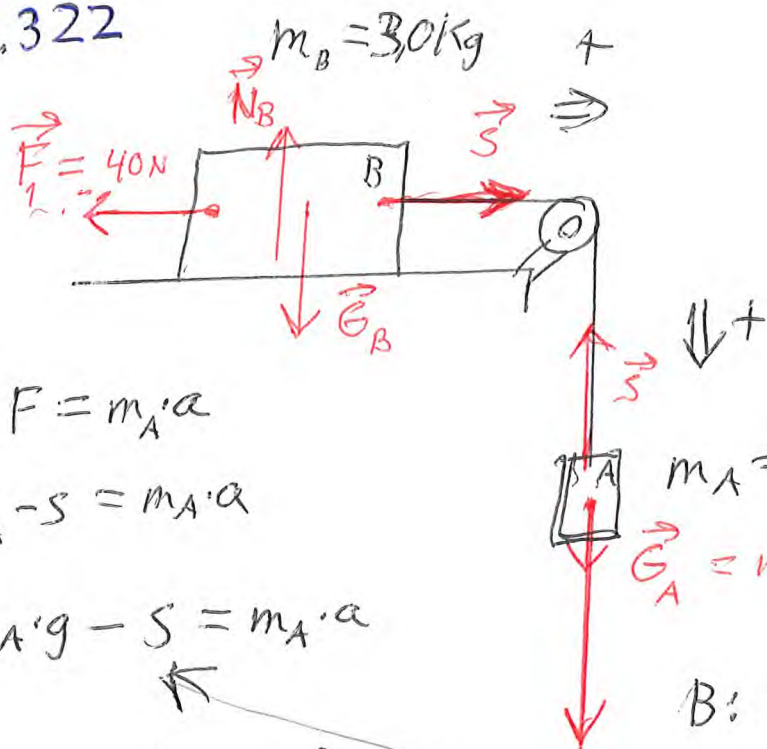
$$a = 0,5605 \frac{\text{m}}{\text{s}^2} = \underline{0,56 \frac{\text{m}}{\text{s}^2}}$$

$$G_L - M(a + \mu g) = ma$$

$$mg - M(a + \mu g) = ma$$

$$mg - Ma - M\mu g = ma$$

13.322



$$A: \sum F = m_A \cdot a$$

$$G_A - S = m_A \cdot a$$

$$m_A \cdot g - S = m_A \cdot a$$

$$m_A \cdot g - (F_1 + m_B \cdot a) = m_A \cdot a$$

$$m_A \cdot g - F_1 - m_B \cdot a = m_A \cdot a$$

$$m_A \cdot g - F_1 = m_A \cdot a + m_B \cdot a$$

$$m_A \cdot g - F_1 = (m_A + m_B) \cdot a$$

$$a = \frac{m_A \cdot g - F_1}{(m_A + m_B)} = \frac{5,0\text{ kg} \cdot 9,81 \frac{\text{m}}{\text{s}^2} - 40\text{ N}}{(5,0 + 3,0)\text{ kg}}$$

$$a = 1,137 \frac{\text{m}}{\text{s}^2}$$

$$\underline{a = 1,1 \frac{\text{m}}{\text{s}^2}}$$

nedover

$$B: \sum F = m_B \cdot a$$

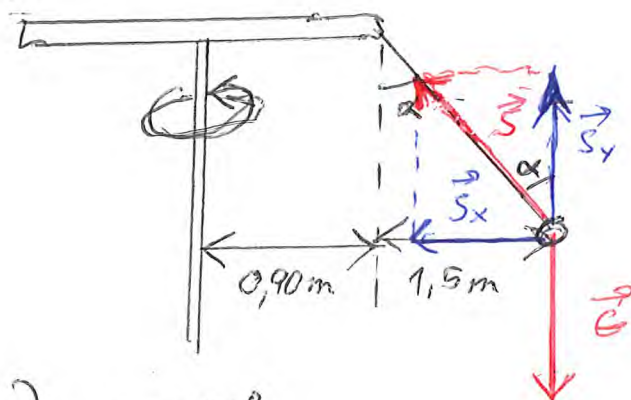
$$S - F_1 = m_B \cdot a$$

$$S = F_1 + m_B \cdot a$$

$$m_A = 5,0\text{ kg}$$

$$G_A = m_A \cdot g = 5,0 \cdot 9,81\text{ N} = 49,05\text{ N}$$

13.326 a)



$$r = 0,90\text{m} + 1,5\text{m} = 2,4\text{m}$$

b) $\alpha = 36^\circ$
 $V = ?$

Y: $\sum F = 0$

$$S_y = G$$

$$S \cdot \cos \alpha = mg$$

$$S = \frac{mg}{\cos \alpha}$$

X: $\sum F = ma$
 $S_x = m \frac{v^2}{r}$

$$S \cdot \sin \alpha = m \frac{v^2}{r}$$

$$\frac{mg \cdot \sin \alpha}{\cos \alpha} = \frac{v^2}{r}$$

$$g \cdot \tan \alpha = \frac{v^2}{r}$$

$$g \cdot \tan \alpha = v^2$$

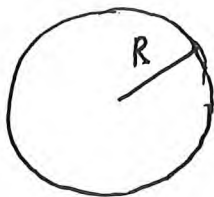
$$v = \sqrt{g \cdot \tan \alpha}$$

$$v = \sqrt{9,81 \frac{\text{m}}{\text{s}^2} \cdot 2,4\text{m} \cdot \tan 36^\circ}$$

$$v = 4,135 \frac{\text{m}}{\text{s}}$$

$$\underline{v = 4,1 \frac{\text{m}}{\text{s}}}$$

13.327 $R = 900\text{m}$



$$a = \frac{v^2}{R} = g \quad \text{or} \quad v^2 = \left(\frac{2\pi R}{T} \right)^2$$

$$\frac{\frac{4\pi^2 R^2}{T^2}}{R} = g$$

$$\frac{4\pi^2 R^2}{T^2} = gR \quad | :R$$

$$4\pi^2 R = gT^2$$

$$\frac{4\pi^2 R}{g} = T^2$$

$$T = \sqrt{\frac{4\pi^2 R}{g}} = 2\pi \sqrt{\frac{R}{g}}$$

$$T = 2\pi \sqrt{\frac{900\text{m}}{9,81\frac{\text{m}}{\text{s}^2}}} = 60,18\text{s}$$

$$\underline{T = 60,2\text{s}}$$

13.330 $r = 0,25\text{m}$

a) 20 omdr. per sek. $a = \frac{v^2}{r}$

$$v = \frac{s}{t} = \frac{20 \cdot 2\pi r}{t} = \frac{40\pi \cdot 0,25\text{m}}{1,00\text{s}} = 31,41\frac{\text{m}}{\text{s}}$$

$$a = \frac{v^2}{r} = \frac{(31,41\frac{\text{m}}{\text{s}})^2}{0,25\text{m}} = 3946\frac{\text{m}}{\text{s}^2}$$

$$\underline{= 3,9 \cdot 10^3 \frac{\text{m}}{\text{s}^2}}$$

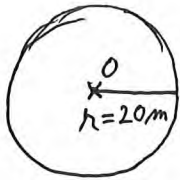
b) $m = 0,100\text{kg}$

$$\Sigma F = ma$$

$$F = m \cdot a = 0,100\text{kg} \cdot 3,946 \cdot 10^3 \frac{\text{m}}{\text{s}^2}$$

