Field Guide to

Illumination

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SPIE Field Guides
Volume FG11

John E. Greivenkamp, Series Editor

SPIE
PRESS

Introduction to the Series

Welcome to the *SPIE Field Guides*—a series of publications written directly for the practicing engineer or scientist. Many textbooks and professional reference books cover optical principles and techniques in depth. The aim of the *SPIE Field Guides* is to distill this information, providing readers with a handy desk or briefcase reference that provides basic, essential information about optical principles, techniques, or phenomena, including definitions and descriptions, key equations, illustrations, application examples, design considerations, and additional resources. A significant effort will be made to provide a consistent notation and style between volumes in the series.

Each *SPIE Field Guide* addresses a major field of optical science and technology. The concept of these *Field Guides* is a format-intensive presentation based on figures and equations supplemented by concise explanations. In most cases, this modular approach places a single topic on a page, and provides full coverage of that topic on that page. Highlights, insights, and rules of thumb are displayed in sidebars to the main text. The appendices at the end of each *Field Guide* provide additional information such as related material outside the main scope of the volume, key mathematical relationships, and alternative methods. While complete in their coverage, the concise presentation may not be appropriate for those new to the field.

The *SPIE Field Guides* are intended to be living documents. The modular page-based presentation format allows them to be easily updated and expanded. We are interested in your suggestions for new *Field Guide* topics as well as what material should be added to an individual volume to make these *Field Guides* more useful to you.

John E. Greivenkamp, *Series Editor*
College of Optical Sciences
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The Field Guide Series

Field Guide to Illumination

In writing this *Field Guide* to Illumination, the first task was to decide what topics to include. Illumination tends to mean different things to different people. Certainly any subject matter under the purview of the CIE, Commission Internationale de l'Eclairage (the International Commission on Illumination) or the Illuminating Engineering Society of North America (IESNA) must be considered. Some particular areas pertaining to imaging systems and nonimaging optics are potentially overlooked. Thus, we chose to address a number of topics that fall under the following three categories: imaging system illumination, nonimaging optics for illumination, and architectural illumination, which all call upon principles of radiometry and photometry. Although this is not a guide to radiometry, enough information on the subject is included to make this manual a self-contained document. Additionally, those optical properties of materials that are pertinent to illumination, such as surface color, scattering, and retroreflection are described.

The content in this Field Guide starts with traditional illumination in imaging systems, followed by the recent advances in computer-aided design of high efficiency nonimaging illumination optics, along with the modern source models that support these techniques. Sections on the illumination of visual displays are included.

There was not enough room for a complete treatment of architectural illumination, but some important topics are included at the end of this *Field Guide* such as indoor and outdoor architectural illumination.

The notation and terminology are consistent throughout this *Guide*, but we do not lose sight of the fact that they may not be consistent in the field. Examples of alternate notation and terminology are presented.

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Table of Contents

Table of Contents (cont.)

Table of Contents (cont.)

Table of Contents (cont.)

Table of Contents (cont.)

Glossary

α	Absorptance
α	Observation angle (in retroreflection)
a_i	Input area to a compound concentrator
a_o	Output area of a compound concentrator
A	Absorbance
A_i	Illuminated area
A_r	Radiating area
a_x	Area of plane x
$a\Omega$	Throughput, étendue
C	Concentration ratio
CCT	Correlated color temperature
CIE	International Commission on Illumination
CRI	Color rendering index
d_i	Diameter of input aperture to a CPC
d_o	Diameter of output aperture of a CPC
d_s	Diameter of small aperture of a CPC
$D(\mu,\lambda)$	Donaldson matrix
E	Irradiance
$E_\perp$	Illuminance normal to the illumination
E_0	Axial irradiance
E_e	Edge irradiance
E_i	Image irradiance
E_{i0}	Axial image irradiance
E_λ	Spectral irradiance
$f/\#$	F-number
$f/\#_w$	Working F-number
F'	Increase factor
$F_{a\,\text{to}\,b}$	Form factor from a to b
f_{skew}	Skew invariant
I	Intensity
$I_{\text{LED A}}$	Averaged LED intensity, CIE condition A
$I_{\text{LED B}}$	Averaged LED intensity, CIE condition B
I_λ	Spectral intensity
L	Radiance
L*,a*,b*	CIE 1976 (L*a*b*) color space; CIELAB
L*,u*,v*	CIE 1976 (L*u*v*) color space; CIELUV
L_i	Image radiance
L_o	Object radiance
L_λ	Spectral radiance

Glossary (cont.)

M	Integrating sphere multiplier
m	Lateral image magnification
n	Index of refraction
NA	Numerical aperture
OD	Optical density
p_e	Purity
pf	Packing fraction
psa	Projected solid angle
R	Reflectance factor
R_A	Coefficient of retroreflection
R_a	General color rendering index
R_I	Coefficient of retroreflected luminous intensity
R_L	Coefficient of retroreflected luminance
$S_\lambda(\lambda)$	Spectral density of a light source
T	Transmittance factor
$T/\#$	T-number
u, v	CIE 1960 UCS chromaticity coordinates
u', v'	CIE 1976 UCS chromaticity coordinates
$V(\lambda)$	Photopic luminous efficiency
$V'(\lambda)$	Scotopic luminous efficiency
$W^*U^*V^*$	CIE 1964 uniform space coordinates
x, y	CIE 1931 chromaticity coordinates
X, Y, Z	CIE tristimulus values
$\bar{x}, \bar{y}, \bar{z}$	CIE color matching functions
β	Entrance angle (in retroreflection)
ξ	Generalized étendue
λ	Wavelength, emission wavelength
$\lambda_{0.5m}$	Center wavelength (for LED)
λ_c	Centroid wavelength (for LED)
λ_d	Dominant wavelength
λ_p	Peak wavelength (for LED)
μ	Excitation wavelength
ρ	Reflectance
$\bar{\rho}$	Average reflectance
τ	Transmittance
υ	Viewing angle (in retroreflection)
Φ	Flux
Φ_λ	Spectral flux
Ω	Projected solid angle (psa)

ω	Solid angle
$\overline{\Omega}_{a\,\text{to}\,b}$	Average projected solid angle from a to b
Ω_i	Input psa to a compound concentrator
Ω_o	Output psa from a compound concentrator
Ω_x	Projected solid angle viewed from plane x
θ_i	Input half-angle of compound concentrator
θ_o	Output half-angle from compound concentrator
$\theta_{\max}$	Maximum output half-angle from CPC
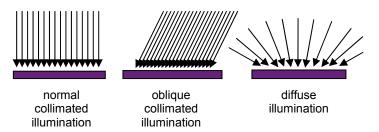
Flux and Irradiance

In examining terminology for illumination, it is useful to separate the spatial considerations from the spectral concerns. In many cases, the spatial and spectral issues are independent and can be separated without losing any generality. In other cases, the spatial and spectral issues cannot be separated physically, but it is useful to separate them conceptually. The commonly used spatial quantities are **flux**, **irradiance**, **intensity**, and **radiance**.

Flux, Φ, is the optical power or rate of flow of radiant energy.

Irradiance, E, is the flux per unit area striking a surface. Occasionally, the flux per unit area leaving a surface, called **exitance**, M, is important. However, the geometry is the same as for irradiance, so it will not be treated separately here. Furthermore, when exitance is used, it is often the flux leaving a nonphysical surface such as the exit port of an integrating sphere or the real image in an imaging system, where it is identical to the irradiance onto the surface.
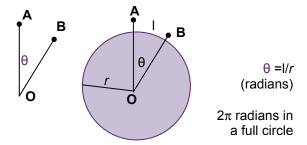
The irradiance quantity itself says absolutely nothing about the directionality of the flux. For example, if the three cases in the figure below all have the same flux per unit area striking the surface, then they all have the same irradiance. Because of this ambiguity, specifications for illumination systems often qualify the irradiance quantity with an added description of the desired directional properties.

normal collimated illumination

oblique collimated illumination

diffuse illumination
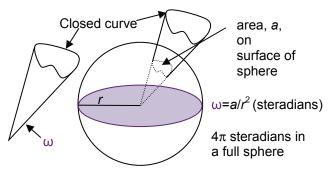
Solid Angle

The definition of intensity involves the concept of a **solid angle**. A solid angle is a 3D angular volume that is defined analogously to the definition of a plane angle in two dimensions.

A **plane angle**, θ, made up of the lines from two points meeting at a vertex, is defined by the arc length of a circle subtended by the lines and by the radius of that circle, as shown below. The dimensionless unit of plane angle is the **radian**, with 2π radians in a full circle.

$$\theta = l/r \text{ (radians)}$$

2π radians in a full circle

A solid angle, ω, made up of all the lines from a closed curve meeting at a vertex, is defined by the surface area of a sphere subtended by the lines and by the radius of that sphere, as shown below. The dimensionless unit of solid angle is the **steradian**, with 4π steradians in a full sphere.

Closed curve

area, *a*, on surface of sphere

$$\omega = a/r^2 \text{ (steradians)}$$

4π steradians in a full sphere

Intensity, Radiance, and Projected Solid Angle

Intensity, *I,* is the flux per unit **solid angle**. It is the amount of flux from a point source contained in a small angular volume. A source can be considered a point source for this application if the irradiance falls off as the **inverse square** of the distance from the source. Intensity, for a given source, can vary with direction.
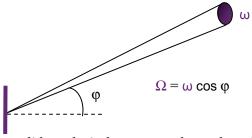
> The term "intensity" is used in many disciplines, some even closely related to optics, to mean things other than flux per unit solid angle. Use caution and rely on context to determine the meaning of the word in a particular situation.

Radiance, *L*, applies to extended sources and surfaces. It is the flux per unit solid angle per unit **projected area** of the source or surface. The projected area is the projection of the area onto a surface normal to the direction of view and is equal to the actual area times the cosine of the angle between the surface normal and the direction of view. Radiance can vary with position on a surface, and like intensity, it can vary with direction. A source or surface with constant radiance in all directions is called Lambertian. A **Lambertian source** or surface has intensity that varies with the cosine of the angle with the surface normal.

In many cases, the angle of view changes over the extent of the receiver. These cases require an alternate definition of radiance: radiance is the flux per unit area per unit **projected solid angle**. (In fact, this is the more general definition and covers the simpler case where the entire surface of the extended source is at essentially the same angle as the direction of view.)
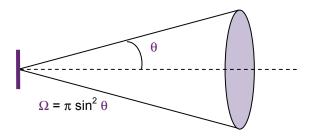
Solid Angle and Projected Solid Angle

The relationship between solid angle and projected solid angle can be confusing. Projected solid angle has meaning primarily for a small Lambertian source, which has intensity that varies as the cosine of the angle with the surface normal. The **projected solid angle**, Ω, is the **solid angle**, ω, weighted by the cosine of the angle with the surface normal.

$$\Omega = \omega \cos \varphi$$

When the solid angle is large enough so that the angle with the surface normal is not the same over the entire solid angle, the total projected solid angle must be computed by integrating the incremental projected solid angles. See the reference by Bartell for a more detailed explanation.

For some special cases, the integration results in simple expressions, such as for a large circular cone that is normal to a surface and subtends a half angle, θ.

$$\Omega = \pi \sin^2 \theta$$

A hemisphere has 2π steradians (solid angle) but π projected steradians (projected solid angle).

Spectroradiometric and Radiometric Quantities

In the spectral dimension of illumination, the most general view looks at the spectral density—the amount of radiation per unit wavelength interval. In terms of the four spatial quantities already considered, the spectral quantities are **spectral flux**, Φ_λ; **spectral irradiance**, E_λ; **spectral intensity**, I_λ; and **spectral radiance**, L_λ. These quantities, usually written with a subscript to indicate that they are integrable, must be integrated to determine the amount of radiation in a particular spectral band. For example, the **total radiant flux**, Φ (in units of watts), in the band between wavelength $\lambda 1$ and wavelength $\lambda 2$ is

$$\Phi\,(\lambda 1,\lambda 2) = \int_{\lambda 1}^{\lambda 2} \Phi_\lambda(\lambda)\cdot d\lambda.$$
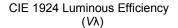
Similar expressions can be written for the **total irradiance**, E (watts/m^2); **total radiant intensity**, I (watts/ sr); and **total radiance**, L (watts/m^2·sr).
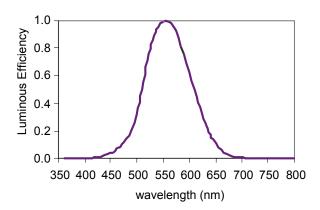
Photometry measures the response of the human eye to light. Although not everyone has exactly the same response, the standardized **CIE 1924 luminous efficiency function** works very well for most people. (The CIE is the International Commission on Illumination.) This function, shown on the following page, is designated $V(\lambda)$. The values for this function, in 5-nm increments, are given in the Appendix. Not coincidentally, this function is identical to the **CIE color matching function**, $\bar{y}$ The unit of **luminous (photopic) flux** is the **lumen**. The luminous flux is found from the spectral flux and the $V(\lambda)$ function from the following relationship:

$$\text{luminous flux} = 683\int \Phi_\lambda(\lambda)\cdot V(\lambda)\cdot d\lambda.$$

The factor of 683 in this equation comes directly from the definition of the fundamental unit of luminous intensity, the **candela**.

Photometric Quantities

CIE 1924 Luminous Efficiency
($V\lambda$)

Notes on notation:

- The photopic quantities of flux, irradiance, intensity, and radiance are called **luminous flux**, **illuminance**, **luminous intensity**, and **luminance**, respectively.
- These quantities are sometimes notated with a subscript "v" (for visual), as Φ_v, E_v, I_v, and L_v. But often the subscript is omitted since the meaning is usually clear from the context, and it could be confused with the subscript notation often reserved for integrable quantities.
- The designations Φ, E, I, and L are common but not universally standard. Another set of symbols sometimes used is P, H, J, and N, respectively, for radiometric quantities; P_λ, H_λ, J_λ, and N_λ for spectral quantities; and F, E, I, and B for the corresponding photometric quantities.
- Solid angle and projected solid angle are not always distinguished by ω and Ω, respectively.

Matrix of Basic Quantities

		SPECTRAL		
		Radio-metric	**Spectral**	**Photopic**
S P A T I A L	**Flux**	Power watts (W)	Power/ wavelength interval watts/nm	Luminous flux lumens (lm)
	Flux/ area	Irradiance W/m²	Spectral irradiance W/m²·nm	Illuminance lm/m² or lux
	Flux/ solid angle	(Radiant) intensity W/sr	Spectral intensity W/sr·nm	(Luminous) intensity lm/sr or candela (cd)
	Flux/ area· solid angle	Radiance W/m²·sr	Spectral radiance W/m²·sr·nm	Luminance lm/m²·sr or cd/m² or nit

The table above shows the four spatial quantities and the three spectral categories that are discussed in the preceding pages. These create 12 distinct cells that cover the vast majority of specifications for illumination systems.

With two exceptions, both used mainly in the United States, work in illumination is almost always done in SI units. The two exceptions (both deprecated) are:

Illuminance
1 footcandle (lm/ft²) = 10.764 lux (lm/m²)
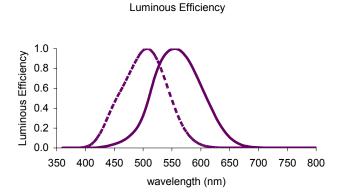
Luminance
1 footlambert (candela/πft²) = 3.426 nit (candela/m²)

Photopic and Scotopic Vision

The human visual system responds to light over a wide dynamic range, in excess of 6 orders of magnitude. To achieve this dynamic range, the mechanisms for high-light-level vision and low-light-level vision are different. The high-level region, called the **photopic region**, is active at luminance levels above about 3 cd/m². The low-level region, called the **scotopic region**, is active below approximately 0.01 cd/m². The region between pure photopic and pure scotopic is called the **mesopic region**, where the visual response is a mixture of the two. The **photopic efficiency**, usually designated $V(\lambda)$, peaks at 555 nm, while the **scotopic efficiency**, usually designated $V'(\lambda)$, peaks at 507 nm.

Scotopic and Photopic
Luminous Efficiency
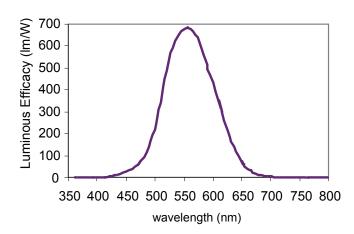
The values for photopic efficiency and scotopic efficiency, both in 5-nm increments, are given in the Appendix.

Luminous Efficacy

Luminous efficacy, quantified in lumens per watt, is a measure of the ability of a light source to produce a visual response from its power. In the photopic region, luminous efficacy peaks at 683 lumens per watt at 555 nm. In fact, the lumen is defined in terms of the power at 555 nm (frequency of 540×10^{12} Hz). Specifically, the definition (adopted in 1979) is in terms of the candela (lumen per steradian).

> The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation at a frequency of 540×10^{12} Hz and that has a radiant intensity in that direction of 1/683 watt per steradian.

Luminous Efficacy
(photopic)

It is usually clear from the context whether the power is the radiated power (as in the discussion above) or, often for lamps, the "wall-plug" power.

Typical Values of Illumination Quantities

Irradiance and Illuminance	
Direct sunlight	1000 W/m² (250–2500 nm)
Direct sunlight	100,000 lux
Shade	10,000 lux
Overcast day	1,000 lux
Office space	300–600 lux
Full moon	0.2 lux
Quarter moon	0.01 lux
Moonless clear night	0.001 lux
Luminous Intensity	
Automobile headlight	5,000–20,000 cd
Household flashlight	100–1,000 cd
100-W tungsten lightbulb	100 cd
LED traffic signal	250–700 cd
Single LED	1 mcd–25 cd
Radiance and Luminance	
Sun	2×10^7 W/m² ·sr (250–2500nm)
Sun	2×10^9 nit
Frosted lightbulb	100,000 nit
Fluorescent lamp	5,000 nit
Computer screen	100 nit

Wavelength Ranges for Illumination	
UV-C*	250 to 280 nm
UV-B	280 to 315 nm
UV-A	315 to 400 nm
Visible	~360–400 to ~760–800 nm
Near-infrared (NIR)†	760 nm to 1.1 µm
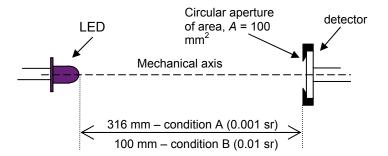
* Actual definition of UV-C is 100 to 280 nm. However, the range from 100 to 250 nm is not of interest for illumination systems.

† Actual definition of NIR is to 1.4 µm. However, 1.1 µm is the upper limit for silicon-based detectors.

Averaged LED Intensity

In 1997 the CIE established a special quantity for light-emitting diodes (LEDs) called the **averaged LED intensity**. This was introduced because, as stated in CIE Publication 127:2007, *Measurement of LEDs*, "There are significant differences between LEDs and other light sources which made it necessary for the CIE to introduce a new quantity for their characterization with precisely defined measurement conditions."

To obtain **averaged LED intensity**, the LED is measured on its mechanical axis (in line with the package) by a circular detector of area 100 mm^2 at a prescribed distance from the front tip of the LED package. Two distances are used: 316 mm **(condition A)** and 100 mm **(condition B)**, with the solid angles defined as 0.001 sr and 0.01 sr, respectively. The measurements made are notated as $I_{LED\ A}$ and $I_{LED\ B}$ in units of intensity (candela or W/sr). Since the entire measurement geometry is completely defined, the measurements should be repeatable.

Light Source Color

The perceived color of a light source is quantified by its **chromaticity**. Chromaticity is calculated from the spectral density of the light source, S_λ, and the **CIE color matching functions**, $\overline{x}, \overline{y},$ and $\overline{z}$ as follows:

$$X = \int S_\lambda(\lambda) \cdot \overline{x}(\lambda) \cdot d\lambda, \qquad x = \frac{X}{X + Y + Z},$$

$$Y = \int S_\lambda(\lambda) \cdot \overline{y}(\lambda) \cdot d\lambda,$$

$$Z = \int S_\lambda(\lambda) \cdot \overline{z}(\lambda) \cdot d\lambda, \qquad y = \frac{Y}{X + Y + Z},$$
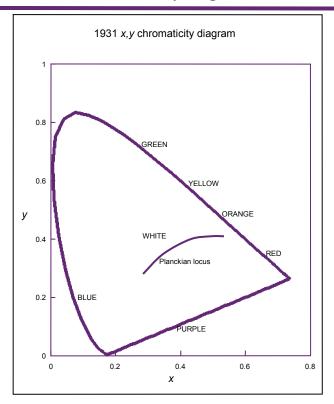
where X, Y, and Z are called **tristimulus values**, and x and y are the **CIE 1931 chromaticity coordinates**. The integrals above are usually calculated as block summations from 360 to 830 nm, generally at 1-nm or 5-nm intervals. A table of $\overline{x}, \overline{y},$ and $\overline{z},$ in 5-nm intervals can be found in the Appendix.

The 1931 chromaticity coordinates (x, y) are common coordinates for light-source colors and are represented graphically by the familiar "horseshoe" graph. All of the possible colors of light are contained inside the horseshoe shape, with the pure monochromatic spectral colors around the curved perimeter, the purples along the straight line at the bottom, and less-saturated colors in the interior. The various shades of white, which are of the most interest in illumination systems, occupy the central region.

Those white lights that have near-blackbody spectra (such as tungsten incandescent lamps) lie along the **Planckian locus**. The lower blackbody temperatures lie toward the red, and the higher temperatures toward the blue.

