ONLINE APPENDIX

The Availability of Injunctions in Standard-Essential Patent Licensing

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A Proofs

A.1 No-FRAND Tolerance Scenario

Proof of Lemma 1

Proof. If $r_L \geq 1/2 + \theta =: \overline{r}(\theta)$, the implementer's counteroffer is always FRAND, and the patent holder's expected payoffs from litigation are r_L. The patent holder accepts the offer (assuming that she accepts an offer when indifferent). Suppose that $r_L \geq r_P$ (violating Assumption 2), the patent holder accepts. To see this, recall from the expression in equation (4), that the patent holder expects r_L for low values of θ_i and r_P for high values of θ_i. This means that, for $r_L \geq r_P$, the convex combination of r_L and r_P is weakly less than r_L. This concludes the *if* part of the proof.

For the *only if* part, we show that if $r_L < r_P$ and $r_L < 1/2 + \theta$ (that means, $r_L < \min\{r_P, 1/2 + \theta\}$, then the patent holder will reject the counteroffer (implying litigation). Let $r_L < 1/2 + \theta$ and $r_L < r_P$, then the convex combination of r_L and r_P (as in equation (3)) is larger than r_L. To see this, note that the patent holder expects r_L for low values of θ_i, $1/2 + \theta_i$ for intermediate values of θ_i, and r_P for high values of θ_i. The intermediate-θ_i payoffs are increasing in θ_i, strictly larger than r_L at the lower bound of θ_i and strictly smaller than r_P at the upper bound of θ_i:

$$\begin{aligned} 1/2 + \theta_i\big|_{\theta_i = \theta_L(r_L)} &= 1/2 + r_L - 1/2 = r_L < r_P \\ 1/2 + \theta_i\big|_{\theta_i = \theta_P(r_P)} &= 1/2 + r_P - 1/2 = r_P > r_L \end{aligned}$$

The intermediate-θ_i payoffs are therefore strictly bounded by r_L and r_P, so that for $r_L < r_P$, the expression in (3) is less than r_L. Q.E.D.

Proof of Proposition 1

Proof. Recall that for any $r_P \geq 1/2 + \theta$, the offer is never FRAND, and ror any $r_L \leq 1/2 - \theta$, the counteroffer is never FRAND. For any (r_P, r_L) within these boundaries, the expected payoffs for the patent holder are $\pi_{P,1}$, the expected payoffs for the implementer are $1 - \pi_{L,1}$. Solving the game by backward induction, we first derive the implementer's best response $r_L^*(r_P)$. The first derivative of $1-\pi_{P,1}$ with respect to the implementer's counteroffer is

$$\left.\frac{\partial\,(1-\pi_{P,1})}{\partial r_L}\right|_{r_L>1/2-\theta} = -\frac{2\theta+2r_L-1}{4\theta} \quad\text{and}\quad \left.\frac{\partial\,(1-\pi_{P,1})}{\partial r_L}\right|_{r_L\le 1/2-\theta} = 0.$$

For all $r_L > 1/2-\theta$, the first derivative is strictly negative; it is equal to zero for $r_L \le 1/2-\theta$. The optimal counteroffer, for any offer r_P, is then $r_L^* \le 1/2-\theta$.

The patent holder anticipates this optimal counteroffer (not a function of r_P). The first derivative $\pi_{P,1}$ with respect to the offer r_P is

$$\left.\frac{\partial \pi_{P,1}}{\partial r_P}\right|_{r_P<1/2+\theta} = \frac{2\theta-2r_P+1}{4\theta} \quad\text{and}\quad \left.\frac{\partial \pi_{P,1}}{\partial r_P}\right|_{r_P\ge 1/2+\theta} = 0.$$

For all $r_P < 1/2+\theta$, the first derivative is strictly positive; it is equal to zero for $r_P \ge 1/2+\theta$. The optimal offer is then $r^*P = 1/2+\theta$.

The implementer rejects all offers for $\theta > 0$ (Lemma 1). The patent holder rejects any counteroffer because the expected outcome of litigation (no injunction; and an expected settlement outcome of $E(1/2+\theta_i) = 1/2$) is higher than the counteroffer. Q.E.D.

