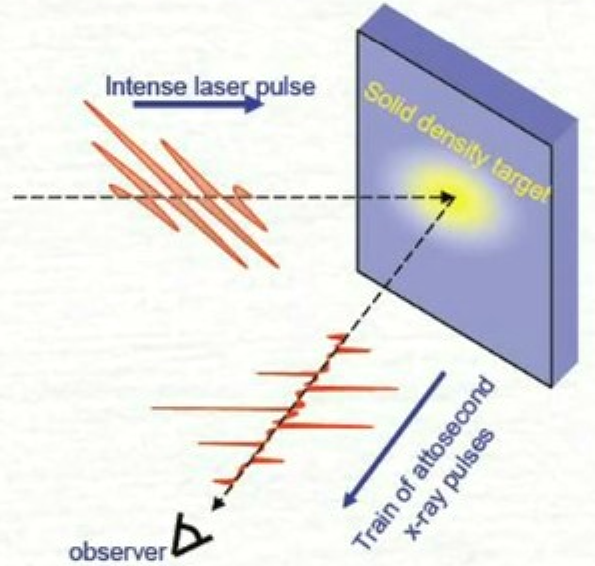
High Harmonics and (sub-)Attosecond Pulses in Relativistic Regime

Alexander Pukhov,

Sergei Gordienko, Teodora Baeva, Daniel an der Brügge

Institut für Theoretische Physik Uni-Düsseldorf



Conjecture of the Talk

- **Theory of HHG in relativistic regime: relativistic γ-spikes and the universal spectrum**
- **Intense (sub-)attosecond pulses**
- **Relativistic plasma control for single attosecond pulse selection**
- **3D effects: (self-)focusing of HHG in vacuum and spectrum modifications**
- **Towards vacuum breakdown intensity limit via relativistic HHG**

Important laser-plasma parameters

Dimensionless laser amplitude $$a = \frac{eA}{mc^2}$$

relativistic when $a \approx 1 \leftrightarrow I\lambda^2 = 1.37 \times 10^{18}$ W μm²/cm²

Critical plasma density $$N_c = \frac{\omega_0^2 m}{4\pi e^2}$$

***S*-number** $$S = \frac{N}{aN_c}$$

Gordienko & Pukhov
Phys. Plasmas **12**, 043109 (2005)

Historical overview (milestones)

- **First observaton of HHG from solid targets:**
  Carman *et al.*, Phys. Rev. Lett. **46**, 29 (1981).
  CO_2 laser, 10^{16} W/cm^2.
- **First theoretical attempt:**
  Bezzerides *et al.*, Phys. Rev. Lett. **49**, 202 (1982).
  suggested $\omega_{\text{cutoff}} = \omega_p$.
- **Doppler effect hypothesis:**
  Bulanov, Naumova, Pegoraro, Phys. Plasmas **1**, 745 (1993).
- **No sharp cutoff, "selection rules":**
  Lichters, MtV, Pukhov, Phys. Plasmas **3**, 3425 (1996).
- **Universal spectrum $I_n/I_0 = n^{-8/3}$ and $\omega_{\text{cutoff}} = 4\gamma^3\omega_0$.**
  Baeva, Gordienko, Pukhov, Phys. Rev. E**74**, 046404 (2006)
- **Experimental observation of HHG up to keV energies:**
  Dromey, Zepf *et al.*, Nat. Phys. **2**, 456 (2006).

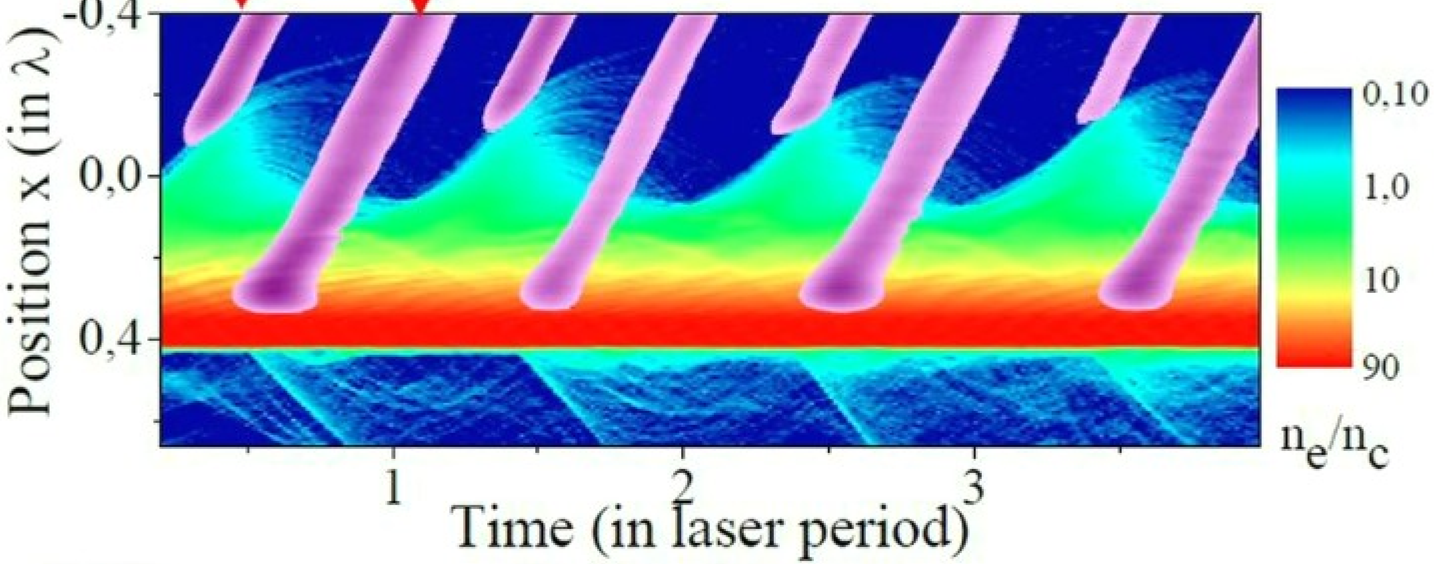
Two mechanims (at least) are involved in HHG

- **Relativistic Oscillations**
  Doppler effect

Talk by Fabiene Quéré

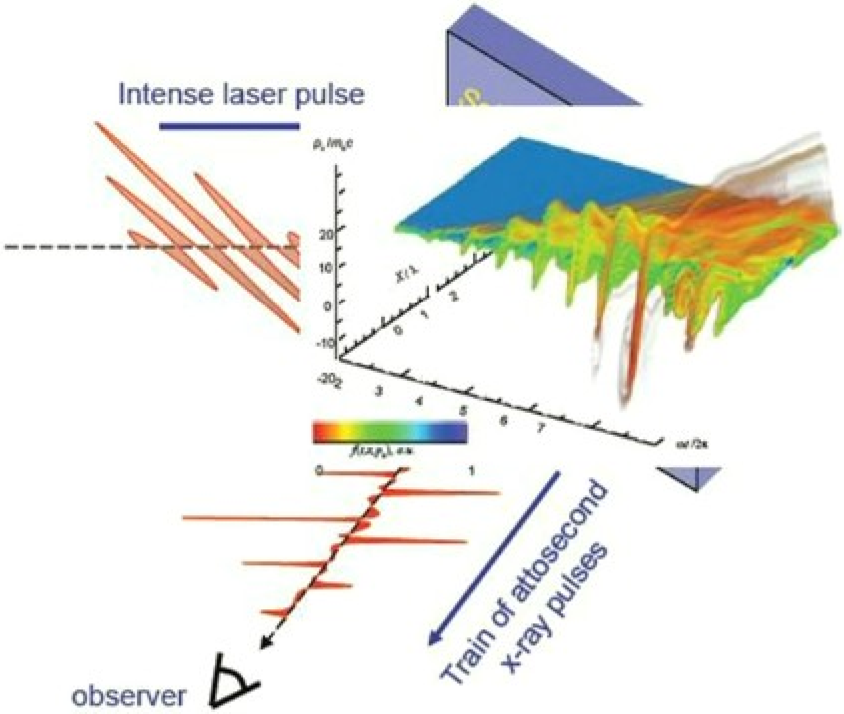
- **Coherent Wake Emission**
  Plasma oscillations

Quéré *et al*, PRL **96** (2006)
Thaury *et al*, Nature Physics **3** (2007)



The apparent reflecting point oscillates at relativistic velocities together with the plasma surface

Baeva, Gordienko, Pukhov, Phys. Rev. E**74**, 046404 (2006),
Pukhov, **NATURE** Physics, Vol. **2**, p. 439 (2006).



1. Boundary Condition: $\mathbf{E}_\tau=0$

- External observer sees the reflection at x(t), where
$$\mathbf{E}_\tau(x(t))=0$$

