Log file for Barrier Option

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Contents

1 Introduction

The goal is to express all barrier option payoffs in terms of a unique "atom" formula. Rebates are not included at the moment.

2 Atom: "up-and-out; down-and-out"

The fundamental payoff of choice is relative to the "up-and-out; down-and-out" payoff. The barriers are L, U and the active window if the continuously monitored interval $[t_s, t_e]$. T_e is the expiry of the option, S the underlying and X the strike.

$$\begin{aligned} \text{payoff} &= \max(S - X, 0)\ \mathbb{1}_{\forall t \in [t_s, t_e]: L < S(t) < U} \\ &\equiv \text{uodo}(S, X, L, U, \{t_s, t_e\}, T_e) \end{aligned} \tag{1}$$

3 The zoo

3.1 vanilla

$$\begin{aligned} \text{payoff} &= \max(S - X, 0) \\ &= \text{uodo}(S, X, 0, \infty, \{0, T_e\}, T_e) \end{aligned} \tag{2}$$

3.2 single barrier, B

"up-and-out" - $S_0 < B$

$$\begin{aligned} \text{payoff} &= \max(S - X, 0)\mathbb{1}_{\forall t \in [t_s, t_e]: S(t) < B} \\ &= \text{uodo}(S, X, 0, B, \{t_s, t_e\}, T_e) \end{aligned} \tag{3}$$

"down-and-out" - $B < S_0$

$$\begin{aligned} \text{payoff} &= \max(S - X, 0)\mathbb{1}_{\forall t \in [t_s, t_e]: B < S(t)} \\ &= \text{uodo}(S, X, B, \infty, \{t_s, t_e\}, T_e) \end{aligned} \tag{4}$$

"down and in" - $B < S_0$

$$\begin{aligned} \text{payoff} &= \max(S - X, 0)\mathbb{1}_{\exists t^* \in [t_s, t_e]: S(t^*) < B} \\ &= \max(S - X, 0)\left(1 - \mathbb{1}_{\forall t \in [t_s, t_e]: B < S(t)}\right) \\ &= \text{“vanilla” - “down-and-out”} \\ &= \text{uodo}(S, X, 0, \infty, \{0, T_e\}, T_e) - \text{uodo}(S, X, B, \infty, \{t_s, t_e\}, T_e) \end{aligned} \tag{5}$$

"up and in" - $S_0 < B$

$$\begin{aligned}\text{payoff} &= \max(S-X,0)\mathbb{1}_{\exists t^*\in[t_s,t_e]:B<S(t^*)} \\ &= \max(S-X,0)\left(1-\mathbb{1}_{\forall t\in[t_s,t_e]:S(t)<B}\right) \\ &= \text{“vanilla” - “up-and-out”} \\ &= \text{uodo}(S,X,0,\infty,\{0,T_e\},T_e) - \text{uodo}(S,X,0,B,\{t_s,t_e\},T_e) \qquad (6)\end{aligned}$$

3.3 double barrier, {L, U}

"double knock-in" - $L < S_0 < U$

$$\begin{aligned}\text{payoff} &= \max(S-X,0)\ \mathbb{1}_{\exists t^*\in[t_s,t_e]:S(t^*)<L||U<S(t^*)} \\ &= \max(S-X,0)\ \left(1-\mathbb{1}_{\forall t\in[t_s,t_e]:L<S(t)<U}\right) \\ &= \text{“vanilla” - “up-and-out;down-and-out”} \\ &= \text{uodo}(S,X,0,\infty,\{0,T_e\},T_e) - \text{uodo}(S,X,L,U,\{t_s,t_e\},T_e) \qquad (7)\end{aligned}$$

References