On Nonlinear Interaction of the Electromagnetic Wave with the Relativistic Mirror

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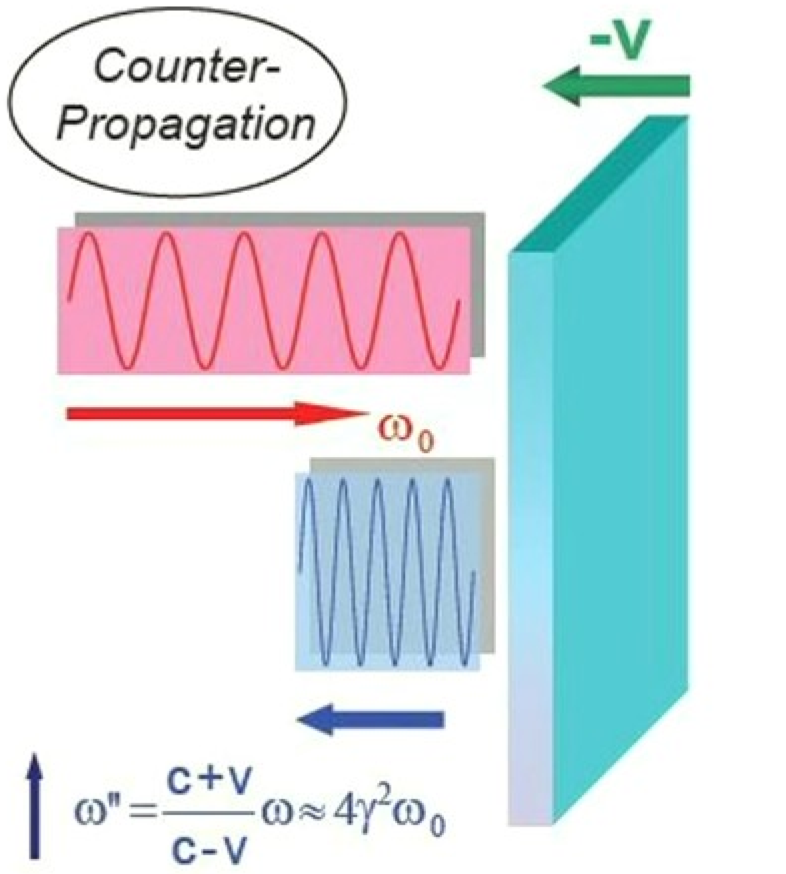
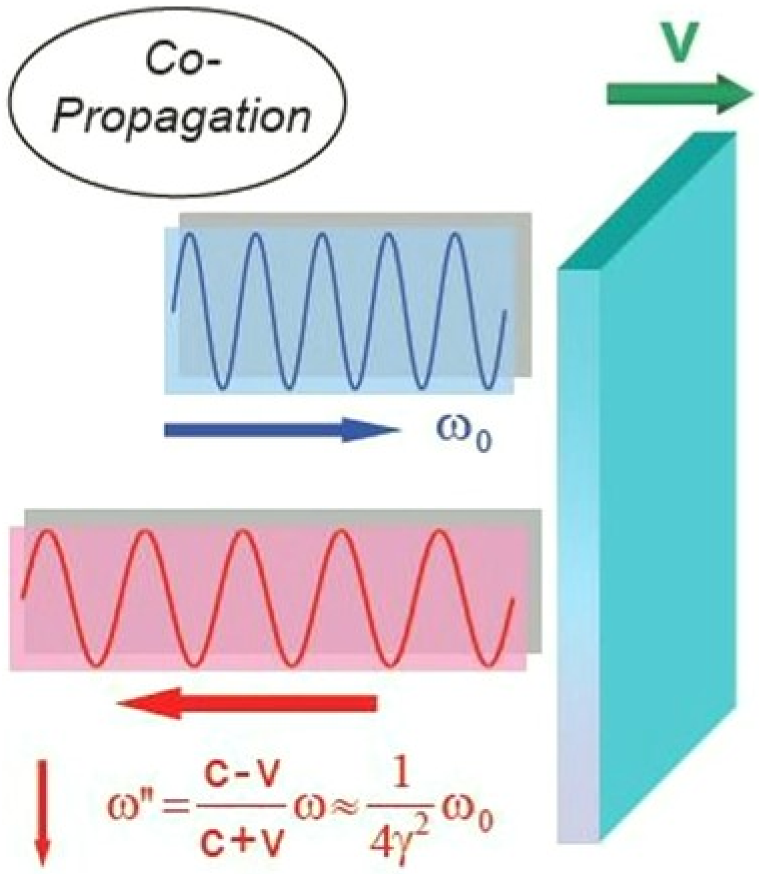
Reflection of EM wave at the relativistic mirror



The mirror "compresses" the EM wave

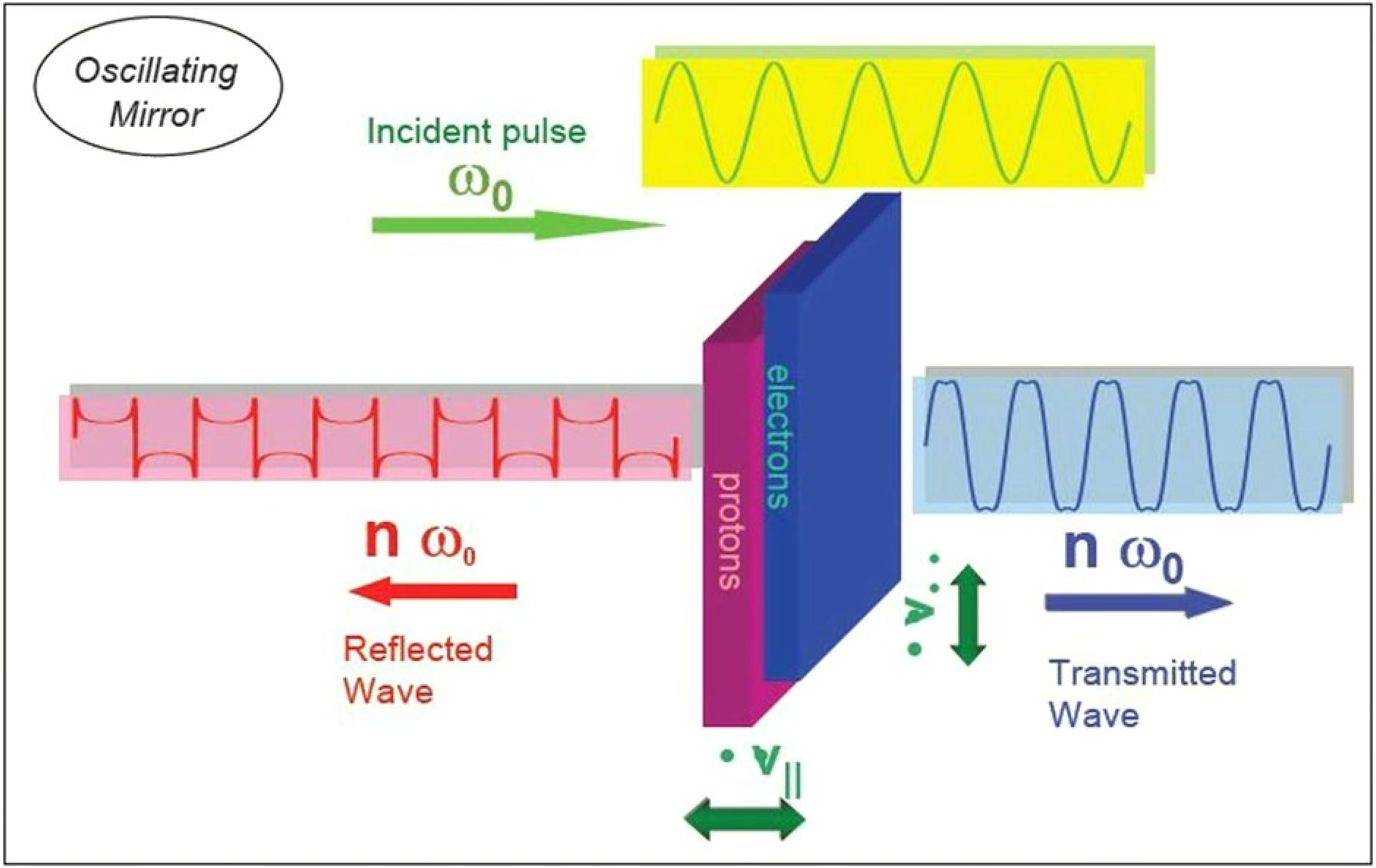


The EM wave pushes the mirror

W. Hicks, Philos. Mag. 3, 9 (1902); M. Abraham, Ann. Phys. 14, 236 (1904);

A. Einstein, Ann. Phys. (Leipzig) 17, 891 (1905); W. Pauli, Theory of Relativity (Pergamon Press, 1958)

High Order Harmonics from the Oscillating Relativistic Mirror



Coherent Nonlinear Thomson Scattering (I)

The electric field in the scattered EM wave is given by virtue of the Lienard-Wiechert potentials by the 1D expression

$$\mathrm{E}_{\perp}(x,t)=\frac{2\pi n_0 e l \mathrm{v}_{\perp}(t')}{c-\mathrm{v}_{\parallel}(t')\mathrm{sign}(x-x(t'))}$$

Here t' is the retarded time: $t'-x(t')/c=t-x/c$

with x and t – the coordinate and time of the observation point and x(t') - the mirror coordinate at t', and the electron density n_0 and l the layer thickness.