Chromaticity Diagram

1931 *x,y* chromaticity diagram

Two other coordinate systems are used to describe the chromaticity of light sources: the **CIE 1960 UCS coordinate system** (*u*, *v*), and the **CIE 1976 UCS coordinate system** (*u′*, *v′*). Both attempt to portray equal perceived color differences by equal distances.

$$u = 4X/(X + 15Y + 3Z) = 4x/(-2x + 12y + 3),$$
$$v = 6Y/(X + 15Y + 3Z) = 6y/(-2x + 12y + 3),$$
$$u' = 4X/(X + 15Y + 3Z) = 4x/(-2x + 12y + 3),$$
$$v' = 9Y/(X + 15Y + 3Z) = 9y/(-2x + 12y + 3).$$

The CIE 1960 UCS coordinate system is obsolete except for calculating **correlated color temperature**.

Color Temperature and CCT

Any light source whose chromaticity coordinates fall directly on the Planckian locus has a **color temperature** equal to the blackbody temperature of the Planckian radiator with those coordinates. Color temperature is usually expressed in Kelvins (K). The concept of color temperature is especially useful for incandescent lamps, which very closely approximate a blackbody spectrum throughout the visible region. For these lamps, the color temperature also defines the spectrum in this region.

For white lights that don't have chromaticity coordinates that fall exactly on the Planckian locus but do lie near it, the **correlated color temperature (CCT)** is used. The CCT of a light source, also expressed in Kelvins, is defined as the temperature of the blackbody source that is closest to the chromaticity of the source in the CIE 1960 UCS (u, v) system. CCT is an essential metric in the general lighting industry to specify the perceived color of fluorescent lights and other nonincandescent white-light sources such as LEDs and high intensity discharge HID lamps.

The difference in perceived color is closely related to the reciprocal of CCT. The reciprocal is expressed in reciprocal megakelvin $(MK)^{-1}$, with one $(MK)^{-1}$ approximately equal to a just-noticeable color difference:
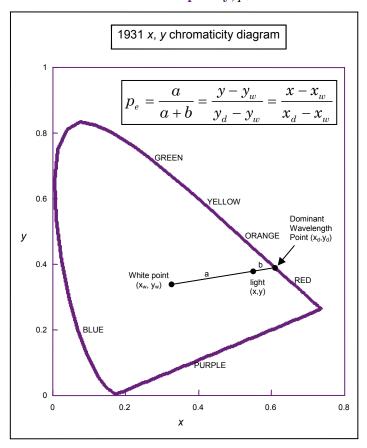
$$(MK)^{-1} = 10^6 / CCT.$$

There are limitless different spectra, all with the same CCT, that may have little or no resemblance to the blackbody curve for that temperature or to each other.

There is no approved method for computing CCT nor is there a simple and accurate closed-form expression. One simple and accurate method is to use a program such as Excel with *solver* to find the blackbody temperature that minimizes the distance between its (u, v) coordinates and those of the light in question.

Dominant Wavelength and Purity

Colored light sources can be modeled as a mixture of a monochromatic source and a white light. The wavelength of this theoretical monochromatic source is called the **dominant wavelength**, λ_d, and is the perceived color of the light. The percent of the total power provided by the monochromatic source is the **purity**, p_e.

1931 *x*, *y* chromaticity diagram

$$p_e = \frac{a}{a+b} = \frac{y - y_w}{y_d - y_w} = \frac{x - x_w}{x_d - x_w}$$

The choice of the "white point" is arbitrary. Very often, the default choice is an "equal energy white" (*x* = 0.3333, *y* = 0.3333).

Surface Color

The color of a surface, like that of a light source, can be quantified. The **reflectance factor**, $R(\lambda)$, of the surface is combined with the spectral density of the illumination, S_λ, and the CIE color matching functions, $\bar{x}$, $\bar{y}$, and $\bar{z}$, in the calculation of tristimulus values:

$$X = k \int S_\lambda(\lambda) \cdot R(\lambda) \cdot \bar{x}(\lambda) \cdot d\lambda,$$

$$Y = k \int S_\lambda(\lambda) \cdot R(\lambda) \cdot \bar{y}(\lambda) \cdot d\lambda,$$

$$Z = k \int S_\lambda(\lambda) \cdot R(\lambda) \cdot \bar{z}(\lambda) \cdot d\lambda,$$

$$k = \frac{100}{\int S_\lambda(\lambda) \cdot \bar{y}(\lambda) \cdot d\lambda}.$$

Or, in summation form:

$$Y = k \sum_{\lambda=380}^{780} S_\lambda(\lambda)\, R(\lambda)\, \bar{y}(\lambda), \qquad X = k \sum_{\lambda=380}^{780} S_\lambda(\lambda)\, R(\lambda)\, \bar{x}(\lambda),$$

$$Z = k \sum_{\lambda=380}^{780} S_\lambda(\lambda)\, R(\lambda)\, \bar{z}(\lambda), \qquad k = 100 \bigg/ \sum_{\lambda=380}^{780} S_\lambda(\lambda)\, \bar{y}(\lambda).$$

The chromaticity of light sources is 2D, with the photometric value of luminous flux, illuminance, luminous intensity, or luminance playing the role of the third "dimension." However, the chromaticity of surfaces is 3D, with a "lightness" dimension included. Common 3D surface-color spaces are derived from the tristimulus values.

x, y, Y – CIE 1931 chromaticity plus Y tristimulus value

L*,u*,v* – CIE 1976 (L*u*v*) color space; CIELUV

L*,a*,b* – CIE 1976 (L*a*b*) color space; CIELAB

W*U*V*– CIE 1964 uniform space coordinates (obsolete except for the calculation of color rendering index)

Color of Fluorescent Surfaces

The phenomenon of **fluorescence** is characterized by the absorption of light at one wavelength and the nearly instantaneous emission at a longer wavelength. Various surfaces of interest in illumination systems, such as road signs designed to convert the blue-rich skylight at twilight to more visible yellow, also exhibit intentional fluorescent properties to make them brighter or more detectable.

The calculation of the chromaticity of fluorescent surfaces has a degree of complexity that is not present for nonfluorescent surfaces. For fluorescent surfaces, the reflectance factor $R(\lambda)$ is replaced by the **Donaldson matrix**, $D(\mu,\lambda)$, where μ is the absorbed or excitation wavelength, and λ is the emission wavelength:

$$X = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_\mu(\mu)\, D(\mu,\lambda)\, \bar{x}(\lambda),$$

$$Y = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_\mu(\mu)\, D(\mu,\lambda)\, \bar{y}(\lambda),$$

$$Z = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_\mu(\mu)\, D(\mu,\lambda)\, \bar{z}(\lambda).$$

Note that for nonfluorescent surfaces, the Donaldson matrix collapses to the reflectance factor for the diagonal elements ($\mu = \lambda$), and is zero everywhere else.

> The normalization constant, k, is the same as for non-fluorescent surfaces.

In fact, the reflectance factor can be thought of as a special case of the Donaldson matrix.

The details of computing the chromaticity of fluorescent surfaces can be found in ASTM Standard E2152-01, *Standard Practice for Computing the Colors of Fluorescent Objects from Bispectral Photometric Data.*

Color Rendering and CRI

The color of an object depends on both the object and the illuminating source. Some sources are better than others at rendering the colors of objects. This ability is quantified in a figure of merit called the **color rendering index (CRI)** for the source. The CRI is calculated by comparing the chromaticity coordinates of eight prescribed nonfluorescent **test-color samples** (specified by their spectral radiance factors) under the source being evaluated with the chromaticity coordinates of the same eight samples under a **reference illuminant**. The reference illuminant is a blackbody radiator (with the same CCT as the source) for sources with a CCT of less than 5000 K, and a phase of daylight (with the same CCT as the source) for sources with a CCT of 5000 K or higher. The distance between the two chromaticity coordinates for a particular test-color sample represents the color difference between the sample illuminated by the source being evaluated and the same sample illuminated by the reference illuminant. The **general color rendering index**, designated R_a, is the average distance (in W^*, U^*, V^* space) between the eight pairs of chromaticity coordinates (a pair for each test-color sample). It is normalized so a source that is identical to its reference illuminant has a CRI of 100, and a "warm white" fluorescent lamp has a CRI of about 50. Here are examples of typical CRI values for several illuminating sources:

Sunlight (CIE D65)	100
Tungsten lamp (CIE A)	100
Xenon	97
White light LED (blue + YAG)	83
Compact fluorescent lamp	80
Daylight fluorescent	75
Metal halide lamp	61
Warm white fluorescent	52
High-pressure sodium lamp	20
White light from RGB LED combination	20 to 65
White light from four LED combination	up to 90

Calculating CRI and Problems with CRI

Complete details for calculating **CRI** are beyond the scope of this guide. The information is available in two CIE publications (both are necessary):

- CIE 13.3-1995, *Method of Measuring and Specifying Color Rendering Properties of Light Sources*
- CIE Publication 15:2004, *Colorimetry, 3rd Edition*

With these publications comes a software disk for calculating CRI from a light source spectral density. The publications and disk can be purchased from the CIE at www.cie.co.at/cie/index.html.

Work on the CRI began in the 1940s when widespread use of fluorescent tubes began for general lighting. The figure of merit has come under criticism lately for several reasons. Among them:

- The eight test-color samples are all moderate in saturation. CRI does not produce numbers that correspond well with observations on highly saturated colors.
- The CRI is considered to be less accurate when the test illuminant and the reference illuminant differ by more than 0.0054 (in u, v space). Many real illuminants are farther away than this from their respective reference illuminants.
- Many different illuminants, such as sunlight and tungsten lamps, have "perfect" CRIs near 100, yet they render colors quite differently.
- New white light sources, such as combinations of LEDs, seem to perceptually render colors far better than is predicted by their rather low CRIs.

The CIE is expected to issue a new recommendation for quantifying color rendering within the next several years.

Typical Source Parameters

Spectral densities for several common light sources are displayed on the following pages. The table below lists some of the relevant parameters for these sources.

Typical values for common illumination sources			
Source	**Approx. CCT (K)**	**CRI (R_a)**	**Luminous Efficacy (lm/W)**
Tungsten†	2800 to 3200	100	12 to 20
Sunlight	6500	100	100‡
Daylight fluorescent	6360	75	55
Warm white fluorescent	3000	52	60
High-pressure sodium	2030	20	115
Metal halide	4020	61	90
Xenon	5000 to 6000	97	10 to 20

† Tungsten lamps allow a great deal of control over these parameters. Guidelines for controlling tungsten lamps are detailed on the next page.

‡ Lumens per radiated watt. All others are "wall-plug."

LEDs are becoming common illumination sources. They produce white light by using a blue LED and one or more phosphors. Depending on the phosphor used, they can have CCTs from less than 3000 K to in excess of 9000 K. The luminous efficacies also vary over a wide range, from 40 or 50 lm/W for high-flux devices and up to and above 100 lm/W for low-flux devices, with CRIs up to about 80 (R_a). White light is also produced from LEDs by mixing the light from several single-color LEDs. LED lighting technology is changing extremely rapidly as newer devices are developed.

Tungsten Lamps

Tungsten filament incandescent lamps, particularly tungsten halogen lamps, are often used in illumination systems. Paired with stable, current-controlled power supplies, they provide extremely stable sources. If the current is altered, the filament temperature changes, with a resulting change in filament resistance. Therefore, the voltage change is not linearly related to the current change. Similar nonlinear relationships hold for other characteristics of the lamp, such as power consumed, luminous flux output, efficacy (lumens/Watt), color temperature, and lifetime.

Tungsten halogen lamps should be operated between 95% and 105% of their **rated current**. Within this operating envelope, the relationships between the operating parameters are approximately exponential. Modifying the current to a value different from the rated current changes the other parameters according to the exponents shown in the table below:

Parameter	Exponent
Color temperature (K)	0.80
Lifetime (hours)	−25
Luminous efficacy (lumens/Watt)	3.6
Luminous flux (lumens)	6.5
Power (Watts)	2.9
Voltage (volts)	1.9
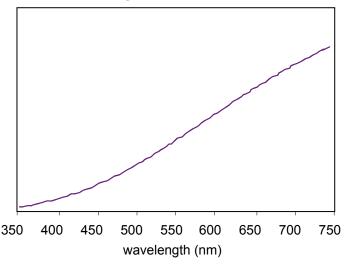
An example of a lamp operated at 5% over its rated current is shown below:

Parameter	Rated	Operated
Current	6.02 A	$6.02 \times 1.05 = 6.32$
Color temperature	2900 K	$2900 \times 1.05^{0.8} = 3015$
Lifetime	1000 hrs	$1000 \times 1.05^{-25} = 295$
Luminous efficacy	16 lm/W	$16 \times 1.05^{3.6} = 19$
Luminous flux	1040 lm	$1040 \times 1.05^{6.5} = 1428$
Power	65 W	$65 \times 1.05^{2.9} = 75$
Voltage	10.8 V	$10.8 \times 1.05^{1.9} = 11.8$
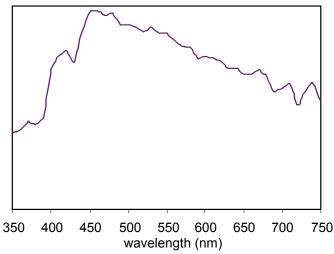
Tungsten and Sunlight

This page and those that follow show typical spectra of several common illumination sources.

Tungsten Lamp (CIE A)

wavelength (nm)

Sunlight (CIE D65)

wavelength (nm)

Fluorescent Lamps

Warm White Fluorescent

wavelength (nm)

Daylight Fluorescent

wavelength (nm)
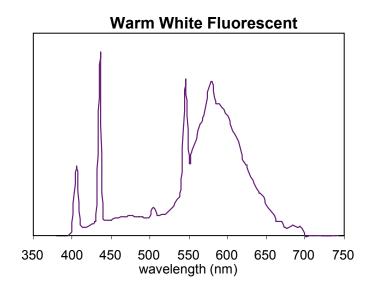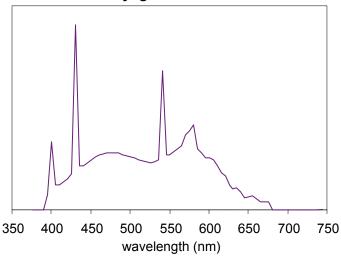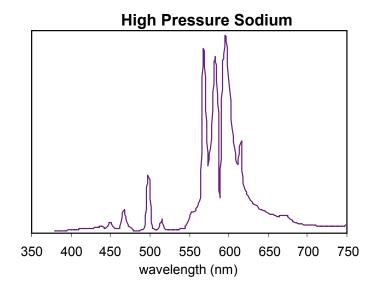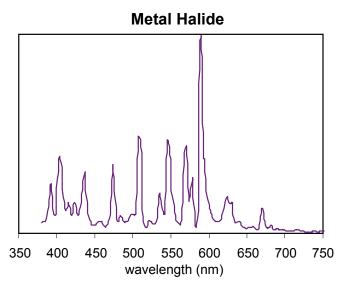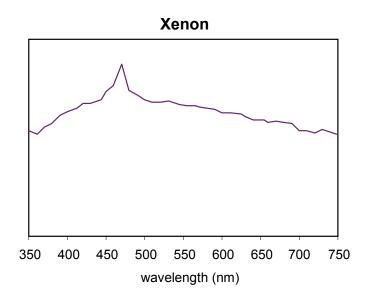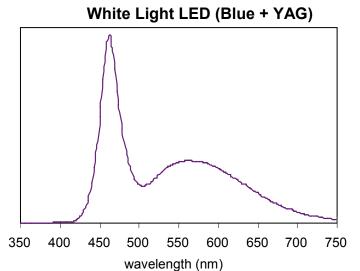
H.P. Sodium and Metal Halide

High Pressure Sodium

wavelength (nm)

Metal Halide

wavelength (nm)

Xenon

wavelength (nm)

White Light LED (Blue + YAG)

wavelength (nm)
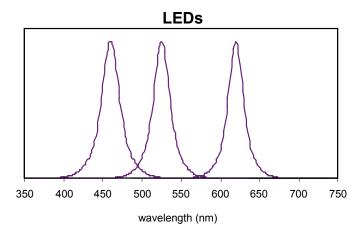
Light Emitting Diodes (LEDs)
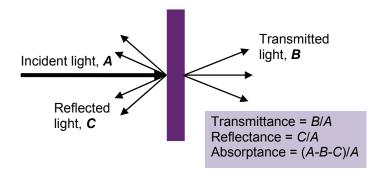
LEDs are moderately narrowband emitters with an approximately Gaussian spectral shape. The spectrum of an LED is often expressed by a single wavelength, with four different single-wavelength descriptions in general use. The most common spectrum-based description is the **peak wavelength**, λ_p, which is the wavelength of the peak of the spectral density curve. Less common is the **center wavelength**, $\lambda_{0.5m}$, which is the wavelength halfway between the two points with a spectral density of 50% of the peak. For a symmetrical spectrum, the peak and center wavelengths are identical. However, many LEDs have slightly asymmetrical spectra. Least common is the **centroid wavelength**, λ_c, which is the mean wavelength. The peak, center, and centroid wavelengths are all derived from a plot of $S_\lambda(\lambda)$ versus λ. The fourth description, the **dominant wavelength**, λ_d, is a colorimetric quantity that is described in the section on **color**. It is the most important description in visual illumination systems because it describes the perceived color of the LED.

Spatially, LEDs, especially those in lens-end packages, are often described by their **viewing angle**, which is the full angle between points at 50% of the peak intensity.

LEDs

wavelength (nm)

Transmittance, Reflectance, and Absorptance
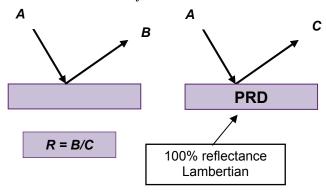
Several alternative methods describe the response of materials to illumination. One common approach is the ratio of the light that is transmitted, reflected, or absorbed to the incident light. This method describes a material by its **transmittance**, τ, its **reflectance**, ρ, or its **absorptance**, α. Do not confuse absorptance with **absorbance**, A, which is equivalent to **optical density (OD)** and is a conversion of transmittance or reflectance to a log scale. For example, 10% transmittance can be described as 1A, 1% as 2A, etc.

Incident light, **A**

Transmitted light, **B**

Reflected light, **C**

Transmittance = *B*/*A*
Reflectance = *C*/*A*
Absorptance = (*A-B-C*)/*A*

A material that produces intensity proportional to the cosine of the angle with the surface normal is called Lambertian. The radiance of a **Lambertian surface** is constant with viewing direction (since the projected area of a viewed surface is also proportional to the cosine of the angle with the surface normal). Furthermore, the directional distribution of scattered light is independent of the directional distribution of the incident illumination. It is impossible to tell, by looking at a Lambertian surface, where the incident light comes from. Perfectly Lambertian surfaces don't really exist, but many materials, such as matte paper, flat paint, and sandblasted metal (in reflection), as well as opal glass and sandblasted quartz (in transmission), are good Lambertian approximations over a wide range of incidence and view angles.

Reflectance Factor and BRDF

A quantity sometimes confused with reflectance is the **reflectance factor**, *R*. The reflectance factor is defined in terms of a hypothetical **perfectly reflecting diffuser (PRD)**, a surface that is perfectly Lambertian and has a 100% reflectance. The reflectance factor is the ratio of the amount of light reflected from the material to the amount of light that would be reflected from a PRD if similarly illuminated and similarly viewed.

Notes on reflectance (ρ) and the reflectance factor (R):

- For a **Lambertian surface**, ρ and R are identical.
- **Reflectance** must be between 0 and 1. The reflectance factor is not similarly bound. A highly polished mirror, for example, has near-zero R for any nonspecular incident and viewing angles, and a very high R (>1.0) for any specular incident and viewing angles.
- The reflectance factor is more closely related to the **bidirectional reflectance distribution function (BRDF)** than to reflectance. The BRDF is defined as the radiance of a surface divided by its irradiance:

$$\text{BRDF} = L/E.$$

- The reflectance factor measures per hemisphere (there are π projected steradians in a hemisphere) what the BRDF measures per projected steradian:

$$R = \text{BRDF} \cdot \pi.$$

Harvey / *ABg* Method
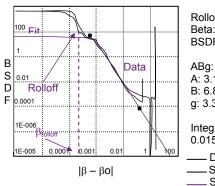
The **Harvey** or ***ABg* method** is used to parameterize scatter from a weakly scattering surface, which is typical for optical surfaces such as lenses and mirrors. It also can be used to model Lambertian surfaces and anisotropic (i.e., asymmetric) scatter. An example for a three-axis polished surface is provided here, which has a **total integrated scatter (TIS** or **TS)** of about 1.6%. The vertical scale represents the **BSDF**, for which an R (reflection) or T (transmission) can be substituted for the S (surface). The horizontal scale represents the absolute difference between $\beta_0 = \sin\theta_0$, or the specular direction, and $\beta = \sin\theta$, or any direction away from specular. Note that both axes are plotted in log space such that the roll-off slope is linear. The *ABg* parameters are:

- g is the slope of the roll-off as shown in the figure whose value of 0 defines a **Lambertian surface**.
- B is the roll-off parameter defined as

$$B = \left|\beta_{\mathrm{rolloff}}\right|^g .$$

- A is the amplitude factor and can be found from

$$BSDF = \frac{A}{B + \left|\beta - \beta_0\right|^g} .$$

Rolloff point:
Beta: 0.00773
BSDF: 45.5

ABg:
A: 3.14e-006
B: 6.89e-008
g: 3.39

Integrated scatter:
0.01596

—— Data
—— Signature
—— Synthetic

Directional Properties of Materials

The reflectance of a material can depend on the direction of the incident light. This dependence is often indicated by a number or letter.

- ρ(0°): reflectance for normal incidence.
- ρ(45°): reflectance for a 45-deg oblique incidence.
- ρ(*d*) or ρ(*h*): reflectance for diffuse illumination.

The **reflectance factor** of a material can depend on both the direction of illumination and the viewing geometry. This is usually indicated by two letters or numbers, the first indicating the incident geometry and the second the viewing geometry.