Proof of Lemma 2

Proof. Suppose an interior-solution best response $\tilde{r}_L^*(r_P)$ that maximizes the expression in equation (6):

$$\tilde{r}_L^*(r_P) = \arg\max_{r_L}\,(1-\tilde{\pi}_{P,1}) = \frac{r_P}{2} + \frac{1-2\theta}{4}. \tag{A.1}$$

For an interior solution, $\tilde{r}_L^*(r_P)$ must be such that (1) $\theta_L(\tilde{r}_L^*(r_P)) > -\theta$ and (2) $\theta_L(\tilde{r}_L^*(r_P)) < \theta$. A violation of (1) implies the counteroffer is never FRAND; a violation of (2) implies the counteroffer is always FRAND. Recall that by Assumption 2, $r_L \le r_P$.

(1) The condition rewrites as $r_L > 1/2-\theta$ and, for $r_L = \tilde{r}_L^*(r_P)$,

$$r_P > \frac{1-2\theta}{2} =: \tilde{R}_1.$$

For r_P satisfying this condition (above the threshold), the solution in equation (A.1) is an interior solution. If r_P is lower than the threshold, then the implementer's expected payoffs are

$$1-\int_{-\theta}^{\theta}\frac{r_P}{2\theta}d\theta_i = 1-r_P,$$

and therefore not a function of the counteroffer. Any offer satisfying the condition $r_L < 1/2-\theta$ is a best response. By Assumption 2, we have $r_L \le r_P$.

(2) The condition rewrites as $r_L < 1/2+\theta-\delta$ and, for $r_L = \tilde{r}_L^*(r_P)$,

$$r_P < \frac{1+6\theta}{2} =: \tilde{R}_2$$

For r_P satisfying this condition (below the threshold), the solution in equation (A.1) is an interior solution, for r_P above the threshold we have $\tilde{r}_L^*(r_P) = 1/2+\theta$ (the value for

r_L satisfying $\theta_L(r_L) = \theta$) as a corner solution. To see the corner solution, note that for $r_L \geq 1/2 + \theta$, the implementer's expected payoffs are

$$1 - \int_{-\theta}^{\theta} \frac{r_L}{2\theta} d\theta_i = 1 - r_L.$$

The best such counteroffer is the lowest value in the allowable range.

Q.E.D.

Proof of Lemma 3

Proof. Let $\tilde{R}_1 := \frac{1-2\theta}{2} > 0$ and $\tilde{R}_2 := \frac{1+6\theta}{2}$ denote the two critical thresholds in Lemma 2. Suppose the patent holder's offer is an interior solution that induces an interior solution in Lemma 7. Note that $r_P \leq 1$. The solution is then, $r_P \in \left[\tilde{R}_1, \min\left\{1, \tilde{R}_2\right\}\right]$. This solution maximizes $\tilde{\pi}_{P,1}$ in equation (6) for $r_L = \frac{r_P}{2} + \frac{1-2(\theta+\delta)}{4}$:

$$r_P^* = \arg\max \int_{-\theta}^{\theta_L(\tilde{r}_L^*(r_P))} \frac{\tilde{r}_L^*(r_P)}{2\theta} d\theta_i + \int_{\theta_L(\tilde{r}_L^*(r_P)))}^{\theta} \frac{r_P}{2\theta} d\theta_i = \frac{1+6\theta}{2}. \tag{A.2}$$

This solution satisfies $r_P^* \geq \tilde{R}_1$ and $r_P^* \leq \tilde{R}_2$ (with strict equality). It satisfies $r_P^* \leq 1$ for $\theta \leq 1/6$. For all θ higher than this threshold, the patent holder's equilibrium offer is equal to 1. Q.E.D.

Proof of Proposition 2

Proof.

1. The equilibrium offer is given by Lemma 3; the equilibrium counteroffer is by Lemmas 2 and 3.

3. By Lemma 2, the implementer never accepts an offer. By Lemma 1 and the counteroffer in the proposition, the patent holder accepts the counteroffer for $\theta < 1/6$ but rejects for higher values of θ (such that $\frac{1+2\theta}{2} > \frac{3-2\theta}{4}$). When the patent holder rejects, the case enters litigation. Because the offer $r_P^* = 1$ is never FRAND, whether or not the court grants an injunction depends on the counteroffer. The counteroffer r_L^* is FRAND (implying no injunction) if $\theta_i \leq \theta_L(r_L^*)$ and not FRAND (implying injunction) if $\theta_i > \theta_L(r_L^*)$.