There is a constraint on the intensity of scattered radiation due to the limiting electric current: because $|j|\leq n_0 ec$

Thin semi-transparent relativistic mirror model was used and it can be used to describe analytically :

a) High-order harmonics;

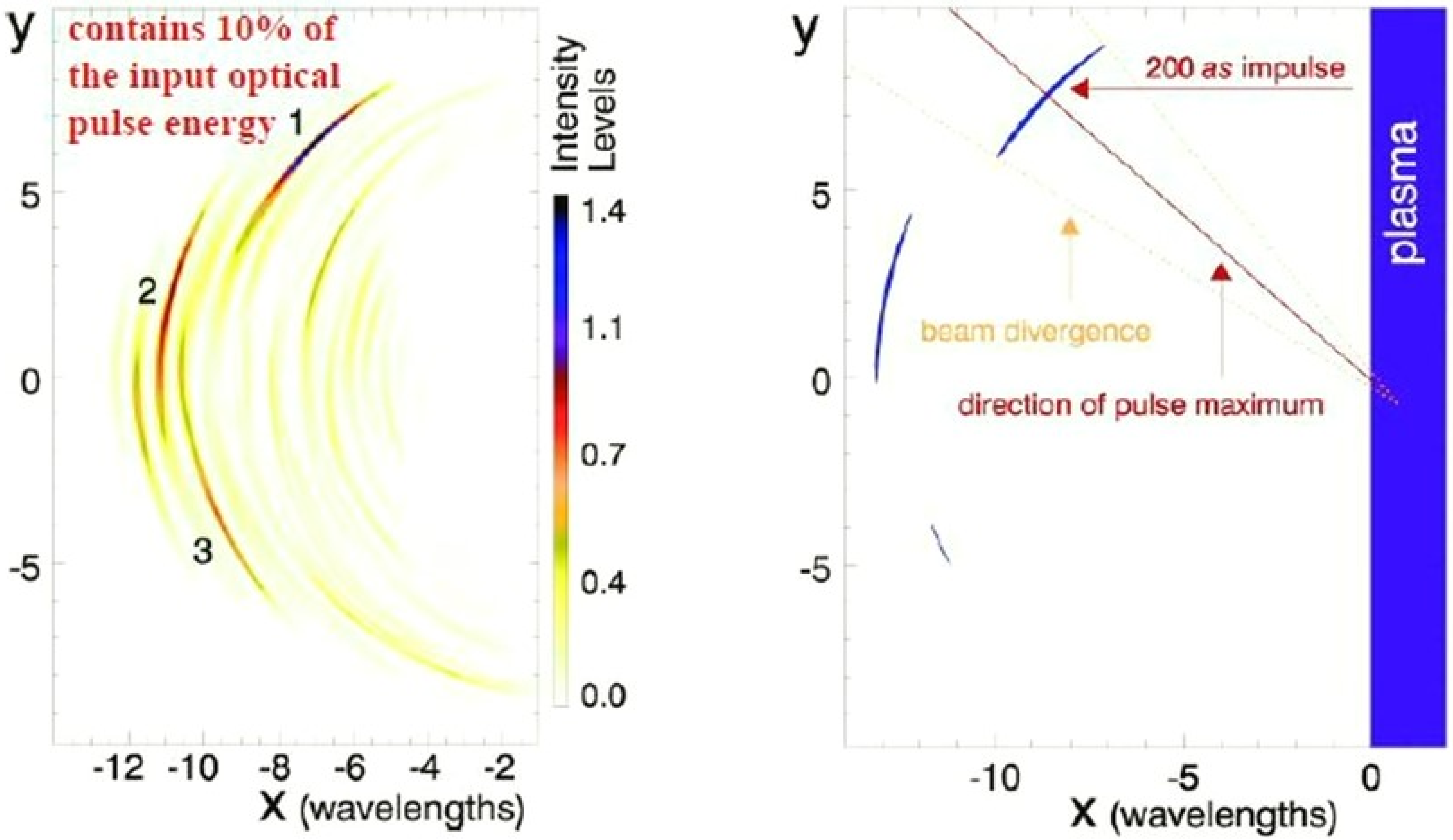
b) Relativistic transparency and laser pulse shaping;

c) “Superluminal” barrier tunneling;

d) Atto-second EM pulse generation.

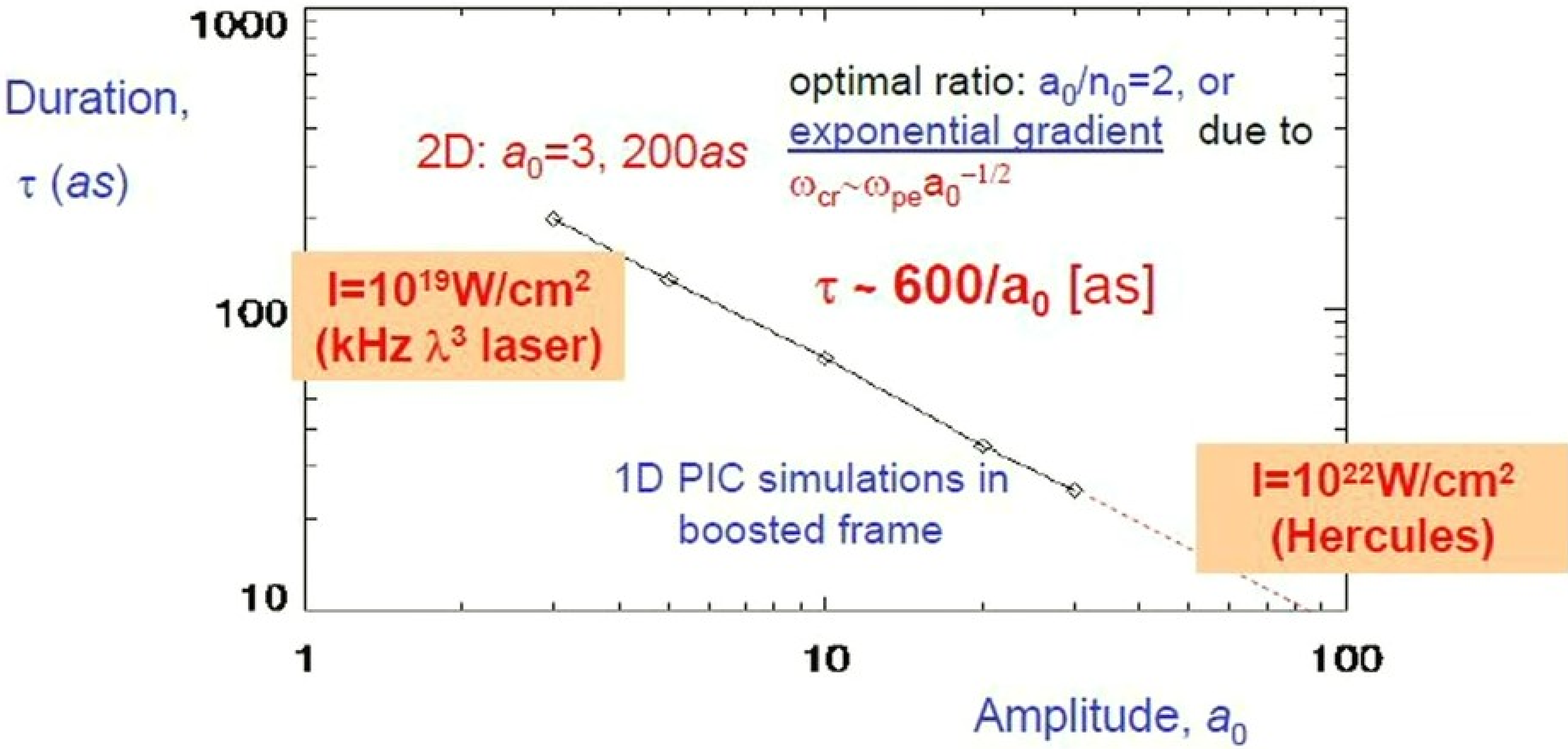
S.V.Bulanov, N.M.Naumova, F.Pegoraro, *Phys. Plasmas* 1, 745 (1994)

Isolated Attosecond Pulses formed by Relativistic Reflection/Deflection/Compression [3D PIC; in the λ^3 regime]



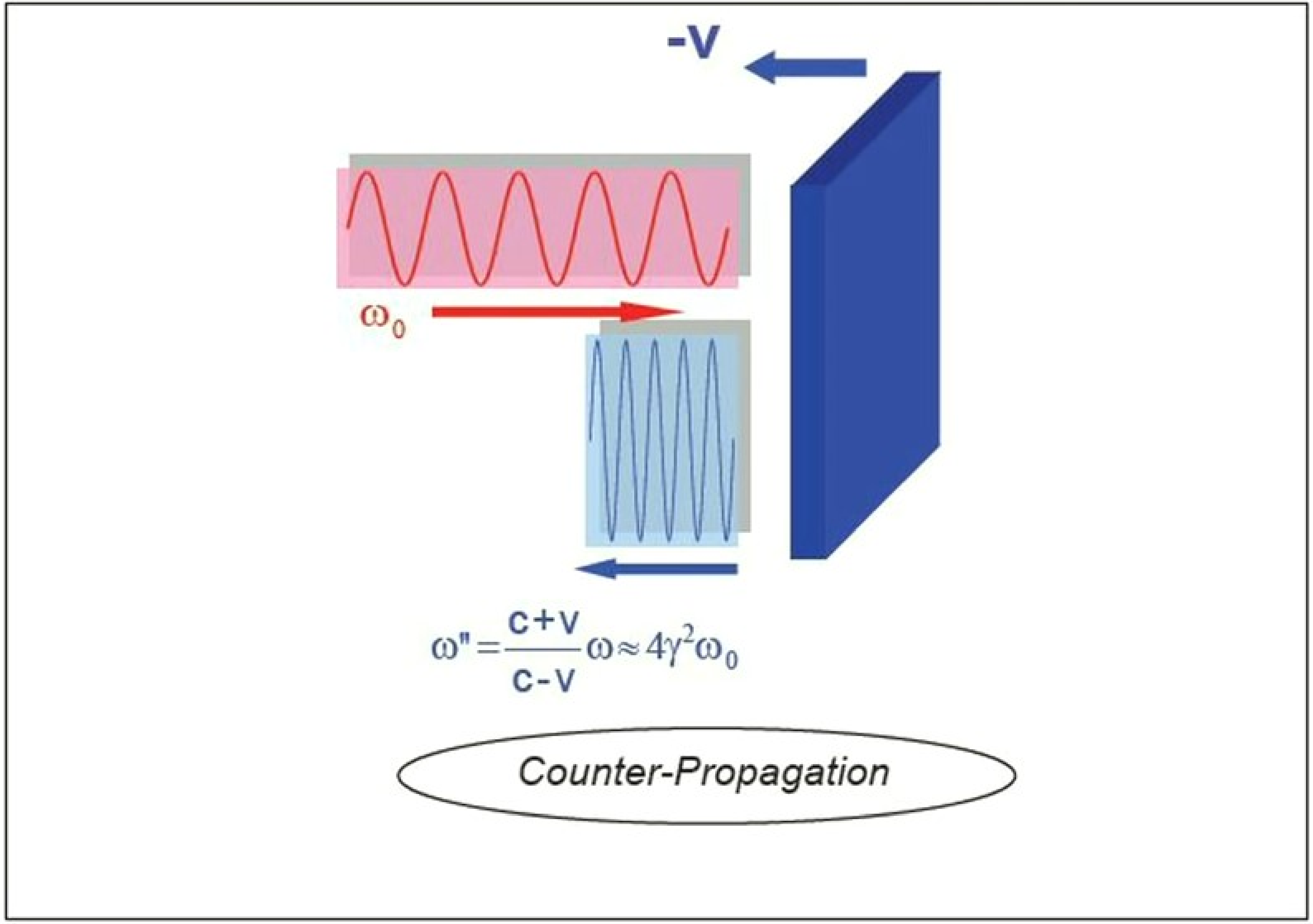
G. Mourou, N. Naumova, J. Nees, I. Sokolov, B. Hou, *Phys. Rev. Lett.; Opt. Lett.* (2004)
http://www.eecs.umich.edu/CUOS/attosecond

Scalable Isolated Attosecond Pulses



http://www.eecs.umich.edu/CUOS/attosecond

Light Intensification: Reflection of the EM wave at the relativistic mirror



Q: Can we achieve intensity 10^{23} W/cm^2 and higher?