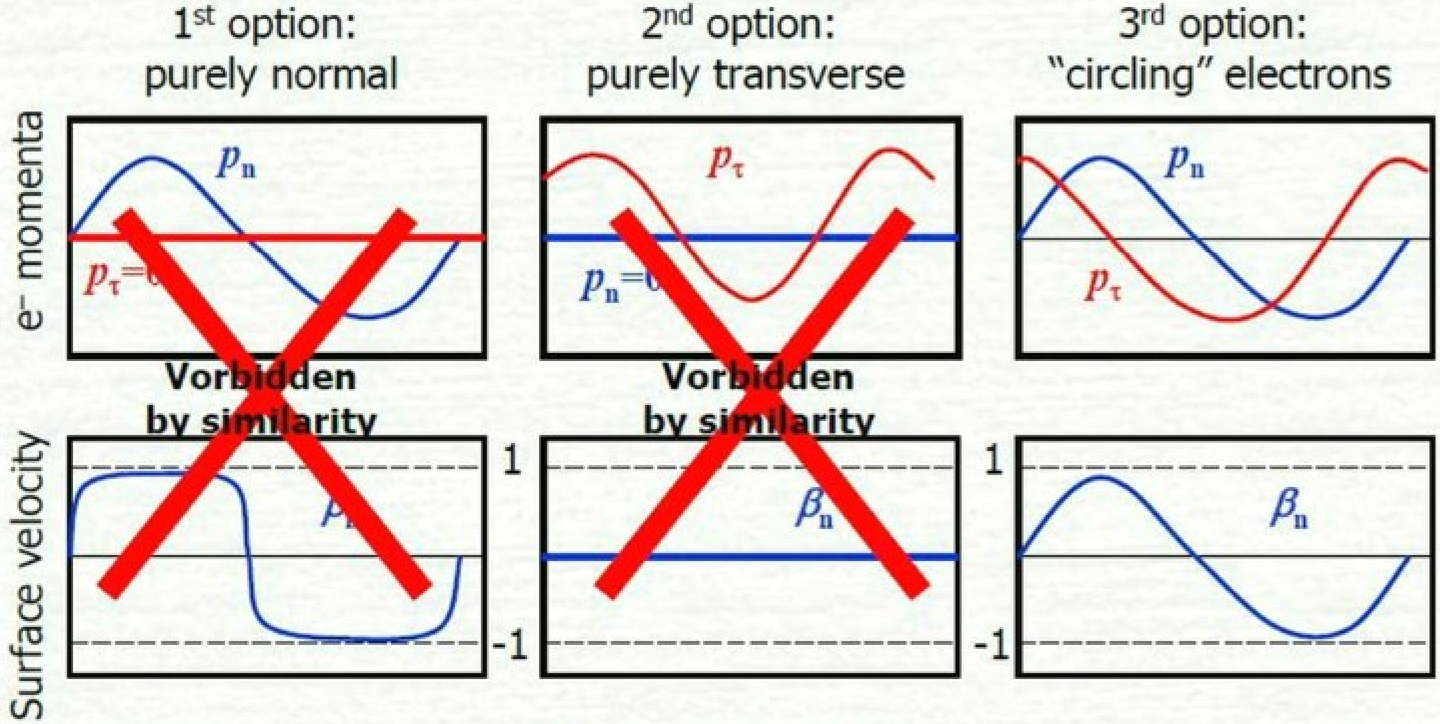
- Equation for the **Apparent Reflection Point** x(t),

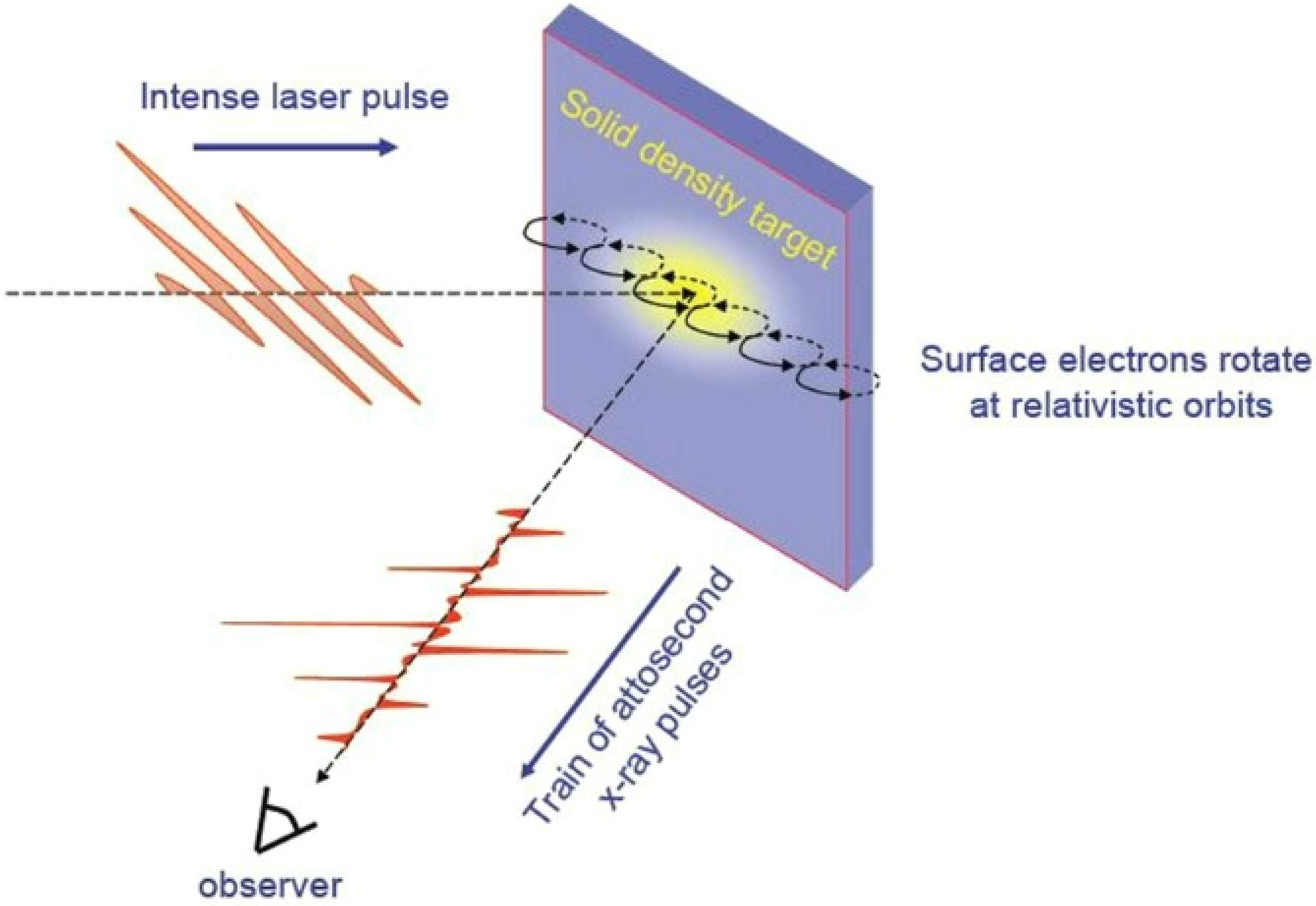
$$\mathbf{E}_\tau^{i}(x-ct)+\mathbf{E}_\tau^{r}(x+ct)=0$$

- $x(t)$ is located within the plasma skin layer

Surface dynamics is defined by relativistic similarity (Gordienko & Pukhov, 2005)

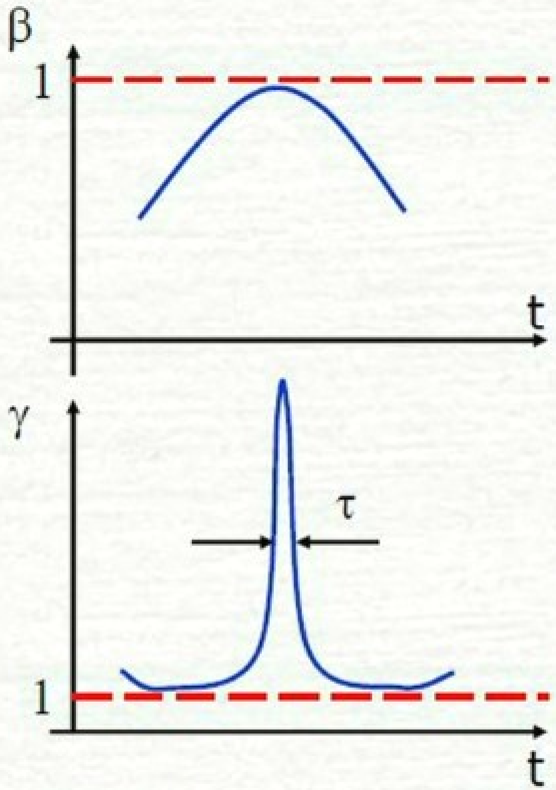
How do the surface electrons move?




Intense laser pulse
Solid density target
Surface electrons rotate
at relativistic orbits
Train of attosecond
x-ray pulses
observer

Relativistic γ-Spikes

Baeva, Gordienko, Pukhov, *Phys. Rev. E* **74**, 046404 (2006)



Plasma surface velocity $\beta = v_n/c$ is a smooth function.
At the maximum it can be approximated by a parabola:

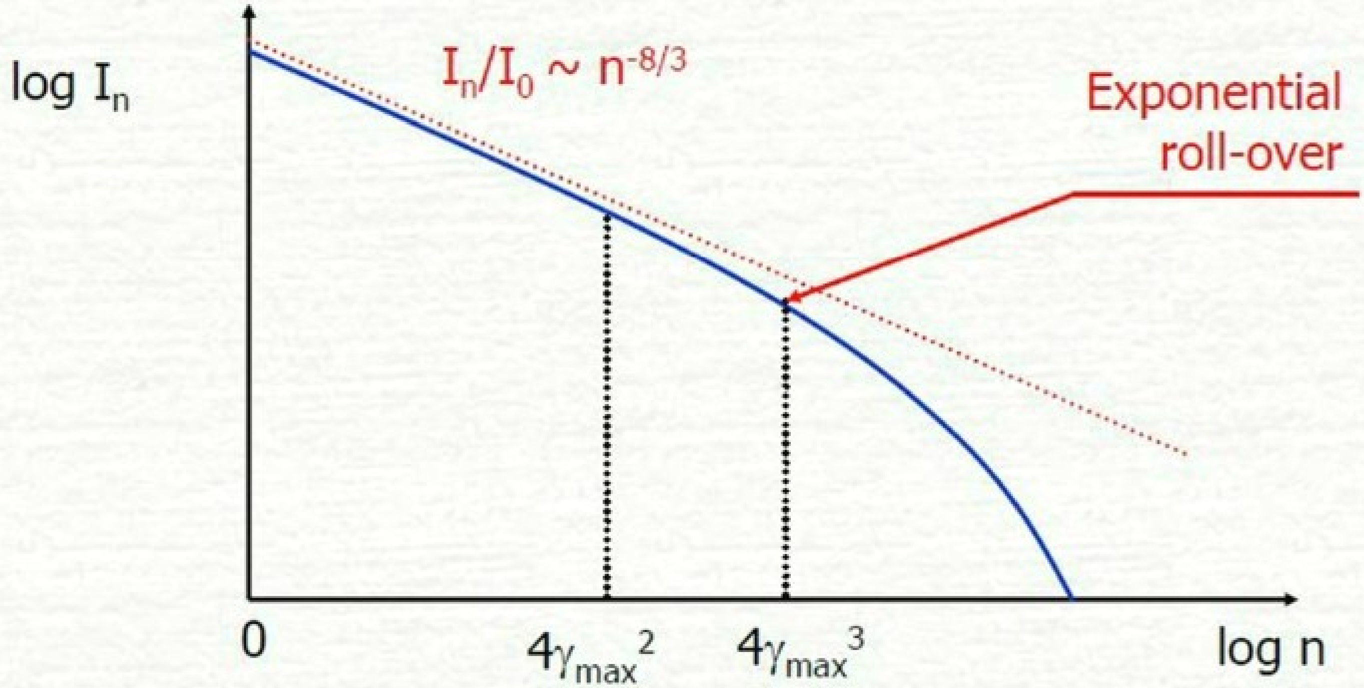
$$\beta(t) \approx \beta_{max}(1 - \omega_0^2\, t^2),$$