2. For low θ, the patent holder accepts the counteroffer, and the royalty rate is r_L^*. For higher values of θ and the patent holder rejecting the counteroffer, the expected royalty is the expected outcome of settlement negotiations. When the realization θ_i is sufficiently low so that the counteroffer is FRAND and the court denies an injunction, the settlement outcome is r_L^*. When the realization θ_i is sufficiently high so that the counteroffer is not FRAND and the court grants an injunction, the settlement outcome is $r_P^* = 1$. The expected settlement outcome is

$$r^*\Big|_{\theta \geq \frac{1}{6}} = \int_{-\theta}^{\theta_L(\tilde{r}_L^*)} \frac{\tilde{r}_L^*}{2\theta} d\theta_i + \int_{\theta_L(\tilde{r}_L^*))}^{\theta} \frac{1}{2\theta} d\theta_i. \tag{A.3}$$

Simplifying this expression yields the expression for the equilibrium royalty.

Q.E.D.

A.1.1 FRAND Tolerance Scenario

Below are the formal proofs of the results for the scenario with FRAND tolerance so that $\delta > 0$ in the Huawei-ZTE litigation framework.

Proof of Proposition 3

Proof. First, recall that any counteroffer $r_L \geq \frac{1-2\delta}{2}$ is FRAND and any offer $r_P \leq \frac{1+2\delta}{2}$ is FRAND.

1. In $t = 3$, the patent holder accepts the counteroffer if $r_L \geq \min\left\{r_P, \frac{1-2\delta}{2}\right\}$. Any counteroffer $r_L \geq \frac{1-2\delta}{2}$ is FRAND. Regardless of the decision, the patent holder's payoffs are r_L. We assume that acceptance breaks the tie. Any counteroffer $r_L < \frac{1-2\delta}{2}$ is non-FRAND. If the patent holder rejects, then the payoffs depend on the litigation framework. In the Huawei-ZTE framework, the patent holder's payoffs are r_P if $r_P \leq \frac{1+2\delta}{2}$ and FRAND and $1/2$ otherwise. Hence, for FRAND r_P, the patent holder accepts the counteroffer if $r_L \geq r_P$ and rejects otherwise. For non-FRAND r_P, the patent holder rejects the counteroffer if $r_L < 1/2$. Because $r_L << \frac{1-2\delta}{2}$, this holds true. In the amended framework, litigation yields payoffs of r_P for the patent holder. The patent holder accepts if $r_L \geq r_P$ and rejects if $r_L < r_P$.

2. In $t = 2$, the implementer anticipates that the patent holder accepts any offer $r_L \geq \frac{1-2\delta}{2}$. Because any higher offer lowers the implementer's payoffs, it is optimal for the implementer to make a counteroffer $r_L \leq \frac{1-2\delta}{2}$. A counteroffer $r_L = \frac{1-2\delta}{2}$ is FRAND and yields payoffs of $1 - \frac{1-2\delta}{2}$, a counteroffer less than that yields payoffs of either $1 - r_P$ or $1 - 1/2$. Let's consider three cases for the initial offer:

   - $r_P > \frac{1+2\delta}{2}$ (non-FRAND). The patent holder rejects a non-FRAND r_L. In the Huawei-ZTE framework, litigation yields a royalty $1/2$; in amended framework, litigation yields a royalty of r_P. In both frameworks, the royalty is higher than the royalty from a FRAND counteroffer, and the implementer makes a FRAND counteroffer $r_L = \frac{1-2\delta}{2}$.
   - $r_P \leq \frac{1+2\delta}{2}$ but $r_P \geq \frac{1-2\delta}{2}$ (FRAND). The patent holder rejects a non-FRAND r_L. In both litigation frameworks, litigation yields a royalty r_P that is higher than the royalty is (weakly) higher than the royalty from a FRAND counteroffer, and the implementer makes a FRAND counteroffer $r_L = \frac{1-2\delta}{2}$.
   - $r_P \leq \frac{1+2\delta}{2}$ and $r_P < \frac{1-2\delta}{2}$ (FRAND). The patent holder rejects a non-FRAND counteroffer if $r_L < r_P$. Any non-FRAND counteroffer $r_L \geq r_P$ the patent holder accepts, and the royalty is r_L. Any non-FRAND counteroffer $r_L < r_P$ the patent holder rejects, and litigation in both framework yields a royalty r_P. The implementer will prefer any non-FRAND counteroffer $r_L \leq r_P$ over a FRAND offer.