THEORY

A: 1) **OPCPA: $10^{23} - 10^{26}$ W/cm^2**

[I. N. Ross et al., Optics Commun. **144**, 125 (1997)]

2) **Huge laser system (Zettawatt): $10^{26} - 10^{28}$ W/cm^2**

[T.Tajima & G.Mourou, Phys. Rev. STAB 5, 031301 (2002)]

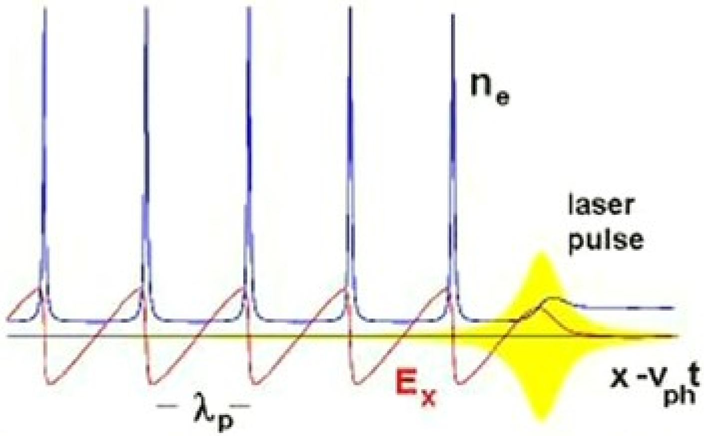
ZW=10^{21} W

3) **Relativistic mirror in plasma wake: $10^{29} - 10^{30}$ W/cm^2**

[S.V.Bulanov,T.Esirkepov,T.Tajima, Phys. Rev. Lett. **91**, 085001 (2003)]

40 km length wake left behind supertanker

Wake Plasma Wave



$$\lambda_p = 2\pi / k_p \qquad k_p v_{ph} = \omega_{pe} \qquad \omega_{pe} = \left(4\pi n e^2 / m_e\right)^{1/2}$$

T. Tajima and J. Dawson, Phys. Rev. Lett. 43, 267 (1979)

Wave Break (Gradient Catastrophe)

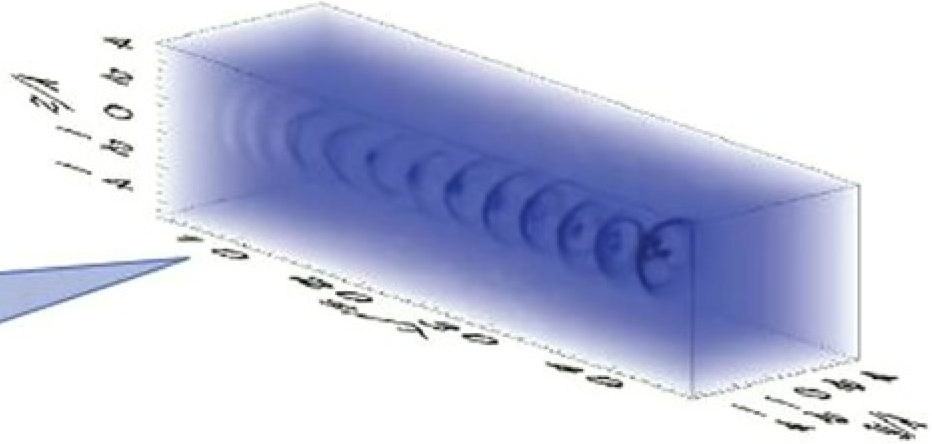
$$J = |\partial x / \partial x_0| \rightarrow 0$$

When the Langmuir wave breaks, i.e. when $v_e \rightarrow v_{ph}$ the electron density becomes infinite at the wave crests

$$n_e\Big|_{x=x_{br}} \rightarrow \infty$$

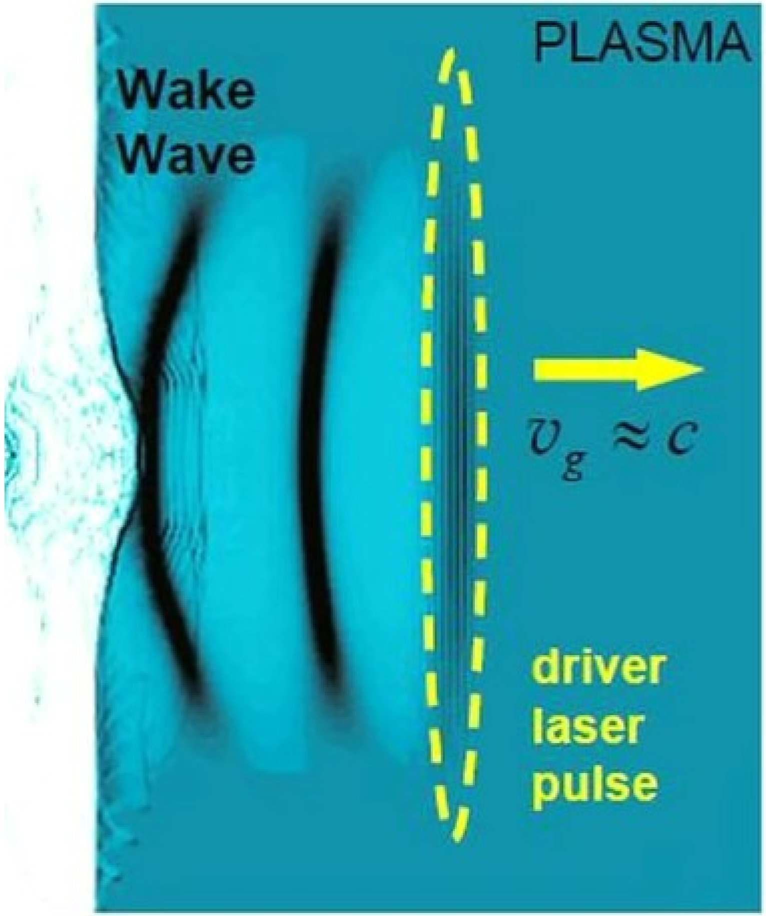
Relativistically strong wake wave has a paraboloidal form

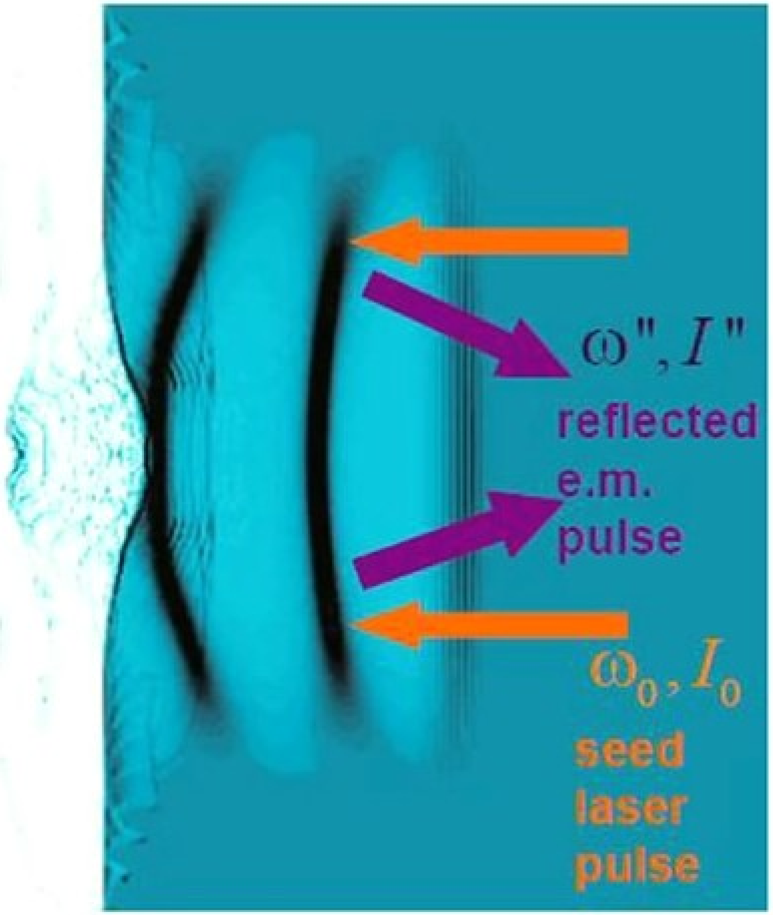
Dependence of the relativistic Langmuir frequency $\omega_{pe}(a)$ on the wave amplitude, which is determined by inhomogeneity of the laser pulse amplitude $a(r_\perp)$ leads to the dependence of the wake wave wavelength on $r_\perp$



3D PIC simulations of the wake field generation reveal its 3D form

E.M. Wave Intensification via Interaction with Breaking Wave Plasma Wave



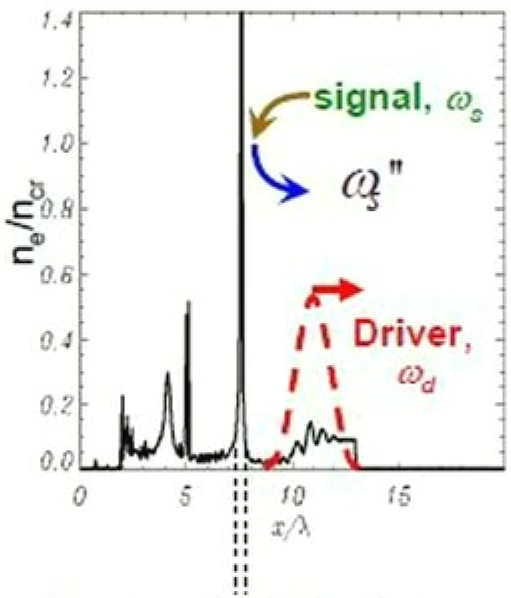


$$\omega'' = \frac{1 + v_{ph}/c}{1 - v_{ph}/c}\,\omega \approx 4\gamma_{ph}^{2}\omega_0$$

$$I''_{max} \approx R(\gamma_{ph})\,\gamma_{ph}^{6}\,I_0$$

Reflection at the "Flying Mirror"

Electron density cusp $\propto (x-x_{peak})^{-2/3}$



Here are ½ of all electrons (in the wake wave period)