Its γ-factor $\gamma = 1/\sqrt{1-\beta^2}$
has a sharp spike of the width

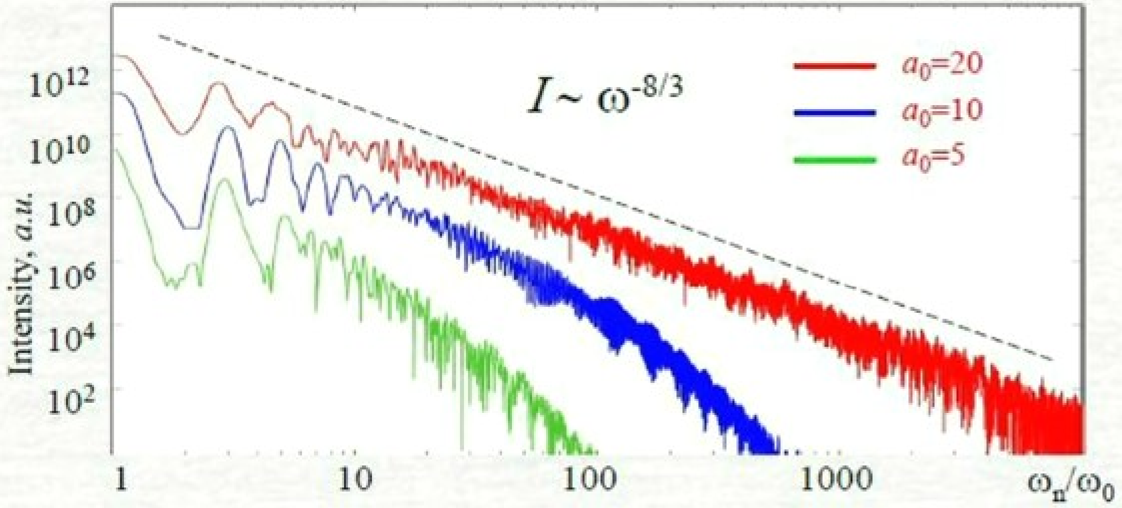
$$\tau \approx 1/\omega_0 \gamma_{max}$$

The Canonical Spectrum

Baeva, Gordienko, Pukhov, *Phys. Rev. E* **74**, 046404 (2006)



Reflected radiation spectra in 1D PIC simulations



The Gaussian laser pulse $a=a_0\exp[-(t/\tau)^2]\cos\omega_0 t$ is incident onto an overdense plasma layer with $n=30n_c$.
The color lines correspond to laser amplitudes a_0=5,10,20.
The broken line marks the analytical scaling $I \sim \omega^{-8/3}$.

VULCAN Experiment: Harmonics down to "Water Window"

B. DROMEY, M. ZEPF, et al., **NATURE** Physics, Vol. **2**, p. 456 (2006).

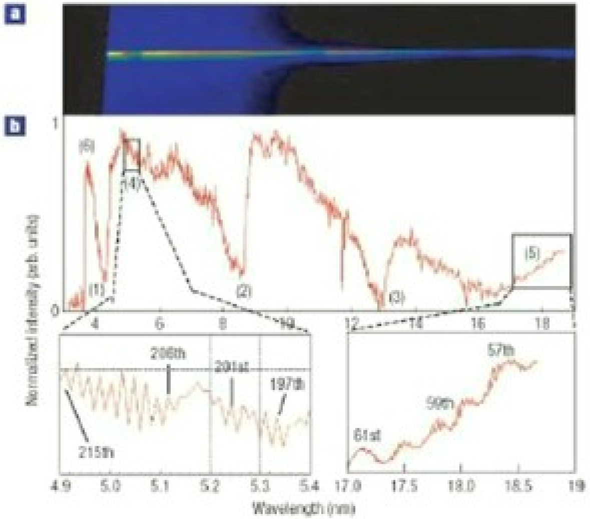


Figure 3 Unprocessed high harmonic spectrum recorded with the extreme-ultraviolet spectrometer. a, Raw CCD image obtained with the double PM setup (E = 70 J on target, false colours). **b**, A lineout of **a**. Spectral features: (1) first-order carbon K-edge (4.36 nm), (2) second-order carbon K-edge, (3) third-order carbon K-edge, (4) region of resolved harmonics around 200th order

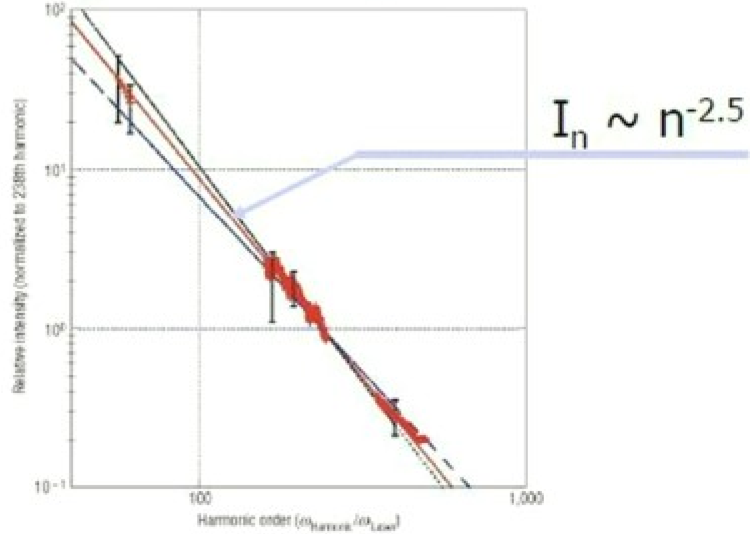
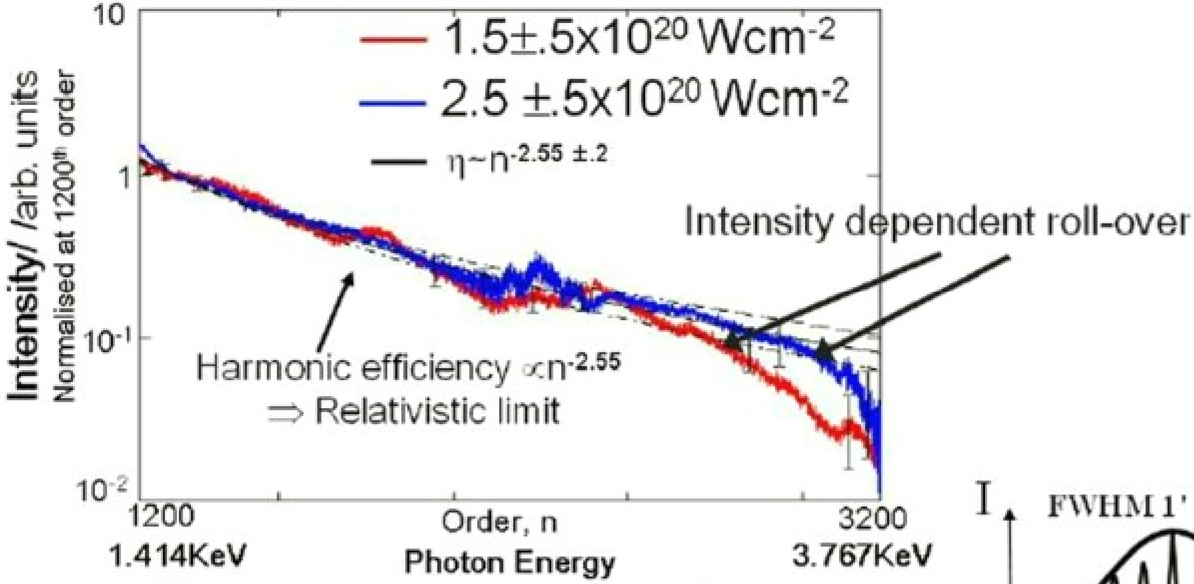
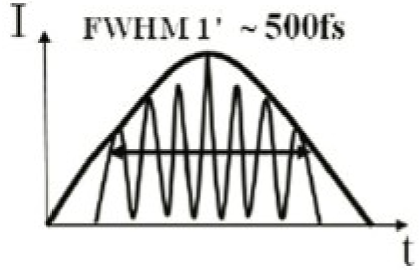


Figure 4 Relative intensity of harmonics normalized to the 238th harmonic (at the carbon K-edge). The lines are fits to the data with the exponent p as a fitting parameter such that $I(n)/I(238) = n^{-p}/238^{-p}$. The best fit (red line) corresponds to a value of $p = 2.5$ confirming harmonic production in the relativistic limit. The error