   To summarize, the counteroffer is

$$
r_L = \begin{cases} \frac{1-2\delta}{2} & \text{if } r_P \geq \frac{1-2\delta}{2} \\ \{r_L : r_L \leq r_P\} & \text{if } r_P < \frac{1-2\delta}{2} \end{cases}
$$

3. In $t = 1$, the patent holder anticipates the implementer's counteroffer in $t = 2$ and her own decision in $t = 3$.

   - Any non-FRAND $r_P > \frac{1+2\delta}{2}$ triggers a FRAND counteroffer $r_L = \frac{1-2\delta}{2}$, and the patent holder accepts. Payoffs are $\frac{1-2\delta}{2}$.

- A FRAND offer $r_P \leq \frac{1+2\delta}{2}$ but $r_P \geq \frac{1-2\delta}{2}$ triggers a FRAND counteroffer $r_L = \frac{1-2\delta}{2}$, and the patent holder accepts. Payoffs are $\frac{1-2\delta}{2}$.
- A FRAND offer $r_P < \frac{1-2\delta}{2}$ triggers a non-FRAND counteroffer. If $r_L < r_P$, the patent holder will reject, with litigation yielding payoffs of r_P; if $r_L = r_P$, the patent holder accepts, yielding payoffs of r_P.

To summarize, any $r_P \geq \frac{1-2\delta}{2}$ dominates any lower r_P. Because any $r_P \geq \frac{1-2\delta}{2}$ yields payoffs of $\frac{1-2\delta}{2}$ (independent of r_P) any such offer is optimal.

Q.E.D.

A.1.2 Equilibrium Analysis of the Huawei-ZTE Framework

Proof of Lemma 4

Proof. If $r_L \geq 1/2 + \theta - \delta$, the implementer's counteroffer is always FRAND, and the patent holder's expected payoffs from litigation in equation (5) are r_L. Assuming acceptance as a tie breaker, she accept any offer such that $r_L \geq 1/2 + \theta - \delta$. Suppose, instead, $r_L < 1/2 + \theta - \delta$. If $r_L \geq r_P$, the patent holder accepts. To see this, recall from the expression in equation (4), that the patent holder expects r_L for low values of θ_i and r_P for high values of θ_i. This means that, for $r_L \geq r_P$, the convex combination of r_L and r_P is weakly less than r_L. This concludes the *if* part of the proof.

For the *only if* part, we show that if $r_L < r_P$ and $r_L < {}^{1+2\theta-2\delta}/_{2}$ (that means, $r_L < \min\{r_P, {}^{1+2\theta-2\delta}/_{2}\}$, then the patent holder will reject the counteroffer (implying litigation). Let $r_L < {}^{1}/_{2+2\theta-2\delta}$ and $r_L < r_P$ but $r_P - r_L < 2\delta$ (i.e., *Case 2*): the patent holder rejects because for $r_L < r_P$ the convex combination of r_L and r_P (as in equation (4)) is larger than r_L. The analogous argument holds for $r_L < r_P$ and $r_P - r_L \geq 2\delta$ (*Case 1*): the convex combination of r_L and r_P (as in equation (3)) is larger than r_L. To see this, note that the patent holder expects r_L for low values of θ_i, $1/2 + \theta_i$ for intermediate values of θ_i, and r_P for high values of θ_i. The intermediate-θ_i payoffs are increasing in θ_i, strictly larger than r_L at the lower bound of θ_i and strictly smaller than r_P at the upper bound of θ_i:

$$\begin{aligned} 1/2 + \theta_i|_{\theta_i=\theta_L(r_L)} &= 1/2 + r_L - 1/2 + \delta = r_L + \delta < r_P \\ 1/2 + \theta_i|_{\theta_i=\theta_P(r_P)} &= 1/2 + r_P - 1/2 - \delta = r_P - \delta > r_L \end{aligned}$$

We have $r_L + \delta < r_P$ and $r_P - \delta > r_L$ because $r_P - r_L \geq 2\delta > \delta$. The intermediate-$\theta_i$ payoffs are therefore strictly bounded by r_L and r_P, so that for $r_L < r_P$, the expression in (3) is less than r_L.