Wave equation for the vector-potential A_z of EM pulse

$$\partial_{tt} A_z - c^2 \Delta A_z + \frac{4\pi e^2 n_e (x - v_{\mathrm{ph}} t)}{m_e \gamma_e} A_z = 0, \quad n_e \approx \frac{1}{2} n_0 \left(1 + \lambda_p \delta(x - v_{\mathrm{ph}} t)\right)$$

In the moving frame we seek solution of this Eq.

$$\frac{d^2 \mathrm{A}_z}{dx'^2} + q^2 \mathrm{A}_z = \chi \delta(x') \mathrm{A}_z \quad \text{with} \quad q^2 = \left(\frac{\omega_s'}{c}\right)^2 - k_\perp'^2 - \frac{\omega_{pe}^2}{2c^2 \gamma_{\mathrm{ph}}} > 0$$

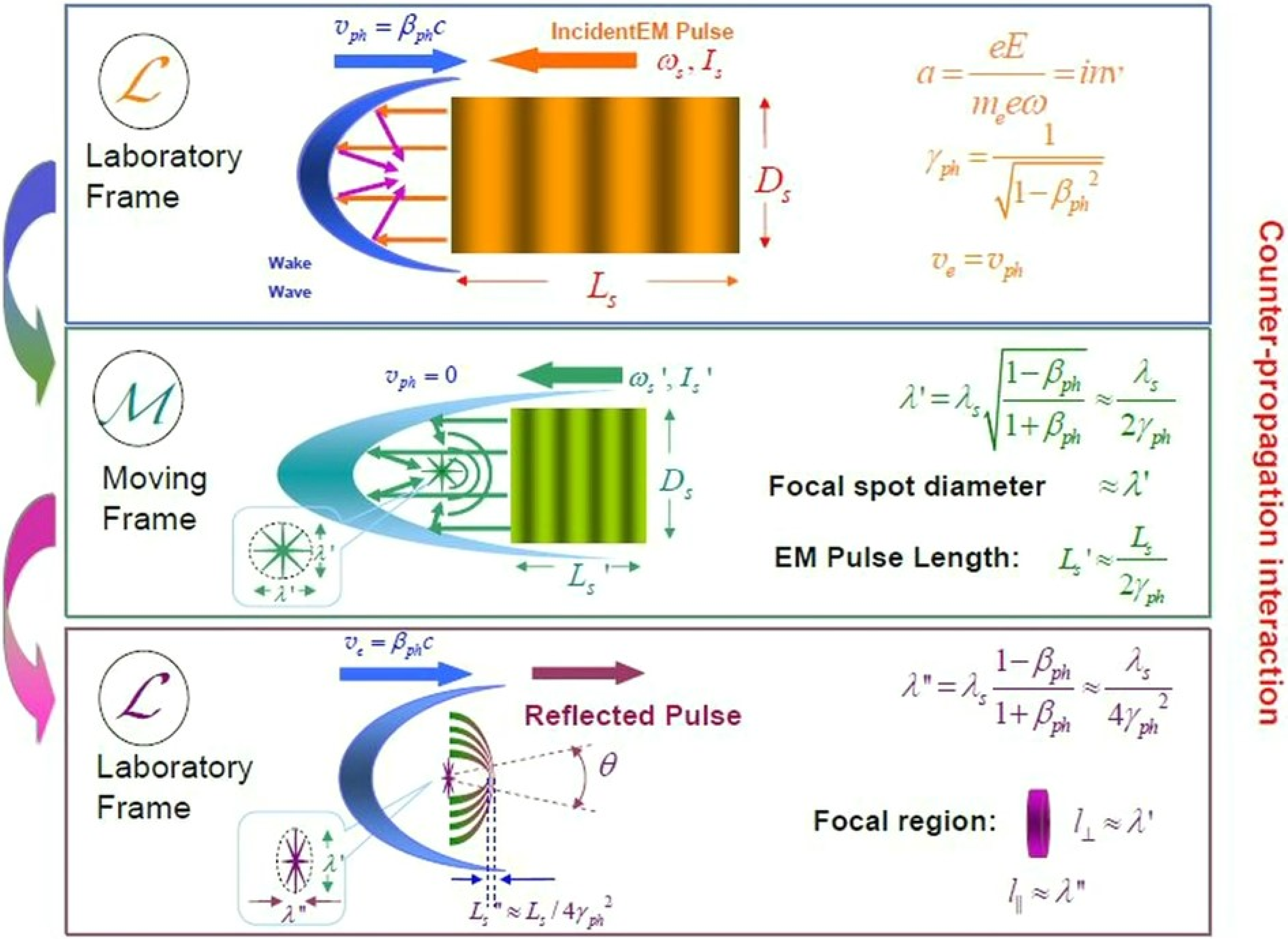
where $A_z = \exp(iqx') + \rho(q) \exp(-iqx')$

It yields $\rho(q) = -\dfrac{\chi}{\chi + 2iq}$

In the strongly nonlinear wake:

$$\lambda_p \approx 4 \frac{c}{\omega_{pe}} (2\gamma_{ph})^{1/2} \Rightarrow \chi \approx 4 \frac{\omega_{pe}}{c} (2\gamma_{ph})^{1/2}$$

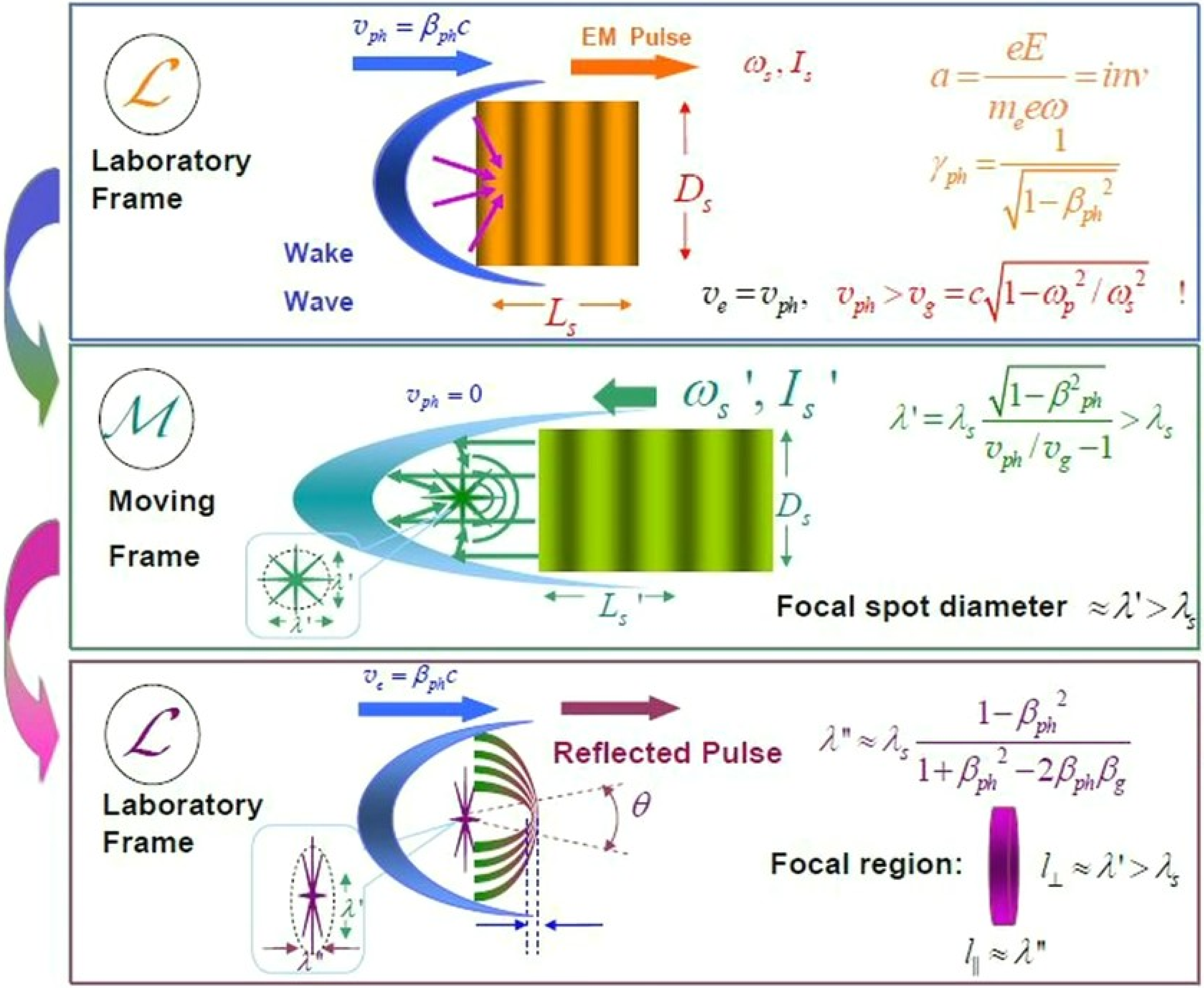
We find the reflection coefficient $R(q) = |\rho(q)|^2 \approx \left(\dfrac{\omega_d}{\omega_s}\right)^2 \dfrac{1}{2\gamma_{\mathrm{ph}}^3}$