keV harmonics

B. Dromey, M. Zepf et. al. (PRL 2007)



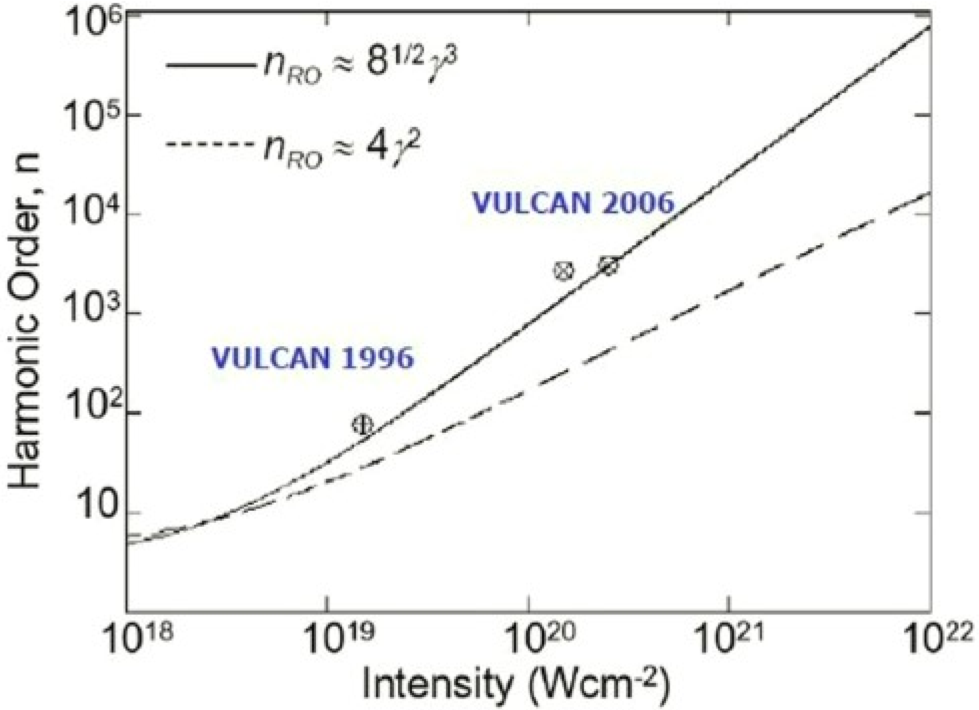


First coherent, femtosecond, sub-nm source

Courtesy of M. Zepf

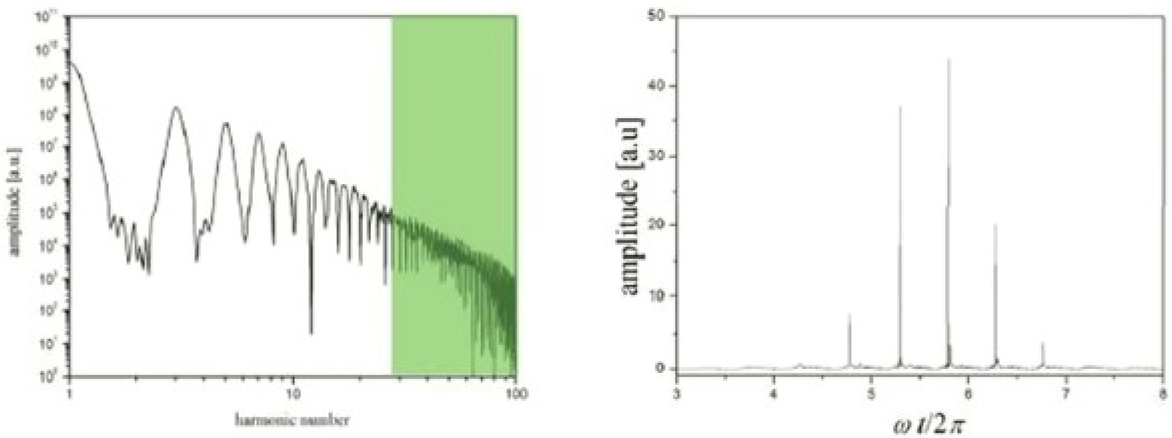
Roll-over scaling confirmed as $\sim\gamma^3$

B. Dromey, M. Zepf et. al. (in press, PRL 2007)



Attosecond pulses

- After proper filtering of HHG one obtains *a train of (sub-)attosecond pulses*



Source: Baeva, Gordienko, Pukhov, Phys. Rev. E**74**, 046404 (2006)

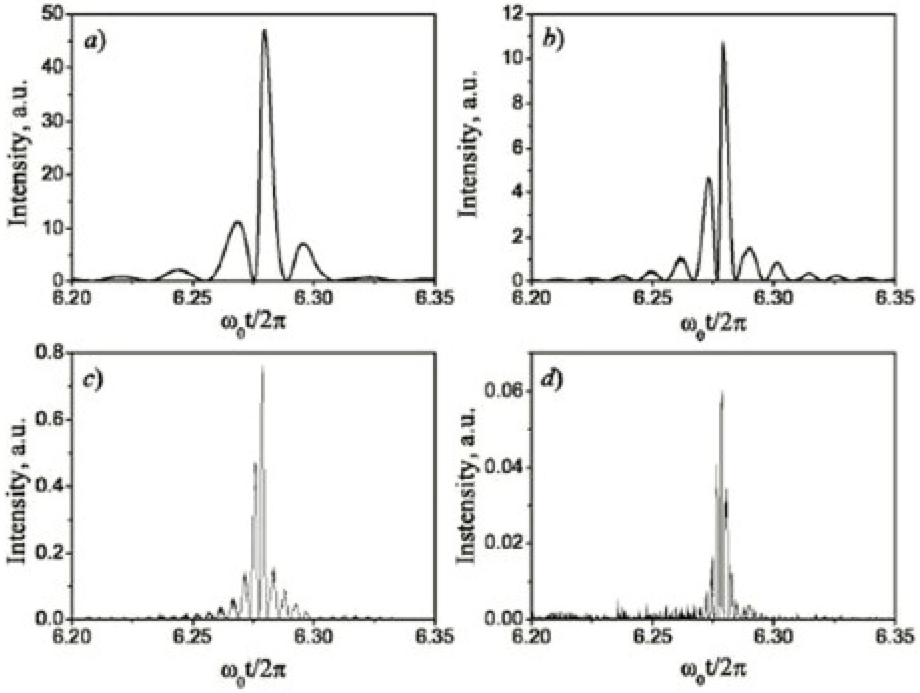
Attosecond Pulse Shape as a Function of Filter Threshold Ω_c

$\Omega_c < 4\gamma^2$

$\Omega_c = 4\gamma^2$

$4\gamma^2 < \Omega_c < 4\gamma^3$

$\Omega_c > 4\gamma^3$



Shortest Pulse Duration

Baeva, Gordienko, Pukhov, Phys. Rev. E**74**, 046404 (2006)

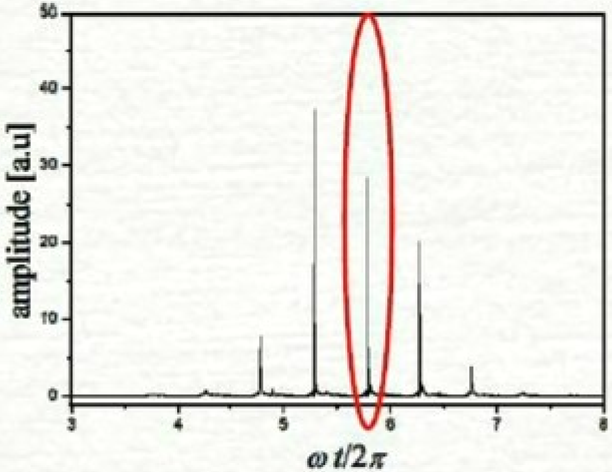
Pulses can be zeptosecond!

$$\omega_0 \tau_{\text{pulse}} \sim \frac{1}{\gamma_{\max}^3} \sim \frac{1}{a^3}$$

$$\gamma_{\max} = a \cdot f(S)$$

High harmonics: train of attosecond pulses

Yet some applications require
single attosecond pulses!
Can we extract one pulse from the train?



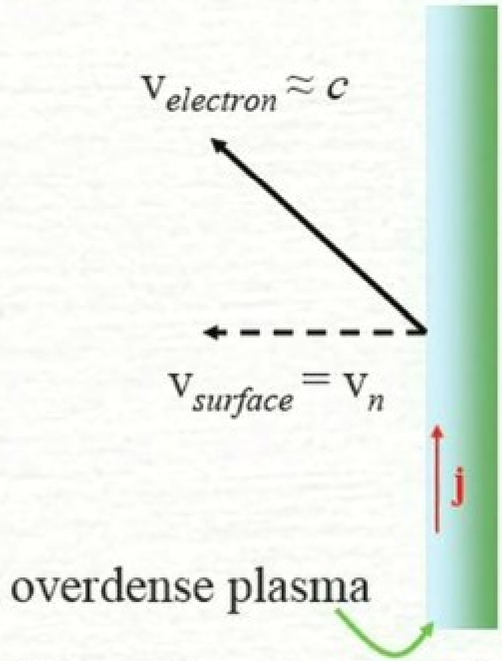
Surface dynamics vs individual elecrons dynamics

Gordienko & Pukhov *Phys. Plasmas* **12**, 043109 (2005)

- Ultra-relativistic similarity theory demands that

$$p_\tau \sim a_0$$

$$p_n \sim a_0$$



Surface dynamics

$$\mathbf{p}_n = a_0 \mathbf{P}_n(S, \omega t)$$
$$\mathbf{p}_\tau = a_0 \mathbf{P}_\tau(S, \omega t)$$

$$\mathbf{P}_n \sim \mathbf{P}_\tau \sim 1$$

Simple algebra shows...

$$\beta_s(t) = \frac{p_n(t)}{\sqrt{m_e^2 c^2 + p_n^2(t) + p_\tau^2(t)}} = \frac{P_n(t)}{\sqrt{P_n^2(t) + P_\tau^2(t)}} - O(a_0^{-2})$$

$$\gamma_s(t) = \frac{1}{\sqrt{1 - \beta_s^2(t)}} = \sqrt{1 + \frac{P_n^2(t)}{P_\tau^2(t)}} + O(a_0^{-2}),\ p_\tau \neq 0$$

$$\gamma_s(t) = \sqrt{\frac{p_n^2 + m_e^2 c^2}{m_e^2 c^2}} \propto a_0,\ p_\tau = 0$$

γ-spike!