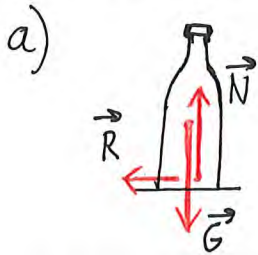
$$\underline{= 3,9 \cdot 10^2 \text{N}}$$

13.331+



$$v = 20 \frac{\text{km}}{\text{h}} = 20 \cdot \frac{1000 \text{ m}}{3600 \text{ s}} = 5,555 \frac{\text{m}}{\text{s}}$$

$$m = 0,50 \text{ kg}$$



b)

$$R = \mu N$$

$$R = \mu mg$$

$$\sum F = ma_x$$

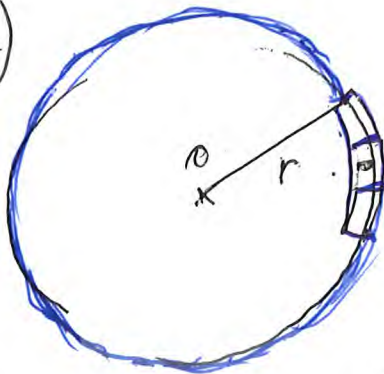
$$R = m \frac{v^2}{r}$$

$$\cancel{\mu} mg = \cancel{m} \frac{v^2}{r}$$

$$\mu = \frac{v^2}{rg} = \frac{(5,555 \frac{\text{m}}{\text{s}})^2}{20 \text{ m} \cdot 9,81 \frac{\text{m}}{\text{s}^2}}$$

$$= 0,16$$

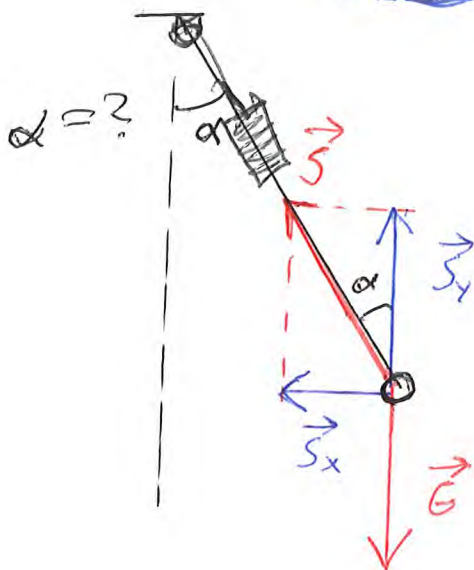
13.332 a)



$$r = 260 \text{ m}$$

$$v = 82 \frac{\text{km}}{\text{h}} = 22,77 \frac{\text{m}}{\text{s}}$$

$$m = 0,50 \text{ kg}$$



$$y: \sum F = 0$$

$$S_y = G$$

$$S \cdot \cos \alpha = mg$$

$$S = \frac{mg}{\cos \alpha}$$

$$x: \sum F = ma$$

$$S_x = m \frac{v^2}{r}$$

$$S \cdot \sin \alpha = m \frac{v^2}{r}$$

$$\downarrow$$

$$\frac{mg \cdot \sin \alpha}{\cos \alpha} = m \frac{v^2}{r}$$

$$g \cdot \tan \alpha = \frac{v^2}{r}$$

$$\tan \alpha = \frac{v^2}{gr}$$

$$\alpha = \tan^{-1} \left(\frac{v^2}{gr} \right)$$

b) Vekta viser snordraget

$$S = \frac{mg}{\cos \alpha}$$

$$S = \frac{0,50 \text{ kg} \cdot 9,81 \frac{\text{N}}{\text{kg}}}{\cos 11,49^\circ}$$

$$S = 5,005 \text{ N}$$

$$\underline{S = 5,0 \text{ N}}$$

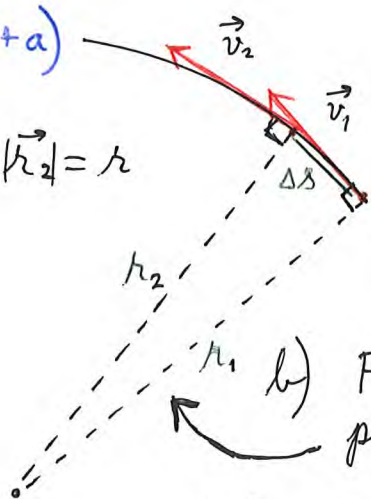
$$\alpha = \tan^{-1} \left(\frac{(22,77 \frac{\text{m}}{\text{s}})^2}{9,81 \frac{\text{m}}{\text{s}^2} \cdot 260 \text{ m}} \right)$$

$$\alpha = 11,49^\circ$$

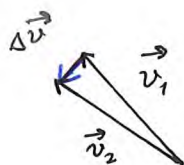
$$\underline{\alpha = 11^\circ}$$

13.333+a)

$$|\vec{r}_1| = |\vec{r}_2| = r$$



$$|\vec{v}_1| = |\vec{v}_2| = v$$



Vi ser at $\Delta \vec{v}$ står nesten vinkelrett på både \vec{v}_1 og \vec{v}_2 . Når Δt går mot null vil $\Delta \vec{v}$ dermed peke mot sentrum i sirkelen.

b) Fartstrekanten er formlik med posisjonstrekanten fordi $\vec{r}_1 \perp \vec{v}_1$ og $\vec{r}_2 \perp \vec{v}_2$

Dette gir

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}$$

$$\Delta v = \frac{v}{r} \cdot \Delta s$$

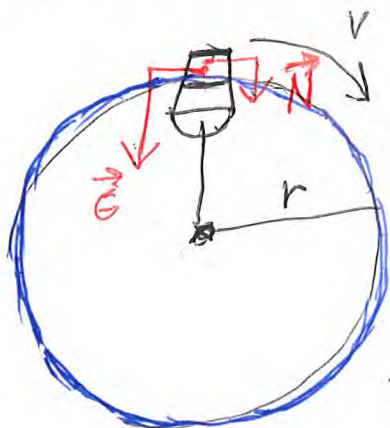
$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \cdot \frac{\Delta s}{\Delta t} \text{ som gir } a = \frac{v}{r} \cdot v = \frac{v^2}{r} \text{ når } \Delta t \rightarrow 0$$