Q.E.D.

A.1.3 Equilibrium Analysis of the Amended Framework

Below are the formal proofs of the results for the scenario with FRAND tolerance so that $\delta > 0$ in the amended litigation framework.

Proof of Lemma 7

Proof. The payoffs for Case 1 and Case 2 are the same, given by equation (6), and not dependent on the critical threshold for the patent holder, $\theta_P(r_P)$. This implies that we can disregard $r_P - r_L \geq 2\delta$ as a condition for our problem. We closely follow the steps for Case 2a in the proof of Lemma 5. What distinguishes Cases 1/2a and Case 2b is the critical threshold for the implementer, where $\theta_L(r_L) < \theta$ for Case 1/2a and $\theta_L(r_L) \geq \theta$ for Case 2b. As we will establish below, Case 2b is associated with a corner solution.

Consider Case 1/2a with an interior-solution best response $\tilde{r}_L^*(r_P)$ that maximizes the expression in equation (6):

$$\begin{aligned}\tilde{r}_L^*(r_P) &= \arg\max\left(1-\tilde{\pi}_{P,1}\right)\\ &= \frac{r_P}{2}+\frac{1-2(\theta+\delta)}{4}\end{aligned} \tag{A.4}$$

For an interior solution, $\tilde{r}_L^*(r_P)$ must be such that (1) $\tilde{r}_L^*(r_P) \le 1$; (2) $\theta_L(\tilde{r}_L^*(r_P)) < \theta$; and (3) $\theta_L(\tilde{r}_L^*(r_P)) > -\theta$. The first condition is by the restriction of the offers and counteroffers to the unit interval ($r_L \in [0,1]$); a violation of (2) implies the counteroffer is never FRAND; a violation of (3) implies the counteroffer is always FRAND.

(1) We have $\tilde{r}_L^*(r_P) \le 1$ for

$$r_P \le \frac{3}{2}+\theta+\delta.$$

This condition is always satisfied because $r_P \in [0,1]$.

(2) The condition rewrites as $r_L < 1/2+\theta-\delta$ and, for $r_L = \tilde{r}_L^*(r_P)$,

$$r_P < 1/2+3\theta-\delta =: \tilde{R}_2$$

For r_P satisfying this condition (below the threshold), the above solution is an interior solution, for r_P above the threshold we have $\tilde{r}_L^*(r_P) = 1/2+\theta-\delta$ (the value for r_L satisfying $\theta_L(r_L)=\theta$) as a corner solution. To see the corner solution, note that for $r_L \ge 1/2+\theta-\delta$, the implementer's expected payoffs are

$$1-\int_{-\theta}^{\theta}\frac{r_L}{2\theta}d\theta_i = 1-r_L.$$

The best such counteroffer is the lowest value in the allowable range.

(3) The condition rewrites as $r_L > 1/2-\theta-\delta$ and, for $r_L = \tilde{r}_L^*(r_P)$,

$$r_P > 1/2-\theta-\delta =: \tilde{R}_1.$$

For r_P satisfying this condition (above the threshold), the above solution is an interior solution. If r_P is lower than the threshold, then the implementer's expected payoffs are

$$1-\int_{-\theta}^{\theta}\frac{r_P}{2\theta}d\theta_i = 1-r_P,$$

and therefore not a function of the counteroffer. Any offer satisfying the condition $r_L < 1/2-\theta-\delta$ is a best response.

Bullet point (2) also subsumes Case 2b. Q.E.D.