(Sub-)attosecond pulse emission

The Condition:

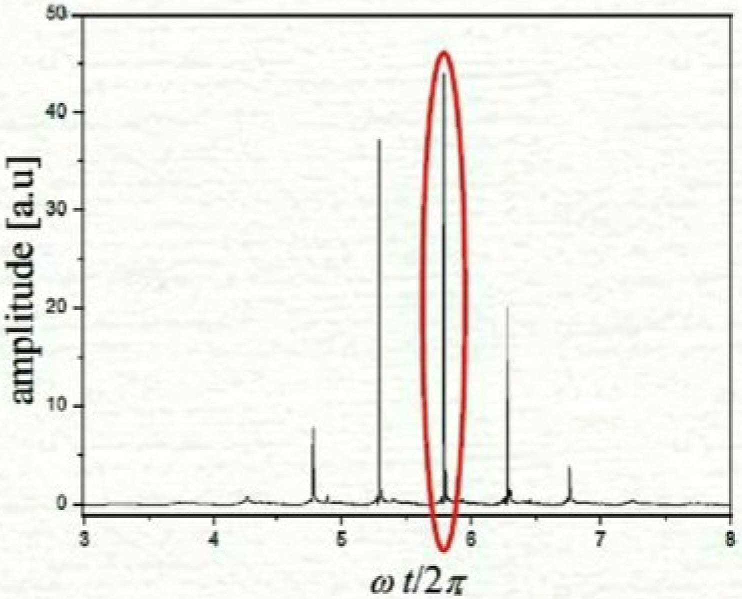
Vector with 2 components!

$$\mathbf{p}_\tau = 0$$

Tangential electron momentum at the surface vanishes

Relativistic plasma control via polarization gating

Canonical momentum conservation: $\mathbf{p}_\tau = \mathbf{A}_\tau$



$\Updownarrow$

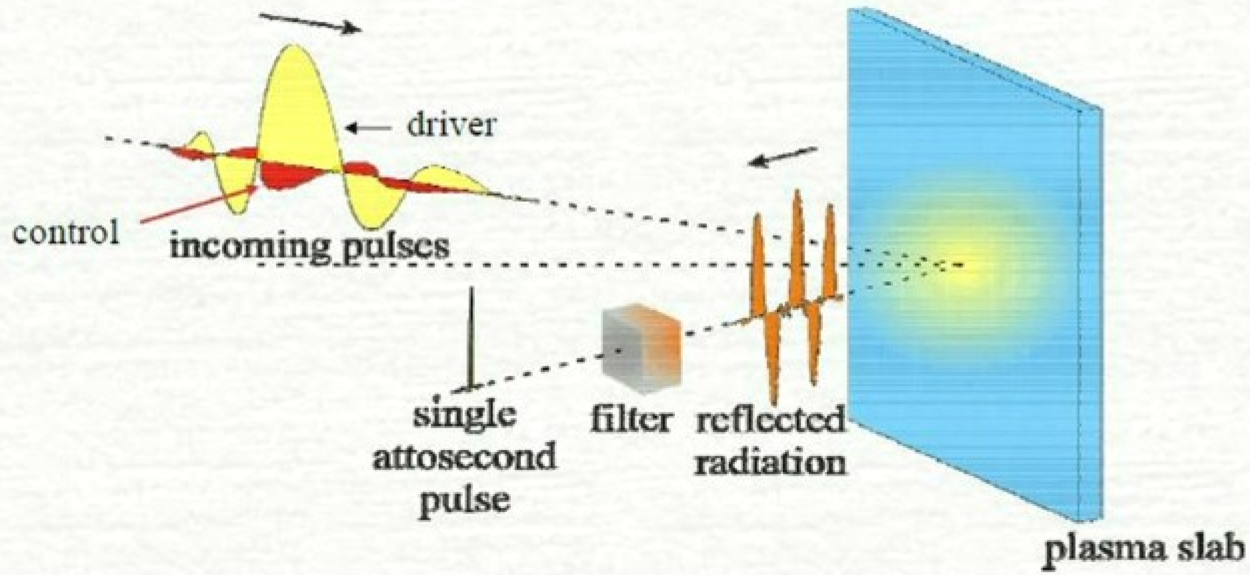
enforce

$\mathbf{A}_\tau = 0$

once!

Relativistic plasma control via polarization gating

Baeva, Gordienko, Pukhov, *Phys. Rev. E*74, 065401R (200(



Relativistic plasma control

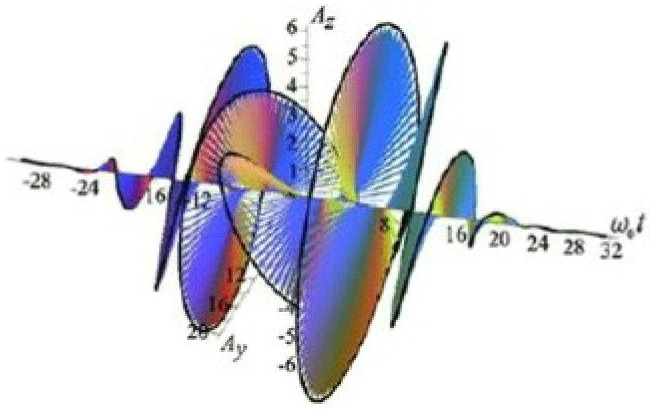
Baeva, Gordienko, Pukhov, *Phys. Rev.* E**74**, 065401R (2006)

Simulation parameters:

Driving polarization: ω_0=1, a_0=20

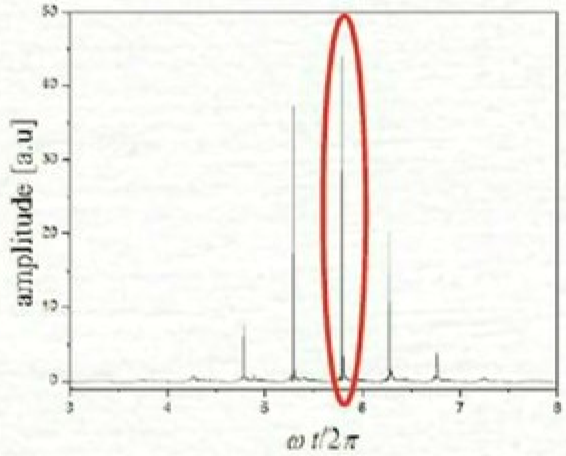
Control polarization: ω_d=1.25, a_d=6

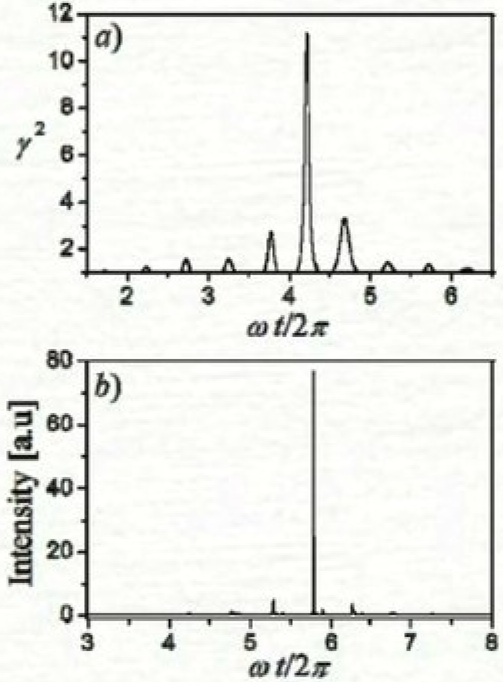
Phase shift: $\Delta\varphi=\pi/8$



Relativistic plasma control

Baeva, Gordienko, Pukhov, *Phys. Rev. E* **74**, 065401R (2006)

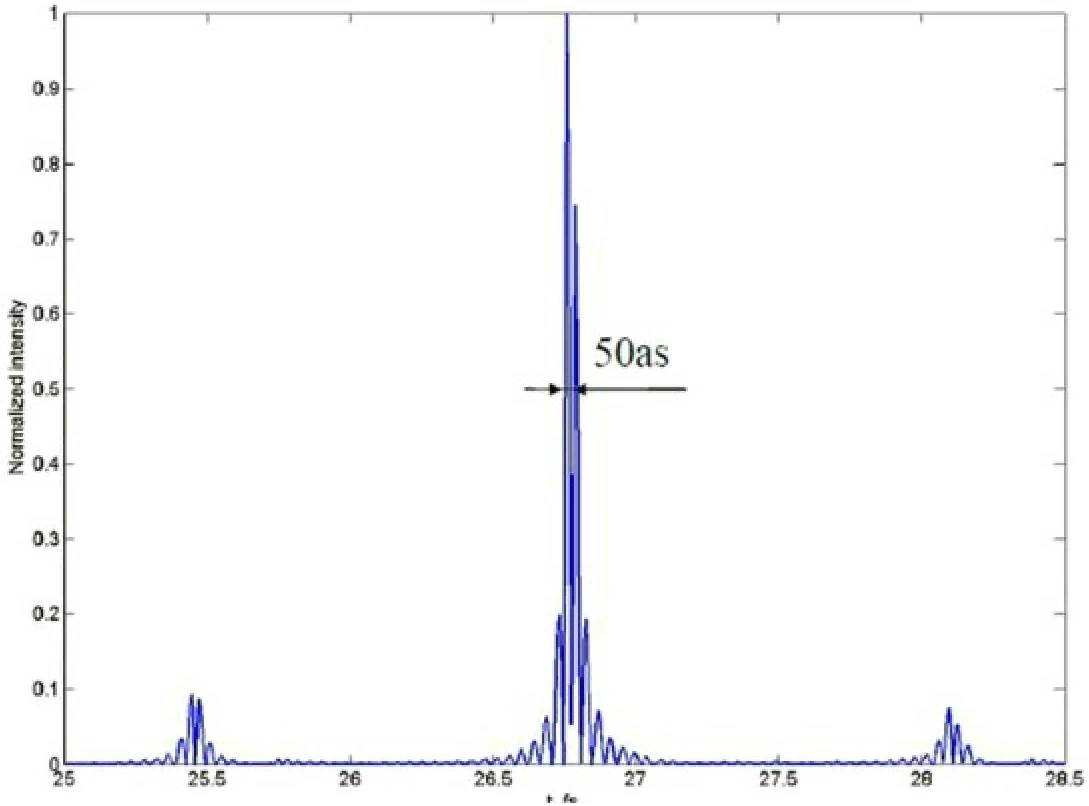




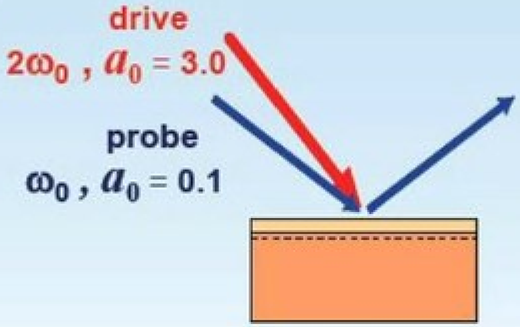
Polarization gating (1D-PIC)

