210C - PS4

I worked with Zorah on the analytical parts + Jack / Grace / Ahmad on the code

1. a) Guess:

$$\hat{y}_t = \eta_{ya}\,\hat{a}_t$$

$$\hat{\pi}_t = \eta_{\pi a}\,\hat{a}_t$$

$$\hat{i}_t = \eta_{ia}\,\hat{a}_t$$

$$\eta_{ia}\hat{a}_t = \phi_\pi \hat{\pi}_t = \phi_\pi \eta_{\pi a}\hat{a}_t \Rightarrow \eta_{ia} = \phi_\pi \eta_{\pi a}$$

$$\eta_{ya}\hat{a}_t = -\sigma[\eta_{ia}\hat{a}_t - \mathbb{E}[\eta_{\pi a}\hat{a}_{t+1}]] + \mathbb{E}[\eta_{ya}\hat{a}_{t+1}] \qquad [\text{assume } \mathbb{E}[\varepsilon_t] = 0]$$

$$= -\sigma\eta_{ia}\hat{a}_t + \sigma\eta_{\pi a}[\rho_a\hat{a}_t] + \eta_{ya}[\rho_a\hat{a}_t]$$

$$\Rightarrow \eta_{ya}(1-\rho_a) = -\sigma(\phi_\pi\eta_{\pi a}) + \sigma\eta_{\pi a}\rho_a$$

$$\Rightarrow \eta_{ya} = \frac{-\sigma\eta_{\pi a}(\phi_\pi - \rho_a)}{1-\rho_a}$$

$$\eta_{\pi a}\hat{a}_t = \kappa\left[\eta_{ya}\hat{a}_t - \frac{1+\varphi}{\sigma+\varphi}\hat{a}_t\right] + \beta\,\mathbb{E}[\eta_{\pi a}\hat{a}_{t+1}] \Rightarrow \eta_{\pi a} = \kappa\left[\eta_{ya} - \frac{1+\varphi}{\sigma+\varphi}\right] + \beta\eta_{\pi a}\rho_a$$

$$\Rightarrow \eta_{\pi a} = \frac{\kappa\left[\eta_{ya} - \frac{1+\varphi}{\sigma+\varphi}\right]}{1-\beta\rho_a}$$

$$\eta_{ya} = \left[\frac{\kappa\left[\eta_{ya} - \frac{1+\varphi}{\sigma+\varphi}\right]}{1-\beta\rho_a}\right]\left[\frac{-\sigma(\phi_\pi-\rho_a)}{1-\rho_a}\right] \Rightarrow \eta_{ya}\left[1 + \left(\frac{\sigma(\phi_\pi-\rho_a)}{1-\rho_a}\right)\left(\frac{\kappa}{1-\beta\rho_a}\right)\right] = \frac{\sigma\kappa\left(\frac{1+\varphi}{\sigma+\varphi}\right)(\phi_\pi-\rho_a)}{(1-\beta\rho_a)(1-\rho_a)}$$

$$\eta_{ya} = \frac{\sigma\left(\frac{1+\varphi}{\sigma+\varphi}\right)(\phi_\pi-\rho_a)\kappa}{(1-\rho_a)(1-\beta\rho_a) + \sigma\kappa(\phi_\pi-\rho_a)}$$