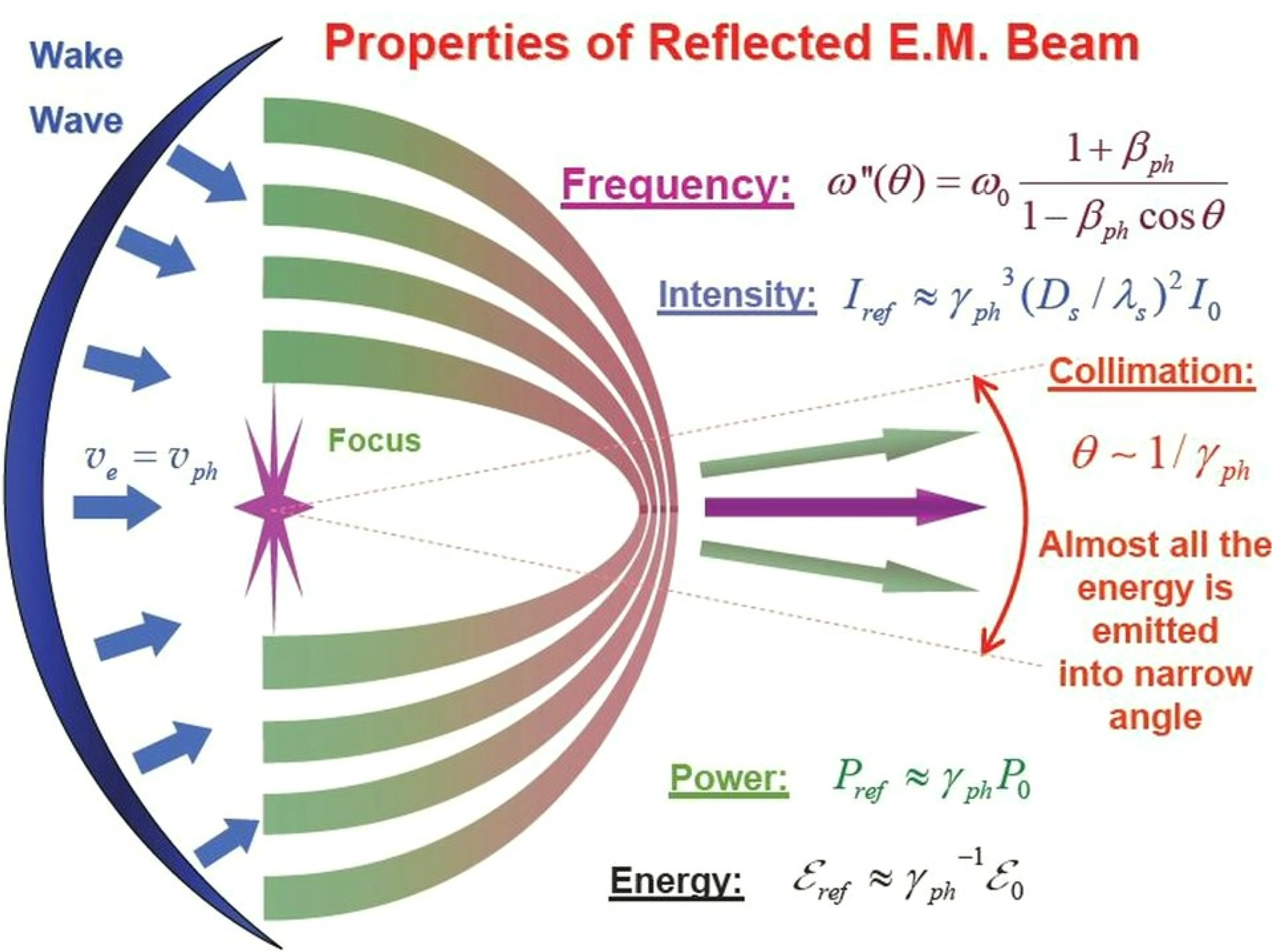


Laboratory Frame
$v_{ph} = \beta_{ph} c$
IncidentEM Pulse
ω_s, I_s
D_s
L_s
Wake Wave
$a = \frac{eE}{m_e e \omega} = inv$
$\gamma_{ph} = \frac{1}{\sqrt{1-\beta_{ph}^2}}$
$v_e = v_{ph}$
Moving Frame
$v_{ph} = 0$
ω_s', I_s'
D_s
L_s'
λ'
$\lambda' = \lambda_s \sqrt{\frac{1-\beta_{ph}}{1+\beta_{ph}}} \approx \frac{\lambda_s}{2\gamma_{ph}}$
Focal spot diameter $\approx \lambda'$
EM Pulse Length: $L_s' \approx \frac{L_s}{2\gamma_{ph}}$
Laboratory Frame
$v_e = \beta_{ph} c$
Reflected Pulse
θ
λ'
λ''
$L_s'' \approx L_s / 4\gamma_{ph}^2$
$\lambda'' = \lambda_s \frac{1-\beta_{ph}}{1+\beta_{ph}} \approx \frac{\lambda_s}{4\gamma_{ph}^2}$
Focal region: $l_\perp \approx \lambda'$
$l_\parallel \approx \lambda''$
Counter-propagation interaction

Co-propagation interaction: photon accelerator

S.C.Wilks *et al.*, Phys. Rev. Lett. 62, 2600 (1989);
S.V. Bulanov and A. S. Sakharov, JETP Lett. 54, 203 (1991); Z.-M. Sheng, Y. Sentoku, K. Mima, K. Nishihara, Phys. Rev. E 62, 7258 (2000)





Properties of Reflected E.M. Beam
Wake Wave
Frequency: $\omega''(\theta)=\omega_0\frac{1+\beta_{ph}}{1-\beta_{ph}\cos\theta}$
Intensity: $I_{ref}\approx\gamma_{ph}{}^3(D_s/\lambda_s)^2 I_0$
Collimation:
$\theta\sim 1/\gamma_{ph}$
Focus
$v_e=v_{ph}$
Almost all the energy is emitted into narrow angle
Power: $P_{ref}\approx\gamma_{ph}P_0$
Energy: $\mathcal{E}_{ref}\approx\gamma_{ph}{}^{-1}\mathcal{E}_0$

Reflected E.M. Beam Parameters

Example:

Laser pulses (*driver&seed*) wavelength	$\lambda_0 = 1\mu m$
Plasma density	$n_e = 10^{17} cm^{-3}$
Lorentz factor, associated with the phase velocity of the wakefield	$\gamma_{ph} = \dfrac{\omega_0}{\omega_{pe}} = 100$
Seed pulse intensity	$I_s = 10^{17} W/cm^2$
Seed beam diameter	$D_s = 200\mu m$
Driver pulse intensity	$I_d = 10^{18} W/cm^2$
Driver beam diameter	$D_d = 800\mu m$
Laser energies	$17J$ & $0.1J$

* * *

Intensity of reflected wave in the focal spot $I_f \approx 10^{29} W/cm^2$

This corresponds to the critical QED field

Upper Limit on the Electric Field Amplitude

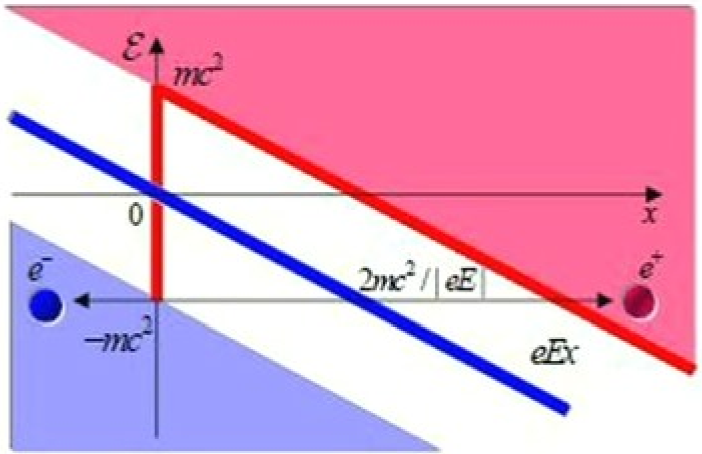
We reach a limit when the nonlinear **QED** with the electron-positron pair creation in the vacuum comes into play, at the critical **QUD** electric field, which corresponds to so strong electric field that it starts to create the electron-positron pairs at the Compton length $\lambda_c = \hbar / m_e c$, i.e.

$$E_{Schw} = \frac{m_e^2 c^3}{e\hbar}$$

It corresponds to the intensity $\approx 10^{29} W / cm^2$

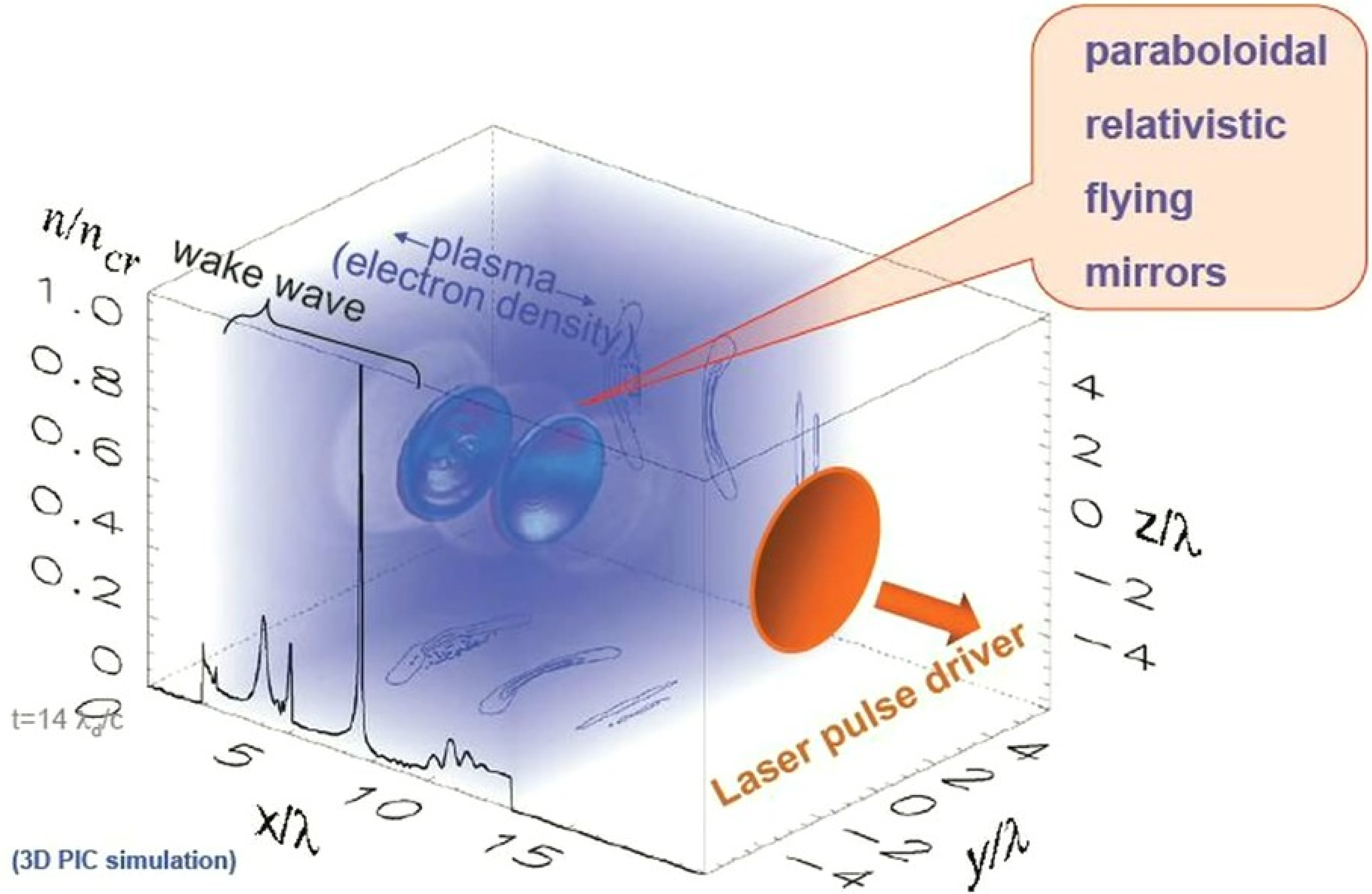
W.Heisenberg, H.Euler (1936)
J. Schwinger (1951)