Dermed blir $\Sigma F = ma$

til $\Sigma F = \frac{mv^2}{r}$

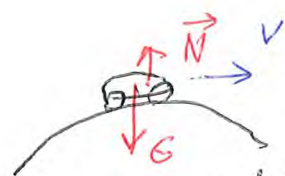
og $v = \frac{\text{omkrets}}{\text{periode}} = \frac{2\pi r}{T}$ gir $\Sigma F = \frac{m}{r} \cdot \left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 m r}{T^2}$

13.336



$$r = 1,00 \text{ m}$$

$N = 0$
 ↓ svever
 vannet faller ut av bøtta



$N = 0 \Rightarrow$ bilen hopper

$$\Sigma F = ma$$

$$N + G = m \frac{v^2}{r}$$

$$0 + mg = m \frac{v^2}{r} \quad | : m$$

$$g = \frac{v^2}{r}$$

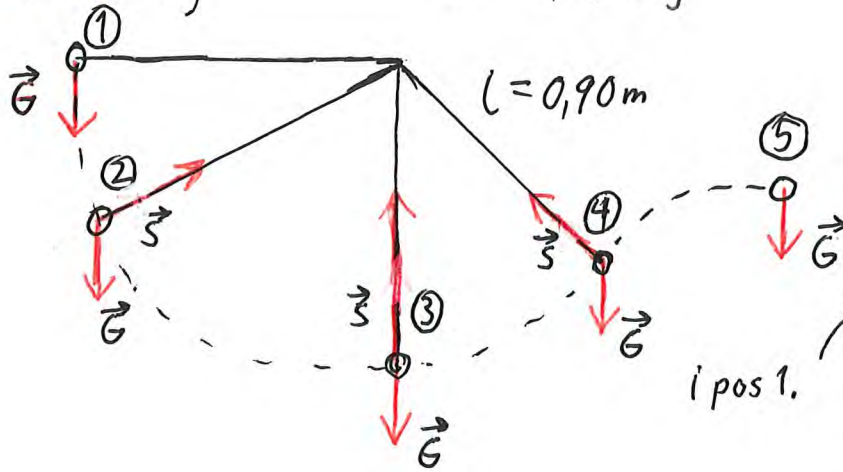
$$gr = v^2$$

$$v = \sqrt{gr} = \sqrt{9,81 \frac{\text{m}}{\text{s}^2} \cdot 1,00 \text{ m}}$$

$$v = 3,13 \frac{\text{m}}{\text{s}}$$

13.338 a)

$$m = 0,210 \text{ kg}$$



b) $S = 0$ i posisjon 1

$E_{po} = E_k$ i bunnen, 3.

$$mgh = \frac{1}{2}mv^2$$

$$mgl = \frac{1}{2}mv^2$$

$$2gl = v^2$$

$$\Sigma F = m \frac{v^2}{R} \text{ i bunnen, 3}$$

$$S - mg = m \frac{v^2}{R}$$

$$S = m \left(g + \frac{v^2}{l} \right)$$

$$= m \left(g + \frac{2gl}{l} \right) = m(g + 2g) = 3mg = 3 \cdot 0,210 \text{ kg} \cdot 9,81 \frac{\text{m}}{\text{s}^2}$$

$$= 6,180 \text{ N} = \underline{6,2 \text{ N}} \text{ i pos (3)}$$

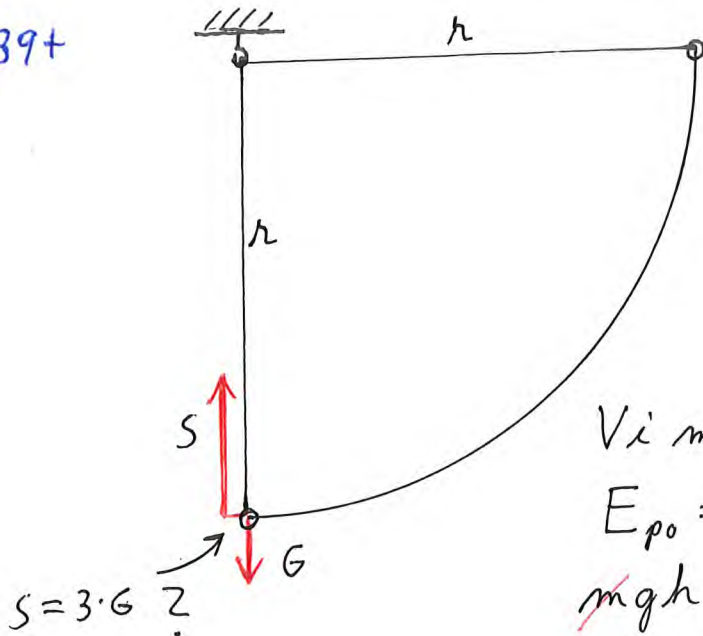
$$G = mg = 0,210 \text{ kg} \cdot 9,81 \frac{\text{m}}{\text{s}^2}$$

$$= 2,060 \text{ N} = \underline{2,1 \text{ N}}$$

rett ned
over alt.

$S = 0$ i pos. (5)

13.339+



$$\Sigma F_y = ma_y \text{ i bunnen}$$

$$S - G = ma_y$$

$$S - mg = m \frac{v^2}{r}$$

$$S = m \left(g + \frac{v^2}{r} \right)$$

Vi må finne v .