Proof of Lemma 8

Proof. Let $\tilde{R}_1 := 1/2-\theta-\delta > 0$ and $\tilde{R}_2 := 1/2+3\theta-\delta$ denote the two critical thresholds in Lemma 7. Suppose the patent holder's offer is an interior solution that induces an interior solution in Lemma 7. Note that $r_P \le 1$. The solution is then, $r_P \in \left[\tilde{R}_1, \min\left\{1,\tilde{R}_2\right\}\right]$. This solution maximizes $\tilde{\pi}_{P,1} = \tilde{\pi}_{p,2a}$ in equation (6) for $r_L = \frac{r_P}{2} + \frac{1-2(\theta+\delta)}{4}$:

$$r_P^* = \arg\max \int_{-\theta}^{\theta_L(\tilde{r}_L^*(r_P))} \frac{\tilde{r}_L^*(r_P))}{2\theta} d\theta_i + \int_{\theta_L(\tilde{r}_L^*(r_P)))}^{\theta} \frac{r_P}{2\theta} d\theta_i = \frac{1+6\theta-2\delta}{2}. \tag{A.5}$$

This solution satisfies $r_P^* \geq \tilde{R}_1$ and $r_P^* \leq \tilde{R}_2$ (with strict equality). It satisfies $r_P^* \leq 1$ for

$$\theta \leq \frac{1+2\delta}{6}.$$

For all θ higher than this threshold, the patent holder's equilibrium offer is equal to 1. Q.E.D.

Proof of Proposition 5

Proof. The patent holder's equilibrium offer is by Lemma 8. We obtain the implementer's counteroffer by plugging $r_P = \tilde{r}_P^*$ into the expression for $\tilde{r}_L^*(r_P)$ in Lemma 7. Recall from the proof of Lemma 8 that $r_P^* \geq \tilde{R}_1$ and $r_P^* \leq \tilde{R}_2$. For low values of θ, the second condition holds with equality, for higher values of θ and $r_P^* = 1$, we have $\tilde{R}_1 < r_P^* = 1 < \tilde{R}_2$. The counteroffer is for the intermediate range of offers.

The royalty rate is r_L when then the patent holder accepts the counteroffer, that is, when $r_L \geq \min\{r_P, 1/2 + \theta - \delta\}$ by Lemma 4. The royalty rate is the expected outcome of settlement negotiations when the patent holder rejects the counteroffer and the case enters the litigation stage. First, note that $r_L^* < r_P^*$ for all $\theta > 0$. For low values of $\theta \leq \frac{1+2\delta}{6}$, $r_L^* > 1/2 + \theta - \delta$ and the patent holder accepts the counteroffer. For high values of $\theta > \frac{1+2\delta}{6}$ so that $r_P^* = 1$, we have $r_L^* < 1/2 + \theta - \delta$ and the patent holder rejects the counteroffer. The expected settlement offer is the expression for $\tilde{\pi}_{P,1}$ in equation (6) for $r_P = 1$:

$$r^*\big|_{\theta > \frac{1+2\delta}{6}} = \int_{-\theta}^{\theta_L(\tilde{r}_L^*)} \frac{\tilde{r}_L^*}{2\theta} d\theta_i + \int_{\theta_L(\tilde{r}_L^*))}^{\theta} \frac{1}{2\theta} d\theta_i. \tag{A.6}$$

Simplifying this expression yields the expression for the equilibrium royalty. Q.E.D.

A.2 Additional Results

Proof of Proposition 6

Proof. **Huawei-ZTE Framework:** The patent holder's equilibrium offer in Proposition 4 never exceeds $\overline{r}(\theta)$.
Amended Framework: In Lemma 8, we used $r_P \leq \min\{1, \tilde{R}_2\}$. For the proof of this proposition, we have $r_P \leq \min\{1, \overline{r}(\theta), \tilde{R}_2\}$. Observe that $\overline{r}(\theta) < 1$ for all $\theta + \delta \leq 1/2$, which holds by Assumption 3. Moreover, $\overline{r}(\theta) < \tilde{R}_2$ for $\theta > \delta$.

Assumption 4 implies $\tilde{r}_P^* \geq \overline{r}(\theta)$ which holds for $\theta \leq \delta$. For all $\theta > \delta$, the equilibrium offer is $\overline{r}(\theta)$. This critical threshold δ for a corner solution under Assumption 4 is more restrictive than the critical threshold in Proposition 5 as long as

$$\delta < \frac{1+2\delta}{6} \qquad \text{or} \qquad \delta < 1/4.$$

For all values of $\delta \geq 1/2$, Assumption 4 is not binding and the equilibrium is as described in Proposition 5. The equilibrium counteroffer and royalty rate follow from Lemma 8. Q.E.D.