- R(0°/45°): the reflectance factor for normal incidence and a 45-deg oblique viewing (a common geometry for measuring the color of a surface).
- R(0°/*d*): the reflectance factor for normal incidence and diffuse (everything except the specular) viewing only.
- R(8°/*h*): the reflectance factor for near-normal incidence and hemispherical (everything, including the specular) viewing.
- R(45°/*h*): the reflectance factor for hemispherical illumination and a 45-deg oblique viewing.

The same notation used for reflecting materials can be applied to transmitting materials, where transmittance τ can be dependent on incident geometry, and the transmittance factor, *T*, on both the incident and transmitting geometries. The use of the transmittance factor is not as common as transmittance, reflectance, and the reflectance factor.

Some materials have reflecting properties that are not the same for every azimuthal angle, even for the same elevation angle, e.g., the specular geometry of mirrorlike surfaces has vastly different reflecting properties than any geometry with the same incident and reflecting elevation angles that are not both in the same plane with the surface normal.

Retroreflectors—Geometry

Retroreflectors reflect incident light back toward the direction of the light source, operating over a wide range of angles of incidence. Typically they are constructed in one of two different forms, 90-deg corner cubes or high index-of-refraction transparent spheres with a reflective backing. Retroreflectors are used in transportation systems as unlighted night-time roadway and waterway markers, as well as in numerous optical systems, including lunar ranging. Some are made of relatively inexpensive plastic pieces or flexible plastic sheeting, and some are made of high-priced precision optics.

The performance of retroreflectors is characterized within a geometrical coordinate system, usually with three angles for the incident and viewing geometries and a fourth orientation angle for prismatic designs like corner cubes, which are not rotationally isotropic in their performance. All the geometric variations are described in detail in ASTM E808-01, *Standard Practice for Describing Retroreflection*, along with expressions for converting from one geometric system to another.

Two angles commonly used to specify the performance of retroreflectors are the **entrance angle**, β, and the **observation angle**, α. The entrance angle is the angle between the illumination direction and the normal to the retroreflector surface. High-quality retroreflectors work over fairly wide entrance angles, up to 45-deg or more (up to 90 deg for pavement marking). The observation angle, the angle between the illumination direction and the viewing direction, is generally very small, often one degree or less.

Another useful angle for interpreting the performance of retroreflectors is the **viewing angle**, υ, the angle between the viewing direction and the normal to the retroreflector surface.

Retroreflectors—Radiometry

The performance of retroreflectors is quantified by several coefficients. These are the most common:

R_I, **coefficient of retroreflected luminous intensity**,

$$R_I = \frac{I}{E_\perp},$$

where $E_\perp$ is the illuminance on a plane normal to the direction of illumination, and I is the intensity of the illuminated retroreflector.

R_A, **coefficient of retroreflection**,

$$R_A = \frac{R_I}{A} = \frac{I/A}{E_\perp},$$

where A is the area of the retroreflector.

R_L, **coefficient of retroreflected luminance**,

$$R_L = \frac{R_A}{\cos \upsilon} = \frac{L}{E_\perp}$$

is the ratio of the luminance in the direction of observation to $E_\perp$.

R_Φ, **coefficient of retroreflected luminous flux**:

$$R_\Phi = \frac{R_A}{\cos \beta}.$$

R_F, **retroreflectance factor**

$$R_F = \frac{\pi \cdot R_I}{A \cdot \cos \beta \cdot \cos \upsilon} = \frac{\pi \cdot R_A}{\cos \beta \cdot \cos \upsilon} = \frac{\pi \cdot R_L}{\cos \beta}.$$

It is the retroreflectance factor, R_F that is numerically equivalent to the reflectance factor, R.

Retroreflectors are often specified by the coefficient of retroreflection, R_A, for various observation angles and entrance angles.

Values for R_A of several hundred (cd/m²)/lux are not uncommon, corresponding to reflectance factors up to and over 1000.

Lambertian and Isotropic Models
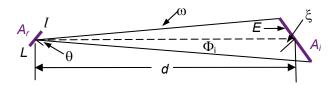
There are no direct "conversion factors" between the four basic quantities in illumination: **flux**, Φ; **irradiance**, E; **intensity**, I; and **radiance**, L. But for many situations, knowledge of one factor allows the calculation of the others. Making this calculation usually requires knowledge of the directional properties of the illuminating source, or at least a fair model of these directional properties. The two most common models are isotropic and Lambertian.

An **isotropic source** is defined here as having intensity independent of direction. For a **Lambertian source**, the radiance is independent of direction and the intensity is therefore proportional to the cosine of the angle with the surface normal. A few nearly isotropic sources exist, such as a round, frosted light bulb, a frosted ball-end on a fiber, and a line filament (in one plane, anyway). However, most flat radiators, diffusely reflecting surfaces, and exit pupils of illuminating optical systems are more nearly Lambertian than isotropic. Reasonable predictions can be made by modeling them as Lambertian.

The model of directional illumination properties need only apply, of course, over the range of angles applicable to your particular situation. In many cases, the mutually contradictory models of an isotropic and a Lambertian source are used simultaneously. This is valid over small angular ranges where the cosine of the angle with the surface normal doesn't change much. This assumption is not all that restricting. For example, for a small Lambertian source illuminating an on-axis circular area, the error in flux caused by using an isotropic model is less than 1% for a subtended full angle of 22 deg [NA = 0.19, f/2.6], less than 5% for a full angle of 50 deg [NA = 0.42, f/1.2], and less than 10% for 70 deg [NA = 0.57, f/0.9]. However, for a full angle of 180 deg (a full hemisphere), the error is 100%!

Known Intensity

Consider a small source at a distance. For a **known intensity** that is essentially constant over all relevant directions, i.e., toward the illuminated area:

where I is the intensity of the radiating area in the direction of the illuminated area;

A_r is the radiating area;

θ is the angle between the normal to the radiating area and the direction of illumination;

$A_r \cos\theta$ is the projected radiating area as viewed from the illuminated area;

A_i is the illuminated area;

ξ (*xi*) is the angle between the normal to the illuminated area and the direction of illumination (assumed constant over this small angular range);

d is the distance between the two areas (assumed to be constant);

ω is the solid angle formed by the illuminated area when viewed from the radiating area (assumed to be small);

$\Omega = \omega \cos\theta$ is the corresponding projected solid angle (for small solid angles);

E is the irradiance at the illuminated area;

Φ_i is the total flux irradiating the illuminated area; and

L is the radiance of the radiating area.

$$E = \frac{I \cos\xi}{d^2}, \qquad \Phi_i = I\omega, \qquad L = \frac{I}{A_r \cos\theta}.$$

Known Flux and Known Radiance

If, in the same situation, the **flux** within the **solid angle** is known, then the intensity is

$$I = \Phi_i / \omega ,$$
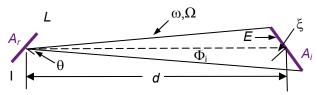
the irradiance is

$$E = \Phi_i / A_i ,$$

and the radiance is

$$L = \frac{\Phi_i}{\omega \, A_r \, \cos\theta} = \frac{\Phi_i}{\Omega \, A_r} .$$

Consider the same situation, but not necessarily with a small radiating area or small illuminated area:

If the radiance is known and the radiating area is small, then

$$I = L \, A_r \, \cos\theta .$$

If $\cos\theta$ is essentially constant from all points on the radiating area to all points on the illuminated area, then

$$\Phi_i = L \, A_r \, \omega \cos\theta = L \, A_r \, \Omega .$$

If $\cos\theta$ varies substantially over the illuminated area, then the second form of this equation, using the projected solid angle, should be used.

Since there are π projected steradians in a hemisphere, the total flux radiated (for a Lambertian radiator) is

$$\Phi_r = L \, A_r \, \pi .$$
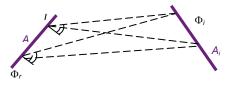
The irradiance at the illuminated area (E) is

$$E = \frac{\Phi_i}{A_i} = \frac{L \, A_r \, \Omega}{A_i} = \frac{L \, A_r \, \cos\theta \cos\xi}{d^2} = L \ \Omega_i ,$$

where Ω_i is the projected solid angle of the radiating area when viewed from the illuminated spot.

Form Factor and Average Projected Solid Angle

Here the approximations of constant cosines cannot be used.

The angles between the normal to the radiating surface and the directions to points on the illuminating surface vary not only with the locations of the points on the illuminated surface, but also with the locations of points on the radiating surface. The concept of projected solid angle takes the former into account, but not the latter. What is needed is an **average projected solid angle**, $\bar{\Omega}_{r \, to \, i}$, which is the projected solid angle subtended by the illuminated area and averaged over all points on the radiating area. Then the illuminating flux, Φ_i, from a **Lambertian radiator** is

$$\Phi_i = L \, A_r \, \bar{\Omega}_{r \, to \, i} = \frac{\Phi_r}{\pi} \, \bar{\Omega}_{r \, to \, i}.$$

In practice, the average projected solid angle is not used. However, its geometrical equivalent, called the **form factor**, $F_{a \, to \, b}$, is used. The only difference between the form factor and the average projected solid angle is a multiplier of π:

$$F_{a \, to \, b} = \bar{\Omega}_{a \, to \, b} \big/ \pi.$$

The form factor measures in hemispheres what the average projected solid angle measures in projected steradians. The form factor also can be interpreted as the portion of the flux leaving a Lambertian radiator, a, that illuminates a surface, b:

$$\Phi_i = \Phi_r \, F_{r \, to \, i}.$$

Note that the form factor is directional, as are the solid and the projected solid angles. $F_{a \, to \, b}$ is not in general equal to $F_{b \, to \, a}$. However, the product of the area and the form factor is constant:

$$A_a \, F_{a \, to \, b} = A_b \, F_{b \, to \, a}.$$
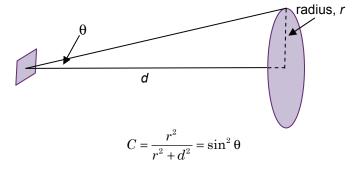
Configuration Factor

The **form factor** and the **average projected solid angle** both link two extended areas. The form factor measures in hemispheres what the average projected solid angle measures in projected steradians. Another term, the **configuration factor**, C, is similarly related to the **projected solid angle**, linking a small area with an extended area. Like the form factor, the configuration factor measures in hemispheres what the projected solid angle measures in projected steradians:

$$C = \Omega/\pi.$$

Tables of configuration factors and form factors for myriad geometries can be found in handbooks on illumination, in books on radiative heat transfer (where the issues are identical to illumination by Lambertian radiators), and on the Internet. Three cases with applicability to many optical situations are listed here:

Case 1: Small area to an extended circular area; both areas parallel and with axial symmetry.

$$C = \frac{r^2}{r^2 + d^2} = \sin^2 \theta$$
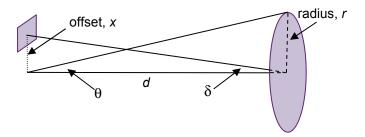
and

$$\Omega = \frac{\pi\, r^2}{r^2 + d^2} = \pi \sin^2 \theta.$$

Useful Configuration Factor

Case 2: Small area to an extended circular area; both areas parallel, but without axial symmetry.

$$C = \frac{1}{2}\left(1 - \frac{1 + \left(\frac{d}{x}\right)^2 - \left(\frac{r}{x}\right)^2}{\left[\left\{1 + \left(\frac{d}{x}\right)^2 + \left(\frac{r}{x}\right)^2\right\}^2 - 4\left(\frac{r}{x}\right)^2\right]^{\frac{1}{2}}}\right)$$

or, equivalently:

$$C = \frac{1}{2}\left(1 - \frac{1 + \tan^2 \delta - \tan^2 \theta}{\left[\tan^4 \delta + \left(2\tan^2 \delta\right)\left(1 - \tan^2 \theta\right) + \sec^4 \theta\right]^{\frac{1}{2}}}\right)$$

and

$$\Omega = \pi \cdot C.$$

These expressions degenerate to the expressions for case 1 above when x, or equivalently, δ, is equal to zero.

Useful Form Factor

Case 3: An extended circular area illuminating another extended circular area; both areas parallel and centered on the same axis.

$$F_{r\,to\,i} = \frac{1}{2}\left\{1 + \frac{1+\left(\frac{r_i}{d}\right)^2}{\left(\frac{r_r}{d}\right)^2} - \left[\left(1 + \frac{1+\left(\frac{r_i}{d}\right)^2}{\left(\frac{r_r}{d}\right)^2}\right)^2 - 4\left(\frac{r_i}{r_r}\right)^2\right]^{\frac{1}{2}}\right\}.$$
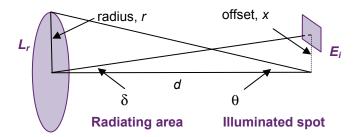
Some numerical values of $F_{r\,to\,i}$ for this case are shown in the table below for several sizes of radiating and illuminated disks (each expressed as a multiple of the distance between the two parallel circular areas that are centered on the same axis).

		Form Factor, $F_{r\,to\,i}$					
		r_i/d					
		0.03	0.10	0.30	1.00	3.00	10.0
	0.03	.001	.010	.083	.500	.900	.990
	0.10	.001	.010	.082	.499	.900	.990
r_r/d	0.30	.001	.009	.077	.489	.899	.990
	1.00	.000	.005	.044	.382	.890	.990
	3.00	.000	.001	.009	.099	.718	.989
	10.0	.000	.000	.001	.010	.089	.905

Irradiance from a Uniform Lambertian Disk

Many illumination situations can be modeled as illumination by a **uniform circular Lambertian disk**, with the illuminated area parallel to the disk and at some distance from it.

The irradiance at the illuminated spot is equal to the radiance of the radiating area times the projected solid angle of the radiating area when viewed from the illuminated spot:

$$E_i = L_r \ \Omega_i .$$

If the illuminated spot is on axis ($x = 0$, $\delta = 0$), then

$$E_i = \pi \ L_r \ \sin^2 \theta = \pi \ L_r \ \frac{r^2}{r^2 + d^2} .$$

If the spot is offset from the axis, it is necessary to use the projected solid angle or the configuration factor discussed previously for case 2:

$$\Omega_i = \frac{\pi}{2} \left\{ 1 - \frac{1 + \tan^2 \delta - \tan^2 \theta}{\left[\tan^4 \delta + \left(2 \tan^2 \delta \right) \left(1 - \tan^2 \theta \right) + \sec^4 \theta \right]^{1/2}} \right\} .$$
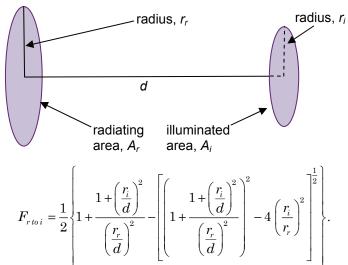
Note: The configuration factor, form factor, and projected solid angle are useful mainly when the radiation pattern is Lambertian or nearly Lambertian.

Cosine Fourth and Increase Factor

Consider the previous case of illumination by a uniform circular Lambertian disk, with the illuminated area parallel to the disk and at some distance from it. For many values of aperture size (θ) and field angle (δ), the irradiance falls off very nearly at $\cos^4\delta$, a phenomenon often referred to as the **cosine-fourth law**.
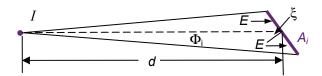
Two of the cosine terms in the $\cos^4$ law are due to the fact that, off axis, the distance increases with the cosine of δ and the inverse square law applies. The third cosine factor comes from the Lambertian source, and the fourth from the fact that the illuminated surface is inclined to the direction of propagation.

In reality, the $\cos^4$ "law" is not exactly true, and is far from true for large values of θ and δ. The table below displays values of the **increase factor**, *F*, which is the multiplier that must be applied to the irradiance calculated by using the axial irradiance and $\cos^4$ falloff. *F* compensates for the inaccuracy in the "cosine-fourth" assumption:
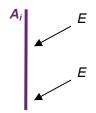
$$E_i = \pi \, L_r \, \sin^2 \theta \cdot \cos^4 \delta \cdot F'$$

				θ			
	(deg)	1.8	3.6	7.2	14.5	30.0	45.0
	NA	0.03	0.06	0.13	0.25	0.50	0.71
	f/#	16	8	4	2	1	0.71
	0	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.01	1.03	1.05
	20	1.00	1.00	1.01	1.03	1.11	1.20
δ(deg)	30	1.00	1.00	1.01	1.05	1.23	1.49
	40	1.00	1.00	1.02	1.08	1.37	1.94
	60	1.00	1.01	1.02	1.09	1.48	2.69

Increase Factor, *F'*

The $\cos^4$ approximation is valid within a few percent up to very large apertures and field angles.

Known Irradiance

If a surface is illuminated by a source of uniform intensity at a distance d and the irradiance on the surface is known, then the intensity of the source is

$$I = \frac{E \cdot d^2}{\cos \xi}.$$

For any surface that is illuminated by uniform irradiance, the total flux illuminating the surface is

$$\Phi = E \cdot A_i.$$

The radiance of the surface, caused by the light reflecting from the surface, depends on the reflecting properties of the surface.

If the surface is Lambertian over all angles of reflection (for this incident geometry), then

$$L = \frac{\rho \cdot E}{\pi},$$

where ρ is the reflectance of the surface for the relevant incident geometry.
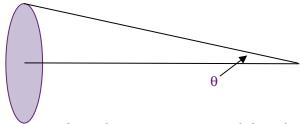
If the surface is not Lambertian over all angles but is Lambertian over the direction of concern, then

$$L = \frac{R \cdot E}{\pi},$$

where R is the reflectance factor of the surface for the relevant incident geometry and for the direction of concern.

ω, Ω, NA, and *f/#* for a Circular Cone

The case of a circular disk subtending a known half-angle, θ, shows up often in illumination situations.

There are at least four common ways of describing the cone: **solid angle** (ω), **projected solid angle** (Ω), **numerical aperture (NA)**, and *f*-number (*f/#*):

$$\omega = 2\pi(1 - \cos\theta) \qquad \Omega = \pi\sin^2\theta$$
$$NA = n \cdot \sin\theta \qquad f/\# = 1/2\sin\theta,$$
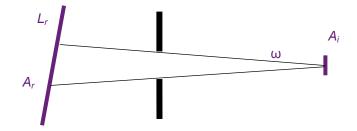
where *n* is the index of refraction.

Cone subtended by a circular disk				
θ(deg)	ω	Ω	NA/n	f/#
1.8	0.003	0.003	0.03	16.00
3.6	0.012	0.012	0.06	8.00
7.2	0.049	0.049	0.13	4.00
12.7	0.154	0.152	0.22	2.27
14.5	0.200	0.196	0.25	2.00
20.0	0.379	0.367	0.34	1.46
25.0	0.589	0.561	0.42	1.18
30.0	0.842	0.785	0.50	1.00
35.0	1.14	1.03	0.57	0.87
40.0	1.47	1.30	0.64	0.78
45.0	1.84	1.57	0.71	0.71
50.0	2.24	1.84	0.77	0.65
60.0	3.14	2.36	0.87	0.58
70.0	4.13	2.77	0.94	0.53
80.0	5.19	3.05	0.98	0.51
90.0	6.28	3.14	1.00	0.50
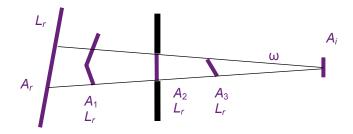
Invariance of Radiance

Unlike intensity, which is associated with a specific point, and irradiance, which is associated with a specific surface, radiance is associated with the propagating light rays themselves. This distinction is not trivial and implies that the radiance of a surface can be considered separate from the actual physical emitter or reflector that produces the radiance.

Consider a uniform Lambertian radiating source, A_r, with radiance, L_r, illuminating an area, A_i, through a limiting aperture that limits the solid angle of the source to ω:
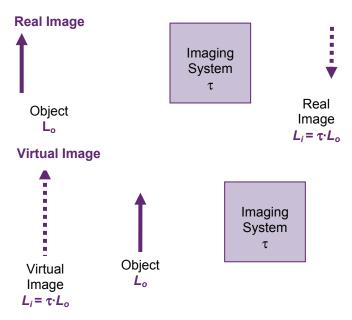
The physical location of the radiating source is irrelevant. Only the solid angle matters. In fact, the physical location (and shape) can be assumed to be anywhere (and any shape) as long as the solid angle is the same. All of the following descriptions of the radiating area, A_1, A_2, and A_3, are equivalent to A_r from an illumination point of view:
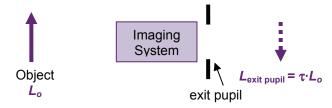
Image Radiance

In an imaging system with no vignetting or significant aberrations, for Lambertian objects, point-by-point, the radiance of an image is equal to the radiance of the object except for losses due to reflection, absorption, and scattering. These losses are usually combined into a single value of **transmittance**, τ. This equivalence of radiance is true for virtual as well as real images, and for reflective or refractive imaging systems.

Real Image

Imaging System τ

Object L_o

Real Image $L_i = \tau \cdot L_o$

Virtual Image

Virtual Image $L_i = \tau \cdot L_o$

Object L_o

Imaging System τ
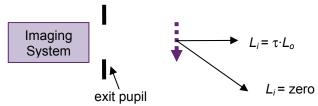
Viewed from any point on a real image, the entire exit pupil of the optical system is also the radiance of the corresponding object point but reduced by τ.

Object L_o

Imaging System

exit pupil

$L_{\text{exit pupil}} = \tau \cdot L_o$

Limitations on Equivalent Radiance

In all cases, the image radiance only exists when the image is viewed through the exit pupil of the imaging system. When viewed in a direction that doesn't include the pupil, the radiance is zero.