plug back in to get $\eta_{\pi a}$

b) see attached

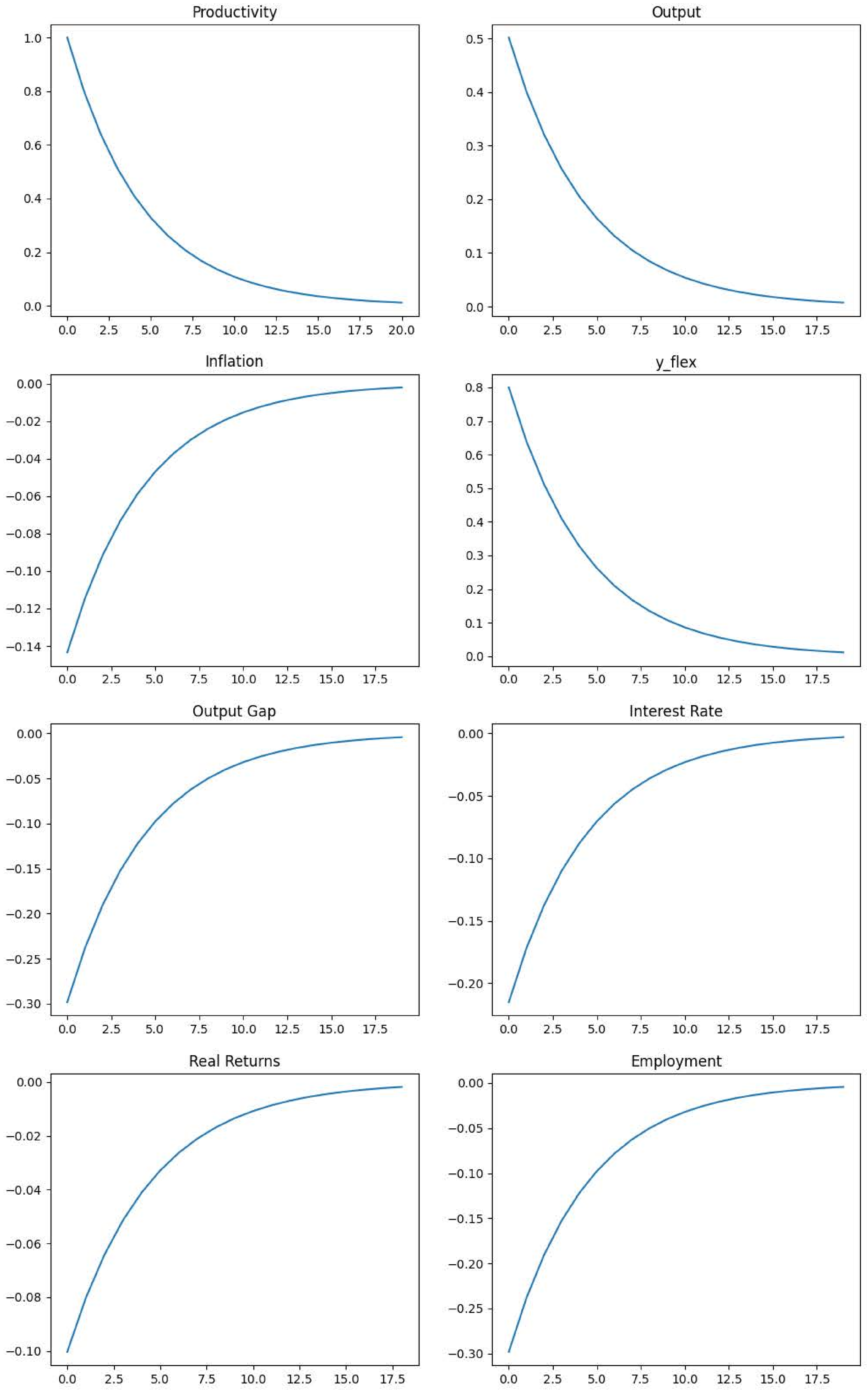
c) A shock to productivity increases output and also decreases labor which makes sense from the firm's perspective since they can produce more with less input.

The optimal y (y-flex) also increases since it is optimal to produce more with higher productivity. The output gap temporarily decreases since firms cannot adjust as they would optimally.

Interest rates and real returns decrease since firms can lower prices and the central bank responds through the taylor rule.

