1. Time 09.03.2023

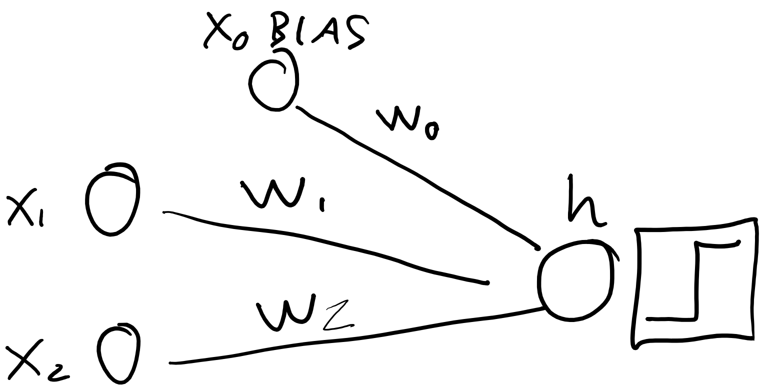
lørdag 4. mars 2023 17:57

MATRICES AND VECTORS

- We have now looked at some motivation and background.

Let's look at the Perceptron:

(but this is similar for Regression and other models)



$\vec{x}$ $\underline{x}$

Calculate input:

$$x_0 \cdot w_0 + x_1 \cdot w_1 + x_2 \cdot w_2 = \underline{x} \cdot \underline{w}$$

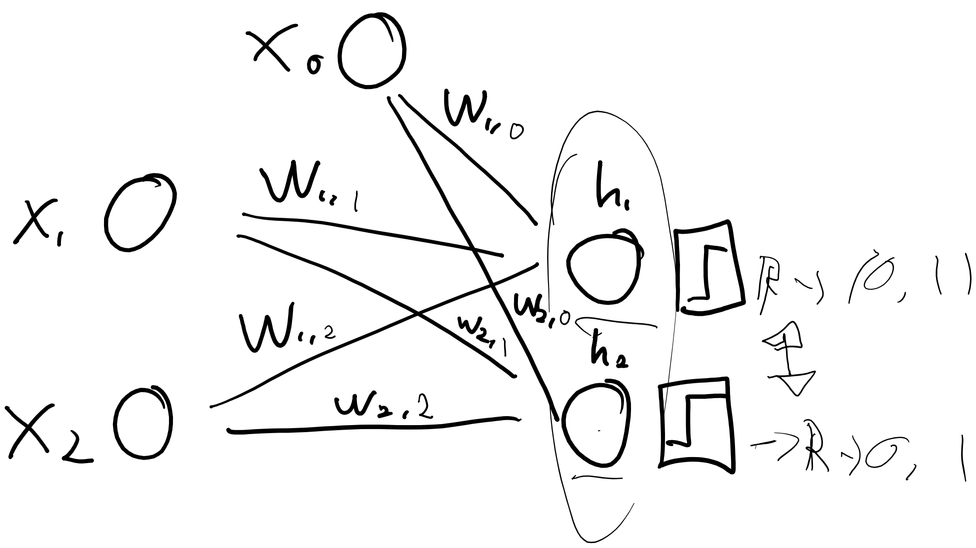
$$= \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \cdot \begin{bmatrix} w_0 \\ w \\ w_2 \end{bmatrix}$$

For example $\underline{x} = (-1, 2, 1)$

$\underline{w} = (1, 4, 3)$

$\underline{x} \cdot \underline{w} = -1 \cdot 1 + 2 \cdot 4 + 3 \cdot 1 = -1 + 8 + 3 = 10$

Multiple outputs:



1 2 3 9

$\underline{W}$ is now a matrix

$$W = \begin{bmatrix} w_{1,1} & w_{1,2} & w_{1,3} \\ w_{2,1} & w_{2,2} & w_{2,3} \end{bmatrix} = \begin{bmatrix} \underline{w}_1 \\ \underline{w}_2 \end{bmatrix}$$

so $\underline{w}_1$ is a vector for h_1-outputweight, $\underline{w}_2$ for h_2

We want output $\underline{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$ – Values in nodes

$$W \underline{x} = \begin{bmatrix} W_{1,0} & W_{1,1} & W_{1,2} \\ W_{2,0} & W_{2,1} & W_{2,2} \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}$$

$$= \begin{bmatrix} W_{1,0} \cdot x_0 + W_{1,1} \cdot x_1 + W_{1,2} \cdot x_2 \\ W_{2,0} \cdot x_0 + W_{2,1} \cdot x_2 + W_{2,2} \cdot x_2 \end{bmatrix} = \begin{bmatrix} \underline{W_1} \cdot \underline{x} \\ \underline{W_2} \cdot \underline{x} \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

Example:

$$\underline{x} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 3 & -2 \\ -5 & 0 & 1 \end{bmatrix} \begin{matrix} (\overset{-1}{1 \cdot -1} + \overset{6}{3 \cdot 2} + \overset{-2}{-2 \cdot 1}) \\ (\overset{5}{-1 \cdot -5} + \overset{0}{0 \cdot 2} + \overset{1}{1 \cdot 1}) \end{matrix} \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

Next step: Multiple inputs at once.

$$X = \begin{bmatrix} \underline{X}_1 & \underline{X}_2 \end{bmatrix} = \begin{bmatrix} X_{1,0} & X_{2,1} \\ X_{1,1} & X_{2,2} \\ X_{1,2} & X_{2,3} \end{bmatrix}$$

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \qquad \underline{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

We get 2 sets of $\underline{h}$, one for each input.