Talk by Michael Geissler

a=20, τ=15fs; HH-Pulse: 30-100harmonic; n_0=90n_c

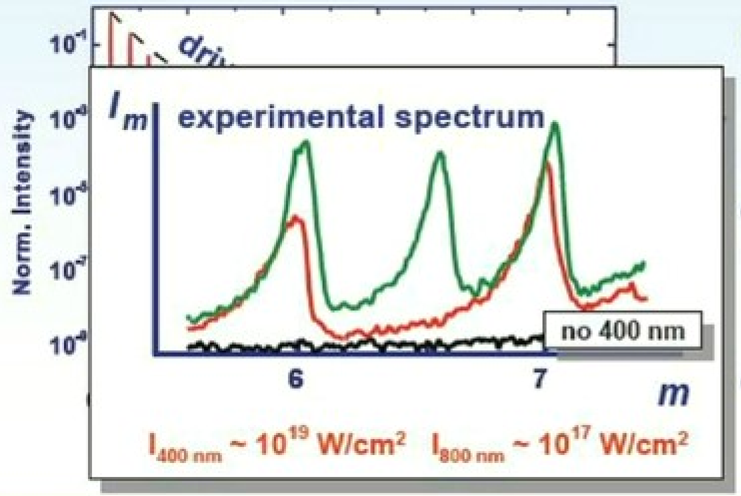


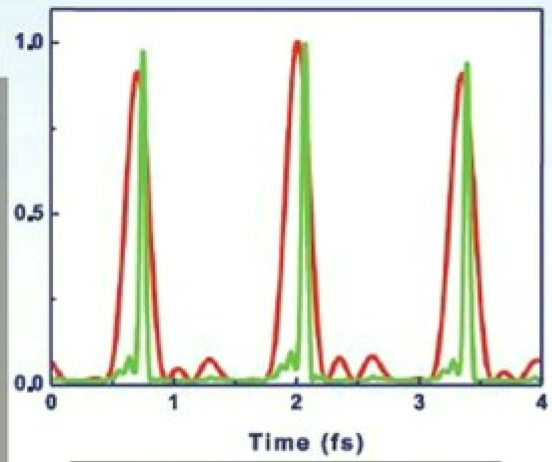
Two-Colour HOHG / Atto-Pulses (PIC)



attosecond pulses produced by the probe pulse

- up to $5\ \omega_0$
- up to $30\ \omega_0$





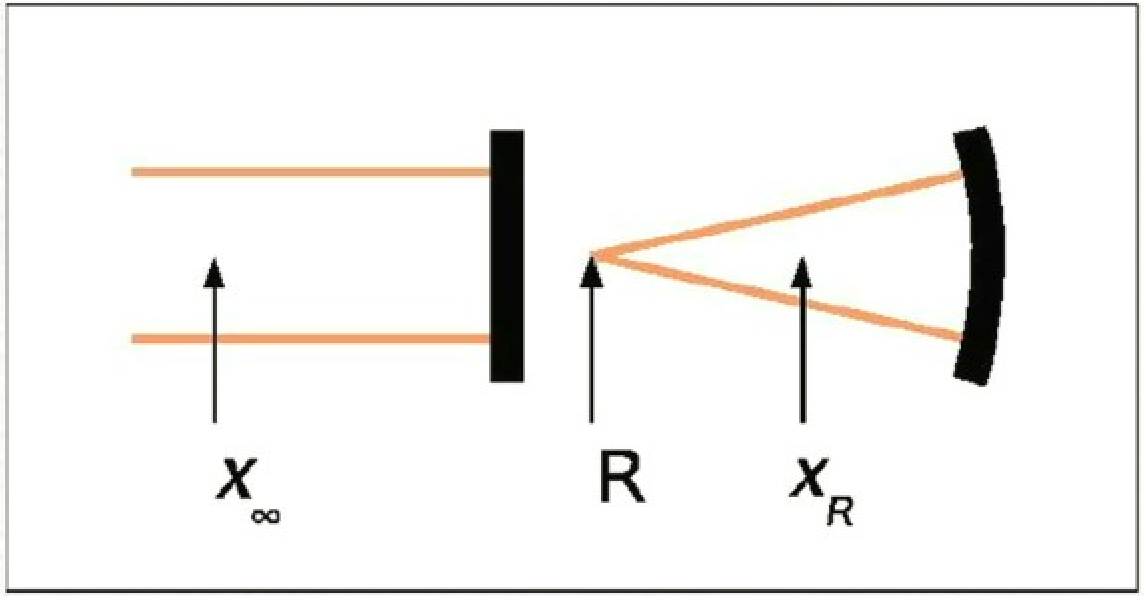
zero-cycle atto-pulses

3D Regimes of Relativistic HHG

Daniel an der Brügge and Alexander Pukhov, Phys. Plasmas **14**, 093104 (2007)

- Is the local HHG quasi-1d?
- How changes the spectrum over vacuum propagation?
- (Self-)focusing of high harmonics

3D geometry used in simulations



Quasi-1D regime of HHG

Daniel an der Brügge and Alexander Pukhov, Phys. Plasmas **14**, 093104 (2007)

- If the focal spot radius $R>\lambda$, the local 1D approximation gives excellent results

Spectrum modification during vacuum propagation

Daniel an der Brügge and Alexander Pukhov, in press, Phys. Plasmas 2007

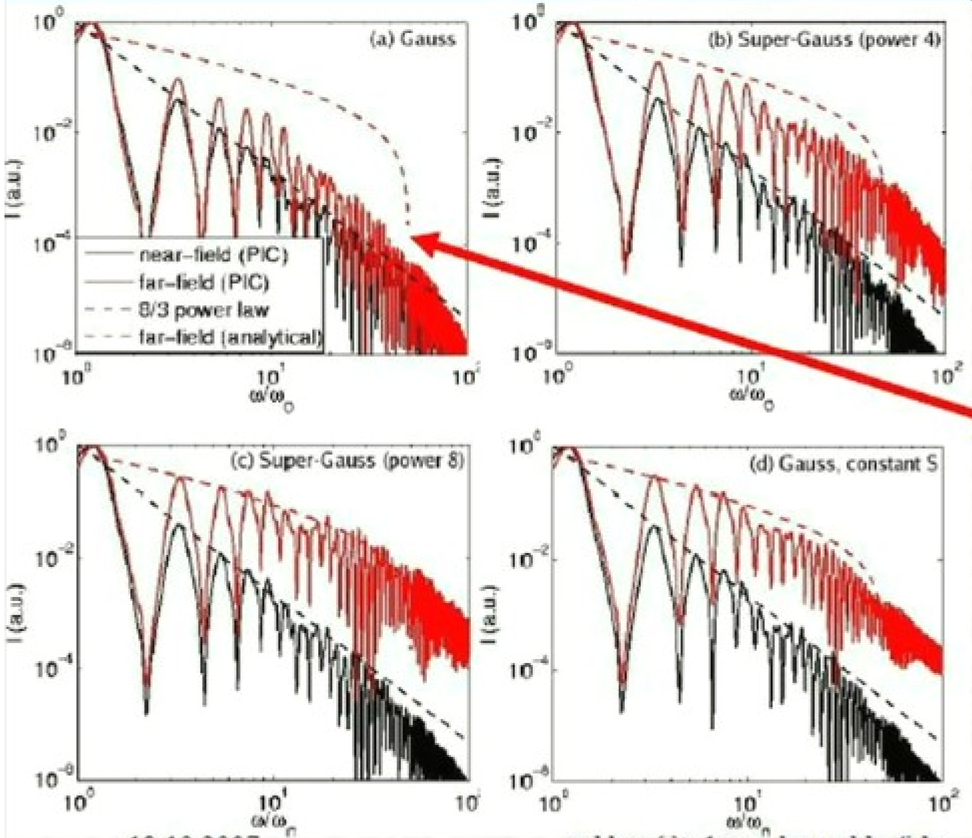
- Lower harmonics diffract faster
  → vacuum propagation works as a natural filter

$$I(x,\omega) = I_0 \frac{\left(\frac{\omega}{\omega_0}\right)^{-p+2}}{\left(\frac{x}{x_{Rl}}\right)^2 + \left(\frac{\omega}{\omega_0}\right)^2} \left(1 - \frac{1}{a_0}\sqrt[q]{\frac{\omega}{\omega_c}}\right)^2$$

$$\underset{x,\, a_0 \to \infty}{\approx} \frac{I_0 x_{Rl}^2}{x^2} \left(\frac{\omega}{\omega_0}\right)^{-p+2}$$

3D PIC simulations

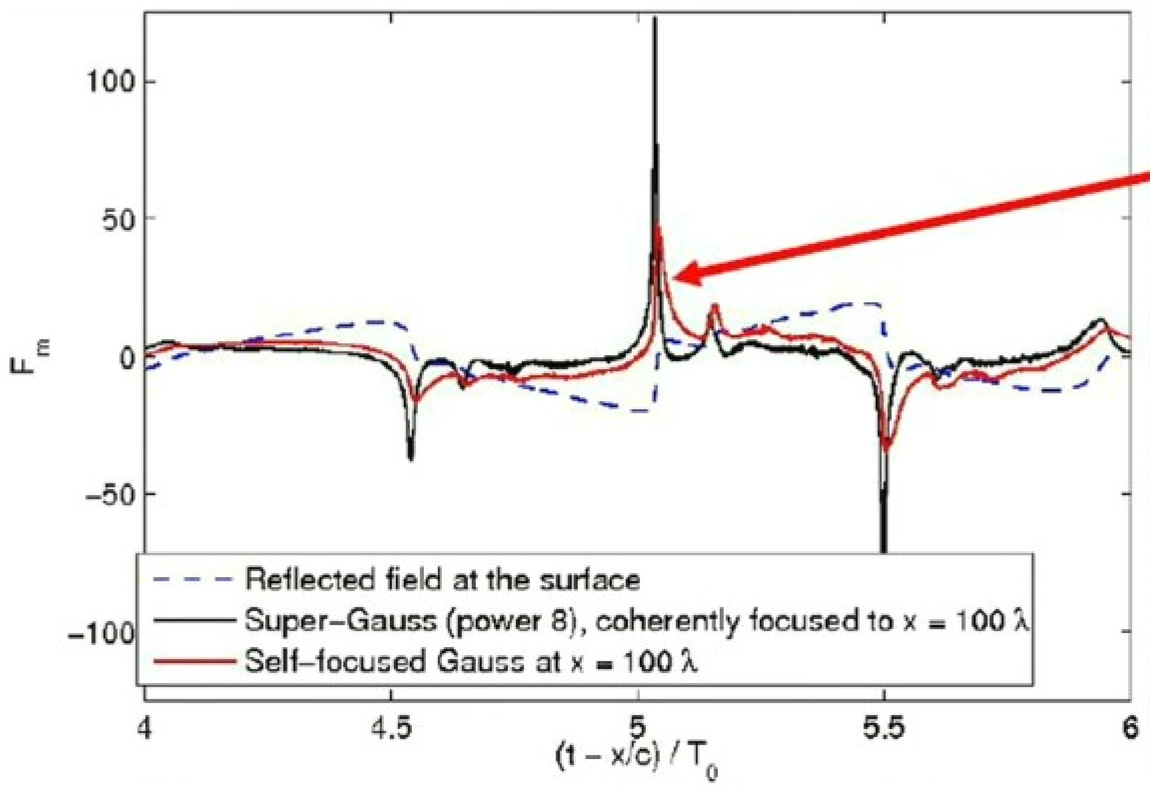
Daniel an der Brügge and Alexander Pukhov, in press, Phys. Plasmas 2007