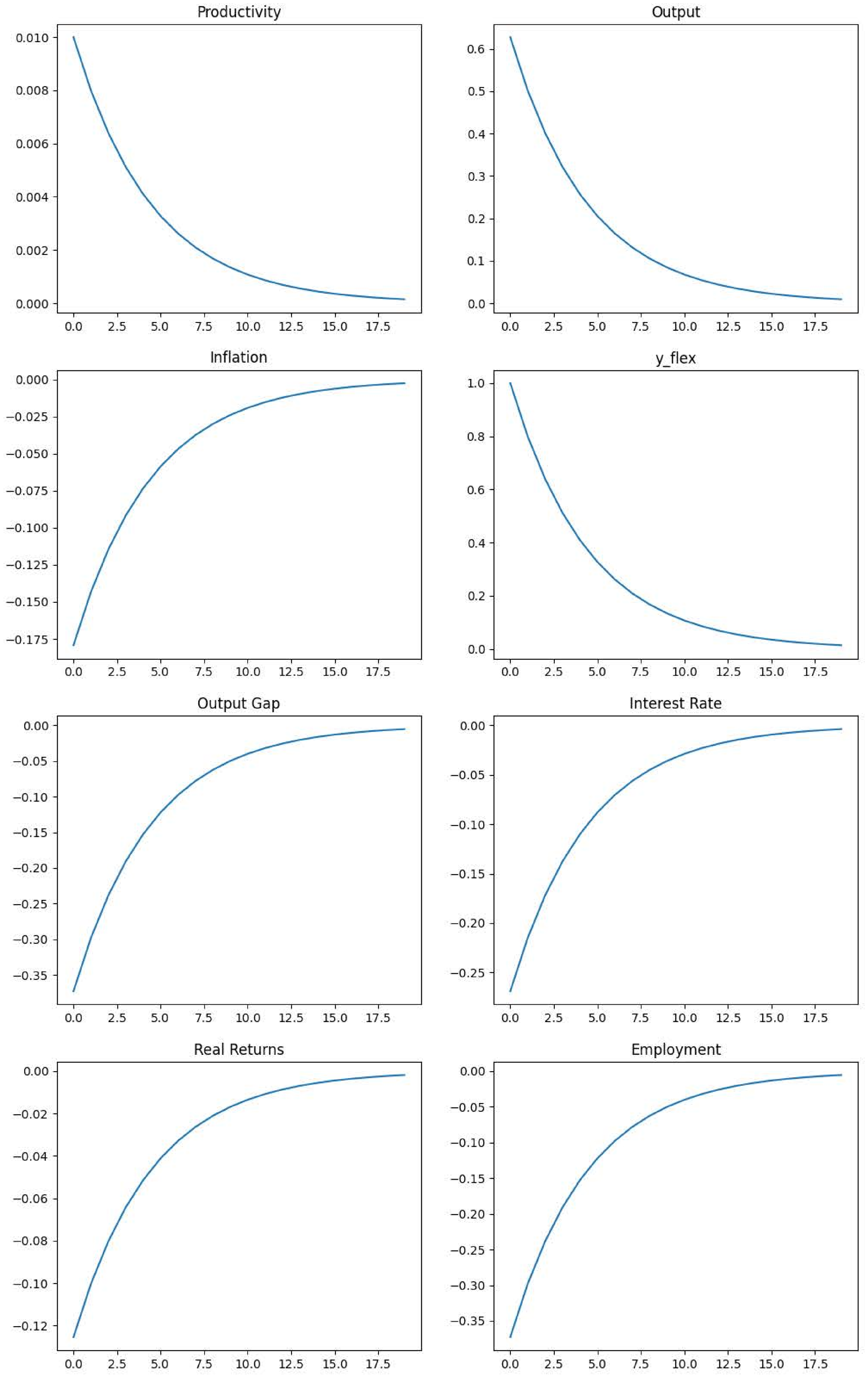
d) See attached

1.b



Productivity
1.0
0.8
0.6
0.4
0.2
0.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
Output
0.5
0.4
0.3
0.2
0.1
0.0
Inflation
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
y_flex
0.8
0.7
0.6
Output Gap
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
Interest Rate
Real Returns
Employment

1.d


Productivity
Output
Inflation
y_flex
Output Gap
Interest Rate
Real Returns
Employment

2. a) The equation is not recursive since we would need $\frac{P_t^*}{P_t}$ on the left and $\frac{P_{t+1}^*}{P_{t+1}}$ on the right. Since P_{t+1}^* isn't on the right, we do not have a recursive equation.

b) $F_{2t} = \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} (P_{t+k}/P_t)^{\varepsilon-1}$

$$\left[F_{2t+1} = \sum_{k=0}^{\infty} \theta^k \Lambda_{t+1,t+1+k} Y_{t+1+k} \left(P_{t+1+k}/P_{t+1}\right)^{\varepsilon-1}\right]$$

$$F_{2t} = \theta^0 \Lambda_{t,t} Y_t \left(\frac{P_t}{P_t}\right)^{\varepsilon-1} + \sum_{k=1}^{\infty} \theta^k \underbrace{\Lambda_{t,t+k}}_{\Lambda_{t,t+1}\Lambda_{t+1,t+k}} Y_{t+k} \left(\frac{P_{t+k}}{P_t}\right)^{\varepsilon-1}$$

$$= Y_t + \sum_{k=0}^{\infty} \theta^{k+1} \Lambda_{t,t+1} \Lambda_{t+1,t+k+1} Y_{t+k+1} \underbrace{\left(\frac{P_{t+k+1}}{P_t}\right)^{\varepsilon-1}}_{\frac{P_{t+k+1}}{P_{t+1}} \cdot \frac{P_{t+1}}{P_t}}$$