Sub-burrier tunneling



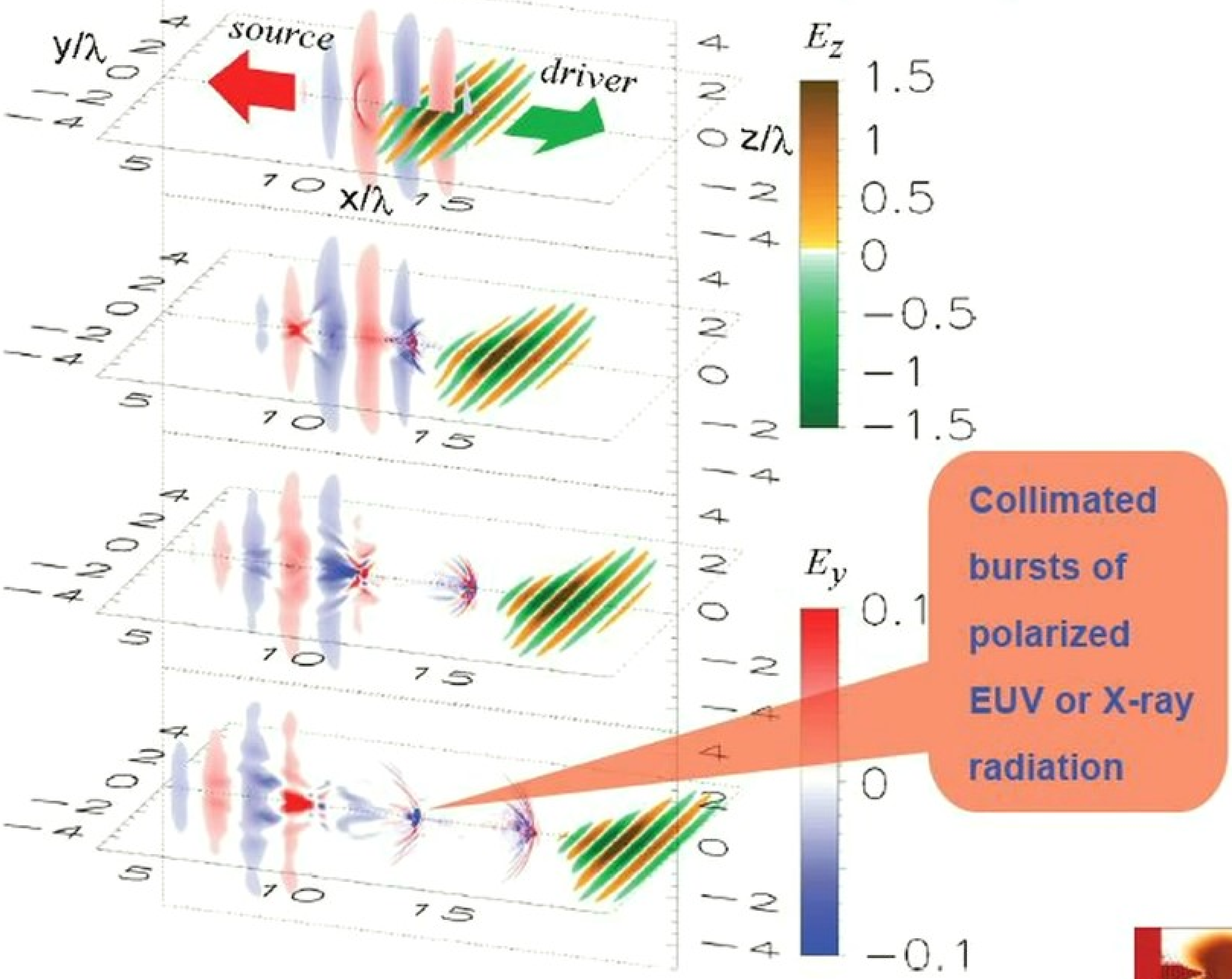
$$w = \frac{1}{\pi^2}\frac{\alpha c}{\lambda_c^{\ 4}}\left(\frac{E}{E_{Schw}}\right)^2 \exp\left(-\frac{\pi E_{Schw}}{E}\right)$$

$$E \ll E_{Schw}, \quad \alpha = \frac{e^2}{\hbar c}, \quad \lambda_c = \frac{\hbar}{m_e c}$$



paraboloidal
relativistic
flying
mirrors
n/n_cr
wake wave
plasma
(electron density)
Laser pulse driver
t=14 λ_d/c
x/λ
y/λ
z/λ
(3D PIC simulation)

3D PIC simulation of counter-propagating pulses



3D Particle-In-Cell Simulation Results

parabolidal mirror velocity $\beta_{ph} \approx 0.87$

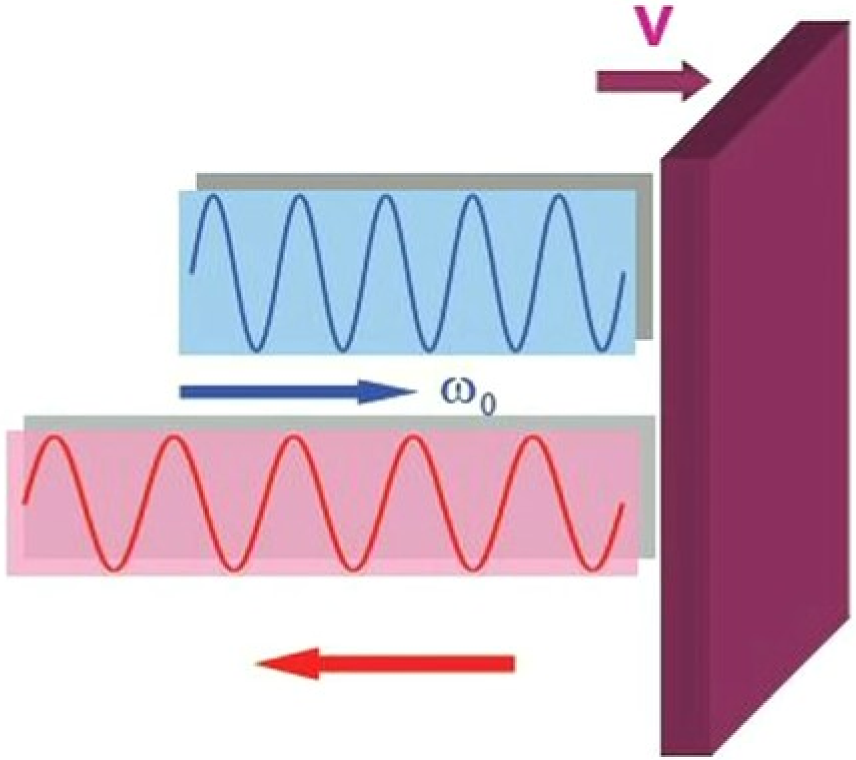
γ_{ph}-factor $\gamma_{ph} \approx 2$

frequency upshift $\frac{\omega_f}{\omega_i} \approx 14 \boxed{= \frac{1+\beta_{ph}}{1-\beta_{ph}}}$

reflected **EM** wave amplitude $\frac{E_{f\max}}{E_0} \approx 16$

reflected **EM** wave intensity $\frac{I_{f\max}}{I_0} \approx 256$

High Efficiency Ion Acceleration



$$\omega'' = \frac{c-v}{c+v}\,\omega \approx \frac{1}{4\gamma^2}\,\omega_0$$

Co-Propagation Interaction

Laser-driven ion with Exawatt

EXPERIMENT

Laser-foil interaction in TW-PW range:

TW=10^{12} W
PW=10^{15} W

Ion max energy … up to 60 MeV per nucleon.

Efficiency at max energy < 10%.

Scaling law when approaching PW: $\mathcal{E}_{i\,\max} \sim I^{1/2}$

Q: What will happen at EW (Exawatt)?

EW=10^{18} W

THEORY

A: High Efficiency Regime.

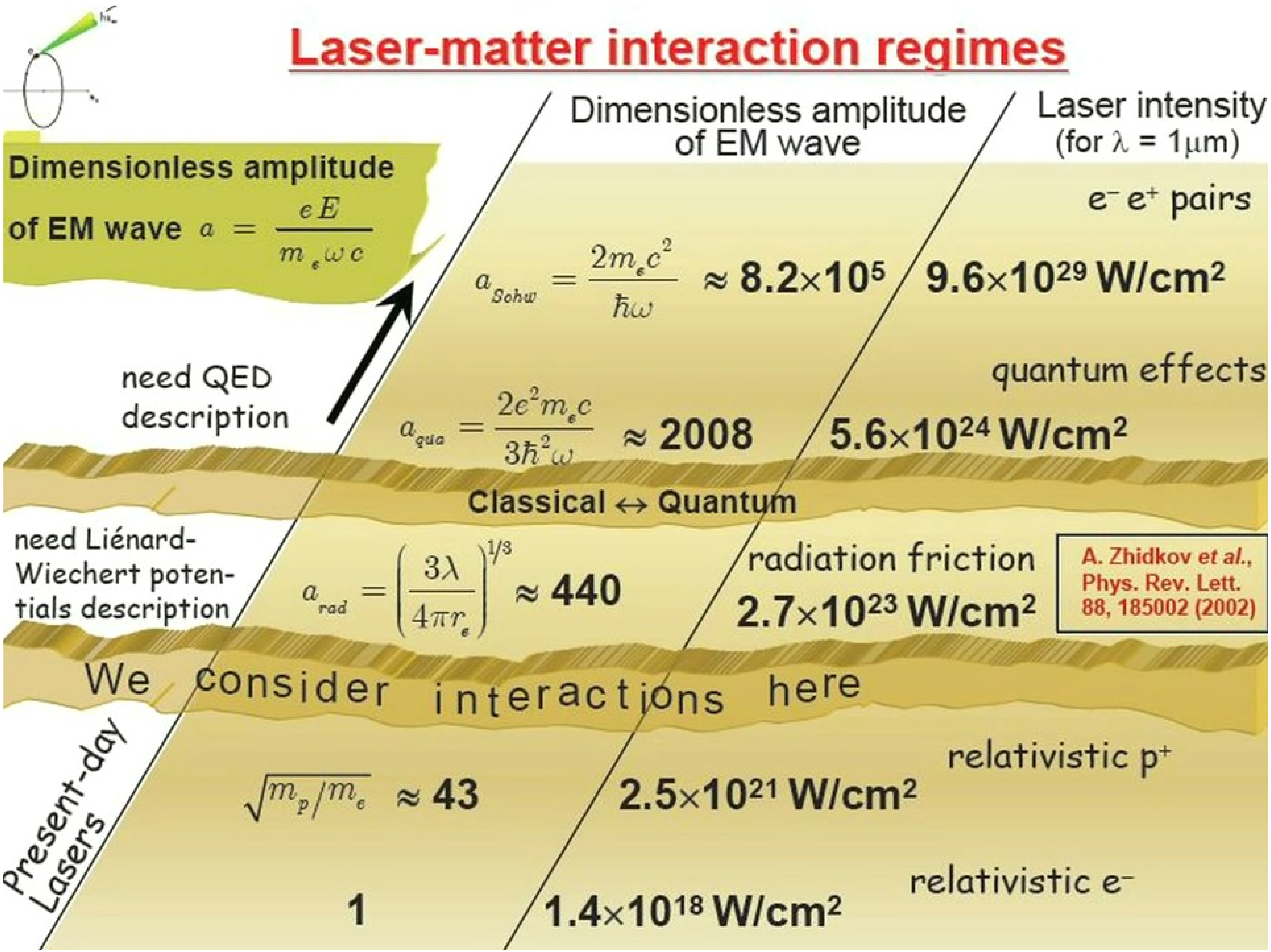
Ion max energy … $\gg$ 1 GeV per nucleon