$$E_{po} = E_k$$

$$mgh = \frac{1}{2}mv^2$$

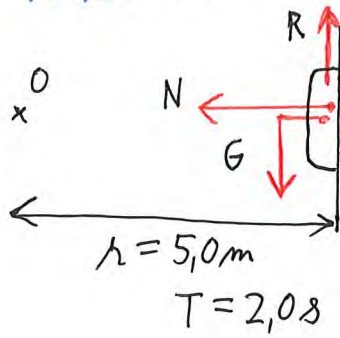
$$gr = \frac{1}{2}v^2$$

$$v^2 = 2gr$$

$$\text{Dette gir: } S = m \left(g + \frac{2gr}{r} \right) = m(g + 2g) = 3mg$$

$$= \underline{3 \cdot G} \text{ i bunnen.}$$

13.340+



$$a) \sum F_y = 0$$

$$R = G$$

$$R = mg$$

$$R = 60 \text{ kg} \cdot 9,81 \frac{\text{N}}{\text{kg}}$$

$$= 588,6 \text{ N}$$

$$= \underline{0,59 \text{ kN}}$$

$$b) \sum F_x = m a_x$$

$$N = m \frac{v^2}{r}$$

$$N = m \frac{v^2}{r}$$

$$\text{or } v = \frac{2\pi r}{T}$$

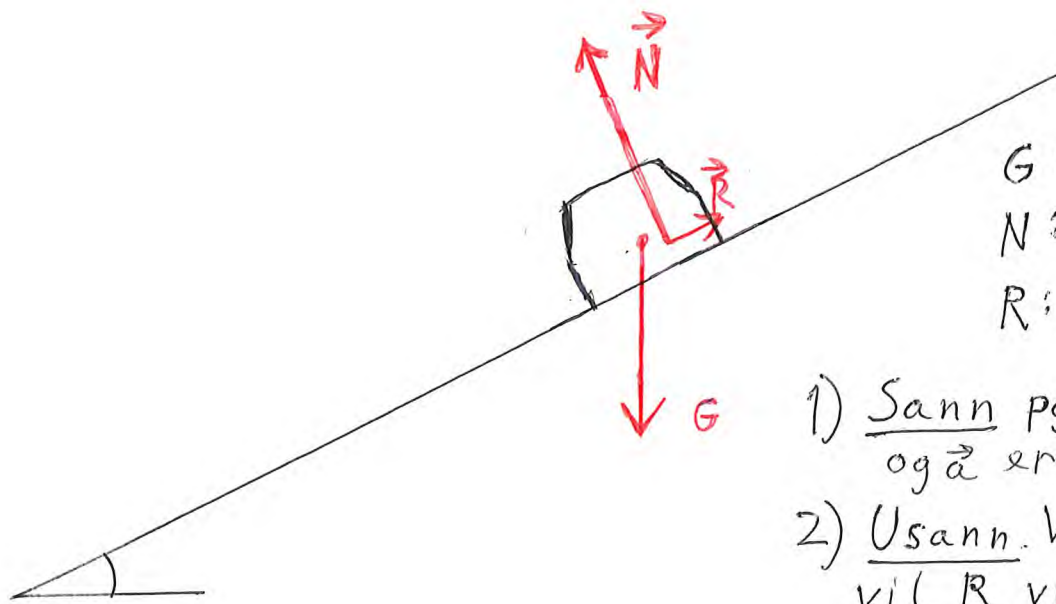
$$v = \frac{2\pi \cdot 5,0 \text{ m}}{2,08} = 15,70 \frac{\text{m}}{\text{s}}$$

$$N = 60 \text{ kg} \cdot \frac{(15,70 \frac{\text{m}}{\text{s}})^2}{5,0 \text{ m}} = 957 \text{ N}$$

$$\mu = \frac{R}{N} = \frac{588,6 \text{ N}}{957 \text{ N}} = \underline{0,20}$$

13.341+

↑ y-retning

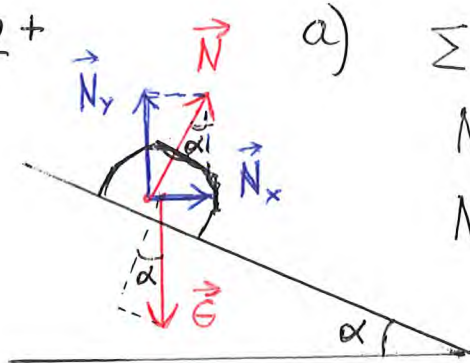


G: tyngdekraft
N: normalkraft
R: friksjonskraft

- 3) Sann. Ved en passe fart vil $R=0$ i tegningen og $N_y = G \Rightarrow N > G$

- 1) Sann pga. $\sum \vec{F} = m\vec{a}$ og \vec{a} er mot sentrum.
2) Usann. Ved stor fart vil R virke ned skråplanet. Ved lav fart vil R virke opp skråplanet.

13.342+



$$a) \quad \sum F = ma$$

$$N_x = m \frac{v^2}{r}$$

$$N \cdot \sin \alpha = m \frac{v^2}{r}$$

$$\frac{mg \cdot \sin \alpha}{\cos \alpha} = m \frac{v^2}{r}$$

$$g \cdot \tan \alpha = \frac{v^2}{r}$$

$$\tan \alpha = \frac{v^2}{gr}$$

$$\sum F_y = 0$$

$$N_y = G$$

$$N \cdot \cos \alpha = mg$$

$$N = \frac{mg}{\cos \alpha}$$

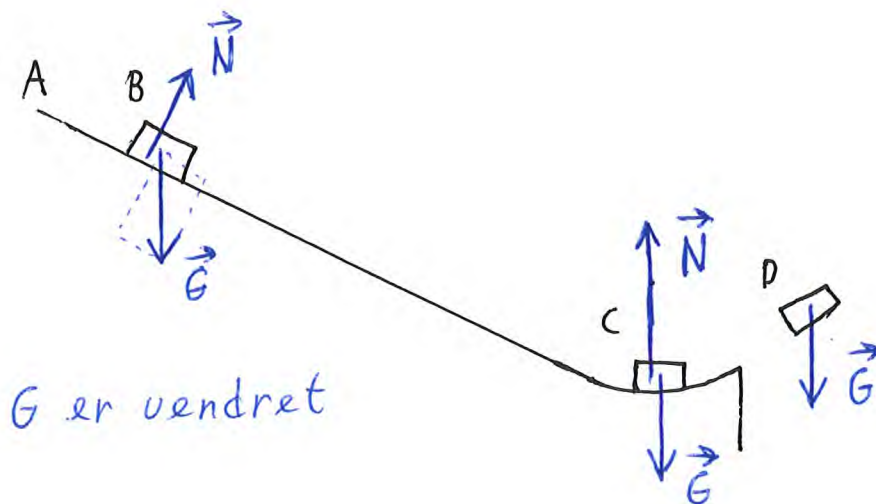
$$b) \quad r = 300 \text{ m} \quad v = 60 \frac{\text{km}}{\text{h}} = 60 \cdot \frac{1000 \text{ m}}{3600 \text{ s}} = 16,66 \frac{\text{m}}{\text{s}}$$

$$\tan \alpha = \frac{v^2}{gr}$$

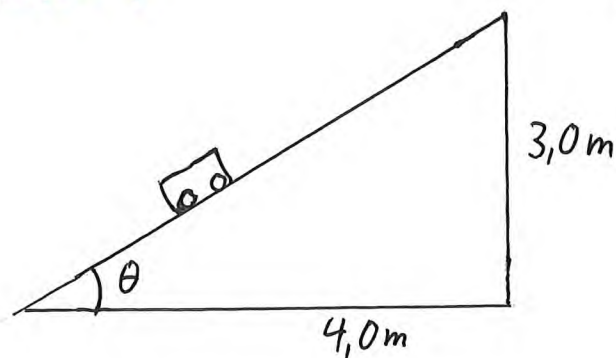
$$\alpha = \tan^{-1} \left(\frac{v^2}{gr} \right)$$

$$\alpha = \tan^{-1} \left(\frac{(16,66 \frac{\text{m}}{\text{s}})^2}{9,81 \frac{\text{m}}{\text{s}^2} \cdot 300 \text{ m}} \right) = 5,4^\circ$$