If the object is not Lambertian, then the angular distribution of radiance of the image is also not Lambertian. The relationship between the angular distributions of object and image radiances is not straightforward and must be determined by ray tracing on the specific system. However, in many practical cases, the entrance pupil of the imaging system subtends a small angle from the object, and the source is essentially Lambertian over this small angle.
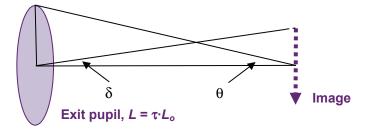
If the object and the image are in media of different refractive indices, n_o for the object and n_i for the image, then the expression for equivalent radiance is

$$\frac{L_i}{n_i^2} = \tau \cdot \frac{L_o}{n_o^2}.$$

The point-by-point equivalence of radiance from object to image is only valid for well-corrected optical systems. For systems that suffer from aberrations or are not in focus, each small point in the object is mapped to a "blur spot" in the image. Thus, the radiance of any small spot in the image is related to the average of the radiances of the corresponding spot in the object and its surrounding area.

Image Irradiance

Since the exit pupil, when viewed from the image, has the radiance of the object, then the irradiance at the image is the same as the irradiance from a source of the same size as the exit pupil and the same radiance as the object (reduced by τ). In most imaging systems, the exit pupil is round and the irradiance is the same as the irradiance from a uniform **Lambertian disk**:

Exit pupil, $L = \tau \cdot L_o$ δ θ **Image**

$$E_i = \pi\,\tau L_o\,\sin^2\theta \cdot \cos^4\delta \cdot F'\,.$$

A table of values for the **increase factor**, F, is presented in the section on illumination transfer. F is very close to 1.0 except for a combination of large field angle (δ) and large aperture (θ), which is not a common combination in imaging systems.

The $\cos^4\delta$ term contributes to substantial field darkening in wide-angle imaging systems—for example, $\cos^4 45$ deg = 0.25.

If the physical aperture stop is not the limiting aperture for all the rays converging to an off-axis image point, the light is vignetted. The irradiance at image points where there is **vignetting** will be lower than predicted.

On axis, $\cos^4\delta = 1.0$ and $F = 1.0$. The image irradiance on axis, E_{i0}, is

$$E_{i0} = \pi\,\tau L_o\,\sin^2\theta\,.$$

f/#, Working *f/#*, *T/#*, NA, Ω

For a camera working at infinite conjugates (distant object, magnification, |m|<<1), the image irradiance can be expressed in terms of the lens' ***f*-number**, *f/#*:

$$E_{i0} = \frac{\pi \, \tau \, L_o}{4 \, (f/\#)^2} \, .$$

This *f/#*, usually associated with a lens, is an "infinite conjugates" quantity. When a lens is used at finite conjugates, the **working *f*-number**, *f/#*_w, describes the cone angle illuminating the image:

$$f/\#_w = (f/\#) \cdot (1 - m),$$

where *m* is the lateral magnification of the image (negative for real images), and the axial image irradiance is:

$$E_{i0} = \frac{\pi \, \tau \, L_o}{4 \, (f/\#_w)^2} \, .$$

Note that *f/#*_w degenerates to the conventional "infinite conjugates" *f/#* when the lens is used at infinite conjugates.

Occasionally, a lens will be designated with a ***T*-number**, T/#, which combines the *f/#* and the transmittance into a single quantity,

$$T/\# = \frac{f/\#}{\sqrt{\tau}} \quad \text{with axial image irradiance:} \quad E_{i0} = \frac{\pi \, L_o}{4 \, (T/\#)^2} \, .$$

Another descriptor of the image illumination cone angle is the **numerical aperture**, NA,

$$NA = \sin\theta \quad \text{with axial image irradiance:} \quad E_{i0} = \pi \, \tau L_o \, NA^2.$$

In all cases, even without circular symmetry, on or off axis, the cone illuminating the image can be described by its **projected solid angle**, Ω, with image irradiance:

$$E_i = \tau L_o \, \Omega \, .$$

Flux and Étendue

The total **flux** reaching the image is the product of the image irradiance and the area of the image. The **image irradiance** is proportional to the projected **solid angle** of the exit pupil when viewed from the image:

$$\Phi_i = \tau L_o \; a_i \Omega_i,$$

where Ω_i is the projected solid angle of the exit pupil viewed from the image, a_i is the area of the image, L_o is the [assumed uniform] radiance of the object, and τL_o is the radiance of the exit pupil.

The flux reaching the image also can be expressed in terms of the radiance of the exit pupil, τL_o, the area of the exit pupil, a_p, and the projected solid angle of the image when viewed from the exit pupil, Ω_p:

$$\Phi_i = \tau L_o \; a_p \Omega_p.$$

The quantity $a\Omega$, representing the area of a plane in the optical system times the projected solid angle of another plane when viewed from it, appears equivalently in both expressions. This **area-solid-angle-product** is a fundamental property of the optical system that determines the amount of light that can get through the system. It is called the **throughput** or **étendue**.

> The radiance of an object is invariant and cannot be increased by an optical system, and the étendue is a fundamental property of an optical system. These two concepts mean that, for a source of given radiance and a given optical system, the maximum flux that can be transmitted through the system is predetermined.
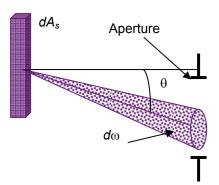
And, without "throwing away" light, the étendue cannot be decreased, but area and solid angle can be traded off.

Generalized Étendue

The terminology for illumination in nonimaging systems is the same as that for imaging systems; however, the range of validity is extended to include all angular space, while that of imaging systems is limited to paraxial systems. With this taken into account, étendue is often called **generalized étendue**. In this domain the étendue cannot be regarded as the simple product of the area and solid angle; it must be integrated per the following equation and figure:

$$\mathcal{E} = n^2 \iint\limits_{\text{aperture}} \cos\theta \, dA_s \, d\omega,$$

where n is the refractive index, θ is the angle from the normal, dA_s is the differential source area, and $d\omega$ is the differential solid angle.

The total flux through the aperture is found by integrating the radiance over the aperture:

$$\Phi = \iint\limits_{\text{aperture}} L(\mathbf{r}, \hat{\mathbf{a}}) \cos\theta \, dA_s \, d\omega,$$

where $\mathbf{r}$ and $\hat{\mathbf{a}}$ denote the positional and directional aspects of source emission. Assuming that the source is Lambertian so radiance is independent of angle, then

$$\Phi = L_s \iint\limits_{\text{aperture}} \cos\theta \, dA_s \, d\omega = \frac{L_s \mathcal{E}}{n^2}.$$

Note that total flux is the product of the radiance and the geometrical étendue factor. This also shows the **conservation of étendue** that follows from the conservations of radiance and energy.

Concentration

Concentration (*C*) is a term associated with the generalized étendue. It represents the ability to transfer more light into a desired area by using the conservation of étendue to alter the angle at the output of an optical system. It is defined as the ratio of the input area (*A*) to the output aperture area (*A′*) that transmits the prescribed flux from area A. For this reason it is called the **concentration ratio**:

$$C = A/A'.$$

This expression, a limit factor of the laws of thermodynamics, is a forbearer of the invariance of radiance and étendue.

In a 2D system, which is analogous to an extruded trough, and a 3D system, which is analogous to a well, we find that the respective concentrations are given by

$$C_{2D} = \frac{a}{a'} = \frac{n' \sin \theta'}{n \sin \theta} \qquad \text{and} \qquad C_{3D} = \frac{A}{A'} = \left(\frac{n' \sin \theta'}{n \sin \theta} \right)^2,$$

where *a* and *a′* are the aperture widths, *A* and *A′* are the aperture areas, *n* is the input index, *n′* is the output index, θ′ is the output angle, and θ is the input angle. **Optimal concentration** is realized when the output angle is π/2, giving

$$C_{2D,opt} = \frac{n'}{n \sin \theta_a} \qquad \text{and} \qquad C_{3D,opt} = \left(\frac{n'}{n \sin \theta_a} \right)^2,$$

where θ_a, the **acceptance angle**, is the prescribed upper input angle over which conservation of étendue is maintained.
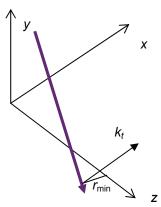
Skew Invariant

The **skew invariant** is another limiting factor in nonimaging system design. Its definition is rather esoteric:

$$f_{\text{skew}}(s) = \frac{d\mathcal{E}(s)}{ds},$$

where $s = r_{\min}k_t$, and $r_{\min}$ is the ray's closest approach to the optical axis (z, as shown), and k_t is the tangential component of the ray's propagation direction.

A simpler way to think about the skew invariant is to recognize that in a rotationally symmetric system (e.g., a lens), loss is introduced from the input to the output if the two spatial distributions are not the same shape. For example, if the object shape is a uniform square but a uniform round output is desired, then transfer losses will be produced.
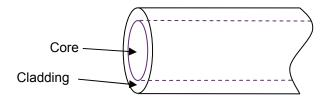
To maximize transfer efficiency with different distributions, the symmetry of the optical system must be broken; or, in other words, there must be a "twist" in the optical components to force rays out of their respective sagittal planes. Many nonimaging optical systems take advantage of this property by including faceted reflectors (e.g., segmented headlights), segmented lenses (e.g., pillow optics for projection displays), or 3D edge-ray concentrators that employ V-wedges near the source (i.e., solar concentrators).

For a rotationally symmetric system, the rotational skewness of each ray is conserved or invariant. This skew invariant is given by the first derivative of the étendue.

Fibers—Basic Description

Optical fibers, **lightpipes**, and **lightguides** are all variations on the same theme. They each contain a central transparent core, usually circular in cross-section, surrounded by an annular cladding. The cladding has a lower index of refraction than the core.
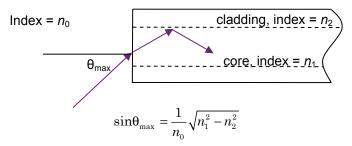
The core can transmit light for long distances with low loss because of total internal reflection at the interface between the core and the cladding. The primary purpose of the cladding is to maintain the integrity of this interface. Without it, total internal reflection would occur at a core-air interface, but dust, nicks, abrasions, oils, and other contamination on the interface would reduce the transmission to unacceptably low levels.

Sometimes layers of buffering and/or jacketing are placed outside the cladding for additional protection.

The core diameter can range from very small, on the order of the wavelength of light, to a centimeter or more. The very thin cores are essentially waveguides and not used for illumination. Flexible glass and quartz fibers have core diameters ranging from approximately 50 microns to about 1 millimeter. If they are thicker than that, they are rigid and called rods or light pipes. Plastic fibers are flexible at thicker core diameters. Sometimes liquid cores and plastic cladding are used to make flexible, high-transmittance lightguides that are over a centimeter in core diameter.

Numerical Aperture and Étendue

The maximum angle that a fiber can accept and transmit depends on the indices of refraction of the core and cladding (as well as the index of the surrounding medium, usually air, $n_0 = 1$).

$$\sin\theta_{max} = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2}$$

and the NA is

$$NA = n_0\sin\theta_{max} = \sqrt{n_1^2 - n_2^2}.$$

The fiber has a maximum acceptance projected **solid angle**, $\Omega = \pi\sin^2\theta_{max}$, and an acceptance area, the cross-sectional area of the core. Together, they define a throughput or étendue for the fiber in air:

$$\text{Étendue} = \frac{\pi^2}{4}d^2 NA^2,$$
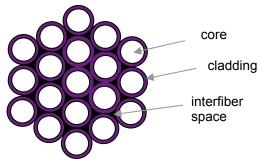
where d is the core diameter.

This étendue defines the maximum flux-carrying capability of the fiber when presented with a source of radiance.

> Note: A fiber illuminated at less than its maximum acceptance angle will, theoretically, preserve the maximum illumination angle at its output. However, bending and scattering at the core-cladding interface broadens this angle toward the maximum allowable. This effect is not important in illumination systems in which it is desirable to utilize the maximum étendue of low-throughput components such as fibers and fill the full input NA.

Fiber Bundles

To achieve high throughput with flexible glass or quartz fibers, multiple fibers are often arranged in a bundle, such as the 19-fiber tightly packed bundle shown below:
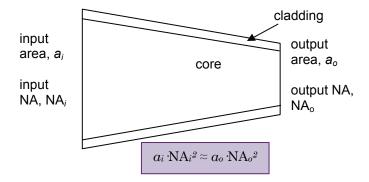
The ratio of the light-carrying core area to the area of the entire bundle is called the **packing fraction (pf)**, and can be as high as 85%. This packing fraction reduces the effective area of the bundle and, correspondingly, its étendue.

In addition to flexibility, fiber bundles have other possible advantages in illumination systems:
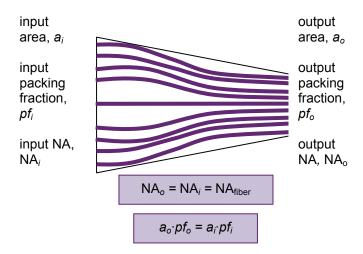
- **Shape Conversion:** In some situations, such as when illuminating a spectrometer, it can be useful to convert a circular cross-section of fibers to a line cross-section to align with, or actually become, the entrance slit to the spectrometer.

- **Splitting the Bundle:** By feeding a large fiber bundle with a single light source and splitting the bundle into two or more branches, it is possible to illuminate multiple locations, from multiple angles, with one source.

- **Mixed Bundle:** When illuminating with light over a wide spectral band, such as the full solar spectrum (~250 to 2500 nm), a mixed bundle of high OH silica fibers for good UV transmission and low OH silica fibers for good IR transmission can compensate for the lack of an adequate single-fiber type.

Tapered Fibers and Bundles

By tapering a single fiber, it is possible to trade off between area and solid angle while keeping the product (étendue) approximately constant.

cladding

input
area, a_i

core

output
area, a_o

input
NA, NA_i

output NA,
NA_o

$$a_i \cdot NA_i{}^2 \approx a_o \cdot NA_o{}^2$$

On the other hand, when a bundle of straight fibers is tapered, the tradeoff is between the area and packing fraction.

input
area, a_i

output
area, a_o

input
packing
fraction,
pf_i

output
packing
fraction,
pf_o

input NA,
NA_i

output
NA, NA_o

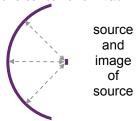$$NA_o = NA_i = NA_{fiber}$$

$$a_o \cdot pf_o = a_i \cdot pf_i$$
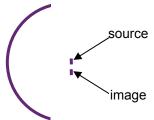
Spherical Reflector

The light emitted from a source in the direction away from the optical system can be redirected toward the optical system by using a **spherical mirror** with the source located at the center of curvature.

source
and
image
of
source
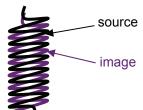
If the source is solid, it is necessary to place the source slightly away from the center of curvature and the image just above, below, or alongside the physical source.

source

image

Ignoring losses on reflection, the image has the same radiance as the source, but the effective source area (source plus image) is doubled.

Sometimes this technique is used to place the image of a source in a location where the physical source itself could not fit because of an obstruction such as a lamp envelope or socket.

If the source is not solid, such as a coiled wire tungsten filament, imaging the source almost directly onto itself can help fill in the area between the coils.

source

image
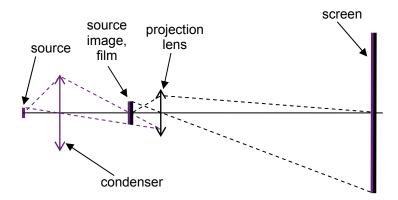
In this case, the effective area of the source is not appreciably increased, but the apparent radiance is nearly doubled.

Abbe Illumination

Abbe illumination is characterized by imaging the source (or imaging an image of the source) directly onto the illuminated area. Since the uniformity of illumination is directly related to the uniformity of source radiance, Abbe illumination requires an extended source of uniform radiance such as a well-controlled arc, a ribbon filament lamp, the output of a clad rod, a frosted bulb, an illuminated diffuser, or the output of an integrating sphere.
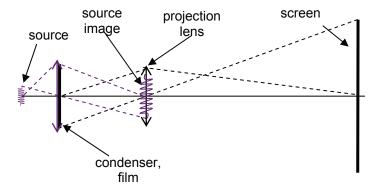
The paraxial layout below shows Abbe illumination used in a projection system. The source is imaged by a condenser onto the film. The projection objective images the film and the image of the source onto the screen. The purple dotted lines show the marginal and chief rays from the source. The black dotted lines show the marginal and chief rays from the film (and the image of the source). The marginal rays go through the on-axis points on the object and image and on the edges of the pupils (which are the lenses in this case). The chief rays go through the edges of the object and image and the on-axis points of the pupils.

Köhler Illumination

Köhler illumination is used when the source is not uniform, such as a coiled tungsten filament. Köhler illumination is characterized by imaging the source through the film onto the projection lens. The film is placed adjacent to the condenser, where the illumination is quite uniform, provided the source has a relative uniform angular distribution of intensity.
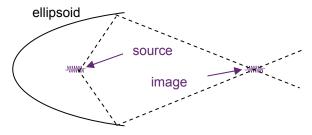
The paraxial layout below shows Köhler illumination used in a projection system. The source is imaged by a condenser onto the projection lens. The projection objective images the film onto the screen. The purple dotted lines show the marginal and chief rays from the source. The black dotted lines show the marginal and chief rays from the film.
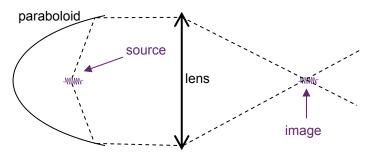
With similar sources, similar condenser NAs, source/condenser étendue as limiting étendue, and similar screen sizes, the average screen irradiance levels are the same for both Abbe and Köhler illumination systems. The choice between the two generally depends upon the type of source available.

Ellipsoidal and Paraboloidal Mirrors

Very efficient collection of light from a source can be achieved using an **ellipsoidal mirror**, placing the source at one of the foci. The source is imaged at the other focus, with light collected over more than a hemisphere.

An alternative is to use a **paraboloidal mirror** to collimate the light from a source and a lens to reimage it. Again, the light from the source is collected over more than a hemisphere.

The forward light is usually ignored in both of these types of designs.

In both cases, the image of the source may not be good quality, but image quality may not be important in illumination systems. Also, obstructions like lamp bases, sockets, and mounting hardware can produce directional anomalies in the radiance of the image.

If the quality of illumination is important, devices such as lenslet arrays or faceted reflectors may be used.

Spectral Control and Heat Management

Specifications for illumination systems often contain spectral requirements. Some of these requirements can be partially met by the selection of lamp type, but usually some sort of filtering is needed. Also, for visual systems, especially those using tungsten lamps, unwanted heat from infrared light may need to be removed. Again, filtering is needed.

The simplest type of filter is the **absorbing filter** placed in front of the light source. Filter glasses with a wide range of spectral characteristics are available from glass manufacturers. The primary concern with absorbing glass filters is cracking from excessive absorbed heat.

> Often a cracked filter will continue to work just fine.

Interference filters use multilayer thin-film coatings that either transmit or reflect light at specific wavelengths. Cracking is generally not a concern unless the filter is made of an absorbing substrate. These filters are available with a much wider variety of spectral properties than absorbing filters, including narrow bandwidth and sharp cut-off, and can be designed and manufactured to achieve specific custom properties. They are also available for different angles of illumination, typically 0 deg and 45 deg.

> Interference filters shift their spectral properties with incident angle and therefore may not be suitable for uncollimated light with a divergence of more than about 10 deg from the axis.

Hot mirrors and **cold mirrors** are excellent ways to manage heat that must be removed from a light source. A hot mirror reflects infrared light and transmits visible light. A cold mirror reflects visible light and transmits infrared light. The reflector behind the light on a dentist's chair is a cold mirror.

Illumination in Visual Afocal Systems

Afocal visual systems, such as binoculars, take the collimated light from an extended distant object and present collimated light to the eye, but with angular magnification. Therefore, the object appears larger. However, the apparent radiance (and therefore perceived brightness) of the object is the same as that of the naked eye, provided the size of the aperture stop is the same with and without the binoculars. Without the binoculars, the aperture stop is merely the eye pupil. With the binoculars, the aperture stop is the smaller of:

- the pupil of the eye magnified by the angular magnification, or
- the aperture of the objective lens.

In other words, if the collimated ray bundle entering the eye from the binoculars is smaller than the eye pupil, the apparent radiance of the object will be less with the binoculars than with the naked eye. If the pupil of the eye is the limiting aperture both with and without the binoculars, the apparent radiance will be the same.

Binoculars are traditionally designated by two numbers, the first being the angular magnification, the second the diameter of the objective lens in mm. A light-adapted eye pupil with a 2-mm diameter would remain the aperture stop for all of the following common sizes of bird-watching binoculars: 8×42, 8×32, 10×42, 6×25, and 10×25. These binoculars are generally used during the day. However, marine binoculars, which are used under all lighting conditions, are typically 7×50 to accommodate a 7-mm-diameter dark-adapted eye pupil.
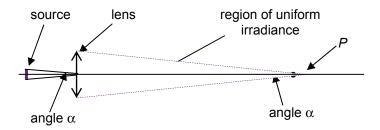
> Note that a true point source, such as a star, will have higher apparent intensity (and therefore appear brighter) with binoculars than with the naked eye because more light is collected with the binoculars, but there is no angular magnification.

Searchlight

A **searchlight** can provide uniform irradiance in three dimensions that is extremely insensitive to the position of the irradiated object.

A searchlight consists of a small circular Lambertian source at the focal point of a collimating lens. Anywhere inside the shaded area in the figure below, the source appears as a circular disk at infinity, subtending a full angle α. The entire extent of the source is visible, because it does not completely fill the collimating lens. Since the view of the source is the same anywhere inside this region, the irradiance is the same.

Outside this region and beyond point *P*, the lens restricts the area of the source that is visible. The lens itself appears as a disk of the same radiance as the source. In this region, the irradiance falls off as the square of the distance from the lens.
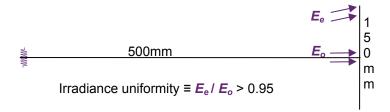

For real searchlights with small sources and large-diameter lenses, the paraxial description above is not exactly valid over the entire shaded region. However, over a relatively small portion of this region, the irradiance is extremely uniform. This region of irradiance uniformity extends not only laterally, but longitudinally as well.