Efficiency at max energy … $\gg$ 10%

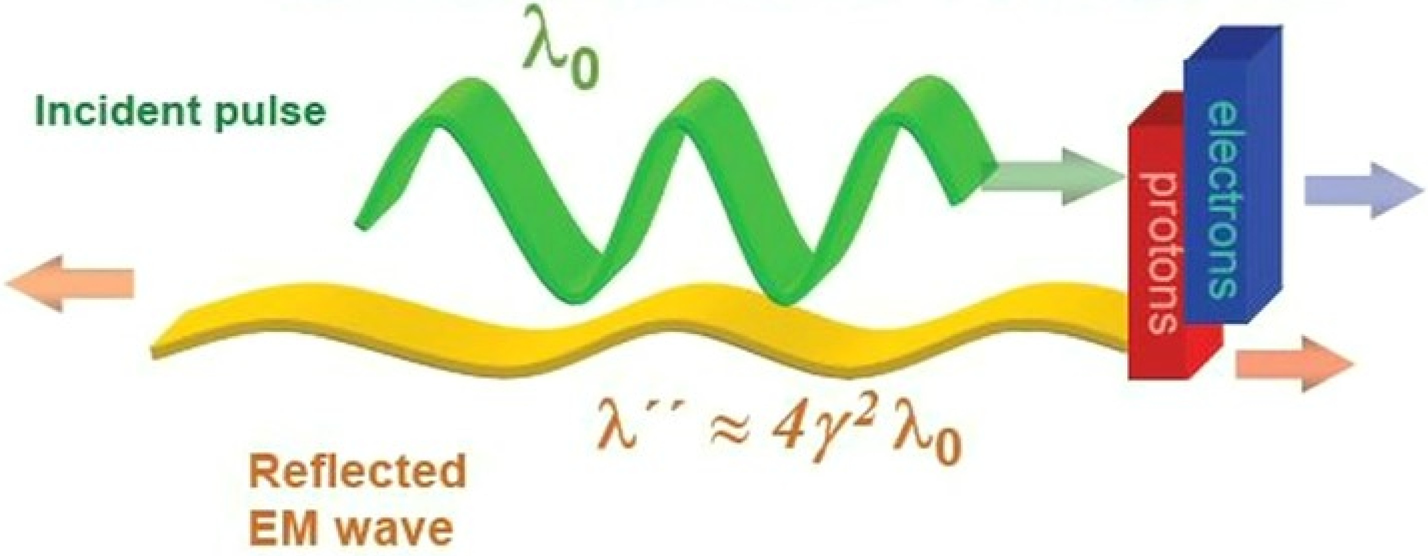
Scaling law : $\mathcal{E}_{i\,\max} \sim I$

S.V.Bulanov,T.Esirkepov, J. Koga, T.Tajima, Plasma Phys. Rep. (2004) in press;
T. Esirkepov, M. Borghesi, S. Bulanov, G. Morou, T. Tajima, Phys. Rev. Lett. Submitted.

Laser-matter interaction regimes



Relativistic PHOTON ENGINE



Light pressure: $$P_{las} = \frac{{E_0}^2}{2\pi}\left(\frac{\omega''}{\omega_0}\right)^2$$

Radiation pressure on the front part of the "cocoon" is equal to

$$\mathcal{P} = \frac{E_0^{\,2}}{2\pi}\left(\frac{\omega''}{\omega_0}\right)^2 = \frac{E_0^{\,2}}{2\pi}\frac{1-\beta_M}{1+\beta_M}$$

In the laboratory reference frame it yields an equation for the proton momentum

$$\frac{dp}{dt} = \frac{E_0^{\,2}}{2\pi n_0 l_0}\left(\frac{\left(m_p^{\,2}c^2 + p^2\right)^{1/2} - p}{\left(m_p^{\,2}c^2 + p^2\right)^{1/2} + p}\right)$$

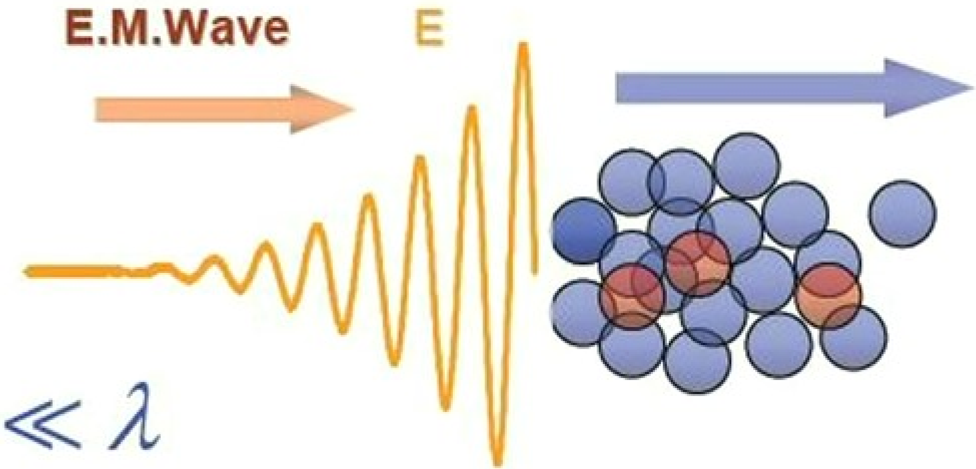
Its solution is given by $\displaystyle \frac{2p^3 + 2\left(m_p^{\,2}c^2 + p^2\right)^{3/2}}{3m_p^{\,2}c^2} + p = \frac{E_0^{\,2}}{2\pi n_0 l_0}t$

Asymptotically at $t \to \infty$ we have $\displaystyle p \approx m_p c\left(\frac{3}{2}a_0^{\,2}\frac{m_e}{m_p}\left(\frac{\omega_0}{\omega_{pe}}\right)^2\frac{ct}{l_0}\right)^{1/3}$

Energy balance: $\displaystyle \frac{p}{m_p c} = \frac{W(W+2)}{2(W+1)}$

here $\displaystyle W(t-x/c) = \int_{-\infty}^{t-x/c}\frac{E_0^{\,2}(\xi)d\xi}{2\pi n_0 l_0 m_i c}$, i.e. at $t \to \infty$ $\displaystyle p = \frac{E_0^{\,2}\tau_{pulse}}{4\pi n_0 l_0}$

Radiation Mechanism of the Ion Acceleration



Radiation pressure

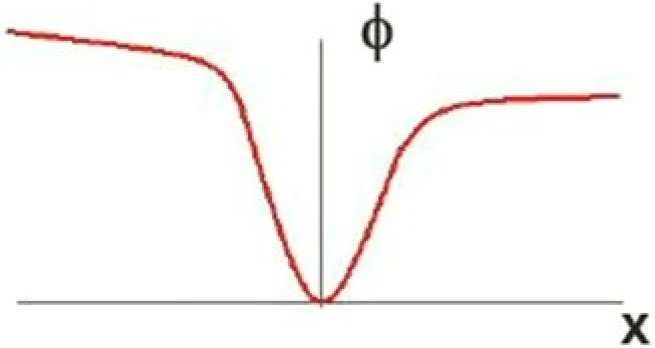
$$F_{rad} = \frac{2e^4N^2E^2}{3m_e^2c^4}$$