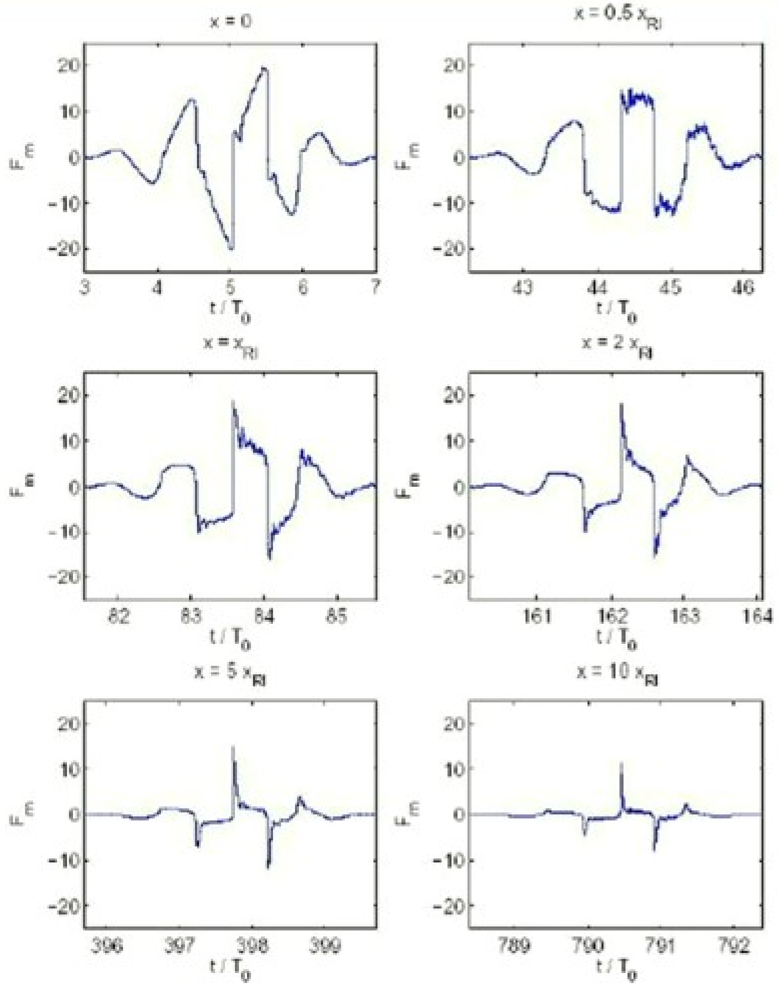
Results indicate natural divergence of HH generated by Gaussian pulses

(Self-)focusing of HH in vacuum

Daniel an der Brügge and Alexander Pukhov, in press, Phys. Plasmas 2007



Attosecond pulses have much higher amplitude at the focus than the driving laser!



3D PIC simulations

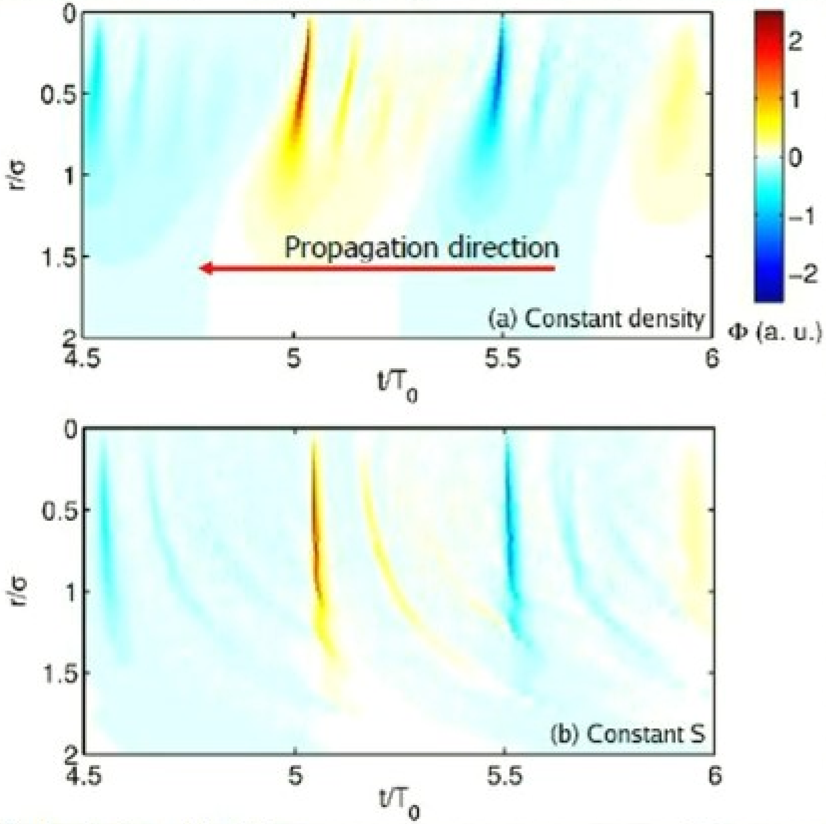
Rectification of attosecond pulses during vacuum propagation: simulation with *S*=const surface

3D attosecond pulse propagation

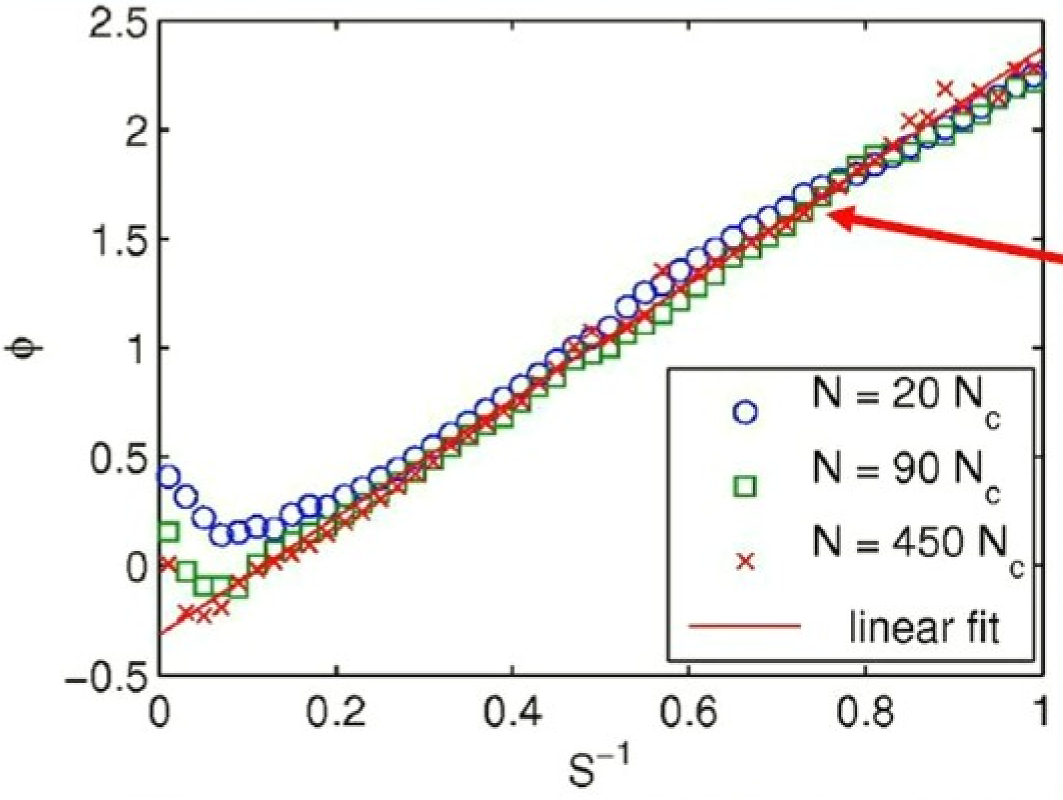
Daniel an der Brügge and Alexander Pukhov, Phys. Plasmas **14**, 093104 (2007)

Gaussian laser spot

Natural self-focusing: the phase of attosecond pulse generation is *S*-dependent!



S-dependence of attosecond pulse generation phase



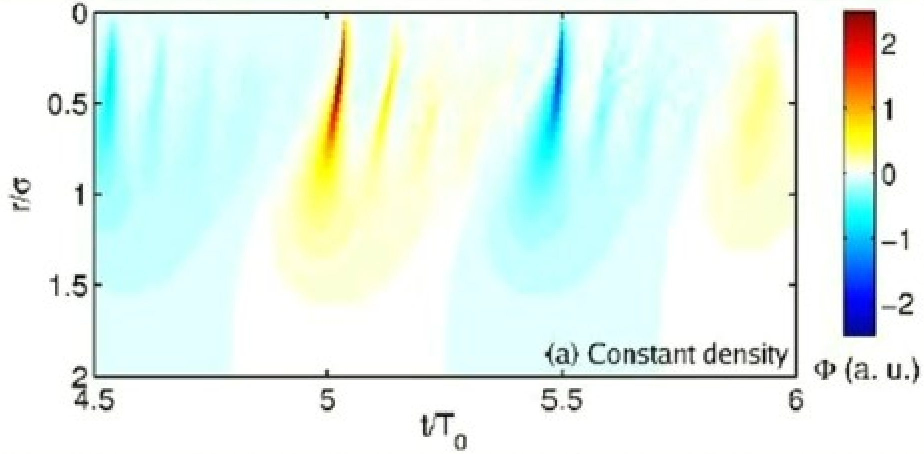
Numerical fit:

$\phi = 2.7S^{-1} - 0.32$

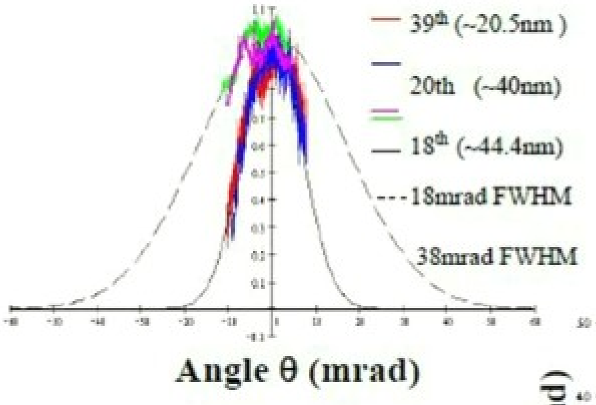
Harmonics Focus Position

Daniel an der Brügge and Alexander Pukhov, in press, Phys. Plasmas 2007

$$x_{\mathrm{sf}} = \frac{S_0}{2.7} x_R$$



Diffraction limited ROM performance

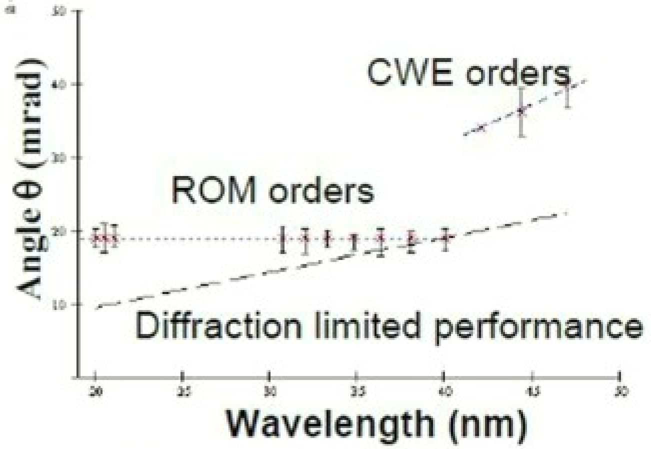


CWE – 2 -3 times diffraction limited

20th order – Diffraction limited

39th order – 2 Diffraction limited

All orders in the interval 20 -40 have identical divergence



Conclusions

1. Theory of high harmonics generation at plasma surfaces is developed.

2. The canonical harmonics spectrum is the universal power law

$$I_n \sim n^{-8/3}$$

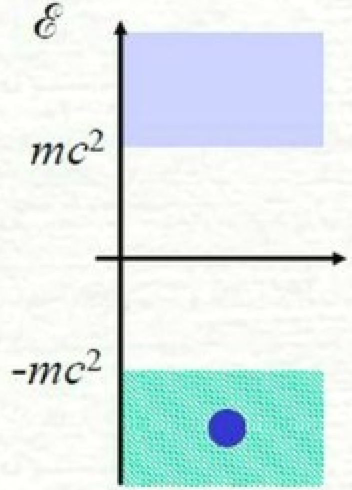
3. Harmonics appear as a train of attosecond pulses

4. Relativistic plasma control allows to select a single attosecond pulse

5. 3D vacuum propagation can work as a natural filter to rectify the attosecond pulses

Vacuum is not empty

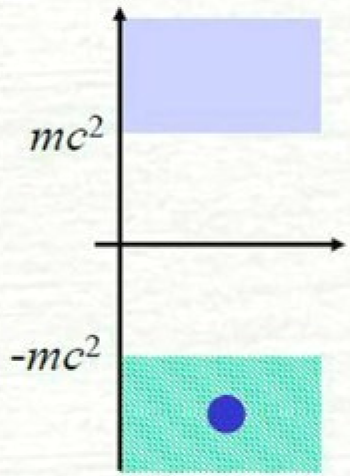
Compton wavelength $\lambda_C = \dfrac{h}{mc}$



Strong EM fields can bring virtual particles into the real world

$$eE_C \lambda_C \cong 2mc^2$$

Critical laser intensity
$I_C \sim 5 \times 10^{29}$ W/cm^2.



Virtual particles appear and disappear on Compton lengths

Vacuum polarization:
$e^- \, e^+$ pairs are created

Required Laser Energy

To reach the critical intensity at the laser fundamental,
λ=1 μm, one needs 64 MJ energy.

However, the energy scales down when the wavelength decreases:

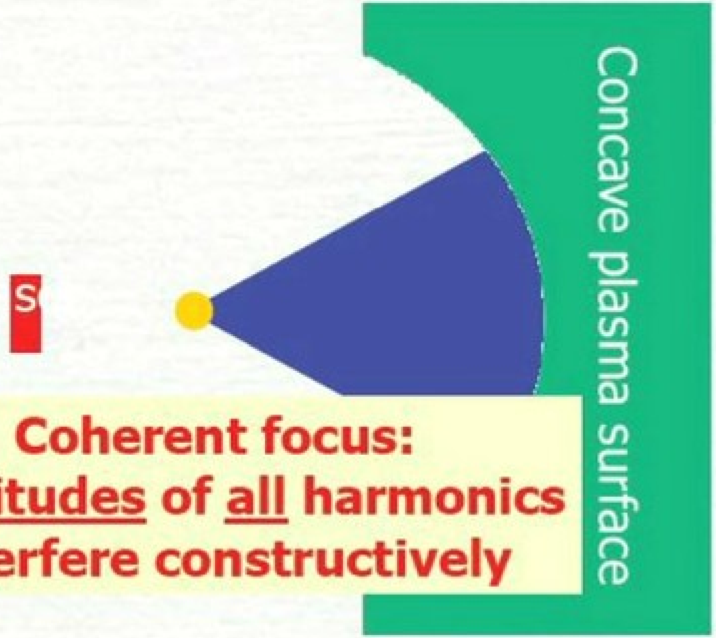
$$W = \frac{4\pi}{3c} I_C \lambda^3$$

Focusing laser harmonics one can reach the critical intensity using moderate energy lasers

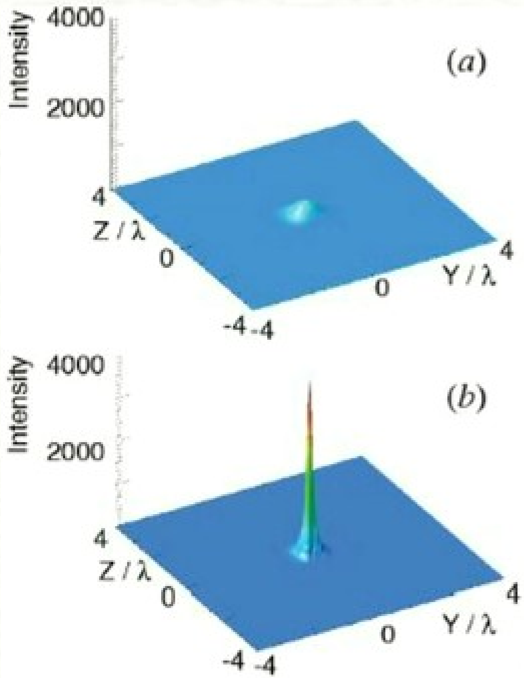
Coherent Harmonics Focusing: plasma harmonics are phase-locked!

VLPL

Gordienko, Pukhov, Shorokhov, Baeva, *Phys. Rev. Lett.* **94**, 103903 (2005)

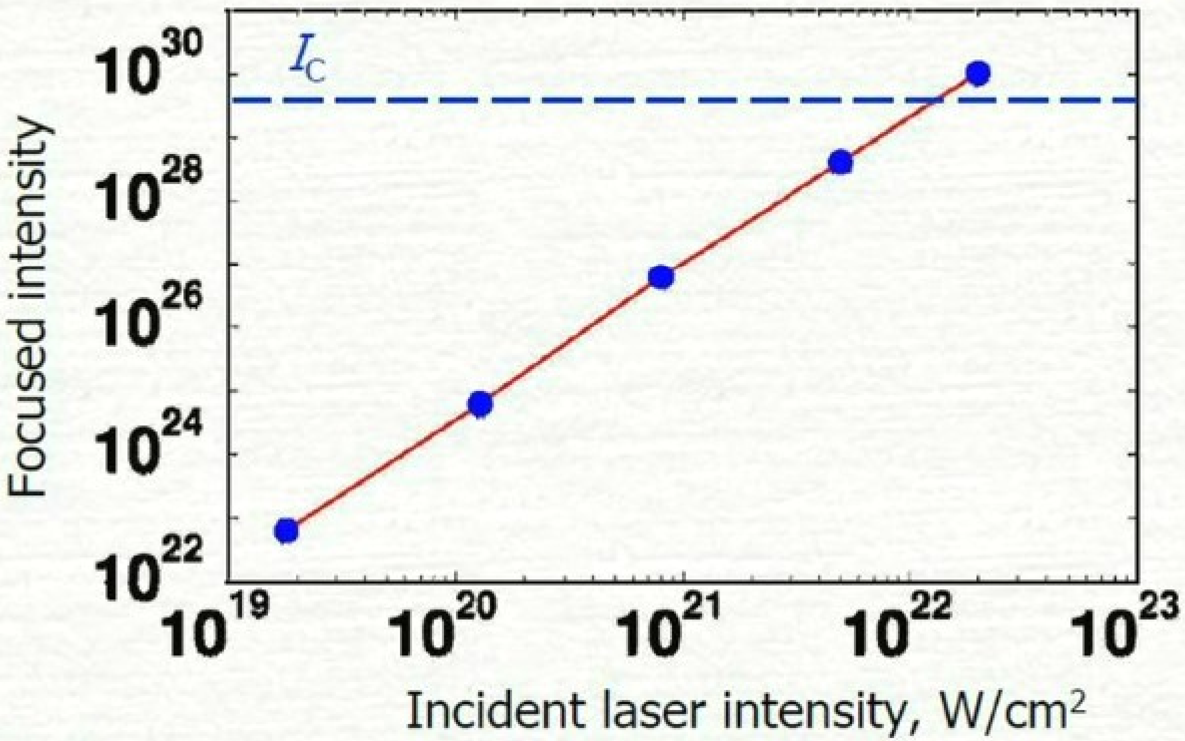


Coherent focus:
<u>amplitudes</u> of <u>all</u> harmonics interfere constructively



Scaling of the CHF intensity

Gordienko, Pukhov, Shorokhov, Baeva, *Phys. Rev. Lett.* **94**, 103903 (2005)



Conclusions

1. Plasma harmonics are phase-locked, coherent, and can be focused to give huge intensities

Universal Plasma Surface Dynamics

Poynting flux $\mathbf{P} = \frac{1}{c}\frac{\mathbf{E}\times\mathbf{B}}{4\pi}$