13.358



13.359



$$\tan \theta = \frac{3,0}{4,0}$$

$$\theta = \tan^{-1}\left(\frac{3,0}{4,0}\right) = 36,86^\circ$$

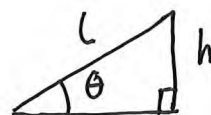
$$a) V_0 = 6,0 \frac{m}{s}$$

$$E_p = E_{k0}$$

$$mgh = \frac{1}{2} m V_0^2$$

$$gh = \frac{1}{2} V_0^2$$

$$h = \frac{V_0^2}{2g} = \frac{(6,0 \frac{m}{s})^2}{2 \cdot 9,81 \frac{m}{s^2}} = 1,834 m$$



$$\sin \theta = \frac{h}{l}$$

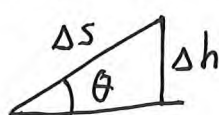
$$l = \frac{h}{\sin \theta} = \frac{1,834 m}{\sin 36,86^\circ}$$

$$l = 3,057 m = \underline{3,1 m}$$

$$b) s = 2,6 m$$

$$\Delta s = l - s$$

$$= 3,057 m - 2,6 m = 0,457 m$$



$$\sin \theta = \frac{\Delta h}{\Delta s}$$

$$\Delta h = \Delta s \cdot \sin \theta$$

$$\Delta h = 0,457 m \cdot \sin 36,86^\circ$$

$$= 0,2741 m$$

$$W_f = R \cdot s$$

$$mg \Delta h = R \cdot s$$

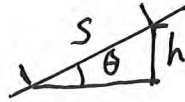
$$R = \frac{mg \Delta h}{s}$$

$$R = \frac{8,0 kg \cdot 9,81 \frac{N}{kg} \cdot 0,2741 m}{2,6 m}$$

$$R = 8,273 N = \underline{8,3 N}$$

13.359 c)

$$s = 1,2 \text{ m}$$



$$h = s \cdot \sin \theta$$

$$= 1,2 \text{ m} \cdot \sin 36,86^\circ$$

$$= 0,7198 \text{ m}$$

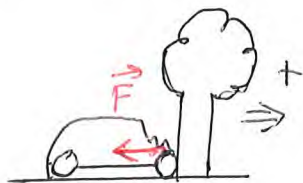
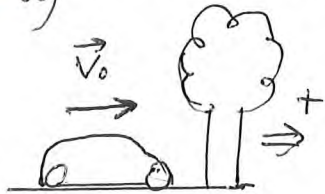
$$W = E_p + E_k$$

$$F \cdot s = mgh + \frac{1}{2}mv^2$$

$$F = \frac{m(gh + \frac{1}{2}v^2)}{s} = \frac{8,0 \text{ kg} (9,81 \frac{\text{m}}{\text{s}^2} \cdot 0,7198 \text{ m} + \frac{1}{2} \cdot (60 \frac{\text{m}}{\text{s}})^2)}{1,2 \text{ m}}$$

$$= 167 \text{ N} = \underline{0,17 \text{ kN}}$$

13.362 a) $m = 1400 \text{ kg}$



$$v_0 = 82 \frac{\text{km}}{\text{h}} = 82 \cdot \frac{1000 \text{ m}}{3600 \text{ s}} = 22,77 \frac{\text{m}}{\text{s}}$$

$$t = 0,40 \text{ s}$$

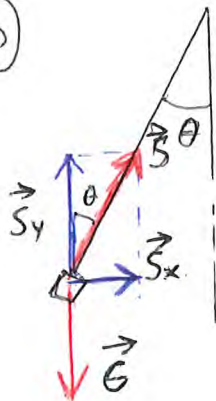
$$\Sigma F = m \bar{a}$$

$$\bar{F} = m \frac{(v - v_0)}{t}$$

$$\bar{F} = 1400 \text{ kg} \cdot \frac{(0 - 22,77 \frac{\text{m}}{\text{s}})}{0,40 \text{ s}} = 79695 \text{ N}$$

$$= \underline{80 \text{ kN}}$$

b)



$$x: \Sigma F = m \cdot a$$

$$S_x = m a$$

$$S \cdot \sin \theta = m a$$

$$\frac{m g \cdot \sin \theta}{\cos \theta} = m a$$

$$a = g \cdot \tan \theta$$

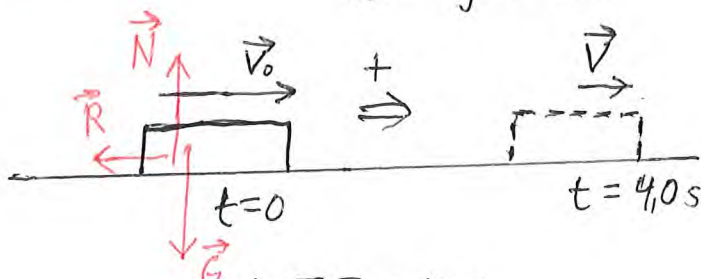
$$y: \Sigma F = 0$$

$$S_y = G$$

$$S \cdot \cos \theta = m g$$

$$S = \frac{m g}{\cos \theta}$$

c)