$R \ll \lambda$

Electron $-e, m_e$

Ion Ze, m_i

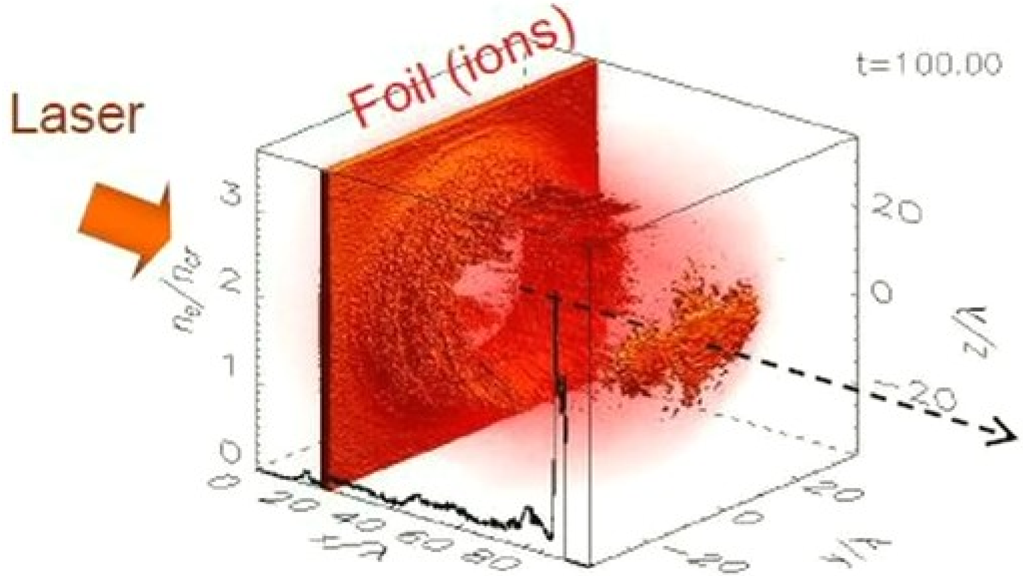
$\sum Z \ll N$

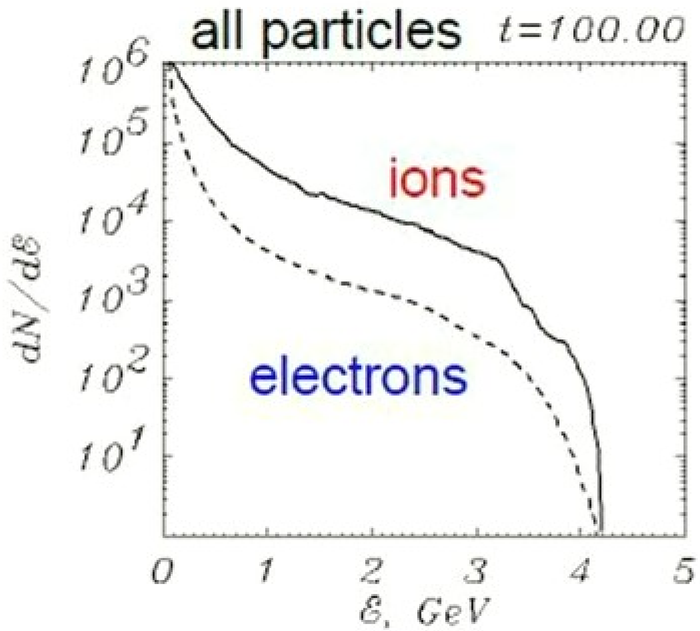


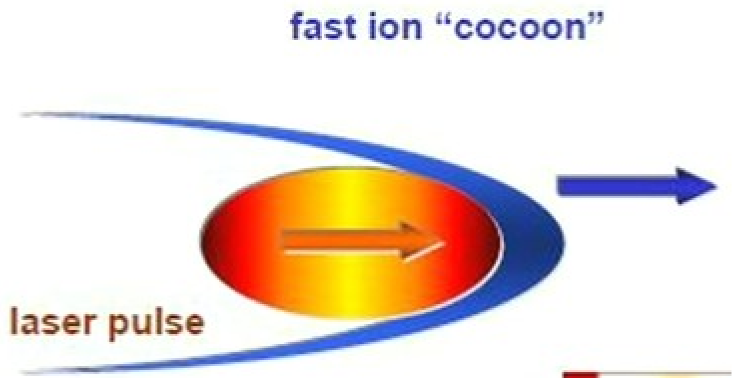
$$\phi \approx eN / R \geq m_i v_i^2 / 2Ze$$

V. I. Veksler, in: *Proceed. of CERN Symposium on High Energy Accelerators and Pion Physics, Geneva, 1956*, Vol. 1, p. 80.

Multi GeV ions from a foil irradiated by Exawatt laser

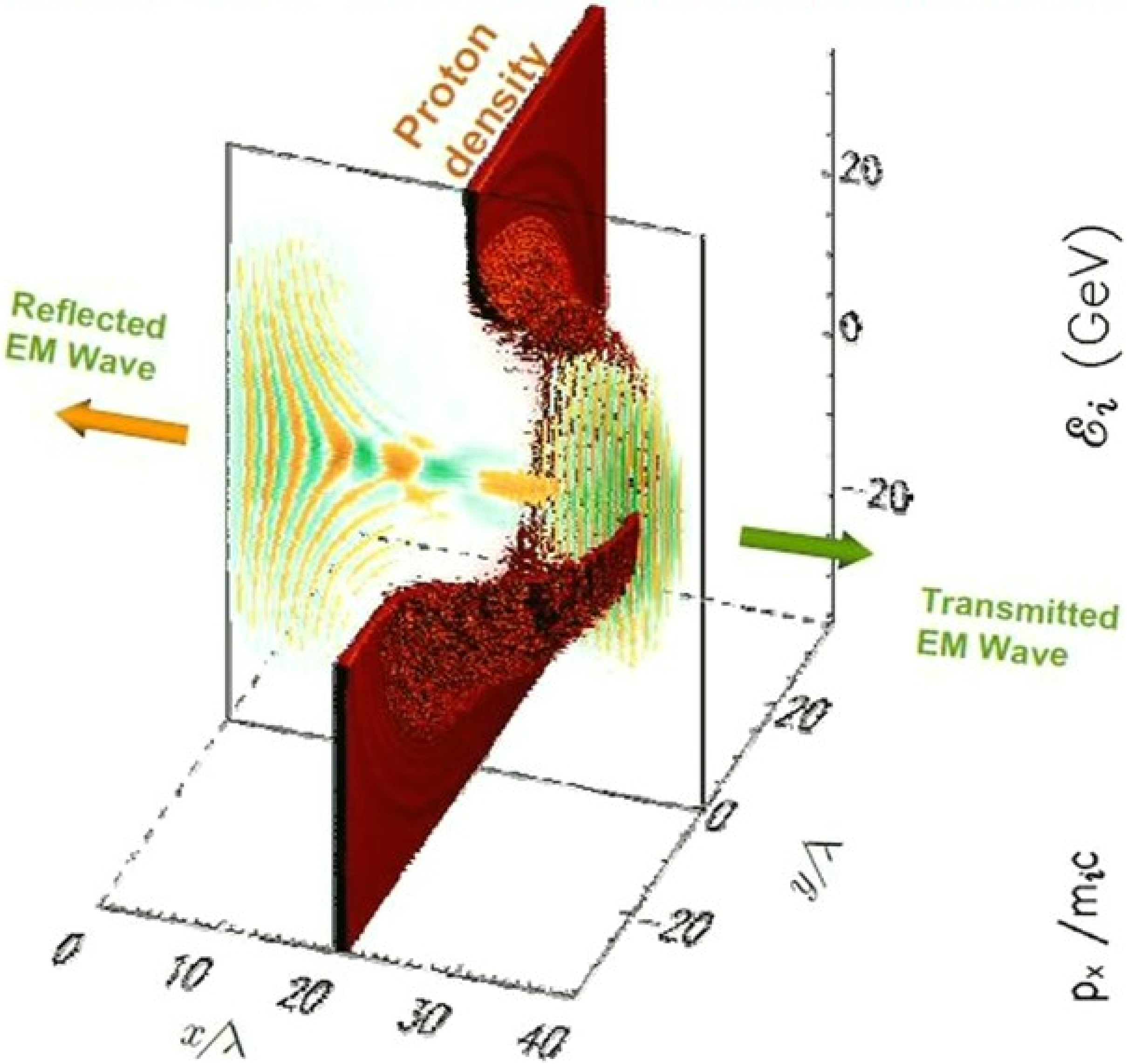


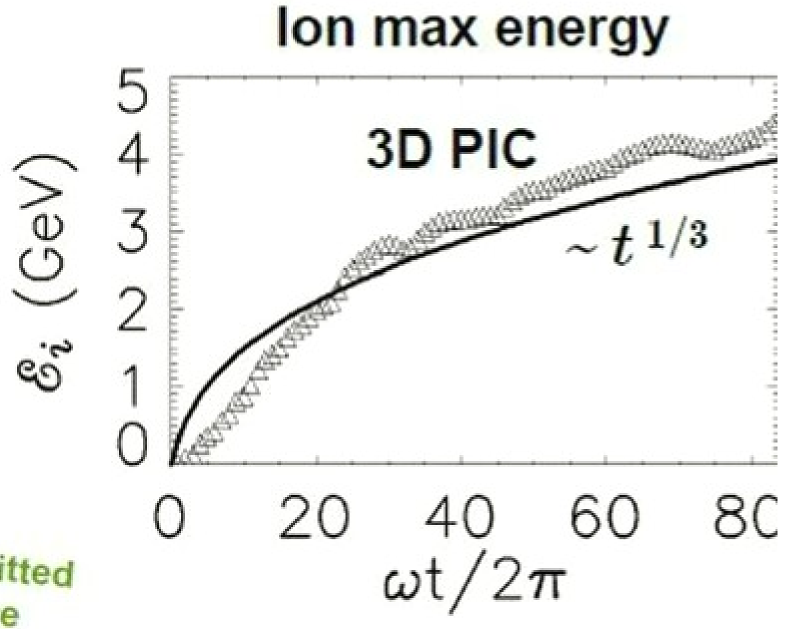


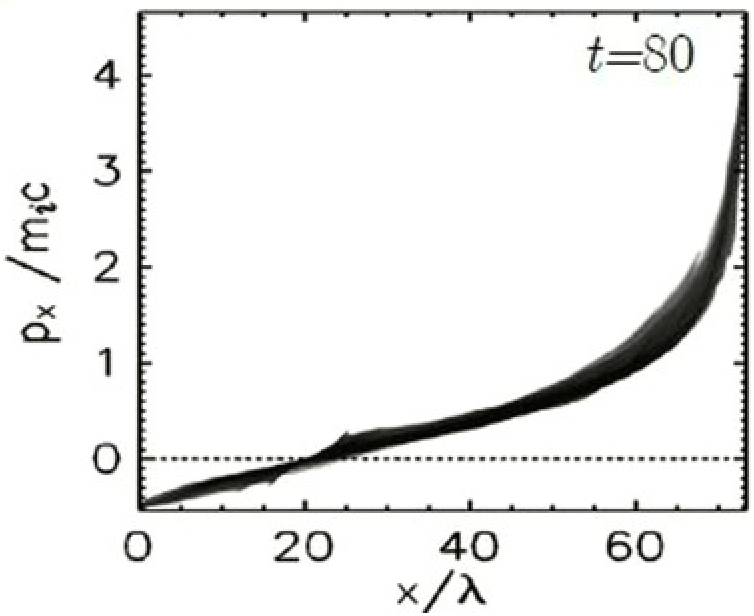


REMP

Multi GeV ions from a foil irradiated by Exawatt laser







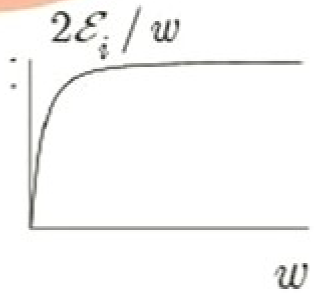
Final ion energy:

$$\mathcal{E}_i = \mathcal{E}_{las} / N_{tot}$$

where $\mathcal{E}_{las}$ is the laser pulse energy and N_{tot} is the ion number

Efficiency of the laser energy conversion into the fast ion energy can be formally up to

100% !



Example:

$1\mathrm{MJ}$ laser pulse can accelerate

10^{14} protons up to the energy **60 *GeV*** per particle

or

10^{12} protons up to **6 *TeV*** per particle

Conclusions

- Oscillating mirrors for high order harmonics and atto-second pulse production
- Counter-propagation interaction of the light pulse with the parabaloidal relativistic mirror for the EM radiation frequency up-shifting (with the atto-, zepto-, … second pulse generation) and intensification up to the QED critical field limit
- Co-propagation interaction of the light pulse with the relativistic mirror for the ultra-relativistic ion acceleration with the high efficiency (up to 100%) of the laser energy conversion into the fast ion energy