A searchlight provides a volume of uniform irradiance.

Source at a Distance

A small source at a distance from an object can provide reasonably uniform irradiance across the object. It is somewhat counterintuitive that a bare lamp filament, with its obviously terrible radiance uniformity, can produce excellent irradiance uniformity. For example, a small (assumed Lambertian) lamp filament at 500 mm from a flat object whose largest dimension is 150 mm will provide irradiance uniformity across the object of better than 95% (considering only $\cos^4$ falloff). The same lamp and object at 1.0-meter distance produces nearly 99% irradiance uniformity.

E_e

150 mm

E_o

500mm

Irradiance uniformity ≡ E_e / E_o > 0.95
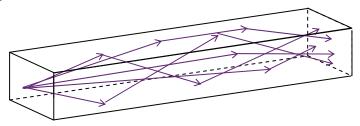
A source of uniform radiance can be created by illuminating a transmission or reflection diffuser with uniform irradiance.

A common calibration laboratory method used to realize a standard of known radiance is to illuminate a reflection diffuser, typically 50 mm in diameter, with a standard of known irradiance, typically a calibrated 1000-W tungsten halogen lamp (ANSI type FEL), at a 500-mm distance. The irradiance uniformity across the diffuser is better than 99.5%. If the reflectance factor of the diffuser is uniform, the radiance uniformity of the standard is also better than 99.5%.

Mixing Rod

A **mixing rod** is a long piece of clear quartz, glass, or plastic. Light entering one face of the rod undergoes multiple total internal reflections emerging from the other parallel face.

Due to the multiple reflections, the **irradiance** at the exit face can be extremely uniform. In a well-designed and illuminated rod, the **radiance** can be quite directionally uniform as well. The directional uniformity of radiance can be enhanced by placing a diffuser at the exit face of the rod or simply frosting the rod-end itself.

Mixing rods can have any shape desired. The rods with plane sides do a better mixing job in most cases.

Typically the rods have an aspect ratio (length to largest transverse dimension) of about 10:1, and are usually about 75- to 150-mm long. They can be clad like an optical fiber, but generally are not. Unlike a fiber, the number of reflections in a mixing rod is quite small, and losses are not a serious problem.
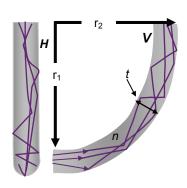
Rather than using a rod with polished faces, it is possible to achieve a similar effect using a mirrored tube with a hollow center.

The combination of a rod and its illuminator are sometimes designed by computer simulation. But the degree of uniformity required doesn't always demand this level of complexity, so simple trial-and-error is often sufficient.

Bent Lightpipes

Complex lightpipes made from straight sections, bends, and tapers are common in many industries. **Bent lightpipes** are components used to mix or collect light from different paths that bend around objects or provide light output over an extended region. An example is automotive dashboard illuminators that employ lightpipes coupled to a small incandescent source or an LED. The lightpipe allows the source light to be directed around dials and knobs. The bends allow simple packaging and lower costs at the expense of design complexity.

Any cross-sectional shape, bend angle, bend shape, and so forth is possible, but the simplest is a single, right-angle bend using common-center, circular bends and an arbitrary cross section. A circular cross-section is shown here. Two important slices are called **principal sections**: the vertical (V), which shows the bend of the lightpipe, and the horizontal (H), which shows the bend going into the page. The vertical slice defines the transmission properties of the lightpipe. For normally incident input light coupled to the lightpipe, there are no propagation losses except Fresnel losses if the **bend ratio**, R, is

$$R = r_2/r_1 = 1 + t/r_1 \leq n,$$

where r_1 and r_2 are the two bend radii, t is the lightpipe thickness in the vertical section, and n is the lightpipe index in air. As the input angle increases, there are losses at the limit of this equation, but the equation is transcendental. By decreasing the thickness of the lightpipe, one can increase the acceptance angle such that there is no loss.

More complex parameterization of lightpipes, including uncommon bend centers, noncircular bends, and arbitrary cross sections, can be found in the literature.

Integrating Sphere

Integrating spheres produce illumination that has extremely uniform radiance and irradiance. An integrating sphere is a hollow spherical shell coated on the inside with a highly reflecting diffuse coating. The projected solid angle from any point on a sphere to any element of area on the sphere is the same, regardless of location. This fact combined with the diffuse coating and the multiple reflections cause any light introduced into the sphere to produce uniform irradiance on and radiance of the wall of the sphere. A hole or "port" in the sphere allows this uniform illumination to be used in an optical system.

> The radiance at the exit of an integrating sphere extends to a full hemisphere (π projected steradians). The irradiance at the wall of an integrating sphere is incident from a full hemisphere.

The radiance, L, of the wall of an integrating sphere generated by flux, Φ, introduced into the sphere is
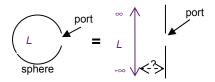
$$L = \frac{\Phi}{\pi \cdot A_s} \cdot M,$$

where A_s is the area of the complete sphere wall, and M is the "sphere multiplier," which is equal to the average numbers of reflections in the sphere. The multiplier, M, is

$$M = \frac{1}{1 - \bar{\rho}},$$

where $\bar{\rho}$ is the **average reflectance** of the wall of the sphere, counting the holes as areas of zero reflectance.

A good working model of an integrating sphere is to consider the port to be a hole in a wall, and, at a totally arbitrary distance behind it, another wall of infinite extent and radiance, L.

Lenslet Arrays

Imaging illumination systems, whether single- or double-lens systems, paraboloidal reflector and lens systems, or single ellipsoidal reflector systems, all suffer from possible nonuniformities in intensity (and consequently also in irradiance). These are due, among other causes, to possible nonuniformities in the source as well as obstructions such as filament support wires, gas discharge electrodes, and LED heat-sink structures.

These nonuniformities can be smoothed out by using a **lenslet array**, an array (usually 2D) of small lenses. Typically, the arrays are 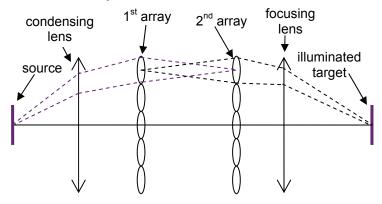used in pairs. In the diagram below, the dotted purple lines show the marginal rays for one of the lenslets in the first array; the black dotted lines show the marginal rays for the corresponding lenslet in the second array.
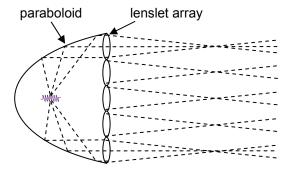
In this configuration, the source is imaged by each lenslet of the first array into the corresponding lenslet of the second array. Each lenslet of the first array is imaged onto the entire target. This overlaying creates uniform illumination of the target. In effect, the lenslet arrays create multiple **Köhler illumination** systems, all superimposed on the target.

Lenslet arrays are generally designed using illumination design software.

Small Reflectors, Lenslet Arrays, and Facets
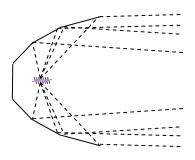
Ellipsoidal and paraboloidal reflectors are often "small" with respect to the lamp dimensions and the distances between the lamp and the reflecting surfaces. In these cases, in addition to the effects of lamp support structures, the size and structure of the lamp itself can produce nonuniformities in illumination.

One method of minimizing these nonuniformities is to include a **lenslet array** in front of the detector. This broadens the beam a little, depending on the *f*# of the lenslets, but it can produce much more uniform illumination than the reflector alone.

Tandem lenslet arrays also can be used to minimize the effects of small reflectors.

Another approach is to break the reflector into small flat **facets**, either radially, circumferentially, or both.

Lenslet arrays and faceted reflectors are usually designed with illumination design software.

Source Modeling Overview

A system software model, whether a simple paraxial design or a detailed design of an illumination system, may fail to agree with experimental results due to the lack of a comprehensive **source model**. For the simplest case, where the optics are far away from the source and collect light over a small solid angle, a point source model or a simple geometrical model of the source may be sufficient. The directional distribution of light from these simple models is usually assumed to be isotropic or Lambertian.

For more efficient designs with optics that are close to the source and collect light over a large solid angle, a more complete model of the source is required to obtain meaningful results. These models must reflect the physical size and shape of the source and should contain directional distributions that account for factors such as filament support wires and lamp envelopes.

Source models are made for all types of sources, including LEDs, incandescent, fluorescent, metal vapor, and high-pressure gas discharge sources. The modeling includes spectral, radiance or luminance distributions, and lifetime aspects. For example, accurate source models for the following have been developed:

- The temperature distribution along an incandescent filament varies from its ends to the center. Additionally, the interior of the filament glows "hotter" due to the re-incident radiation.
- Arc emission sources such as metal halide and HID lamps change their radiance distribution and power output over time due to ablation of the electrodes. These lamps have a deposited material to capture this ablation, called the "salt lake" in continuous sources and the "getter" in a pulsed one.

There are essentially four ways of creating complex source models. Three are described on the next page, while the fourth, not presented here, is based on the physics of emission. This method is outside the confines of this text.

Source Modeling Methods

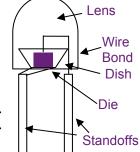There are three **source modeling methods**, where the accuracy of the model typically increases with number:

1. "Bottom-up" (**geometrical model**): the source geometry starting with the electrodes, supports, and envelope; finishes with the packaging. Emission is assigned to the radiative components.
   - Benefits: No complex measurements; handles re-incident light; provides tolerancing capabilities.
   - Limitations: Emission characteristics assumed; approximate surface and material properties; can include tedious CAD development.

2. "Top-down" (**radiance model**): the optical output of a representative sample of the lamp. These measurements are made with a **goniometer**, which moves a detector around a lamp on two axes. A camera measures the 2D radiance distribution of the lamp from each of many goniometer positions. The resulting 4D model represents a complete description of the lamp that can be used in a computer optical design program.
   - Benefits: Emission is based on physical measurements.
   - Limitations: Does not handle reincident light; is limited by the variance of the number of source samples measured and aligned; and their complex measurement.

3. "Bottom to top" (**system model**): Integrates the bottom-up and top-down approaches to develop a more thorough source model.
   - Benefits: Complete geometrical and radiative models.
   - Limitations: Integration of two submethods.

There are many hybrid methods and methods based on applying the physics of the emission process of a prescribed source. Loosely, the first two methods show agreement to within 25% of experimental results, while the bottom-to-top method shows agreement within 10%. In all cases, rays are assigned, typically in a Monte Carlo approach, to the emission areas.

LED Modeling

The components of an **LED** include the emitting die(s), the lens, the reflecting dish, wire bond and pad, and standoffs. Other components can include phosphors and included detectors.

Geometrical modeling is useful to develop LED sources; however, it is difficult to obtain or measure the shapes and sizes of the components within the lens. Radiance modeling suffers because of the large amount of variance between LED samples of one model. The primary issues

Lens

Wire Bond

Dish

Die

Standoffs

are the die position within the reflecting dish, the axial position of the die and dish with respect to the lens vertex, and the size and shape of the reflecting dish. Four distinct methods are available for LED modeling:
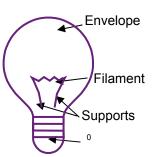
1. Develop a flat object and assign rays to the surface based upon the intensity distribution provided by the manufacturer. This method ignores spatial variation of the emission.
2. Develop a geometrical model of the LED and assign rays to the emitting surfaces of the die. Optimize the dish shape (typically a cone), size, and the axial offset of the die-dish to the lens vertex. The lens shape must be measured and the die and dish placed at the transverse center of the lens. The model is complete when the intensity pattern from the manufacturer agrees with the ray-trace model.
3. Same as method #2, except develop the layer structure within the die to generate Monte Carlo rays within the active layer(s). This method is tedious for ray tracing due to the index of refraction discontinuity between the die (n = 2.5+) and the epoxy lens (n = 1.45+).
4. Radiance by itself or a system: integrated into #2 or #3.

Incandescent Lamp Modeling

The components of an **incandescent lamp** include the base, filament(s), supports, and the envelope. Other components can include coatings and envelope faceting. Note that the shapes and sizes of components depend on the application. Sources developed for the automotive headlight industry provide the highest level of tolerance from one sample to another.
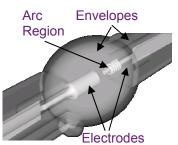

Both geometrical and radiance methods are useful for incandescent source modeling. Radiance modeling is better suited to this source since a goniometer can be focused on the filament source, while the glass envelope supplies little effect on overall optical ray paths. Only light rays that are re-incident (approaching grazing incidence) on the envelope show adverse effects. Geometric modeling involves breaking the glass envelope to gain access to the internal components. This process requires the use of calipers to measure the coil spacing, the thicknesses and lengths of the components, and the number of coils. Provided parameters can help with this process:

- **Maximum overall length (MOL):** Overall distance that includes the base and pins.
- **Light center length (LCL):** Distance between the center of the emitter and a defined reference plane.
- **Filament type:** Designated by @-#, where @ is a series of letters (e.g., C = coiled, CC = coiled coil, and SR = straight ribbon), and # is a number providing an arbitrary pattern for the filament supports.
- **Bulb type:** Designated by @-#, where @ is the bulb shape (e.g., T = tubular), and # is the diameter in eighths of an inch.
- **Base type:** Innumerable types that have no shorthand notation to describe them. Examples include screw, mogul, bipin, and prong.

Arc and Fluorescent Lamp Modeling

The components of an **arc lamp** include the base(s), electrodes, and envelope(s). Other components can include coatings, salt lake (continuous) or getter (pulsed), and ignition wire (flashlamp). The optical radiation is represented by a virtual object called the arc. Note that the aspects of components depend on the application. Automotive head-light arcs provide the highest level of accuracy from one sample to another.
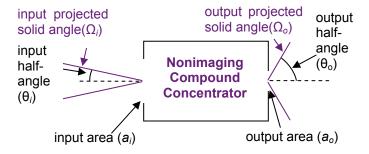

Radiance modeling is especially suited to this source since geometrical modeling cannot effectively represent the arc. The arc must be approximated with a cylinder, tube, or some other geometric shape. Radiance modeling is also suited to this source because a goniometer can be focused on the arc. Due to the typical smaller sizes of these sources compared to incandescent sources, the effect of re-incident rays is more pronounced. Thus, methods to integrate a simplified measurement of the radiance distribution into the geometrical model have been employed. One such method uses the **Abel transform** based on a single image capture of the arc. The Abel transform assumes symmetry of the arc shape and revolves it around a localized centroid of the arc source. Such system models are the most effective way to model such sources.

Fluorescent lamps include the tube and base(s). These are the simplest sources to model other than the complex geometry of compact fluorescent lamps now available. After the geometry is entered, the inner surface of the tube acts as the emitter. Internally, mercury vapor is excited, releasing UV radiation, which is then converted into visible light upon being incident on the phosphor. Geometrical modeling is better suited to this source due to the large size and simplicity of the configurations.

Nonimaging Compound Concentrators

Nonimaging compound concentrators were first developed for solar energy collection to concentrate the **irradiance** from the sun. In solar collection, depending on the degree of sophistication of the sun tracking system, the range of sun input angles can be fairly small. The collectors (solar cells, water pipes, etc.) respond essentially to irradiance and can be illuminated at any angle. The compound concentrator trades off between area and solid angle, presenting a large collection area to the sun (collecting over a narrow solid angle) and delivering the energy to a smaller area (and over a wider solid angle). These devices come close to achieving the theoretical maximum concentration (in three dimensions).
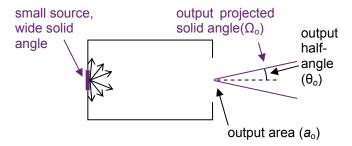
If the output projected **solid angle** is the maximum, π, ($\theta o = 90$ deg), the concentration is maximum:

$$\frac{a_i}{a_o}(\text{max}) = \frac{1}{\sin^2 \theta_i}.$$

Nonimaging compound concentrators are designed using the **edge-ray principle**, which directs all rays that are at the maximum input angle (θ_i for $\theta_o = 90$ deg in the drawing above) to the edge of the output aperture. All rays at input angles less than this maximum are directed inside the output aperture with no concern for image quality. Often this angle, θ_i, is called the **acceptance angle**, θ_a.

Concentrators as Luminaires

Nonimaging compound concentrators are used in illumination as **luminaires**—devices used to direct the light from a source for illumination. For illumination, they are used in the reverse direction from their configuration in solar collection; they collect light from as large an angle as possible from a small source and direct it over a smaller angle through a larger aperture. For solar collection, they collect energy over a large area and a small angle, delivering it to a small area.

Nonimaging compound concentrators are efficient because:
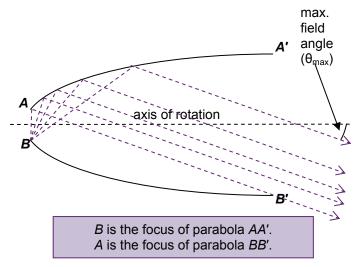
•   They collect light from the source over a very large solid angle.
•   They are designed using the **edge-ray principle**, keeping all the energy within the intended field.

Imaging systems are designed to be best on axis, with the edges of the field "spilling over." Nonimaging compound concentrators are designed to be best at the edges of the field, keeping all the energy inside the design boundaries.

Nonimaging concentrators used as luminaires are usually composed of an internal mirror surface with the figure of a **compound parabolic concentrator (CPC)**, **compound elliptical concentrator (CEC)**, or **compound hyperbolic concentrator (CHC)**. Dielectric filled concentrators that employ total internal reflection are also used.

Compound Parabolic Concentrators

The **compound parabolic concentrator (CPC)** is a common shape of nonimaging concentrator used for illumination. A CPC is formed by a parabola with its focus at one edge of the entrance (small) aperture, rotated around an axis that is perpendicular to and through the center of both apertures. CPCs can be quite long.

B is the focus of parabola *AA'*.
A is the focus of parabola *BB'*.

The complete equation for the surface of a CPC can be found in the equation summary in the Appendix.

The ratio of the diameter of the small and large apertures is determined by maximum field angle

$$\frac{d_o}{d_i} = \frac{1}{\sin\theta_{max}},$$

where d_o and d_i are the diameters of the output (large) and input (small) apertures, respectively.
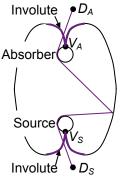
The length of the concentrator is

$$\text{Length} = \frac{d_o + d_i}{2\tan\theta_{max}}.$$

Compound Elliptical and Hyperbolic Concentrators

The **compound elliptical concentrator (CEC)** and **compound hyperbolic concentrator (CHC)** work in the domains of finite and diverging conjugates, while the CPC worked at infinite conjugates. For CECs, the CPC-development methods can be used. To visualize their shapes, consider what is called the **string method**, where the string acts as the edge ray:

- Choose two points, D_A and D_S.
- Select a length of string that allows the pen, P, to read points V_A and V_S.
- While pulling the string taut with P, sweep out the shape on one side of the reflector.
- Flip the string to the other side and repeat.

For a CPC, the absorber points D_A and V_A are located at infinity such that a constant angle, θ_a, is obtained. This method is adaptable to handle nonplanar sources. If the source, as shown in the figure, impedes on the string path, then a secondary region called the **involute** is formed. The involute ensures that rays with output angles less than θ_a are transferred by the reflector. Such reflectors with nonplanar sources or absorbers are better denoted as edge-ray reflectors. Note that the terms "absorber" and "source" are swapped for collector design.

The CHC is designed with the **flow-line method**, which treats light rays as fluid flow. Due to the conservation of étendue, we find vectors that define the geometrical flux through the amplitude and their directions define the flow line. The flow lines are hyperbolae. A reflector can be placed along one of the rotationally symmetric flow lines, and due to invariance there is no adverse effect on light emission from a Lambertian source located at the flow line origin.
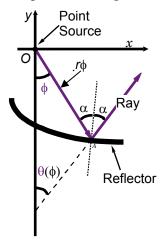
Tailored-Edge-Ray Design

The involute sections of edge-ray luminaires for nonplanar sources or absorbers indicate that the acceptance angle does not need to be constant over the extent of the reflector. Designs with a functional acceptance angle are called **tailored-edge-ray reflectors**. Using the figure, the equation that governs their shape for a point source design is

$$r(\phi) = r_1 \exp\left[\int_{\phi_1}^{\phi} \tan\left(\frac{s - \theta(s)}{2}\right) ds\right],$$

where r is the distance from the point source to the reflector, r_1 is the distance for the polar angle ϕ_1, and θ is the desired output intensity from the reflector. θ is the variable acceptance angle (note that "acceptance" is a holdover from solar concentrator design). For uniformity at the target,

$$\theta(\phi) = \arctan\left[\tan\theta_1 + \int_{\phi_1}^{\phi} I_{\text{src}}(v)\,dv\right],$$

where I_{src} is the intensity distribution emitted by the point source.

To allow for finite-extent sources, the first equation can be modified, and the reader is encouraged to consult the literature.

Though the formalism presented here might appear daunting, tailored-edge-ray design is a powerful tool to design optimal optics around both the emission aspects of the source and the desired irradiance distribution at the target. The one caveat is that tolerances are quite demanding unless one places sufficient leeway into the source intensity distribution (I_{src}).

Faceted Reflector Design

Essentially there are two design procedures for **faceted reflectors**: those based on the **tailored-edge-ray** method and those that provide a **stippled illumination pattern**. Stippling means that the target irradiance distribution is created from the overlap of the light from
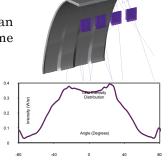
different segments of the reflector. This washes out any structure that could be imaged from the sources, such as a filament and its supports. Thus, the designer builds a basic reflector shape, such as parabolic, and then replaces the one smooth reflector with a series of flat, areal segments. This type of faceted reflector can be found in LCD and overhead projectors.

Tailored-edge-ray reflectors can also use this effect with some added benefits:

*   Energy conservation restrictions mean the reflectors grow large, but faceting allows the shape to be "restarted" to minimize the overall volume.
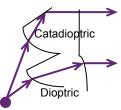
*   Facets can individually address different por-tions of the desired target distribution.