$$x: \Sigma F = m a$$

$$R = m a$$

$$R = m \cdot \frac{v - v_0}{t}$$

$$\mu = ?$$

$$\mu = \frac{R}{N}$$

$$v_0 = 5,0 \frac{\text{m}}{\text{s}}$$

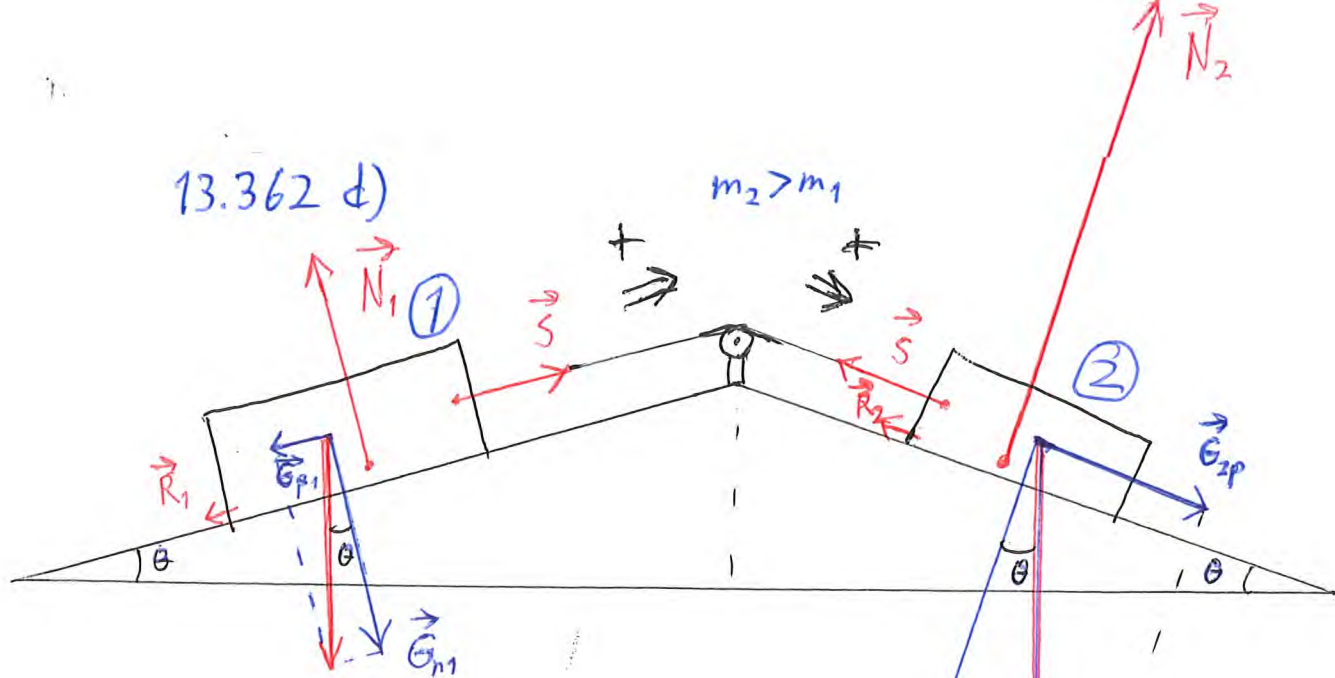
$$v = 1,0 \frac{\text{m}}{\text{s}}$$

og $N = G$ fordi $\Sigma F = 0$ i y-retning.

$$N = m g$$

$$\mu = \frac{|R|}{N} = \frac{|m \cdot (\frac{v - v_0}{t})|}{m g} = \frac{|v - v_0|}{t \cdot g}$$

$$\mu = \frac{|1,0 - 5,0| \frac{\text{m}}{\text{s}}}{4,0 \text{ s} \cdot 9,81 \frac{\text{m}}{\text{s}^2}} = \underline{0,10}$$



$$1p: \sum F = m_1 a$$

$$S - R_1 - G_{p1} = m_1 a$$

$$S - \mu N_1 - G_1 \sin \theta = m_1 a$$

$$S - \mu G_1 \cos \theta - G_1 \sin \theta = m_1 a$$

$$S = m_1 a + G_1 (\sin \theta + \mu \cos \theta)$$

$$S = m_1 a + m_1 g (\sin \theta + \mu \cos \theta)$$

$$1n: \sum F = 0$$

$$N_1 = G_{1n}$$

$$N_1 = G_1 \cos \theta$$

$$2n: \sum F = 0$$

$$N_2 = G_{2n}$$

$$N_2 = G_2 \cos \theta$$

$$2p: \sum F = m_2 a$$

$$G_{p2} - S - R_2 = m_2 a$$

$$G_2 \sin \theta - S - \mu N_2 = m_2 a$$

$$G_2 \sin \theta - S - \mu G_2 \cos \theta = m_2 a$$

$$m_2 g \sin \theta - [m_1 a + m_1 g (\sin \theta + \mu \cos \theta)] - \mu m_2 g \cos \theta = m_2 a$$

$$m_2 g \sin \theta - m_1 a - m_1 g \sin \theta - m_1 g \mu \cos \theta - \mu m_2 g \cos \theta = m_2 a$$

$$\sin \theta [m_2 g - m_1 g] - \cos \theta [m_1 g \mu + m_2 g \mu] = [m_2 + m_1] \cdot a$$

$$g \sin \theta (m_2 - m_1) - \mu g \cos \theta (m_2 + m_1) = (m_2 + m_1) \cdot a$$

$$\left(\frac{m_2 - m_1}{m_2 + m_1} \right) \cdot g \sin \theta - \mu g \cos \theta = a$$

$$m_1 = 0 \Rightarrow a = \left(\frac{m_2 - 0}{m_2 + 0} \right) \cdot g \sin \theta - \mu g \cos \theta$$

$$a = g \sin \theta - \mu g \cos \theta$$

$$a = g (\sin \theta - \mu \cos \theta)$$

13.364

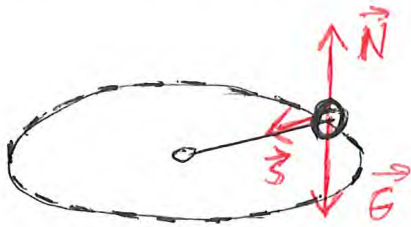
$m = 0,060 \text{ kg}$

$S_{\max} = 10 \text{ N}$

$r = 0,84 \text{ m}$

T

a)



$$b) T = 2\pi \sqrt{\frac{mr}{S}}$$

$$T = 2\pi \sqrt{\frac{0,060 \cdot 0,84 \text{ m}}{10 \text{ N}}} = 0,4460 \text{ s}$$

$$= \underline{0,45 \text{ s}}$$

$$v = \frac{s}{t} = \frac{2\pi r}{T} = \frac{2\pi \cdot 0,84 \text{ m}}{0,4460 \text{ s}} = 11,83 \frac{\text{m}}{\text{s}}$$

$$= \underline{12 \frac{\text{m}}{\text{s}}}$$

$$c) v = \frac{s}{t} \text{ gir } v_b = \frac{2\pi r}{T} \text{ for banefarten.}$$

$$T = \frac{2\pi r}{v_b} \quad \text{og } \sum F_x = m a_x \text{ gir}$$

$$S = m \frac{v_b^2}{r}$$

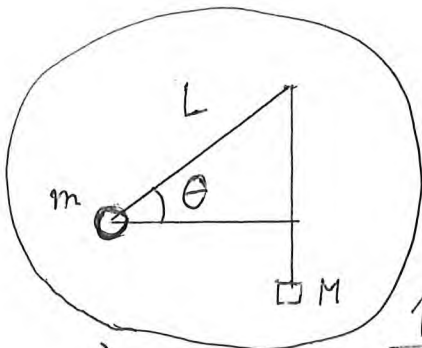
$$\frac{S \cdot r}{m} = v_b^2$$

$$v_b = \sqrt{\frac{S \cdot r}{m}}$$

$$T = 2\pi \cdot \frac{r}{\sqrt{\frac{S \cdot r}{m}}} = 2\pi r \cdot \sqrt{\frac{m}{S \cdot r}}$$