$$= Y_t + \theta \Lambda_{t,t+1} \left(\frac{P_{t+1}}{P_t}\right)^{\varepsilon-1} F_{2t+1}$$

c) $F_{1t} = (1+\mu) \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} (P_{t+s}/P_t)^{\varepsilon-1} \left(\frac{W_{t+s}/P_t}{A_{t+s}}\right)$

$$\left[F_{1t+1} = (1+\mu) \sum_{s=1}^{\infty} \theta^s \Lambda_{t+1,t+1+s} Y_{t+1+s} \left(\frac{P_{t+1+s}}{P_{t+1}}\right)^{\varepsilon-1} \left(\frac{W_{t+1+s}/P_{t+1}}{A_{t+1+s}}\right)\right]$$

$$F_{1t} = (1+\mu)\theta^0 \Lambda_{t,t} Y_t \left(\frac{P_t}{P_t}\right)^{\varepsilon-1} \left(\frac{W_t/P_t}{A_t}\right) + (1+\mu) \sum_{s=1}^{\infty} \theta^s \underbrace{\Lambda_{t,t+s}}_{\Lambda_{t,t+1}\Lambda_{t+1,t+s}} Y_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\varepsilon-1} \left(\frac{W_{t+s}/P_t}{A_{t+s}}\right)$$

$$= (1+\mu) Y_t \left(\frac{W_t/P_t}{A_t}\right) + (1+\mu) \sum_{s=0}^{\infty} \theta^{s+1} \Lambda_{t,t+1} \Lambda_{t+1,t+s+1} Y_{t+s+1} \underbrace{\left(\frac{P_{t+s+1}}{P_t}\right)^{\varepsilon-1}}_{\frac{P_{t+s+1}}{P_{t+1}} \cdot \frac{P_{t+1}}{P_t}} \underbrace{\left(\frac{W_{t+s+1}/P_t}{A_{t+s+1}}\right)}_{\frac{W_{t+s+1}/P_{t+1}}{A_{t+s+1}} \cdot \frac{P_{t+1}}{P_t}}$$

$$= (1+\mu) Y_t \left(\frac{W_t/P_t}{A_t}\right) + \theta \Lambda_{t,t+1} \left(\frac{P_{t+1}}{P_t}\right)^{\varepsilon} F_{1,t+1}$$

d) $P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*\,1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$

$$\frac{P_t}{P_t} = \frac{1}{P_t}\left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*\,1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$= \left[\theta \left(\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon} + (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$1 = \left[\theta \Pi_t^{\varepsilon-1} + (1-\theta) p_t^{*\,1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \Rightarrow 1 = \theta \Pi_t^{\varepsilon-1} + (1-\theta) p_t^{*\,1-\varepsilon}$$

e) If $p_t^* > 1$ that means the optimal price relative to the price the firm is actually charging is high. Since the current price level is a combination of previous prices and P^*, prices are growing when $P^* > P$ so inflation will also be > 1.

2. f) + g) see attached

SS calcs:

Normalize $Y=1$, $A=1$

Let $v=1$

$Q = \frac{1}{\beta}\left(\frac{P}{P}\right)^{\phi_\pi} \cdot 1 \Rightarrow Q = \frac{1}{\beta}$

$\Lambda = \beta\left(\frac{c}{c}\right)^{-\gamma} = \beta$

$R = \frac{1}{\Lambda} = \frac{1}{\beta}$

$\Pi = \frac{P}{P} = 1$

$Y = AN \Rightarrow N = 1$

$C = Y \Rightarrow C = 1$

$W/P = \frac{\chi N^{\varphi}}{C^{-\gamma}} = \frac{\chi Y^{\varphi}}{Y^{-\gamma}} = \chi$

$F_2 = Y + \theta \Lambda \Pi^{\varepsilon-1} F_2 \Rightarrow F_2 = \frac{Y}{(1-\beta\theta)} = \frac{1}{1-\beta\theta}$

$F_1 = (1+\mu) Y \frac{W/P}{A} + \theta \Lambda \Pi^{\varepsilon} F \Rightarrow F_1 = \frac{(1+\mu)\chi}{1-\beta\theta}$

(Not sure what it would mean to have an IRF for markup?)

h) When θ is higher, prices are more sticky. As such, the output gap is much larger when θ is higher. Similarly, N will adjust since prices cannot adjust in response to greater productivity.

Consumption and output are stickier when p is stickier since demand is not responsive to the change in productivity.