Same logic, now matrix multiplication:

$$W X = \begin{bmatrix} W_{1,0} & W_{1,1} & W_{1,2} \\ W_{2,0} & W_{2,1} & W_{2,2} \end{bmatrix} \begin{bmatrix} X_{1,0} & X_{2,0} \\ X_{1,1} & X_{2,1} \\ X_{1,2} & X_{2,2} \end{bmatrix} = \begin{bmatrix} \underline{W}_1 \cdot \underline{X}_1 & \underline{W}_1 \cdot \underline{X}_2 \\ \underline{W}_2 \cdot \underline{X}_1 & \underline{W}_2 \cdot \underline{W}_2 \end{bmatrix}$$

$$
W = \begin{bmatrix} 1 & 3 & -2 \\ -5 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & -1 \\ 2 & -3 \\ 1 & 5 \end{bmatrix}
$$

$$
\left[\begin{array}{c|c}
\overset{-1}{1\cdot -1} + \overset{6}{3\cdot 2} + \overset{-2}{-2\cdot 1} & \overset{-1}{1\cdot -1} + \overset{-9}{3\cdot -3} + \overset{-10}{-2\cdot 5} \\
\hline
\overset{5}{-5\cdot -1} + \overset{0}{2\cdot 0} + \overset{1}{1\cdot 1} & \overset{+5}{-5\cdot -1} + \overset{0}{0\cdot -3} + \overset{5}{1\cdot 5}
\end{array}\right]
\qquad
\left[\begin{array}{c|c} 3 & -20 \\ \hline 6 & 10 \end{array}\right]
$$

Watch out! Row vs Colums, which is which?

Convention: <u>Rows</u> then Colums

$W_{1,2}$ $\qquad$ $W(1,2]$ $\qquad$ $W[1][2]$

1st row $\qquad$ 2nd colum

$$
\begin{bmatrix} W_{1,1} & W_{1,2} & W_{1,3}, & \cdots \\ W_{2,1} & W_{2,2} & W_{2,2} & \cdots \end{bmatrix}
$$

However, the book sometimes swithes.

Our notation for X was **bad**.

We want one row to be one input-vector.

Change from $X = \begin{bmatrix} x_{1,0} & x_{2,0} \\ x_{1,1} & x_{2,1} \\ x_{1,2} & x_{2,2} \end{bmatrix} = \begin{bmatrix} \underline{x_1} & \underline{x_2} \end{bmatrix}$

$x(1, 2)$

to $X = \begin{bmatrix} x_{1,0} & x_{1,1} & x_{1,2} \\ x_{2,0} & x_{2,1} & x_{2,2} \end{bmatrix} = \begin{bmatrix} \underline{x_1} \\ \underline{x_2} \end{bmatrix}$ ✓

$\begin{bmatrix} w_{1,0} & w_{1,1} & w_{1,2} \end{bmatrix}$

But now, how do we calculate $W \cdot X$?

Dimension does not line up.

This is where we use the <u>transpose</u>

x.T

We define:

$$X^T = \left(\begin{bmatrix} X_{1,0} & X_{1,1} & X_{1,2} \\ X_{2,0} & X_{2,1} & X_{2,2} \end{bmatrix}\right)^T = \begin{bmatrix} X_{1,0} & X_{2,0} \\ X_{1,1} & X_{2,1} \\ X_{1,2} & X_{2,2} \end{bmatrix}$$

Transposing matrices swaps columns and rows.

We often need both X and X^T in calculations.

$W X^T$ x.T

We can write dot product $\underline{W} \cdot \underline{X}$ as $\underline{W}^T \underline{X}$.

Vectors can be both column-vectors and row-vectors.

In machine learning, we sometimes also use elementwise or Hamard multiplication.

$$\underline{w} \odot \underline{x} = \begin{bmatrix} w_0 \cdot x_0 \\ w_1 \cdot x_1 \\ w_2 \cdot x_2 \end{bmatrix}$$

$x \circledast y$

Last thing, dimensions:

$$\underbrace{\underset{(m)}{\underline{w}} \;,\; \underset{(m)}{\underline{x}}}_{eq} \;,\; \underset{(1)}{\underline{w} \cdot \underline{x}}$$

dimension, (rows, cols)

$$\underbrace{\underset{(c \times m)}{W} \;,\; \underset{(m)}{\underline{x}}}_{=} \;,\; \underset{(c)}{W \cdot \underline{x}}$$

~~$W \cdot x$ $m \times c$ m~~

$$\underbrace{\underset{(c \times m)}{W} \quad \underset{(m \times n)}{X^T}}_{=} \;,\; \underset{(c \times n)}{WX^T}$$

w @ x.T

STATQUEST

Please watch 3Blue1Brown for

the best videos about Linear Algebra!