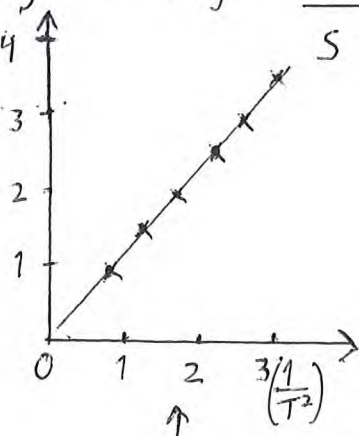
$$T = 2\pi \cdot \sqrt{r^2} \cdot \sqrt{\frac{m}{S \cdot r}} = 2\pi \cdot \sqrt{\frac{m \cdot r^2}{S \cdot r}}$$

$$T = 2\pi \sqrt{\frac{mr}{S}}$$



$$d) S = Mg$$

(S)



$$\frac{S}{\left(\frac{1}{T^2}\right)} = \text{konst}$$

$\frac{1}{T^2}$	$0,8416 \text{ s}^{-2}$	$1,2345 \text{ s}^{-2}$	$1,6865 \text{ s}^{-2}$	$2,1005 \text{ s}^{-2}$	$2,5195 \text{ s}^{-2}$	$2,9725 \text{ s}^{-2}$
S	0,981 N	1,471 N	1,962 N	2,452 N	2,943 N	3,433 N

$$e) \sum F = m \frac{v^2}{r}$$

$$S_x = m \cdot \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

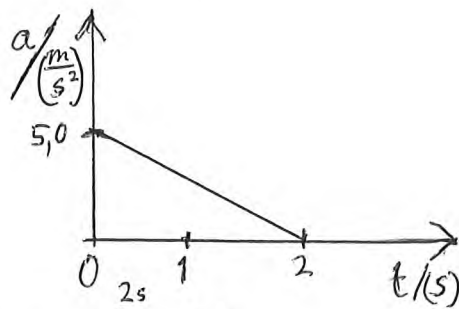
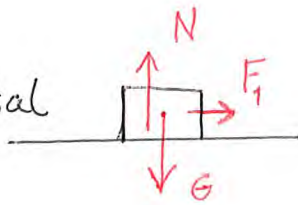
$$S \cdot \cos \theta = 4\pi^2 \cdot m \cdot \frac{r}{T^2}$$

$$S \cdot \cos \theta = 4\pi^2 \cdot m \cdot \frac{L \cdot \cos \theta}{T^2}$$

$$\Leftrightarrow \frac{S}{\left(\frac{1}{T^2}\right)} = \text{konst.} \quad \underline{\text{stemmer}}$$

13.366 a) $m = 10 \text{ kg}$

Antar F_1 horisontal



$$a(t=0) = \frac{50 \text{ N}}{10 \text{ kg}} = 5,0 \frac{\text{m}}{\text{s}^2}$$

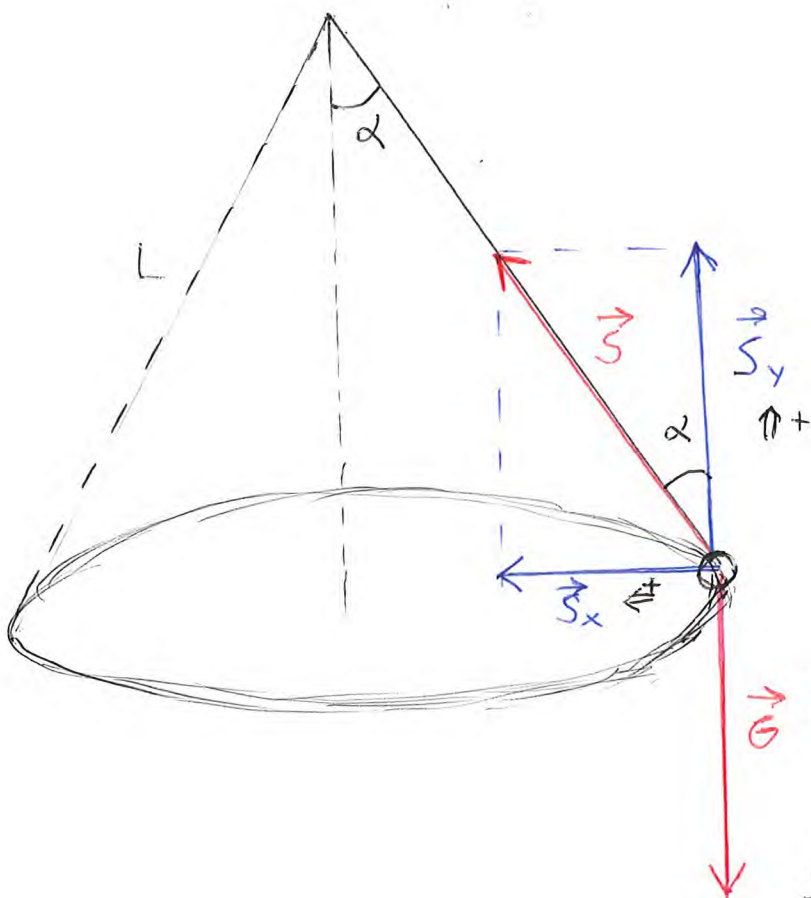
$$F_1(t) \\ F_1(0) = 50 \text{ N} \\ F_1(2,0 \text{ s}) = 0$$

$$\sum F = ma \\ F_1 = ma \\ a = \frac{F_1}{m}$$

$$v = \int_0^t a dt = \text{Areal av trekant} = \frac{(a+a_0)}{2} \cdot t \\ = \frac{(0 + 5,0 \frac{\text{m}}{\text{s}^2})}{2} \cdot 2,0 \text{ s} = 5,0 \frac{\text{m}}{\text{s}}$$

b) $m = 0,20 \text{ kg}$
 $L = 1,20 \text{ m}$
 $\alpha = 30^\circ$

$$x: \sum F = ma \\ S_x = m \frac{v^2}{r} \\ S \cdot \sin \alpha = \sum F$$



$$y: \sum F = 0 \\ S_y - G = 0 \\ S_y = G$$

$$S \cdot \cos \alpha = mg \\ S = \frac{mg}{\cos \alpha}$$

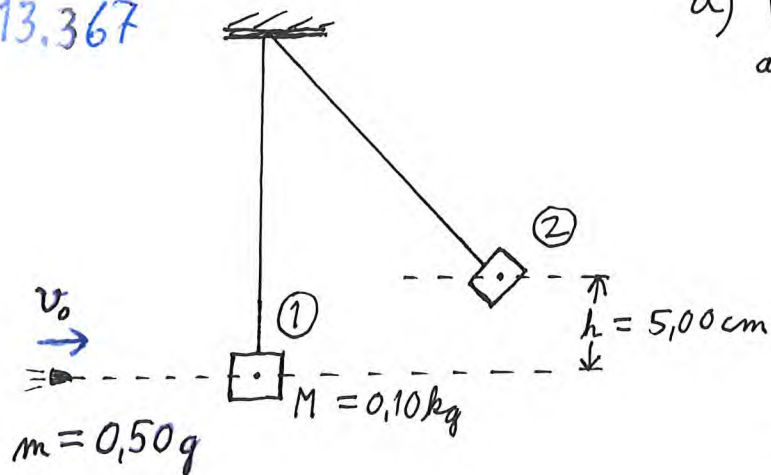
$$S \cdot \sin \alpha = \frac{mg}{\cos \alpha} \cdot \sin \alpha$$

$$\sum F = mg \cdot \tan \alpha$$

$$\sum F = 0,20 \text{ kg} \cdot 9,81 \frac{\text{N}}{\text{kg}} \cdot \tan 30^\circ \\ = 1,1 \text{ N} \quad (1,132 \text{ N})$$

$$S = \frac{mg}{\cos \alpha} = \frac{0,20 \cdot 9,81 \text{ N}}{\cos 30^\circ} \\ = 2,265 \text{ N} = 2,3 \text{ N}$$