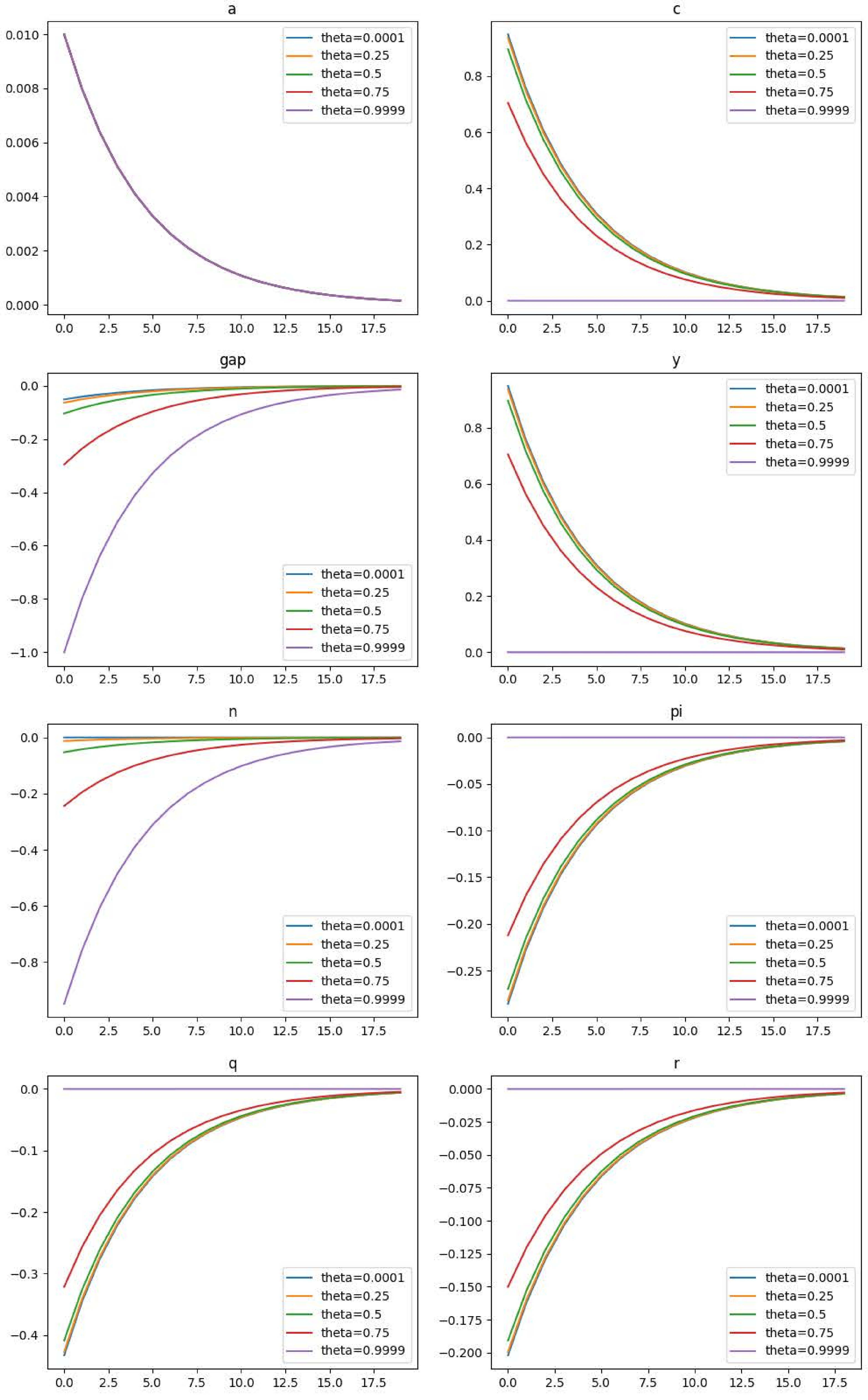
Interest rates and inflation are also less responsive when θ is higher since prices cannot move in response to the change in productivity.

i) In the RBC model since prices aren't sticky, Y and C can adjust perfectly in response to a change in a. Resultingly there will be no output gap.

The case as $\theta \to 0$ is most similar to the RBC.

Without market power, firms in RBC cannot sustain markups.

2.f+g


a
c
gap
y
n
pi
q
r
theta=0.0001
theta=0.25
theta=0.5
theta=0.75
theta=0.9999
0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5