*   Tolerancing is improved since various allowances can be incorporated as a function of segment position.

These reflectors are typical in the automotive headlight industry and are increasingly used in other applications. This example (LED) shows the utility of faceting. The LEDs' intensity distribution pattern, along with uniformity at the target, gives the reflector shape shown here.

Advanced Nonimaging Optic Design

There are a number of advanced nonimaging design algorithms, such as **nonimaging Fresnel lens design**, **nonedge-ray design**, and **simultaneous multiple surfaces method (SMS)**. Nonimaging Fresnel lens design is used in lighthouses, solar concentrators, traffic lights, and automotive lamps. The in-expensive, small-volume optics are thin dielectrics, plastic or glass, with two types of Fresnel elements:

- **Catadioptric:** uses two refractions and one TIR to bend the light in the desired direction.
- **Dioptric:** uses two refractions to bend the light in the desired direction.

Dioptrics are used toward the lens center while catadioptrics are used once Fresnel losses become large. The TIR condition reduces the angles of incidence on the two refractive surfaces.

Nonedge-ray design follows the equations of tailored-edge ray design but adds two additional factors:

- System performance criteria drive optimization; and
- Multiple extended-size sources are allowed.

This design method trades between system performance and transfer efficiency from the source to the absorber. It is used in multiple small-source applications, such as LED lighting and diode-laser pumping.

SMS provides for multiple ray paths from the source to points on the to-be-generated optical surfaces of the

LED Hybrid Optic

device. Refraction, reflection, and TIR are used in conjunction to generate the multiple surfaces and provide the optimal output angular spread from the optic. SMS is part of a family of optics called **hybrid optics** that use many different optical phenomena for their operation. A primary example is the pseudo-collimating lenses used for high-brightness LEDs.

Displays—Overview

A multitude of existing **displays** incorporate different illumination strategies to provide a lit screen. Optical display technologies include **backlighting**, **projection**, and **organic LED (OLED)**.

Backlit displays use large liquid crystal (LC) modules that are lit from the rear by small sources coupled to a TIR element that spans the extent of the screen. The TIR is frustrated by structures placed on a surface of this element. Sources used in backlit displays include **cold-cathode fluorescent lamps (CCFL)** and LEDs, in which the ejected light proceeds through many additional layers, including polarizers, the LC, and diffusers. Additional layers may include a **brightness enhancement film (BEF)**, which recirculates ejected light until it is in the desired angular range. The next few pages describe the components of a backlit display in more detail.
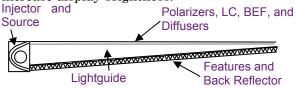
Projection displays use smaller **SLMs** in different spectral ranges to multiplex a full-color image. The illumination components include a broadband source (e.g., a narrow-gap arc lamp or LED), a reflector to capture the emitted radiation, lenslet arrays (often called fly's eyes), and dichroic filters to separate the light into the desired spectral ranges (typically red, green, and blue). There are both **front-projection displays** and **rear-projection displays**. Front-projection displays use distinct spectral channels to illuminate the screen; however, this increases cost and can reduce tolerances. Rear-projection displays fold the system in order to maintain a smaller display depth. Projection displays are discussed in more detail later.

Unlike backlit and projection displays, **OLED displays** deposit pixel emitters onto a substrate. These emitters provide both the illumination and display information, so the design demands for the illumination engineer are negligible. OLED modules can be used in projection displays.

Backlit Display Components

Standard components of a **backlit LCD** include:

- **Source:** Typically CCFL, LED, or **electro-luminescent (EL)**.
- **Injector:** A specular or diffuse reflector that captures and injects the light into the lightguide.
- **Lightguide:** A dielectric, typically acrylic, that captures the injected light via TIR. Features are placed on the backside of the lightguide to break the TIR condition. The lightguide is also wedged using decreasing thickness with increasing distance from the injector.
- **Features:** Paint patterns or geometric structures to frustrate the TIR. The density and/or depth of the features increases with distance from the injector to provide uniform illumination over the screen. The geometric structures can be holes (extending into the lightguide) or bumps (extending out of the lightguide).
- **Back reflector:** A diffuse or specular reflector placed below the features to capture and recirculate any light that is emitted from the lightguide backside.
- **Polarizers:** Two crossed linear polarizers placed on the display output side with an LC placed in between.
- **Liquid crystal:** Sandwiched between the two crossed linear polarizers to rotate the polarization by 90 deg for a pixel that has information content. Closely placed pixels provide for the color content (e.g., three pixels to provide red, green, and blue).
- **Diffusers:** Sometimes placed on the output side of the lightguide to provide better angular uniformity from the display.
- **Brightness enhancement film:** A microstructure, such as a prism, to select a desired angular output range while the higher angular content is recirculated to increase display brightness.

Injector and Source

Polarizers, LC, BEF, and Diffusers

Lightguide

Features and Back Reflector

Backlit Display: Source and Injector

Small sources are preferable for backlights to reduce the overall display volume. The source can be located to the side of a lightguide or placed directly behind the polarizers and LC. The former allows for thin displays at the expense of lightguide complexity; the latter increases the depth due to the removal of the lightguide, and careful design is required to provide uniform luminance. Three standard sources are used: **CCFLs**, **LEDs**, or **EL** films. CCFLs are small-diameter lamps that run the length of one or more sides of the display. For LED backlights, multiple LEDs are used to provide the required luminance level. They can be white-light emitters, a combination of multiple colors (e.g., red, green, and blue), or a combination of color-emitting LEDs and white-light emitters. EL films provide single-color background displays with information shown in black. Examples include watch faces and automotive dashboards. They work by passing current through the EL material, which then emits spatially uniform Lambertian light. EL backlights have no need for a lightguide because the EL is mated to the back of the LC-polarizer module.
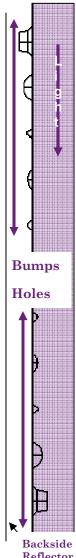
An **injector** is standard for any type of backlighting scheme. For a backlight whose source is located to the side of a lightguide, either diffuse or specular reflectors are placed around the source to better capture the emitted radiation. Standard shapes for a CCFL include spherical, parabolic, and elliptical troughs. For LEDs, dielectric (especially acrylic) couplers akin to the hybrid optics presented are used. The output aperture of the injector is mated to the input aperture of the lightguide. For backlighting without a lightguide, reflectors are often placed around the sources to assist in directing the light and to provide uniform luminance from the display. The simplest case is the **lightbox**, which is a highly reflecting, diffuse material placed around the sources over the extent of the screen backside. Lightboxes are analogous to integrating spheres.

Backlit Display: Lightguides, Features, Reflectors

Plastics are best suited for **backlight lightguides** because they can take advantage of **injection molding**. The thickness at the injector end of the lightguide depends on the screen size, with larger displays requiring thicker lightguides. The lightguide is thinned with increasing distance from the injector. This thinning assists with ejection of the trapped TIR light, because as the lightguide cross-sectional area decreases, the conservation of étendue demands an increase in the angular extent.

Structure or **features** are added to the backside of the lightguide, i.e., on the wedged surfaceside because the backside has more distance for the light to spread over the spatial extent of the display. Initially, paint patterns were used to cause the ejection; however, the paint must undergo a separate and costly process, and the paint spots provide little direct control of the resulting angular distribution. For this reason, replicated geometrical structures are added during injection molding either as **holes**, which extend into the lightguide, or **bumps**, which extend out of the lightguide. Geometrical shapes, as shown in the figure, include hip roofs, spheres, and ellipsoids. The density and/or depth of these features increases with distance from the injector. The design of such feature patterns is from the **diffusion equation**, followed by optimization for improved performance.
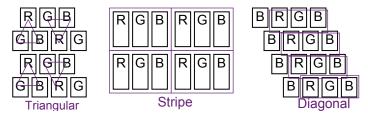
Bumps

Holes

Backside Reflector

Light leaked through the back feature-side of the lightguide is caught with a reflector, diffuse, or specular, which is placed below the lightguide. The **backside reflector** provides recirculation and better efficiency.

Backlit Display: Polarizers, LC, and BEF

There are a number of additional components that comprise an LCD, including two **linear polarizers** and the **twisted-nematic LC module**. The first polarizer passes linear polarization at one orientation, while the second passes linear polarization orthogonal to the first. The LC is sandwiched in between these two polarizers, and in a pixel's transmissive state, it rotates the light that exits the first polarizer by 90 deg. A pixel in a nontransmissive state absorbs the incident radiation. The LC module also has glass substrates on both sides of the LC, and on each glass substrate there are transmissive **indium-tin oxide (ITO)** electrodes. A **spectral filter mask** is inserted to provide color output from the display. Typically, a three-color mask is used, where neighboring subpixels pass red, green, or blue. The combination of these pixels forms a display through resolution considerations of the viewer. There are a number of **color-pixel patterns** including:

- **Triangular** or **delta:** better for motion pictures;
- **Stripes:** better for television; and
- **Diagonal:** better for motion pictures.

A **BEF**, a replicated structure of microprisms, recirculates emitted light until it is in the desired angular range. The BEF is situated just below the polarizers and LC. A **dual brightness enhancement film (DBEF)** incorporates the polarization into the optic such that the first linear polarizer can be removed.
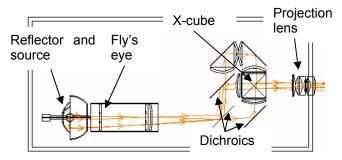
Projection Displays

Projection displays typically use three channels— red, green, and blue—to develop the object to be displayed by the projection lens. Each of the channels uses a **spatial light modulator (SLM)** to generate this object. One-channel systems use color filter wheels to temporally generate the scene. There are essentially three options for the SLM:

• Transmissive LCs akin to those used in backlighting;

• **Digital light processing (DLP)** modules, which incorporate millions of micromirrors over their surface area; or

• Reflective LCs, such as **liquid crystal on silicon**, **(LCoS)**, which integrate the LC with the circuitry.

The SLMs are microdisplays that use magnification from the projection lens to generate the screen image. An X-cube combines the three spectral channels, and the resulting "object" is projected onto the screen.

The illumination components of a projection display include the source, a reflector, and fly's eye lenses and/or straight lightpipes. The source is typically a narrow-gap arc or even LEDs. The reflector (conic, edge-ray, or faceted) is specular, and it captures most of the source emission. The fly's eye provides better spatial uniformity over the SLMs by creating several images of the source. The lightpipe mixes the light to provide better spatial uniformity. The overall illumination system is typically arranged in a Köhler scheme to hide the source structure.
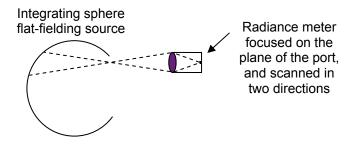
Mapping Flat-Fielding Sources

High-performance camera systems such as airborne and satellite cameras generally go through a process known as **flat-fielding**. The camera is presented with a large-sized extended light source that has nearly perfectly uniform radiance. Since the flat-field source is uniform, any pixel-to-pixel nonuniformities in the camera are inherent to the camera and can be remedied with image processing.

Generally, flat-fielding sources are realized by internally illuminated integrating spheres. Spheres with exit ports of about a 50-cm diameter are common. Ports of over a meter in diameter are sometimes needed depending on the aperture of a single large camera or the combined apertures of an array of smaller cameras. Radiance uniformities of 98% or 99% or better are the norm.

To verify that the flat-fielding sources have been designed properly and that there are no deficiencies in their manufacture, they are mapped for radiance uniformity. The mapping is done with a radiance meter, which is often photopically filtered for no other reason than commercial availability and the desire to band-limit the silicon detector to a region of good sensitivity.

The radiance meter is operated either in a collimated mode or is focused on a small spot in the plane of the exit port. Keeping the viewing direction constant, the meter is scanned in two directions to create a radiance map of the source.

Integrating sphere flat-fielding source

Radiance meter focused on the plane of the port, and scanned in two directions

Goniophotometers

Light sources designed to produce useable irradiance (automobile headlamps, roadway luminaires, and interior lighting fixtures) as well as those designed to produce useable intensity (automobile tail lights, traffic signals, aircraft and marine running lights) are all characterized by **goniophotometers**—devices used to measure the directional distribution of light from sources.

A goniophotometer consists of a small detector placed at a distance from the source where **intensity** is meaningful, (i.e., the inverse-square law applies). Except for highly collimated sources such as searchlights, a distance from the source of five to ten times the largest dimension of the source is usually sufficient. The lamp or detector (or a combination of the two) is moved to map the intensity distribution of the source.

Goniophotometers are classified as type A, B, or C depending on how they are constructed. This can be confusing because, in addition to three types of physical construction, there are three variations of spherical coordinates for reporting data that are also called types A, B, and C. These usually, but not always, match the type of goniophotometer used. Details of the three coordinate systems are shown on the next page.

Types A and B goniophotometers are similar in that the luminaire is mounted on a device with horizontal and vertical axes and a distant fixed detector.

Type C goniophotometers move the detector around the luminaire on a horizontal axis and rotate the luminaire on a vertical axis. Sometimes, for large luminaires, involving large distances, the detector is fixed and a large high-quality mirror moves on a horizontal axis, directing the light to the detector.

Type C goniophotometers are necessary for measuring lamps that are sensitive to the burning position.

Types A, B, C Goniometer Coordinate Systems

All are spherical coordinate systems.

Type A spherical coordinates:
Polar axis: vertical
Label on vertical angles: *Y*
Label on horizontal angles: *X*
Range of Y: –90 (nadir) to +90 (zenith)
Range of X: –180 (left, from luminaire) to +180
Straight ahead: $Y = 0$, $X = 0$
Primary uses: optical systems, automotive lighting

Type B spherical coordinates:
Polar axis: horizontal
Label on vertical angles: *V*
Label on horizontal angles: *H*
Range of *V*: –180 to +180
Range of *X*: –90 (left, from luminaire) to +90
Straight ahead: $V = 0$, $H = 0$
Primary uses: floodlights

Type C spherical coordinates:
Polar axis: vertical
Label on vertical angles: *V*
Label on horizontal angles: *L* (lateral)
Range of *V*: 0 (nadir) to 180 (zenith)
Range of *L*: 0 (along primary axis of luminaire) to 360
Straight down: $Y = 0$, $X = 0$
Primary uses: indoor lighting, roadway lighting

"Snapshot" Goniophotometers

Conventional **goniophotometers** take a long time to produce an **intensity** mapping. In addition, they must have precise motion control to achieve the desired angular resolution. As such, they are well suited for characterizing luminaire designs, but not really useful for quality control or sorting of LEDs, for example. For these applications, several versions of rapid "snapshot" goniophotometers have been developed:

- Rapid-scan goniophotometers
- Multiple-detector goniophotometers
- Tapered fiber bundle goniophotometers
- Camera-based goniophotometers

Rapid-scan goniophotometers are small devices used to characterize LEDs and the output of optical fibers. They operate on similar principles to the conventional type C goniophotometers, but motions are much faster, making measurements in seconds rather than minutes.

Multiple-detector goniophotometers place numerous discrete detectors in the intensity field of interest and capture the entire intensity distribution at one time. The angular resolution is restricted to the spacing of the detectors.

A tapered fiber optic bundle can be manufactured with one concave spherical face with all the fibers directed toward the source. At the other end of the bundle, the fibers can be aligned with the pixels of a detector array. The detector array captures the entire intensity distribution at one time.

Camera-based goniophotometers place a diffuse reflecting surface (flat or concave) at an appropriate distance from the source, and view the light reflected from the surface with an imaging photometer that, together with the reflecting surface, is calibrated to capture the entire intensity distribution in one "snapshot."

Software Modeling Discussion

Outside the laboratory, software programs are used to model, optimize, and tolerance optical systems. Two types of codes exist in the optical design arena: **lens design codes** and **optics analysis codes**. The former are used primarily to design the lenses used in optical systems. They include robust analysis tools such as point spread function graphs, spot diagrams, and modular transfer function curves; optimization tools to improve upon the performance of the imaging system; and tolerancing tools to ensure manufacturability. Increasingly, these lens design codes include **nonsequential ray tracing**. Nonsequential ray tracing is required for a number of illumination systems, especially those based on nonimaging optics. In standard lens design, rays follow a prescribed sequence of optical interfaces. Thus, the traced rays know the sequence of surface intercepts, which reduces the computation load since the algorithm does not need to determine which surface is struck next by a ray.

Optics analysis codes are based around nonsequential ray tracing such that computation time must be spent to determine which surfaces are struck by each ray. Nonsequential ray tracing is inherently slower than sequential ray tracing. Analysis codes are further broken down into two geometry types: **surface-based geometry** and **solid-based geometry**. Surface-based codes require the user to generate each surface, assigning the optical properties on the two sides of each interface. Solid-based codes develop enclosed objects that allow the user to assign volume-based properties such as the type of material (e.g., BK7) and surface-based properties (e.g., a silver mirror).

Optical design codes incorporate more **computer-aided design (CAD)** into their capabilities. This feature allows the codes to import mechanical design formats such as IGES and STEP. Certain industries such as the automotive and architectural industries have specialized codes. The list of codes is extensive and always changing.

Role of Light in Architecture

The illumination of buildings is a design process aimed at orchestrating light for the user's well-being. The layering and patterning of light is considered successful when complex physiological and psychological responses are satisfied. Such responses are centrally conditioned by vision: the medium through which information and perceptions about a given space are recorded and interpreted. Economics and energy efficiency play a critical role in design decisions, but the satisfaction of vision requirements is of overriding importance.

The characteristic features of an architectural space only come to life with light. Hence, no light no architecture. At the same time, light is not neutral: The way it is arranged gives a particular appreciation of the space and generates specific emotive and aesthetic responses.

The electric illumination of an architectural space is simply the result of transmitted or reflected light emanating from distant and immediate surrounding surfaces. Therefore, the lighting designer can influence the interface between light and matter to meet these visual requirements and sensations. Hence, only with a proper understanding of physiological and psychological factors and a familiarity with available technologies can lighting decisions be made for proper effect.

Despite some setbacks in the 1970s caused by an advocacy for windowless buildings to save energy, light available from the sun and sky has regained the attention of lighting designers for the many benefits it brings to users. When available and well controlled, daylight is by far the preferred source of illumination. Today, the common design approach combines the contribution of both electric and natural lights for increased work productivity, and reduced absenteeism or visual fatigue.

Lighting Design—Layering of Light

For both energy conservation and visual variety, lighting design is implemented in layers to properly distribute light throughout the architectural space.

The **horizontal ambient layer** is maintained to 1/3 to 2/3 the task illumination level. Lower bound levels (1/3) for horizontal ambient light may be appropriate for a museum or boutique store to emphasize a display. Upper bound levels (2/3) are more relaxing for most casual activities where a 25- to 35-fc ambient light level is sufficient and relates well to tasks requiring 50 to 60 fc.

The **vertical ambient layer** is critical in keeping vertical tasks glare free, such as washout of video display terminals (VDTs). In addition, when people look away from a task, the line of sight is then the vertical average luminance from the walls and ceiling. **Wall washing** and **grazing** are some of the techniques used to reinforce the sensation of spaciousness, clarity, and pleasantness.

A **task layer** supplements the ambient illumination to fulfill lighting requirements for critical activities. Energy is saved by (1) locating the source near the task to provide the light level recommended by the Illuminating Engineering Society of North America (IESNA), (2) reducing ambient light levels, and (3) turning off the task light when not in use. The scene presents varied lighting instead of the monotonous atmosphere, resulting from the **general illumination approach**.

The **accent** or **focal layer** gives the space its identity and mood by highlighting or spotlighting certain architectural elements and objects, such as paintings, sculptures, and landscapes. **Downlighting**, **accent lighting**, and **backlighting** are some techniques used to produce such effects on various elements in the space.
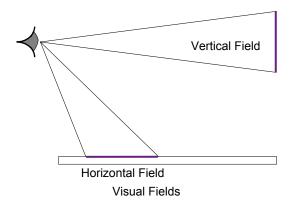
The **ornamental layer** introduces elements that add sparkle to the space with effects similar to those of Christmas lights. Chandeliers, candles, and sconces can be considered for this purpose.

Eye Adaptation and Visual Fields

Eye adaptation to the visual environment is the eye's response and sensitivity to the ambient light level as the person moves from one environment to the next, such as walking from the bright and sunny outdoors to the dark indoors. If the difference between the two light levels is extreme, the person may feel like he or she has moved into a totally black environment. Slowly, the sensitivity of the eye attunes itself to the dark environment and details become increasingly distinguished. It takes 20 to 30 minutes for the eyes to completely adapt to a dark environment and grasp the details. Conversely, eyes adapt to a sunny environment in 2 to 3 minutes.

Transient adaptation is the ability of the visual system to adapt in short intervals to the different luminances prevailing in a fixed visual field, for instance, when looking through a bright window and down to a desk. Due to such variations, the iris constantly adjusts the aperture to control the light entering the eye. Large variations between luminances in a scene are considered detrimental to visual comfort and lead to eye fatigue.
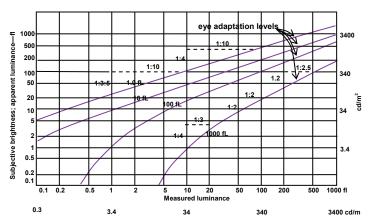
Visual fields refer to the direction of the eyes' line of sight. When looking down, the viewer apprehends a **horizontal field**, and when looking up, a **vertical field**.