13.367



a) Vis at $v = 1,0 \frac{m}{s}$ rett etter at kula har festet seg.

$$E_{p2} = E_{k1}$$

$$(m+M)gh = (m+M)v^2$$

$$2gh = v^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \cdot 9,81 \cdot 0,0500} \frac{m}{s} = 0,9904 \frac{m}{s} = \underline{1,0 \frac{m}{s}}$$

b) $(\sum mv)_{for} = (\sum mv)_{etter}$

$$mv + 0 = (m+M)U$$

U er v fra a)

$$= \left(\frac{m+M}{m} \right) U = \left(\frac{0,00050 + 0,10}{0,00050} \right) \cdot 1,0 \frac{m}{s} = 201 \frac{m}{s} = \underline{0,20 \frac{km}{s}}$$

c) Man kan filme kula med et høyhastighetskamera.

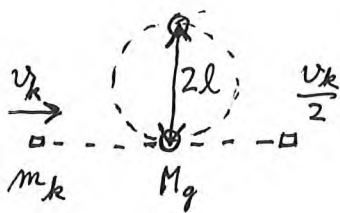
Fordel: Pålitelig resultat hele veien.

Ulempe: Trenger dyrt utstyr.

Man kan bruke lasersensorer og la kula passere to stykker og måle tida mellom passeringene når man kjenner avstanden.

Fordel: Pålitelig gjennomsnittsmåling av farten.

Ulempe: Mye utstyr.



d) $(\sum mv)_{for} = (\sum mv)_{etter}$

$$m_k v_k + 0 = \frac{1}{2} m_k v_k + M_g V_g$$

$$\frac{1}{2} m_k v_k = M_g V_g$$

$$\frac{1}{2} m_k v_k = M_g \cdot 2\sqrt{gl}$$

$$m_k v_k = M_g \cdot 4\sqrt{gl}$$

$$\underline{v_k = \frac{4M_g}{m_k} \sqrt{gl}}$$

topp \rightarrow $E_p = E_k$ \leftarrow bunn

$$E_p = E_k$$

$$M_g gh = \frac{1}{2} M_g V_g^2$$

$$g \cdot 2l = \frac{1}{2} V_g^2$$

$$4gl = V_g^2$$

$$V_g = \sqrt{4gl}$$

$$V_g = 2\sqrt{gl}$$

13.368 20 omdr./min.

$$A = \pi r^2$$

$$r = 33 \text{ m}$$

$$V > V_{\min}$$

$$P_{el} = 1,65 \cdot 10^6 \text{ W}$$

$$E_{\text{tot}} = 4,8 \cdot 10^9 \text{ Wh}$$

$$P_1 = \frac{1}{2} A \rho v^3$$

$$P_{el1} = 0,40 \cdot P_1$$

$$V < V_{\min}$$

$$P_{el} = 0$$

$$\rho = 1,29 \frac{\text{kg}}{\text{m}^3}$$

a) $v = ?$ hvis $P_{el1} = P_{el}$

$$0,40 \cdot \frac{1}{2} A \rho v^3 = P_{el}$$

$$0,40 \cdot \frac{1}{2} \cdot \pi r^2 \cdot \rho v^3 = P_{el}$$

$$v^3 = \frac{2 P_{el}}{0,40 \pi r^2 \cdot \rho}$$

$$v = \sqrt[3]{\frac{2 P_{el}}{0,40 \pi r^2 \rho}}$$

$$v = \sqrt[3]{\frac{2 \cdot 1,65 \cdot 10^6 \frac{\text{J}}{\text{s}}}{0,40 \cdot \pi \cdot (33 \text{ m})^2 \cdot 1,29 \frac{\text{kg}}{\text{m}^3}}} = 12,31 \frac{\text{m}}{\text{s}} = \underline{12 \frac{\text{m}}{\text{s}}}$$

$$P_{el1} \cdot t = E_{\text{tot}}$$

$$t = \frac{E_{\text{tot}}}{P_{el1}} = \frac{4,8 \cdot 10^9 \frac{\text{Wh}}{\text{s}} \cdot h}{1,65 \cdot 10^6 \frac{\text{W}}{\text{s}}} = 2909 h = 121 \text{ døgn}$$

$$= 0,33 \text{ år} = \underline{33\% \text{ av året}}$$

b) Antall omdreiningen gir $h = 67 \text{ m} + 33 \text{ m} = 100 \text{ m}$

$$s = v_0 t + \frac{1}{2} a t^2$$

$$h = \frac{1}{2} g t^2 \quad (v_{0y} = 0)$$

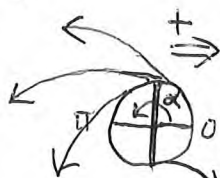
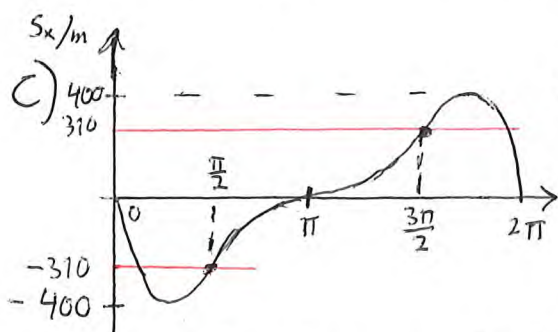
$$\frac{2h}{g} = t^2$$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 100 \text{ m}}{9,81 \frac{\text{m}}{\text{s}^2}}} = 4,515 \text{ s}$$

$$v = \frac{20 \cdot 2 \pi r}{60 \text{ s}} = \frac{40 \pi \cdot 33 \text{ m}}{60 \text{ s}} = 69,11 \frac{\text{m}}{\text{s}}$$

$$s_x = v_x \cdot t = 69,11 \frac{\text{m}}{\text{s}} \cdot 4,515 \text{ s}$$

$$= 312 \text{ m} = \underline{0,31 \text{ km}}$$



Avstand $s_x = 0$ for $\alpha = 0, \pi, 2\pi$ fordi møllebladet da kaster isklumpen rett opp eller rett ned.

s_x blir som for et horisontalt kast for $\alpha = \frac{\pi}{2}$ og $\frac{3\pi}{2}$, men kastet blir

enda lenger litt før toppen og litt etter bunnen pga. lenger tid i lufta kombinert med stor $|v_x|$.