Vertical Field

Horizontal Field

Visual Fields

Apparent Brightness

Vision is stimulated by brightness mapped on the retina as a byproduct of light reflected from an opaque surface or transmitted through a transparent medium (the glass bulb of a lamp, for example). A distinction must be made, however, between **photometric brightness** or luminance and **apparent brightness**. Luminance or photometric brightness is calibrated in relation to the eye's sensitivity to various wavelengths, while apparent brightness is perceived in the context of the ambient light level to which the eye is adapted. Hence, the brightness of an object relative to the retinal image is by no means a complete specification of its visual appearance. This may be understood by considering the blinding brightness of a car's headlights during the night that is barely perceivable during the day, even though a light meter will register the same photometric brightness.
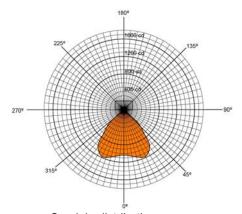
At an ambient surrounding of 3.4 cd/m^2 (1 fL), a measured luminance of 34 cd/m^2 (10 fL) appears to be 340 cd/m^2 (100 fL). At a low ambient level, the difference perceived between two surfaces is also reduced from a difference of 1:10 to 1:4.

Subjective brightness versus measured luminance. (Reprinted with permission from Stein and Reynolds, copyright Wiley & Sons, 2005.)

Photometric Report and VCP

Manufacturers provide a **photometric report** that details the optical performance and characteristic light distribution patterns of a luminaire. The **candela distribution curve (CDC)**, presented in either polar (figure below) or rectilinear plots and in tables, shows the luminous intensity distribution measured at different angles, from 0 to 360 deg in increments of 5 deg. Using the plot below, the luminous intensity can be found for a specific direction. Rectangular luminaires (2 x 4 or 1 x 4) require candela distribution curves in at least three planes: crosswise, longitudinal, and 45 deg. These luminous intensities can quickly reveal the potential for glare.
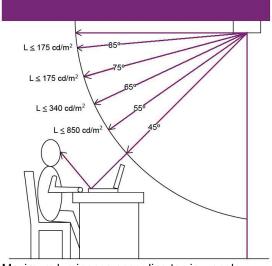

Candela distribution curve.

The **visual comfort probability (VCP)**, a rating system for evaluating direct discomfort glare, is expressed as the percent of occupants of a space who will be bothered by direct glare. Standard data provided for a luminaire specification include tables of its VCP ratings for various room geometries, based on IESNA standard conditions. These include a uniformly distributed illumination level of 1000 lux (~100 fc), luminaire height, observer position, and room surface reflectances (ceiling, 80%; walls, 50%; and floor, 20%). In general, a minimum VCP of 70 is the established limit for the viable use of a luminaire.

The Layered Approach

Spacing criteria (SC) to achieve uniform ambient illumination is the ratio of the spacing (S) distance between the respective axis of parallel luminaires and their mounting height (MH), i.e., SC = S/MH. For a rectangular luminaire, SC is given along both axes, lengthwise and crosswise. The distance between walls and adjacent light fixtures is set at no more than one-half S.

The **coefficient of utilization (CU)** specifies the proportion of lumens that reach the workplane from the fixture for given room geometries and surface reflectances. The CU gives some indication of the luminaire's efficiency.

When located in the field of view, bright light sources can cause discomfort and disability glare. However, the severity of glare depends on the angle at which the luminaire is seen. The *IESNA Handbook* provides as maxima the following **luminances from a direct luminaire** according to the view or cut-off angle:

Maximum luminance according to view angle.

Daylight Factor

Windows and skylights admit daylight as free illumination, and the viewers have visual contact with the outside. However, only a portion of outdoor light is received inside a building. The sky's condition, along with the size, placement, and orientation of the window(s) opening, the glazing type and transmittance, and shading as well as room proportions, affect the quantity and quality of received light.

The **daylight factor (DF)** accounts for these parameters and is used to determine the percentage of outside light that can be received inside a room with a specific configuration. DF is the ratio of interior illuminance at a given point on a given plane (usually the work plane) to the exterior illuminance under minimal light conditions of an overcast sky, e.g., the C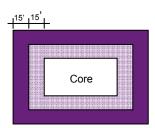IE overcast sky distribution. For parallelepipedic-room buildings with windows on one side only, a minimum DF of 2% is generally recommended throughout the work plane.

The DF general equation is

$$DF = Ei/Eo,$$

where *Ei* and *Eo* are the indoor and outdoor horizontal illuminances, respectively.

Daylight should be allowed to reach through side windows for the tasks performed at the rear of the space. Shelves in side windows can help reflect light deep into interior spaces and shade the vision portion of the window.

Office tasks can be lit at a depth of up to 15 feet from the window. The next 15 feet may need to be supplemented by electric light. Beyond 30 feet, little or no daylight is available unless the space has windows on opposite sides. Evenly distributed roof skylights provide uniform light but for one story only, assuming proper glazing transmittance and sunlight control.

Daylight Strategies

IESNA RP-5-99 Recommended Practice of **daylighting** emphasizes the following illumination practices:

1. Block direct sunlight in the vicinity of tasks. Blocking direct sunlight at the window (e.g., with louvers) is the first step toward glare control by allowing only light from the sky and that reflected from the ground to pass through the window glass.

2. Design windows to minimize direct glare. East, west, or south facing windows can have too much glare if excessive sunlight strikes the glass. With 100,000 lux of light available on the outside on a sunny day, a glass pane, with typically only 2% transmittance, can reach a luminance up to 2000 cd/m2, which exceeds the tolerated average of 850 cd/m2. Internal blinds (structurally stable external shading devices with adjustable louvers) can block direct sunlight and reduce the luminance of the window or skylight. And glazing that has a visible light transmittance of 25% can be an acceptable trade-off of daylight availability, the view to the outside, and minimize glare.

3. Zone electric lighting for daylight responsive control. The electric light distribution system should be zoned according to daylight availability inside an open-plan office. Daylight zoning depends on the room configuration, sky condition, and solar exposure. Large open-plan offices are often subdivided into the perimeter zones, the intermediate zone, and the core zones based on daylight availability as indicated in item 1.

4. Provide responsive lighting controls. Controls are at the heart of efficient electric light operation and daylight harvesting, specifically to accommodate the time-dependent electric light demand. The variables governing control strategies include the space layout, configuration, orientation, the occupancy patterns, lighting usage, and daylight availability. Controls include tuning to reduce electric power while still meeting each user's needs, and adaptive compensation to lower the light levels at night.

Nighttime Visibility Criteria

The eye is capable of adapting to a wide range of light levels but not at the same time. To function well, it must be adapted to the prevailing light conditions. As previously indicated for daytime conditions, our eyes use **photopic vision**, which utilizes the eye's cones and the center of the visual field. The eye works differently when it is adapted to low light levels. Under very dark, moonlit conditions, our eyes use **scotopic vision**, which primarily utilizes the eye's rods, resulting in greater acuity in the peripheral visual field.

For **nighttime visibility** in most urban and suburban environments, our eyes use **mesopic vision**, which is a combination of both photopic and scotopic. In nighttime environments, the goal of the lighting design is to keep the eye adapted to mesopic or scotopic vision, and not to introduce high light levels that will create an imbalance in the visual field and cause the eye to try to use photopic vision. Recent research indicates that light sources rich in blue and green (metal halide or fluorescent) improve peripheral mesopic vision, clarity, and depth of field better than sources rich in red and yellow, such as incandescent and high-pressure sodium.

Extreme **glare** leads to loss of visibility. Glare is caused by a high luminance ratio between the glare source and the prevailing light conditions to which the eye is adapted. In other words, insufficiently shielded light sources generate direct glare. The following measures reduce the luminance ratio and control nighttime glare:

- **Uniform light distribution** in a visual scene and brightness ratios kept to 1:5 between average and maximum luminance;
- **Reduction of light levels** and **source brightness** using fixtures with low wattage;
- **Shielding the light source** and **locating fixtures** to avoid glare. Fixtures near the property line should have "house-side shielding" to prevent glare to residential neighbors.

Recommended Illuminance for Façades

Façade illumination aims to reproduce at night a building's aesthetic and formal characteristics that are perceived during the day for the purpose of attracting attention and creating a good impression. To this effect, **floodlighting** is one technique employed, which treats a building as a giant piece of sculpture for visual display. Luminaires are typically mounted in close proximity to buildings and are aimed to illuminate the structure. Lighting the building from the top down reduces stray uplight, and precisely aimed fixtures minimize trespass light.
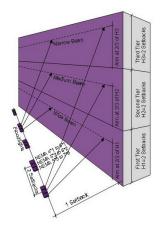
Effective illumination of facades is a complex and subjective task. Results depend heavily upon surrounding light levels, the surface finish of the intended target, the spectral color distribution of the lamp source, mounting location allowances, and viewers' perceptions.

The following table lists the IESNA's recommended illuminance levels for the floodlighting of buildings and monuments.

Area Description	Target Surface Finish	Average Target Illuminance
Bright	Light	5 (50)
Bright	Medium light	7(70)
Bright	Dark	10(100)
Dark	Light	2(20)
Dark	Medium light	3(30)
Dark	Medium dark	4(40)
Dark	Dark	5(50)

Adapted from *IESNA Hand-book*, 9th Edition, copyright IESNA, 2000.

Façade Floodlighting for Uniform Illumination

Adapted from *IESNA Handbook*, 9th Edition, copyright IESNA, 2000.

Floodlighting fixtures can be mounted at ground level, or on stands and poles. They can also be attached to the building itself or to adjacent structures. The key lights are set up for a modeling effect but should be combined with other color sources to soften the strong effects of shadows.

Floodlight categories are **narrow beam** (types 1, 2, 3), **medium beam** (types 4, 5) and **wide beam** (types 6, 7) (IESNA RP-33). The further away the luminaire is from the facade, the narrower the light beam must be. Aiming and positioning ground-mounted floodlights for uniform illumination depend first on the available **setback** in relation to the building height. If the height is 2 times the setback dimension, the center of a "wide beam" floodlight aimed at 2/3 the height of the building is recommended. If a building is 30 feet high, the recommended aiming point is 20 feet high. Floodlight **spacing** along the facade should not exceed 2 times the setback distance. If the setback is 22.5 feet, the floodlights should be placed no more than 45 feet apart, with the first floodlight at ½ to 1 of the setback dimension. As the building height increases to 4 times the setback, a medium-beam floodlight with the same aiming elevation is recommended. Buildings with up to 6 times the setback require more narrow-beam floodlights. Thus, one location on the ground may hold multiple floodlights, each aimed at different building elevations. The illumination from the ground of façades with more than 6 times the setback is not recommended due to the difficulty of achieving uniformity.

Illumination of Outdoor Areas

Lighting **building entries** at night provides (1) vertical illumination to comfortably light people's faces, and (2) horizontal illumination to light the pathway and any changes in the light level. Such a pool of light comes from a mounting position high on the building, on a pedestrian-scaled post lantern, or on the underside of a canopy.

Emergency egress doors are provided with lighting on the outside of the door threshold and extended for a distance at least equal to the width of the door opening.

Softscape lighting is for private yards, patios, parks, gardens, boulevards, entry markers, and other natural features such as water. They are softly illuminated and emit a minimum of glare, contrast, or spill light to the neighbors. Some techniques used to light trees to achieve the desired effect are **frontlighting** to highlight details, texture, and color; **backlighting** to show form and separate the plant from the background; **sidelighting** to emphasize plant texture and create shadows; **uplighting** to make branches glow; and **downlighting** for accent details, colors, and texture. The illumination of tree trunks along with canopies helps anchor them to the landscape.

Hardscape lighting is for outdoor sculptures, fountains, or vertical displays. A 3D sculpture is illuminated from two directions to provide highlights and soften shadows. The key light is focused on the mass of the sculpture with light added to relieve shadows.

Stairs and **ramps** are hazardous in low light, so contrast is essential for their safe use. Illuminated handrails, step lights, or small fixtures in the balustrade provide light differentiation between the step risers and threads. Other techniques to complement light effects are coloring of the step nosing and color differentiation between threads and risers.

Walkways, **sidewalks**, and **bikeways** are illuminated at levels recommended by the IESNA with lights placed to provide visual information.

Special Considerations for Outdoor Fixtures

Controls using astronomical time switches and/or photosensors are deployed to ensure that exterior lighting is not operated when sufficient daylight is available or during nighttime except those fixtures for security.
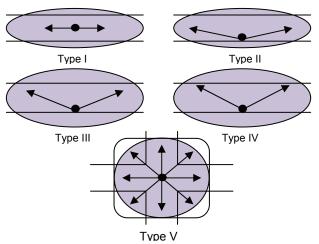
Special considerations must be given to proper installation of luminaires exposed to the outdoor environment. If installed on ground they must have the "**wet location rated**" label and if placed under canopies but still exposed to the elements, they must be "**damp location rated**." In addition, durable with vandal-resistant components and regularly maintained luminaires to minimize dirt accumulation or to prevent obstruction by grass, leaves, mud, and other debris, ensure steady operation of exterior lighting. Separate security fixtures are exclusively used to provide low light levels for security cameras (.01 footcandle). The table below summarizes the illuminance and luminance ratios for various outdoor areas.

Outdoor space type	Horiz. Avg. Illuminance (fc)	Vert. Avg. Illuminance (fc)	
Building entrance (Active/Inactive)	5.0 / 3.0	3.0 / 3.0	
Emergency lighting: Egress Path	1.0		
Roadside sidewalks & Type A bikeways: commercial, intermediate, residential areas	1.0; 0.6; 0.	2.2; 1.1; 0.5	
Walkways distant from roadside & Type B bikeways	0.5	0.5	
Loading dock	10.0 The plane of the task may be horizontal, inclined, or vertical.		
Storage yards,active-inactive	10.0 / 1.0	3.0 / 0.3	

(Adapted from *IESNA Lighting Handbook, Ninth Edition.*)

Outdoor Luminaire—Transverse Light Distribution

Luminaires' **beam pattern distributions** are classified by IESNA according to transverse and lateral projections. (see the next two figures). Five types, shown below, are illustrated according to the maximum candlepower and the trace of the half-maximum value. A **luminaire's transverse reach** is expressed in MH units: **type I** (1 MH), **type II** (1.75 MH), **type III** (2.75 MH), **type IV** (6 MH), and **type V** (symmetric distribution in four quadrants). Type V is usually best at the center of parking lots. Type IV or forward-throw distribution is best for wide, multilane roads and parking lot perimeters. Type I has a long and narrow distribution that can be applied to narrow roadways, walkways, or bike paths. It also can be located at or near the center of a pathway, approximately two MHs in width, or used as overhead lighting in areas such as parking lots, plazas, courtyards, and along walkways. Types II and III distribute light to one side of the light source. These luminaires should generally be used for street lighting to direct light to the street side of the lamp but not shining into the building side.
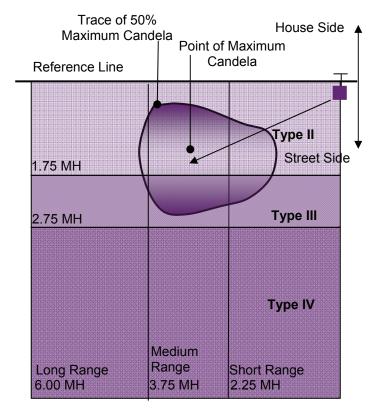
Type I   Type II

Type III   Type IV

Type V

Adapted from *IESNA Handbook*, 9th Edition, copyright IESNA, 2000.

Outdoor Luminaire—Lateral Light Distribution

Fixtures for roadway and parking applications are further classified as **short**, **medium**, or **long lateral distribution**. This classification relates the types of fixtures, the spacing between them according to the point of maximum candelas, and the MH. For a short-range lateral throw, the maximum luminaire spacing is generally less than 4.5 times the MH. A medium throw allows a maximum spacing of generally less than 7.5 times the MH, and a long throw is generally less than 12 times the MH (see figure below).

Type III distribution of a luminaire. Half-maximum candelas trace falls within 2.75MH. (Adapted from *IESNA Handbook*, 9th Edition, copyright IESNA, 2000.)

Criteria For Roadway Lighting

Three principal criteria are used to design major roadway lighting systems: **luminance**, **illuminance**, and the newer concept of **small target visibility (STV)**. The illuminance **inverse square law calculations** are well known. It was found, however, that illuminance levels do not correlate well with visibility or driver performance. IESNA Standard RP-8-00 addresses one shortcoming of the illuminance method by adding a maximum **veiling luminance ratio (VLR)** that is specifically intended to limit glare from a luminaire. The luminance determination is necessary to calculate the VLR. Luminance describes the reflected light from the pavement as seen when driving, so evaluating the quality of a lighting system by how it looks at night is actually the same as evaluating its luminance.

In reference to the figure on the next page, luminance at point *P* is determined as the sum of contributions from all *n* luminaires:

$$L_P = \Sigma r(\beta_i, \gamma_i) I(\varphi_i, \gamma_i)/10{,}000\ h^2,$$

where r is the **reflectance coefficient** at angles β and γ.
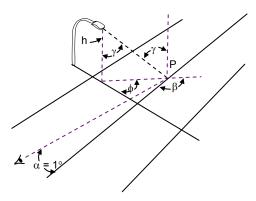
The veiling luminance, *Lv*, and the VLR are also necessary to limit the glare effect. The *Lv* can be determined as follows:

$$L_v = \Sigma\ 10\ E_v\ /\ (\theta^2 + 1.5\ \theta),$$

where: L_v is the veiling luminance at the observer's location, in cd/m^2; E_v is the vertical illuminance on the plane of the observer's eye; and θ is the angle between the line of sight and the luminaires in degrees.

Values recommended for luminance and VLR are found in the 9th Edition of the *IESNA Handbook*.

Small Target Visibility

Single fixture for luminance determination. (Adapted from *IESNA Hand-book*, 9th Edition, copyright IESNA, 2000.)

The **STV** method was developed to account for the contrast that must be present to allow drivers moving at high speed to quickly detect hazards and react to them. Indeed, a roadway lighting system may provide a high and uniform road surface luminance, yet the visibility threshold may be low due to the absence of contrast. Three luminance components influence the visibility of a target: the target luminance itself, the luminance of the background, and the veiling luminance or glare. Given these three luminances, all of which can be calculated, the visibility of each target in the array can be determined in terms of the **visibility level (VL)**. VL is the ratio of target contrast to the contrast of a similar target at threshold, a measure of visibility that has been widely used.

STV predicts the visibility of a standard object (18 × 18 cm) located on the roadway at a specific distance from the driver, and accounts for the contrast between the standard target and its background by considering the driver's age, viewing time, pavement reflectance, and glare from the luminaire. The larger the STV number, the more visible the object. For more details, go to IESNA RP-8-00.

Recommended Roadway Luminaires

After the desired minimum illuminance and the pole height are initially set based on light spill, road cross-section, pavement type, and roadway category (*IESNA Handbook*), a few luminaires and sources can be tested to achieve a design that meets the recommended light level, uniformity, and acceptable glare. Common layouts include luminaires on one side of the road, luminaires on both sides, or luminaires in a center median. One-side and median configurations often have the additional advantage of requiring less wire and conduit, resulting in lower construction costs.

Starting with any of the five previously described types of luminaires is a convenient way to facilitate selection according to roadway width and light spill control. **Full cutoff** luminaires should be specified wherever possible to prevent light pollution. Typically, Types I, II, and III are appropriate for narrow roadways, while type IV is proper for multilane roads. Lateral-medium-throw luminaires are preferred over short-throw types because fewer poles are required and over long-throw types because then **semicutoffs** are not needed. Combining transversal and longitudinal distributions helps the designer select luminaires for even light distribution based on roadway widths and pole spacing.
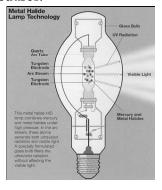
The architecture of such luminaires comes in many shapes. Two are shown below. The most prevalent is the "**cobra head**" luminaire, typically mounted on a 6-ft arm. This fixture with a flat lens is recommended to minimize disability glare and light trespass. Reflectors may be formed or faceted aluminum. The **rectilinear** "**shoebox**" luminaire, designed as a full cutoff, is also popular. The reflector in this design is usually larger than that in a cobra head to achieve better optical control.

(Reprinted with permission of Acuity Brands Lighting.)

Recommended Lamps for Roadway Luminaires

Metal halide, **high-pressure sodium**, and **induction lamps** are the most common lamps, ranging from 100 to 400 W depending on the MH and roadway cross-section. Metal halide is used where color rendition is a concern or white light is desired, but this lamp also emits light in the blue-green portion of the spectrum, which enhances the driver's peripheral vision. A **light-loss factor (LLF)** accounts for the depreciation in lamp output and luminaire performance over time. Typical LLFs are 0.60 to 0.70 for **high-pressure sodium** lamps and 0.45 to 0.55 for **metal halide lamps**. Note that the LLF varies depending on environmental conditions and maintenance procedures.

Left: metal halide lamp. Above: 400W high pressure sodium BT-37. (Images reprinted with permission from Sylvania.)

The isofootcandle procedure, not covered here, can be used to determine luminaire spacing based on needed illuminance and uniformity. With the help of software programs, many iterations can be performed quickly to compare various lighting systems and determine the safest and most efficient solution.

Equation Summary

Basic Quantities in Illumination

$$\text{photopic luminous flux} = 683 \int \Phi_\lambda(\lambda) \cdot V(\lambda) \cdot d\lambda$$

Color

Light source color:

$$X = \int S_\lambda(\lambda) \cdot \overline{x}(\lambda) \cdot d\lambda \qquad\qquad x = \frac{X}{X+Y+Z}$$

$$Y = \int S_\lambda(\lambda) \cdot \overline{y}(\lambda) \cdot d\lambda \qquad\qquad y = \frac{Y}{X+Y+Z}$$

$$Z = \int S_\lambda(\lambda) \cdot \overline{z}(\lambda) \cdot d\lambda$$

$$u = 4X/(X+15Y+3Z) = 4x/(-2x+12y+3)$$
$$v = 6Y/(X+15Y+3Z) = 6y/(-2x+12y+3)$$
$$u' = 4X/(X+15Y+3Z) = 4x/(-2x+12y+3)$$
$$v' = 9Y/(X+15Y+3Z) = 9y/(-2x+12y+3)$$

Color temperature and CCT:

$$(MK)^{-1} = {10^6}\Big/{CCT}$$

Surface color:

$$X = k \int S_\lambda(\lambda) \cdot R(\lambda) \cdot \overline{x}(\lambda) \cdot d\lambda$$

$$Y = k \int S_\lambda(\lambda) \cdot R(\lambda) \cdot \overline{y}(\lambda) \cdot d\lambda$$

$$Z = k \int S_\lambda(\lambda) \cdot R(\lambda) \cdot \overline{z}(\lambda) \cdot d\lambda$$

$$k = \frac{100}{\int S_\lambda(\lambda) \cdot \overline{y}(\lambda) \cdot d\lambda}$$

Equation Summary (cont.)

Color of fluorescent surfaces:

$$X = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_{\mu}(\mu) \ D(\mu,\lambda) \ \overline{x}(\lambda)$$

$$Y = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_{\mu}(\mu) \ D(\mu,\lambda) \ \overline{y}(\lambda)$$

$$Z = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_{\mu}(\mu) \ D(\mu,\lambda) \ \overline{z}(\lambda)$$

$$k = 100 \Big/ \sum_{\lambda=380}^{780} S_{\lambda}(\lambda) \ \overline{y}(\lambda)$$

Illumination Properties of Materials

BRDF and reflectance factor:

BRDF=L/E   R=BRDF·π

Harvey/*ABg* scatter model:

$$B = \left|\beta_{rolloff}\right|^{g} \qquad \text{BSDF} = \frac{A}{B + \left|\beta - \beta_0\right|^{g}}$$

Retroreflectors:

$$R_I = \frac{I}{E_{\perp}} \qquad R_A = \frac{R_I}{A} \qquad R_L = \frac{R_A}{\cos \upsilon} = \frac{L}{E_{\perp}}$$

Illumination Transfer

Known intensity:

$$E = \frac{I \cos \xi}{d^2} \qquad \Phi_i = I\omega \qquad L = \frac{I}{A_r \cos \theta}$$

Equation Summary (cont.)

Known flux:

$$I = \Phi_i / \omega \qquad E = \Phi_i / A_i \qquad L = \frac{\Phi_i}{\omega\, A_r\, \cos\theta} = \frac{\Phi_i}{\Omega\, A_r}$$

Known radiance:

$$E_0 = \frac{\Phi_i}{A_i} = \frac{L\, A_r\, \Omega}{A_i} = \frac{L\, A_r\, \cos\theta\, \cos\xi}{d^2} = L\;\Omega_i$$

Known irradiance:

$$I = \frac{E \cdot d^2}{\cos\xi} \qquad \Phi = E \cdot A_i \qquad L = \frac{R \cdot E}{\pi}$$

Relationship of configuration factor to projected solid angle and of form factor to average projected solid angle:

$$C = \Omega / \pi \qquad F_{a\,to\,b} = \overline{\Omega}_{a\,to\,b} / \pi$$

Projected solid angle of circular area from on-axis point:

$$\Omega = \frac{\pi\, r^2}{r^2 + d^2} = \pi \sin^2\theta$$

Projected solid angle of circular area from off-axis point:

$$\Omega = \frac{\pi}{2}\left(1 - \frac{1 + \tan^2\delta - \tan^2\theta}{\left[\tan^4\delta + \left(2\tan^2\delta\right)\left(1 - \tan^2\theta\right) + \sec^4\theta\right]^{\frac{1}{2}}}\right)$$

$$\Omega = \frac{\pi}{2}\left(1 - \frac{1 + \left(d/x\right)^2 - \left(r/x\right)^2}{\left[\left\{1 + \left(d/x\right)^2 + \left(r/x\right)^2\right\}^2 - 4\left(r/x\right)^2\right]^{\frac{1}{2}}}\right)$$

Equation Summary (cont.)

Average projected solid angle from one circular area to another, with both areas parallel and centered on the same axis:

$$\bar{\Omega}_{r\,to\,i} = \frac{\pi}{2}\left[1 + \frac{1+\left(\dfrac{r_i}{d}\right)^2}{\left(\dfrac{r_r}{d}\right)^2} - \left\{ \left(1 + \frac{1+\left(\dfrac{r_i}{d}\right)^2}{\left(\dfrac{r_r}{d}\right)^2}\right)^2 - 4\left(\dfrac{r_i}{r_r}\right)^2 \right\}^{\frac{1}{2}} \right]$$

Cosine fourth and increase factor:
$$E_i = \pi\, L_r\, \sin^2\theta \cdot \cos^4\delta \cdot F'$$

ω, Ω, NA, and *f*/# for a circular cone:
$$\omega = 2\pi(1 - \cos\theta) \qquad \Omega = \pi\sin^2\theta$$

$$NA = n \cdot \sin\theta \qquad f/\# = 1/2\sin\theta$$

Illumination in Imaging Systems

Object and image radiance:
$$\frac{L_i}{n_i^2} = \tau \cdot \frac{L_o}{n_o^2}$$

Image irradiance off axis:
$$E_i = \pi\, \tau L_o\, \sin^2\theta \cdot \cos^4\delta \cdot F'$$

Image irradiance on axis:
$$E_{i0} = \pi\, \tau L_o\, \sin^2\theta = \frac{\pi\, \tau\, L_0}{4(f/\#)^2} = \frac{\pi\, L_0}{4(T/\#)^2}$$
$$= \pi\, \tau\, L_0\, NA^2 = \tau\, L_0\, \Omega$$

Image flux:
$$\Phi_i = \tau\, L_o\, a_i\Omega_i = \tau\, L_o\, a_p\Omega_p$$

Equation Summary (cont.)

Illumination in Nonimaging Systems

Generalized étendue:
$$\mathcal{E} = n^2 \iint\limits_{\text{aperture}} \cos\theta \, dA_s \, d\omega$$

$$\Phi = \iint\limits_{\text{aperture}} L(\mathbf{r}, \hat{\mathbf{a}}) \cos\theta \, dA_s \, d\omega,$$

$$\Phi = L_s \iint\limits_{\text{aperture}} \cos\theta \, dA_s \, d\omega$$
$$= \frac{L_s \mathcal{E}}{n^2}$$

Concentration:
$$C = A/A'$$

$$C_{2D} = \frac{a}{a'} = \frac{n' \sin\theta'}{n \sin\theta} \quad C_{3D} = \frac{A}{A'} = \left(\frac{n' \sin\theta'}{n \sin\theta}\right)^2$$

$$C_{\text{2D,opt}} = \frac{n'}{n \sin\theta_a} \quad C_{\text{3D,opt}} = \left(\frac{n'}{n \sin\theta_a}\right)^2$$

Skew invariant:
$$f_{\text{skew}}(s) = \frac{d\mathcal{E}(s)}{ds},$$
$$\text{where } s = r_{\min} k_t$$

Fibers, Lightpipes, and Lightguides

Maximum acceptance angle:
$$\sin\theta_{\max} = \frac{1}{n_0} \sqrt{n_1^2 - n_2^2}$$

Numerical aperture:
$$\text{NA} = n_0 \sin\theta_{\max} = \sqrt{n_1^2 - n_2^2}$$

Equation Summary (cont.)

Étendue:

$$\text{E'tendue} = \frac{\pi^2}{4} d^2 NA^2$$

Tapered single fiber:

$$a_i \cdot NA_i{}^2 \approx a_o \cdot NA_o{}^2$$

Tapered bundle:

$$NA_o = NA_i = NA_{\text{fiber}}$$
$$a_o \cdot pf_o = a_i \cdot pf_i$$

Uniform Illumination

Bent lightpipes:

$$R = r_2/r_1 = 1 + t/r_1 \le n$$

Integrating sphere radiance:

$$L = \frac{\Phi}{\pi \cdot A_s} \cdot M \quad M = \frac{1}{1 - \overline{\rho}}$$

$$\theta(\phi) = \arctan\left[\tan\theta_1 + \int_{\phi_1}^{\phi} I_{\text{src}}(v)\, dv \right]$$

$$r(\phi) = r_1 \exp\left[\int_{\phi_1}^{\phi} \tan\left(\frac{s - \theta(s)}{2} \right) ds \right]$$

Nonimaging Compound Concentrators

Preservation of étendue:

$$a_i \Omega_i = a_o \Omega_o$$

Maximum concentration ratio:

$$\frac{a_i}{a_o}(\text{max}) = \frac{1}{\sin^2 \theta_i}$$

Equation Summary (cont.)

Diameter of CPC used as a collimator:

$$\frac{d_o}{d_i} = \frac{1}{\sin \theta_{max}}$$

Length of CPC:

$$\text{Length} = \frac{d_o + d_i}{2 \tan \theta_{max}}$$

Equation for surface of CPC:

$$(r \cos \theta_{max} + z \sin \theta_{max})^2$$
$$+r \, d_s (1 + \sin \theta_{max})^2$$
$$-z \, d_s \cos \theta_{max} (2 + \sin \theta_{max})$$
$$-\frac{d_s^2}{4}(1 + \sin \theta_{max})(3 + \sin \theta_{max})$$
$$= 0,$$

where:

r is the radius of the cone, perpendicular to the axis,
z is the axial position measured from the small end,
d_s is the diameter of the small end, and
θ_{max} is the maximum field angle at the large end.

Tailored edge ray design:

$$\theta(\phi) = \arctan\left[\tan \theta_1 + \int_{\phi_1}^{\phi} I_{src}(v) dv \right]$$

$$r(\phi) = r_1 \exp\left\{ \int_{\phi_1}^{\phi} \tan\left[\frac{s - \theta(s)}{2} \right] ds \right\}$$

CIE Illuminants A and D65

λ (nm)	CIE Ill. A Tungsten at 2856 K	CIE Ill. D65 Sunlight
300	0.930483	0.03410
305	1.128210	1.66430
310	1.357690	3.29450
315	1.622190	11.76520
320	1.925080	20.23600
325	2.269800	28.64470
330	2.659810	37.05350
335	3.098610	38.50110
340	3.589680	39.94880
345	4.136480	42.43020
350	4.742380	44.91170
355	5.410700	45.77500
360	6.144620	46.63830
365	6.947200	49.36370
370	7.821350	52.08910
375	8.769800	51.03230
380	9.795100	49.97550
385	10.899600	52.31180
390	12.085300	54.64820
395	13.354300	68.70150
400	14.708000	82.75490
405	16.148000	87.12040
410	17.675300	91.48600
415	19.290700	92.45890
420	20.995000	93.43180
425	22.788300	90.05700
430	24.670900	86.68230
435	26.642500	95.77360
440	28.702700	104.86500
445	30.850800	110.93600
450	33.085900	117.00800
455	35.406800	117.41000
460	37.812100	117.81200
465	40.300200	116.33600
470	42.869300	114.86100

CIE Illuminants A and D65 (cont.)

λ (nm)	CIE Ill. A	CIE Ill. D65
475	45.517400	115.39200
480	48.242300	115.92300
485	51.041800	112.36700
490	53.913200	108.81100
495	56.853900	109.08200
500	59.861100	109.35400
505	62.932000	108.57800
510	66.063500	107.80200
515	69.252500	106.29600
520	72.495900	104.79000
525	75.790300	106.23900
530	79.132600	107.68900
535	82.519300	106.04700
540	85.947000	104.40500
545	89.412400	104.22500
550	92.912000	104.04600
555	96.442300	102.02300
560	100.000000	100.00000
565	103.582000	98.16710
570	107.184000	96.33420
575	110.803000	96.06110
580	114.436000	95.78800
585	118.080000	92.23680
590	121.731000	88.68560
595	125.386000	89.34590
600	129.043000	90.00620
605	132.697000	89.80260
610	136.346000	89.59910
615	139.988000	88.64890
620	143.618000	87.69870
625	147.235000	85.49360
630	150.836000	83.28860
635	154.418000	83.49390
640	157.979000	83.69920
645	161.516000	81.86300
650	165.028000	80.02680

CIE Illuminants A and D65 (cont.)

λ (nm)	CIE Ill. A	CIE Ill. D65
655	168.510000	80.12070
660	171.963000	80.21460
665	175.383000	81.24620
670	178.769000	82.27780
675	182.118000	80.28100
680	185.429000	78.28420
685	188.701000	74.00270
690	191.931000	69.72130
695	195.118000	70.66520
700	198.261000	71.60910
705	201.359000	72.97900
710	204.409000	74.34900
715	207.411000	67.97650
720	210.365000	61.60400
725	213.268000	65.74480
730	216.120000	69.88560
735	218.920000	72.48630
740	221.667000	75.08700
745	224.361000	69.33980
750	227.000000	63.59270
755	229.585000	55.00540
760	232.115000	46.41820
765	234.589000	56.61180
770	237.008000	66.80540
775	239.370000	65.09410
780	241.675000	63.38280
785	243.924000	63.84340
790	246.116000	64.30400
795	248.251000	61.87790
800	250.329000	59.45190
805	252.350000	55.70540
810	254.314000	51.95900
815	256.221000	54.69980
820	258.071000	57.44060
825	259.865000	58.87650
830	261.602000	60.31250

$$\bar{x}, \bar{y}, \bar{z}, V(\lambda), \text{ and } V'(\lambda)$$

Note: The photopic efficiency function, $V(\lambda)$, is identical to the $\bar{y}$ standard observer function.

λ (nm)	$\bar{x}$	$\bar{y}, V$	$\bar{z}$	V'
360	0.000130	0.000004	0.000606	0.000000
365	0.000232	0.000007	0.001086	0.000000
370	0.000415	0.000012	0.001946	0.000000
375	0.000742	0.000022	0.003486	0.000000
380	0.001368	0.000039	0.006450	0.000000
385	0.002236	0.000064	0.010550	0.001108
390	0.004243	0.000120	0.020050	0.002209
395	0.007650	0.000217	0.036210	0.004530
400	0.014310	0.000396	0.067850	0.009290
405	0.023190	0.000640	0.110200	0.018520
410	0.043510	0.001210	0.207400	0.034840
415	0.077630	0.002180	0.371300	0.060400
420	0.134380	0.004000	0.645600	0.096600
425	0.214770	0.007300	1.039050	0.143600
430	0.283900	0.011600	1.385600	0.199800
435	0.328500	0.016840	1.622960	0.262500
440	0.348280	0.023000	1.747060	0.328100
445	0.348060	0.029800	1.782600	0.393100
450	0.336200	0.038000	1.772110	0.455000
455	0.318700	0.048000	1.744100	0.513000
460	0.290800	0.060000	1.669200	0.567000
465	0.251100	0.073900	1.528100	0.620000
470	0.195360	0.090980	1.287640	0.676000
475	0.142100	0.112600	1.041900	0.734000
480	0.095640	0.139020	0.812950	0.793000
485	0.057950	0.169300	0.616200	0.851000
490	0.032010	0.208020	0.465180	0.904000
495	0.014700	0.258600	0.353300	0.949000
500	0.004900	0.323000	0.272000	0.982000
505	0.002400	0.407300	0.212300	0.998000
510	0.009300	0.503000	0.158200	0.997000

$\bar{x}, \bar{y}, \bar{z}, V(\lambda),$ and $V'(\lambda)$ (cont.)

λ (nm)	$\bar{x}$	$\bar{y}, V$	$\bar{z}$	V'
515	0.029100	0.608200	0.111700	0.975000
520	0.063270	0.710000	0.078250	0.935000
525	0.109600	0.793200	0.057250	0.880000
530	0.165500	0.862000	0.042160	0.811000
535	0.225750	0.914850	0.029840	0.733000
540	0.290400	0.954000	0.020300	0.650000
545	0.359700	0.980300	0.013400	0.564000
550	0.433450	0.994950	0.008750	0.481000
555	0.512050	1.000000	0.005750	0.402000
560	0.594500	0.995000	0.003900	0.328800
565	0.678400	0.978600	0.002750	0.263900
570	0.762100	0.952000	0.002100	0.207600
575	0.842500	0.915400	0.001800	0.160200
580	0.916300	0.870000	0.001650	0.121200
585	0.978600	0.816300	0.001400	0.089900
590	1.026300	0.757000	0.001100	0.065500
595	1.056700	0.694900	0.001000	0.046900
600	1.062200	0.631000	0.000800	0.033150
605	1.045600	0.566800	0.000600	0.023120
610	1.002600	0.503000	0.000340	0.015930
615	0.938400	0.441200	0.000240	0.010880
620	0.854450	0.381000	0.000190	0.007370
625	0.751400	0.321000	0.000100	0.004970
630	0.642400	0.265000	0.000050	0.003335
635	0.541900	0.217000	0.000030	0.002235
640	0.447900	0.175000	0.000020	0.001497
645	0.360800	0.138200	0.000010	0.001005
650	0.283500	0.107000	0.000000	0.000677
655	0.218700	0.081600	0.000000	0.000459
660	0.164900	0.061000	0.000000	0.000313
665	0.121200	0.044580	0.000000	0.000215
670	0.087400	0.032000	0.000000	0.000148
675	0.063600	0.023200	0.000000	0.000103
680	0.046770	0.017000	0.000000	0.000072

$\bar{x}, \bar{y}, \bar{z}, V(\lambda)$, and $V'(\lambda)$ (cont.)

λ (nm)	$\bar{x}$	$\bar{y}, V$	$\bar{z}$	V'
685	0.032900	0.011920	0.000000	0.000050
690	0.022700	0.008210	0.000000	0.000035
695	0.015840	0.005723	0.000000	0.000025
700	0.011359	0.004102	0.000000	0.000018
705	0.008111	0.002929	0.000000	0.000013
710	0.005790	0.002091	0.000000	0.000009
715	0.004106	0.001484	0.000000	0.000007
720	0.002899	0.001047	0.000000	0.000005
725	0.002049	0.000740	0.000000	0.000003
730	0.001440	0.000520	0.000000	0.000003
735	0.001000	0.000361	0.000000	0.000002
740	0.000690	0.000249	0.000000	0.000001
745	0.000476	0.000172	0.000000	0.000001
750	0.000332	0.000120	0.000000	0.000001
755	0.000235	0.000085	0.000000	0.000001
760	0.000166	0.000060	0.000000	0.000000
765	0.000117	0.000042	0.000000	0.000000
770	0.000083	0.000030	0.000000	0.000000
775	0.000059	0.000021	0.000000	0.000000
780	0.000042	0.000015	0.000000	0.000000
785	0.000029	0.000011	0.000000	0.000000
790	0.000021	0.000007	0.000000	0.000000
795	0.000015	0.000005	0.000000	0.000000
800	0.000010	0.000004	0.000000	0.000000
805	0.000007	0.000003	0.000000	0.000000
810	0.000005	0.000002	0.000000	0.000000
815	0.000004	0.000001	0.000000	0.000000
820	0.000003	0.000001	0.000000	0.000000
825	0.000002	0.000001	0.000000	0.000000
830	0.000001	0.000000	0.000000	0.000000

Archaic and Arcane Units of Illumination

Luminous Intensity

The unit of luminous intensity is, and has been, the base unit of photometry. Until recently, it suffered from a lack of stable, reproducible standards. Over the past century and a half, standards have ranged from actual candles (wax or whale fat), gas lamps (pentane, isopropyl acetate), vegetable oil lamps (colza, i.e. canola oil), carbon filament lamps, blackbody furnaces, and finally, in 1979, a radiometric standard. Because of the past difficulty in realizing the standard, most of these expressions for intensity are approximate at best.

1 Hefner candle ≈ 0.9 cd
1 candlepower* = 1 candela (cd)
1 new candle = 1 bougie nouvelle = 1 cd
1 candle (UK) ≈ 1 cd
1 decimal candle = 1 bougie decimal = 1.02 cd
1 international candle = 1.02 cd
1 Vereinskerze (German candle) ≈ 1.1 cd
1 pentane candle ≈ 10 cd
1 Munich candle ≈ 1.2 cd
1 carcel unit ≈ 9.8 cd
1 Violle ≈ 20.4 cd

Luminous Flux
1 spherical candlepower* (SCP) = 4π lumens (lm)
1 mean spherical candlepower* (MSCP) = 4π lm

Illuminance
1 nox = 0.001 lux
1 milliphot = 10 lux
1 footcandle* = 1 lm/ft^2 = 10.764 lux
1 flame = 43.06 lux
1 cm-candle = 1 phot = 10,000 lux

* still in occasional use

Archaic and Arcane Units of Illumination (cont.)

Luminance

Several units of luminance have the number π in the denominator. This was done before the availability of calculators and computers to facilitate the calculation of the luminance of a **Lambertian** surface, which radiates uniform **luminance** over a projected solid angle of π steradians. For example, a Lambertian surface with **reflectance**, ρ, receiving an **illuminance** of x lux has a **luminance** of ρx apostilbs.

1 bril = 3.183×10^{-8} nit
1 skot = 3.183×10^{-4} nit
1 apostilb = 1 Blondel = 1 cd/πm^2 = 0.3183 nit
1 millilambert = 3.183 nit
1 foot-Lambert***** = 1 cd/πft^2 = 3.426 cd/m^2 = 3.426 nit
1 cd/ft2 = 10.76 nit
1 Lambert = 1 cd/πcm^2 = 3183 nit
1 stilb = 1 cd/cm^2 = 10,000 nit

CCT

1 mired* (microreciprocal degree) = 1 reciprocal megaKelvin (MK)$^{-1}$
1 mirek* (microreciprocal Kelvin) = 1 reciprocal megaKelvin (MK)$^{-1}$

Photons

1 Einstein**†** = Avagadro's number of photons
1 Einstein**†** = 6.022×10^{23} photons
1 micromole**†** = 6.022×10^{17} photons in the 400- to 700-nm band

Photon Radiance

1 Rayleigh**†** = 7.96×10^{-8} photons/sec·m^2·sr

Wavelength

1 millimicron* = 1 nanometer

* still in occasional use
† still in occasional use in fields other than illumination
  or optics

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Index

Index

Index

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